The Quark-Sector Closure Ledger in VERSF

A Projected-Hessian Account of CKM Amplitudes, the C_3 Triangle Residue, Phase Provenance, and the Remaining Empirical Tensions

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General-reader summary

Quarks are the particles that make up protons and neutrons. They come in three families, and a quark in one family can occasionally turn into a quark in another. How readily each of these changes happens is captured by a small set of numbers, which experiments have measured very precisely. The Standard Model — physics' current best theory — uses these numbers but does not explain why they take the values they do; it simply accepts them as given.

The VERSF programme tries to go one step deeper and *calculate* these numbers from a single underlying mathematical structure, rather than putting them in by hand. This paper takes stock of how far that attempt has got for the quarks.

The honest answer is: a real step forward, but not the finish line. Some of the mixing numbers were already known and are carried over unchanged. What the calculation genuinely produces on its own is the rare change between the first and third families, together with a quantity that measures the tiny imbalance between matter and antimatter (physicists call this CP violation). On the scorecard, that matter–antimatter amount comes out close to experiment, which is encouraging. But one of the mixing angles (the one labelled γ) comes out about 7 degrees too large, and the calculation's strongest success rests on a single ingredient whose origin has not yet been independently pinned down. If that ingredient were chosen just to make the answer fit, the success would not count — so the paper is careful to flag it as the most exposed part of the result.

The paper also records a promising lead: a natural adjustment to the calculation would shrink that 7-degree gap to under 2 degrees. It is not yet accepted, both because it slightly worsens the agreement elsewhere and, more importantly, because it must fall out of the deeper theory on its own rather than be tuned to match the data. For now it is logged as a candidate, not a result.

So this is neither a finished derivation nor a mere fit. The quark-mixing problem has gone from an open wish-list of desired numbers to a short, precise checklist: a clear account of what has genuinely been derived, what was already known, what matches experiment, and exactly where the programme could still fail.

Contents

Abstract

The VERSF weak-doublet Hessian programme reduces the quark-flavour problem to a finite set of projected-Hessian returns and their algebraic consequences. The companion C_3 manuscript computes the role-even quark curvature in the minimal committed-quark C_3 branch cell, returning a scalar six-coordinate branch Hessian

$G^B\_C_3 = \kappa\_C_3 I_6$,

hence — all inside that minimal cell — C_3 covariance, democratic branch weights, the total common-mode amplitude $|\zeta| = b/\sqrt{3}$ under role-isotype norm transfer, and a $2\leftrightarrow3$ readout

survivor under no-direct-insertion branch selection. With the Hermitian minimal half-transport and the +i frame rotation, the branch returned in the minimal committed-quark C_3 branch cell is

$$\zeta\_C_3 = b/(\sqrt{3})\ e^{(5\pi i/6)}.$$

The exact commutator with the inherited quark generator then gives the leading CKM triangle residue

$$\Delta_{13} = \tfrac{1}{2}\, a\, \zeta\_C_3.$$

This manuscript assembles those results into a quark-sector closure ledger. The inherited generator is the anti-Hermitian

$$\Omega\_q = \begin{bmatrix} 0 & a & c\cdot e^{(i\phi)} \\ -a & 0 & b \\ -c\cdot e^{(-i\phi)} & -b & 0 \end{bmatrix}$$

with $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\phi = 2\pi/3$.

The returned C_3 curvature gives, in exact rational-radical form,

$$\zeta\_C_3 = -81/4000 + i\cdot 27\sqrt{3}/4000 = -0.0202500 + 0.0116913i,$$

$$\Delta_{13} = -729/320000 + i\cdot 243\sqrt{3}/320000 = -0.0022781 + 0.0013153i,\ |\Delta_{13}| = 0.0026306.$$

Exponentiating the assembled unitary returns a full CKM triangle. Against current benchmark CKM data, $|V\_ub|$ is slightly low, α and β are good, the Jarlskog invariant is close, and γ remains high. The CP agreement is explicitly graded contingent: it is predictive only if the three phase legs — the C_3 half-transport target, the Hermitian minimal-length rule, and the +i frame-rotation sign — are fixed independently of CKM data.

The conclusion is bounded. VERSF has not derived the full Standard Model. It has moved the quark mixing sector to a finite closure ledger: inherited amplitudes, a returned C_3 curvature, an exact commutator-generated residue, a full empirical confrontation, and a short list of named remaining exposures. A conditional candidate refinement of the returned amplitude ($\rho = \sqrt{3}/2$), graded Candidate/Conditional and not part of the closed ledger, is recorded separately in §18 and Appendix D.

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0. Purpose and claim level

The companion C_3 extraction theorem has the form

projected Hessian returns the C_3 branch $\Longrightarrow \zeta\_C_3 \Longrightarrow \Delta_{13}$.

The companion projected-Hessian manuscript supplies the minimal-cell calculation behind the antecedent: it computes the second variation on the six branch coordinates of the committed-quark C_3 cell and finds a scalar branch Hessian, converting the C_3 claim from a bare conditional into a minimal-cell return.

The present manuscript asks the next question:

What is the programme-level status of the entire quark mixing sector once that return is granted?

The answer is neither *finished* nor *unchanged*. The quark sector reaches a more mature stage: a finite ledger in which each ingredient carries a grade.

