The Projected C_3 Hessian Return in VERSF

A minimal-cell derivation of the role-even quark curvature, the C_3 democratic branch, and the CKM triangle residue

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Status. The conditional C_3 extraction theorem reduced the remaining quark-side proof debt to five projected-Hessian returns. This manuscript discharges those returns within the minimal committed-quark C_3 branch cell, conditional on the named structural premises PH-CYC–PH-GAP. A full su(8) calculation can still overturn the result by violating one of the local premises (falsifiers H1–H13). The calculation is local to that cell: it computes the second variation on the branch coordinates, carries the result through the role-even projection and the mass/readout projection, and closes on the previously verified commutator to obtain the CKM residue. Every return is graded against a named premise; the structural premises are the negations of the microscopic falsifiers H1–H13, with empirical and provenance exposures recorded separately as E1–E3. A final section confronts the result with measured quark mixing, separating internal returns (graded Exact / Audit / Conditional) from the empirical comparison (graded Empirical, conditional on the inherited rational inputs).

General-reader summary

The Standard Model of particle physics contains a handful of numbers that describe how the three "generations" of quarks blend into one another. These mixing numbers are normally measured in experiments and entered into the theory by hand; nothing in the theory explains why they take the values they do. This paper takes one of the smallest of them — the tiny amount of blending between the first and third generations — and asks whether it can instead be *calculated* from the underlying structure of the theory rather than fitted to data.

The starting point is a symmetry. In the relevant setting the three generations cycle into one another on an equal footing, like three identical spokes of a wheel. A direct calculation shows that this equal footing forces the blending to be shared equally among the three pairings of generations. Two of those three pairings are already spoken for: one is the well-known mixing between the first two generations (the Cabibbo angle, which the theory inherits rather than predicts), and the other is the very quantity we are trying to explain. That leaves exactly one pairing free — and it is this leftover pairing that produces the answer, by interacting with the already-known one.

The theory must also account for a subtler effect: a kind of twist, or phase, that is ultimately responsible for the small imbalance between matter and antimatter. The paper fixes this twist to a specific value using a "shortest-path" argument, rather than choosing it to match the data.

When the calculated piece is combined with the inherited ingredients and assembled into the actual mixing pattern measured in experiments, the outcome is encouraging but not a clean victory. It reproduces the observed amount of matter–antimatter imbalance to within a few percent, and the derived twist is precisely what makes that work. But one of the angles describing the mixing comes out about eight degrees too large — a real, recorded disagreement — and the most impressive-looking agreement still rests on that twist, whose deeper origin has not yet been fully secured. Throughout, the paper separates what is genuinely derived, what is inherited from earlier work, and what remains owed, so that none of the agreement is claimed as more than it is.

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Abstract

The conditional C_3 extraction theorem reduced the remaining quark-side proof debt to five projected-Hessian returns: C_3 covariance, democratic branch weights, total common-mode norm b, Cabibbo-protected 2–3 readout survival, and positive phase-root sign. This manuscript computes those returns in the minimal committed-quark generation-completion cell.

The inherited closure Hamiltonian is $H_{cl} = G|_{su(8)}$, where G is the Hessian of the admissibility functional $\mathcal{F}$ at the admissible background. After the radial $u(1)$ contribution is removed by the $su(8)$ restriction, the branch-relevant Hessian is the sum $G_C + G_T + G_R$ of the closure, transport, and record-composition second variations. On the branch coordinate vector $x = (x_{12}, x_{23}, x_{31})$, with each x_{ij} in the two-real-dimensional Killing branch plane B_{ij}, the three quadratic returns are

$$G_C^B = \kappa_C\cdot I_6,\quad G_T^B = \kappa_T\cdot I_6,\quad G_R^B = \kappa_R\cdot I_6.$$

Hence

$$G_{C_3}^B = (\kappa_C + \kappa_T + \kappa_R)\cdot I_6,$$

and the pre-readout role-even quark block obeys

$$[K_{\text{even,pre}}^{q}, R_3] = 0.$$

The block is therefore circulant: $K_{\text{even,pre}}^{q} = d\cdot I_3 + h\cdot R_3 + h^*\cdot R_3^\dagger$. Its three branch weights are equal. The committed-quark role-isotype metric identifies the total role-even common-mode branch budget with the inherited b^2 norm of the role-odd 2–3 transport coefficient, so each branch carries weight $b^2/3$ and $|\zeta| = b/\sqrt{3}$.

The generation-depth readout removes the occupied Cabibbo 1–2 branch and the direct target 3–1 branch, leaving the 2–3 common-mode survivor. The Hermitian-side minimal half-transport root selects $h/|h| = e^{i\pi/3}$, hence $\arg(\zeta) = 5\pi/6$ after the frame i-rotation. Therefore

$$\zeta_{C_3} = (b/\sqrt{3})\cdot e^{5\pi i/6},$$

and the exact leading BCH commutator gives

$$\Delta_{13} = \tfrac{1}{2}\cdot a\cdot \zeta_{C_3}.$$

With $a = 9/40$ and $b = 81/2000$, this gives $\zeta_{C_3} = -0.0202500 + 0.0116913\cdot i$ and $\Delta_{13} = -0.0022781 + 0.0013153\cdot i$, with $|\Delta_{13}| = 0.0026306$.

Combined with the inherited direct 1–3 entry $c\cdot e^{i\varphi}$ and exponentiated to the unitary group, the returned curvature yields a full CKM triangle. It reproduces the CP observables — $\beta = 22.4°$ (measured ~22.2°) and Jarlskog $J = 3.0 \times 10^{-5}$ (measured ~3.1×10^{-5}) — with $\alpha = 84.6°$ at the lower edge of the measured band and $|V_{ub}| = 0.00356$ about 7% low, against a single residual

tension γ = 72.9° versus a measured ~63–66°. The derived residue carries the CP sector: removing it distorts every CP-sensitive entry. This CP agreement is contingent rather than secured: J and β are produced almost entirely by the phase arg ζ = 5π/6, whose provenance is audited in §15. It counts as independent evidence only if the Hessian selection fixing that phase is itself free of mixing-data input.

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0. Inherited inputs, named premises, and the derivation ledger

The purpose of this manuscript is not a further reduction. It is to write the projected-Hessian return as an actual second-variation calculation on the minimal C_3 branch coordinates, and to grade each step against an explicit premise.

0.1 Inherited inputs

Tag	Inherited object	Role here
IN-HCL	H_cl = G	_su(8), the substrate admissibility Hessian
IN-RAD	Radial u(1) removal under the su(8) restriction	Restricts the branch calculation to G_C + G_T + G_R
IN-OMQ	Role-odd quark generator Ω_q with a = 9/40, b = 81/2000, c = 243/100000, φ = 2π/3	Supplies the doorway and the inherited 2–3 norm b
IN-BCH	Symmetric split-BCH convention; $\Delta_{13} = \frac{1}{2}\cdot a\cdot\zeta$ exact	Used after the Hessian return

0.2 Named premises (this manuscript)

Each premise is the structural condition that makes one step of the minimal-cell calculation go through. Its negation is the corresponding microscopic falsifier (§17).

Tag	Premise	Negation
PH-CYC	The closure, transport, and record second variations are C_3-cyclic on the committed cell, and each branch-plane operator commutes with the branch-plane U(1)	H1, H4
PH-EQV	P_C is R_3-equivariant on the committed cell; P_R, P_q, and the role trace are generation-blind before mass/readout	H2, H3
PH-EMB	The minimal committed-quark C_3 cell exhausts the branch-relevant C_3-isotypic support of the full compressed su(8) Hessian — no additional C_3-isotypic copies couple to the role-even common mode	H13

Tag	Premise	Negation
PH-ISO	The committed role Hessian metric is isotropic, G_role = κ_role·I (so the role-even and role-odd lines of a committed isotype carry equal weight — Prop. 1)	H5
PH-SAME	The role-even three-branch common mode and the inherited role-odd 2–3 transport rest on the *same* committed closure profile Z in the $\mathbb{C}^8$ closure factor (§2), so they are σ-partners of one direction	H12
PH-RDO	The mass/readout has no weight on the occupied Cabibbo plane and no weight on the direct-target plane, with no rescaling of the survivor. The 1–2 exclusion is structural (the plane is occupied by the doorway); the 3–1 exclusion is the choice consistent with the residue being generated rather than inserted, not a structural impossibility (§9)	H6, H7
PH-HERM	Minimal-Hermitian-lift principle: admissibility length is minimised on the Hermitian side before the frame i-rotation, so the half-transport selects the principal root rather than the antipodal root. *Established upstream* as the Minimal-Hermitian-Lift theorem (predecessor paper, *The C_3 Substrate Reduction and the Minimal-Hermitian-Lift CKM Branch in VERSF*, Thm 2); that theorem itself rests on the side-choice premise ML-SIDE, which is the data-independence condition audited in §15.3	H8
PH-ROOT	The half-transport phase target on the survivor is h^2/	h
PH-ROT	The frame rotation sign is +i (Ω_W = +i·ε_W·H_W, ε_W > 0), fixing arg ζ = 5π/6 rather than −π/6. Quantitatively load-bearing, not a sign convention: the −i choice changes	J
PH-GAP	The role-mixing block Ω_mix is gapped, giving a clean role-even reading	H10

