The Quantum Gauge Completion of the VERSF Standard Model

▲ Programme Milestone — Quantum Gauge Consistency, Renormalisation and Observable-Reconstruction Series

Gate QGC-1 / Substrate Measure Descent, Faddeev–Popov–BRST–BV Closure, Anomaly Audit, One-Loop Running Census, Algebraic Renormalisation and Conditional Perturbative Unitarity

Keith Taylor VERSF Theoretical Physics Programme — Quantum Gauge Consistency, Renormalisation and Observable-Reconstruction Series Tier A conditional quantum-completion theorem paper — working manuscript

Primary predecessors: *The Gauge-Closure Selection Theorem in VERSF*; *The Substrate Anomaly-Inadmissibility Theorem in VERSF*; *From Substrate CAR Algebra to Relativistic Fermion Fields in VERSF*; *The Gauge Action and Coupling Theorem in VERSF*; *QCD Running and the Conditional Triality-Gap Confinement Criterion in VERSF*; *The Closure-Operator Yukawa Construction Theorem in VERSF*; *The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF*; *The Strong CP and Global θ-Sector Theorem in VERSF*; and the VERSF gauge, matter, closure-interface, flavour and faithful-global-group programmes.

General Reader Summary

The earlier VERSF papers assemble the visible architecture of the Standard Model: the three forces, the particles that feel them, the Higgs field that gives them mass, and the rules governing how the strengths of the forces change with energy.

But a blueprint is not yet a working quantum theory.

The obstacle is redundancy. A gauge field is like a landscape that can be drawn in many different map projections. The maps look different on paper, yet they all describe one and the same terrain. A quantum theory works by adding up possibilities — and if it naively adds up every map instead of every landscape, it counts the same physical situation endlessly many times and produces nonsense.

Removing the redundancy honestly creates four linked problems, and this paper works through all four.

First, the duplicate copies must be divided out. Doing this carefully produces a correction factor in every calculation. The famous "ghost" fields of particle physics are nothing more than a compact way of writing that correction factor down. They are bookkeeping entries, not new particles. This paper derives that status rather than assuming it: within VERSF, ghosts record how much redundant description is being discarded at each point, and they can never appear in a detector.

Second, a precise residue of the original symmetry must survive the surgery. That residue is called BRST symmetry. Its signature is a single compact rule,

$s^2 = 0$,

which says that applying the redundancy-check twice reveals nothing new. This rule sorts the enlarged mathematical state space into genuinely physical states and pure bookkeeping, and only the physical states carry probability. The paper builds this structure for the full Standard Model and proves the rule holds.

Third, quantum effects can break a classical symmetry. Such a breaking is called an anomaly. If an anomaly strikes a gauge symmetry, the construction collapses: probabilities stop adding up to one. The Standard Model's particle table has a remarkable property — every dangerous anomaly cancels, generation by generation, like a ledger whose entries sum exactly to zero. VERSF reads this as a deep statement: only a balanced source ledger can persist at all. And one anomaly deliberately survives. The axial anomaly is physical rather than dangerous; it is the quantum effect behind the strong-CP phase $\bar{\theta}$, one of the open puzzles the wider programme addresses. A correct quantum completion must cancel the dangerous anomalies while keeping this one, at exactly the right strength. This paper shows the inherited VERSF particle census does precisely that.

Fourth, quantum corrections make the strengths of the forces drift with energy. The paper computes this drift from the derived VERSF field content and finds exactly the known Standard Model behaviour — the strong force weakening at high energy, hypercharge strengthening — with the textbook coefficients 41/6, −19/6 and −7 emerging from the particle census rather than being inserted by hand, valid in the energy window where no additional VERSF carrier is light. The starting values of the couplings remain what the earlier gauge-action paper derived; this paper transports them across energy scales.

Two further results complete the quantum picture.

The renormalisable Standard Model core is shown to close on its existing operator basis: every infinity it generates lands in a slot already present. Higher-dimensional VERSF corrections close order by order once the complete admissible operator basis at that order has been supplied — a census the programme still owes (Debt 18).

And under stated conditions, the surviving physical states scatter with total probability one — the theory is unitary. The paper is careful about what "scattering" can mean: under the separate VERSF confinement gate quarks and gluons are not admitted as isolated asymptotic states, charged particles drag clouds of soft photons, and the W, Z, Higgs and top quark are fleeting resonances rather than stable arrivals. Honest predictions are framed in the quantities experiments actually measure.

The paper is equally explicit about what it does not do. All of its results are local and perturbative. The global construction — controlling the notorious Gribov copies, defining the chiral measure across all topological sectors, and proving positivity beyond perturbation theory — is catalogued as a precise list of open debts rather than quietly assumed.

The strongest honest result is therefore:

Conditional on a gauge-basic substrate measure, an admissible regulator, the inherited anomaly-free representation census, local causal field emergence, BRST-positive physical cohomology and the stated infrared conditions, the VERSF Standard Model possesses a gauge-independent, perturbatively renormalisable and perturbatively unitary quantum completion whose one-loop flow is the Standard Model flow and whose axial anomaly has the required topological form.

The remaining nonperturbative target is equally exact:

Construct the global quotient-compatible chiral measure, control Gribov copies and determinant-line topology, prove physical positivity and causal reconstruction beyond perturbation theory, and recover the same quantum identities without relying on a local gauge-orbit patch.

In one sentence:

VERSF becomes a quantum Standard Model when its substrate measure descends to gauge-orbit space, its local quotient is represented by the Faddeev–Popov–BRST/BV complex, its gauge-anomaly class vanishes while the physical axial anomaly survives, its divergences close on the declared operator ledger, and its BRST cohomology supports gauge-independent unitary observables.

Scope and Closure Box

Claim	QGC-1 status
Local Standard Model gauge algebra and matter representations	Inherited
Faithful $\mathbb{Z}_6$ quotient	Inherited conditionally
Yang–Mills action and gauge-coupling boundary operators	Inherited conditionally from GAC-1
Substrate CAR/Fock construction	Inherited conditionally

Claim	QGC-1 status
Complete light-cone microcausality	Open inherited premise
Substrate measure pushes forward to continuum fields	New microscopic premise/target
Gauge-equivalent continuum fields represent one physical orbit	Inherited gauge interpretation
Orbit descent of the bosonic measure and absolute fermion determinant	Exact under measure premises
Equivariant trivialisation of the anomaly-line phase	Curvature obstruction removed by the local audit; holonomies open
Local orbit quotient produces Faddeev–Popov determinant	Exact under local-slice hypotheses
Ghost fields represent the orbit Jacobian	Exact
Ghosts are physical substrate particles	Rejected
Standard Model BRST differential is nilpotent	Exact
Classical BV master equation	Exact for the closed inherited gauge algebra
Gauge-fixed action differs by a BRST-exact term	Exact
Slavnov–Taylor identity at tree level	Exact
All-order Slavnov–Taylor restoration	Conditional on regulator locality and vanishing gauge-anomaly class
Perturbative local gauge anomalies cancel	Exact from inherited representation census
Ordinary global SU(2) sign anomaly is absent	Exact for the inherited doublet census
Mixed gravitational–U(1) census row	Conditional on the emergent-geometry background premise (P25)
Full faithful-quotient global anomaly classification	Open
Singlet axial anomaly survives	Standard quantum result; substrate-native derivation open
Strong-phase sector-weight invariance	Derived structurally
Positive trivialisation giving $\bar{\theta} = 0$	Not supplied by this paper
One-loop Standard Model gauge beta coefficients	Exact given field census within the spectrum window (P26)
Numerical gauge-coupling boundary values	Open in GAC-1
Renormalisable Standard Model counterterm closure	Conditional all-order theorem
Higher-dimensional VERSF operators	EFT-ordered, not discarded
Gauge-parameter independence of physical observables	Exact perturbatively under BRST assumptions
Positivity of the full gauge-fixed Euclidean integrand	Rejected

Claim	QGC-1 status
Positive physical BRST cohomology	Conditional
Perturbative unitarity on physical states	Conditional theorem
Naive S-matrix for all elementary fields	Rejected
Infrared-safe physical observable reconstruction	Conditional
Global Gribov-free gauge slice	Open
Nonperturbative chiral gauge measure	Open
Constructive four-dimensional quantum field theory	Not established
Empirical confirmation	Not established

Abstract

The VERSF Standard Model programme derives a local gauge algebra

$$\mathfrak{g}_{SM} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y,$$

a conditionally faithful global group

$$G_{SM}^{faithful} = [SU(3)_C \times SU(2)_L \times U(1)_Y] / \mathbb{Z}_6,$$

the associated connection and curvature carriers, an auxiliary-reduced Yang–Mills kinetic operator, chiral matter and scalar representations, closure-derived Yukawa operators, and a substrate interpretation of gauge-anomaly cancellation as persistent source-ledger closure. These results do not by themselves provide a quantum gauge theory. Gauge-orbit overcounting, the regulated measure, ghost determinants, quantum gauge identities, anomaly descent, loop renormalisation, physical-state positivity and observable unitarity remain a distinct completion gate.

This paper establishes the Quantum Gauge Completion Theorem at perturbative theorem-architecture level.

Let Ω_{sub} be the VERSF history space with primitive measure μ_{sub}, and let

$$\mathfrak{Q} : \Omega_{sub} \longrightarrow \mathscr{F}_{SM}$$

be the declared coarse-graining map into Standard Model continuum fields

$$\varphi = \{A_\mu^A, \Phi, \Psi, \bar{\Psi}\}.$$

Assume that the pushforward

$$\mu\_cont = \mathfrak{Q}\_* \mu\_sub$$

is local or quasi-local in the declared Wilsonian sense, constant on gauge orbits in its absolute part — the chiral determinant phase is typed as a determinant-line section whose descent is derived, not assumed — and compatible with the inherited action. In a perturbative neighbourhood where the gauge action is free after stabilisers and zero modes are treated, choose a local gauge condition $F^A[\varphi] = 0$. The quotient integral then admits the exact local representation

$$\int_{\mathcal{F}\_SM/\mathcal{G}} d\mu\_phys \cdot \mathcal{O} \cdot e^{-S\_E} = (1/\mathrm{Vol}\,\mathcal{G}) \int_{\mathcal{F}\_SM} d\mu\_cont \cdot \delta[F] \cdot \det'\mathcal{M}\_FP \cdot \mathcal{O} \cdot e^{-S\_E},$$

where

$$\mathcal{M}\_FP^{AB} = \delta F^A[\varphi^g] / \delta\alpha^B |_{\alpha=0}.$$

The determinant has the Berezin representation

$$\det'\mathcal{M}\_FP = \int D\bar{c}\, Dc \cdot \exp[-\int \bar{c}\_A\, \mathcal{M}\_FP^{AB}\, c\_B].$$

The ghost fields are therefore a representation of the orbit-volume Jacobian and carry no independent physical substrate spectrum.

For the inherited closed gauge algebra define

$$s A\_\mu^A = (D\_\mu c)^A, \quad s c^A = -\tfrac{1}{2} f^A\_{BC} c^B c^C,$$

$$s \bar{c}^A = B^A, \quad s B^A = 0,$$

$$s \Psi = +i c^A T\_A \Psi, \quad s \Phi = +i c^A t\_A \Phi.$$

The Jacobi identity and representation homomorphism imply

$$s^2 = 0.$$

For gauge-fixing fermion

$$\Psi\_gf = \int d^4x \cdot \bar{c}\_A (F^A - (\xi\_A/2) B^A),$$

the complete gauge-fixing and ghost action is

$$S\_gf+gh = s \Psi\_gf.$$

Introducing antifields φ^*_I, the minimal BV action

$$S\_BV = S\_cl + \int \varphi^*\_I\, s\, \varphi^I$$

satisfies

$$(S_{BV}, S_{BV}) = 0.$$

After regularisation, a quantum breaking of the corresponding master or Slavnov–Taylor equation is a ghost-number-one local cocycle. If the regulated breaking has vanishing class in $H^1(s|d)$ — the census condition established below — it is BRST-exact and removable by a local counterterm. Induction over loop order yields a renormalised effective action obeying

$$\mathcal{S}(\Gamma_R) = 0$$

to all perturbative orders. This is the algebraic quantum-completion theorem.

The inherited Standard Model representation census passes the local anomaly audit generation by generation. In left-handed Weyl notation,

$$Q_L : (3, 2)_{1/6},\ u_L^c : (\bar{3}, 1)_{-2/3},\ d_L^c : (\bar{3}, 1)_{1/3},$$

$$L_L : (1, 2)_{-1/2},\ e_L^c : (1, 1)_1,$$

with optional

$$\nu_L^c : (1, 1)_0.$$

The $SU(3)^3$, $SU(3)^2U(1)$, $SU(2)^2U(1)$, $U(1)^3$ and mixed gravitational–U(1) anomaly coefficients vanish. The weak-doublet multiplicity is $3 + 1 = 4$ per generation, so the ordinary global SU(2) sign obstruction is absent. Full global anomaly freedom for every bundle of the faithful quotient remains a separate determinant-line and bordism audit.

The singlet axial current is not a gauge redundancy. Its regulated fermion measure therefore retains the Fujikawa Jacobian

$$\mathcal{D}\Psi\, \mathcal{D}\bar{\Psi} \mapsto \mathcal{D}\Psi\, \mathcal{D}\bar{\Psi} \cdot \exp\left[-i\, \delta\alpha \cdot (g_s^2/16\pi^2) \int G^a_{\mu\nu}\, \tilde{G}^{a\mu\nu} + \cdots \right],$$

with the coefficient — including the representation and flavour multiplicity suppressed in this schematic form — fixed by the index density (§20). The quantum completion consequently preserves the strong-phase invariant

$$\bar{\theta} = \theta_3 + \arg\det(M_u M_d).$$

The topological character line and quark determinant line combine into one anomaly-covariant physical line whose holonomy is $e^{i\bar{\theta}}$. QGC-1 derives the covariance and Jacobian; it does not assume the positive trivialisation required to set the holonomy to unity.

In background-field gauge the one-loop effective action is

$$\Gamma_E^{(1)} = \tfrac{1}{2}\,\mathrm{STr}\log \mathbb{H}_{gf},$$

where the supertrace includes gauge, ghost, fermion and scalar fluctuations with their statistics. The logarithmic curvature-square terms reproduce the Standard Model one-loop coefficients, in the convention

$$\mu\, dg_A/d\mu = (b_A/16\pi^2) \cdot g_A^3 + \cdots :$$

$$\mathbf{(b_Y, b_2, b_3) = (41/6, -19/6, -7).}$$

With the alternative grand-unified definition $g_1 = \sqrt{5/3} \cdot g'$, the first coefficient is $b_1 = 41/10$. No grand-unified normalisation is used elsewhere unless independently derived.

The background Ward identity gives

$$Z_{g_A} \cdot Z_{\bar{A}_A}^{1/2} = 1,$$

so the scale dependence of the canonically normalised coupling is read directly from the background two-point function. The values $g_s(\mu\star)$, $g(\mu\star)$, $g'(\mu\star)$ remain the VERSF response-boundary data of GAC-1; the quantum completion provides their loop transport, threshold matching and scheme conversion.

The all-order counterterm classification follows from BRST cohomology. At fixed effective order,

$$\Gamma_R = S_{SM}^{(4)} + s\,\Psi_R + \Sigma_{d>4,\, i} \left[C_i^{(d)}(\mu) / M\star^{d-4} \right] \cdot \mathcal{O}_i^{(d)}.$$

Ghost-number-zero nontrivial cocycles renormalise gauge-invariant physical operators; BRST-exact cocycles change gauge fixing, field coordinates or unphysical normalisations. The Standard Model renormalisable core is perturbatively closed, while the higher-dimensional closure and bath operators form an explicitly ordered EFT ledger.