Quark-sector closure ledger claim. Given the inherited quark generator Ω_q, the minimal-cell projected C_3 Hessian return, the role-isotype norm transfer, the generation-depth readout, and the phase-firewall premises, the CKM sector reduces to a finite ledger: a, b, c, ϕ are inherited; ζ_C_3 is returned; Δ_{13} is commutator-generated; $|V\_ub|$, α, β, γ, J are empirically confronted; and the unresolved exposures are exactly **PH-EMB**, **PH-SAME**, **PH-RDO$_{31}$**, **PH-ROOT**, **PH-HERM**, **PH-ROT**, the amplitude firewall **PH-AMP-FIRE**, and the γ tension (with **PH-ISO** absorbed into PH-SAME, and **PH-GAP** tracked under EMB-5).

This is the intended level of the paper. It is stronger than a local C_3 computation, because it assembles the full quark-sector result. It is weaker than final Standard Model closure, because it keeps every remaining structural and provenance debt explicit.

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1. Epistemic grading

The grading vocabulary is retained as the principal protection against overclaiming.

Grade	Meaning
Inherited	Imported from earlier VERSF work or the quark-generator construction. Not rederived here.
Returned	Computed inside the committed-quark C_3 branch cell.
Exact	Follows algebraically from the stated objects, with no further premise.
Audit condition	A precise, checkable structural premise the result is conditioned on.
Conditional	Holds if its named audit conditions hold.
Empirical	Comparison with measured CKM quantities. Does not revise any internal derivation.
Owed	Required for full closure, not supplied here.
Exposure	A precise point at which the programme can fail.

The load-bearing distinction is between **sector-level ledger closure** and **microscopic closure**. This paper targets the former. The latter still requires full su(8) embedding exhaustion and independent phase provenance.

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2. Inherited quark generator

The inherited role-odd quark generator is the anti-Hermitian matrix

$$\Omega_q = \begin{bmatrix} 0 & a & c\cdot e^{i\phi} \\ -a & 0 & b \\ -c\cdot e^{-i\phi} & -b & 0 \end{bmatrix}$$

with $\Omega_q^\dagger = -\Omega_q$ and

$a = 9/40 = 0.225$, $b = 81/2000 = 0.0405$, $c = 243/100000 = 0.00243$, $\phi = 2\pi/3$.

The entries carry different statuses.

Quantity	Role	Status
a	inherited Cabibbo doorway	**Inherited**; reproduces $\lvert V_{us}\rvert$
b	inherited 2↔3 transport scale	**Inherited**; reproduces $\lvert V_{cb}\rvert$
$c\cdot e^{i\phi}$	inherited direct 1↔3 seed	**Inherited**; participates in the triangle
ζ_{C_3}	role-even C_3 common curvature	**Returned** in the minimal C_3 cell, **Conditional** on named audit conditions
Δ_{13}	generated triangle residue	**Exact** commutator consequence of a and ζ_{C_3}

This distinction is enforced throughout. The paper must not count a and b as independent successes in the CKM comparison, because they are already aligned with $\lvert V_{us}\rvert$ and $\lvert V_{cb}\rvert$ by construction. The genuine empirical test lives in the 1↔3 and CP sector.

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3. The projected object and the C_3 return

The weak-doublet projected Hamiltonian is

$H_W = P_W^\dagger H_{cl} P_W$, $P_W = P_Y P_R P_C$.

The daggered compression keeps Hermiticity explicit even when the projectors are ordered rather than mutually commuting: since H_{cl} is self-adjoint, H_W is self-adjoint. The weak-frame generator

$$\Omega_W = i\epsilon_W H_W$$

is anti-Hermitian, so its off-diagonal branches take the form

$$A_{ij}(\eta) = \eta E_{ij} - \eta^* E_{ji}.$$

Before mass/readout, the role-even quark block is the generation-blind role trace and quark compression of H_W. After readout, P_Y selects the visible branch component.

The minimal C_3 branch cell uses the six real coordinates

$$x = (s_{12}, a_{12}, s_{23}, a_{23}, s_{31}, a_{31})^T,$$

corresponding to the three Hermitian branch planes B_{12}, B_{23}, B_{31}. The companion projected-Hessian return computes the closure, transport, and record second variations as

$$G^B_C = \kappa_C I_6,\ G^B_T = \kappa_T I_6,\ G^B_R = \kappa_R I_6,$$

hence

$$G^B_{C_3} = G^B_C + G^B_T + G^B_R = \kappa_{C_3} I_6.$$

Because this scalar branch Hessian commutes with the generation cycle R_3, the role-even pre-readout quark block $\mathcal{K}$ satisfies

$$[\mathcal{K}, R_3] = 0,$$

so it is circulant,

$$\mathcal{K} = dI_3 + hR_3 + h^* R_3^\dagger,$$

and the branch weights obey

$$w_{12} = w_{23} = w_{31}.$$

This is the first pillar of the closure ledger: **C_3 democracy is returned in the minimal branch cell, not fitted from CKM data.**

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4. The amplitude ledger

C_3 democracy fixes the *split*, not the *scale*. The scale comes from the role-isotype norm transfer, which rests on two distinct audit conditions:

- **PH-ISO** — $G\_role = \kappa\_role \cdot I$ on the committed role support (role-isotype isotropy);
- **PH-SAME** — the role-even common mode and the inherited role-odd 2↔3 transport use the *same* closure profile Z.

Under these two conditions the role-even and role-odd insertions are σ-partners of one committed closure profile. The inherited role-odd 2↔3 transport has norm

$\|Z\|\_K^2 = b^2$,

so the total role-even common-mode branch budget is

$\|Z\_C_3\|\_K^2 = b^2$.

Since the C_3 branch decomposition is orthogonal and democratic,

$\|Z\_C_3\|\_K^2 = \|Z_{12}\|\_K^2 + \|Z_{23}\|\_K^2 + \|Z_{31}\|\_K^2 = 3|\zeta|^2$,

hence

$|\zeta| = b/(\sqrt{3})$.

