The Primitive Full-Carrier Provenance, Recurrent Fisher Pullback, Source-Projected Bath Factorisation and Conditional Doob–Kraus Descent Theorem in VERSF

Exact sector-resolved capacity, a four-regulator 102-dimensional marked-action lift, the physical-provenance boundary, and the conditionally constructed source-projected bath, factorisation and marked-descent architecture

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Designation: PFHBA-7 - Recurrent Pullback Execution and Conditional Source-Projected Provenance Programme

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Lineage: PFHBA-7 is the integrative successor to PFHBA-1: it retains the PFHBA-1 machine-certified pullback calculation unchanged - Theorem A, the four-regulator execution, and every numerical certificate carry over verbatim - and adds the source-projected bath, factorisation and conditional Doob–Kraus descent architecture. References to "the PFHBA-1 stack" and "the PFHBA-1 execution" throughout denote that retained calculation inside this paper, not a separate document; PFHBA-2 through PFHBA-6 were successive integrative revisions of this same paper, which PFHBA-7 supersedes.

Research-draft status: Complete theorem and four-regulator execution at the population edge-score layer, together with an analytically derived quadratic-tangent factorisation theorem and a conditionally constructed source-projected bath and Doob descent architecture. All descent-layer objects are conditional, typed or source-open exactly as graded in the audit matrix; no descent calculation has been numerically executed. Physical HFB admission is not claimed.

Headline result. SECTOR-RESOLVED RECURRENT PULLBACK EXISTENCE PASS / EXACT-LIFT IDENTIFIABILITY FAIL / QUADRATIC-TANGENT SOURCE-DEFECT FACTORISATION THEOREM PASS / PROJECTED EUCLIDEAN BATH AND UNMARKED DOOB POPULATION DESCENT CONDITIONAL CONSTRUCTION / MARKED DECOMPOSITION, KRAUS AMPLITUDE RETURN, NUMERICAL PRIMITIVE PROVENANCE, PHYSICAL CLOCK, BRANCH ORIENTATION AND HFB-1 ADMISSION NOT YET ISSUED. The record-preserving $K=7$ × neutral renewal process has 930 positive edge-logit coordinates and Fisher rank 867. That rank decomposes exactly and orthogonally as 216 source-local history-destination main-effect + 147 source-local neutral-destination main-effect + 504 destination-interaction directions. The stricter product-preserving tangent has only 24 shared-history and 21 shared-neutral directions, so the 24-dimensional HCMR neutral block is three dimensions too large for a globally shared neutral factor and must use history-conditioned neutral response, interaction response or a non-product extension. The complete HCMR-MR1 102-dimensional parent, including its six exact record nulls, admits an exact sector-typed recurrent Fisher pullback at regulator levels 3, 4, 5 and 6 on both surviving conjugate branches. The maximum relative reconstruction residual is 8.85×10^{-16}. This closes the capacity and existence question more strongly than the earlier total-rank test. It does not establish primitive derivation: even after broad source-local history/neutral/interaction typing, the minimum-null exact-lift family has real dimension 34,152; two independently chosen typed lifts reconstruct the same parent while producing nearly orthogonal mark-score responses, and exact branch conjugacy lifts through the recurrent Fisher law. Corpus integration now derives the candidate forward architecture that must replace inverse lifting: the compressed candidate bath operator on the whitened transverse carrier, the quadratic-tangent defect-source factorisation theorem with its HM4 residual $R\_HM4 = J\_s(I - J\_D^{+}J\_D)$ and minimum-norm coupling $C\^min = J\_s J\_D^{+}$, the conditionally positive Euclidean history transfer, and the unmarked Doob descent with Perron-ratio rates $q(y|x) = -\mathscr{H}\_xy\ r(y)/r(x)$ - each stated in full, with its hypotheses and grade, in the abstract and Parts IX-XII. The source-derived mark decomposition and the Kraus amplitudes remain source-open. These are analytical constructions and conditional architectures with decisive numerical tests that have not been run; the remaining physical debts are the normalised invariance defect and carrier principal-angle certificate, the covariance-compatibility spectrum, the multi-regulator numerical J_D, J_s, ε_HM4 and C_n, the unmarked Doob kernel, the mark decomposition and amplitude-level return, the neutral interaction, completion residue, orientation-odd coupling, physical clock and HFB-1 admission.

Audit matrix

Audit item	Verdict	Meaning
K=7 × recurrent working law	PASS / IMPORTED	Record-preserving renewal restores nonzero long-history neutral information.
Product edge-coordinate census	EXACT PASS	31 positive history marks × 30 positive renewal marks = 930 coordinates.
Product Fisher rank	EXACT PASS	Rank 867 with exactly 63 source-normalisation nulls.
Source-local score decomposition	EXACT PASS	867 = 216 history-destination main-effect + 147 neutral-destination main-effect + 504 destination interaction.
Broad source-local capacity	EXACT PASS	Required dimensions 12, 24 and 60 fit inside 216, 147 and 504 respectively.
Strict shared-neutral factor capacity	FAIL BY 3	The factor-preserving neutral tangent has rank 21 against the 24-dimensional neutral demand.
Four-regulator typed pullback	PASS	H_full is reconstructed at levels 3-6 and branches 1,6 to $\leq 8.85 \times 10^{-16}$ relative residual.
Robust rather than tail-only capacity	PASS	495 positive Fisher modes remain above 1% of the maximum at every regulator.

Audit item	Verdict	Meaning
Exact lift uniqueness	FAIL BY THEOREM	Minimum-null untyped family dimension 78,576; broad typed family dimension 34,152.
Mark-level output invariance	FAIL	Two exact typed lifts have relative Jacobian difference 1.72-1.95 and near-zero Fisher score cosine.
Branch orientation	FAIL FOR SELECTION	$H\_6 = C^T H\_1 C$ and $J\_6 = J\_1 C$; the recurrent positive quadratic law preserves the exact pair.
Physical clock	OPEN	$p = 1 − e^{-1}$ fixes only rate × step; Fold-to-traversal and TPB compensator remain unreturned.
Completion scale	OPEN	Static H_full does not return $K\_comp(p^2)$, Z_comp or $\Lambda\star$.
Reduced executable gauge carrier	FAIL / IMPLEMENTATION LIMIT	Scalar factor phases do not carry the full $su(3) \oplus su(2) \oplus u(1)$ tangent.
Full corpus gauge carrier	CONDITIONAL STRUCTURAL PASS	Matrix-valued SU(3), SU(2), U(1) transport exists in the corpus and should replace the reduced scalar implementation.
Commitment-event process	CONDITIONAL PASS	Poisson arrivals, event covariance, spectral response and memory kernel are supplied.
Native marked jump skeleton	CONDITIONAL PASS	A marked Poisson–Kraus construction can be written once source rates and partial isometries are fixed.
Bath-carrier equivalence	COMPRESSION / INVARIANCE OPEN	Ω_Q,n^2 is a compression; restriction requires the normalised invariance defect ε_inv to vanish and a principal-angle certificate that the closure and commitment carriers coincide.
Projected-covariance compatibility	TEST DEFINED / SPECTRUM OPEN	Transport Σ_η to whitened coordinates; require $\Sigma\_Q \geq 0$ and correct transformation; report the spectrum - whitening-induced anisotropy is not a failure.
Defect-bath factorisation	DERIVED TEST	Exact iff $\ker J\_D \subseteq \ker J\_s$ on the typed defect space E_D,n.
Canonical coupling	DERIVED CONDITIONAL FORM	$C\_n\text{\^{}}min = J\_s,n\ J\_D,n^+$: the minimum-norm representative of the defect-quotient coupling, admissible only when the residual vanishes.
HM4 provenance residual	NOT YET EXECUTED	$R\_HM4 = J\_s\ (I − J\_D^+ J\_D)$.

Audit item	Verdict	Meaning
Euclidean bath transfer	CONDITIONAL CONSTRUCTION	Positivity requires the declared coercivity bound (P1) and strict kernel-positivity hypothesis (P2).
Native Doob–Kraus law	UNMARKED POPULATIONS CONDITIONAL / MARKS SOURCE-OPEN	Fixed-step unmarked Doob kernel under (P1)-(P2); rates q(y
Marked decomposition and Kraus isometries	TYPED / SOURCE-OPEN	Source amplitude decomposition $A = \Sigma_\alpha A_\alpha$ or record projectors R_α; then $M_{\alpha,yx} = P_y A_\alpha P_x$, $V_{\alpha,yx} = \mathrm{Pol}(M_{\alpha,yx})$, consistent with the populations.
Neutral rank-three repair	PRECISE TEST DEFINED	Surjectivity and stable surjectivity of $P_{miss} C_{neutral}^\dagger \Omega_Q^{-2} C_{hist}$ onto the three missing directions, with a projection-leakage check.
Completion residue	FORM DERIVED / NUMBER OPEN	$Z_{comp} = Z_{core,H} + c_H^\dagger \Omega_Q^{-4} c_H$.
Numerical source descent	OPEN	Requires automatic differentiation of the full faithful source.
HFB-1 admission	NOT ADMITTED	Capacity closed; ε_{HM4} is the gateway provenance test, followed by the invariance, compatibility, positivity, mark-decomposition, amplitude, neutral, scale and branch gates.

General reader summary

The immediate question was whether the latest recurrent VERSF history law contains enough genuine information to support the complete 102-dimensional Standard Model carrier used in the four-regulator HCMR calculation. Earlier work had found an obstruction: when completed neutral records were treated as absorbing, the long-history neutral information vanished and the marked history process did not have enough rank to generate the nonterminal full-carrier Hessian. K7-CBR-1 repaired that defect by preserving completed records in a ledger while reopening the working carrier for the next fact.

This paper performs the next calculation. It does not merely compare the total recurrent rank, 867, with the required rank, 96. It separates the recurrent information into three exact kinds. Some score directions depend only on the K=7 history transition. Some depend only on the neutral renewal transition. The rest measure genuine interaction between history and neutral destination. Because the transition law is a product at this retained layer, these three classes are orthogonal in the Fisher metric.

The resulting count is unexpectedly generous: each block of the Standard Model carrier has many more recurrent directions of the right kind than it needs, and 495 Fisher modes remain within two orders of magnitude of the largest, so the target need not hide in numerically tiny directions. A stricter audit, however, finds a genuine shortfall: only 21 globally shared neutral-factor directions exist against a 24-dimensional neutral demand, so the neutral block is short by three dimensions unless the primitive law makes neutral response depend on the history source, uses destination interactions, or ceases to factorise.

Using that exact decomposition, the paper builds an explicit 930×102 field-to-mark Jacobian whose Fisher pullback is the complete HCMR parent, repeated at four regulator levels and on both surviving conjugate branches, every reconstruction passing at floating-point residual. In the narrow mathematical sense, the recurrent path law can carry the HCMR quadratic object.

That success creates a sharper problem. The Jacobian is far from unique: tens of thousands of continuously different sector-typed embeddings reproduce exactly the same HCMR matrix. The paper constructs two such lifts independently; their quadratic pullbacks are indistinguishable, but their predictions for how a given field perturbation changes the individual history marks are almost orthogonal. The result is therefore both a pass and a refusal: the capacity and exact-pullback questions pass, while primitive provenance is refused, because a Jacobian built to factor the already frozen parent - rather than obtained by differentiating one primitive marked action - cannot count as evidence of derivation once the recurrent Fisher rank is this large.

The same discipline applies to branch orientation, time and scale. The two nonzero branches remain exactly related by complex conjugation, and the relation lifts through the recurrent score law. The benchmark event probability fixes only a dimensionless rate-step product, not seconds. The static parent contains no momentum derivative from which the completion residue and matching scale could be read. None of those gaps can be repaired by choosing a more attractive factorisation.

What has changed since the original execution is that the missing object is no longer vague. It is not "some Jacobian somewhere". It has been reduced to a specific forward chain, each stage of which is now written down: start from the primitive fields; take the six basic mismatch quantities of the master substrate; project their combined output onto the working part of the closure wheel to obtain the bath source; feed that source into the smoothing evolution that respects the direction of commitment; convert the smoothed evolution into a normalised stepping rule; and read off from that rule the law by which physical records are stamped. Each arrow in this chain is now an explicit construction rather than a hope, with two honest qualifications: a small number of the arrows come with stated conditions that must be checked rather than assumed, and the final ingredient - the piece that attaches the record labels themselves to the stepping rule - is specified in shape only: what has been fixed is the architecture and the tests it must pass, not the working machinery itself, which still has to be built from the source.

The story so far can therefore be told plainly. The history geometry has enough room to hold the Standard Model carrier, but room alone cannot tell us which occupant is the real one; the earlier working shortcut kept only a handful of simple phase quantities where the true gauge structure needs far more; and the completed corpus already contains the missing machinery - the full gauge transport, the commitment-event process and the bath criterion. The new factorisation theorem tells us exactly how to test whether those structures genuinely descend from the source rather than being fitted to the answer. The first and most decisive test is the leftover quantity described above: exactly zero releases the coupling calculation; anything else refutes the current candidate at the source. But a zero does not by itself discharge the later checks - the health of the smoothing evolution, the still-unbuilt record-stamping ingredient, the neutral repair, the scale, and the mirror-image orientation each have their own test. The residual is the gateway, not the whole toll; and none of these tests has yet been run.

Abstract

We test whether the record-preserving recurrent K=7 closure-Born-TPB construction can support the complete four-regulator HCMR full-carrier parent and whether such support is sufficient for primitive provenance. At regulator levels n=3,4,5,6, the retained population law is the product of the frozen seven-state D36/K=7 history kernel and a nine-state active-singlet-record renewal kernel. The history factor has 31 positive transitions and edge-logit rank 24; the renewal factor has 30 positive transitions and edge-logit rank 21. The product law has 930 positive edge coordinates and rank 867.

We prove an exact product-score decomposition. At each source, the zero-mean product score space is the orthogonal direct sum of history-destination main effects, neutral-destination main effects and destination-interaction scores. Summed over all 63 sources, their source-local ranks are 216, 147 and 504. These dominate the HCMR requirements 12, 24 and 60 under broad source-local typing. If the perturbation is required to preserve one globally shared history factor and one globally shared neutral factor, the corresponding ranks reduce to 24 and 21; the latter is three short of the neutral demand. Using Fisher-orthonormal frames in those three subspaces, we construct an explicit sector-typed Jacobian $J_{n,k} \in \mathbb{R}^{930\times 102}$ satisfying

$$J_{n,k}^{\mathsf{T}} I_n J_{n,k} = H_{\mathrm{full}}^{(n,k)}$$

at every regulator and for k=1,6. The six completed-record columns are represented by the exact score-normalisation null class. The largest relative reconstruction residual over sixteen independently verified reference and alternative lifts is 8.85×10^{-16}. The positive Fisher spectrum is regulator stable; 495 eigenvalues remain above 10^{-2} of the maximum at every level.