0.3 Derivation ledger

Object	Prior status	Status here	Grade
G_C on branch orbit	Owed	Computed: κ_C·I_6	Audit condition: PH-CYC
G_T on branch orbit	Owed	Computed: κ_T·I_6	Audit condition: PH-CYC
G_R on branch orbit	Owed	Computed: κ_R·I_6	Audit condition: PH-CYC
[K_even,pre^q, R_3] = 0	Owed	Returned by scalar branch Hessian	Audit condition: PH-CYC, PH-EQV
$w_{12} = w_{23} = w_{31}$	Owed	Returned by circulant block	Exact, given the row above
Total common-mode norm b	Owed	Returned by role-isotype norm matching (Prop. 1)	Conditional: PH-ISO ∧ PH-SAME + IN-OMQ

Object	Prior status	Status here	Grade
2–3 readout survivor	Owed	Returned by branch-complement readout	Audit condition: PH-RDO (3–1 exclusion non-question-begging, §9)
Minimal Hermitian lift (principal vs antipodal root)	Owed	Established by the Minimal-Hermitian-Lift theorem (predecessor, Thm 2): $h/\|h\| = e^{i\pi/3}$	Inherited; firewall relocates to upstream ML-SIDE (§15.3)
Projected sign $\sigma_\varphi^q = +1$	Owed	Follows from §10 given the inherited lift and the target	Inherited via PH-HERM ∧ PH-ROOT
Frame rotation sign $+i$	Owed	Sets $\arg \zeta = 5\pi/6$ not $-\pi/6$	Conditional: PH-ROT (data-contact risk, §15.3)
Phase branch: $\arg \zeta = 5\pi/6$	Owed	Combined from the three rows above	Conditional on PH-HERM ∧ PH-ROOT ∧ PH-ROT (see §15.3)
$[\Omega_0, \Omega_q]_{13} = -a\zeta$	—	Verified by direct multiplication (App. B)	Exact
$\Delta_{13} = \frac{1}{2}\cdot a\cdot\zeta$ (leading)	Previously exact	Used after Hessian return	Exact at leading order: IN-BCH
Δ_{13} BCH tail	—	Converged residue 0.0026332 vs leading 0.0026306 (App. C)	Discharged: stable to ~0.1%
Diagonal Cartan term	Owed	Small, traceless, imaginary (App. C)	Live structural condition: removability by allowed rephasing

Scope of the conditional returns. The amplitude branch is conditional on the orbit/norm returns (PH-CYC, PH-EQV) and on *two* norm premises, not one: role-metric isotropy (PH-ISO) and the shared-closure-profile identification (PH-SAME). The directional branch is conditional on readout survival (PH-RDO), of which the 1–2 exclusion is structural and the 3–1 exclusion is the non-question-begging choice (§9). The phase branch is conditional on three legs: the minimal-Hermitian-lift principle (PH-HERM) and the half-transport target (PH-ROOT), both established upstream by the Minimal-Hermitian-Lift theorem, and the frame rotation sign (PH-ROT), owed here. The upstream establishment is conditional on the predecessor's side-choice premise ML-SIDE; the phase debt of the microscopic paper is therefore the data-independence of ML-SIDE together with the rotation sense PH-ROT, not the root selection itself.

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1. The exact projected object

The object to be computed is the weak-doublet closure Hamiltonian after generation completion, role projection, and mass/readout projection. To keep Hermiticity explicit even when the projectors are ordered rather than mutually commuting, the compression is written as

$$H_W = P_W^\dagger \cdot H_{cl} \cdot P_W, \quad P_W = P_Y \cdot P_R \cdot P_C.$$

If P_C, P_R, and P_Y commute on the admissible support, this reduces to the symmetric form $P_W \cdot H_{cl} \cdot P_W$. If they do not commute, the daggered form is the correct ordered Hermitian compression. Since H_{cl} is self-adjoint (IN-HCL), H_W is self-adjoint, and the weak-frame generator

$$\Omega_W = i \cdot \varepsilon_W \cdot H_W$$

is anti-Hermitian. This closes the frame-consistency requirement: every off-diagonal branch of Ω_W has the form

$$A_{ij}(\eta) = \eta \cdot E_{ij} - \eta^* \cdot E_{ji}.$$

Before mass/readout, the role-even quark block is

$$K_{\text{even,pre}}^q = \tfrac{1}{2} \cdot \mathrm{Tr}_{\text{role}}(P_q \cdot P_R \cdot P_C \cdot H_{cl} \cdot P_C \cdot P_R \cdot P_q).$$

After readout, the visible role-even quark block is

$$K_{\text{even}}^q = \tfrac{1}{2} \cdot \mathrm{Tr}_{\text{role}}(P_q \cdot P_Y \cdot P_R \cdot P_C \cdot H_{cl} \cdot P_C \cdot P_R \cdot P_Y \cdot P_q).$$

The C_3 amplitude and equal weights are pre-readout results; the visible 2–3 survivor is a post-readout result. The two stages are kept distinct because the C_3 orbit is constituted prior to the readout in the projection ordering — readout breaks branch democracy on an orbit that is already democratic.

2. The minimal committed-quark C_3 Hessian cell

The minimal cell contains exactly the degrees of freedom needed to test the C_3 branch return: three generations, two weak-role readings, and the closure support inherited from su(8).

$$\mathcal{H}_{\text{CKM}}^{\text{min}} = \ell^2(G_3) \otimes \mathbb{C}^2_{\text{role}} \otimes \mathbb{C}^8_{\text{closure}}, \quad G_3 = \{1, 2, 3\}.$$

The generation cycle R_3 acts by

$$R_3 : 1 \to 2,\ 2 \to 3,\ 3 \to 1,$$

and on matrix units by

$$R_3 \cdot E_{12} \cdot R_3^{-1} = E_{23},\quad R_3 \cdot E_{23} \cdot R_3^{-1} = E_{31},\quad R_3 \cdot E_{31} \cdot R_3^{-1} = E_{12}.$$

For each unordered generation plane define the Hermitian branch quadratures

$S\_ij = (E\_ij + E\_ji)/\sqrt{2}$, $A\_ij = i\cdot(E\_ij - E\_ji)/\sqrt{2}$.

The real two-plane B_ij = span{S_ij, A_ij} is the Hermitian branch support associated with the anti-Hermitian frame branch after multiplication by i. A general pre-readout role-even branch perturbation is

$X = X_{12} + X_{23} + X_{31}$, $X\_ij \in B\_ij$.

Write $X\_ij = s\_ij\cdot S\_ij + a\_ij\cdot A\_ij$ and collect the six real coordinates into

$x = (s_{12}, a_{12}, s_{23}, a_{23}, s_{31}, a_{31})^T$.

With the Killing-normalised inner product $\langle X, Y\rangle\_K = \tfrac{1}{2}\cdot\mathrm{Tr}(XY)$ on the Hermitian side, the branch planes are mutually orthogonal, so

$\|X\|\_K^2 = \|X_{12}\|\_K^2 + \|X_{23}\|\_K^2 + \|X_{31}\|\_K^2 = x^Tx$.

Key point. The minimal Hessian calculation is a six-coordinate second-variation calculation. If the Hessian on x is scalar, then C_3 covariance and equal branch weights are *returned* rather than postulated.

A caveat that must be named (PH-EMB). The calculation proves democracy *within* this minimal cell. It does not by itself prove that the full compressed su(8) Hessian has no additional C_3-isotypic copies coupled to the role-even common mode. If such ambient copies exist, the minimal-cell democracy need not exhaust the full projected Hessian, and a critic may fairly say the result proves democracy after restricting to the democratic cell. The manuscript therefore carries PH-EMB as an explicit premise: the minimal cell exhausts the branch-relevant C_3-isotypic support of the full su(8) compression. Its negation is falsifier H13, and discharging it is owed to the full su(8) calculation, not supplied here.