With $b = 81/2000$,

$|\zeta| = 27\sqrt{3}/2000 = 0.0233827$.

Step	Status
C_3 equal weights	**Returned** (minimal cell)
total branch budget b^2	**Conditional** on PH-ISO ∧ PH-SAME
branch amplitude $b/\sqrt{3}$	**Exact** from equal weights + norm budget
independence from CKM $\|V\_ub\|$	valid only if b and PH-SAME are independent of the 1↔3 fit

The amplitude exposure is no longer democracy. It is the conjunction **PH-ISO ∧ PH-SAME**. If the role metric is anisotropic, the even/odd norm transfer fails. If the even common mode and odd transport do not share the closure profile, the inherited b^2 budget does not transfer.

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5. Readout-selection ledger

Before readout the C_3 orbit is democratic. After readout the visible CKM correction is not. The mass/readout projection P_Y must act on the branch orbit as

$P\_Y^B\Pi_{12} \approx 0$, $P\_Y^B\Pi_{23} \approx \Pi_{23}$, $P\_Y^B\Pi_{31} \approx 0$,

up to controlled Ω_mix and small-c effects.

Branch	Desired readout	Status
1↔2	suppressed	structural — plane occupied by the Cabibbo doorway
2↔3	survives	**Returned** in the minimal-cell readout model; **Owed** as a full-embedding fact (see below)
3↔1	suppressed	**Exposure** — no-direct-insertion condition, not yet structural

The 2↔3 survivor carries two distinct grades that should not be conflated:

Claim	Grade
2↔3 survivor inside the minimal-cell readout model	**Returned**
2↔3 survivor as a forced full su(8) readout fact	**Owed** / **Conditional** on PH-RDO and PH-EMB

The 1↔2 exclusion is solid: that plane already carries the Cabibbo doorway a, and a leading role-even common-mode survivor there would duplicate or disturb the inherited Cabibbo structure.

The 3↔1 exclusion is more delicate. It is *required* by the no-direct-insertion principle — the residue should be generated by the commutator, not inserted directly — but unless the readout projection structurally annihilates Π_{31}, this is a methodological exclusion rather than a forced projection result.

This is not a double standard with the inherited direct seed $c \cdot e^{(i\phi)}$, which occupies the same 1↔3 plane. The seed is an inherited object that the commutator acts *on*; the survivor is the *returned* correction, which must be generated by the commutator rather than inserted — inserting it directly would make the residue circular. Seed and survivor share the plane but play distinct roles.

The safe statement:

The 2↔3 survivor is the unique no-direct-insertion readout once the occupied Cabibbo branch is excluded and the direct-target branch is disallowed by the selection principle.

The stronger statement still owed:

$P\_Y^B \Pi_{31} = 0$ as an actual structural consequence of the generation-depth / mass readout.

This is one of the central remaining exposures of the quark-sector ledger.

6. The phase firewall

The amplitude is only half of the returned curvature. The phase is the most exposed part of the result, and it earns its own audit.

The Hermitian-side 2↔3 survivor has coefficient h, and the half-transport condition reads

$$h^2/(|h|^2) = e^{(2\pi i/3)}.$$

The two roots are

$$h/(|h|) = e^{(i\pi/3)} \text{ and } h/(|h|) = e^{(i4\pi/3)} = e^{(-i2\pi/3)}.$$

The minimal-Hermitian-lift rule selects the shorter Hermitian-side phase, $e^{(i\pi/3)}$. The frame coefficient is then obtained by the +i frame rotation,

$$\zeta = +i|\zeta|h/(|h|),$$

so that

$$\arg \zeta = \pi/2 + \pi/3 = 5\pi/6, \quad \zeta_{C_3} = b/(\sqrt{3})e^{(5\pi i/6)}.$$

The phase return rests on three legs.

Leg	Statement	Status
PH-ROOT	$h^2/|h|^2 = e^{(2\pi i/3)}$ (half-transport target)	**Audit condition**
PH-HERM	root selected by minimal Hermitian-side length → $e^{(i\pi/3)}$	**Audit condition** (inherited only if established upstream)
PH-ROT	frame rotation is +i, not −i	**Audit condition** — load-bearing, must be fixed without CP-data input

This is the **phase firewall**. The CP agreement is predictive only if all three legs are fixed without reference to J, $|V_{td}|$, β, γ, or the observed CP sign. The phase 5π/6 is exactly what makes the CP sector work; if it were selected *because* it works, the apparent CP success would be circular. If it is *forced* by closure structure, the CP success is genuine evidence. The firewall is the line between those two readings, and it is the single most important epistemic warning in the paper.

7. Returned C_3 curvature

Combining amplitude and phase, and using

$e^{(5\pi i/6)} = -\sqrt{3}/2 + i/2$,

gives

$\zeta\_C_3 = (b/\sqrt{3})(-\sqrt{3}/2 + i/2) = -b/2 + i\cdot b/(2\sqrt{3}) = -b/2 + i\cdot\sqrt{3}\, b/6$.

With $b = 81/2000$,

$\zeta\_C_3 = -81/4000 + i\cdot 27\sqrt{3}/4000 = -0.0202500 + 0.0116913i$.

Grade. ζ_C_3 is **Returned** in the minimal C_3 Hessian cell, **Conditional** on PH-ISO, PH-SAME, PH-RDO, PH-ROOT, PH-HERM, PH-ROT.

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8. CKM residue from the returned curvature

Let

$\Omega_0(\zeta) = \zeta E_{23} - \zeta^* E_{32}$.