We then prove that this is an existence theorem, not a provenance theorem. For a rank-867 Fisher form and rank-96 nonterminal parent, the minimum-null exact-lift class is a real Stiefel family of dimension 78,576. Allowing the 63 source-normalisation nulls for all 102 columns enlarges the affine family to dimension 85,002. Even after enforcing the broad source-local history/neutral/interaction typing used by the construction, the family dimension is 34,152. Two frozen typed lifts reproduce the same HCMR parent while differing in Frobenius norm by 1.72-1.95 relative and returning sample mark-score Fisher cosines between -0.067 and +0.011.

The exact HCMR branch relation $H_6 = C^T H_1 C$ lifts as $J_6 = J_1 C$, so the positive recurrent quadratic law cannot orient the conjugate pair. The benchmark renewal probability fixes only $\nu\tau = 1$, not an absolute clock, and the static pullback supplies neither $K_{comp}(p^2)$ nor Z_{comp}.

Corpus integration then supplies four new analytical claims, each stated at its true strength. First, bath-carrier identification. On the whitened closure space, with

$$\tilde{u}_0 = M_n^{1/2} u_0 / \|M_n^{1/2} u_0\|, \quad P_{Q,n} = I - \tilde{u}_0 \tilde{u}_0^\dagger,$$

the compressed candidate bath operator is

$$\Omega_{Q,n}^2 = P_{Q,n} M_n^{-1/2} L_n M_n^{-1/2} P_{Q,n} \text{ on ran } P_{Q,n}.$$

This is always a valid compression; it is a genuine restriction, and hence the physical identification, only when the whitened closure operator leaves ran $P_{Q,n}$ invariant - an exact numerical obligation, the invariance defect $R_{inv}^{(n)} = (I - P_{Q,n}) M_n^{-1/2} L_n M_n^{-1/2} P_{Q,n}$, reported in the normalised form ε_{inv} - and when the projected closure carrier and the commitment-bath carrier are proved to be the same invariant six-dimensional subspace. Second, the defect-source factorisation theorem on the typed defect space $E_{D,n} = E_C \oplus E_J \oplus E_T \oplus E_R \oplus E_B \oplus E_H$: with the source carried into whitened bath coordinates by the declared transport W_n and both W_n and $P_{Q,n}$ frozen at the background, for the MASD defect Jacobian $J_{D,n} : T_{X\star}X_n \to E_{D,n}$ and the projected commitment-source Jacobian $J_{s,n} : T_{X\star}X_n \to Q_n$,

$$J_s = C J_D \text{ if and only if } \ker J_D \subseteq \ker J_s.$$

This is a quadratic tangent theorem at $X\star$: it certifies first-order descent through the defect quotient, and the Euclidean generator built from it is the quadratic local generator in the linearised defect coordinates, not a nonlinear bath law.

Third, the canonical minimum-norm representative of the defect-quotient coupling on the reachable defect image,

$$C^{min} = J_s J_D^+.$$

Fourth, the unmarked Doob descent, graded in two steps: under an explicit coercivity bound and a strict kernel-positivity hypothesis the fixed-step unmarked kernel exists; under the additional Doob-gauge stoquasticity condition the continuous-time Markov-rate limit follows -

$$\mathscr{H}_n \to e^{-a_t \mathscr{H}_n} \to K_n^{Doob} \to q(y|x) = -\mathscr{H}_{xy}\, r(y)/r(x),$$

which converts the Euclidean history transfer into a normalised unmarked Doob kernel, the continuous-time rates carrying the Perron-vector ratio. The unmarked kernel determines neither the marked decomposition $K_n = \Sigma_\alpha K_{n,\alpha}$ nor the partial isometries: the mark labels themselves must come from a source-derived amplitude decomposition $A_n = \Sigma_\alpha A_{n,\alpha}$, orthogonal record projectors R_α, or an explicitly imported marked-support law, and the Kraus amplitudes $L_\alpha = \sqrt{q_\alpha} V_\alpha$ remain source-open until that return is made. These claims are analytical constructions and conditional architectures, not yet a multi-regulator numerical certificate: no numerical J_D, J_s, ε_{HM4}, C_n, Doob kernel, amplitude array or completion residue is returned here.

Final grade: sector-resolved recurrent pullback existence pass; exact-lift identifiability fail; source-defect factorisation theorem pass; Euclidean bath and marked-rate descent conditional construction; Kraus amplitude return, numerical primitive provenance, physical clock, branch orientation and HFB-1 admission not yet issued.

Principal theorem

Theorem A - Machine-certified recurrent pullback theorem (PFHBA-1). Fix the frozen D36/K=7 history kernel, the record-preserving nine-state neutral renewal law built from the FCHI polar support maps, and the HCMR-MR1 four-regulator parent matrices. Then: 1. the product population edge-logit Fisher form has coordinate dimension 930, rank 867 and nullity 63; 2. its positive score space decomposes orthogonally into source-local history-destination, source-local neutral-destination and destination-interaction spaces $S_H \oplus S_N \oplus S_X$ with dimensions 216, 147 and 504; 3. the stricter factor-preserving tangent has only 24 shared-history and 21 shared-neutral directions, so a 24-dimensional neutral lift necessarily uses history-conditioned neutral response, destination interaction or a non-product extension; 4. the HCMR nonterminal carrier decomposes as 12 history + 6 gauge + 18 completion + 36 charged + 24 neutral dimensions, with six exact completed-record nulls; 5. at every n=3,4,5,6 and k=1,6 there exists a broadly sector-typed $J_{n,k}$ in $\mathbb{R}^{930\times102}$ such that $J_{n,k}^T I_n J_{n,k} = H_{full}^{(n,k)}$; 6. the exact-lift class is not unique: before score-gauge additions it contains St(96,867), and under the broad source-local typing it contains St(12,216) × St(24,147) × St(60,504); 7. two explicitly frozen typed representatives reconstruct the same parent and yield inequivalent mark-level score responses; 8. if $H_6 = C^T H_1 C$, then $J_6 = J_1 C$ is an exact lift on the same Fisher law, so branch conjugacy is not broken by positive quadratic pullback; 9. no conclusion about the physical field-to-mark derivative, Yukawa readout, clock normalisation, completion residue or HFB boundary follows from items 1-8 alone. Items 1-8 are exact theorems or machine-certified finite-regulator returns at the declared population edge-score layer. Item 9 is the physical-admission boundary.

Theorem B - Source-projected bath-descent theorem (conditional). Type the six MASD defect classes as maps into the direct-sum defect space

$$E_{D,n} = E_C \oplus E_J \oplus E_T \oplus E_R \oplus E_B \oplus E_H,\quad D_n : X_n \to E_{D,n},$$

let $s_n(X) = P_{Q,n} W_n S_n[\Phi(X)]$ be the projected commitment source, with $W_n = M_n^{(1/2)}$ the declared transport into the whitened bath coordinates, values in the bath carrier $Q_n = \operatorname{ran} P_{Q,n}$, and W_n, $P_{Q,n}$ frozen at the background, and at the frozen background $X^\star$ define the two Jacobians

$$J_{D,n} = DD_n|_{X^\star} : T_{X^\star}X_n \to E_{D,n},\quad J_{s,n} = Ds_n|_{X^\star} : T_{X^\star}X_n \to Q_n.$$

Then:

1. **Existence.** A coupling $C_n : E_{D,n} \to Q_n$ with $J_{s,n} = C_n J_{D,n}$ exists if and only if

$\ker J_{D,n} \subseteq \ker J_{s,n}$.

2. **Residual criterion.** The kernel inclusion holds if and only if the HM4 provenance residual vanishes,

$R_{HM4}^{(n)} = J_{s,n} (I - J_{D,n}^{+} J_{D,n}) = 0$.

3. **Minimum-norm representative.** When the residual vanishes, the unique solution vanishing on $(\operatorname{ran} J_{D,n})^{\perp}$ - equivalently the unique minimal-Frobenius-norm solution - is

$C_n^{min} = J_{s,n} J_{D,n}^{+}$.

This representative is canonical relative to the five declared conventions of Section 28; physical canonicity additionally requires invariance under admissible reparameterisations and regulator refinement, a separate unproved condition.

4. **Conditional descent architecture.** Given an admissible C_n, the quadratic local generator in the linearised defect coordinates, its transfer, and the unmarked Doob kernel are defined by Parts XI and XII conditional on the coercivity, kernel-positivity and generator hypotheses stated there. Items 1-3 are tangent statements at $X^\star$; no nonlinear bath law is implied. The marked decomposition and the Kraus partial isometries V_α are typed by the amplitude return of Part XII and are not determined by the unmarked Doob kernel.
5. **No certificate.** No physical certificate follows until the objects of items 1-4 are numerically evaluated on the faithful source and shown to converge across regulators.

Items 1-3 are exact theorems proved in Appendix E. Items 4-5 delimit the conditional architecture derived in Parts XI and XII.

Contents

Part I - Question, source boundary and grades

1. The exact question

The July Master Ledger asked for a primitive full-carrier provenance theorem: one unextended history law should return the retained instrument, its field embedding, every Hessian block, the physical clock and the branch used by HFB-1. The later papers divide that demand into two logically different questions.

Capacity question. Does the recurrent marked process possess enough independently measurable score directions, in the correct broad sectors, to support the HCMR retained parent?

Provenance question. Does the primitive master action itself differentiate to one physically distinguished field-to-mark Jacobian and hence to the HCMR parent, without reading the target parent or observed Standard Model data?

K7-CBR-1 answered the first question only at total-rank level: 867 recurrent score directions are available for a rank-96 nonterminal target. FCIP-A1 and USTU-1 showed why total capacity does not answer the second question. The PFHBA-1 execution performed the stronger sector-resolved capacity calculation, constructed the exact pullback, and then determined what that construction does and does not prove. The present paper retains that execution unchanged, integrates the later corpus advances - the faithful gauge carrier, the commitment-event bath, the projected closure operator and the positive-transfer/Doob architecture - and derives the source-projected factorisation and Doob–Kraus descent that convert the provenance question into one executable numerical test.

2. Imported source boundary

The original PFHBA-1 stack:

Source	Object consumed	Grade retained
D36-R1	Frozen row-normalised K=7 one-Fold kernel and stationary law	Reproducible conditional clock branch
FCHI-1	Levels 3-6 neutral polar support maps and 102-dimensional carrier census	Supplied structural candidate
K7-CBR-1	Record-preserving renewal rule and finite-mean benchmark $p = 1 - e^{-1}$	Conditional recurrent construction
HCMR-MR1	Four-regulator H_full matrices for branches k=1,6 and exact block ordering	Complete candidate execution; physical certificate refused
FCIP-A1 / STEP2	Instrument, clock, cross-block and ledger non-uniqueness theorems	Exact no-go for unextended provenance
CMHRE-1	Absorbing marked-law rank failure and diagnostic lift warning	Exact no-go for one-shot long-history provenance
RRHF-1	Representation-resolved charged/gauge action-level extension	Conditional action-level pass
PRLU-1	Charged/scalar ledger and neutral-terminal provenance boundary	Conditional ledger result
FRDT-1	Failure of current unextended derivative to reproduce representation-resolved target	Executed structural falsifier
ASAC-1	Conditional length/traversal law and action/completion-scale no-goes	Major conditional advance

The later sources integrated by the present revision, classified by what each supplies:

Source	Object consumed	Supply class
EPMD-1	Reduced executable source deficit: 36 scalar gauge-edge coordinates against a 144-dimensional faithful gauge tangent demand	Executed reduced-source falsifier; implementation limit, not a corpus theorem
PFHBA-1 calculation package	Frozen arrays, scripts, manifests and independent verification of the four-regulator pullback	Machine-certified numerical certificates

Source	Object consumed	Supply class
Exact Projection Coordinates	Exact coordinates of the projection onto the working closure carrier	Theorem
Full Projected Closure Operator	The pair (L_n, M_n) and the generalized eigenproblem returning the physical mode frequencies	Theorem
Origin of the Closure Scale	Structural account of the closure-scale mechanism	Conditional architecture and source clue
Spectral Density of the Commitment-Event Bath	Poisson arrivals, event covariance, spectral response and real-time memory kernel	Conditional architecture
Three Admissibility Classes and Maxwell-Form Persistent Transport	Admissibility classification and persistent-transport structure	Theorem plus conditional architecture
Bath or Ledger	Discrimination between bath and ledger realisations of completed records	Source clue
Bath Criterion	Criterion for admissible bath coupling	Conditional architecture
Non-Abelian Bath-Transport Theorem	Matrix-valued SU(3), SU(2), U(1) bath transport	Theorem
Gauge-Closure Selection Theorem	Selection of the gauge closure structure	Theorem
Carrier Theorem	Full corpus gauge carrier	Theorem; supplies the matrix-valued structure the reduced implementation lacked
HMGBC-1	Positive-transfer and Doob normalisation architecture	Conditional architecture; still-open premises on the physical branch

The present calculation consumes no measured mass, coupling, mixing angle, strong phase, neutrino datum or confinement observable. It does consume the frozen HCMR matrix as a **post-source pullback target** - a load-bearing distinction, since reconstructing a supplied matrix proves embeddability, not derivation. No later source is consumed as a numerical descent certificate, because no such certificate yet exists.

3. Claim-grade firewall

The following implications are forbidden:

- rank ≥ 96 does not imply that the field embedding is known;

- an exact factorisation $H = J^T I J$ does not imply that J was differentiated from the primitive action;
- a Fisher-orthogonal sector split does not uniquely identify gauge, completion or charged field coordinates inside the interaction sector;
- regulator stability of a factorisation does not promote a supplied target;
- a zero terminal Hessian block does not by itself derive the physical persistent-record dilation;
- a benchmark probability does not set seconds or GeV;
- a static Hessian does not determine a momentum derivative or wave-function residue;
- a valid Fisher isometry does not define a physical event law;
- a covariance does not determine a full probability measure;
- a Poisson total rate does not uniquely determine three internal stage hazards;
- a real-time memory kernel is not automatically a positive Euclidean covariance;
- a gauge group theorem does not by itself supply a finite-regulator matrix-valued source;
- $C = J_s J_D^+$ is physically admissible only if the HM4 residual vanishes;
- a formal expression for Z_comp is not a numerical scale prediction;
- a scalar Doob kernel determines the marked population law, not the representation-bearing amplitudes; partial isometries require a separate amplitude-level return;
- operator positivity does not imply a positivity-preserving or positivity-improving semigroup; the Doob construction consumes strict kernel positivity, not the inequality $T \geq 0$;
- a pseudoinverse coupling is canonical only relative to declared inner products, coordinates, projections and the source functional; physical canonicity requires invariance under admissible reparameterisations and regulator refinement;
- a compression P A P of an operator is not its restriction unless the invariance defect $(I - P)AP$ vanishes;
- a tangent factorisation at a frozen background does not imply a nonlinear law away from it; $Ds = C\,DD$ at $X\star$ does not give $s(X) = C(D(X))$;
- an unmarked stochastic kernel does not determine its marked decomposition; mark labels must be returned from the source, not read off the kernel;
- an unnormalised residual norm is not a cross-regulator test; vanishing must be declared on a normalised residual with a reported pseudoinverse cutoff;
- a conditional source theorem must not be labelled machine-certified.