Finally, let Q_B be the asymptotic BRST charge. If

$$Q_B^2 = 0,\ Q_B^\dagger = Q_B,\ [Q_B, S] = 0,$$

and the cohomology

$$\mathscr{H}_{phys} = \ker Q_B / \mathrm{im}\, Q_B$$

has positive norm, then S descends to a unitary operator on $\mathscr{H}_{phys}$. The optical theorem follows on physical states. This completion is perturbative and local in orbit space. It does not prove a global Gribov-free BRST construction, a nonperturbative chiral measure, constructive four-dimensional existence, or a naive elementary-particle S-matrix in confining and infrared-singular sectors.

QGC-1 therefore closes the local orbit quotient, ghost typing, BRST/BV algebra, perturbative anomaly audit, one-loop gauge-flow census, all-order algebraic-renormalisation criterion and conditional physical-state unitarity. It registers the global quotient measure, Gribov problem, determinant-line topology, complete microcausality, nonperturbative positivity and physical infrared reconstruction as ordered open debts.

Contents

Part I — Foundations

Part II — From the Substrate Measure to Gauge-Orbit Space

Part III — BRST and BV Quantum Closure

Part IV — Quantum Anomaly Closure

Part V — One-Loop Effective Action and Running

Part VI — All-Order Renormalisation

Part VII — Physical States, Unitarity and Observables

Part VIII — Global and Nonperturbative Completion

Part IX — Closure

Part I — Foundations

1. Notation, Conventions and Firewalls

1.1 Gauge group and algebra

The inherited local gauge algebra is

$$\mathfrak{g}_{SM} = \mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y.$$

The conditionally inherited faithful global group is

$$G_{SM}^{faithful} = [SU(3)_C \times SU(2)_L \times U(1)_Y] / \mathbb{Z}_6.$$

Local adjoint indices are denoted A, B, C. Factorwise indices are

$$a, b, c = 1, \ldots, 8$$

for colour,

$$i, j, k = 1, 2, 3$$

for weak isospin, and the Abelian index is denoted Y.

The generators are Hermitian:

$$[T_A, T_B] = i f_{AB}{}^{C} T_C.$$

For the simple factors,

$$\mathrm{tr}(T_A T_B) = \tfrac{1}{2} \delta_{AB}$$

in the fundamental convention.

In this plain-text edition, sub- and superscripts are attached with the underscore and caret where Unicode glyphs are unavailable; braces group multi-symbol scripts. Overbars denote conjugate or background objects ($\bar{c}$, $\bar{\Psi}$, $\bar{A}$), tildes denote duals ($\tilde{F}$, $\tilde{G}$), and primes on determinants ($\det'$) denote removal of zero modes.

1.2 Field ledger

The continuum field ledger is

$$\varphi^I = \{ A_\mu^A, \Phi, \Psi, \bar{\Psi}, c^A, \bar{c}^A, B^A \}.$$

Here:

- A_μ^A are gauge connections;
- Φ is the closure-interface scalar carrier;
- Ψ denotes all chiral and completed fermion carriers;
- c^A, $\bar{c}^A$ are ghost and antighost fields;
- B^A is the Nakanishi–Lautrup auxiliary field.

BV antifields are denoted

$$\varphi^*_I.$$

Ghost numbers are

$$\mathrm{gh}(c) = 1,\ \mathrm{gh}(\bar{c}) = -1,\ \mathrm{gh}(B) = 0,$$

$$\mathrm{gh}(\varphi^*_I) = -1 - \mathrm{gh}(\varphi^I).$$

1.3 Gauge redundancy versus physical symmetry firewall

A gauge transformation changes the local description of an internal frame. It is a redundancy of the continuum representation, not a physical operation relating distinct observable states.

A global physical symmetry can act nontrivially on states and charges. A gauge redundancy must act trivially on physical observables.

The two may not be interchanged.

1.4 Ghost-versus-particle firewall

The ghost fields represent the Faddeev–Popov determinant. They are not additional physical bath sectors, particles, energy carriers or asymptotic states.

Their internal loop contributions are required because gauge-orbit overcounting is field dependent.

1.5 Local-versus-global gauge firewall

BRST transformations depend on the local Lie algebra. They do not determine:

- the faithful global quotient;
- admissible principal bundles;
- line operators;
- monopole sectors;
- topological charge lattices;
- discrete θ-terms;
- global anomalies.

Those require the full group and bundle lift.

1.6 Perturbative-versus-nonperturbative firewall

The Faddeev–Popov construction is exact inside a gauge-orbit patch satisfying its slice hypotheses. It is not automatically a global quotient, because non-Abelian gauge conditions can intersect one orbit more than once.

Perturbative BRST closure is not a proof of a global Gribov-free construction.

1.7 Gauge-anomaly versus axial-anomaly firewall

A gauge anomaly breaks the redundancy required to define the theory and must vanish.

A global axial anomaly may be physical. Its survival is required for:

- the QCD index structure;
- instanton-induced chiral selection;
- phase transfer between θ_3 and the quark determinant;
- the physical definition of $\bar{\theta}$.

The quantum completion must cancel one and retain the other.

1.8 Gauge-fixed measure versus primitive positivity firewall

The gauge-fixed functional integral contains:

- Grassmann ghost fields;
- chiral fermion determinants;
- phase-carrying topological sectors.

Its integrand is not generally a nonnegative probability density.

Any primitive-sector positivity premise used by the strong-CP programme must be established before or independently of gauge fixing. It cannot be inferred from the ghost-completed Euclidean integrand.

1.9 Off-shell versus physical firewall

The gauge-fixed effective action, elementary Green functions and effective potential away from stationary points may depend on the gauge parameter.

Physical pole locations, properly defined extrema, S-matrix elements and gauge-invariant observables must not.

1.10 Renormalisable-core versus complete-VERSF firewall

The dimension-four Standard Model action is perturbatively renormalisable.

If the VERSF coarse-graining generates dimension-six and higher operators, the full theory is an effective field theory ordered in powers of the substrate or completion scale. It is not legitimate to describe the complete higher-operator theory as finite-parameter renormalisable.

1.11 Parton-versus-asymptotic-state firewall

Quarks and gluons can appear in short-distance amplitudes and factorisation formulae. They are not isolated physical asymptotic states in the confining theory.

A partonic amplitude is not by itself a physical S-matrix element.

1.12 Infrared firewall

In theories with massless photons, scattering between undressed charged Fock states has infrared divergences. Physical predictions require:

- inclusive probabilities;
- coherent or dressed asymptotic states;
- or another explicitly declared infrared prescription.

1.13 Unstable-particle firewall

The W, Z, Higgs boson and top quark are unstable. They are described through resonance poles, spectral functions and decay amplitudes, not as exact incoming or outgoing asymptotic states.

1.14 Microcausality firewall

The VERSF free-fermion bridge establishes operator-valued fields conditionally but has separately ledgered complete light-cone-respecting microcausality as open.

The use of LSZ, causal perturbation theory and standard all-order analyticity is therefore conditional on the emergent causal-field gate.

1.15 Regulator firewall

No regulator may be called admissible merely because it makes an integral finite.

An admissible regulator must preserve or permit restoration of:

- locality or declared quasi-locality;
- gauge or BRST identities;
- the chiral representation ledger;
- the correct anomaly;
- the faithful quotient where globally relevant;
- unitarity and physical-state positivity after removal.

1.16 γ_5 firewall

Chiral gauge theories require particular care in dimensional continuation. A formal use of an anticommuting four-dimensional γ_5 inside arbitrary d-dimensional loop traces is not a complete regulator prescription.

Any symmetry breaking generated by the chosen scheme must be classified and removed by allowed counterterms, except for genuine anomalies.

1.17 Numerical non-insertion firewall

No measured coupling, beta coefficient, pole mass, mixing angle, decay rate, anomaly coefficient or cross section may be used to select:

- the substrate measure;
- the regulator;
- the gauge-fixing functional;
- the counterterm branch;
- the determinant-line trivialisation;
- the matching scale;

- the hidden field census.

1.18 Signature and Wick-rotation convention

Measure-theoretic and heat-kernel statements (Parts II and V) are written in Euclidean signature with weight e^{-S_E}. Operator, BV and scattering statements (Parts III, VI and VII) are written in Lorentzian signature with weight e^{iS}; the quantum master equation is quoted in its Lorentzian form $\tfrac{1}{2}(W, W) - i\hbar\, \Delta_{reg} W = 0$. Wick rotation between the two is applied patchwise, and its analytic admissibility is part of P1 and P23. One deliberate exception to the Part-III assignment: the gauge-fixing sign derivation of §13 is performed in Euclidean form, because its output feeds the Euclidean fluctuation determinants of Part V; the Lorentzian gauge-fixed action is obtained from it by the declared rotation. No statement in this paper depends on rotating a quantity whose analytic continuation is unestablished.

---

2. Predictive-Content Ledger

Object	Grade	Status
Local gauge algebra	Inherited	Gauge-census programme
Faithful global quotient	Inherited conditional	Gauge-closure programme
Gauge kinetic action and Z_A response operators	Inherited conditional	GAC-1
Fermion Fock carriers	Inherited conditional	Matter programme
Closure-derived Yukawa operators	Inherited conditional	CMSY-1
Substrate-to-continuum measure map	Defined target	Microscopic derivation open
Gauge-basic pushforward measure	Conditional premise	P4–5
Local orbit disintegration	Exact	Theorem 1
Faddeev–Popov determinant	Exact in a valid local slice	Theorem 2
Ghost determinant representation	Exact	Theorem 3
BRST nilpotency	Exact	Theorem 4
Gauge-fixing action $s\Psi_{gf}$	Exact	Theorem 5
Classical BV master equation	Exact	Theorem 6
Tree-level Slavnov–Taylor identity	Exact	Theorem 7
Gauge-fixing independence	Exact given order-by-order ST restoration	Theorems 8, 18
Nielsen identity	Exact conditional identity	Theorem 9
Local gauge-anomaly cancellation	Exact from census	Theorem 11

Object	Grade	Status
Standard Witten SU(2) audit	Exact from census	Theorem 12
Full quotient global anomaly freedom	Open	Debt 12
Axial Jacobian	Standard quantum result	Theorem 13
Strong-phase sector-weight invariance	Derived	Theorem 14
One-loop gauge beta coefficients	Exact given census within spectrum window (P26)	Theorem 15
Coupling boundary values	Open numerically	GAC-1 debt
All-order ST restoration	Conditional	Theorem 18
Renormalisable-core counterterm closure	Conditional	Theorem 19
EFT operator closure	Conditional order by order	Theorem 20
Positive BRST physical cohomology	Conditional premise	P20
Perturbative physical-state unitarity	Conditional	Theorem 24
Global Gribov-free quantisation	Open	Debt 20
Nonperturbative chiral measure	Open	Debt 21
Complete physical S-matrix	Conditional and sector-specific	Open completion
Empirical agreement	Not established	Blind comparison required

3. Inherited VERSF Infrastructure

3.1 Gauge connection and curvature

The connection is inherited as the local comparison rule required to transport internal distinctions across the emergent geometric structure. Its curvature is the persistent holonomy residual around infinitesimal loops.

3.2 Gauge action and couplings

GAC-1 identifies the unbroken gauge kinetic action as

$$S_{gauge} = -\tfrac{1}{4} \int d^4x \, [Z_3 G^a_{\mu\nu} G^{a\mu\nu} + Z_2 W^i_{\mu\nu} W^{i\mu\nu} + Z_1 B_{\mu\nu} B^{\mu\nu}] + \cdots$$

and gives

$$g_s = Z_3^{-1/2}, \; g = Z_2^{-1/2}, \; g' = Z_1^{-1/2}$$

in a fixed generator convention.

QGC-1 does not replace those boundary values. It supplies their quantum evolution and consistency.

3.3 Chiral matter ledger

The inherited one-generation left-handed Weyl ledger is

$Q_L : (3, 2)_{1/6}$, $u_L^c : (\bar{3}, 1)_{-2/3}$, $d_L^c : (\bar{3}, 1)_{1/3}$,

$L_L : (1, 2)_{-1/2}$, $e_L^c : (1, 1)_1$,

with optional sterile

$\nu_L^c : (1, 1)_0$.

Three-generation replication is inherited conditionally from the species and generation census.

3.4 Substrate anomaly inadmissibility

A nontrivial gauge-anomaly class is interpreted as a persistent source-ledger leak. The earlier theorem supplies the VERSF admissibility principle:

$[\Omega_{gauge}] \neq 0 \Longrightarrow$ no admissible persistent source sector.

QGC-1 embeds that principle into BRST and BV cohomology.

3.5 Fermion and scalar interactions

The fermionic kinetic carriers and closure-derived Yukawa operators are inherited in canonical form:

$Y_f^c = \mathcal{E}_{fL}^\dagger \, \mathcal{C}_Y \, \mathcal{E}_{fR}$.

The scalar and Yukawa values remain boundary data for the quantum RG problem.

3.6 Strong-CP architecture

The physical invariant is inherited as

$\bar{\theta} = \theta_3 + \arg\det(M_u M_d)$.

The strong-CP gate requires a correct fermion-measure Jacobian and RG/threshold stability. QGC-1 supplies their quantum typing.

3.7 QCD running and confinement

The QCD running paper inherits the background-field coefficient and gauge–ghost determinant as an open quantum gate. QGC-1 supplies that gate.

Confinement remains a nonperturbative physical-state problem and is not replaced by BRST perturbation theory.

---

4. Purpose and Claim Levels

QGC-1 addresses nine questions.

1. How does the substrate measure become a continuum quantum measure?
2. How is gauge-orbit overcounting removed?
3. Why do ghost fields appear?
4. What algebra replaces gauge invariance after gauge fixing?
5. Which anomalies must vanish and which must survive?
6. How are the universal loop coefficients recovered?
7. Why do divergences remain inside a controlled operator ledger?
8. How are physical positive-norm states selected?
9. Under what conditions is the observable theory unitary and gauge independent?

The paper separates three levels.

Level P — Perturbative local quantum completion

This includes:

- local gauge slicing;
- the Faddeev–Popov determinant;
- the BRST/BV algebra;
- local anomaly cancellation;
- Slavnov–Taylor identities;
- algebraic renormalisation;
- one-loop beta coefficients;
- perturbative physical-state unitarity.

This level is the main theorem of the paper.

Level G — Global nonperturbative gauge completion

This requires:

- quotient-compatible bundle sums;
- complete global-anomaly cancellation;
- determinant-line trivialisation;
- Gribov control;
- a nonperturbative chiral measure;
- regulator removal;
- physical positivity and reconstruction.

This level remains open.

Level O — Observable completion

This requires:

- infrared-safe asymptotic states;
- confinement-compatible QCD observables;
- dressed or inclusive QED observables;
- unstable-particle resonance treatment;
- threshold and scheme matching;
- blind comparison with experiment.

This level is conditional and sector dependent.

---

5. Named Premises

Premises are labelled P1–P26. The label QGC-1 refers throughout to this gate paper, never to a premise.

P1 — Emergent local causal layer. The VERSF coarse-grained record layer supports local or controlled quasi-local relativistic fields, causal propagators and the analytic structure required for perturbative renormalisation.

P2 — Gauge-structure inheritance. The local gauge algebra, faithful global quotient and representation ledger are those inherited from the VERSF gauge and matter programmes.

P3 — Classical action inheritance. The declared classical action contains the derived gauge, matter, scalar, Yukawa and topological operators with independently fixed normalisations.

P4 — Measure pushforward. There exists a coarse-graining map

$\mathfrak{Q} : \Omega\_sub \rightarrow \mathscr{F}\_SM$

whose pushforward measure is well defined on the retained continuum histories.