Lemma 1 (branch-plane Schur isotropy)

Let R(θ) be the rotation of the real branch two-plane B_ij = span{S_ij, A_ij} given in coordinates by

$(s\_ij, a\_ij) \mapsto (\cos\theta\cdot s\_ij - \sin\theta\cdot a\_ij, \sin\theta\cdot s\_ij + \cos\theta\cdot a\_ij)$.

Let L be a self-adjoint operator on B_ij commuting with R(θ) for all θ. Then $L = \mu\cdot I_2$ *for some real μ.*

Proof. The family {R(θ)} acts irreducibly on the real two-plane B_ij (equivalently, on the complex line $B\_ij \cong \mathbb{C}$). A self-adjoint operator commuting with every rotation of the plane has both eigenspaces rotation-invariant; since no proper subspace is invariant under all R(θ), the two eigenvalues coincide, so $L = \mu\cdot I_2$ with μ real. ■

Lemma 1 is the engine of §§3–5: under PH-CYC every branch-plane second-variation operator is scalar, and under the C_3-cyclic hypothesis the same scalar recurs on each of the three planes.

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3. Closure second variation

The closure defect around the generation triangle is the deviation of the cyclic product from the identity. For small Hermitian branch transports $U_{ij} = \exp(i \cdot \varepsilon \cdot X_{ij})$, the triangle product is

$$U_{12} \cdot U_{23} \cdot U_{31} = I + i \cdot \varepsilon \cdot (X_{12} + X_{23} + X_{31}) + O(\varepsilon^2),$$

so the first-order closure defect projected to the branch support is

$$C_\Delta^{(1)} = X_{12} + X_{23} + X_{31}.$$

Take the standard quadratic normalisation $A_C^{(2)} = (\kappa_C/2) \cdot \|C_\Delta^{(1)}\|_K^2$. Because the branch planes are Killing-orthogonal, all cross terms vanish,

$$\langle X_{12}, X_{23} \rangle_K = \langle X_{23}, X_{31} \rangle_K = \langle X_{31}, X_{12} \rangle_K = 0,$$

hence

$$A_C^{(2)} = (\kappa_C/2) \cdot (\|X_{12}\|_K^2 + \|X_{23}\|_K^2 + \|X_{31}\|_K^2) = \tfrac{1}{2} \cdot x^T \cdot (\kappa_C \cdot I_6) \cdot x,$$

and the closure Hessian returned on the branch orbit is

$G_C^B = \kappa_C \cdot I_6$. *[Audit condition: PH-CYC]*

Closure does not bias one generation branch against another in the committed C_3 cell.

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4. Transport second variation

The transport term measures the failure of local generation transport to preserve the committed quark reading. In the minimal cyclic cell the same local transport linearisation L_T acts on each branch before mass/readout. With $T_{ij}^{(1)} = L_T \cdot X_{ij}$ the cyclic penalty is

$$A_T^{(2)} = (\kappa_T/2) \cdot \Sigma_{cyc} \|L_T \cdot X_{ij}\|_K^2.$$

Before P_Y the committed-quark cell has no distinguished generation branch (PH-EQV), so L_T commutes with the branch-plane phase rotation and with the C_3 permutation. Lemma 1 then gives, on each two-real-dimensional branch plane,

$L\_T^\dagger \cdot L\_T = \mu\_T \cdot I_2$.

Absorbing μ_T into κ_T,

$A\_T^{(2)} = (\kappa\_T/2)\cdot(\|X_{12}\|\_K^2 + \|X_{23}\|\_K^2 + \|X_{31}\|\_K^2)$,

so

$G\_T\text{\^{}}B = \kappa\_T \cdot I_6$. *[Audit condition: PH-CYC]*

Transport contributes the same quadratic curvature to all three C_3 branches.

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5. Record-composition second variation

The record-composition term measures the failure of local records to compose consistently around the generation triangle. Linearising the record morphism around the admissible background gives

$R\_\Delta^{(1)} = L\_R \cdot X_{12} + L\_R \cdot X_{23} + L\_R \cdot X_{31}$,

with quadratic penalty $A\_R^{(2)} = (\kappa\_R/2)\cdot\|R\_\Delta^{(1)}\|\_K^2$. As with closure, the branch images are orthogonal after removal of the radial and trace directions (IN-RAD); as with transport, the same record-linearisation acts on each branch in the minimal cyclic cell (PH-CYC). Therefore

$\langle L\_R \cdot X\_ij, L\_R \cdot X\_kl\rangle\_K = 0$ for $ij \neq kl$, $L\_R^\dagger \cdot L\_R = \mu\_R \cdot I_2$ on each B_ij,

and after absorbing μ_R into κ_R,

$A\_R^{(2)} = (\kappa\_R/2)\cdot(\|X_{12}\|\_K^2 + \|X_{23}\|\_K^2 + \|X_{31}\|\_K^2)$,

so

$G\_R\text{\^{}}B = \kappa\_R \cdot I_6$. *[Audit condition: PH-CYC]*

Record composition also supplies a scalar Hessian on the C_3 branch orbit.

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6. The combined projected-Hessian return

The branch-relevant closure Hamiltonian is the sum of the three second variations after radial u(1) removal:

$H_{cl}^{B} = G_{C}^{B} + G_{T}^{B} + G_{R}^{B} = (\kappa_C + \kappa_T + \kappa_R)\cdot I_6$.

Define $\kappa_{C_3} = \kappa_C + \kappa_T + \kappa_R$. Then the missing pre-readout Hessian return is a scalar operator on the six-dimensional C_3 branch coordinate space:

$G_{C_3}^{B} = \kappa_{C_3}\cdot I_6$.

Since R_3 is represented on x by a permutation of the three branch two-planes, I_6 commutes with R_3, so

$[G_{C_3}^{B}, R_3] = 0$.

Because P_C is generation-completion equivariant and P_R, P_q, and the role trace are generation-blind before mass/readout (PH-EQV), the commutator carries into the pre-readout quark block:

$[K_{\text{even,pre}}^{q}, R_3] = 0$. *[Audit condition: PH-CYC, PH-EQV]*

This is the first owed return, now supplied by the Hessian calculation.

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7. Circulant form and democratic weights

Every Hermitian 3×3 generation block commuting with R_3 is circulant, so

$K_{\text{even,pre}}^{q} = d\cdot I_3 + h\cdot R_3 + h^{*}\cdot R_3^{\dagger} = \begin{bmatrix} d & h & h^{*} \\ h^{*} & d & h \\ h & h^{*} & d \end{bmatrix}$.

The three forward branch coefficients are equal,

$(K_{\text{even,pre}}^{q})_{12} = (K_{\text{even,pre}}^{q})_{23} = (K_{\text{even,pre}}^{q})_{31} = h$,

and the reverse coefficients are their conjugates. Therefore the branch weights are

$w_{12} = |h|^2$, $w_{23} = |h|^2$, $w_{31} = |h|^2$,

hence

$w_{12} = w_{23} = w_{31}$. *[Exact, given §6]*

This is the matrix consequence of the scalar C_3 Hessian — not an inference from CKM data.

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8. The norm return: why the total branch budget is b^2

Equality of branch weights fixes the *split* but not the *total scale*. The scale comes from the committed-quark role-isotype metric (PH-ISO). The aim of this section is to make the identification $\|Z_{C_3}\|_K^2 = b^2$ *derived* from isotropy of the role Hessian metric, rather than assigned.

Proposition 1 (committed role-isotype norm equivalence)

Let the committed role Hessian metric be isotropic, $G_{role} = \kappa_{role}\cdot I$, on the role space $\mathbb{C}^2_{role}$ with Killing-orthonormal up/down basis $\{e_u, e_d\}$. Let σ be the role-swap involution $e_u \leftrightarrow e_d$, with even/odd eigenlines

$$V_+ = \mathbb{R}\{(e_u + e_d)/\surd 2\},\ V_- = \mathbb{R}\{(e_u - e_d)/\surd 2\}.$$

Then:

(i) σ is G_{role}-orthogonal, and G_{role} restricts to the same scalar on each eigenline,

$$G_{role}|\{V_+\} = G_{role}|\{V_-\} = \kappa_{role}.$$

(ii) For any single committed closure profile $Z \in B$, the even-role and odd-role insertions of Z carry equal norm under the combined Hessian–Killing metric $G_{role} \otimes \langle\cdot,\cdot\rangle_K$:

$$\|(e_u + e_d)/\surd 2 \otimes Z\|^2 = \|(e_u - e_d)/\surd 2 \otimes Z\|^2 = \kappa_{role}\cdot\|Z\|_K^2.$$

Proof. (i) Because $G_{role} = \kappa_{role}\cdot I$, $\langle\sigma X, \sigma Y\rangle_G = \kappa_{role}\cdot\langle\sigma X, \sigma Y\rangle_K = \kappa_{role}\cdot\langle X, Y\rangle_K = \langle X, Y\rangle_G$, since σ is Killing-orthogonal; and each eigenvector $(e_u \pm e_d)/\sqrt{2}$ has unit Killing norm, so its G-length is κ_{role}. (ii) Both insertions factor as (unit role vector) $\otimes$ Z, and by (i) the role factor has the same G-weight κ_{role} on V_+ and V_-, leaving the common Killing factor $\|Z\|_K^2$. ■

The operative statement is (ii), *not* the per-vector identity $\|X_{even}\|_G^2 = \|X_{odd}\|_G^2$ for an arbitrary perturbation, which fails by the cross term $2\cdot\mathrm{Re}\langle X_u, X_d\rangle_K$. The even common-mode branch and the odd transport branch are the two σ-partners of a *single* committed closure profile Z; (ii) is exactly the equality of their norms.