4. Vocabulary

EDGE-SCORE RETURN means a result derived from the retained population transition probabilities and their logit Fisher metric. **TYPED PULLBACK** means a Jacobian whose columns are restricted to the source-local history-destination main-effect, source-local neutral-destination main-effect or destination-interaction score spaces according to the broad HCMR block ledger. It is weaker than a strict factor-preserving tangent, because the local main effects may depend on the opposite source state. **REFERENCE LIFT** means one deterministic representative constructed before any comparison with a second representative. It is not called canonical physical. **PRIMITIVE RETURN** means an automatic derivative of the common marked action with primitive variables, source hashes and no target visibility. No such return is supplied here. **PHYSICAL EQUIVALENCE** removes exact score-normalisation gauges, field-basis conventions that leave all declared observables invariant, and the accepted branch equivalence if separately proved. **MACHINE CLOSED** means an exact theorem or machine-certified finite-regulator return with frozen inputs and independent verification. **ANALYTICALLY REDUCED** means a question converted into an explicitly stated executable test whose decisive numbers have not been calculated. **CONDITIONALLY CONSTRUCTED** means an architecture derived in full under explicitly stated hypotheses that have not yet been proved or numerically verified on the faithful source. **CONDITIONAL STRUCTURAL PASS** means an architecture that exists in the corpus at theorem or construction level but has not been executed at finite regulator on the faithful source. **STRUCTURALLY OPEN** means a question for which no executable reduction yet exists.

Part II - The recurrent product law

5. History factor

Let P_H be the seven-state frozen D36/K=7 kernel. Its hub row has seven positive destinations. Each of the six rim rows has four positive destinations. Therefore

$$\Sigma_h r_h = 7 + 6 \times 4 = 31,$$

and the edge-logit Fisher rank of the history factor is

$$\Sigma_h (r_h - 1) = 6 + 6 \times 3 = 24.$$

The source-normalisation gauge contributes one null direction per history source.

6. Neutral renewal factor

At every regulator, the recurrent neutral working carrier contains three active states A_i, three singlet states S_i and three completed-record working states C_i. The retained benchmark transition law has:

- four positive outcomes from each A_i: no jump plus three Born-weighted A-to-S destinations;
- four positive outcomes from each S_i: no jump plus three Born-weighted S-to-C destinations;
- two positive outcomes from each C_i: no jump and record-and-renew C_i-to-A_i.

Thus

$$\Sigma_a s_a = 6 \times 4 + 3 \times 2 = 30,$$

and

$\Sigma_a (s_a - 1) = 6 \times 3 + 3 \times 1 = 21.$

The level-3 to level-6 Born-map drift remains between 8.44×10^{-8} and 1.61×10^{-7} in maximum entry, while the transition and stationarity residuals stay below 1.2×10^{-15}.

7. Product transition and edge coordinates

The retained product law is

$$P^{(n)} = P_H \otimes P_N^{(n)}$$

on $7 \times 9 = 63$ source states. At source (h,a), the positive destination count is $r_h s_a$. The full positive edge-coordinate count is

$$\Sigma_{(h,a)} r_h s_a = (\Sigma_h r_h)(\Sigma_a s_a) = 31 \times 30 = 930.$$

For source stationary weight $\pi_h \pi_a$ and product destination vector $w_{(h,a)} = p_h \otimes q_a$, the edge-logit Fisher block is

$$I_{(h,a)} = \pi_h \pi_a [\mathrm{diag}(w_{(h,a)}) - w_{(h,a)} w_{(h,a)}^T].$$

Its rank is $r_h s_a - 1$. The single null is the common addition of a constant to every outgoing logit at that source.

Part III - Exact edge-logit Fisher decomposition

8. Sourcewise ANOVA theorem

Define $L_0^2(p_h)$ as the real functions of the outgoing history destination with p_h-weighted mean zero, and similarly $L_0^2(q_a)$. Under the product measure p_h tensor q_a,

$$L_0^2(p_h \otimes q_a) = [L_0^2(p_h) \otimes 1] \oplus [1 \otimes L_0^2(q_a)] \oplus [L_0^2(p_h) \otimes L_0^2(q_a)].$$

Theorem 1 - Exact sourcewise score decomposition

The three summands are mutually orthogonal in the covariance/Fisher inner product and have dimensions

$$r_h - 1,\ s_a - 1,\ (r_h - 1)(s_a - 1).$$

Proof

Every vector in the first summand depends only on the history destination and has zero p_h mean. Every vector in the second depends only on the neutral destination and has zero q_a mean. Their product expectation factorises and vanishes. The same factorisation makes each main-effect summand orthogonal to every tensor product of two zero-mean contrasts. The dimensions multiply in the interaction term. Their sum is

$$(r_h - 1) + (s_a - 1) + (r_h - 1)(s_a - 1) = r_h s_a - 1,$$

which equals the full positive Fisher rank at that source. Hence the decomposition is complete. QED.

9. Global rank identity

Summing Theorem 1 over all 63 sources gives:

Source-local history-destination main-effect rank

$$9 \Sigma_h(r_h - 1) = 9 \times 24 = 216.$$

Source-local neutral-destination main-effect rank

$$7 \Sigma_a(s_a - 1) = 7 \times 21 = 147.$$

Interaction rank

$$[\Sigma_h(r_h - 1)][\Sigma_a(s_a - 1)] = 24 \times 21 = 504.$$

Therefore

$$867 = 216 + 147 + 504,$$

and

$$930 = 867 + 63.$$

This identity is regulator independent as long as the positive-support pattern does not change. The four-level execution confirms that it does not.

10. Orthonormal certificate

For every positive destination vector p, the calculation constructs a matrix A satisfying

$$p^T A = 0, A^T \operatorname{diag}(p) A = I.$$

Tensoring these contrast bases gives explicit matrices Q_H, Q_N and Q_X with

$Q\_H^T I Q\_H = I\_216, Q\_N^T I Q\_N = I\_147, Q\_X^T I Q\_X = I\_504,$

and every cross product zero. Across all four regulators, the maximum orthogonality or cross-sector residual is 1.78×10^{-15}.

Part IV - Sector-resolved capacity and the 102-dimensional pullback

11. HCMR carrier ledger

The frozen HCMR parent is ordered as:

Block	Real dimension	Required score class
History defect plus bath	12	Source-local history-destination main effect
Gauge direct plus auxiliary	6	Interaction
Generation-address completion	18	Interaction
Charged left-right parent	36	Interaction
Neutral Nambu parent	24	Source-local neutral-destination main effect
Completed persistent record	6	Exact score-null representative
Total	**102**	Rank 96 plus six nulls

The choice to place gauge, completion and charged blocks in the interaction sector is broad rather than microscopic. These blocks require both history and retained destination typing; the current population law does not further distinguish their representation content. The distinction is carried as an open debt rather than hidden. Part IX identifies the corpus structures that resolve the gauge portion of that debt, and Part XI states where the gauge dependence must enter the forward construction.

12. Sector-capacity theorem

Theorem 2 - Broad source-local typed capacity

The recurrent score law has enough source-local rank to support every major nonterminal HCMR block without borrowing between the three broad destination-score classes:

$12 \leq 216, 24 \leq 147, 6 + 18 + 36 = 60 \leq 504.$

This is stronger than the earlier total inequality $96 \leq 867$, but it does not yet impose globally shared factor parameters.

The positive spectrum is also robust. At every level, 495 eigenvalues exceed 10^{-2} times the maximum eigenvalue. Thus a rank-96 pullback can be supported without relying exclusively on the 10^{-11} tail of the Fisher spectrum.

Corollary 2.1 - Strict factor-preserving neutral deficit

If admissible perturbations must preserve one history kernel shared across all nine neutral source states and one neutral kernel shared across all seven history source states, then the tangent ranks are only

24 shared-history, 21 shared-neutral.

The 12-dimensional HCMR history block fits inside the shared-history tangent. The 24-dimensional neutral block does not fit inside the shared-neutral tangent: it has an exact deficit of three. Therefore any primitive lift of the current neutral parent must do at least one of the following before target access:

- make the neutral transition response depend on the K=7 history source;
- place at least three neutral directions in a genuine history-neutral destination-interaction tangent;
- enlarge the recurrent neutral law; or
- replace the product law by a non-product primitive transition.

This is a structural requirement, not a numerical shortfall. It also shows why the 147-dimensional source-local neutral class must not be mistaken for the tangent of one globally shared neutral factor. Part XIII converts this requirement into an exact numerical release test on the common-bath history-neutral cross response.

Tangent class	Available rank	HCMR demand	Margin	Verdict
Shared history factor	24	12	+12	PASS
Shared neutral factor	21	24	-3	FAIL BY 3
Source-local history-destination main effect	216	12	+204	PASS - broad typing
Source-local neutral-destination main effect	147	24	+123	PASS - broad typing
Destination interaction	504	60	+444	PASS

13. Explicit typed lift

Choose Fisher-orthonormal frames

E_H in St(12,216), E_N in St(24,147), E_X in St(60,504), and split E_X into consecutive frames E_g, E_c and E_Y of widths 6, 18, 36. Let H_H, H_g, H_c, H_Y and H_N be the corresponding positive diagonal blocks of H_full. Define

$$J = [E\_H\ H\_H^{(1/2)}, E\_g\ H\_g^{(1/2)}, E\_c\ H\_c^{(1/2)}, E\_Y\ H\_Y^{(1/2)}, E\_N\ H\_N^{(1/2)}, 0\_6].$$

Theorem 3 - Exact typed pullback

Assume the frozen HCMR parent has the declared block-diagonal major-sector structure in the stated ordering - history, gauge, completion, charged, neutral, record-null - with vanishing inter-sector cross blocks. Then for every frozen HCMR level and branch,

$$J^T I J = H\_full.$$

Proof

Within each selected frame, $E^T I E = I$. Frames from different score classes are Fisher orthogonal, and the three interaction subframes are mutually orthogonal columns of one 60-frame. Hence each diagonal pullback is $H\_block^{(1/2)} I H\_block^{(1/2)} = H\_block$ and every major cross block vanishes. The terminal columns are zero, reproducing the declared record null block. QED.

The block assumption is doing real work and is therefore stated in the hypotheses rather than left implicit. The displayed construction takes independent sector square roots; it would not reproduce a general 102-dimensional positive parent with nonzero inter-sector cross blocks. If such cross blocks existed, the lift would instead require a global square root of the full parent embedded into a single sufficiently large combined score frame. The four-regulator residuals of Part V, at machine precision on every level and branch, certify that the supplied parent satisfies the block assumption; the theorem now exposes that fact rather than relying on it silently.

The theorem should not be misread. H_full is an input to $H\_block^{(1/2)}$; the theorem proves exact embeddability in the recurrent Fisher geometry. It does not prove that the primitive action differentiates to this J.

Part V - Four-regulator execution

14. Numerical protocol

For n=3,4,5,6:

freeze the inputs (history kernel, polar support maps, branch-specific parent); construct the recurrent population law with $p\_D = p\_R = p\_B = 1 - e^{-1}$, the 63-state product support and all 930 edge coordinates; build the sourcewise Fisher blocks and the three contrast bases before loading the HCMR parent; form the typed reference Jacobian and an independently seeded typed alternative; verify both pullbacks, the score decomposition and branch conjugacy; and hash every frozen input and output. The full thirteen-step executable sequence, with commands, is Appendix C. Two disciplines matter: the score bases are fixed before the parent is loaded, and the alternative frame is a fixed-seed orthonormal frame in the same three exact score sectors, not tuned to enlarge the difference.

15. Fisher certificate

Level	Edge dim.	Rank	Nullity	Min positive eigenvalue	Max eigenvalue	Modes >1% max
3	930	867	63	5.51048×10^{-12}	4.03696×10^{-3}	495
4	930	867	63	5.68850×10^{-12}	4.03696×10^{-3}	495
5	930	867	63	5.73096×10^{-12}	4.03696×10^{-3}	495
6	930	867	63	5.74157×10^{-12}	4.03696×10^{-3}	495

The positive condition number is approximately 7.0×10^{8}. That wide spectrum is a real property of the nearly key-preserving Born maps. It does not undermine rank-96 capacity because the upper 495 modes remain well separated from the smallest tail.

Figure 1. Sector-resolved recurrent Fisher rank compared with the HCMR dimensions assigned to each score class.

Figure 2. Positive recurrent Fisher spectra at levels 3-6. The first 96 modes lie in the robust upper part of the spectrum.

16. Pullback certificate

Level	Branch	H rank/nullity	Reference relative residual	Alternative relative residual	Relative J difference	Sample Fisher cosine
3	1	96 / 6	6.02×10^{-16}	4.87×10^{-16}	1.853	-0.0042
3	6	96 / 6	6.02×10^{-16}	4.39×10^{-16}	1.851	-0.0534
4	1	96 / 6	3.34×10^{-16}	8.78×10^{-16}	1.724	-0.0291
4	6	96 / 6	3.34×10^{-16}	3.16×10^{-16}	1.755	-0.0045
5	1	96 / 6	3.56×10^{-16}	7.23×10^{-16}	1.720	-0.0672
5	6	96 / 6	3.56×10^{-16}	2.43×10^{-16}	1.842	-0.0091
6	1	96 / 6	8.84×10^{-16}	8.61×10^{-16}	1.951	-0.0490
6	6	96 / 6	8.85×10^{-16}	4.70×10^{-16}	1.815	+0.0107

The identical sample score norms on each pair are required by the common pullback:

$\|J\_ref\ x\|\_I^2 = x^T H x = \|J\_alt\ x\|\_I^2$.

The near-zero cross cosine shows that equal quadratic norm does not imply equal mark-level response.

Figure 3. Two independently selected typed lifts reconstruct the same level-dependent HCMR parent at floating residual.

Figure 4. Relative Jacobian separation remains order one while the Fisher overlap of a frozen sample response is near zero.

Part VI - Exact lift-family and non-identifiability theorem

17. General parameterisation

Let I be the 930-dimensional Fisher form with rank r = 867. Let Q be any 930 × 867 matrix satisfying

$Q^T I Q = I\_867$.

Let H_nt be the positive 96 × 96 nonterminal HCMR parent. Then every minimum-score-gauge exact lift may be written as

J_nt = Q K H_nt^(1/2),

where

$K^T K = I\_96$.

Thus K belongs to the real Stiefel manifold St(96,867).

Conversely, if J_nt^T I J_nt = H_nt, then Q^T I J_nt H_nt^(−1/2) is a 96-frame in the positive score space, so the representation above is complete after choosing Q.

18. Dimension theorem

The real dimension of St(p,n) is

np - p(p+1)/2.

Therefore

dim St(96,867) = 867 × 96 - 96 × 97 / 2 = 78,576.

The Fisher null space has dimension 63. Adding any N with I N = 0 changes no pullback. For all 102 columns this score-gauge family has dimension

$63 \times 102 = 6{,}426.$

The full affine exact-lift family therefore has dimension

$78{,}576 + 6{,}426 = 85{,}002.$

Theorem 4 - Exact-lift non-identifiability

The recurrent Fisher form and the HCMR parent do not determine a unique field-to-mark Jacobian, even up to source-normalisation gauge. The minimum-null physical-score family alone has dimension 78,576.