P5 — Gauge-basic absolute measure. The continuum action, the bosonic measure, the absolute value of the regulated fermion determinant and physical observables are constant along gauge orbits. The phase of the chiral determinant is exempted: it is typed as a determinant-line section whose descent is derived by the anomaly audit (Definition 6.2), not declared.

P6 — Local regular orbit patch. In every perturbative neighbourhood used for loop expansion, stabilisers and exact zero modes are separately treated, the gauge action is free and proper on the retained configurations, the regulated measure admits disintegration along the gauge orbits, and a valid local gauge slice exists.

P7 — Regulator admissibility. A regulator exists that preserves the background gauge structure or permits all non-anomalous symmetry breakings to be removed by local counterterms.

P8 — Closed irreducible gauge algebra. The retained Standard Model gauge transformations close under the inherited Lie bracket with no untyped open-algebra terms.

P9 — Quantum action principle for the regulated measure. After gauge fixing, any breaking of the classical master or Slavnov–Taylor identity produced by the regulated measure is, at each loop order, an integrated local insertion of bounded canonical dimension and definite ghost number. The validity of the quantum master equation is not assumed; it is the output of the anomaly-class analysis (Theorems 10 and 18).

P10 — Frozen chiral census. The complete light chiral representation ledger is the inherited Standard Model census of §3.3, with no uncounted charged chiral carriers. Local gauge-anomaly cancellation is not assumed: it is derived from this census in Theorem 11.

P11 — Global anomaly compatibility. For the global completion claim, the fermion determinant line is consistently defined on every admissible faithful-group bundle and has no uncancelled global gauge anomaly.

P12 — Axial-Jacobian inheritance. The regulated chiral fermion measure reproduces the standard index-density Jacobian for physical axial transformations.

P13 — Background-field closure. A background-field or equivalent gauge-consistent scheme exists for extracting running gauge operators.

P14 — Wilsonian locality and power counting. At every declared effective order, divergences are local and organised by canonical dimension or the derived VERSF derivative hierarchy.

P15 — Counterterm completeness. Every local BRST-invariant divergence has a representative in the declared renormalisable or EFT operator ledger. For the dimension-≤4 core this is discharged by the finite enumerated basis of §31 — the complete set of gauge-invariant operators of dimension at most four on the inherited representations — so the premise carries independent weight only for the higher-dimensional tower, whose census is Debt 18.

P16 — Threshold consistency. Integrating out heavy fields preserves the low-energy gauge identities through matching operators, including Wess–Zumino terms where anomaly bookkeeping requires them.

P17 — Global bundle sum. For the global theorem, the functional integral sums over the correct quotient-compatible bundle and topological sectors.

P18 — Gribov control. For the global theorem, gauge-orbit copies are handled by a derived modular region, gauge-invariant formulation or other nonperturbative construction.

P19 — Asymptotic BRST charge. In sectors admitting asymptotic scattering, the BRST current defines a conserved nilpotent charge Q_B.

P20 — Positive physical cohomology. The cohomology of Q_B carries a positive inner product after null and exact vectors are quotiented.

P21 — Infrared-admissible observables. Scattering and decay observables are defined using gauge-invariant, dressed, inclusive or factorised constructions appropriate to each sector.

P22 — No hidden anomaly reservoir. No uncounted chiral sector is silently used to restore gauge consistency.

P23 — Causal reconstruction. The required microcausality, clustering and spectral conditions hold in the domain where LSZ or equivalent observable reconstruction is used.

P24 — Numerical non-insertion. No observed quantum coefficient or amplitude is used to choose upstream measure, regulator, branch or counterterm data.

P25 — Emergent-geometry background participation. For the mixed gravitational–U(1) row of the anomaly census, the emergent 4-manifold sector couples as a gauged background in the declared sense: the coarse-grained record layer supplies a background metric whose diffeomorphism and local-Lorentz redundancies are identifications rather than physical operations, so that mixed gauge–gravitational anomaly freedom is load-bearing for consistency. This premise is conditional on the emergent-geometry gate and is used only for that census row.

P26 — Running-spectrum window. Between the matching scale and the scale at which the one-loop census is applied, the propagating charged spectrum is exactly the three-generation Standard Model census with one Higgs doublet: no additional vectorlike fermion, charged scalar, bath excitation or completion carrier is light in the window. Any additional light carrier changes

(b_Y, b_2, b_3) through the universal formula and must enter through the declared threshold matching of §28.

Part II — From the Substrate Measure to Gauge-Orbit Space

6. The Substrate-to-Continuum Measure Pushforward

Let

$(\Omega_{sub}, \Sigma_{sub}, \mu_{sub})$

be the regulated VERSF history space.

The continuum reconstruction map is

$\mathfrak{Q} : \Omega_{sub} \longrightarrow \mathscr{F}_{SM},$

where $\mathscr{F}_{SM}$ is the field configuration space of the retained gauge, matter and scalar carriers.

The pushforward is defined by

$\mu_{cont}(E) = \mu_{sub}(\mathfrak{Q}^{-1}(E))$

for measurable $E \subset \mathscr{F}_{SM}$. Equivalently,

$\mu_{cont} = \mathfrak{Q}_* \mu_{sub}.$

This statement does not assume that μ_{cont} has a flat formal density. Where a density exists relative to a reference measure $D\varphi$, write

$d\mu_{cont}[\varphi] = \mathcal{J}_{sub\to cont}[\varphi] \cdot D\varphi.$

The functional

$\mathcal{J}_{sub\to cont}$

is a derived Jacobian. It may contribute local counterterms, anomalies or higher-order operators. It cannot be set to one by convention unless the microscopic map proves it.

Definition 6.1 — Gauge-basic pushforward

The pushforward is gauge basic when

$$d\mu\_cont[\varphi^g] = d\mu\_cont[\varphi]$$

and

$$S[\varphi^g] = S[\varphi]$$

for every admissible gauge transformation connected to the identity. The chiral determinant phase is treated separately in Definition 6.2.

A gauge-basic absolute measure descends to the orbit space.

Definition 6.2 — Absolute/phase split of the fermionic measure

After integration over the Grassmann carriers at fixed bosonic background, the regulated effective measure factorises as

$$d\mu\_cont = d\mu\_bos \cdot |\mathrm{Det}\ D[\varphi]| \cdot e^{i\vartheta[\varphi]},$$

where $d\mu\_bos$ is the bosonic part, $|\mathrm{Det}\ D|$ is the absolute value of the regulated chiral fermion determinant, and $e^{i\vartheta}$ is its phase. Write

$$d\mu\_cont^{abs} \equiv d\mu\_bos \cdot |\mathrm{Det}\ D[\varphi]|$$

for the absolute part. Under P5 the absolute part is exactly constant on gauge orbits. The phase is not automatically a function that descends: it is a section of the anomaly (determinant) line, naturally formulated gauge-equivariantly over $\mathscr{F}\_reg$ — equivalently over the quotient stack $[\mathscr{F}\_reg/\mathcal{G}]$ (§42). The local anomaly is the curvature, i.e. the infinitesimal gauge variation, of the anomaly-line connection; the global anomaly is its holonomy. The phase descends when the line admits a gauge-equivariant trivialisation compatible with its connection: vanishing of the local gauge-anomaly class removes the curvature obstruction (Theorems 10–13), and the global audit tests the remaining holonomies (Debt 12).

This split removes a circularity that would otherwise sit between Part II and Part IV: gauge-basicity is assumed only for the absolute part of the measure, and the descent of the phase — the object whose consistency the anomaly audit exists to establish — is derived downstream, never declared upstream.

Theorem 1 — Substrate Measure Descent (absolute part)

Assume P4, P5 and P6. Then the absolute measure $d\mu\_bos \cdot |\mathrm{Det}\, D|$ descends: there exists a physical measure $d\mu\_phys$ on the regular quotient

$\mathscr{F}\_reg / \mathcal{G}$

such that, locally,

$d\mu\_cont\hat{}abs = d\mu\_orbit \cdot d\mu\_phys$.

Physical expectation values of gauge-invariant $\mathcal{O}$ depend only on the quotient data:

$\langle \mathcal{O} \rangle = [\ \int\{\mathscr{F}\_reg/\mathcal{G}\}\ d\mu\_phys \cdot \mathcal{O} \cdot e\hat{}\{-S\_E\}\ ] \,/\, [\ \int\{\mathscr{F}\_reg/\mathcal{G}\}\ d\mu\_phys \cdot e\hat{}\{-S\_E\}\ ]$.

As stated, this is an absolute-measure identity. The phase factor descends when the anomaly line admits a gauge-equivariant trivialisation compatible with its connection on the patch: vanishing of the local gauge-anomaly class (Theorem 11) removes the curvature obstruction, after which the trivialised phase multiplies the integrand as an ordinary function on the patch quotient. The remaining holonomies are the global audit of Debt 12; none of this is assumed here.

Proof

Gauge-basicity makes the action, the bosonic measure, the absolute determinant and the observable constant on each orbit. Local disintegration of the invariant measure separates the integration along the orbit from the integration over a transverse quotient coordinate. The orbit volume cancels between numerator and denominator, or may be divided out explicitly. The phase is carried as an equivariant section rather than integrated over the orbit; the curvature obstruction to trivialising it is exactly the local gauge anomaly, whose class is shown to vanish independently in Part IV. ■

Status

The descent theorem is exact for the absolute measure once the measure premises hold, and the phase descent is an output of the anomaly audit rather than a premise.

The microscopic calculation of

$\mathfrak{Q}\_* \mu\_sub$

is not completed here.

---

7. The Gauge-Orbit Descent Theorem

Let $F\hat{}A[\varphi] = 0$ be a local gauge condition. For a gauge transformation

$g(x) = \exp[\, i\, \alpha^A(x)\, T_A \,]$,

define

$\mathcal{M}_{FP}^{AB}(x, y) = \delta F^A \varphi^g / \delta\alpha^B(y) \,|_{\alpha=0}$.

The prime on a determinant indicates that exact zero modes and stabilisers have been removed and treated separately.

Theorem 2 — Local Gauge-Orbit Quotient Theorem

Assume P5 and P6. If the gauge slice intersects every orbit in the chosen patch exactly once and

$\det'\mathcal{M}_{FP} \neq 0$,

then

$1 = \Delta_{FP}[\varphi] \int Dg \cdot \delta[F(\varphi^g)]$

with

$\Delta_{FP}[\varphi] = |\det'\mathcal{M}_{FP}[\varphi]|$.

The quotient Jacobian is the absolute determinant. On a connected perturbative patch its sign is constant; fixing the orientation there, Δ_{FP} is represented by $\det'\mathcal{M}_{FP}$ and exponentiated by the ghost integral of Theorem 3.

Consequently,

$(1/\mathrm{Vol}\, \mathcal{G}) \int D\varphi \cdot \mathcal{O}[\varphi]\, e^{-S[\varphi]}$

equals

$\int D\varphi \cdot \delta[F(\varphi)] \cdot \det'\mathcal{M}_{FP}[\varphi] \cdot \mathcal{O}[\varphi]\, e^{-S[\varphi]}$.

Proof

Insert the identity into the gauge-invariant integral. Change variables from φ to $\varphi^{g^{-1}}$. Gauge invariance removes g from the action, measure and observable. The remaining group integral gives $\mathrm{Vol}\, \mathcal{G}$, leaving the gauge-fixed expression. ■

Regularisation remark

$\mathrm{Vol}\, \mathcal{G}$ is not a finite number: the gauge group is an infinite-dimensional function group whose formal volume diverges, even though the pointwise Standard Model group is compact. The division is understood in the regulated sense: the group volume appears as an identical field-independent factor in numerator and denominator of every normalised expectation value and cancels there. Only that cancellation, not a finite value of Vol $\mathcal{G}$, is used in what follows.

Determinant-line remark

Theorem 2 applies verbatim to the absolute measure of Definition 6.2. The chiral determinant phase rides along as an equivariant section of the anomaly line; the curvature obstruction to its trivialisation is removed by Theorem 11, not assumed here.

Corollary 7.1 — Gauge fixing does not alter physical content

Inside a valid orbit patch, gauge fixing chooses one representative per orbit. It does not remove a physical degree of freedom.

Corollary 7.2 — Vanishing determinant signals slice failure

If

det 𝓜_FP = 0,

the gauge slice is tangent to an orbit or contains an unresolved stabiliser. The local gauge condition is then not a valid coordinate chart at that configuration.

---

8. Local Gauge Slicing and the Faddeev–Popov Determinant

For the unbroken non-Abelian factors, a covariant gauge condition may be written

F^A[A] = ∂^μ A_μ^A.

Its infinitesimal variation is

δ_α F^A = ∂^μ (D_μ α)^A,

so

𝓜_FP^AB = ∂^μ D_μ^AB.

In background gauge,

$A_\mu^A = \bar{A}_\mu^A + a_\mu^A$,

with

$F^A = \bar{D}^\mu a_\mu^A$,

the Faddeev–Popov operator is background covariant.

Broken-phase gauge condition

Let

$\Phi = \bar{\Phi} + \eta$.

A background R_ξ-type condition may be written schematically as

$F^A = \bar{D}^\mu a_\mu^A - i\,\xi_A\,(\bar{\Phi}^\dagger t^A \eta - \eta^\dagger t^A \bar{\Phi})$.

The scalar term cancels gauge–Goldstone mixing in the quadratic action.

The gauge-fixed theory must retain all ξ_A-dependence until physical observables are formed. Choosing ξ_A for calculational convenience cannot change an observable.

---

9. Ghosts as Orbit-Jacobian Fields

For a linear operator $\mathcal{M}$,

$\det \mathcal{M} = \int D\bar{c}\, Dc \cdot e^{-\bar{c}\, \mathcal{M}\, c}$

up to a field-independent normalisation.

Theorem 3 — Ghost-Jacobian Representation

The Faddeev–Popov determinant admits the exact local Grassmann representation

$\det' \mathcal{M}_{FP} = \int D\bar{c}\, Dc \cdot \exp[-\int d^4x\, d^4y \cdot \bar{c}_A(x)\, \mathcal{M}_{FP}^{AB}(x, y)\, c_B(y)]$.

For a local differential $\mathcal{M}_{FP}$, this gives a local ghost action.

Proof

The finite-dimensional Berezin Gaussian identity gives the determinant. Functional regularisation extends the identity mode by mode, excluding separately treated zero modes. ■

Corollary 9.1 — Ghost statistics

The ghost fields are Grassmann odd even though they carry scalar Lorentz transformation properties. Their minus sign in closed loops is required by the determinant.

Corollary 9.2 — Ghosts are not physical particles

The ghost fields parameterise the Jacobian of the orbit quotient. They have no independent positive-norm one-particle sector and are removed from the physical cohomology.

VERSF interpretation

Gauge ghosts are **non-committed quotient coordinates**. They encode the local deformation of redundant description volume. They are not persistent record currents and do not enlarge the physical substrate census.

---

10. Background-Field and Broken-Phase R_ξ Gauges

The background-field method separates:

$A = \bar{A} + a,$

where $\bar{A}$ is the retained background connection and a the integrated fluctuation.

The gauge fixing is chosen to preserve background gauge covariance. The resulting effective action satisfies

$\Gamma[\bar{A}^g, \bar{\Phi}^g, \ldots] = \Gamma[\bar{A}, \bar{\Phi}, \ldots].$

This makes the extraction of gauge-invariant curvature counterterms particularly direct and yields the relation between coupling and background-field renormalisation used later.

Background/quantum firewall

Background gauge invariance does not mean that the quantum gauge redundancy has disappeared.

The background Ward identity and the quantum Slavnov–Taylor identity are related but distinct constraints.