Application

In the committed quark doublet the role metric before readout is isotropic (PH-ISO), so in the up/down role basis the quadratic role form is

$$Q_{role} = \kappa_{role}\cdot(\|X_u\|_K^2 + \|X_d\|_K^2) = \kappa_{role}\cdot(\|X_{even}\|_K^2 + \|X_{odd}\|_K^2),$$

the last equality being the parallelogram identity for the orthogonal even/odd change of basis. The inherited role-odd quark generator (IN-OMQ) places the committed transport on the odd line with closure profile Z carrying

$(\Omega\_q)_{23} = b$, $b = 81/2000$, $\|Z\|\_K^2 = b^2$.

By Proposition 1(ii) the role-even partner of that *same* committed closure profile Z carries the identical norm. This step has a premise that must be named, because Proposition 1(ii) equates the even and odd insertions of *one and the same* profile Z, whereas the object surviving readout is a three-branch democratic common mode and the inherited b is a single 2–3 transport. The chain closes only if the role-even common mode is the σ-even image of *precisely* the inherited role-odd 2–3 transport profile — that they rest on a single shared closure direction in the $\mathbb{C}^8$ closure factor of §2. This is the premise PH-SAME, and it is a statement about the closure factor, *not* about G_role. If the committed cell pins one shared closure direction across the role-even and role-odd sectors, PH-SAME holds and the identification is automatic; if it pins different directions, $|\zeta| \neq b/\sqrt{3}$ even under perfect isotropy. Granting PH-SAME, the two role insertions differ only by the role line (even vs odd), Proposition 1 equates their weights, and the role-even common-mode profile inherits the Killing norm of Z. The total pre-readout role-even common-mode budget is therefore

$$\|Z\_C_3\|\_K^2 = \|Z\|\_K^2 = b^2.$$

The branch decomposition is Killing-orthogonal and democratic (§§6–7),

$$\|Z\_C_3\|\_K^2 = \|Z_{12}\|\_K^2 + \|Z_{23}\|\_K^2 + \|Z_{31}\|\_K^2 = 3\cdot|\zeta|^2,$$

so

$3\cdot|\zeta|^2 = b^2$, hence $\boldsymbol{|\zeta| = b/\sqrt{3}}$**.** *[Conditional: PH-ISO ∧ PH-SAME + IN-OMQ]*

The scale is not fitted to the CKM triangle entry; it is inherited from the already-returned committed 2–3 norm b and split over the three C_3-orthogonal branches.

Narrowed exposure. After the scalar C_3 Hessian has been returned, the amplitude $b/\sqrt{3}$ has *two* remaining exposures, not one. The first is role-metric isotropy: if G_role is anisotropic on committed support, the even and odd partners need not carry equal weight (PH-ISO, falsifier H5). The second is the shared-closure-profile identification: even with perfect isotropy, if the role-even common mode and the inherited role-odd 2–3 transport rest on *different* closure directions, the norm budget does not transfer and $|\zeta| \neq b/\sqrt{3}$ (PH-SAME, falsifier H12). The amplitude vulnerability is therefore not C_3 democracy but the conjunction of role-isotype norm equivalence (a statement about G_role) and closure-profile sharing (a statement about the $\mathbb{C}^8$ factor) — two transparent, separately falsifiable conditions, neither reducible to the other.

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9. The mass/readout branch projection

The pre-readout Hessian is democratic; the visible CKM correction is not, because P_Y is applied to the constituted C_3 orbit. The readout does not create the C_3 norm — it selects the visible component of that norm. The aim of this section is to write the readout explicitly and to be honest about which of its two exclusions is structural and which is the non-question-begging choice.

The readout projector and the status of its two exclusions

Let Π_{12}, Π_{23}, Π_{31} be the branch projectors on the C_3 orbit, with $I\_B = \Pi_{12} + \Pi_{23} + \Pi_{31}$. Two of the three branch planes already carry inherited assignments, and are therefore *not available* as the leading common-mode readout:

$\Pi_{12} = \Pi\_\text{Cabibbo}$ — the 1–2 plane already holds the role-odd doorway $(\Omega\_q)_{12} = a$ (IN-OMQ);
$\Pi_{31} = \Pi\_\text{direct target}$ — the 3–1 plane is the slot of the residue Δ_{13} being computed.

The two exclusions do not have the same status, and the section should not pretend they do. The 1–2 exclusion is structural: that plane is genuinely occupied by the protected Cabibbo doorway $(\Omega\_q)_{12} = a$, so a common-mode survivor there would duplicate inherited structure. The 3–1 exclusion is weaker — methodological rather than structural. A survivor on Π_{31} would *insert* the target correction directly rather than have it generated through the commutator of §12; excluding it is the choice consistent with the residue being generated rather than inserted, which is what the derivation is for. It is not a proof that the readout cannot weight 3–1. Absent an independent structural reason forbidding weight on 3–1, this exclusion presupposes part of what is being derived, and the honesty is carried by falsifier H7 rather than by the readout being "forced." With both planes set aside,

$$\boldsymbol{\Pi\_Y\hat{}B = I\_B - \Pi\_\text{Cabibbo} - \Pi\_\text{direct target} = I\_B - \Pi_{12} - \Pi_{31} + O(\Omega\_\text{mix},c).}$$

Since $I\_B = \Pi_{12} + \Pi_{23} + \Pi_{31}$, this reduces to

$\Pi\_Y\hat{}B = \Pi_{23} + O(\Omega\_\text{mix},c)$.

Consequence

Applied to the democratic pre-readout orbit $Z\_C_3 = Z_{12} + Z_{23} + Z_{31}$, the survivor is

$$\boldsymbol{\Pi\_Y\hat{}B\cdot Z\_C_3 = Z_{23} + O(\Omega\_\text{mix},c).}$$

So the leading post-readout role-even quark branch is

$K\_\text{even}\hat{}\{q,(23)\} = h\cdot E_{23} + h^*\cdot E_{32} + O(\Omega\_\text{mix},c)$

on the Hermitian side, and

$\Omega\_\text{even}\hat{}\{q,(23)\} = \zeta\cdot E_{23} - \zeta^*\cdot E_{32} + O(\Omega\_\text{mix},c)$

on the anti-Hermitian frame side. The 2–3 survivor is the unique *no-direct-insertion* readout: it is what remains once the occupied Cabibbo branch is excluded (structural) and the direct-target branch is disallowed as a selection principle (PH-RDO). The phrase makes explicit that PH-RDO is doing real work — the survivor is not forced by structure alone. *[Audit condition: PH-RDO; the 3–1 exclusion is the no-direct-insertion selection principle, not a structural impossibility (H7); O(Ω_mix,c) controlled by PH-GAP]*

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10. Phase-root return

The phase is fixed before the frame i-rotation. Let h be the Hermitian coefficient of the 2–3 survivor, K_even^{q,(23)} = $h\cdot E_{23} + h^*\cdot E_{32}$. The C_3 half-transport condition on the Hermitian side (PH-ROOT) is

$h^2/|h|^2$ = $\mathcal{O}$_q, $\mathcal{O}$_q = e^(2πi/3),

with the two roots

h/|h| = e^(iπ/3) or h/|h| = e^(i4π/3).

The Hessian H_cl is self-adjoint, being a second variation of a real admissibility functional (IN-HCL), so admissibility length is measured on the Hermitian side (PH-HERM). The principal Hermitian phase lengths of the two roots are π/3 and 2π/3 (since e^(i4π/3) = e^(−i2π/3) has principal angle 2π/3). Minimal length selects

h/|h| = e^(iπ/3).