19. Typed family

Under the broad typing used in Theorem 3, the minimum-null family contains

$\mathrm{St}(12,216) \times \mathrm{St}(24,147) \times \mathrm{St}(60,504).$

Its dimension is

$2{,}514 + 3{,}228 + 28{,}410 = 34{,}152.$

This count is a lower bound on unresolved microscopic freedom because it freezes the broad assignment of all gauge, completion and charged coordinates to the interaction sector and forbids cross-sector mixtures.

Corollary 4.1

Sector-resolved capacity does not become sector-resolved identifiability. A direct primitive derivative or an independent field-to-mark typing theorem remains mandatory.

20. Why two exact lifts matter

A purely abstract dimension count could be dismissed as gauge redundancy. The two-lift execution rules that out. The reference and alternative lifts:

- obey the same broad score typing;
- use the same recurrent Fisher law;
- reproduce the same HCMR parent;
- have the same quadratic response for every field perturbation;
- nevertheless map those perturbations to different individual edge scores.

The relative Jacobian separation is larger than 1.7 at every level. For the frozen sample vector, the Fisher cosine lies close to zero. These differences concern marked transition responses, not a rotation of the Kraus list describing one fixed marked channel.

Part VII - Conjugate-branch lift theorem

21. Exact branch operator

Define

$J_m = \mathrm{diag}(I_m, -I_m)$

on the realification of an m-dimensional complex space, and

$C = I_{12} \oplus I_6 \oplus J_9 \oplus I_{36} \oplus J_6 \oplus J_6 \oplus I_6.$

The four-regulator arrays satisfy

$H_{full}^{(n,6)} = C^T H_{full}^{(n,1)} C$

with zero matrix-entry residual.

22. Lifted conjugacy

Theorem 5 - Recurrent branch-conjugacy lift

If $J_{(n,1)}^T I_n J_{(n,1)} = H_{full}^{(n,1)}$, then

$J_{(n,6)} = J_{(n,1)} C$

satisfies

$J_{(n,6)}^T I_n J_{(n,6)} = H_{full}^{(n,6)}.$

Proof

Substitution gives

$C^T J_{(n,1)}^T I_n J_{(n,1)} C = C^T H_{full}^{(n,1)} C = H_{full}^{(n,6)}.$

QED.

The maximum numerical residual of this lifted identity is 3.41×10^{-13} in absolute matrix entries, set by multiplication of the large gauge block and floating square roots. The underlying HCMR conjugacy is exact.

Consequence

No positive quadratic weighting built only from I_n and H_full can prefer k=1 over k=6. A physical orientation requires an action-derived odd coupling or an accepted exact-equivalence rule. Constructing a field-to-mark factorisation does not lift the tie.

Implication for the source-projected calculation

The forward architecture of Parts X-XII inherits this constraint rather than escaping it automatically. The Doob construction alone will not orient the branch if the complete Euclidean generator is conjugation-even: if $H_6 = C^T H_1 C$ at generator level, then the transfer, Perron pair, Doob kernel and marked rates on the two branches are exactly conjugate as well. Orientation therefore requires either

$$H_6 \neq C^T H_1 C$$

through an action-derived odd term entering the generator, or a proof that the two branches are physically output-equivalent. The HM4 and Doob tests of the next-execution protocol must consequently be evaluated on both branches, and the branch gap reported alongside the residuals.

Part VIII - The reduced-source failure and the faithful gauge carrier

23. The EPMD-1 deficit

The first attempt to execute a forward source calculation used a reduced executable truncation of the master substrate. In that truncation the gauge content of the source was carried by 36 scalar gauge-edge coordinates: one scalar factor phase per gauge factor per retained edge family. EPMD-1 executed the resulting derivative and returned a structural failure.

The failure is a counting theorem, not a numerical accident. A faithful twelve-edge Standard Model connection carries, at each edge, the full tangent of the gauge group, and the corpus gauge algebra is $su(3) \oplus su(2) \oplus u(1)$ with real dimension $8 + 3 + 1 = 12$. The faithful gauge tangent demand is therefore

$$12 \times (8 + 3 + 1) = 144$$

gauge tangents. Against the 36 scalar coordinates actually implemented, the deficit was

$$144 - 36 = 108.$$

This was a failure of the reduced executable truncation, not proof that the full theoretical corpus lacked the gauge carrier. Scalar factor phases can never carry the non-Abelian $su(3) \oplus su(2)$ tangent, whatever their number; the truncation was structurally incapable of the target, and its refusal certifies the implementation limit rather than a corpus gap.

24. The corpus repair

The later gauge papers supply exactly the matrix-valued structure the truncation lacked. The Non-Abelian Bath-Transport Theorem constructs matrix-valued SU(3), SU(2) and U(1) transport on the retained carrier. The Gauge-Closure Selection Theorem fixes the closure structure that selects it. The Carrier Theorem establishes that the full corpus gauge carrier exists at the required dimension. Together these grade the audit row "full corpus gauge carrier" as a conditional structural pass: the object exists in the corpus at theorem level and must replace the reduced scalar implementation before the source derivative of Part X is executed.

The statement this Part contributes is threefold: the earlier implementation was provably too small by 108 gauge tangents; the corpus already contains the faithful carrier; and the faithful carrier has not yet been differentiated at any finite regulator.

Part IX - The six-mode bath carrier

25. Bath-carrier typing and conditional identification

Introduce the uniform vector on the seven-state K7 wheel,

$$u_0 = (1/\sqrt{7})\,(1, \ldots, 1)^T.$$

Both the commitment-event bath construction and the full projected closure operator act on a six-dimensional nonuniform K7 carrier, though they provide different data. The commitment-event process supplies the stochastic driving: Poisson arrivals of completed commitment events, their covariance, spectral response and real-time memory kernel. The projected closure operator supplies the restoring frequencies: the pair (L_n, M_n) and the generalized eigenproblem

$$L_n v = \omega^2 M_n v$$

restricted away from the uniform mode.

One point must be typed with care, because the closure modes are explicitly non-orthogonal: the correct nonuniform subspace is not automatically the ordinary Euclidean complement of u_0. The mass operator M_n defines the whitening, so the projection must be made after whitening, using the whitened uniform vector

$$\tilde{u}_0 = M_n^{1/2} u_0 \,/\, \|M_n^{1/2} u_0\|.$$

Define the transverse projector and the bath frequency operator as

$$P_{Q,n} = I - \tilde{u}_0\tilde{u}_0^\dagger,\quad \Omega_{Q,n}^2 = P_{Q,n} M_n^{-1/2} L_n M_n^{-1/2} P_{Q,n} \text{ on ran } P_{Q,n},$$

and write $Q_n = \mathrm{ran}\, P_{Q,n}$ for the six-mode bath carrier. Until invariance is proved, $\Omega_{Q,n}^2$ is a **compressed candidate bath operator**, not a restriction: the compression is always well defined, but it agrees with the restriction of the whitened closure operator only if

$$[M_n^{-1/2} L_n M_n^{-1/2}, P_{Q,n}] = 0,$$

equivalently only if the invariance defect

$$R_{inv}^{(n)} = (I - P_{Q,n}) A_n P_{Q,n}, \quad A_n = M_n^{-1/2} L_n M_n^{-1/2},$$

vanishes. For cross-regulator comparability the protocol reports the normalised defect

$$\varepsilon_{inv}^{(n)} = \| (I - P_{Q,n}) A_n P_{Q,n} \| / \max(\|A_n P_{Q,n}\|, \varepsilon_0),$$

and requires it to tend to zero. The equality of the closure carrier and the commitment-bath carrier is certified separately by a principal-angle test: with $P_{closure}$ and $P_{commitment}$ the orthogonal projectors onto the two candidate six-dimensional carriers, report $\|P_{closure} - P_{commitment}\|$, or equivalently the six singular values of the overlap between orthonormal frames of the two subspaces, and require the singular values to approach one. Together, ε_{inv} and the principal-angle certificate give the invariant-subspace obligation an exact numerical form. The definition is convention-safe: if u_0 is a common eigenvector of L_n and M_n, or if the carrier is defined as the M_n-orthogonal complement of the uniform mode, the same operator is obtained; the whitened-projector form is the general statement containing those special cases.

The identification claim is graded precisely. $\Omega_{Q,n}^2$ is the canonical **compressed candidate**: canonical, because the whitened transverse projection is the unique convention-independent construction from (L_n, M_n, u_0); a candidate, because it becomes the physical bath frequency operator only when the two obligations just stated - the vanishing invariance defect and the carrier principal-angle equality - verify in the protocol.

The event covariance requires the same coordinate discipline. The commitment-event process supplies a compound-symmetry covariance on the unwhitened Euclidean K7 coordinates,

$$\Sigma_\eta = \nu_\eta^2 [(1 - \rho) I + \rho\, \mathbb{1}\mathbb{1}^\dagger],$$

where ν_η^2 is the per-vertex event variance and ρ the single-step event correlation. For this to be a genuine positive-semidefinite covariance on the seven coordinates before any transport, the compound-symmetry parameter must lie in the admissible range

$$-1/6 \le \rho \le 1,$$

since the eigenvalues of Σ_η are $\nu_\eta^2(1 + 6\rho)$ on the uniform mode and $\nu_\eta^2(1 - \rho)$ with multiplicity six on its complement; the declared domain is recorded here where the covariance is introduced, and positivity of the transported projected covariance is then the separate downstream check of the protocol. Further, the event rate is a separate object and is written Γ throughout - ν_η is a variance scale, not a rate. On the ordinary Euclidean nonuniform carrier, projecting with $I - u_0 u_0^\dagger$ kills the rank-one correlated component exactly and leaves the isotropic form $\nu_\eta^2(1 - \rho)$ on that carrier. But the bath carrier of this paper is the whitened transverse carrier, and $P_{Q,n}$ removes the whitened uniform vector $\tilde{u}_0$, which need not be parallel to $\mathbb{1}$; hence $P_{Q,n} \mathbb{1}$ need not vanish and the correlated component may survive the projection. The covariance must therefore be transported into the same coordinates as the frequencies before the two are combined. With the declared whitening $\tilde{q} = M_n^{(1/2)} q$, the transported covariance is

$$\tilde{\Sigma}_{\eta,n} = M_n^{(1/2)} \Sigma_\eta M_n^{(1/2)},$$

and the physical projected covariance is

$$\Sigma_{Q,n} = P_{Q,n} \tilde{\Sigma}_{\eta,n} P_{Q,n}.$$

Scalar isotropy on the whitened carrier - $\Sigma_{Q,n}$ proportional to $P_{Q,n}$ - is **not** required and is not an admission gate. If M_n has different eigenvalues on the nonuniform modes, whitening naturally turns an initially isotropic covariance into a diagonal but anisotropic one; that is a coordinate effect, not a physical failure. The meaningful compatibility conditions are:

positivity, $\Sigma_{Q,n} \succeq 0$;

correct transformation under the declared coordinate map, so that frequencies and covariance are combined in one coordinate system;

and, where the symmetry of the construction requires it, compatibility with the bath operator - for example $[\Sigma_{Q,n}, \Omega_{Q,n}^2] = 0$, or the appropriate conjugate-pair block structure.

The protocol reports the full spectrum of $\Sigma_{Q,n}$ rather than testing scalar isotropy. The compatibility question and the non-degeneracy of $\Omega_{Q,n}^2$ remain independent statements; the first concerns the driving noise, the second the mode frequencies. No numerical value of ν_η, ρ, Γ or the spectra is certified here; the content of this Part is the exact typing of the carrier and its two compatibility obligations - invariance and covariance compatibility.

Part X - The source-defect factorisation theorem

26. Defect classes, the typed defect space and the projected source

This is the central new structure of the revision. The master substrate supplies six MASD defect classes,

$D_C, D_J, D_T, D_R, D_B, D_H,$

evaluated on the primitive configuration X. These are six defect **sectors**, not six scalar coordinates: each class is in general vector-, matrix- or operator-valued on the retained finite-regulator carrier. Type them jointly as one map into the direct-sum defect space

$$E_{D,n} = E_C \oplus E_J \oplus E_T \oplus E_R \oplus E_B \oplus E_H, \quad D_n : X_n \to E_{D,n},$$

where X_n is the primitive configuration space at regulator n and each summand carries the natural finite-regulator inner product of its class. Define the projected commitment source in the canonical bath coordinates. The bath carrier Q_n and projector $P_{Q,n}$ live in whitened coordinates, so the source functional must be transported into those coordinates before projection:

$$s_n(X) = P_{Q,n} W_n S_n[\Phi(X)], \quad s_n : X_n \to Q_n,$$

where $S_n[\Phi]$ is the commitment source functional of the faithful field map Φ, $W_n = M_n^{1/2}$ is the declared whitening transport - or, where a different pairing is declared, the exact dual-basis map that carries the source into the canonical bath coordinates - and $P_{Q,n}$ is the whitened transverse projector of Part IX. Mixing coordinate systems here would silently misalign the source against the frequencies and covariance, which are already expressed on the whitened carrier.

The carrier convention is frozen: W_n and $P_{Q,n}$ are evaluated at the frozen background $X^\star$ and held fixed. If either depended on the primitive fields, the derivative would contain additional terms,

$$D(P_Q W S) = (DP_Q) W S + P_Q (DW) S + P_Q W (DS),$$

and ignoring them would silently convert a moving-carrier problem into a fixed-carrier one. The present paper declares the fixed-carrier convention; a moving-carrier extension is a separate structure recorded as open.

At the frozen background $X^\star$, define the two Jacobians

$$J_{D,n} = DD_n|_{X^\star} : T_{X^\star}X_n \to E_{D,n}, \quad J_{s,n} = Ds_n|_{X^\star} : T_{X^\star}X_n \to Q_n.$$

$J_{D,n}$ is the defect Jacobian: it measures how primitive perturbations move the six defect sectors. $J_{s,n}$ is the projected source Jacobian: it measures how the same perturbations move the bath source. A candidate coupling is a linear map

$$C_n : E_{D,n} \to Q_n.$$

With this typing, every statement of Theorem B is dimensionally rigorous whatever the internal dimensions of the six classes.

27. The factorisation theorem

Theorem 6 - Defect-source factorisation

There exists a coupling $C : E_{D,n} \to Q_n$ with

$J_s = C J_D$

if and only if

$\ker J_D \subseteq \ker J_s$.

The proof is given in Appendix E. The content of the theorem is that the bath source descends from the physical defect quotient exactly when every primitive perturbation invisible to the defects is also invisible to the source. If some primitive direction moves the source without moving any defect, no defect-mediated coupling can reproduce the source, and the bath is not generated by the defect channel.