Direct multiplication gives the exact commutator entry

$[\Omega_0, \Omega\_q]_{13} = -a\zeta$.

The symmetric split composition is

$V\_CKM = \exp(-\Omega_0 + \frac{1}{2}\,\Omega\_q)\exp(+\Omega_0 + \frac{1}{2}\,\Omega\_q)$,

with leading BCH logarithm

$\log V\_CKM = \Omega\_q - \frac{1}{2}[\Omega_0, \Omega\_q] + \cdots$,

so the generated 1↔3 residue beyond the inherited seed is

$\Delta_{13} = \frac{1}{2}\, a\, \zeta\_C_3$.

Substituting ζ_C_3,

$\Delta_{13} = \frac{1}{2}\, a(-b/2 + i\cdot\sqrt{3}\, b/6) = -a\, b/4 + i\cdot\sqrt{3}\, a\, b/12$.

With $a = 9/40$, $b = 81/2000$,

$\Delta_{13} = -729/320000 + i\cdot 243\sqrt{3}/320000 = -0.0022781 + 0.0013153i$, $|\Delta_{13}| = 0.0026306$.

The exact content is *not* that the observable |V_ub| equals 0.0026306. The exact content is that the generator-level residue beyond the inherited direct seed is $\Delta_{13} = \frac{1}{2} a \zeta_{C_3}$. The physical CKM matrix must still be assembled and exponentiated (§11).

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9. BCH stability and the diagonal Cartan term

The 1↔3 BCH tail is already discharged numerically inside the two-factor composition. The leading residue magnitude is

$|\Delta_{13}^{(lead)}| = 0.0026306,$

while the converged two-factor logarithmic residue is

$|\Delta_{13}^{(conv)}| = 0.0026332,$

a relative difference of about 0.1%. The 1↔3 residue channel is therefore stable, and the leading expression is a reliable residue calculation inside the inherited two-factor composition.

The diagonal Cartan term is a separate issue. The split product generates an imaginary traceless diagonal contribution whose nonzero entries are, in the converged logarithm, of order

4.7×10^{-4}.

This is smaller than $|\Delta_{13}|$ but not negligible by hierarchy alone. Its harmlessness rests on structural removability as an allowed quark-field rephasing direction. The live condition is therefore not BCH convergence but the **rephasing audit**: the diagonal Cartan term must be removable by allowed quark rephasing. This remains a named microscopic **Exposure**.

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10. Cabibbo back-reaction

The leading commutator also produces a correction in the Cabibbo plane. The leading estimate is

$|\Delta_{12}|_{lead} = \frac{1}{2} c\, |\zeta_{C_3}| = c\, b/(2\sqrt{3}) = \sqrt{3}\, c\, b/6.$

Relative to the Cabibbo doorway,

$|\Delta_{12}|_{lead}/a = \sqrt{3}\, c\, b/(6\, a) = 1.26 \times 10^{-4}.$

The 1↔2 channel converges less rapidly than the 1↔3 residue channel; the converged two-factor value is

$|\Delta_{12}|$_conv/a ≈ 2.11 × 10⁻⁴.

So the leading estimate understates the back-reaction by roughly a factor of 1.7, but the converged value remains deeply protective.

Quantity	Leading	Converged two-factor
$\|\Delta_{12}\|/a$	1.26×10^{-4}	2.11×10^{-4}

The conclusion is unchanged: the C_3 residue generation does not spoil the Cabibbo entry.

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11. Full CKM assembly

The physical CKM object is the unitary matrix obtained from the assembled generator. The generator-level 1↔3 entry is *not* the observable.

The inherited direct seed is

$(\Omega\_q)_{13}$ = ce^(iϕ) = 0.00243 e^(2πi/3) = −0.001215 + 0.002104i.

Adding the generated residue gives the generator entry

g_{13} = ce^(iϕ) + Δ_{13} = −0.0034931 + 0.0034197i, $|g_{13}|$ = 0.0048883.

This is still a generator entry, not the physical |V_ub|. Exponentiation mixes the 1↔2 and 2↔3 entries into the 1↔3 slot; in particular the term ½ a b ≈ 0.00456 is itself comparable to |V_ub|, so the full unitary must be exponentiated rather than read off g_{13}.

Using the inherited signs and the returned ζ_C_3, the exponentiated CKM output is

|V_us| = 0.2231, |V_cb| = 0.0405, |V_ub| = 0.00356, α = 84.5°, β = 22.5°, γ = 73.0°, |J| = 3.00 × 10^{-5}.

The first two are not new predictions — they reproduce inherited inputs. The 1↔3 and CP sector is the genuine empirical confrontation.

Internal numerical audit. The assembled unitary satisfies ‖V†V − I‖ ≈ 2.5×10^{-16}. The angles sum to exactly 180°. Every closed form in §§7–8 (ζ_C_3, Δ_{13}, the commutator entry −aζ) and the exponentiated triangle above reproduce to full displayed precision. The values α/β/γ here are the directly computed magnitudes; they reproduce the companion-cell readout to within ~0.1°, which is convention-level.

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12. Empirical confrontation

This section is empirical, not internal. The Particle Data Group 2025 CKM review treats the CKM elements as fundamental Standard Model parameters and describes the unitarity triangle and Jarlskog invariant as the standard phase-convention-independent way to organize CKM CP violation.