The frame coefficient is obtained by the i-rotation ζ = +i·ε_W·h with ε_W > 0 (PH-ROT), so

arg(ζ) = π/2 + π/3 = 5π/6, equivalently **σ_φ^q = +1.** *[Audit condition: PH-HERM ∧ PH-ROOT ∧ PH-ROT]*

The phase return decomposes into *three* distinct debts, which should not be bundled. The first is the *minimal-Hermitian-lift principle* (PH-HERM): that length is measured on the Hermitian side, so the principal root e^(iπ/3) is selected over the antipodal e^(i4π/3). The second is the *half-transport target* (PH-ROOT): that $\mathcal{O}$_q = e^(2πi/3). The third is the *frame rotation sign* (PH-ROT): that the rotation is +i rather than −i. The rotation sign is not a free convention here, because it controls a physical observable: the −i rotation sends ζ → −ζ (arg −π/6), and with the inherited direct seed present this does not merely conjugate the triangle — it changes |J| from 3.0×10^{-5} to 0.7×10^{-5} (§15.3), spoiling the agreement of §14. Only +i reproduces the measured magnitude.

The first two legs are *established upstream*, not owed here: the Minimal-Hermitian-Lift theorem (predecessor, Thm 2) fixes both the target and the principal-root selection, returning h/|h| = e^(iπ/3) and σ_φ^q = +1. What remains genuinely open in this manuscript is the rotation sense PH-ROT, together with the data-independence of the upstream selection — the predecessor's side-choice premise ML-SIDE — which §15.3 audits.

The sign is not chosen by comparison with CKM data; it is selected because the Hessian minimises Hermitian generator length before the frame rotation. This claimed independence from mixing data is itself the firewall condition audited in §15.3: it holds only if the upstream side-choice (ML-SIDE, underlying PH-HERM and PH-ROOT) and the rotation sense (PH-ROT) are each fixed by closure structure rather than by the phase or sign that reproduces J. Until that is shown, $\arg \zeta = 5\pi/6$ is established up to a data-independence premise that the predecessor paper itself flags as owed, not an independent result.

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11. The returned C_3 curvature

Combining amplitude and phase, the minimal projected-Hessian return is

$$\zeta_{C_3} = (b/\sqrt{3})\cdot e^{(5\pi i/6)}.$$

Since $e^{(5\pi i/6)} = -\sqrt{3}/2 + i/2$,

$$\zeta_{C_3} = -b/2 + i\cdot b/(2\sqrt{3}).$$

With $b = 81/2000$,

$$\zeta_{C_3} = -81/4000 + i\cdot 27\sqrt{3}/4000 = -0.0202500 + 0.0116913\cdot i.$$

---

12. CKM residue from the returned curvature

The inherited role-odd quark generator (IN-OMQ) is

$$\Omega_q = \begin{bmatrix} 0 & a & c\cdot e^{(i\varphi)} \\ -a & 0 & b \\ -c\cdot e^{(-i\varphi)} & -b & 0 \end{bmatrix},$$

with $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\varphi = 2\pi/3$. For the returned survivor $\Omega_0(\zeta) = \zeta\cdot E_{23} - \zeta^*\cdot E_{32}$, direct multiplication gives

$$[\Omega_0(\zeta), \Omega_q]_{13} = -a\cdot\zeta.$$

The convention-invariant symmetric split-BCH form (IN-BCH) is

$$V_{CKM} = \exp(-\Omega_0 + \Omega_q/2)\cdot\exp(+\Omega_0 + \Omega_q/2),$$

so

$$\log V_{CKM} = \Omega_q - \tfrac{1}{2}\cdot[\Omega_0, \Omega_q] + \text{higher order}.$$

With ζ_C_3 carried inside $\Omega_0 = \zeta\_C_3 \cdot E_{23} - \zeta\_C_3^* \cdot E_{32}$, there is no free rescaling: the 1–3 entry follows directly,

$(\log V\_CKM)_{13} = (\Omega\_q)_{13} - \frac{1}{2} \cdot [\Omega_0, \Omega\_q]_{13} = c \cdot e\hat{}(i\varphi) + \frac{1}{2} \cdot a \cdot \zeta\_C_3$,

using $[\Omega_0, \Omega\_q]_{13} = -a \cdot \zeta\_C_3$ (Appendix B). The residue beyond the inherited direct seed is therefore

$\Delta_{13} = \frac{1}{2} \cdot a \cdot \zeta\_C_3$. *[Exact at leading order: IN-BCH]*

Substitution gives

$\Delta_{13} = -a \cdot b/4 + i \cdot a \cdot b/(4\sqrt{3}) = -729/320000 + i \cdot 243\sqrt{3}/320000$,

$\Delta_{13} = -0.0022781 + 0.0013153 \cdot i$, $|\Delta_{13}| = 0.0026306$.

Two clauses of the BCH treatment are kept separate, because they have different status. The *convergence* of the higher-order BCH tail in the 1–3 residue channel is verified: the converged two-factor value is $|\Delta_{13}| = 0.0026332$ against the leading 0.0026306, a difference of ~0.1% (Appendix C), so the leading expression is stable and that clause is discharged. The *removability* of the residual diagonal Cartan phase generated by the split product is a separate, live structural condition: the diagonal terms of $\log V\_CKM$ are traceless and imaginary — consistent with an allowed quark-rephasing direction — but at ~18% of $|\Delta_{13}|$ (Appendix C) they are not negligible by hierarchy, so their harmlessness rests on structural removability, not smallness. That removability is a structural claim, not a convergence statement, and is recorded as such (falsifier H11).

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13. Cabibbo back-reaction

The same commutator produces a much smaller correction in the Cabibbo plane. The leading estimate is

$|\Delta_{12}| = \frac{1}{2} \cdot c \cdot |\zeta\_C_3| = \frac{1}{2} \cdot c \cdot b/\sqrt{3}$, so $|\Delta_{12}|/a = c \cdot b/(2\sqrt{3} \cdot a) = 1.26 \times 10^{-4}$.

As in the 1–3 channel, the leading number understates the converged value, and the section should give both rather than revert to the leading estimate alone. Forming the full split product and reading the converged (1,2) correction beyond the inherited doorway a:

Quantity	Leading	Converged two-factor
$\lvert\Delta_{12}\rvert/a$	1.26×10^{-4}	2.11×10^{-4}

The leading term understates the Cabibbo back-reaction by about 1.7×, but the converged value remains deeply protective: at ~2×10^{-4} of the doorway it leaves the leading Cabibbo angle essentially unchanged. This is the honest two-factor treatment that the 1–3 tail discussion of §17 refers back to.

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14. Physical 1–3 element and empirical confrontation

The returns above are internal: they fix the curvature ζ_C$_3$ and the residue Δ$_{13}$ as graded consequences of the named premises. This section is of a different epistemic kind — it confronts the result with measured quark mixing. The grade throughout is therefore **Empirical**, conditional on the inherited rational inputs IN-OMQ; no internal return is revised.

14.1 The residue is not the observable

The physical 1–3 mixing element is |V_ub|, the (1,3) entry of the *unitary* matrix V_CKM = exp(Ω_total), not an entry of the generator. Two corrections separate Δ$_{13}$ from |V_ub|.

First, the inherited generator already carries a direct 1–3 entry (IN-OMQ),

(Ω_q)$_{13}$ = c·e^(iφ) = 0.00243·e^(2πi/3) = −0.001215 + 0.002104·i,

so the generator 1–3 entry is the sum

g$_{13}$ = c·e^(iφ) + Δ$_{13}$ = −0.0034931 + 0.0034197·i, |g$_{13}$| = 0.0048883.

Second, exponentiation to the unitary group mixes the Cabibbo and 2–3 entries into the 1–3 slot at second order: [exp Ω]$_{13}$ ≈ Ω$_{13}$ + ½·Ω$_{12}$·Ω$_{23}$ + …, and ½·a·b ≈ 0.00456 is itself the size of |V_ub|. The generator magnitude |g$_{13}$| = 0.00489 is therefore *not* the observable, and must not be quoted as the prediction. Building the full anti-Hermitian generator with the inherited signs (+a, +b) and exponentiating gives

|V_ub| = 0.00356. *[Empirical: IN-OMQ]*

14.2 The full unitarity triangle

From the same unitary V_CKM the rephasing-invariant unitarity-triangle angles α, β, γ and the Jarlskog invariant J follow directly. The contrast with and without the derived residue is the central empirical observation: the inherited direct term alone produces a badly distorted triangle, and it is the residue Δ$_{13}$ — specifically its phase, arg ζ_C$_3$ = 5π/6 — that restores agreement.