The scope of the theorem is local. It is a statement about derivatives at the frozen background: it proves first-order descent through the defect quotient. From $Ds|_{X\star} = C\, DD|_{X\star}$ it does not follow that $s(X) = C\, D(X)$ away from $X\star$. Two grades are therefore distinguished. The **quadratic local theorem** - the grade this paper claims - uses only the linearised defects, $\delta D_n = J_{D,n}\, \delta X$, and defines the quadratic generator of Part XI around $X\star$. The **nonlinear descent theorem** - not claimed here - would require the stronger condition $s_n(X) = C_n(D_n(X))$ on a neighbourhood of $X\star$, or verified higher-order jet factorisation $D^k s_n = D^k (C_n \circ D_n)$ for $k \geq 2$. The HM4 theorem supplies a quadratic tangent coupling, not yet a full nonlinear bath law.

28. The HM4 residual and the minimum-norm coupling

Introduce the HM4 provenance residual

$R_{HM4} = J_s (I - J_D^+ J_D)$,

where J_D^+ is the Moore–Penrose pseudoinverse. Since $I - J_D^+ J_D$ is the orthogonal projector onto $\ker J_D$, the kernel-inclusion condition of Theorem 6 holds if and only if

$R_{HM4} = 0$.

When the residual vanishes, define

$C^{min} = J_s J_D^+$.

Appendix E proves that this choice reproduces the source exactly on the reachable image $\mathrm{ran}\, J_D$, is the unique minimal-Frobenius-norm solution, and that all remaining freedom acts only on the unreachable complement of $\mathrm{ran}\, J_D$, where no physical defect motion ever arrives.

The name is chosen deliberately: C^{min} is the **canonical minimum-norm representative of the defect-quotient coupling**, canonical only relative to the five declared choices listed in Theorem

B item 3; physical canonicity - invariance under admissible reparameterisations and regulator refinement - is a separate condition to be established, not a consequence of the pseudoinverse formula.

The residual is stated in finite-dimensional Hilbert spaces with the declared inner products, where the pseudoinverse formula is exact. For the numerical execution, the raw norm of R_HM4 is not comparable across regulators and source normalisations, so the reported quantity is the normalised residual

$$\varepsilon\_HM4 = \| J\_s (I - J\_D^{+} J\_D) \| / \max(\|J\_s\|, \varepsilon_0),$$

with ε_0 a declared floor preventing division by a vanishing source Jacobian, together with the singular-value cutoff used to construct J_D^{+}. "Vanishes" means ε_HM4 below the declared regulator-stable tolerance, reported against the singular gap of J_D.

Two further boundaries must be stated. First, this is a theorem about factorisation through the physical defect quotient, not a numerical result: neither J_D,n nor J_s,n has been differentiated on the faithful source at any regulator, so ε_HM4 is a defined test, not an executed one. Second, C^min is physically admissible only if the residual vanishes; writing the formula does not license its use on a source whose residual is unknown.

Part XI - The Euclidean history generator

29. Conditional positivity and the history Schur complement

Given an admissible coupling C_n : E_D,n → Q_n, define the enlarged Euclidean history generator in the quadratic local grade of Section 27: the defect entry is the linearised defect coordinate

$$\delta D\_n = J\_D,n \; \delta X,$$

evaluated on the tangent fluctuation δX about the frozen background, and

$$\mathscr{H}\_n = -\Delta\_n + V\_loc\text{^}(n) + (1/2)\, q^{T}\, \Omega\_Q,n^2\, q + q^{T}\, C\_n\, \delta D\_n,$$

where q is the six-mode bath coordinate on Q_n, Δ_n and V_loc^(n) are the retained kinetic and local terms of the history sector, and the last term couples the bath linearly to the linearised defect sectors through C_n. Using the full nonlinear defect map D_n(X) here would exceed the evidential strength of Theorem 6, which is a tangent theorem; the quadratic generator around X⋆ is exactly what the factorisation licenses. The Euclidean transfer is

$$T\_n = e\text{^}(-a\_t \mathscr{H}\_n).$$

Positivity is not automatic and is imposed as two explicit hypotheses.

(P1) Coercivity. Completing the square in the bath variables produces the effective shift

$$-(1/2)\, D_n^\dagger C_n^\dagger \Omega_{Q,n}^{-2} C_n D_n,$$

which is a bounded modification of $V_{loc}^{(n)}$ only if the defect sectors are bounded on the retained carrier or the core action supplies enough positive curvature. The declared condition is

$$W_{core}^{(n)} - C_n^\dagger \Omega_{Q,n}^{-2} C_n \succeq 0,$$

or, more generally, semiboundedness $\mathscr{H}_n \geq -c_n I$ on the declared regulator domain. Without (P1) the transfer need not exist as a bounded positive operator.

(P2) Kernel positivity. A positive self-adjoint operator is not automatically positivity-preserving in a chosen basis; the Doob construction of Part XII consumes strict kernel positivity and the positivity-improving property, not merely $T \succeq 0$. The declared condition is that, in the physical mark basis of the retained carrier, the transfer kernel is strictly positive - automatic for the scalar quadratic-plus-local reduction by Feynman–Kac, an explicit hypothesis to be verified for the matrix-valued faithful carrier, on which Feynman–Kac does not act directly. Appendix F.1 separates the four positivity notions in full.

Both hypotheses are recorded in the audit matrix and the execution protocol; neither is assumed silently. Positivity must in any case be established from this Euclidean transfer, not by inserting the oscillatory real-time memory kernel of the commitment-event process directly. The real-time kernel is a response function; it oscillates and is not a covariance. Any construction that injects the real-time kernel as if it were a positive Euclidean covariance violates the firewall of Section 3.

Integrating out the quadratic bath yields the history response as an exact Schur complement:

$$W_{hist}^{(n)}(p^2) = W_{core}^{(n)}(p^2) - C_n^\dagger (p^2 I + \Omega_{Q,n}^2)^{-1} C_n.$$

Appendix F proves, under (P1)-(P2), positivity of the transfer for the quadratic-plus-local generator and shows that the subtracted term is a Stieltjes function of p^2: monotone, completely monotone in the resolvent variable, and therefore incapable of introducing spurious poles on the physical axis. The same structure supplies the completion response and residue used in Part XV.

Part XII - The marked Doob–Kraus descent

30. From transfer to marked population law and typed amplitudes

Let T_n be the Euclidean transfer of Part XI, assumed under (P1)-(P2) to have a strictly positive kernel and hence to be irreducible on the retained state space. Define the Perron pair

$T_n r_n = \lambda_{0,n} r_n$, $l_n^\dagger T_n = \lambda_{0,n} l_n^\dagger$,

with r_n and l_n strictly positive. The Doob kernel is

$$K_n(y|x) = T_n(x,y)\, r_n(y) / (\lambda_{0,n}\, r_n(x)).$$

K_n is exactly stochastic: its rows sum to one by the eigenvalue equation. It is the normalised stepping law generated by the transfer.

What this construction returns is the **unmarked Doob kernel** K_n. The marked decomposition

$$K_n = \Sigma_\alpha K_{n,\alpha},$$

by the physical record mark α - the type of completed record the transition stamps - is not determined by K_n itself: an unmarked stochastic kernel admits many decompositions. The mark labels must come from the source, by one of three declared routes: a source-derived amplitude decomposition $A_n = \Sigma_\alpha A_{n,\alpha}$; orthogonal record projectors R_α supplied by the source dilation; or an explicitly imported marked-support law, which would then carry an import grade rather than a descent grade. Until one of these returns is made, every marked object below is typed, not derived.

Fixed-step marked instrument (primary form). The mathematically safe form of the descent is defined at a fixed finite transfer step a_t, without a continuum limit. The marked population law at step a_t is $K_{n,\alpha}(y|x)$, and the discrete-time Kraus instrument is

$$L_{\alpha,yx} = \sqrt{K_\alpha(y|x)}\, V_{\alpha,yx},$$

with the completeness condition in its general form

$$\Sigma_{\{\alpha,y\}} K_\alpha(y|x)\, V_{\alpha,yx}^\dagger V_{\alpha,yx} = P_x.$$

If every partial isometry has initial projector exactly P_x - that is, $V_{\alpha,yx}^\dagger V_{\alpha,yx} = P_x$ for all outgoing transitions - this reduces, by $\Sigma_{\{\alpha,y\}} K_\alpha(y|x) = 1$, to the simpler statement $\Sigma_{\{\alpha,y\}} L_{\alpha,yx}^\dagger L_{\alpha,yx} = P_x$. The general form is retained because different transitions may be supported on different initial subspaces; which case holds is decided by the amplitude return, not assumed. Existence must be graded carefully: under (P1)-(P2) the unmarked Doob kernel exists, and an abstract classical dilation of it exists, but the physical representation-bearing marked instrument remains open until the source-derived mark decomposition and the amplitude-level arrays are returned with their completeness relation. The fixed-step form requires no assumption about the sign structure of the generator.

Generator limit (conditional form). The continuous-time rates carry the Perron-vector ratio; the Doob normalisation is not a diagonal-order correction. The unmarked rate law is

$$q_n(y|x) = -\mathscr{H}_{xy}\, r_n(y)/r_n(x), \quad y \neq x,$$

equivalently the off-diagonal part of the ground-state (Doob-transformed) generator

$$G_n = -r_n^{-1} (\mathscr{H}_n - E_{0,n})\, r_n, \quad q_n(y|x) = (G_n)_{xy},$$

with r_n the diagonal multiplication operator; the a_t-expansion behind this, and why the ratio $r_n(y)/r_n(x)$ is order one rather than a diagonal-order correction, are derived in Appendix G.2. Nonnegativity requires an additional structural condition, stated on the Doob-transformed generator:

(P3) Doob-gauge stoquasticity. In the physical mark basis, $-\mathscr{H}_{xy}\, r_n(y)/r_n(x) \geq 0$ for all $x \neq y$.

Because $r_n > 0$ this has the same sign content as $-\mathscr{H}_{xy} \geq 0$, but the rate magnitudes carry the ratio. A generic Hamiltonian need not satisfy it, and positivity of the transfer at finite a_t does not by itself deliver a classical Markov generator, so (P3) is declared as an explicit hypothesis; if it fails, the fixed-step form above remains the physical descent object and no continuous-time rate law is claimed. The fixed-step construction is unaffected: the Doob kernel contains the ratio exactly at every finite a_t.

Amplitude-level return (typed, not derived). The scalar Doob kernel determines the marked population law only. It does not determine the representation-bearing maps $V_{\alpha,yx}$. Those require a separate amplitude-level construction from the faithful source: with A_α the amplitude operator of mark α supplied by the source dilation and P_x, P_y the local support projectors,

$$M_{\alpha,yx} = P_y A_\alpha P_x, \quad V_{\alpha,yx} = \mathrm{Pol}(M_{\alpha,yx}),$$

where Pol denotes the polar-decomposition partial isometry. The physical consistency requirement is that the amplitudes and the populations descend from the same source: the polar moduli of $M_{\alpha,yx}$ must reproduce the Doob populations. The execution protocol returns M_α, V_α and this consistency test as separate frozen arrays.

The grade of this Part is therefore: **fixed-step unmarked Doob kernel conditional on (P1)-(P2); continuous-time Markov-rate limit conditional additionally on (P3); physical marked instrument source-open.** The chain

$$\mathscr{H}_n \to e^{(-a_t \mathscr{H}_n)} \to K_n^{\mathrm{Doob}} \to \{\, K \text{ at fixed } a_t, \text{ or } q = -\mathscr{H}_{xy}\, r(y)/r(x) \text{ under (P3)} \,\} \to [\text{source-derived mark decomposition: } A = \Sigma_\alpha A_\alpha \text{ or } R_\alpha] \to K_\alpha,\ L_\alpha = \sqrt{q_\alpha}\, V_\alpha$$

is explicit at every stage - generator, transfer, normalised kernel, marked populations or rates, typed amplitudes - and replaces the earlier informal instruction to "differentiate forward to the mark law". What it does not yet contain is numbers; its dependence on an admissible C_n, and hence on a vanishing ε_{HM4}, is stated once in Section 28 and holds for every object here.

Part XIII - The history-neutral interaction and the rank-three release test

31. The interaction residual

Define the joint marked rate $q\_X(y,j|x,i)$: the rate at which the process at history state x and neutral state i completes a marked transition to history destination y and neutral destination j. Define its marginals by summation and the interaction residual

$\Xi\_X(x,i; y,j)$ = log of [$q\_X(y,j|x,i)\ \Gamma\_X(x,i)$] divided by [$(\Sigma\_\{j'\}\ q\_X(y,j'|x,i))\ (\Sigma\_\{y'\}\ q\_X(y',j|x,i))$],

where $\Gamma\_X(x,i)$ is the total marked rate at (x,i). Then

$\Xi\_X = 0$

if and only if the history and neutral destinations factorise at every source: the joint rate is the product of its marginals divided by the total rate exactly when the log-ratio vanishes identically.

This continuous-time form, written $\Xi\_X\^(q)$, exists only when (P3) holds; its Γ_X factor normalises the joint rate against the marginals. Because (P3) may fail, a fixed-step form is defined on the Doob kernel itself, with the analogous normalisation: K_X is in general a subkernel with class weight

$p\_X(x,i) = \Sigma\_\{y,j\}\ K\_X(y,j|x,i)$,

and the correctly normalised fixed-step residual is

$\Xi\_X\^(K)(x,i; y,j) = \log [\ K\_X(y,j|x,i)\ p\_X(x,i) / (\ K\_X,H(y|x,i)\ K\_X,N(j|x,i)\ )\ ]$

- equivalently, normalise $\bar{K}\_X = K\_X / p\_X$ first and form the unweighted log-ratio; the unnormalised form is correct only when $p\_X = 1$. $\Xi\_X\^(K)$ vanishes identically if and only if the one-step marked law, conditioned on the event class, factorises at every source. It requires only (P1)-(P2), so the neutral release programme remains executable without a classical generator; under (P3) the two forms carry the same factorisation content as $a\_t \to 0$.

32. The rank-three release test

Corollary 2.1 proved that a globally shared neutral factor is three dimensions short of the 24-dimensional neutral demand, and that the deficit must be repaired by history-conditioned neutral response, genuine interaction, an enlarged neutral law or a non-product primitive transition. The common-bath architecture of Parts IX-XI supplies a specific candidate mechanism: both sectors couple to the same six-mode bath, so integrating out the bath generates a history-neutral cross block

$$W_{hist,neutral} = W_{core,hist,neutral} - C_{hist}^{\dagger} \Omega_Q^{-2} C_{neutral},$$

where C_{hist} and $C_{neutral}$ are the history and neutral columns of the coupling. The typing is fixed first, because the test is meaningless without it:

$$C_{hist} : H_{12} \to Q_6,\ C_{neutral} : N_{24} \to Q_6,$$

so the bath-mediated cross response

$$B = C_{neutral}^{\dagger} \Omega_Q^{-2} C_{hist} : H_{12} \to N_{24}$$

maps history perturbations into the 24-dimensional neutral demand space.

Let P_{miss} be the orthogonal projector onto the three-dimensional complement of the strict shared-neutral tangent inside the neutral demand, constructed in Appendix H. Because ran P_{miss} has dimension exactly three, rank and range conditions coincide and no independent fourth-singular-value test exists (Appendix H.2); the release criterion is therefore stated without redundancy:

(T1) Surjectivity. $\mathrm{ran}(P_{miss} B) = \mathrm{ran}\, P_{miss}$ - equivalently $\mathrm{rank}[P_{miss} B] = 3$: the projected response reaches all three missing directions.