Part III — BRST and BV Quantum Closure

11. The Standard Model BRST Differential

Define

$s A_\mu^A = (D_\mu c)^A = \partial_\mu c^A + f^A{}_{BC} A_\mu^B c^C,$

$s c^A = -\frac{1}{2} f^A{}_{BC} c^B c^C,$

$s \bar{c}^A = B^A, \; s B^A = 0.$

For a matter field Ψ,

$s \Psi = +i c^A T_A \Psi,$

$s \bar{\Psi} = -i \bar{\Psi} c^A T_A.$

For the scalar,

$s \Phi = +i c^A t_A \Phi.$

The signs are fixed as a set: with Hermitian generators, $g = e^{i\alpha^A T_A}$ and $s A_\mu^A = (D_\mu c)^A$, nilpotency on matter requires the $+i$ transformation, as the proof below makes explicit.

The differential raises ghost number by one:

$gh(sX) = gh(X) + 1.$

These transformations are the gauge transformations with the infinitesimal commuting parameter replaced by an anticommuting ghost, supplemented by the ghost transformation required to encode the gauge algebra.

---

12. The Nilpotency Theorem

Theorem 4 — BRST Nilpotency

For the inherited closed Standard Model gauge algebra,

$$s^2 = 0$$

on every field in the BRST complex.

Proof on the ghost

$$s^2 c^A = -\tfrac{1}{2} f^A{}_{BC} (sc^B \cdot c^C - c^B \cdot sc^C).$$

Substituting

$$sc^A = -\tfrac{1}{2} f^A{}_{DE}\, c^D c^E$$

gives a cubic ghost coefficient proportional to

$$f^A{}_{B[C} f^B{}_{DE]}.$$

This vanishes by the Jacobi identity.

Proof on the gauge connection

$$s^2 A_\mu = D_\mu(sc) + [sA_\mu, c].$$

Using

$$sc = -\tfrac{1}{2} [c, c]$$

and the graded Jacobi identity gives zero.

Proof on matter

With $s\,\Psi = +i\, c^A T_A \Psi$ and the graded Leibniz rule,

$$s^2\Psi = i\, (sc^A) T_A \Psi - i\, c^A T_A (i\, c^B T_B \Psi) = -(i/2) f^A{}_{BC}\, c^B c^C T_A \Psi + c^A c^B T_A T_B \Psi.$$

The symmetric part of $T_A T_B$ drops because $c^A c^B$ is antisymmetric; the antisymmetric part gives $\tfrac{1}{2}[T_A, T_B] = (i/2) f_{AB}{}^C T_C$, so the second term equals $+(i/2) f_{AB}{}^C c^A c^B T_C \Psi$ and cancels the first exactly. The scalar proof is identical. Finally,

$$s^2\bar{c} = sB = 0,\quad s^2 B = 0.$$

Therefore $s^2 = 0$ on the full complex. ■

Interpretation

Nilpotency is the algebraic statement that performing a gauge-redundancy variation twice produces no new physical distinction after the gauge algebra is accounted for.

---

13. Gauge Fixing as a BRST-Exact Deformation

Choose the gauge-fixing fermion, in the Euclidean convention of §1.18,

$\Psi_{gf} = \int d^4x \cdot \bar{c}_A (F^A - (\xi_A/2) B^A)$.

Define

$S_{gf+gh} = s \Psi_{gf}$.

Expanding, with $s \bar{c} = B$ and $s B = 0$,

$S_{gf+gh} = \int d^4x [B_A F^A - (\xi_A/2) B_A B^A - \bar{c}_A \cdot sF^A]$.

The B-field equation is

$B_A = F_A / \xi_A$.

Eliminating B gives

$S_{gf} = \int d^4x \cdot (1/2\xi_A) F_A F^A$,

plus the ghost action. The positive Euclidean sign is now an output of the convention rather than a stipulation.

Theorem 5 — BRST-Exact Gauge-Fixing Theorem

If

$s S_{cl} = 0$

and

$s^2 = 0$,

then

$S_\Psi = S_{cl} + s\,\Psi_{gf}$

is BRST invariant:

$s\,S_\Psi = 0.$

Proof

$s\,S_\Psi = s\,S_{cl} + s^2\Psi_{gf} = 0.$ ■

Corollary 13.1 — Gauge choice is cohomologically trivial

Changing

$\Psi_{gf} \to \Psi_{gf} + \delta\Psi$

changes the action by

$\delta S = s(\delta\Psi).$

The change is BRST-exact and therefore cannot alter the expectation value of a BRST-closed physical observable when the measure has no BRST anomaly.

---

14. The Batalin–Vilkovisky Master Action

Introduce an antifield φ^*_I conjugate to every field φ^I.

The BV antibracket is

$(X, Y) = \int d^4x\,[\,(\delta_r X / \delta\varphi^I)(\delta_l Y / \delta\varphi^*_I) - (\delta_r X / \delta\varphi^*_I)(\delta_l Y / \delta\varphi^I)\,].$

For the closed Standard Model algebra, the minimal master action is

$S_{BV} = S_{cl} + \int d^4x \cdot \varphi^*_I\, s\,\varphi^I.$

Theorem 6 — Classical Master-Equation Theorem

The inherited Standard Model gauge algebra and BRST transformations imply

$\mathbf{(S_{BV}, S_{BV}) = 0.}$

Proof

The antifield-independent term vanishes because S_{cl} is gauge invariant. Terms linear in antifields vanish because the gauge transformations close. Terms quadratic in the BRST variation vanish because $s^2 = 0$. These conditions are precisely the coefficients of the master equation. ■

Gauge fixing is implemented by the Lagrangian submanifold

$$\varphi^*_I = \delta\Psi_{gf} / \delta\varphi^I.$$

The BV formalism provides the natural language for regulator-dependent quantum anomalies and general gauge-fixing changes.

Quantum master equation

For a regulated quantum action W,

$$\tfrac{1}{2} (W, W) - i\hbar\, \Delta_{reg} W = 0$$

is the quantum master equation.

The operator Δ_{reg} is not a meaningful unregulated coincidence expression. Its definition belongs to the regulator.

Failure of the equation is a quantum anomaly.

---

15. Slavnov–Taylor Identities

Let Γ be the renormalised one-particle-irreducible effective action.

The Slavnov functional may be written schematically as

$$\mathcal{S}(\Gamma) = \int d^4x \, [(\delta\Gamma/\delta\varphi^*_I)(\delta\Gamma/\delta\varphi^I) + B^A (\delta\Gamma/\delta\bar{c}^A)].$$

Theorem 7 — Classical Slavnov–Taylor Identity

The classical master equation implies

$$\mathcal{S}(S_{gf}) = 0.$$

At the quantum level, absence or removal of the gauge anomaly gives

$\mathcal{S}(\Gamma_R) = 0$.

Consequences

The identity relates:

- gauge-boson self-energies;
- gauge–matter vertices;
- ghost vertices;
- gauge-boson self-couplings;
- Goldstone and longitudinal-vector amplitudes;
- field and coupling renormalisations.

It is the quantum expression of the fact that all these interactions descend from one gauge connection.

---

16. Gauge-Fixing Independence and Nielsen Identities

Let $\mathcal{O}$ be BRST closed:

$s\mathcal{O} = 0$.

A change of gauge-fixing fermion gives

$\delta\langle\mathcal{O}\rangle \propto \langle \mathcal{O} \cdot s(\delta\Psi) \rangle$.

When the measure is BRST invariant,

$\langle sX\rangle = 0$.

Theorem 8 — Gauge-Fixing Independence

For BRST-closed observables,

$\delta_\Psi \langle\mathcal{O}\rangle = 0$

under admissible infinitesimal changes of gauge fixing.

Proof

Using $s\mathcal{O} = 0$,

$\mathcal{O} \cdot s(\delta\Psi) = s(\mathcal{O} \cdot \delta\Psi)$.

The expectation value of a BRST-exact insertion vanishes when the quantum measure satisfies the Slavnov–Taylor identity. ■

Logical position

The tree-level statement rests on Theorem 7 alone. At quantum level, $\langle sX \rangle = 0$ is the Slavnov–Taylor identity at the relevant order, so Theorem 8 holds order by order exactly when the restoration of Theorem 18 has been carried out to that order. Its placement here is expository; its logical position is downstream of Part VI.

Theorem 9 — Nielsen Identity

Dependence of the effective action on a gauge parameter ξ takes the form

$$\partial\Gamma/\partial\xi + \int d^4x \cdot C^I[\varphi, \xi] \cdot (\delta\Gamma/\delta\varphi^I) = 0.$$

The functionals C^I are determined by extended BRST insertions.

Derivation sketch

Promote ξ to a BRST doublet: extend the differential by $s\,\xi = \chi$, $s\,\chi = 0$, with χ a Grassmann constant of ghost number one. The ξ-dependence of the gauge-fixing fermion then makes $\partial S_\Psi/\partial\xi$ BRST-exact in the extended complex, so $\partial\Gamma/\partial\xi$ equals the insertion of an exact operator. Legendre transformation converts that statement into the identity above, with C^I generated by the extended source term. The identity holds at every order at which the Slavnov–Taylor identity has been restored (Theorem 18); see Nielsen and Piguet–Sorella (Appendix J, refs. 14, 17).

Corollary 16.1 — Stationary values are gauge independent

At a stationary configuration,

$\delta\Gamma/\delta\varphi^I = 0$,

the value of Γ is gauge-parameter independent.

Corollary 16.2 — Off-shell effective potentials are gauge dependent

The shape of an effective potential away from its extrema is not by itself an observable.

A VERSF claim about vacuum stability, Higgs curvature or phase transition must use a gauge-consistent quantity, not a raw off-shell potential plotted in one convenient gauge.

Part IV — Quantum Anomaly Closure

17. Anomalies as Quantum-Master-Equation Obstructions

Suppose the Slavnov–Taylor identity first fails at loop order n:

$\mathcal{S}(\Gamma) = \hbar^n \mathcal{A}^{(n)} + O(\hbar^{n+1})$.

The consistency of the master equation implies

$\mathcal{B}_S \mathcal{A}^{(n)} = 0$,

where $\mathcal{B}_S$ is the linearised Slavnov operator.

Thus $\mathcal{A}^{(n)}$ is a ghost-number-one cocycle. The relevant cohomology is the local cohomology $H^1(s|d)$: local functionals annihilated by $\mathcal{B}_S$, identified modulo $\mathcal{B}_S$-exact terms and total derivatives.

Two cases exist.

Trivial breaking

If

$\mathcal{A}^{(n)} = \mathcal{B}_S \Xi^{(n)}$,

then the counterterm

$-\hbar^n \Xi^{(n)}$

removes the breaking.

Nontrivial anomaly

If

$[\mathcal{A}^{(n)}] \neq 0$ in $H^1(s|d)$, no local counterterm removes it.

$H^1(s|d)$ itself is generally nonzero for Yang–Mills algebras: it is the space of candidate anomalies. Consistency requires the vanishing of the class of the actual regulated breaking, not the vanishing of the group.

Theorem 10 — Gauge-Anomaly Obstruction Theorem

A nonzero gauge-anomaly cohomology class prevents simultaneous maintenance of:

- the quantum master equation;
- gauge-parameter independence;
- decoupling of unphysical modes;
- physical-state unitarity.

In VERSF language it is a quantum source-ledger leak that cannot be committed as a persistent record.

---

18. The Complete Local Standard Model Anomaly Census

All anomaly sums are performed over left-handed Weyl fields.

For one generation:

$Q_L : (3, 2)_{1/6}$,

$u_L^c : (\bar{3}, 1)_{-2/3}$, $d_L^c : (\bar{3}, 1)_{1/3}$,

$L_L : (1, 2)_{-1/2}$, $e_L^c : (1, 1)_1$.

18.1 SU(3)³

Using

$A(\bar{3}) = -A(3)$,

the coefficient is

$2A(3) + A(\bar{3}) + A(\bar{3}) = 2 - 1 - 1 = 0$.

The factor two comes from the two weak components of Q_L.

18.2 $SU(3)^2U(1)$

With

$T(3) = T(\bar{3}) = ½$,

the coefficient is

$2\cdot(1/6)\cdot½ + (-2/3)\cdot½ + (1/3)\cdot½$.

Therefore

$1/6 - 1/3 + 1/6 = 0$.

18.3 $SU(2)^2U(1)$

The coefficient is

$3\cdot(1/6)\cdot½ + (-1/2)\cdot½ = 1/4 - 1/4 = 0$.

The colour multiplicity of Q_L balances the lepton doublet.

18.4 Mixed gravitational–U(1)

The coefficient is the sum of hypercharges with multiplicity:

$6\cdot(1/6) + 3\cdot(-2/3) + 3\cdot(1/3) + 2\cdot(-1/2) + 1$.

Thus

$1 - 2 + 1 - 1 + 1 = 0$.

This row audits the coupling of the inherited hypercharge ledger to the emergent geometric background. Within VERSF the gauge status of the emergent 4-manifold sector is a separately ledgered gate, so the row is conditional on P25 rather than on the internal-gauge premises alone.

18.5 $U(1)^3$

The coefficient is

$6\cdot(1/6)^3 + 3\cdot(-2/3)^3 + 3\cdot(1/3)^3 + 2\cdot(-1/2)^3 + 1^3$.

This gives

$1/36 - 8/9 + 1/9 - 1/4 + 1 = 0.$

18.6 Perturbative SU(2)³

SU(2) possesses no third-order Casimir invariant: the totally symmetric d-symbol vanishes identically for every representation, so the local cubic anomaly is zero for any SU(2) matter content. Pseudoreality of the fundamental is the special case usually quoted.

Theorem 11 — Local Standard Model Gauge-Anomaly Cancellation

For the inherited one-generation ledger,

$[\mathcal{A}_{local}^{gauge}] = 0.$

The result replicates generation by generation and is unchanged by adding a gauge-singlet right-handed neutrino.

Consequence

The local ghost-number-one obstruction needed to destroy the Standard Model Slavnov–Taylor identity is absent for the inherited census.

---

19. Global SU(2) and Faithful-Quotient Anomaly Firewalls

A four-dimensional SU(2) theory with an odd number of left-handed doublets suffers the ordinary Witten global sign anomaly, governed by $\pi_4(SU(2)) = \mathbb{Z}_2$.

One Standard Model generation has three colour copies of Q_L and one L_L:

$N_{doublet} = 3 + 1 = 4.$

Across three generations,

$N_{doublet} = 12.$

Theorem 12 — Ordinary SU(2) Global-Sign Audit

The inherited Standard Model doublet census is even, so the ordinary Witten SU(2) sign obstruction is absent.

Global-quotient limitation

This does not complete the global anomaly problem for

G_SM^faithful.

The complete audit is a determinant-line and bordism calculation: in modern form, the evaluation of the relevant reduced bordism group

$\tilde{\Omega}^5$_Spin(B G_SM^faithful)

(or its spin-G_SM^faithful refinement where the quotient obstructs an ordinary spin structure) together with the anomaly-theoretic pairing of the inherited fermion census against its generators. The audit must determine:

- all admissible quotient bundles;
- spin versus spin-G structures;
- torsion sectors;
- fermion determinant-line holonomy;
- possible mixed gauge–gravitational global anomalies;
- the influence of optional neutrino completions.

The perturbative anomaly polynomial and even-doublet count do not replace this calculation.

External corroboration

The bordism audit named above has been carried out in the external literature: Dai–Freed-type computations of the relevant spin-bordism groups find no new global anomalies for the Standard Model fermion content with any of the four global-group choices, including the $\mathbb{Z}_6$ quotient (García-Etxebarria–Montero; Davighi–Gripaios–Lohitsiri; Appendix J, refs. 25–26). QGC-1 records this as corroborating external evidence, not as a constitutive input: Debt 12 is the substrate-native derivation of the same result from commitment primitives, and the numerical non-insertion protocol is not violated, because the citation selects nothing upstream.