Quantity	Direct term only (no Δ$_{13}$)	Direct + Δ$_{13}$ (full)	Measured
α	128.0°	84.6°	~85–92°
β	19.6°	22.4°	~22.2°
γ	32.4°	72.9°	~63–66°

Quantity	Direct term only (no Δ_{13})	Direct + Δ_{13} (full)	Measured
Jarlskog $|J|$	1.84×10^{-5}	3.00×10^{-5}	$\sim 3.08 \times 10^{-5}$
$|V\_ub|$	0.00393	0.00356	~0.0038

The triangle closes to 180° identically (a unitarity check, not a fit). With the residue included, β matches to ~1% and the Jarlskog invariant to ~3%, while $\alpha = 84.6°$ sits just at the lower lip of the measured band rather than comfortably inside it. Under ordinary quark-field rephasings, both $|V\_ub|$ and $|J|$ are invariant. The sign sensitivities of §14.1 and §15.3 are a different operation — changes in generator orientation (the $\pm a$, $\pm b$ signs) or in the physical frame-rotation branch ($\pm i$), not rephasings. $|J|$ is invariant under those orientation changes as well; $|V\_ub|$ is not. That is the distinction at issue.

The γ comparison is benchmark-dependent and is made here against the conservative target. The tree-level *direct* determination is ~63–66°; some *indirect* global fits place γ higher, near the predicted 72.9°. Comparing against the direct value — as done above — is the conservative and correct call, because the indirect fits already assume CKM unitarity and partly fold in the same physics being tested. A reader scoring the prediction against an indirect global fit would therefore wrongly record γ as a clean success; the ~8° tension is specific to the tree-level benchmark, and is neither a success nor a failure against the indirect fits.

14.3 Honest empirical status

Three statements fix the scope of this confrontation and must not be softened.

The derived residue is load-bearing for CP. Without Δ_{13} the triangle is wrong in every CP-sensitive entry ($\gamma = 32°$, $\alpha = 128°$, J nearly halved). The phase content derived in §10 is what reproduces β and J. The piece that mildly *lowers* the $|V\_ub|$ magnitude is the same piece that makes the CP structure come out right.

γ is a genuine residual tension. The full prediction $\gamma = 72.9°$ lies about 8° above the measured ~63–66°, a 2–3σ discrepancy at present precision, correlated with $|V\_ub|$ coming out ~7% low — the triangle apex sits slightly too far over. The CP *amount* (β, J) is near-exact; the triangle *shape* in γ is not. This is a specific, falsifiable deviation, recorded as such.

a and b are effectively inputs, not predictions. The same matrix returns $|V\_us| = 0.223$ (measured 0.224) and $|V\_cb| = 0.0405$ (measured 0.0405) — the Cabibbo angle and V_cb. These are reproductions of measured quantities, so the genuine test lies entirely in the 1–3 / CP sector: β and J near-exact, α and $|V\_ub|$ within ~7%, γ high by ~8°. The strength of the β and J agreement is contingent on the inherited rationals a, b, c, φ, arg ζ having been fixed by the substrate reduction independently of CKM data; if any were tuned to mixing data upstream, the agreement is correspondingly less independent. That provenance is an inherited-corpus question, not settled within this manuscript.

15. Provenance audit of the inherited inputs

The empirical confrontation of §14 is only as independent as the inputs it runs on: a, b, c, φ, and arg ζ. This section grades each by provenance and isolates the single exposure on which the empirical agreement most depends. It is an audit, not a return — it revises no internal result, but it fixes how much of §14 may be claimed as prediction.

15.1 Provenance by input

Input	Provenance	Status for predictive claims
$a = 9/40$	Inherited from the hexagonal nearest-neighbour closure-overlap (Cabibbo scale); not rederived here	Inherited input — not evidence in this paper
$b = 81/2000$	Structurally assembled: $b = a\cdot\eta^2/(D_3-D_2)$	Conditional structural assembly; reproduces measured |V_cb|
$c = 243/100000$	Structurally assembled: $c = \chi\cdot a\cdot[a\cdot\eta^2/(D_3-D_1)]$	Conditional structural assembly; genuine multi-entry effect
$\varphi = 2\pi/3$	Discrete C_3 holonomy candidate; microscopic transport-loop derivation open	Candidate, not independently derived
$\arg \zeta = 5\pi/6$	Established upstream (Minimal-Hermitian-Lift theorem) for the root; PH-ROT added here — see §15.3	Independent only if the firewall of §15.3 (ML-SIDE + PH-ROT) holds

15.2 The amplitude sector is decently sourced

The structural factors $D = \mathrm{diag}(1, 2, 4)$ (binary closure-depth stiffness), $\eta = 3/5$ (attenuation), and $\chi = 2/5$ (non-adjacent projection) are inherited from the flavour-transport framework, not from mixing data. From them, with a, the off-diagonal amplitudes are assembled arithmetically:

$b = a\cdot\eta^2/(D_3-D_2) = (9/40)\cdot(9/25)/2 = 81/2000$, $c = \chi\cdot a\cdot[a\cdot\eta^2/(D_3-D_1)] = (2/5)\cdot(9/40)\cdot[(9/40)\cdot(9/25)/3] = 243/100000$.

These are not free knobs: once a and the structural factors are fixed, b and c follow. The Cabibbo-scale input a, however, is itself inherited — it reproduces |V_us| (a measured quantity, §14.3) and must not be advertised as a prediction of this sequence. The amplitude sector therefore carries respectable provenance, with its one inherited anchor (a) named as such rather than hidden.

15.3 The exposure is the phase, and the firewall is not yet secured

The two phases are the live exposures. The holonomy $\varphi = 2\pi/3$ is a discrete C_3 candidate whose microscopic transport-loop derivation remains open. The more sensitive item is $\arg \zeta = 5\pi/6$, because the CP observables of §14 — the Jarlskog invariant J and the angle β — are produced almost entirely by this phase. In the corpus, arg ζ entered first as a motivated candidate, and the amplitude and phase of the C_3 curvature were noted to be effectively pinned by J and |V_td|. That is the circularity hazard stated plainly: if 5π/6 was originally selected because it reproduces J, then its later agreement with J is not independent evidence.

The present manuscript derives arg ζ = 5π/6 from the projected Hessian (§10) rather than asserting it, which is the necessary move. But a derivation can relocate a circularity without discharging it. The §10 derivation rests on *three* internal selection points, not two:

PH-ROOT — the half-transport target $h^2/|h|^2 = e^{(2\pi i/3)}$; PH-HERM — the minimal-length tiebreak selecting $h/|h| = e^{(i\pi/3)}$ over $e^{(i4\pi/3)}$; PH-ROT — the frame rotation sign +i, giving arg ζ = 5π/6 rather than −π/6.

The first two are *established upstream* by the Minimal-Hermitian-Lift theorem (predecessor, Thm 2), so within the present series they are inherited rather than owed. That does not close the firewall — it relocates it. The predecessor establishes the principal-root selection only on its side-choice premise ML-SIDE: that minimality is measured on the self-adjoint admissibility generator, before the i-rotation. The predecessor paper itself names ML-SIDE as "the one claim that must be justified from VERSF admissibility rather than from CKM data." So the data-independence burden for PH-HERM and PH-ROOT is carried entirely by the upstream ML-SIDE premise, and it remains open there.

The third leg is easy to overlook because it can be mistaken for a sign convention, but it is not. With the inherited direct seed present, the −i rotation (ζ → −ζ) does not merely conjugate the triangle: it changes the Jarlskog invariant from $|J| = 3.0 \times 10^{-5}$ to 0.7×10^{-5}, a factor of ~4, so only +i reproduces the measured magnitude. The sign of CP violation is physical, not convention-free, in the comparison to data; the rotation sense is therefore a genuine data-contact point, and if it was ever fixed by landing on the observed CP sector it carries the same circularity hazard as the phase itself.

The firewall holds only if the upstream side-choice (ML-SIDE, underlying PH-HERM and PH-ROOT) and the rotation sense (PH-ROT) are each motivated independently of the mixing data. If the side-choice or the rotation sign was selected — even implicitly — because it lands on the J-reproducing branch, then the Hessian/upstream argument moves the circularity one layer down rather than removing it. The owed work is therefore precise: show that the predecessor's ML-SIDE and the frame rotation sign are each fixed by closure structure with no contact with J, |V_td|, or the observed sign of CP violation.

15.4 Consequence for the empirical claim

The amplitude agreement of §14 is decently sourced; the CP agreement is contingent. The Jarlskog invariant landing at 3.0×10^{-5} and β at 22.4° are not yet independent successes — they wait on the firewall of §15.3. By the same token the residual γ tension and the J agreement occupy a single epistemic bucket: both are downstream of whether arg ζ = 5π/6 is forced or fitted. Until the upstream side-choice (ML-SIDE, underlying PH-HERM and PH-ROOT) and the rotation sense (PH-ROT) are secured without reference to mixing data, the honest status of the entire CP confrontation is *contingent on the phase firewall*, and the strength of §14 must be stated at that level and no higher.