(T2) Stable surjectivity. $\sigma_3(P_{miss} B) > \varepsilon_{reg}$: the smallest of the three singular values clears the regulator-stable tolerance, so the surjectivity decision is stable against the singular gap rather than machine epsilon.

The release conditions are (T1) and (T2) alone. A third quantity, the projection leakage $\|(I - P_{miss})P_{miss} B\|$, vanishes algebraically for an exact projector; it is implementation hygiene, not a release condition, and is reported with the numerical-verification outputs of Appendix H and the protocol rather than gating the physics.

If (T1)-(T2) hold, the common bath supplies the deficit and the neutral block is released at tangent level. If they fail, the bath route fails and one of the other repairs of Corollary 2.1 becomes mandatory. Two boundaries apply. First, passing the test proves that three neutral tangent directions are reached from history perturbations; it does not by itself show that the full 24-dimensional neutral physical Hessian is correctly reproduced - that remains a same-action return. Second, the test repairs the tangent-capacity deficit of the recurrent law; it does not supply NSTC-1's missing physical matrices, as Part XIV states. The test is precisely defined, requires only the arrays already listed in the execution protocol, and has not been run. Its output is one of the decisive returns of the next execution.

Part XIV - Readout, independent descent and neutral implications

33. Yukawa selector

HCMR-MR1 returns $P_Y = I_9$ to floating precision. The pullback theorem does not improve that result. The interaction score space has dimension 504, but no primitive rule inside the retained population law distinguishes the 6 gauge, 18 completion and 36 charged directions, much less a physical three-by-three Yukawa readout.

An exact pullback can be post-composed with many field-space choices while preserving H. Therefore P_Y must be returned by one of two source-clean routes:

1. a nontrivial, gapped, regulator-stable readout projector fixed before mass comparison; or
2. a direct mixed derivative of the common master action that makes a separate selector unnecessary.

34. Gauge dependence in the forward architecture

The broad placement of the gauge, completion and charged blocks in the 504-dimensional interaction sector remains valid for the original pullback theorem: the retained population law genuinely does not resolve their representation content, and the capacity accounting of Part IV is unaffected. What the later corpus supplies is the microscopic route by which that content must enter the forward construction.

The full matrix-valued carrier of Part VIII fixes where the gauge dependence lives in the source: it enters through the MASD defects D_C, D_J and D_B, whose evaluation on the faithful configuration carries the $su(3) \oplus su(2) \oplus u(1)$ tangent edge by edge. After source descent, the gauge dependence propagates through C_n into the generator, the transfer and the Doob kernel, so the Doob score can acquire a genuine gauge derivative:

$$\partial \log K_n / \partial A_e^a \neq 0.$$

The physical gauge response must come from this full transfer, not from gauge phases inserted by hand into a scalar Poisson intensity - that would repeat the reduced-source error of Part VIII at the stochastic layer, decorating the process with gauge labels while carrying none of the non-Abelian tangent.

35. Independent CY3-prime

RRHF-1 proves that two functorial lifts of one representation-resolved parent agree to approximately 10^{-14}. FRDT-1 and HCMR-MR1 show that the genuinely independent completion derivative and closure-Hessian routes disagree by approximately 0.37-0.49 and do not converge together.

The pullback cannot repair that disagreement by choosing J: changing the mark embedding leaves $J^T I J = H$ untouched, so independent descent must be repaired upstream in the action itself. The forward architecture of Parts X-XII is the candidate repair route: a clean same-action descent would give the completion-derivative and closure-Hessian routes a common microscopic origin, turning their disagreement into a computable residual.

36. Neutral sector

The source-local neutral-destination main-effect rank 147 comfortably exceeds the 24-dimensional neutral Nambu parent. That result removes a broad capacity obstruction, but the strict shared-neutral tangent has rank only 21 and therefore fails by three dimensions. It does not supply direct Y_nu, M_R or M_L. NSTC-1's 30-parameter non-closure theorem therefore remains intact.

In particular, the neutral support maps used to construct the renewal probabilities specify allowed coherent routes. Their squared moduli do not determine the complex mixed and Majorana Hessians. A physical neutral certificate still requires the direct same-action arrays and absolute scale. The bath-mediated repair candidate for the rank-three deficit and its release test are stated in Part XIII; NSTC-1's non-closure is unaffected either way, since the test concerns the recurrent tangent deficit, not the thirty free neutral parameters.

Part XV - Clock, action and completion-scale audit

37. Benchmark event probability

The recurrent execution uses

$$p_D = p_R = p_B = 1 - e^{-1}.$$

For a continuous hazard ν observed over a step τ,

$$p = 1 - e^{(-\nu\tau)}.$$

Thus the benchmark fixes only

$$\nu\tau = 1.$$

The family $(\nu, \tau) = (a, 1/a)$, $a > 0$, gives the same retained one-step law. D36-R1 separately proves that the row-normalised Fold and continuous-rate clocks are inequivalent conventions. The present paper therefore inherits, rather than closes, the physical-clock debt.

The commitment-event process sharpens what the debt is. The physically observable object may be the completed record-event rate Γ alone: the internal D, R and B stages may be alternative Markovian dilations of one resolved fact channel, and separate hazards are physical only if the intermediate stages generate independently accessible records. This narrows the clock non-identifiability from three hazards to one rate but does not eliminate it, because Γ still carries the undetermined $\nu\tau$ scaling until the Fold-to-traversal map and TPB compensator are returned.

38. Traversal law

ASAC-1 derives the conditional geometric relation

$$\Delta t_0 = (8\pi/21)\ \xi / c$$

on the Route-M branch. The present paper does not derive that one numerical Fold is one oriented traversal, a fixed number of traversals or a state-dependent stopping time. The map from recurrent marks to seconds remains open.

39. Action normalisation

The common rescaling

$$A\star \to a\ A\star,\ S \to a\ S,\ \text{threshold energies} \to a \text{ times threshold energies}$$

leaves $S/A\star$, Born weights and dimensionless Fisher ratios invariant. An exact edge-score pullback is also invariant under this orbit. It therefore cannot supply the numerical action unit.

40. Completion response

The physical completion scale requires

$$Z\_comp = [\partial K\_comp(p^2)/\partial p^2] \text{ at } p^2 = 0,\ \Lambda\star^2 = K\_comp(0) / Z\_comp,$$

subject to the declared sign and normalisation conventions of the action; the kernel is evaluated at zero momentum, since K_comp is a momentum-dependent function and the scale is set by its static value against its curvature.

H_full in the retained execution is a static finite-regulator Hessian. The equality $H = J^T I J$ at one retained point supplies neither the p^2-dependent completion kernel nor its derivative. Setting Z_comp = 1 would remain identity completion.

Theorem 7 - Static-pullback scale no-go

No factorisation of the static HCMR parent through the retained recurrent Fisher form can determine $\Lambda\star$ without an independently calculated momentum-dependent completion response and action normalisation.

The theorem remains correct and is retained unchanged in content. The source-projected bath descent now supplies the structural formula the static pullback could not. Applying the Schur complement of Part XI to the completion sector, with c_H the completion column of the coupling,

$$K_{comp}(p^2) = K_{core,H}(p^2) - c_H^\dagger (p^2 I + \Omega_Q^2)^{-1} c_H,$$

and differentiating at $p^2 = 0$,

$$Z_{comp} = Z_{core,H} + c_H^\dagger \Omega_Q^{-4} c_H.$$

The revised conclusion is therefore sharper on both sides. The static PFHBA-1 pullback cannot determine Z_{comp}; that no-go stands. The source-projected bath descent supplies its formal same-action expression; that form is derived. Numerical evaluation still requires c_H, Ω_Q, the physical momentum normalisation and the absolute action unit, and by the firewall a formal expression for Z_{comp} is not a numerical scale prediction.

Part XVI - Physical-admission decision

41. What is closed

The machine-certified execution closes the following questions:

- the exact recurrent product edge-coordinate count;
- the exact product Fisher rank and nullity;
- the source-local history-destination, neutral-destination and destination-interaction rank decomposition;
- the strict shared-factor tangent audit and its three-dimensional neutral deficit;
- the sector-level capacity of the recurrent process;
- the existence of exact sector-typed 102-dimensional HCMR pullbacks at four regulators;
- the robust upper-spectrum capacity for a rank-96 target;
- the exact parameterisation and dimension of the lift family;
- explicit mark-level inequivalence of two exact typed lifts;
- preservation of the HCMR conjugate branch relation through recurrent pullback.

The analytical integration delivers, at the analytically-reduced or conditionally-constructed grade: the compressed candidate bath operator with its invariance-defect and principal-angle certificates; the transported covariance and its compatibility conditions; the defect-source factorisation theorem, HM4 residual criterion and minimum-norm representative (exact theorems relative to the declared conventions); the Euclidean Schur-complement history response under (P1)-(P2); the fixed-step unmarked Doob kernel under (P1)-(P2) with the Markov-rate limit additionally under (P3), the mark decomposition and Kraus amplitudes source-open; the neutral release criterion (T1)-(T2), a test rather than a repair; and the formal completion-residue expression. The first list is machine closed; this second group is analytically reduced or conditionally constructed, and numerically unexecuted throughout - each item's precise grade is the corresponding row of the audit matrix and Section 43 ledger.

42. What is not closed

The paper does not establish: the numerical J_D, J_s, R_{HM4} or C_n; the executed $\varepsilon_{inv}^{(n)}$, carrier principal-angle certificate or covariance-compatibility spectrum; the source-derived mark decomposition ($A = \Sigma_\alpha A_\alpha$ or record projectors R_α); verification of (P1)-(P3); the amplitude-level return M_α, V_α with its population consistency; the transfer, Perron pair, or regulator convergence of any descent object; the actual marked jump matrices, interaction residual Ξ_n, neutral rank repair or Z_{comp}; the CY3-prime repair; branch orientation; the physical clock and action unit; or HFB-1 admission.

43. HFB admission ledger

The ledger is graded in three classes: machine closed; analytically reduced to an executable test; still structurally open.

HFB gate	Revised status	Release condition
Recurrent capacity	MACHINE CLOSED	Retain renewal support and finite-mean physical law.
Broad sector-resolved capacity	MACHINE CLOSED	Exact 216/147/504 source-local decomposition.
HCMR embeddability	MACHINE CLOSED AS EXISTENCE	Do not promote to provenance.
Lift identifiability	MACHINE FAIL	Typed family dimension 34,152; two inequivalent explicit lifts.
Strict shared-neutral tangent	MACHINE FAIL BY 3	Rank 21 against the 24-dimensional neutral demand.
Full faithful gauge carrier	THEORETICALLY SUPPLIED / NOT EXECUTED	Replace the reduced scalar implementation with the matrix-valued corpus carrier.
Bath-carrier equivalence	COMPRESSION / INVARIANCE OPEN	Calculate $\varepsilon_{inv}^{(n)}$ and the principal-angle certificate between the closure and commitment carriers; require both to close.
Projected-covariance compatibility	TEST DEFINED / SPECTRUM OPEN	$\Sigma_Q \succeq 0$, correct transformation, optional $[\Sigma_Q, \Omega_Q^2] = 0$ where symmetry requires; report the spectrum.
Source-defect factorisation	ANALYTICAL TEST DERIVED	$\ker J_D \subseteq \ker J_s$ on the typed defect space, evaluated on the faithful source.

HFB gate	Revised status	Release condition
HM4 residual	NOT EXECUTED	Calculate $\varepsilon\_HM4 = \|J\_s(I - J\_D^{+}J\_D)\| / \max(\|J\_s\|, \varepsilon_0)$ at each regulator, reporting the pseudoinverse cutoff.
Bath coupling C_n	FORM DERIVED / VALUES OPEN	$C\_n\text{^}min = J\_s,n\ J\_D,n^{+}$, the minimum-norm representative, conditional on a vanishing residual.
Euclidean positivity	HYPOTHESES DECLARED / UNVERIFIED	Coercivity (P1), kernel positivity (P2), Doob-gauge stoquasticity (P3) on the faithful carrier.
Native marked law	UNMARKED POPULATIONS CONDITIONAL / VALUES OPEN	Fixed-step unmarked Doob kernel under (P1)-(P2); Markov-rate limit $q = -\mathscr{H}\_xy\ r(y)/r(x)$ additionally under (P3); marked instrument source-open.
Mark decomposition and Kraus isometries	TYPED / SOURCE-OPEN	Source return $A = \Sigma\_\alpha\ A\_\alpha$ or R_α; then $M\_\alpha,yx = P\_y\ A\_\alpha\ P\_x$, $V\_\alpha,yx = \mathrm{Pol}(M\_\alpha,yx)$, consistent with the populations.
Neutral deficit	RELEASE TEST DERIVED / RESULT OPEN	(T1) surjectivity onto ran P_miss and (T2) $\sigma_3 > \varepsilon\_reg$; leakage reported as implementation hygiene.
Completion response	FORM DERIVED / NUMBER OPEN	$Z\_comp = Z\_core,H + c\_H^{\dagger}\Omega\_Q^{-4}c\_H$, with scale and unit.
CY3-prime	FAIL	Repair common-action descent.
Branch orientation	OPEN EXACT PAIR	Odd generator term or accepted output-equivalence theorem; evaluate on both branches.
Physical clock and action unit	OPEN	Fold/traversal, TPB compensator and absolute coefficient.
HFB-1	NOT ADMITTED	All load-bearing rows must close together.

44. Final verdict

PFHBA-7 final certificate. The recurrent K=7 × neutral law has machine-certified capacity to carry the HCMR parent, but the exact lift remains non-identifying. Later corpus integration now supplies a faithful matrix-valued gauge carrier, a Poisson commitment-bath process, a compressed candidate bath operator with a normalised invariance-defect and principal-angle test, an exact source-defect factorisation theorem with its HM4 residual criterion, the canonical minimum-norm coupling representative, a conditionally constructed Euclidean transfer and unmarked Doob descent, and a typed - but source-open - mark decomposition and amplitude-level return. These advances reduce primitive provenance to an explicit finite-regulator calculation: J_D, J_s, ε_HM4, C_n, the invariance and covariance-compatibility tests, the positivity hypotheses, the unmarked Doob kernel with its Perron-ratio rates, the source-derived mark decomposition and amplitude-level return, the neutral release test and the completion residue. None of those numerical returns is claimed here. HFB-1 therefore remains unadmitted, but the provenance problem is no longer unspecified: ε_HM4 is the gateway source-clean test - a zero releases the coupling calculation but does not by itself discharge the subsequent invariance, compatibility, positivity, mark-decomposition, amplitude-return, neutral, scale and branch gates, each of which now has its own declared numerical form.

This is a stronger conclusion than either "the recurrent law cannot support the Standard Model carrier" or "a factorisation proves the carrier is derived." The first statement is false; the second is also false. The precise truth is that the recurrent law supplies a large enough metric space, the microscopic field-to-mark map remains the missing physical object, and the required forward architecture and its falsifiable numerical obligations - not the executable source map itself - are now fully specified.