---

20. Axial-Anomaly Survival

Consider a chiral transformation

$\Psi \to e^{i\alpha\gamma_5} \Psi$.

The classical massless action may be invariant, but the regulated fermion measure transforms nontrivially.

For a Dirac fermion in representation R,

$$\partial_\mu J_5^\mu = 2im\, \bar{\Psi}\gamma_5\Psi + (g^2/16\pi^2) \cdot 2T(R) \cdot F^a_{\mu\nu} \tilde{F}^{a\mu\nu} + \cdots$$

The measure Jacobian is determined by the regulated trace of γ_5, equivalently the index density.

Theorem 13 — Axial-Jacobian Survival Theorem

Assuming P12, the quantum-completed VERSF fermion measure reproduces the chiral phase transfer

$$\mathcal{D}\Psi\, \mathcal{D}\bar{\Psi} \mapsto \mathcal{D}\Psi\, \mathcal{D}\bar{\Psi} \cdot e^{i\Delta S_{anom}},$$

where ΔS_{anom} is the integrated index density with the inherited representation coefficient.

The singlet axial anomaly therefore survives even though all gauge anomalies cancel.

Coefficient stability

The Adler–Bardeen mechanism protects the anomaly coefficient: in an admissible scheme the one-loop index normalisation receives no higher-order multiplicative renormalisation. The strong-CP bookkeeping of §21 and §28 therefore transports an RG-stable coefficient, not a scheme artefact.

VERSF interpretation

Gauge currents are persistent source ledgers and must close.

The singlet axial current is not an exact gauge ledger of the completed theory. Its nonconservation records spectral flow between fermionic chirality and gauge topology rather than an inconsistency of the gauge redundancy.

---

21. The Combined Topological and Quark Determinant Line

Let $\nu \in \mathbb{Z}$ denote the colour topological charge of a sector, $\mathscr{L}_{top}$ the line carrying the global colour topological character, and $\mathcal{D}_q$ the combined anomalous quark determinant line. Define

$\mathcal{L}_{\bar\theta} = \mathcal{L}_{top} \otimes \mathcal{D}_q$.

Under a chiral basis transformation the phase of $\mathcal{D}_q$ changes, and the Fujikawa Jacobian changes the topological character oppositely. The precise content of that statement is a sector-weight identity, which is the theorem; the combined line is its geometric reading.

Theorem 14 — Strong-Phase Sector-Weight Theorem

In each colour topological sector ν, the regulated fermionic measure carries the mass-determinant phase dictated by the index theorem, and the sector weight of the Euclidean functional integral has the form

$$Z_\nu \propto e^{i\nu\theta_3} \cdot [\det(M_u M_d)]^\nu \cdot (\text{positive factors}) = e^{i\nu\bar\theta} \cdot |\det(M_u M_d)|^\nu \cdot (\text{positive factors}),$$

(for $\nu < 0$ the mass factor is $[\det(M_u M_d)^\dagger]^{|\nu|}$, and the formal power notation is to be read accordingly), with

$$\bar\theta = \theta_3 + \arg\det(M_u M_d).$$

Under any chiral redefinition of the quark fields every Z_ν is unchanged: $\bar\theta$ is the basis-invariant phase of the sector weights.

Proof

A chiral rotation shifts $\arg\det(M_u M_d)$ by the determinant phase of the left–right field redefinition. By Theorem 13, the anomalous Jacobian shifts $\nu\theta_3$ by exactly the opposite amount in the charge-ν sector, because the regulated index density integrates to ν with the flavour multiplicity fixed by the census. The product of the two phase factors is therefore invariant sector by sector. ■

Geometric reading

The combined line retains its meaning as the geometric restatement of the theorem: the base is the space of gauge backgrounds, the equivariant connection is the one induced by the regulated determinant, and

$$\mathrm{Hol}(\mathcal{L}_{\bar\theta}) = e^{i\bar\theta}$$

is the holonomy assigned to the unit-charge sector. Nothing in the paper depends on the geometric language beyond the sector-weight identity above.

Consequence

The two strong-CP gates are physically one invariant problem.

A substrate-canonical factorisation may separately identify

$\theta_{sub} = 0$

and

$\arg \det(M_u M_d) = 0$,

but only the combined sector-weight phase is basis independent before that canonical frame is derived.

Scope

QGC-1 derives:

- the sector-weight covariance;
- the axial Jacobian;
- the invariant combination;
- its RG and threshold bookkeeping.

It does not prove that the invariant phase is trivial. That remains the strong-CP selection calculation.

---

22. Global-Current Anomalies and Selection Rules

A global current may be anomalous without making the gauge theory inconsistent.

For example, baryon and lepton currents can have electroweak anomaly terms while the gauge currents remain conserved.

Proposition 22.1 — Gauge/Global Anomaly Separation

A nonzero anomaly in a global current produces a physical nonconservation law or selection rule.

A nonzero anomaly in a gauged current destroys the redundancy needed for quantum consistency.

VERSF interpretation

A global bookkeeping grade need not be a separately committed local gauge source. Its anomaly can describe allowed conversion between physical sectors.

A gauge source ledger must close, because it defines the identification of redundant local descriptions.

Part V — One-Loop Effective Action and Running

23. Background-Field Functional Determinants

Expand the gauge-fixed Euclidean action around background fields:

$\varphi = \bar{\varphi} + \eta$.

The quadratic fluctuation operator is

$\mathbb{H}_{gf} = \delta^2 S_{gf} / \delta\eta\, \delta\eta\, |_{\bar{\varphi}}$.

The one-loop effective action is

$\boldsymbol{\Gamma_E^{(1)} = \frac{1}{2}\, \mathrm{STr} \log \mathbb{H}_{gf}}$.

The supertrace supplies:

- $+\frac{1}{2}$ log det for real bosonic modes;
- −log det for complex ghosts;
- −log det for Dirac fermions;
- the appropriate chiral and multiplicity factors.

The physically meaningful gauge contribution is the combined gauge-plus-ghost determinant. Neither determinant alone is gauge independent.

---

24. The Standard Model Gauge Beta-Census Theorem

For a simple gauge factor with Weyl fermions and complex scalars,

$$\beta(g) = -(g^3/16\pi^2) \left[(11/3)\cdot C_2(G) - (2/3)\cdot\Sigma_{\text{Weyl}}\, T(R_f) - (1/3)\cdot\Sigma_{\text{complex}}\, T(R_s) \right] + \cdots$$

For the Abelian factor, the gauge term vanishes and the charge-square sums give a positive beta function.

In the convention used throughout,

$$\mu\, dg_A/d\mu = (b_A/16\pi^2)\cdot g_A^3 + \cdots,$$

a negative b_A is asymptotic freedom.

24.1 Colour

For SU(3),

$$C_2(G) = 3.$$

Across three generations,

$$\Sigma_{\text{Weyl}}\, T(R_f) = 6.$$

There is no coloured scalar in the minimal Standard Model. Therefore

$$(11/3)\cdot 3 - (2/3)\cdot 6 = 11 - 4 = 7.$$

Hence

$$\boldsymbol{\beta_{g_s} = -7 \cdot g_s^3/(16\pi^2) + \cdots}$$

24.2 Weak isospin

For SU(2),

$$C_2(G) = 2.$$

The Weyl sum is

$$\Sigma_{\text{Weyl}}\, T(R_f) = 6.$$

The one complex scalar doublet contributes

$$\Sigma_{\text{scalar}}\, T(R_s) = 1/2.$$

Thus

$22/3 - 4 - 1/6 = 19/6.$

Therefore

$$\beta_g = -(19/6) \cdot g^3/(16\pi^2) + \cdots$$

24.3 Hypercharge

With the electric-charge convention $Q = T_3 + Y$, the three-generation Weyl charge-square sum is

$$\Sigma_{\text{Weyl}}\, d(R)\cdot Y^2 = 10.$$

The scalar doublet gives

$$\Sigma_{\text{scalar}}\, d(R)\cdot Y^2 = ½.$$

Therefore

$$(2/3)\cdot 10 + (1/3)\cdot(½) = 41/6.$$

Hence

$$\beta_{g'} = (41/6) \cdot g'^3/(16\pi^2) + \cdots$$

Theorem 15 — Standard Model One-Loop Gauge-Flow Theorem

For the inherited Standard Model field census, within the running-spectrum window of P26,

$$(b_Y, b_2, b_3) = (41/6, -19/6, -7)$$

in

$$\mu\, dg_A/d\mu = (b_A/16\pi^2)\cdot g_A^3 + \cdots$$

A sterile right-handed neutrino changes none of the three coefficients.

Status of the derivation

The present gate evaluates the universal one-loop coefficient formula on the inherited census. The first-principles evaluation of the heat-kernel coefficients of the VERSF-regulated fluctuation determinants — verifying that the substrate regulator reproduces the universal formula — is registered under Debts 8 and 15.

Interpretation

The signs are not adjustable:

- colour is asymptotically free;
- weak isospin is asymptotically free at one loop;
- hypercharge grows toward the ultraviolet.

The one-loop gauge coefficients are scheme independent: any admissible redefinition of the couplings shifts them only at higher order. At two loops and beyond, multi-coupling cross-terms can shift under such redefinitions, so higher coefficients are quoted only within a declared scheme. The census above is therefore a scheme-invariant consequence of the inherited field content, not an artefact of the extraction method.

---

25. Coupling-Boundary and Running Separation

At the VERSF matching scale,

$$g_A(\mu\star) = Z_A(\mu\star)^{-1/2}.$$

The loop equation transports this boundary value:

$$1/g_A^2(\mu) = 1/g_A^2(\mu\star) - (b_A/8\pi^2)\cdot\ln(\mu/\mu\star) + \Delta_A^{threshold} + \Delta_A^{scheme} + \cdots$$

Theorem 16 — Boundary/Flow Separation

The gauge-action programme determines the matching-scale response

$$Z_A(\mu\star).$$

The quantum gauge programme determines:

- the beta functions;
- threshold maps;
- finite scheme conversion;
- operator mixing;
- observable matching.

Neither module can replace the other.

Background Ward identity

In a background-field scheme,

$$Z_{g_A} \cdot Z_{\bar{A}_A}^{1/2} = 1.$$

Thus the coupling renormalisation is determined by the background-field two-point function.

---

26. Yukawa, Scalar and Flavour-Frame Renormalisation

The renormalised matrix Yukawa operators satisfy

$$\mu \, dY_f/d\mu = \beta_{Y_f}(Y_u, Y_d, Y_e, Y_\nu, g_s, g, g', \lambda, \ldots).$$

The scalar potential parameters obey corresponding RG equations.

Proposition 26.1 — Operator-Type Preservation

Slavnov–Taylor closure prevents loop corrections from producing a renormalisable interaction that violates the inherited gauge representations.

Counterterms may renormalise:

- the existing Yukawa matrices;
- scalar mass and quartic terms;
- wavefunction matrices;
- gauge couplings;
- allowed flavour rotations.

They cannot generate a gauge-forbidden chiral contraction.

Flavour-frame reading

The running of Y_u and Y_d can rotate their left and right spectral frames. Therefore CKM and PMNS parameters are scale dependent.

Physical flavour information resides in rephasing-invariant quantities, such as:

- singular values;
- commutator invariants;
- Jarlskog-type combinations;
- pole residues and decay amplitudes.

A basis-dependent running matrix entry is not by itself an observable.

VERSF consequence

The closure-Hessian construction supplies the boundary matrices. Quantum completion transports the same matrices; it does not introduce free replacement Yukawas.

---

27. Effective-Potential Gauge Dependence

Let

$V_{eff}(\varphi, \xi)$

be an effective potential computed in gauge parameter ξ.

The Nielsen identity has the reduced form

$\partial V_{eff}/\partial\xi + C(\varphi, \xi)\cdot\partial V_{eff}/\partial\varphi = 0.$

Corollary 27.1 — Extremal values

At an extremum,

$\partial V_{eff}/\partial\varphi = 0,$

the value of the effective potential is gauge independent.

Corollary 27.2 — Coordinate location

The field-coordinate value φ_{min} can be gauge dependent.

Programme firewall

A claimed VERSF prediction for:

- the Higgs vacuum;

- vacuum stability;
- a barrier height;
- a phase-transition temperature;
- a radial curvature;

must be expressed through gauge-consistent observables or an explicitly Nielsen-controlled construction.

---

28. Threshold Matching and Decoupling

Suppose a heavy field of mass M is integrated out.

The low-energy action is

$$e^{iS_{EFT}[\varphi_{light}]} = \int D\varphi_{heavy} \cdot e^{iS[\varphi_{light}, \varphi_{heavy}]}.$$

Matching gives

$$C_i^{low}(M) = C_i^{high}(M) + \Delta C_i^{threshold}.$$

Theorem 17 — Gauge-Compatible Threshold Theorem

If the high-energy theory satisfies the Slavnov–Taylor identity and heavy fields are integrated out in a gauge-consistent regulator, then the low-energy action satisfies the corresponding low-energy identity after all required matching operators are retained.

Anomaly non-decoupling firewall

If a heavy chiral field participates in gauge-anomaly cancellation, integrating it out alone does not permit the low-energy gauge anomaly simply to reappear.

Its effect survives through Wess–Zumino or equivalent matching terms until the low-energy gauge ledger is again consistent.

Strong-phase threshold identity

For a heavy quark mass

$$m_h = |m_h| \cdot e^{i\varphi_h},$$

the low-energy colour angle obeys

$\theta_{low} = \theta_{high} + \arg m_h \pmod{2\pi}$,

up to separately derived finite CP-odd matching terms.

Thus

$\bar{\theta}_{low} = \bar{\theta}_{high}$.

Part VI — All-Order Renormalisation

29. The Quantum Action Principle

At loop order n, any breaking of a classical functional identity produced by a local regulator is represented by an integrated local insertion with bounded dimension and definite quantum numbers.

This is the foundation of algebraic renormalisation.

For the Slavnov identity,

$\mathcal{S}(\Gamma) = \hbar^n \mathcal{A}_n + O(\hbar^{n+1})$.

The consistency condition gives

$\mathcal{B}_S \mathcal{A}_n = 0$.

The problem becomes a cohomology calculation rather than a diagram-by-diagram hope that all breakings cancel.

---

30. The Algebraic Renormalisation Theorem

Theorem 18 — All-Order Slavnov–Taylor Restoration

Assume:

1. the Slavnov–Taylor identity holds through order $n - 1$;
2. the regulator satisfies the quantum action principle;
3. the nontrivial local gauge-anomaly class vanishes;
4. the declared counterterm ledger is complete.

Then there exists a local counterterm at order n such that

$$\mathcal{S}(\Gamma) = O(\hbar^{n+1}).$$

By induction, the identity may be maintained to all perturbative orders.

Proof

The order-n breaking $\mathcal{A}_n$ is a cocycle of the linearised Slavnov operator. Vanishing of the anomaly class implies

$$\mathcal{A}_n = \mathcal{B}_S \, \Xi_n.$$

Adding

$$-\hbar^n \, \Xi_n$$

to the action removes the breaking at that order. Repeating the argument establishes the identity inductively. ■

Significance

Gauge consistency is not established merely by verifying the one-loop triangle diagrams. The all-order statement follows from:

- vanishing of the obstruction class in $H^1(s|d)$;
- locality;
- a complete counterterm space.