16. Numerical ledger

Quantity	Exact expression	Value
a	$9/40$	0.225
b	$81/2000$	0.0405
c	$243/100000$	0.00243
$\lvert\zeta_{C_3}\rvert$	$b/\sqrt{3}$	0.0233827
$\arg(\zeta_{C_3})$	$5\pi/6$	150°
ζ_{C_3}	$-81/4000 + i\cdot 27\sqrt{3}/4000$	$-0.0202500 + 0.0116913\cdot i$
Δ_{13}	$-729/320000 + i\cdot 243\sqrt{3}/320000$	$-0.0022781 + 0.0013153\cdot i$
$\lvert\Delta_{13}\rvert$	$a\cdot b/(2\sqrt{3})$	0.0026306
$\lvert\Delta_{12}\rvert/a$ (leading)	$c\cdot b/(2\sqrt{3}\cdot a)$	1.26×10^{-4}
$\lvert\Delta_{12}\rvert/a$ (converged)	two-factor	2.11×10^{-4}
$\lvert V_{ub}\rvert$ (full, exponentiated)	—	0.00356
α, β, γ (full)	—	84.6°, 22.4°, 72.9°
Jarlskog $\lvert J\rvert$ (full)	—	3.00×10^{-5}

17. Microscopic falsifiers

This paper computes the missing Hessian return in the minimal committed-quark C_3 cell. A larger microscopic model can still overturn the result by violating one of the local inputs that made the six-coordinate Hessian scalar or the readout branch-selective. Each falsifier is the negation of a named premise (§0.2).

#	Failure condition	Premise broken	Consequence
H1	A_C, A_T, or A_R is not cyclic before P_Y	PH-CYC	No scalar C_3 Hessian
H2	P_C fails R_3 equivariance on the committed cell	PH-EQV	Covariance does not reach $K_{\text{even,pre}}^q$
H3	P_R, P_q, or role trace carries generation bias before readout	PH-EQV	Pre-readout branch weights can split
H4	$L_T^\dagger L_T$ or $L_R^\dagger L_R$ is not scalar on B_{ij}	PH-CYC (Lemma 1)	Transport/record branch anisotropy
H5	Committed role metric is anisotropic, $G_{\text{role}} \neq \kappa_{\text{role}}\cdot I$	PH-ISO (Prop. 1)	Total norm need not be b^2

#	Failure condition	Premise broken	Consequence
H6	P_Y rescales the surviving branch	PH-RDO	Amplitude changes
H7	P_Y weights 1–2 or 3–1 as the leading common branch	PH-RDO	Cabibbo protection fails / residue inserted not generated
H8	Half-transport length is minimised on the frame side, not the Hermitian side	PH-HERM	Phase root flips
H9	$h^2/\vert h\vert^2 \neq e^{(2\pi i/3)}$	PH-ROOT	Phase target fails
H10	Ω_mix is not gapped	PH-GAP	Clean role-even reading fails
H11	Diagonal Cartan commutator term is not removable by allowed quark rephasing	(structural)	Rephasing audit of the split product
H12	Role-even common mode and inherited role-odd 2–3 transport rest on different closure profiles	PH-SAME	$\vert\zeta\vert \neq b/\sqrt{3}$ even under perfect isotropy
H13	Full su(8) compressed support contains additional C_3-isotypic copies coupled to the role-even common mode	PH-EMB	Minimal-cell democracy does not exhaust the full projected Hessian
E1	γ remains ~8° high after higher-order or improved inputs	(empirical)	1–3 sector shape disfavoured
E2	Upstream ML-SIDE (underlying PH-HERM/PH-ROOT) or PH-ROT rotation sign cannot be fixed without reference to J, |V_td|, or the observed CP sign	(provenance)	CP agreement (J, β) is circular, not predictive
E3	Frame rotation sign was fixed by landing on the observed CP sector	PH-ROT	The +i selection (and hence |J|) is data-fitted

The burden has moved: a full microscopic calculation must now exhibit *which* premise fails, rather than merely objecting that the branch was assumed. Falsifiers E1–E3 are not structural but empirical and provenance-level: E1 records the residual γ tension already visible in §14; E2 records that the CP agreement remains contingent until the three-leg phase firewall of §15.3 is secured; E3 isolates the rotation-sign leg, which is quantitatively load-bearing (it moves |J| by a factor ~4) and therefore a genuine data-contact point rather than a convention.

A stability clause that was once bundled with H11 is now discharged rather than left live. The 1–3 BCH-tail convergence — whether higher BCH orders make the residue channel comparable to Δ_{13} — is settled by the converged two-factor calculation of Appendix C: leading and converged values differ by only ~0.1%. The remaining live content of H11 is therefore not convergence but the structural removability of the diagonal Cartan phase by an allowed rephasing, mirroring the honest treatment of the 1–2 tail in §13.

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18. Theorems

The Hessian democracy and the CKM residue are related but distinct derivational stages, so they are stated separately.

Theorem 1 (minimal C_3 Hessian return)

On the minimal committed-quark generation-completion cell, assume the Hermitian ordered compression $H_W = P_W^\dagger \cdot H_{cl} \cdot P_W$ *(IN-HCL), cyclic closure/transport/record second variations with branch-plane Schur isotropy (PH-CYC), and generation-blind equivariant pre-readout projections (PH-EQV). Then the pre-readout role-even quark Hessian is scalar on the branch orbit, and democracy follows:*

$$G_{C_3}^B = \kappa_{C_3} \cdot I_6 \Longrightarrow [K_{\mathrm{even,pre}}^q, R_3] = 0 \Longrightarrow w_{12} = w_{23} = w_{31}.$$

Proof. The closure, transport, and record second variations on the six branch coordinates are respectively $\kappa_C \cdot I_6$, $\kappa_T \cdot I_6$, and $\kappa_R \cdot I_6$ (§§3–5, via Lemma 1 under PH-CYC). Their sum is $\kappa_{C_3} \cdot I_6$, which commutes with R_3 (§6). The generation-blind equivariant pre-readout projections (PH-EQV) carry this covariance into $K_{\mathrm{even,pre}}^q$, forcing the block to be circulant; equal forward coefficients give equal branch weights (§7). ■

Scope. The conclusion is internal to the minimal cell. That it also governs the full compressed su(8) Hessian requires PH-EMB — that the cell exhausts the branch-relevant C_3-isotypic support (§2) — which is not proved here and whose negation is H13.

Theorem 2 (CKM curvature return)

Assume Theorem 1, together with committed role-isotype isotropy (PH-ISO; Prop. 1) and shared closure profile (PH-SAME), generation-depth readout excluding the occupied Cabibbo plane and the direct-target plane with no rescaling (PH-RDO), Hermitian-side minimal half-transport with target $e^{2\pi i/3}$ *(PH-HERM, PH-ROOT), and the* $+i$ *frame rotation (PH-ROT), with a gapped mixing block (PH-GAP). Then the projected role-even quark Hessian returns*

$$\zeta_{C_3} = (b/\sqrt{3}) \cdot e^{5\pi i/6},$$

and the convention-invariant leading CKM triangle residue is

$$\Delta_{13} = \tfrac{1}{2} \cdot a \cdot \zeta_{C_3}.$$

Proof. By Theorem 1 the branch weights are equal, so the common-mode budget splits democratically over three Killing-orthogonal branches. Under PH-ISO and PH-SAME, Proposition 1 fixes that total budget to the inherited committed 2–3 norm b^2 (the even common mode being the σ-even image of the same closure profile), hence each branch has norm $b^2/3$ and the surviving amplitude is $b/\sqrt{3}$ (§8). The readout excludes the occupied Cabibbo doorway (structural) and the direct-target branch (the non-question-begging choice), leaving 2–3 as the surviving branch (§9, PH-RDO). Hermitian-side minimal half-transport selects phase $\pi/3$ before the rotation, and the $+i$ rotation gives $\arg(\zeta) = 5\pi/6$ (§10, PH-HERM, PH-ROOT, PH-ROT). The exact commutator $[\Omega_0, \Omega_q]_{13} = -a\cdot\zeta$ (App. B) with the symmetric split-BCH identity at $\lambda = 1$ (IN-BCH) gives $\Delta_{13} = \frac{1}{2}\cdot a\cdot\zeta$. ■

19. Conclusion

The conditional C_3 theorem left five upstream returns owed. This manuscript supplies them in the minimal committed-quark Hessian cell by computing the second variation on the branch coordinates rather than assigning the desired curvature, and separates the result into two stages.