For reference, PDG 2025 quotes direct/averaged magnitudes

$|V_{us}| = 0.22431 \pm 0.00085$, $|V_{cb}| = (41.1 \pm 1.2)\times10^{-3}$, $|V_{ub}| = (3.82 \pm 0.20)\times10^{-3}$,

global-fit magnitudes

$|V_{ub}| = 0.003732\ (+0.000090/-0.000085)$, $|V_{cb}| = 0.04183\ (+0.00079/-0.00069)$,

a Jarlskog invariant

$J = (3.12\ {+0.13}/{-0.12})\times10^{-5}$,

and unitarity-triangle angles

$\sin 2\beta = 0.709 \pm 0.011$ ($\beta \simeq 22.6°$), $\alpha = 84.1\ (+4.5/-3.8)°$, $\gamma = 65.7 \pm 3.0°$.

The comparison below uses PDG 2025 direct/averaged values for $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, direct angle determinations for α and γ, and sin 2β for β. Global-fit CKM values are quoted only as secondary references, because a global fit already imposes Standard Model unitarity and would partly launder the very structure under test. This choice is material: the γ tension reported here is measured against the conservative direct benchmark, and would shrink against a global fit.

The ledger comparison is therefore:

Quantity	VERSF ledger output	Current benchmark	Status
$\|V_{us}\|$	0.2231	0.22431 ± 0.00085	reproduces inherited input
$\|V_{cb}\|$	0.0405	$(41.1 \pm 1.2)\times10^{-3}$	reproduces inherited input
$\|V_{ub}\|$	0.00356	$(3.82 \pm 0.20)\times10^{-3}$	low
α	84.5°	84.1 (+4.5/−3.8)°	good
β	22.5°	≈ 22.6° from sin 2β	good
γ	73.0°	65.7 ± 3.0°	high by ≈ 7.3°
$\|J\|$	3.00×10^{-5}	$(3.12\ {+0.13}/{-0.12})\times10^{-5}$	close

The empirical picture is clear. The CP *amount* is good; the triangle *shape* is not perfect. The high-γ value and the low $|V_{ub}|$ are correlated — the apex sits slightly too far over.

The correct empirical status:

The ledger produces a promising but exposed CKM sector: β and J are close, α is good, |V_ub| is somewhat low, and γ is the main residual tension.

A sharper statement of the γ tension: the returned amplitude $\rho = 1$ is the worst-fitting point in its own allowed band. Across the isotypic band $0.816 \leq \rho \leq 1$ (§18), the total triangle tension is maximal *exactly* at the minimal-cell value $\rho = 1$ (2.47σ) — higher even than the opposite floor $\rho = 0.816$ (2.39σ) — with the interior optimum (1.75σ) near $\rho \approx 0.90$. Closing the γ tension therefore requires the full projection to move ρ *away* from what the minimal cell returns, not merely to confirm it.

The strongest success, J, is not the safest claim. It is the *most* exposed claim, because it depends most heavily on the phase firewall — and, because |J| scales with the inherited a, b, c (§14, with elasticities ≈ 1.3, 1.4, 0.6), on the amplitude-side conjunction PH-AMP-FIRE ∧ PH-SAME ∧ PH-EMB-FIRE (§14) as well. Its magnitude is independent evidence only if the phase firewall and that amplitude conjunction all hold.

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13. Direct-term-only comparison

The role of the generated residue is seen by removing it. Without Δ_{13}, the inherited direct seed alone gives a badly distorted unitarity triangle.

Quantity	Direct seed only	Direct seed + C_3 residue
α	128.0°	84.5°
β	19.6°	22.5°
γ	32.4°	73.0°
|J|	1.84×10^{-5}	3.00×10^{-5}
|V_ub|	0.00393	0.00356

The direct-seed-only column is obtained by setting $\Omega_0 = 0$ and exponentiating the inherited generator Ω_q.

The generated residue is therefore **load-bearing**: it does not merely improve the fit — it radically rotates the triangle into the correct CP sector, moving α by ≈ 43°, γ by ≈ 41°, and nearly doubling J. It moves the triangle from a badly distorted CP shape to a near-realistic one. The price is equally explicit — the same residue that repairs the CP amount lowers |V_ub| and pushes γ high against the conservative tree-level benchmark.

The converse reading is equally load-bearing. Because the residue-free triangle is not merely worse but nonsensical (α = 128°), there is no partial-credit fallback: essentially all realistic-CKM structure rests on ζ_{C_3}, and therefore on the still-Owed amplitude and phase premises of §§15–17. If the firewall fails, the result is not a slightly worse triangle but a badly distorted one. §13 is thus a two-sided statement — the residue's success and the programme's exposure are the same fact.

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14. Provenance ledger

The empirical comparison is only as independent as its ingredients.

Input	Provenance	Ledger status
$a = 9/40$	inherited Cabibbo closure-overlap scale	not a prediction here
$b = 81/2000$	inherited / structurally assembled from earlier quark transport inputs	partly structural, but reproduces $\|V_{cb}\|$
$c = 243/100000$	inherited / structurally assembled direct seed	genuine multi-entry effect, but inherited
$\phi = 2\pi/3$	C_3 holonomy candidate	still **Owed** a microscopic loop derivation
$\|\zeta\| = b/\sqrt{3}$	minimal-cell norm transfer	**Conditional** on PH-ISO ∧ PH-SAME
$\arg \zeta = 5\pi/6$	phase-root return	**Conditional** on the phase firewall
Δ_{13}	commutator-generated	**Exact** once ζ is granted

Amplitude firewall. The provenance table protects the *phase* provenance of ζ_C_3, but the magnitude of the strongest empirical success, $|J|$, is not phase-dominated — it rides on the inherited base amplitudes: numerically $d \ln|J|/d \ln a \approx 1.3$, $d \ln|J|/d \ln c \approx 0.6$, and $d \ln|J|/d \ln b \approx 1.4$. The b elasticity decomposes cleanly: ≈ 1.0 from the direct $2\leftrightarrow3$ entry of Ω_q, and ≈ 0.38 from the C_3 amplitude itself, because $|\zeta| = b/\sqrt{3}$ ties the return to the inherited b through **PH-SAME**. The genuinely independent C_3 freedom — the projection's freedom to return $\rho \neq 1$ — carries an elasticity of only ≈ 0.38 (it is the *same* channel, since $|\zeta| = \rho\cdot b/\sqrt{3}$), and it is already firewalled by PH-EMB-FIRE (§18.5).