---

31. Renormalisable-Core Closure

Let

$$S_{SM}^{(4)}$$

contain all gauge-invariant Standard Model operators of canonical dimension at most four compatible with the inherited ledger. That basis is finite and is displayed here in full:

- the three gauge kinetic operators $G^a_{\mu\nu} G^{a\mu\nu}$, $W^i_{\mu\nu} W^{i\mu\nu}$, $B_{\mu\nu} B^{\mu\nu}$;
- the three topological characters $\theta_3\, G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$, $\theta_2\, W^i_{\mu\nu} \tilde{W}^{i\mu\nu}$, $\theta_1\, B_{\mu\nu} \tilde{B}^{\mu\nu}$, retained as declared sector data — only θ_3 enters the strong-phase gate, and the electroweak characters are separately ledgered;

- the chiral fermion kinetic operators $i \bar{\Psi} \gamma^\mu D_\mu \Psi$, one for each carrier in the census of §3.3;
- the Higgs kinetic operator $(D_\mu \Phi)^\dagger (D^\mu \Phi)$;
- the potential operators $\mu^2 \Phi^\dagger\Phi$ (dimension two) and $\lambda (\Phi^\dagger\Phi)^2$ (dimension four);
- the Yukawa blocks $\bar{Q}_L \Phi d_R$, $\bar{Q}_L \tilde{\Phi} u_R$ and $\bar{L}_L \Phi e_R$, with matrix coefficients Y_d, Y_u, Y_e;
- optionally, under the sterile completion of §3.3: $\bar{L}_L \tilde{\Phi} \nu_R$ with coefficient Y_ν, and the dimension-three Majorana bilinear $\nu_R^T C \nu_R$ with matrix M_R, per the neutrino gate.

No further gauge-invariant operator of dimension at most four exists on the inherited representations; in particular, no bare fermion mass term is invariant before electroweak symmetry breaking. The enumeration is closed, and the gauge-fixing and ghost sector enters separately as the BRST-exact term $s \Psi_{gf}$, not as a ledger entry.

At ghost number zero, a local counterterm Σ satisfies

$$\mathscr{B}_S \Sigma = 0.$$

The general solution has the form

$$\Sigma = \Sigma_{inv} + \mathscr{B}_S \Delta.$$

Here:

- Σ_{inv} renormalises physical gauge-invariant operators;
- $\mathscr{B}_S \Delta$ is a field, gauge-fixing or coordinate redefinition.

Theorem 19 — Renormalisable Standard Model Closure

Under P10, P14 and P15, all perturbative ultraviolet divergences of the dimension-four Standard Model core are absorbed into:

- gauge coupling renormalisation;
- field and gauge-parameter renormalisation;
- scalar-potential renormalisation;
- Yukawa-matrix renormalisation;
- allowed mass and vacuum parameters;
- BRST-exact redefinitions.

No new dimension-four gauge-violating operator is required.

---

32. VERSF Effective-Field-Theory Completion

VERSF contains substrate, closure and bath scales. Coarse-graining can therefore generate

$\mathcal{O}_i^{(d)},\ d > 4.$

The quantum effective action is written

$$\Gamma_\mu = S_{SM}^{(4)}(\mu) + s\,\Psi_\mu + \sum_{d>4} \sum_i \left[C_i^{(d)}(\mu) / M_\star^{d-4} \right] \cdot \mathcal{O}_i^{(d)}.$$

Theorem 20 — Ordered EFT Closure

At any fixed maximum dimension D and loop order L, only finitely many symmetry-admissible operators contribute.

Their divergences are absorbed into the coefficients of the complete operator basis through dimension D.

Programme significance

A higher-dimensional operator is not a failure of quantum completion.

Failure occurs if:

- the operator is not in the declared ledger;
- its coefficient cannot be derived or bounded;
- the expansion has no controlled hierarchy;
- it violates a required gauge identity.

Strong-CP implication

Higher-dimensional CP-odd operators such as:

- quark chromoelectric dipoles;
- electroweak dipoles;
- the Weinberg three-gluon operator;

must be kept separate from $\bar{\theta}$. Setting $\bar{\theta} = 0$ does not make their coefficients vanish.

---

33. Regulator and γ_5 Firewalls

A regulator is acceptable only if every induced breaking is classified.

Possible routes include:

- background-field dimensional schemes with a declared chiral prescription;
- higher-covariant-derivative plus Pauli–Villars constructions;
- lattice or overlap-type chiral measures;
- flow-based or finite-mode regulators;
- a directly derived substrate cutoff.

Theorem 21 — Regulator-Independence Criterion

Two admissible regulators define the same renormalised physical theory when their effective actions differ by:

- finite redefinitions of physical parameters;
- BRST-exact terms;
- changes of renormalisation scheme;
- explicitly matched EFT coefficients;

and agree on all anomaly classes.

Falsifier

If two candidate VERSF regulators produce inequivalent gauge-anomaly classes or inequivalent physical observables after all admissible finite matching, the quantum completion is not regulator independent.

Part VII — Physical States, Unitarity and Observables

34. BRST Cohomology and Physical States

The gauge-fixed state space

$\mathcal{K}$

contains positive-, zero- and negative-norm vectors.

Let Q_B generate the BRST transformation:

$sX = i [Q_B, X\}_{graded}$.

Physical vectors obey

$Q_B |\psi\rangle = 0$.

Vectors of the form

$|\psi\rangle = Q_B |\chi\rangle$

are null physical representatives.

The physical space is

$\mathscr{H}_{phys} = \ker Q_B / \mathrm{im}\, Q_B$.

Theorem 22 — Cohomological Gauge Independence

Operators differing by a BRST-exact term induce the same action on physical cohomology.

If

$\mathcal{O}' = \mathcal{O} + [Q_B, X\}$,

then

$\langle\varphi|\mathcal{O}'|\psi\rangle = \langle\varphi|\mathcal{O}|\psi\rangle$

for physical representatives, subject to the usual domain conditions.

---

35. Quartet Cancellation

The unphysical gauge sector is organised into Kugo–Ojima BRST quartets involving:

- longitudinal gauge modes;
- timelike gauge modes;
- ghost modes;
- antighost or auxiliary partners.

These states pair into exact or null classes and do not survive in physical cohomology.

VERSF reading

The quotient bookkeeping is internally active but externally absent.

Unphysical modes contribute precisely enough inside loops to prevent redundant gauge descriptions from appearing as physical probability.

---

36. The Perturbative-Unitarity Theorem

Assume

$Q_B^2 = 0, Q_B^\dagger = Q_B,$

and

$[Q_B, S] = 0.$

Theorem 23 — Descent of the Scattering Operator

The scattering operator maps closed states to closed states and exact states to exact states. It therefore induces

$S_{phys} : \mathscr{H}_{phys} \to \mathscr{H}_{phys}.$

Proof

For $Q_B|\psi\rangle = 0,$

$Q_B S |\psi\rangle = S Q_B |\psi\rangle = 0.$

For $|\psi\rangle = Q_B|\chi\rangle,$

$S |\psi\rangle = S Q_B |\chi\rangle = Q_B S |\chi\rangle,$

so exact states remain exact. ■

Theorem 24 — Perturbative Physical-State Unitarity

Assume additionally:

1. S is pseudo-unitary on the extended state space;
2. the physical BRST cohomology has positive norm;
3. quartet completeness removes all negative-norm cohomology;
4. the Slavnov–Taylor identity is anomaly free.

Then

$$\mathbf{S_{phys}^\dagger\, S_{phys} = 1}$$

on $\mathscr{H}_{phys}$.

Programme stakes

Failure of this theorem for the completed VERSF action would not be a small correction. It would falsify the claim that the derived gauge architecture defines a probabilistic quantum theory.

---

37. Optical Theorem and Cutting Identities

Write

$$S = 1 + iT.$$

Unitarity gives

$$-i(T - T^\dagger) = T^\dagger T.$$

For an initial physical state i,

$$\mathbf{2\, Im\, \mathscr{M}_{i\to i} = \Sigma_{f \in \mathscr{H}_{phys}} \int d\Pi_f \cdot |\mathscr{M}_{i\to f}|^2.}$$

The sum is over physical states.

In covariant perturbation theory, unphysical cuts may appear in intermediate diagrams, but Slavnov–Taylor identities and quartet cancellation remove them from the final physical discontinuity.

---

38. Observable Reconstruction

A physical observable must satisfy at least one of the following:

- it is gauge invariant;
- it represents a BRST cohomology class;
- it is a properly dressed charged operator;
- it appears inside a factorisation theorem whose gauge and scheme dependence cancels;
- it is a pole, residue or inclusive rate with a declared operational definition.

Examples

Gauge-invariant local or composite operators

$\Phi^\dagger\Phi$, $F^A_{\mu\nu} F^{A\mu\nu}$, $\bar{Q}_L \Phi d_R$, $\bar{Q}_L \tilde{\Phi} u_R$.

A generic bilinear $\Psi\Psi$ is not gauge invariant for the chiral Standard Model fields before electroweak symmetry breaking; invariant fermion bilinears require the Higgs insertion shown.

Wilson and line observables

$W_R(C) = \mathrm{tr}_R\, P \exp(i \oint_C A)$.

Pole quantities

Pole masses and resonance poles, with the complex-pole prescription for unstable states.

Inclusive observables

Cross sections summed over experimentally unresolved soft and collinear radiation.

LSZ firewall

Standard LSZ reconstruction requires:

- stable one-particle poles;
- locality and microcausality;
- spectral and clustering properties;
- asymptotic completeness in the relevant sector.

Where these are not established, the paper claims only the perturbative Green-function and cohomological architecture, not a completed rigorous scattering theorem.

---

39. QED Infrared and QCD Confinement Firewalls

39.1 QED

A charged particle carries a long-range electromagnetic field. A finite-particle Fock state without its soft photon cloud is not a complete physical asymptotic state.

Physical predictions use:

- inclusive Bloch–Nordsieck/KLN-type probabilities;
- coherent-state dressings;
- finite detector resolution;
- infrared-safe observables.

39.2 QCD

The perturbative BRST cohomology of the local gauge-fixed theory does not by itself prove confinement.

At short distances, quark and gluon amplitudes are useful ingredients. At long distances, physical asymptotic states must be gauge-invariant colour singlets or admissibly dressed composites.

The QCD confinement gate supplies the separate triality, transfer-matrix and flux-sector analysis.

Corollary 39.1 — No contradiction

Perturbative quark/gluon Green functions and absence of free coloured asymptotic states are compatible, because they concern different stages of observable construction.

---

40. Unstable Particles and Resonance Poles

An unstable particle is not an exact asymptotic eigenstate.

Its physical information lies in the complex pole

$s_\star = M_\star^2 - i\, M_\star\, \Gamma_\star$

of the appropriate analytically continued amplitude.

Proposition 40.1 — Gauge-Consistent Resonance Definition

The position of the properly defined complex pole is gauge independent, even though individual self-energy functions and off-shell propagator components may be gauge dependent. For the Standard Model this has been established to all orders through extended Nielsen-identity arguments (Gambino–Grassi; Appendix J, ref. 27).

VERSF consequence

Predictions for W, Z, Higgs and top properties must be formulated through:

- complex poles;
- residues;
- partial widths;
- gauge-invariant production and decay amplitudes.

Part VIII — Global and Nonperturbative Completion

41. Local Lie Algebra Versus Faithful Global Quotient

The BRST differential uses:

$\mathfrak{g}$_SM.

It does not determine which principal

G_SM^faithful

bundles are summed over.

Theorem 25 — Local/Global Quantum Separation

A locally anomaly-free BRST theory can still fail globally if:

- the determinant line has nontrivial holonomy;
- a large gauge transformation changes the fermion partition-function sign or phase;
- a non-liftable quotient bundle is omitted;
- the line-operator spectrum is inconsistent;
- a torsion anomaly survives.

Therefore local BRST closure is necessary but not sufficient for global quantum completion.

---

42. Bundle Sums and Determinant-Line Topology

For a fixed bundle P,

$$Z_P = \int_{\mathscr{F}(P)/\mathcal{G}(P)} d\mu_P \cdot e^{-S_P}.$$

The complete partition function is schematically

$$Z = \Sigma_{[P] \in \mathfrak{B}_G(X)} \chi_{top}([P]) \cdot Z_P,$$

where $\mathfrak{B}_G(X)$ is the admissible bundle census and χ_{top} denotes allowed continuous or discrete topological data.

The chiral fermion determinant is generally a section of a line bundle over gauge-orbit space rather than an ordinary globally defined function.

Global completion requirement

The tensor product of all chiral determinant lines and required counterterm lines must possess a consistent gauge-equivariant trivialisation over the full admissible configuration space.

This is the global form of anomaly cancellation.

---

43. Gribov Copies and the Failure of Global Gauge Slicing

A non-Abelian gauge condition such as

$$\partial^\mu A_\mu = 0$$

can intersect one gauge orbit more than once.

These multiple intersections are Gribov copies.

The Faddeev–Popov determinant can vanish on the Gribov horizon, marking the failure of the local slice as a global coordinate.

Theorem 26 — Perturbative-Patch Limitation

The ordinary Faddeev–Popov and BRST construction is valid in a local regular orbit patch. It does not prove that one globally defined gauge-fixing fermion intersects every non-Abelian gauge orbit exactly once.

Possible VERSF completion routes

1. Derive a canonical fundamental modular region — or a Gribov–Zwanziger-type horizon restriction — from the substrate record metric.
2. Quantise directly in gauge-invariant holonomy and flux variables.
3. Use overlapping BRST/BV charts with a consistent global descent.
4. Construct a nonperturbative lattice or transfer-matrix measure and recover BRST only in the continuum physical sector.

No route is assumed complete here.

---

44. Lattice BRST and the Neuberger Obstruction

On a compact lattice gauge group, a direct BRST-exact gauge-fixing partition function can exhibit exact cancellations among Gribov copies, producing the Neuberger 0/0 problem.

Thus simply discretising the continuum BRST action does not automatically produce a valid nonperturbative measure.

Firewall

This does not invalidate perturbative BRST.

It shows that nonperturbative global gauge fixing requires additional structure beyond the naive lattice transcription of the local complex.

---

45. Nonperturbative Chiral-Measure Gate

A full nonperturbative Standard Model construction must define the phase of the chiral fermion measure consistently across all gauge backgrounds.

The required object must:

- reproduce local anomaly cancellation;
- reproduce global anomalies where present;
- respect the faithful quotient;
- handle topological sectors;
- maintain locality;
- possess the correct continuum limit.

QGC-1 status

This paper classifies the gate. It does not construct the complete measure from VERSF commitment primitives.

---

46. Microcausality, Reflection Positivity and Reconstruction

A Euclidean functional integral becomes a unitary Lorentzian theory only under additional reconstruction conditions.

These include appropriate forms of:

- reflection positivity;
- Euclidean covariance;
- regularity;
- clustering;
- spectral positivity.

The earlier VERSF free-field programme has not yet established full light-cone-respecting microcausality from substrate first principles.

Theorem 27 — Reconstruction Limitation

Perturbative BRST unitarity does not by itself establish a nonperturbative positive Hilbert space obtained from the complete Euclidean VERSF measure.

The latter requires an independent reconstruction theorem.

47. Relation to Strong CP, Running and Confinement

47.1 Strong CP

QGC-1 supplies:

- the axial Jacobian;
- the sector-weight identity and its combined-line reading;
- the RG identity

  $\mu \, d\bar{\theta}/d\mu = \beta_\theta + \mathrm{Im}\,\mathrm{Tr}(M_u^{-1} \beta_{M_u} + M_d^{-1} \beta_{M_d});$

- complex-mass threshold matching.

It does not choose the trivial holonomy of the combined line.

47.2 Running

QGC-1 supplies the gauge–ghost determinant and background Ward identity required by the gauge and QCD running papers.

It does not supply numerical matching-scale response coefficients.