Figure 5. Old inverse lift versus new forward descent. Left: the PFHBA-1 route chooses a typed Jacobian to factor the frozen parent - existence, non-identifying. Right: the source-clean provenance route generated forward from the primitive fields through the defect and source Jacobians, the minimum-norm coupling, the Euclidean transfer, the Doob kernel and the marked law.

Figure 6. Status map. Green: original machine-closed capacity results. Amber: analytical reductions and unexecuted tests. Red: CY3-prime, branch, clock and HFB gates still unresolved.

Part XVII - Revised programme consequence and next execution target

45. Programme consequence

The previous first-paper target bundled together capacity, provenance, clock and admission. The machine-certified execution separates them and closes the capacity branch completely: rank, sector placement and representability of the HCMR parent as a recurrent Fisher pullback are settled questions.

The analytical integration then converts the provenance branch from an open demand into a specified calculation. The next execution must not construct another Hessian and then factor it. It must begin from the faithful primitive source, differentiate forward, and let the HM4 residual decide.

46. Next execution target

Next execution - Multi-regulator Source-Projected Bath and Doob Descent. At levels n=3,4,5,6, automatically differentiate the full faithful MASD source to obtain J_D,n and J_s,n; calculate

$R\_HM4^{(n)} = J\_s,n \, (I - J\_D,n^{+} J\_D,n)$;

if the residual vanishes, return the minimum-norm representative

$C\_n^{min} = J\_s,n \, J\_D,n^{+}$;

then execute the remaining stages in the order fixed by Appendix I: the compressed candidate bath operator with its normalised invariance defect and principal-angle certificate; the transported covariance and its compatibility spectrum; the (P1)-(P3) verifications or the fixed-step declaration; the quadratic generator, transfer, Perron pair and unmarked Doob kernel; the source-derived mark decomposition and amplitude-level arrays; the interaction residual and neutral release test; the completion response; and the branch gap on both conjugate branches. Only after these arrays are frozen may they be compared with HCMR. The pass condition is not that a coupling exists: it is that ε_HM4 vanishes at every regulator, that the declared hypotheses verify, that the descent arrays converge, that the neutral release test passes in full or a certified alternative repair is supplied, and that the frozen forward arrays land in or decisively outside the PFHBA-1 equivalence class without target access.

If the forward descent lands in the pullback class and closes the remaining tests, the HCMR candidate is promoted. If it lands elsewhere, the primitive return supersedes the candidate. If ε_HM4 does not vanish and no repair of the source functional restores the kernel inclusion, the defect-mediated bath route is refuted at the source and the architecture of Parts IX-XII must be replaced.

Appendix A - Proof details

A.1 Product-score orthogonality

Let $a(h')$ have p-weighted mean zero, $b(n')$ have q-weighted mean zero, and $c(h',n') = a\_1(h') \, b\_1(n')$ with both factors zero mean. Then

$$E\_(p \otimes q)[a\, b] = E\_p[a] \, E\_q[b] = 0,$$

$$E\_(p \otimes q)[a\, c] = E\_p[a\, a\_1] \, E\_q[b\_1] = 0,$$

and similarly for b against c. This proves mutual orthogonality. Completeness follows from the dimension identity.

A.2 Fisher-orthonormal contrasts

Let z be an orthonormal Euclidean basis for the complement of $\sqrt{p}$. Set $A = \mathrm{diag}(p)^{(-1/2)} \, z$. Then

$$p^{T} A = \sqrt{p}^{T} z = 0,$$

$A^T \operatorname{diag}(p) A = z^T z = I$.

After multiplication by the inverse square root of the source stationary weight, the embedded source contrasts are orthonormal under the full stationary Fisher block.

A.3 Pullback construction

For a Fisher-orthonormal frame E and positive symmetric H_b,

$(E H_b^{1/2})^T I (E H_b^{1/2}) = H_b^{1/2} E^T I E H_b^{1/2} = H_b$.

Orthogonality of the selected frames makes the cross terms vanish.

A.4 Lift-family parameterisation

Let Q span the positive Fisher range with $Q^T I Q = I$. If $K^T K = I$, then $J = Q K H^{1/2}$ is an exact lift. Conversely, for an exact full-rank lift, $K = Q^T I J H^{-1/2}$ obeys $K^T K = I$. A term N with $I N = 0$ may be added freely. This gives the Stiefel and score-gauge dimensions quoted in Part VI.

A.5 Terminal columns

For a positive Fisher form, a zero diagonal pullback for a terminal column z implies

$z^T I z = 0$,

hence $I^{1/2} z = 0$ and z belongs to ker I. The zero terminal representative used in the execution is therefore the minimum-null choice, not the only exact coordinate array.

Appendix B - Numerical certificate

Quantity	Return
Product working states	63
Positive edge coordinates	930
Fisher rank / nullity	867 / 63
Source-local history-destination rank	216
Source-local neutral-destination rank	147
Shared history-factor tangent rank	24
Shared neutral-factor tangent rank	21
Strict neutral-factor deficit	3
Destination-interaction rank	504

Quantity	Return
HCMR rank / nullity	96 / 6
Maximum orthogonality residual	1.78×10^{-15}
Maximum relative pullback residual	8.85×10^{-16}
Maximum lifted branch residual	3.41×10^{-13}
Minimum positive Fisher eigenvalue, level 3	5.51048×10^{-12}
Minimum positive Fisher eigenvalue, level 6	5.74157×10^{-12}
Maximum Fisher eigenvalue	4.03696×10^{-3}
Modes above 1% maximum	495
Untyped minimum-null family dimension	78,576
Broad typed family dimension	34,152
Full affine family including score gauge	85,002
Independent verifier	PASS

No numerical certificate is issued for any object of Parts IX-XIII. J_D, J_s, R_HM4, C_n, Ω_Q,n², the Perron pair, the Doob kernel, the marked jump operators, Ξ_n and Z_comp are defined tests and derived forms, not returns.

Appendix C - Reproduction algorithm

1. Verify every file in results/frozen_input_manifest.json.
2. Load the frozen 7 × 7 K=7 history kernel.
3. For levels 3-6, load the FCHI Dirac and Majorana polar support maps and construct the 9 × 9 renewal population law.
4. Form the 63-state product support.
5. Enumerate only positive outgoing edges; create one coordinate for each source-destination edge.
6. Build each source Fisher covariance with stationary weighting.
7. Construct probability-orthonormal history and neutral contrast bases.
8. Tensor the contrasts to produce Q_H, Q_N and Q_X; verify all self and cross Gram matrices.
9. Load the branch-specific HCMR H_full only after the score bases have been fixed.
10. Construct the deterministic typed reference lift and fixed-seed alternative lift.
11. Verify $J^T I J = H$ for levels 3-6 and branches 1,6.
12. Construct C and verify $H\_6 = C^T H\_1 C$ and the lifted relation.
13. Publish arrays, CSV tables, figures, source hashes, the calculation certificate and independent verification.

Reproduction commands:

```
python scripts/run_pfhba1.py
```

python scripts/verify_pfhba1.py

Appendix D - Source ledger

1. Keith Taylor, *The VERSF Master Substrate Action and Constitutive-Descent Theorem* (MASD-1).
2. Keith Taylor, *The VERSF History-Measure, Generation-Geometry and Branch-Closure Theorem* (HMGBC-1).
3. Keith Taylor, *MASD-HMGBC Reduced Physical Evaluation Attempt* (MHPE-1).
4. Keith Taylor, *Full-Carrier Finite-Regulator Hamiltonian and Primitive Terminal Instrument* (FCHI-1).
5. Keith Taylor, *Representation-Resolved Master Kernel and Conditional Boundary Synthesis* (RRMK-1).
6. Keith Taylor, *Finite-Regulator Differentiation Test of RRMK-1* (FRDT-1).
7. Keith Taylor, *Representation-Resolved History Functional Extension and Frozen Test* (RRHF-1).
8. Keith Taylor, *Primitive-Transfer Representation-Ledger Uniqueness Attempt* (PRLU-1).
9. Keith Taylor, *The D36 Clock-Convention, Reproducibility and Manifest-Currency Theorem in VERSF* (D36-R1).
10. Keith Taylor, *Full-Carrier Instrument Provenance Audit and Minimal-Extension Theorem* (FCIP-A1 / STEP2).
11. Keith Taylor, *Complete Marked HMGBC Return Attempt* (CMHRE-1).
12. Keith Taylor, *The K=7 Closure-Born-TPB Renewal Construction in VERSF* (K7-CBR-1).
13. Keith Taylor, *The Absolute-Scale, Action-Normalisation and Closure-Clock Conditional Theorem in VERSF* (ASAC-1).
14. Keith Taylor, *The Four-Regulator Commitment-Maintenance Execution and Physical-Admission No-Go Theorem in VERSF* (HCMR-MR1).
15. Keith Taylor, *NSTC-1 - The Same-Branch Neutral-Sector Return and Exact Current-Corpus Physical-Certificate Boundary*.
16. Keith Taylor, *The Faithful Gauge-Product Quotient, Bundle Census and Topological-Charge-Lattice Theorem in VERSF* (FGQT-1).
17. Keith Taylor, *The Faithful-Quotient Retained Global Measure and Strong-Phase Confinement Theorem in VERSF* (GQSC-1R).
18. Keith Taylor, *Exact Projection-Mapped Derivative Execution* (EPMD-1).
19. Keith Taylor, *Exact Projection Coordinates*.
20. Keith Taylor, *Full Projected Closure Operator*.
21. Keith Taylor, *Origin of the Closure Scale*.
22. Keith Taylor, *Spectral Density of the Commitment-Event Bath*.
23. Keith Taylor, *Three Admissibility Classes and Maxwell-Form Persistent Transport*.
24. Keith Taylor, *Bath or Ledger*.
25. Keith Taylor, *Bath Criterion*.
26. Keith Taylor, *Non-Abelian Bath-Transport Theorem*.
27. Keith Taylor, *Gauge-Closure Selection Theorem*.
28. Keith Taylor, *Carrier Theorem*.

29. Keith Taylor, *VERSF Standard Model Derivation Master Ledger*, July 2026.
30. PFHBA-1 calculation and reproducibility package, frozen arrays, scripts, manifests and independent verification, 24 July 2026.

Appendix E - Defect-source factorisation proof

E.1 Kernel-inclusion theorem

Claim. There exists C with $J_s = C J_D$ if and only if $\ker J_D \subseteq \ker J_s$.

Proof. Necessity: if $J_s = C J_D$ and $J_D v = 0$, then $J_s v = C J_D v = 0$, so $\ker J_D \subseteq \ker J_s$.

Sufficiency: suppose $\ker J_D \subseteq \ker J_s$. Set

$$C = J_s J_D^+.$$

Then

$$C J_D = J_s J_D^+ J_D.$$

The operator $J_D^+ J_D$ is the orthogonal projector onto $(\ker J_D)^\perp$, so $I - J_D^+ J_D$ is the orthogonal projector onto $\ker J_D$. Decompose any v as $v = v_1 + v_0$ with v_0 in $\ker J_D$ and v_1 in its orthogonal complement. Then

$$J_s J_D^+ J_D v = J_s v_1 = J_s v_1 + J_s v_0 = J_s v,$$

where $J_s v_0 = 0$ by the kernel inclusion. Hence $C J_D = J_s$. QED.

E.2 Residual equivalence

The kernel inclusion holds if and only if

$$R_{HM4} = J_s (I - J_D^+ J_D) = 0,$$

because $I - J_D^+ J_D$ projects exactly onto $\ker J_D$: the residual is the action of J_s on the defect-invisible subspace, and it vanishes precisely when that subspace is source-invisible as well.

E.3 Minimum-norm construction and uniqueness on the reachable image

Suppose $R_{HM4} = 0$ and let C_1, C_2 both satisfy $J_s = C J_D$. Then $(C_1 - C_2) J_D = 0$, so C_1 and C_2 agree on ran J_D, the physically reachable defect image. Every solution therefore coincides with $C^{min} = J_s J_D^+$ on ran J_D, and the residual freedom acts only on the orthogonal complement of ran J_D, which no defect motion ever reaches. Among all solutions, C^{min} is the unique one vanishing on that complement, and hence the unique minimal-Frobenius-norm solution: for any solution C, writing $C = C^{min} + N$ with $N J_D = 0$ and N supported on $(\mathrm{ran}\, J_D)^\perp$ gives $\|C\|_F^2 = \|C^{min}\|_F^2 + \|N\|_F^2$. QED.

Uniqueness is relative to the declared inner products on $E_{D,n}$ and Q_n and the declared coordinates; a change of admissible convention conjugates C^{min} covariantly. Physical canonicity - invariance of the represented coupling under such reparameterisations and under regulator refinement - is a separate condition, stated in Section 28, to be established by the execution.

Appendix F - Euclidean bath and Stieltjes response

F.1 Conditional positivity of the transfer

The generator

$$\mathscr{H} = -\Delta + V_{loc} + (1/2)\, q^T \Omega_Q^2\, q + q^T C D$$

is analysed under the two declared hypotheses of Part XI.

(P1) Coercivity. Completing the square in q shifts the bath minimum by $-(1/2)\, D^\dagger C^\dagger \Omega_Q^{-2} C D$. This is a bounded modification of V_{loc} only when the defect sectors are bounded on the retained finite-regulator carrier or the core action dominates the shift. The declared sufficient condition is

$$W_{core} - C^\dagger \Omega_Q^{-2} C \geq 0,$$

or more generally semiboundedness $\mathscr{H} \geq -c\, I$ on the declared regulator domain. Under (P1) the transfer $T = e^{(-a_t \mathscr{H})}$ is a well-defined bounded positive operator.

(P2) Kernel positivity. Operator positivity $T \geq 0$ is basis-independent, but the Doob construction consumes basis-dependent properties: a positivity-preserving semigroup, a strictly positive integral kernel, and the positivity-improving property. For the scalar quadratic-plus-local reduction, strict kernel positivity follows from the Feynman–Kac representation: the kernel is an integral of strictly positive Gaussian path weights against the bounded local factor, and strict positivity supplies the irreducibility and Perron structure consumed in Appendix G. For the matrix-valued faithful carrier, Feynman–Kac does not apply directly and a strictly positive scalar kernel in the physical mark basis is an explicit hypothesis to be verified - for example by proving the semigroup is positivity improving with respect to a declared self-dual cone. The four notions

- operator positivity, positivity preservation, strict kernel positivity, positivity improvement - are distinct and are never conflated in this paper.

The real-time memory kernel of the commitment-event process plays no role in this argument; positivity is a property of the Euclidean generator, and the oscillatory real-time kernel must not be substituted for the Euclidean covariance.

F.2 Schur complement

Integrating the Gaussian bath variables out of the quadratic form at Euclidean momentum p gives, exactly,

$$W_{hist}(p^2) = W_{core}(p^2) - C^\dagger (p^2 I + \Omega_Q^2)^{-1} C.$$

The subtracted term is a matrix Stieltjes function of p^2: writing the spectral decomposition $\Omega_Q^2 = \Sigma_m \omega_m^2 P_m$,

$$C^\dagger (p^2 I + \Omega_Q^2)^{-1} C = \Sigma_m (C^\dagger P_m C) / (p^2 + \omega_m^2),$$

a sum of positive semidefinite residues over simple poles at $p^2 = -\omega_m^2$, all off the physical axis. The function is therefore positive, monotonically decreasing and completely monotone in $p^2 > 0$, and the bath subtraction cannot introduce spurious physical-axis poles.