The consequence is that PH-EMB-FIRE alone does not protect the $|J|$ magnitude; it protects only the 0.38-elasticity ρ channel. Everything else in $|J|$ is pinned by the inherited base amplitudes, which the provenance table grades only softly. A parallel firewall is therefore required, with PH-SAME as a named co-premise:

PH-AMP-FIRE (audit condition). The inherited base amplitudes a, b, c are fixed by upstream transport and seed structure with no reference to $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, or J. Together with PH-SAME (which ties $|\zeta| \propto b$) and PH-EMB-FIRE (which fixes ρ), this pins the entire $|J|$ magnitude; the phase firewall (§17) then fixes its placement.

Until PH-AMP-FIRE is discharged, the $|J|$ agreement is contingent on the full conjunction PH-AMP-FIRE ∧ PH-SAME ∧ PH-EMB-FIRE ∧ (phase firewall). If "aligned by construction" (§2) anywhere means "aligned to data," the |J|-magnitude success is partly laundered through a, b, c, and |J| must be read as Empirical-contingent, not as independent evidence.

The central ledger discipline: inherited reproductions are *not* counted as new predictions. The meaningful advances are exactly four —

1. the minimal-cell projected-Hessian return of the C_3 common curvature;
2. the exact generation of Δ_{13} through the Cabibbo doorway;
3. the full CKM empirical confrontation;
4. the reduction of all remaining vulnerabilities to named exposures.

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15. Embedding-exhaustion audit

The current derivation is minimal-cell. A full su(8) microscopic closure requires one further structural result.

PH-EMB. The minimal committed-quark C_3 branch cell exhausts the branch-relevant C_3-isotypic support of the full compressed su(8) Hessian.

Without PH-EMB a critic may object: *the calculation proves democracy after restricting to the democratic cell, but the full projected Hessian may contain additional C_3 copies, invariant Cartan directions, or branch-relevant leakage.* The required full-embedding statement is

$$H^{(q,even)}_{proj} = H^{(branch)}_{C_3} \oplus H_{decoupled},$$

with exactly one branch-relevant regular C_3 copy coupled to the CKM residue channel; equivalently,

$$m^{(branch)}_{reg}(C_3) = 1.$$

The falsifiers are:

Exposure	Failure
EMB-1	extra C_3 regular copy couples to the role-even common mode
EMB-2	C_3-invariant Cartan direction couples to the branch
EMB-3	nonminimal su(8) support changes the norm budget
EMB-4	full projection breaks the minimal-cell cyclic scalar form
EMB-5	Ω_{mix} is not gapped and contaminates the role-even reading

The ledger records **PH-EMB** as **Owed**. It does not pretend the minimal-cell proof automatically exhausts the full su(8) support.

16. Readout-selection audit

The second major structural exposure is readout. The necessary branch readout is

P_Y^B: $\Pi_{12} \mapsto 0$, $\Pi_{23} \mapsto \Pi_{23}$, $\Pi_{31} \mapsto 0$.

The 1↔2 suppression is structural — the Cabibbo doorway already occupies that plane. The 3↔1 suppression is not yet structural; it is required to prevent direct insertion of the target. Full closure must eventually either prove $P\_Y^B \Pi_{31} = 0$ from the mass/readout structure, or else admit that the 2↔3 survivor is a selection principle rather than a forced return.

Readout claim	Grade
$P\_Y^B \Pi_{12} = 0$	structural — Cabibbo occupied
$P\_Y^B \Pi_{23} = \Pi_{23}$	**Returned** in minimal cell; **Owed** as full-embedding fact (PH-RDO ∧ PH-EMB)
$P\_Y^B \Pi_{31} = 0$	**Owed** as structural proof; currently a no-direct-insertion condition

This statement prevents the reader from treating the direct-target exclusion as a smuggled-in theorem.

17. Phase-firewall audit

The phase firewall is the most important provenance exposure, because the CP agreement depends heavily on $\arg \zeta = 5\pi/6$. The three required legs are:

PH-ROOT: $h^2/|h|^2 = e^{(2\pi i/3)}$, **PH-HERM:** principal Hermitian root selected by minimal length → $e^{(i\pi/3)}$, **PH-ROT:** $\Omega\_W = +i\epsilon\_W H\_W$, $\epsilon\_W > 0 \Rightarrow$ branch factor +i, not −i.

Firewall condition. None of PH-ROOT, PH-HERM, PH-ROT may be selected by reference to J, |V_td|, β, γ, or the observed CP sign. If any one of them is selected *because* it lands on the observed CP sector, the J and β agreement is not independent evidence — it is a recoding of data contact.

The owed phase paper must therefore prove:

1. the half-transport target $e^{(2\pi i/3)}$ from closure structure;
2. the Hermitian-side minimal-length rule from the admissibility functional;
3. the +i frame-rotation sense from the orientation of the weak-frame map.

Only once these three are secured can the J agreement be promoted from contingent evidence to genuine prediction.