47.3 Confinement

BRST removes gauge-redundant polarisations. It does not prove an area law, mass gap or absence of isolated colour.

The triality-gap and transfer-matrix programme remains separately load bearing.

Part IX — Closure

48. The Quantum Gauge Completion Theorem

Theorem 28 — Quantum Gauge Completion of the VERSF Standard Model: Perturbative Form

Assume P1–P10, P12–P16 and P19–P24, together with P25 for the mixed gravitational row of the census and P26 for the one-loop flow window.

Then, in every regular perturbative orbit patch around an admissible VERSF background:

1. the substrate pushforward measure descends locally to gauge-orbit space;
2. the orbit quotient has a Faddeev–Popov representation;
3. the determinant is represented by a typed ghost sector carrying no physical particle states;
4. the inherited gauge algebra generates a nilpotent BRST differential;
5. the gauge-fixed theory admits a BV master action satisfying the classical master equation;
6. the inherited Standard Model matter census has vanishing local gauge-anomaly class;
7. all local quantum Slavnov–Taylor breakings are removable order by order;
8. the singlet axial anomaly survives with the index-theorem coefficient;
9. the one-loop gauge flow is

$(b_Y, b_2, b_3) = (41/6, -19/6, -7)$;

10. the renormalisable Standard Model operator core is perturbatively closed;
11. higher-dimensional VERSF operators form an ordered EFT ledger;
12. BRST-closed physical observables are gauge-fixing independent;
13. the scattering operator descends to physical cohomology;
14. under positive-cohomology and infrared premises, physical transition probabilities are perturbatively unitary.

Proof

Items 1–3 follow from Theorems 1–3. Items 4–5 follow from Theorems 4–6. The anomaly statement follows from Theorems 10–14 and the explicit census. All-order restoration follows from Theorem 18. The beta coefficients follow from Theorem 15. Counterterm closure follows from Theorems 19–20. Gauge independence follows from Theorems 8–9. Physical-state descent and unitarity follow from Theorems 22–24. ■

Corollary 48.1 — Global quantum completion

If P11, P17 and P18 are also discharged, and the complete regulated measure satisfies the required positivity and reconstruction conditions, the local completion extends to the full faithful-group configuration space.

Status

The perturbative theorem is conditionally closed at architecture level.

The global corollary is open.

---

49. What Has Actually Been Derived

QGC-1 derives or fixes:

1. the exact local relation between gauge-orbit integration and Faddeev–Popov gauge fixing;
2. the ghost sector as the Grassmann representation of the orbit Jacobian;
3. the rejection of ghosts as physical substrate particles;
4. the full Standard Model BRST differential;
5. off-shell nilpotency from the Jacobi identity and representation closure;
6. the BRST-exact gauge-fixing action;
7. the classical BV master equation;
8. the Slavnov–Taylor functional;
9. gauge-fixing independence of BRST observables;
10. the Nielsen identity and its physical interpretation;
11. anomaly breaking as ghost-number-one BRST cohomology;
12. explicit cancellation of every local Standard Model gauge anomaly;
13. absence of the ordinary SU(2) Witten obstruction;
14. survival and correct typing of the axial anomaly;
15. the strong-phase sector-weight identity Z_ν ∝ e^{iνθ}, with the combined anomaly line as its geometric reading;
16. the one-loop coefficients 41/6, −19/6, −7;
17. separation of coupling boundary values from quantum running;
18. all-order restoration of the Slavnov–Taylor identity under the declared cohomological premises;
19. renormalisable-core counterterm closure;
20. ordered EFT closure for higher VERSF operators;
21. BRST physical-state cohomology;
22. conditional perturbative unitarity and optical-theorem closure;
23. the correct observable firewalls for QED, QCD and unstable particles;
24. the exact boundary between perturbative and global nonperturbative completion.

QGC-1 does **not** derive:

- the microscopic substrate-to-continuum measure;
- a proof that its Jacobian is local;
- the complete light-cone microcausality theorem;
- a global Gribov-free gauge slice;
- a nonperturbative chiral fermion measure;

- the full global anomaly classification of the faithful quotient;
- reflection positivity of the complete Euclidean VERSF measure;
- constructive existence of four-dimensional Yang–Mills or the full Standard Model;
- nonperturbative confinement;
- asymptotic completeness;
- numerical gauge couplings;
- numerical masses or mixing parameters;
- numerical decay rates or cross sections;
- empirical confirmation.

---

50. Open Debts

Debt 1 — Microscopic measure construction. Construct μ_sub and the map $\mathfrak{Q}$ sufficiently explicitly to evaluate $\mathfrak{Q}_* \mu$_sub.

Debt 2 — Continuum Jacobian. Calculate $\mathcal{J}$_sub→cont and classify its local, topological and anomalous contributions.

Debt 3 — Gauge-basicity theorem. Prove that gauge-equivalent continuum representatives arise from one substrate physical history rather than assuming it.

Debt 4 — Locality and derivative hierarchy. Derive the locality or controlled quasi-locality required by the quantum action principle.

Debt 5 — Complete microcausality. Prove light-cone-respecting (anti)commutativity for the interacting emergent field algebra.

Debt 6 — Substrate-native BRST derivation. Construct the BRST complex directly from the substrate quotient rather than importing its continuum algebra after the orbit map is assumed.

Debt 7 — Substrate-native BV Laplacian. Derive the regulated Δ_reg operator and quantum master equation from the microscopic measure.

Debt 8 — Regulator selection. Identify the regulator naturally induced by VERSF and prove its admissibility.

Debt 9 — Chiral γ_5 completion. Complete the all-loop chiral prescription and symmetry-restoring counterterm audit.

Debt 10 — Local anomaly descent. Derive the anomaly polynomial and descent sequence from commitment complexes rather than importing the record-layer result.

Debt 11 — Axial index derivation. Derive the Fujikawa/index coefficient from the VERSF fermionic measure.

Debt 12 — Global anomaly classification. Derive from commitment primitives the determinant-line and bordism obstruction — the pairing of the inherited census against $\tilde{\Omega}^5$_Spin(B G_SM^faithful) or its spin-G refinement — for all admissible faithful-quotient bundles. The external bordism result (Appendix J, refs. 25–26) is corroborating, not constitutive.

Debt 13 — Hidden-sector exclusion. Prove that no additional anomaly-carrying chiral states are required.

Debt 14 — Full coupling boundary values. Evaluate Z_3, Z_2, Z_1 and provide the quantum matching scheme.

Debt 15 — Two-loop and higher beta functions. Recover the complete multi-loop gauge, Yukawa and scalar flow from the frozen VERSF boundary action.

Debt 16 — Matrix RG completion. Calculate the full closure-derived Yukawa running and threshold rotation of CKM and PMNS frames.

Debt 17 — Higgs effective-action audit. Perform a Nielsen-consistent vacuum and pole-mass calculation.

Debt 18 — EFT operator census. Enumerate all substrate-generated higher-dimensional operators and their power counting.

Debt 19 — Complex-threshold audit. Carry out CP-sensitive matching across every heavy fermion and scalar threshold.

Debt 20 — Gribov completion. Construct a canonical modular region or gauge-invariant substitute from VERSF structure.

Debt 21 — Nonperturbative chiral measure. Define the full measure over all global sectors without gauge or global anomaly.

Debt 22 — Reflection positivity. Prove positivity for gauge-invariant physical correlators in the regulated Euclidean theory.

Debt 23 — Physical Hilbert reconstruction. Construct the nonperturbative positive physical Hilbert space.

Debt 24 — Confinement compatibility. Join the BRST local state complex to the triality-resolved confining Hilbert space.

Debt 25 — QED dressing. Construct the admissible charged asymptotic record with its infrared photon dressing.

Debt 26 — Asymptotic completeness. Establish the stable-state and scattering completeness conditions in each physical sector.

Debt 27 — Observable matching. Map the renormalised VERSF scheme to experimentally used observables.

Debt 28 — Blind numerical calculation. Publish the frozen boundary data, RG flow and amplitudes before empirical comparison.

---

51. Falsification Conditions

F1 — No measure pushforward. The substrate history measure cannot be mapped consistently to continuum fields.

F2 — Non-basic gauge measure. Gauge-equivalent representatives receive inequivalent physical weights without a classified anomaly.

F3 — Orbit multiplicity. The perturbative gauge slice contains uncontrolled copies even in the claimed local domain.

F4 — Singular Faddeev–Popov operator. Unremoved zero modes invalidate the determinant.

F5 — Physical ghost failure. The construction requires ghosts to appear as positive-norm asymptotic particles.

F6 — BRST non-nilpotency. The derived gauge transformations fail $s^2 = 0$.

F7 — Master-equation failure. The inherited action cannot satisfy the classical BV master equation.

F8 — Gauge-anomaly residue. The complete inherited chiral census has a nonzero local gauge-anomaly class.

F9 — Hidden anomaly reservoir. Gauge consistency requires uncounted chiral exotics.

F10 — Global anomaly. A faithful-quotient bundle produces a nontrivial determinant-line obstruction.

F11 — Axial-anomaly loss. The regulated VERSF measure fails to reproduce the QCD axial Jacobian.

F12 — Axial/gauge conflation. The theory cancels the physical axial anomaly together with the gauge anomalies.

F13 — Wrong beta census. The one-loop determinants do not return 41/6, −19/6, −7 for the declared field content and convention.

F14 — Coupling insertion. Measured couplings are used as substrate response values and then called predictions.

F15 — Slavnov–Taylor failure. A non-anomalous breaking cannot be removed by an allowed local counterterm.

F16 — Counterterm incompleteness. Loops generate an admissible operator missing from the declared ledger.

F17 — Uncontrolled EFT. Higher-dimensional operators appear without a suppressing hierarchy.

F18 — Gauge-dependent observable. A claimed physical prediction changes with the gauge-fixing parameter.

F19 — Off-shell laundering. A gauge-dependent effective-potential feature is reported as a physical result without Nielsen control.

F20 — Threshold mismatch. Heavy fields are removed without the operators required to preserve gauge and anomaly identities.

F21 — Strong-phase mismatch. Complex heavy masses are removed without the compensating θ-shift.

F22 — Asymptotic-charge failure. No conserved nilpotent asymptotic BRST charge exists in a sector where scattering observables are claimed.

F23 — Negative physical cohomology. A negative-norm class survives in $\mathscr{H}$_phys.

F24 — Unitarity failure. The physical optical theorem fails after all required states and cuts are included.

F25 — Free-colour overclaim. A partonic state is presented as an isolated physical asymptotic state.

F26 — QED infrared overclaim. An undressed exclusive charged-particle Fock amplitude is reported as a finite physical observable.

F27 — Unstable-state overclaim. A resonance is treated as an exact asymptotic particle.

F28 — Gribov silence. A local BRST patch is described as a global nonperturbative quotient without further construction.

F29 — Neuberger silence. A naive lattice BRST transcription is treated as a valid nonperturbative measure despite exact copy cancellations.

F30 — Global-group erasure. The local Lie algebra is used to replace the faithful bundle and line-operator audit.

F31 — Microcausality failure. The interacting emergent fields violate causal locality in the regime where the standard quantum reconstruction is claimed.

F32 — Regulator dependence. Different admissible regulators produce inequivalent physical predictions after finite matching.

F33 — Numerical branch selection. A regulator, counterterm or determinant-line frame is chosen after consulting observation.

F34 — Empirical disagreement. After blind boundary calculation, running and matching, physical predictions disagree with observation beyond the declared uncertainty and EFT freedom.

---

52. Numerical Non-Insertion Audit

The following operations are forbidden.

1. Inserting the measured beta coefficients into the VERSF action rather than deriving them from the inherited field census.
2. Selecting a regulator because it reproduces a preferred measured number.
3. Adding hidden fermions solely to cancel an anomaly discovered after the fact.
4. Choosing finite counterterms to fit masses, couplings or cross sections.
5. Using measured g_s, g, g′ to normalise the substrate measure.
6. Choosing the matching scale by minimising disagreement with experiment.
7. Treating the MS-bar scheme as a physical law rather than a declared convention.
8. Ignoring threshold corrections because they complicate a preferred result.
9. Rephasing quark masses to make det M_q positive while ignoring the anomalous θ-shift.
10. Choosing a gauge because its off-shell potential most closely resembles a desired VERSF picture.
11. Reporting a gauge-parameter-dependent vacuum coordinate as an observable.
12. Removing higher-dimensional operators after seeing that they spoil agreement.
13. Treating ghosts as extra physical bath states to repair a degree-of-freedom count.
14. Using a local BRST proof as a substitute for the global $\mathbb{Z}_6$ anomaly audit.
15. Ignoring Gribov copies in a claimed nonperturbative result.
16. Comparing a bare VERSF coupling directly with a measured running coupling.
17. Reporting partonic amplitudes as physical QCD cross sections without factorisation and hadronisation control.

18. Reporting infrared-divergent exclusive QED amplitudes as observables.
19. Treating a resonance-field mass parameter as a gauge-independent pole mass.
20. Calling post-fit renormalisation conditions a first-principles prediction.

Blind protocol

A valid calculation proceeds in this order:

1. freeze the substrate measure and continuum map;
2. freeze the faithful global group and field census;
3. choose the regulator from structural criteria;
4. derive the orbit quotient and BRST/BV complex;
5. calculate the anomaly class;
6. freeze all symmetry-restoring counterterms;
7. calculate the gauge, Yukawa and scalar boundary operators;
8. derive their loop flow;
9. perform threshold matching;
10. define physical infrared-safe observables;
11. publish the result and uncertainty;
12. only then compare with experiment.

---

53. QGC-1 Closure Certificate

Gate question

Has VERSF converted its derived classical Standard Model architecture into a quantum gauge theory with controlled redundancy, anomaly closure, renormalisation, physical-state selection and observable unitarity?

Closure answer

Perturbative local quantum architecture: yes, conditionally. Local gauge-anomaly audit: yes. One-loop Standard Model running census: yes. All-order algebraic-renormalisation route: yes, conditionally. Physical-state unitarity: conditional on positive BRST cohomology and infrared completion. Global nonperturbative quantum measure: not yet. Numerical Standard Model prediction: not yet.

Local certificate

Given

$\{ \mu_{cont}, F^A, \mathscr{M}_{FP}, S_{cl}, \Psi_{gf} \}$,

the local quantum action is

$S_{QGC} = S_{cl} + s\,\Psi_{gf}$.

Its BV extension satisfies

$(S_{BV}, S_{BV}) = 0$.

Anomaly certificate

Given the inherited field census,

$[\mathcal{A}_{gauge}] = 0$,

while

$\partial_\mu J_5^\mu \propto F\,\tilde{F}$

survives.

Running certificate

Given the background-field quantum gate,

$\mathbf{(b_Y, b_2, b_3) = (41/6, -19/6, -7).}$

Renormalisation certificate

Given locality, power counting and anomaly cancellation,

$\mathcal{S}(\Gamma_R) = 0$

can be maintained to all perturbative orders, with divergences absorbed into the complete renormalisable-plus-EFT operator ledger.

Physical-state certificate

Given

$Q_B^2 = 0$, $[Q_B, S] = 0$,

and positive cohomology,

$S\_phys^\dagger S\_phys = 1$.

Global certificate still required

The complete programme must additionally supply

{ quotient-compatible bundle sum, global determinant-line trivialisation, Gribov control, nonperturbative chiral measure, physical positivity }.

---

54. Conclusion

The VERSF Standard Model programme has now reached the boundary between architecture and quantum existence.

The earlier papers derive the internal gauge pattern, the matter representations, the closure interface, the gauge kinetic response, the Yukawa operators, the flavour frames, the running architecture, the confinement criterion and the global strong-phase problem.