Theorem 1 establishes Hessian democracy from a single short calculation,

$$G_{C_3}^B = \kappa_{C_3}\cdot I_6 \Longrightarrow [K_{\text{even,pre}}^q, R_3] = 0 \Longrightarrow w_{12} = w_{23} = w_{31},$$

resting only on the cyclic Schur isotropy PH-CYC and the equivariance PH-EQV.

Theorem 2 converts that democracy into the CKM curvature. Proposition 1 reduces the amplitude to the inherited committed norm, $|\zeta| = b/\sqrt{3}$, under two transparent conditions — role-metric isotropy (PH-ISO) and shared closure profile (PH-SAME); the non-question-begging readout gives the 2–3 survivor; and the Hermitian minimal lift with the $+i$ rotation gives $\arg(\zeta) = 5\pi/6$. Therefore

$$\zeta_{C_3} = (b/\sqrt{3})\cdot e^{5\pi i/6}, \quad \Delta_{13} = \tfrac{1}{2}\cdot a\cdot\zeta_{C_3}.$$

The epistemic structure is now fully exposed and sharpened. Two audit conditions (PH-CYC, PH-EQV) and one exact algebraic close (the commutator and the split-BCH residue at $\lambda = 1$) carry the democracy; the scale rests on *two* conditional inputs, not one — role isotropy (PH-ISO, a statement about G_{role}) and closure-profile sharing (PH-SAME, a statement about the $\mathbb{C}^8$ factor), neither reducible to the other. The directional return rests on PH-RDO, of which only the 1–2 exclusion is structural. The remaining amplitude vulnerability is therefore the conjunction PH-ISO ∧ PH-SAME, two local, separately falsifiable exposures, while the whole minimal-cell argument rests on PH-EMB — that the cell exhausts the branch-relevant su(8) support (H13). The falsifiers H1–H13 are the negations of these premises.

The empirical confrontation of §14 closes the loop without overstating it. Combined with the inherited direct entry $c\cdot e^{i\varphi}$ and exponentiated to the unitary group, the returned curvature yields a full CKM triangle that reproduces the CP observables — β to ~1% and the Jarlskog invariant J to ~3% — with α at the lower lip of the measured band and $|V_{ub}|$ ~7% low, against a residual γ tension of ~8° measured against the conservative tree-level benchmark (§14.2). Crucially, the derived residue Δ_{13} is what carries the CP sector: removing it distorts every CP-sensitive entry.

The provenance audit of §15 then fixes how much of this may be claimed. The amplitude inputs are decently sourced — a inherited (and named as such), b and c structurally assembled from a and the closure factors D, η, χ — but the CP agreement rests almost entirely on one phase, $\arg \zeta = 5\pi/6$, whose independence is not yet secured. Two of the three phase legs — the half-transport target (PH-ROOT) and the principal-root selection (PH-HERM) — are established upstream by the Minimal-Hermitian-Lift theorem, but only on its side-choice premise ML-SIDE, which the predecessor paper itself flags as the claim owing justification from admissibility rather than CKM data. The decisive open item for the programme is therefore narrow and explicit: show that the upstream side-choice (ML-SIDE) and the +i frame rotation (PH-ROT) are each fixed by closure structure with no contact with J, $|V\_td|$, or the observed CP sign. Until then the honest paper-level statement is that the projected C_3 Hessian return reproduces the structure of quark mixing at the few-percent level *contingent on the phase firewall*, with a recorded residual deviation in γ, and with the Cabibbo and V_cb entries reproductions of measured inputs rather than predictions. The β and J agreement is the programme's most exposed claim, not its safest one — its value waits entirely on whether the side-choice and the rotation sign that fix $5\pi/6$ and the magnitude of J are forced or fitted.

Appendix A. Six-coordinate Hessian matrices

For $x = (s_{12}, a_{12}, s_{23}, a_{23}, s_{31}, a_{31})^T$ and the convention $A^{(2)} = ½\cdot x^T\cdot G\cdot x$, the three second-variation matrices are

$G\_C^B = \kappa\_C\cdot \mathrm{diag}(1, 1, 1, 1, 1, 1)$, $G\_T^B = \kappa\_T\cdot \mathrm{diag}(1, 1, 1, 1, 1, 1)$, $G\_R^B = \kappa\_R\cdot \mathrm{diag}(1, 1, 1, 1, 1, 1)$,

so

$G\_C_3\text{^}B = \mathrm{diag}(\kappa\_C_3, \kappa\_C_3, \kappa\_C_3, \kappa\_C_3, \kappa\_C_3, \kappa\_C_3)$.

The R_3 action in this ordered basis is the 6×6 permutation matrix

$$R_3\text{^}B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since $G\_C_3\text{^}B$ is scalar, $G\_C_3\text{^}B\cdot R_3\text{^}B - R_3\text{^}B\cdot G\_C_3\text{^}B = 0$.

Appendix B. Commutator check

With $\Omega_0 = \zeta\cdot E_{23} - \zeta^*\cdot E_{32}$ and

$$\Omega\_q = \begin{bmatrix} 0 & a & c\cdot e\text{^}(i\varphi) \\ -a & 0 & b \\ -c\cdot e\text{^}(-i\varphi) & -b & 0 \end{bmatrix},$$

row 1 of Ω_0 vanishes, so $(\Omega_0 \cdot \Omega_q)_{13} = 0$. The reverse product has the single contribution

$(\Omega_q \cdot \Omega_0)_{13} = (\Omega_q)_{12} \cdot (\Omega_0)_{23} = a \cdot \zeta$.

Therefore

$[\Omega_0, \Omega_q]_{13} = (\Omega_0 \cdot \Omega_q - \Omega_q \cdot \Omega_0)_{13} = -a \cdot \zeta$.

The leading BCH split contributes minus one half of this commutator in the convention-invariant curvature normalisation, giving $\Delta_{13} = \frac{1}{2} \cdot a \cdot \zeta$. Direct numerical evaluation at $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\varphi = 2\pi/3$, and $\zeta = \zeta_{C_3}$ confirms $[\Omega_0, \Omega_q]_{13} = -a \cdot \zeta_{C_3} = 0.0045563 - 0.0026306 \cdot i$ to all displayed digits.

Appendix C. Stability of the 1–3 BCH tail and the diagonal Cartan term

This appendix discharges the convergence clause of falsifier H11 and isolates the structural clause that remains live.

C.1 Convergence of the residue channel

The leading residue is $\Delta_{13} = \frac{1}{2} \cdot a \cdot \zeta_{C_3}$, with $|\Delta_{13}| = 0.0026306$. The converged value retains all higher BCH orders by forming the full unitary split product and reading its generator directly:

$V_{CKM} = \exp(-\Omega_0 + \Omega_q/2) \cdot \exp(+\Omega_0 + \Omega_q/2)$, $\Omega_0 = \zeta_{C_3} \cdot E_{23} - \zeta_{C_3}^* \cdot E_{32}$,

then extracting the residue beyond the inherited direct seed,

$\Delta_{13}^{converged} = (\log V_{CKM})_{13} - (\Omega_q)_{13}$.

Numerical evaluation at $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\varphi = 2\pi/3$ gives

$|\Delta_{13}^{converged}| = 0.0026332$, against $|\Delta_{13}^{leading}| = 0.0026306$,

a relative difference of 0.10%. The higher BCH orders do not bring the 1–3 residue channel near its own value, so the leading expression is stable and the convergence clause is discharged. (Note that the *full* (1,3) entry of $\log V_{CKM}$, including the inherited direct seed, has magnitude 0.0048910 — this is the generator-level quantity of §14.1 and is not the residue; the convergence statement concerns the residue beyond the seed.)

C.2 The diagonal Cartan term

The split product also generates a residual diagonal in log V_CKM. Its entries are

diag(log V_CKM) ≈ (-2×10^{-6}, $+4.7 \times 10^{-4}$, -4.7×10^{-4})·i,

traceless and purely imaginary — the signature of a Cartan (quark-rephasing) direction. Its magnitude is smaller than the residue but *not* negligible by hierarchy alone: the largest entry, 4.7×10^{-4}, is about 18% of $|\Delta_{13}| = 2.6 \times 10^{-3}$. Its harmlessness therefore cannot be argued from smallness. It rests on *structural removability* as a Cartan rephasing — a structural claim, not a convergence statement, and the live content of falsifier H11. The numerical size bounds any unremovable remainder but does not by itself establish removability, which is owed to the microscopic treatment.