The stake attached to PH-ROT is large and directly computable, which is why it is the load-bearing leg. Replacing the +i frame rotation with −i sends arg $\zeta \to -\pi/6$ and collapses the CP output: the triangle angle γ falls to ≈ 8° and $|J|$ drops to $\approx 6.9 \times 10^{-6}$, roughly a quarter of its +i value. The collapse is in magnitude and shape, not sign — J does not change sign in this convention — so −i suppresses the CP amount rather than reversing it. PH-ROT is therefore the single binary on which the entire CP sector turns.

One caution on the natural proof route for PH-ROT. Tying the +i sense to the same orientation that fixes a > 0 and the anti-Hermiticity of Ω_q secures the leg only if those conventions are genuinely the *same* orientation as the weak-frame map, not merely both fixed elsewhere. If the quark-generator conventions were set upstream without reference to the weak-frame orientation, flipping the weak-frame i leaves Ω_q untouched and the route is circular. The owed proof must therefore exhibit a shared orientation, not two independently-fixed ones.

This turns the caution into a status question the construction must answer explicitly. As built, are the quark-generator conventions (a > 0, Ω_q anti-Hermitian, ϕ from C_3 holonomy) and the weak-frame orientation *the same* orientation, or two independently-fixed ones? The two answers carry different grades. If shared, PH-ROT is provable by the orientation route and its **Audit condition** grade stands. If independent — the natural reading, since a, b, c, ϕ are all graded Inherited from the upstream construction — then the orientation route is circular, and PH-ROT is not an Audit condition awaiting proof but an exposure with *no known non-circular proof route*, a strictly worse status. The paper cannot leave this open: until the construction declares which case obtains, PH-ROT's own grade is conditional, and the |J| and β independence claims inherit that conditionality.

In the sharpest terms: until the weak-frame orientation is shown to be the *same* orientation as the inherited quark-generator convention, PH-ROT remains the most exposed phase leg. The paper makes no claim to have escaped the obvious criticism — that the sign giving the observed CP sector was selected rather than derived — and states plainly that this escape is owed, not achieved.

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18. Candidate full-projection refinement (Conditional)

This section records a conditional candidate refinement of the returned amplitude. It is **not part of the closed ledger** of §§2–17, does not revise the main theorem (§20), and does not alter any graded result above. Its status is **Candidate** throughout: it holds only if the full su(8) projection independently returns the stated norm factor under the firewall condition of §18.5.

18.1 The renormalisation parameter

The minimal-cell return fixes $|\zeta| = b/\sqrt{3}$ ($\rho = 1$). A full-projection correction is carried by a single scalar,

$|\zeta| = \rho b/(\sqrt{3}),\ 0 \le \rho \le 1,$

where $1 - \rho^2$ is the fraction of the transferred budget lost to C_3-invariant (Cartan) or externally decoupled directions. Leakage can only reduce the retained fraction, so the sign of the correction is structurally fixed while its magnitude is **Owed** to the full embedding.

18.2 The candidate value and its algebra

The candidate value is the discrete factor

$\rho = \sqrt{3}/2 = 0.866025$,

corresponding to a three-quarter norm retention, $\|P\_full\, Z\|^2 = \frac{3}{4}\|Z\|^2$. It simplifies the curvature:

$\zeta\_full = (\sqrt{3}/2)\cdot(b/\sqrt{3})\cdot e^{(5\pi i/6)} = (b/2)\cdot e^{(5\pi i/6)}$, $|\zeta\_full| = b/2$.

In exact form,

$\zeta\_full = -\sqrt{3}\, b/4 + i\cdot b/4 = -81\sqrt{3}/8000 + i\cdot 81/8000 = -0.0175370 + 0.0101250i$,

and the residue is

$\Delta_{13}\hat{}(full) = \frac{1}{2}\, a\, \zeta\_full = -729\sqrt{3}/640000 + i\cdot 729/640000 = -0.0019729 + 0.0011391i$,

with the compact magnitude

$|\Delta_{13}\hat{}(full)| = ab/4 = 0.0022781$.

This is algebraically cleaner than the minimal-cell $|\Delta_{13}| = ab/(2\sqrt{3}) = 0.0026306$, but cleaner form is not stronger evidence; see §18.4.

18.3 Empirical impact

Exponentiating the assembled unitary with $|\zeta| = b/2$ gives $\alpha = 90.4°$, $\beta = 22.2°$, $\gamma = 67.4°$, $|J| = 2.84\times10^{-5}$, $|V\_ub| = 0.00350$. Placed against the direct benchmark in σ units ($\alpha = 84.1 \pm 4.15$, $\beta = 22.6 \pm 0.44$, $\gamma = 65.7 \pm 3.0$):

ρ	**γ**	**α**	**β**	**total tension**
1.000 (closed ledger)	73.0°	84.5°	22.5°	2.47σ
0.904 (empirical optimum)	69.0°	88.7°	22.3°	1.75σ
0.866 (√3/2 candidate)	67.4°	90.4°	22.2°	1.88σ
0.816 (isotropic floor)	65.3°	92.7°	22.1°	2.39σ

The candidate reduces the total tension ($2.47\sigma \rightarrow 1.88\sigma$), but does so by **relocating** the γ tension into α: at $\rho = \sqrt{3}/2$ the α residual is $+1.52\sigma$, larger than the γ residual it removes ($+0.56\sigma$). It also sits past the unconstrained σ-optimum near $\rho \approx 0.90$. The correct empirical statement is therefore a genuine but partial improvement in triangle shape, bought against a growing α tension, and slightly short of the best in-band amplitude.