None of those results alone guarantees that the fields form a consistent quantum gauge theory.

The missing structure begins with redundancy.

A gauge connection contains more mathematical variables than physical degrees of freedom, because local changes of internal frame do not create new physical worlds. The quantum measure must therefore descend from continuum field space to gauge-orbit space.

In a regular local patch, that descent is exact:

orbit quotient $\longrightarrow \delta[F] \cdot \det \mathscr{M}\_FP$.

The ghost fields are the Grassmann representation of the determinant. They are not new particles and not hidden substrate carriers.

The local gauge algebra then produces the BRST differential:

$s^2 = 0$.

Gauge fixing becomes a BRST-exact deformation:

$S\_gf+gh = s\ \Psi\_gf$.

The BV master equation packages the complete gauge algebra:

$(S\_BV, S\_BV) = 0$.

At the quantum level, every possible failure of gauge consistency becomes a cohomology class. A trivial breaking is removable. A nontrivial gauge anomaly is not.

The inherited Standard Model charge table passes that test. Its local gauge-anomaly class vanishes, and its weak-doublet count avoids the ordinary global SU(2) obstruction. This confirms, at the quantum-master-equation level, the earlier VERSF interpretation that an anomaly-free ledger is the only source ledger that can persist.

But the quantum completion does not erase every anomaly.

The axial anomaly survives. It records real spectral flow between fermion chirality and gauge topology and constructs the invariant strong phase:

$\bar{\theta} = \theta_3 + \arg\det(M\_u\ M\_d)$.

The topological phase and the determinant phase are not two unrelated parameters. They are two coordinates on one invariant sector-weight phase — geometrically, one combined anomaly line. This paper supplies that invariance. The strong-CP paper must still calculate whether the substrate trivialises it on the positive real ray.

Loop corrections are no longer an external addition. The background-field determinants of the derived gauge, ghost, matter and scalar carriers return the Standard Model flow:

$(\ 41/6, -19/6, -7\ )$.

The boundary values remain the substrate response coefficients. The quantum gate carries them across scale.

Renormalisation is likewise typed rather than hoped for. Gauge-anomaly cancellation and BRST cohomology imply that the dimension-four Standard Model core closes under perturbative counterterms. Higher-dimensional closure and bath operators are not ignored; they enter an ordered effective-field-theory ledger.

Finally, the enlarged gauge-fixed state space is not mistaken for the physical universe.

The physical Hilbert space is the BRST cohomology:

$\mathscr{H}\_phys = \ker Q\_B / \mathrm{im}\ Q\_B$.

Longitudinal gauge modes, timelike modes and ghosts cancel in quartets. If the cohomology is positive and the scattering operator commutes with the BRST charge, physical amplitudes are unitary.

The observable statement is necessarily more careful than "every Standard Model field has an S-matrix."

Quarks and gluons enter short-distance amplitudes but not isolated asymptotic states. Charged particles require infrared dressing or inclusive probabilities. Unstable particles appear through complex poles. Physical predictions are therefore gauge-invariant, infrared-admissible observables — not arbitrary gauge-fixed Green functions.

The strongest completed chain is:

substrate histories ⟶ continuum pushforward measure ⟶ gauge-orbit quotient ⟶ Faddeev–Popov determinant ⟶ BRST/BV complex ⟶ anomaly audit ⟶ Slavnov–Taylor closure ⟶ renormalised effective action ⟶ physical BRST cohomology ⟶ unitary observables.

This paper closes that chain conditionally in perturbation theory.

It does not hide the final global debts. The full faithful-group bundle census, Gribov problem, chiral determinant-line measure, regulator removal, reflection positivity, microcausality and nonperturbative physical Hilbert space remain to be constructed.

That limitation is not a weakness of the paper. It is the line that separates a real quantum-completion programme from a collection of formal loop equations.

The next and final programme-level calculation is exact:

Construct the VERSF substrate measure on the full faithful-group orbit space, prove its quotient-compatible chiral determinant line globally consistent, control Gribov copies without losing physical positivity, and recover the BRST, anomaly, renormalisation and unitarity identities as the local limit of that nonperturbative measure.

If that construction succeeds, VERSF will no longer merely reproduce the fields and parameters of the Standard Model.

It will possess the quantum mechanism that makes those fields a consistent physical theory.

Appendix A — Faddeev–Popov Orbit-Disintegration Proof

Let $\pi : \mathscr{F}\_reg \to \mathscr{F}\_reg/\mathcal{G}$ be the local principal-bundle projection. Let F = 0 define a transverse local section. For a fixed φ,

I[φ] = ∫ Dg · δ[F(φ^g)].

Near the unique intersection $g\star$,

$$F^A(\varphi^g) = \mathscr{M}\_FP^{AB} \cdot (\alpha - \alpha\star)\_B + \cdots$$

The functional delta identity gives

$$I[\varphi] = 1 / |\det'\mathscr{M}\_FP[\varphi]|.$$

Thus

$$1 = |\det'\mathscr{M}\_FP| \int Dg \cdot \delta[F(\varphi^g)].$$

In the perturbative orientation branch the determinant sign is fixed, yielding the standard determinant. Inserting the identity and changing variables along the orbit proves Theorem 2.

If multiple intersections occur, the integral counts copies and the simple formula no longer represents a one-to-one quotient.

Appendix B — BRST Nilpotency and Gauge-Fixing Proof

In matrix notation,

$$s A\_\mu = D\_\mu c, \; s c = -\tfrac{1}{2} [c, c].$$

Then

$$s^2A\_\mu = D\_\mu sc + [D\_\mu c, c].$$

Using

$$D\_\mu [c, c] = [D\_\mu c, c] + [c, D\_\mu c] = 2 [D\_\mu c, c]$$

for Grassmann-odd c,

$$s^2A\_\mu = -\tfrac{1}{2} D\_\mu [c, c] + [D\_\mu c, c] = 0.$$

Similarly,

$$s^2c = \tfrac{1}{4} ([[c, c], c] - [c, [c, c]]) = 0$$

by the graded Jacobi identity.

For the gauge-fixing action,

$s(S_{cl} + s\Psi) = s S_{cl} + s^2\Psi = 0.$

Appendix C — BV Antibracket and Slavnov–Taylor Identity

For

$S_{BV} = S_{cl} + \varphi^*_I R^I(\varphi),$

the master equation expands as:

$(S_{cl}, S_{cl}) = 0,$

$2 (S_{cl}, \varphi^* R) = 2 \cdot (\delta S_{cl}/\delta\varphi^I) \cdot R^I = 0,$

by gauge invariance, while

$(\varphi^* R, \varphi^* R) = 0$

is the closure and nilpotency condition.

After gauge fixing and Legendre transformation, the same identity becomes the Zinn–Justin or Slavnov–Taylor functional equation for Γ.

Appendix D — Standard Model Anomaly Arithmetic

For one generation:

Sector	Multiplicity	Y
Q_L	6	1/6
u_L^c	3	−2/3
d_L^c	3	1/3
L_L	2	−1/2
e_L^c	1	1

The linear sum is

$6\cdot(1/6) + 3\cdot(-2/3) + 3\cdot(1/3) + 2\cdot(-1/2) + 1 = 0.$

The cubic sum is

$6\cdot(1/6)^3 + 3\cdot(-2/3)^3 + 3\cdot(1/3)^3 + 2\cdot(-1/2)^3 + 1 = 0.$

The non-Abelian mixed sums are those given in §18.

A sterile $(1, 1)_0$ contributes zero to every gauge anomaly.

Appendix E — Standard Model One-Loop Beta Arithmetic

The general coefficient is

$B\_G = (11/3)\cdot C_2(G) - (2/3)\cdot\Sigma\_\text{Weyl}\ T(R\_f) - (1/3)\cdot\Sigma\_\text{complex}\ T(R\_s).$

For SU(3):

$B_3 = 11 - (2/3)\cdot 6 = 7$, $b_3 = -7$.

For SU(2):

$B_2 = 22/3 - (2/3)\cdot 6 - (1/3)\cdot(½) = 19/6$,

$b_2 = -19/6$.

For U(1)_Y:

$\Sigma\_\text{Weyl}\ d\cdot Y^2 = 10$, $\Sigma\_\text{scalar}\ d\cdot Y^2 = ½$,

$b\_Y = (2/3)\cdot 10 + (1/3)\cdot(½) = 41/6.$

Appendix F — Algebraic-Renormalisation Induction

Assume

$\mathcal{S}(\Gamma) = O(\hbar^n)$.

At order n,

$\mathcal{S}(\Gamma) = \hbar^n \mathcal{A}_n$.

Nilpotency of the linearised Slavnov operator gives

$\mathcal{B}_S \mathcal{A}_n = 0$.

If the breaking has vanishing class,

$[\mathcal{A}_n] = 0$ in $H^1(\mathcal{B}_S \mid d)$,

as the census guarantees (Theorem 11), then

$\mathcal{A}_n = \mathcal{B}_S \Xi_n + dK_n$.

The integrated total derivative drops under the declared boundary conditions. Adding

$-\hbar^n \Xi_n$

to Γ cancels the breaking. The procedure iterates.

At ghost number zero,

$\mathcal{B}_S \Sigma = 0$

has solutions

$\Sigma = \Sigma_{inv} + \mathcal{B}_S \Delta$,

giving the physical/exact counterterm split.

Appendix G — BRST Cohomology and Unitarity Proof

Let

$\mathcal{K} = \ker Q_B \oplus \mathcal{K}_{nonclosed}$

be the extended state space.

Nilpotency implies

$\mathrm{im}\, Q\_B \subset \ker Q\_B$.

Hermiticity gives, for a closed state $|\psi\rangle$,

$\langle\psi| Q\_B |\chi\rangle = \langle Q\_B\, \psi \mid \chi\rangle = 0$.

Thus exact states are orthogonal to closed states and represent null physical vectors.

If $[S, Q\_B] = 0$, S acts on the quotient. If the inherited inner product is positive on the quotient and the extended-space pseudo-unitarity relation holds, the induced quotient operator is unitary.

Appendix H — Global Quotient, Gribov and Regulator Audit Protocol

A complete nonperturbative QGC calculation must:

1. specify the 4-manifold category and spin structure;
2. freeze G_SM^faithful;
3. classify all admissible principal bundles;
4. derive the substrate-to-bundle map;
5. define the chiral determinant line on each sector;
6. calculate all local and global anomaly classes;
7. specify topological characters and discrete terms;
8. define the orbit measure without uncontrolled copies;
9. prove regulator locality;
10. prove gauge-invariant reflection positivity;
11. establish the continuum limit;
12. reconstruct the physical Hilbert space;
13. recover the perturbative BRST/BV identities in local charts;
14. join the result to the confinement and infrared sectors;
15. only then claim global quantum completion.

Appendix I — Dependency and Falsifier Map

Result	Principal dependencies	Principal falsifiers
Measure descent	P4–5	F1–F2
FP quotient	P6	F3–F4
Ghost typing	FP determinant	F5
BRST nilpotency	P8	F6
BV master equation	Classical gauge invariance	F7
Local anomaly cancellation	P10	F8–F9
Global anomaly freedom	P11, 17	F10, F30
Axial anomaly	P12	F11–F12
Gauge beta census	P13, field census	F13–F14
All-order ST identity	P7, 10, 14, 15	F15–F17
Gauge independence	ST identity	F18–F19
Threshold consistency	P16	F20–F21
Physical cohomology	P19–20	F22–F23
Physical unitarity	Positive cohomology, ST	F24
Observable reconstruction	P21, 23	F25–F27, F31
Global completion	P11, 17–18	F28–F30, F32
Numerical prediction	All relevant rows	F33–F34

Appendix J — Internal and External References

Standard external foundations

1. L. D. Faddeev and V. N. Popov, "Feynman Diagrams for the Yang–Mills Field," *Physics Letters B* **25**, 29–30 (1967).
2. C. Becchi, A. Rouet and R. Stora, "Renormalization of Gauge Theories," *Annals of Physics* **98**, 287–321 (1976).
3. I. V. Tyutin, "Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism," Lebedev preprint (1975), later published in arXiv translation.
4. A. A. Slavnov, "Ward Identities in Gauge Theories," *Theoretical and Mathematical Physics* **10**, 99–107 (1972).
5. J. C. Taylor, "Ward Identities and Charge Renormalization of the Yang–Mills Field," *Nuclear Physics B* **33**, 436–444 (1971).
6. I. A. Batalin and G. A. Vilkovisky, "Gauge Algebra and Quantization," *Physics Letters B* **102**, 27–31 (1981).
7. L. F. Abbott, "Introduction to the Background Field Method," *Acta Physica Polonica B* **13**, 33 (1982).

8. T. Kugo and I. Ojima, "Local Covariant Operator Formalism of Non-Abelian Gauge Theories and Quark Confinement Problem," *Progress of Theoretical Physics Supplement* **66**, 1–130 (1979).
9. K. Fujikawa, "Path-Integral Measure for Gauge-Invariant Fermion Theories," *Physical Review Letters* **42**, 1195 (1979).
10. K. Fujikawa, "Path Integral for Gauge Theories with Fermions," *Physical Review D* **21**, 2848 (1980).
11. J. Wess and B. Zumino, "Consequences of Anomalous Ward Identities," *Physics Letters B* **37**, 95–97 (1971).
12. S. L. Adler and W. A. Bardeen, "Absence of Higher-Order Corrections in the Anomalous Axial-Vector Divergence Equation," *Physical Review* **182**, 1517 (1969).
13. E. Witten, "An SU(2) Anomaly," *Physics Letters B* **117**, 324–328 (1982).
14. N. K. Nielsen, "On the Gauge Dependence of Spontaneous Symmetry Breaking in Gauge Theories," *Nuclear Physics B* **101**, 173–188 (1975).
15. V. N. Gribov, "Quantization of Non-Abelian Gauge Theories," *Nuclear Physics B* **139**, 1–19 (1978).
16. H. Neuberger, "Nonperturbative BRS Invariance and the Gribov Problem," *Physics Letters B* **183**, 337–340 (1987).
17. O. Piguet and S. P. Sorella, *Algebraic Renormalization*, Springer Lecture Notes in Physics Monographs.
18. T. Appelquist and J. Carazzone, "Infrared Singularities and Massive Fields," *Physical Review D* **11**, 2856 (1975).
19. O. Aharony, N. Seiberg and Y. Tachikawa, "Reading between the Lines of Four-Dimensional Gauge Theories," *Journal of High Energy Physics* **08** (2013) 115.
20. D. Tong, "Line Operators in the Standard Model," *Journal of High Energy Physics* **07** (2017) 104.
21. J. Zinn-Justin, "Renormalization of Gauge Theories," in *Trends in Elementary Particle Theory*, Lecture Notes in Physics **37**, Springer (1975).
22. G. 't Hooft and M. Veltman, "Regularization and Renormalization of Gauge Fields," *Nuclear Physics B* **44**, 189–213 (1972).
23. P. Breitenlohner and D. Maison, "Dimensional Renormalization and the Action Principle," *Communications in Mathematical Physics* **52**, 11–38 (1977).
24. G. Barnich, F. Brandt and M. Henneaux, "Local BRST Cohomology in Gauge Theories," *Physics Reports* **338**, 439–569 (2000).
25. I. García-Etxebarria and M. Montero, "Dai–Freed Anomalies in Particle Physics," *Journal of High Energy Physics* **08** (2019) 003.
26. J. Davighi, B. Gripaios and N. Lohitsiri, "Global Anomalies in the Standard Model(s) and Beyond," *Journal of High Energy Physics* **07** (2020) 232.
27. P. Gambino and P. A. Grassi, "The Nielsen Identities of the SM and the Definition of Mass," *Physical Review D* **62**, 076002 (2000).