F.3 Completion residue

Applying F.2 to the completion sector with coupling column c_H and differentiating at $p^2 = 0$:

$$K_{comp}(p^2) = K_{core,H}(p^2) - c_H^\dagger (p^2 I + \Omega_Q^2)^{-1} c_H,$$

$$d/dp^2 [-c_H^\dagger (p^2 I + \Omega_Q^2)^{-1} c_H] \text{ at } p^2 = 0 = c_H^\dagger \Omega_Q^{-4} c_H,$$

hence

$$Z_{comp} = Z_{core,H} + c_H^\dagger \Omega_Q^{-4} c_H.$$

The correction is manifestly nonnegative, so the bath can only increase the completion residue. This is a formal Schur-complement contribution, not yet a full same-action return; the numerical value awaits c_H, Ω_Q and the absolute normalisations.

Appendix G - Doob and Kraus descent

G.1 Stochasticity of the Doob kernel

Let T be the transfer, strictly positive under (P1)-(P2), with Perron pair $T r = \lambda_0 r$, $l^\dagger T = \lambda_0 l^\dagger$, r and l strictly positive by Perron–Frobenius on the strictly positive kernel of Appendix F.1. Then

$$\Sigma_y K(y|x) = \Sigma_y T(x,y) r(y) / (\lambda_0 r(x)) = (T r)(x) / (\lambda_0 r(x)) = 1,$$

so K is exactly stochastic, and its stationary law is proportional to l(x) r(x). This step consumes strict kernel positivity - hypothesis (P2) - and not merely $T \geq 0$.

G.2 Marked rates: the Perron-ratio expansion and the stoquasticity hypothesis

Expand the transfer inside the Doob kernel. With $T(a_t) = e^{-a_t\mathscr{H}} = I - a_t\mathscr{H} + O(a_t^2)$ and $K(y|x) = T(x,y) r(y) / (\lambda_0 r(x))$, the off-diagonal entries obey, for $y \neq x$,

$$K(y|x) = -a_t \mathscr{H}_{xy} r(y)/r(x) + O(a_t^2),$$

since $\lambda_0 = 1 - a_t E_0 + O(a_t^2)$ contributes only at diagonal order but the Perron ratio r(y)/r(x) is generally order one and does not tend to 1 as $a_t \to 0$: the Perron vector is a fixed positive eigenvector of $\mathscr{H}$, not an a_t-dependent perturbation of the constant vector. The generator limit is therefore

$$q_\alpha(y|x) = \lim_{a_t\to0} K_\alpha(y|x) / a_t = (-\mathscr{H}_{xy})_\alpha r(y)/r(x), \; y \neq x,$$

which is exactly the off-diagonal part of the ground-state (Doob-transformed) generator

$$G = -r^{-1} (\mathscr{H} - E_0) r, \; q(y|x) = G_{xy},$$

with r the diagonal multiplication operator. G is conservative - its rows sum to zero - because $\mathscr{H} r = E_0 r$, so the diagonal compensator is fixed by the off-diagonal rates.

This expansion alone does not deliver nonnegative rates. The declared hypothesis, stated on the Doob-transformed generator, is:

(P3) Doob-gauge stoquasticity. In the physical mark basis, $-\mathscr{H}_{xy} r(y)/r(x) \geq 0$ for all $x \neq y$ - equivalently, since $r > 0$, the sign condition $-\mathscr{H}_{xy} \geq 0$ off the diagonal, with the rate magnitudes carrying the ratio.

Under (P3) the unmarked rates exist and are nonnegative; the marked components q_α inherit this structure once the source mark decomposition of G.4 is returned. If (P3) fails or is not yet verified, no continuous-time rate law is claimed; the descent object is the fixed-step instrument of G.3, which contains the Perron ratio exactly at every finite a_t and needs no expansion.

G.3 Fixed-step instrument and completeness

At fixed transfer step a_t, define the discrete-time Kraus instrument

$L_{\alpha,yx} = \sqrt{K_\alpha(y|x)}\, V_{\alpha,yx}$,

with $V_{\alpha,yx}$ partial isometries. The completeness condition in its general form is

$\Sigma_{\{\alpha,y\}} K_\alpha(y|x)\, V_{\alpha,yx}^\dagger V_{\alpha,yx} = P_x$,

with P_x the support projector at x. If every initial projector coincides with P_x - $V_{\alpha,yx}^\dagger V_{\alpha,yx} = P_x$ for all outgoing transitions - then, since $\Sigma_{\{\alpha,y\}} K_\alpha(y|x) = 1$ for each x, this reduces to $\Sigma_{\{\alpha,y\}} L_{\alpha,yx}^\dagger L_{\alpha,yx} = P_x$. Different transitions may however be supported on different initial subspaces, in which case only the general form holds; which case obtains is decided by the amplitude return of G.4, not assumed. Under (P1)-(P2) the unmarked Doob kernel and an abstract classical dilation exist; the physical representation-bearing marked instrument remains open until the source-derived mark decomposition and the amplitude-level arrays are returned (G.4). In the continuous-time regime under (P3), the corresponding statement is $\Sigma_{\{\alpha,y\}} L_{\alpha,yx}^\dagger L_{\alpha,yx} = \Gamma(x)\,\Pi(x)$ with $L_{\alpha,yx} = \sqrt{q_\alpha(y|x)}\, V_{\alpha,yx}$, where $\Gamma(x) = \Sigma_{\{\alpha,y\}} q_\alpha(y|x)$ is the total completed-event rate at x and $\Pi(x)$ the support projector: the diagonal compensator is fixed by the off-diagonal rates.

G.4 The mark decomposition and the amplitude-level return

Nothing in G.1-G.3 determines the marked structure. Two distinct objects are open. First, the mark decomposition itself: the unmarked kernel K admits many decompositions $K = \Sigma_\alpha K_\alpha$, and the physical one must come from a source-derived amplitude decomposition $A = \Sigma_\alpha A_\alpha$, orthogonal record projectors R_α, or an explicitly imported marked-support law. Second, the partial isometries: the scalar Doob kernel is a population law; the $V_{\alpha,yx}$ carry the internal representation content and must be returned separately from the faithful source. With A_α the amplitude operator of mark α supplied by the source dilation and P_x, P_y the local support projectors, the amplitude return is

$M_{\alpha,yx} = P_y A_\alpha P_x$, $V_{\alpha,yx} = \mathrm{Pol}(M_{\alpha,yx})$,

with Pol the polar-decomposition partial isometry. The consistency test - that the polar moduli of $M_{\alpha,yx}$ reproduce the Doob populations, so that amplitudes and populations descend from one source - is a separate frozen output of the execution protocol. The grade of this appendix is therefore: fixed-step unmarked Doob kernel conditional on (P1)-(P2); continuous-time Markov-rate limit conditional additionally on (P3); mark decomposition and partial isometries typed but source-open.

Appendix H - Neutral rank-three release test

H.1 The missing projector

Let N_{24} be the 24-dimensional neutral demand space of the HCMR carrier and let S_{21} be its intersection with the strict shared-neutral tangent, of dimension 21 by Corollary 2.1. Define

P_miss = orthogonal projector onto the 3-dimensional orthogonal complement of S_21 inside N_24,

constructed explicitly from the frozen shared-neutral tangent frame by Gram–Schmidt within N_24. P_miss depends only on machine-closed objects and is regulator stable because the positive-support pattern is.

H.2 The cross-response conditions

The common-bath cross block of Part XIII, evaluated at $p^2 = 0$, is

$B = C\_neutral^\dagger\ \Omega\_Q^{-2}\ C\_hist : H_{12} \to N_{24}$,

with the typing $C\_hist : H_{12} \to Q_6$ and $C\_neutral : N_{24} \to Q_6$ fixed in Section 32. Because dim ran $P\_miss = 3$, the conditions rank ≥ 3, rank = 3 and surjectivity onto ran P_miss are algebraically equivalent, and $P\_miss\ B$ has at most three nonzero singular values, so no independent σ_4 test exists. The release criterion is stated without redundancy:

(T1) Surjectivity: ran($P\_miss\ B$) = ran P_miss, equivalently rank[$P\_miss\ B$] = 3;

(T2) Stable surjectivity: σ_3($P\_miss\ B$) > ε_reg, i.e. σ_min of $P\_miss\ B$ restricted to (ker P_miss B)^⊥ clears the regulator-stable tolerance;

The release conditions are (T1)-(T2). The projection leakage $\|(I - P\_miss)P\_miss\ B\| \leq \varepsilon\_proj$ vanishes algebraically for an exact P_miss; it is recorded as a numerical-implementation check alongside the frozen singular spectrum, not as a release condition.

The physical interpretation and boundaries of the test - what passing does and does not repair - are stated once in Section 32 and are not repeated here. The protocol of Appendix I requires the full singular spectrum of $P\_miss\ B$ to be frozen and reported.

Appendix I - Updated execution protocol

At each regulator level n = 3, 4, 5, 6 and on both conjugate branches, calculate and freeze, in order:

J_D, J_s, ε_HM4, C_n, Ω_Q^2, ε_inv, principal angles, Σ_Q spectrum, (P1)-(P3) verifications, $\mathscr{H}\_n$, T_n, λ_0, r, l, K_n, the source-derived mark decomposition, K_α or q_α, M_α, V_α, Ξ_n^(K) or Ξ_n^(q), Z_comp.

Protocol:

1. Replace the reduced scalar gauge implementation with the full matrix-valued corpus carrier of Part VIII; verify the 144-dimensional gauge tangent count edge by edge.

2. Construct the whitened uniform vector $\tilde{u}_0$ and transverse projector $P_{Q,n}$; construct the compressed candidate $\Omega_{Q,n}^2$ on ran $P_{Q,n}$; calculate the normalised invariance defect

$$\varepsilon_{inv}^{(n)} = \| (I - P_{Q,n}) A_n P_{Q,n} \| / \max(\|A_n P_{Q,n}\|, \varepsilon_0),\ A_n = M_n^{(-1/2)} L_n M_n^{(-1/2)},$$

and require $\varepsilon_{inv}^{(n)} \to 0$ across regulators; and certify the bath-carrier equivalence by principal angles - report $\|P_{closure} - P_{commitment}\|$, or the six singular values of the overlap between orthonormal frames of the two carriers, and require the singular values to approach one. If either fails, stop and report; the candidate identification of Part IX is then refuted.

2a. Transport the event covariance to the whitened coordinates, $\tilde{\Sigma}_{\eta,n} = M_n^{(1/2)} \Sigma_\eta M_n^{(1/2)}$, form $\Sigma_{Q,n} = P_{Q,n} \tilde{\Sigma}_{\eta,n} P_{Q,n}$; verify $\Sigma_{Q,n} \geq 0$ and the declared transformation law; report the full spectrum of $\Sigma_{Q,n}$, and any required operator compatibility such as $[\Sigma_{Q,n}, \Omega_{Q,n}^2] = 0$ where the symmetry of the construction demands it. Scalar isotropy is not required.

2b. Declare the source transport $W_n = M_n^{(1/2)}$ (or the declared dual-basis map) and freeze W_n and $P_{Q,n}$ at $X\star$; record that the fixed-carrier convention is in force and that any field dependence of W_n or $P_{Q,n}$ would add $(DP_Q)WS + P_Q(DW)S$ terms to J_s.

3. Automatically differentiate the faithful MASD defect map to obtain $J_{D,n} : T_{X\star}X_n \to E_{D,n}$ and the transported projected commitment source $s_n = P_{Q,n} W_n S_n[\Phi]$ to obtain $J_{s,n} : T_{X\star}X_n \to Q_n$, with frozen source hashes and no HCMR target visibility.
4. Calculate the normalised residual

$$\varepsilon_{HM4}^{(n)} = \| J_{s,n} (I - J_{D,n}^{+} J_{D,n}) \| / \max(\|J_{s,n}\|, \varepsilon_0),$$

reporting the singular-value cutoff used to construct $J_{D,n}^{+}$ and the regulator-stable singular gap of $J_{D,n}$. If $\varepsilon_{HM4}^{(n)}$ does not vanish to the declared tolerance, stop and report the source-clean failure; do not proceed to a fitted coupling.

5. If the residual vanishes, return $C_n^{min} = J_{s,n} J_{D,n}^{+}$ and freeze it.
6. Verify the coercivity condition (P1), $W_{core} - C_n^\dagger \Omega_{Q,n}^{-2} C_n \geq 0$ or $\mathcal{H}_n \geq -c_n I$, and the kernel-positivity condition (P2) in the physical mark basis. Test the stoquasticity condition (P3); if it fails, declare the fixed-step instrument at the frozen a_t and record that no continuous-time rate law is claimed.
7. Construct the quadratic local generator $\mathcal{H}_n$ from the linearised defects $\delta D_n = J_{D,n} \delta X$, then T_n, the Perron pair $(\lambda_{0,n}, r_n, l_n)$ and the unmarked Doob kernel K_n; extract the unmarked rates $q(y|x) = -\mathcal{H}_{xy} r_n(y)/r_n(x)$ - equivalently the off-diagonal part of $G_n = -r_n^{-1}(\mathcal{H}_n - E_{0,n})r_n$ - if (P3) holds; verify stochasticity at each a_t.

7a. Return the source-derived mark decomposition - the amplitude decomposition $A_n = \Sigma_\alpha A_{n,\alpha}$ or the record projectors R_α - and only then form the marked populations K_α and rates q_α; verify the general completeness condition $\Sigma_{\{\alpha,y\}} K_\alpha(y|x) V_{\alpha,yx}^\dagger V_{\alpha,yx} = P_x$ at each a_t.

8. Return the amplitude arrays $M_{\alpha,yx} = P_y A_\alpha P_x$ and $V_{\alpha,yx} = \mathrm{Pol}(M_{\alpha,yx})$ from the same frozen source, and test that the polar moduli reproduce the Doob populations.
9. Calculate the history-neutral interaction residual - $\Xi_n^{(q)}$ from the rates under (P3), the class-normalised $\Xi_n^{(K)}$ with weight p_X from the fixed-step kernel otherwise - and the full singular spectrum of $P_{miss} C_{neutral}^\dagger \Omega_Q^{-2} C_{hist}$; report the release test - surjectivity (T1) and stable surjectivity (T2) - with the projection leakage recorded as an implementation check.
10. Calculate the completion response $K_{comp}(p^2)$ and residue $Z_{comp}^{(n)}$.
11. Repeat every stage on branch $k = 6$; report the branch gap in each frozen array.
12. Verify regulator convergence across $n = 3$-6; hash and freeze all arrays.
13. Only after all arrays are frozen, open the HCMR parent and report whether the forward descent lands in or outside the PFHBA-1 equivalence class.