18.4 Why $\sqrt{3}/2$ is not forced

Three distinct structural readings each land on ¾, but they are not one convergent argument:

- **(A)** an orthogonal survivor projection removing one component of the branch norm;
- **(B)** a 30° readout misalignment, $\rho = \cos 30° = \sqrt{3}/2$;
- **(C)** a full-cell norm transfer returning $\|Z\_C_3\text{^(full)}\|^2 = ¾\, b^2$.

These act at three different stages — readout, survivor orientation, and norm transfer — and coincide only in the resulting decimal. Multiple independent stories for one number are a warning, not a corroboration: at most one can be what the projection actually returns.

Moreover, ¾ is **not** the value produced by removing the C_3-invariant Cartan channel. The isotypic decomposition of the branch cell (2 Cartan ⊕ 4 regular; cf. §15) gives an isotropic Cartan-removal retention of 4/6 = ⅔, i.e. $\rho = 0.816$ — not ¾. The candidate value corresponds instead to a mild partial Cartan gap, $\lambda\_inv/\lambda\_reg \approx 1.22$ in the simplest isotropic-response model, against ≈ 1.66 for the empirical optimum. The isotypic structure supplies only the band $0.816 \le \rho \le 1$; it privileges no interior point. The cleanliness of $\sqrt{3}/2$ comes from cos 30° and the ¾ algebra, not from the embedding selecting it.

18.5 Target premise and firewall

The candidate is governed by a target premise and a firewall, neither yet discharged.

PH-ρ (target premise). $\|P\_full\ Z\_C_3\|^2 = ¾\|Z\_C_3\text{^(min)}\|^2$, equivalently $\rho = \sqrt{3}/2$. This names the target; it is a restatement of the candidate value, not a derivation. It is **Owed** to the full su(8) projection.

PH-EMB-FIRE (audit condition). The renormalisation ρ, equivalently the Cartan curvature ratio λ_inv/λ_reg, must be computed from the su(8) embedding and readout normalisation for reasons internal to the embedding, with no reference to γ, α, β, |V_ub|, or J. The triangle is checked after ρ is fixed, never used to fix it. Reading ρ backwards off the target γ is fitting the amplitude — exactly the circularity the phase firewall (§17) forbids for the phase.

The candidate inherits the full phase firewall unchanged: $\arg \zeta = 5\pi/6$ still rests on PH-ROOT, PH-HERM, PH-ROT (§17). The refinement changes only $|\zeta|$, not $\arg \zeta$.

18.6 Candidate grade summary

Item	Statement	Grade
renormalisation band	$0.816 \le \rho \le 1$	**Exact** (isotypic)

Item	Statement	Grade
candidate value	$\rho = \sqrt{3}/2$	**Candidate** — in-band, not forced
curvature	$\zeta\_full = (b/2)\, e^{(5\pi i/6)}$	**Exact** once ρ granted
residue	$\Delta_{13}^{(full)} = ½\, a\, \zeta\_full$, $\lvert\Delta_{13}^{(full)}\rvert = ab/4$	**Exact** once ρ granted
empirical effect	γ tension → +1.7°, α → +1.52σ	**Empirical** — partial, past optimum
PH-ρ	$\lVert P\_full\ Z\rVert^2 = ¾\lVert Z\rVert^2$	**Target premise** — Owed
PH-EMB-FIRE	ρ computed independently of the triangle	**Audit condition**

The candidate does not enter the closed ledger of §19 or the main theorem of §20. It is recorded as a forward branch whose acceptance is contingent on PH-ρ being returned by the full projection under PH-EMB-FIRE.

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19. Quark-sector closure ledger

Quantity / claim	Value / statement	Grade
Ω_q	inherited role-odd quark generator	**Inherited**
a	9/40	**Inherited**; Cabibbo reproduction
b	81/2000	**Inherited**; $\lvert V\_cb\rvert$ reproduction
c	243/100000	**Inherited** direct seed
ϕ	2π/3	**Inherited** C_3 holonomy candidate
$G^B\_C_3$	$\kappa\_C_3\, I_6$	**Returned** (minimal cell)
C_3 covariance	$[\mathcal{K}, R_3] = 0$	**Returned** (minimal cell)
democratic weights	$w_{12} = w_{23} = w_{31}$	**Exact** from covariance
norm transfer	total budget b^2	**Conditional** on PH-ISO ∧ PH-SAME
amplitude	$\lvert\zeta\rvert = b/\sqrt{3}$	**Exact** from democracy + budget
readout survivor	2↔3	**Conditional** on PH-RDO; direct-target suppression **Exposed**
phase	5π/6	**Conditional** on PH-ROOT, PH-HERM, PH-ROT
curvature	$\zeta\_C_3 = -0.0202500 + 0.0116913\, i$	**Conditional** return
residue	$\Delta_{13} = ½\, a\, \zeta\_C_3$	**Exact** once ζ is granted
1↔3 BCH tail	stable at ~0.1%	discharged inside two-factor composition

Quantity / claim	Value / statement	Grade
diagonal Cartan term	removable rephasing direction	rephasing audit still live
$\lvert V_{ub}\rvert$	0.00356	**Empirical**, low
α	84.5°	**Empirical**, good
β	22.5°	**Empirical**, good
γ	73.0°	**Empirical**, high
$\lvert J\rvert$	3.00×10^{-5}	**Empirical**, close; magnitude **Conditional** on PH-AMP-FIRE ∧ PH-SAME ∧ PH-EMB-FIRE and the phase firewall

This is the quark-sector closure ledger. It is a genuine programme milestone because the CKM sector is no longer an open list of desired numbers — it is a finite set of inherited, returned, exact, empirical, and owed entries. The conditional candidate refinement of §18 ($\rho = \sqrt{3}/2$) does not enter this closed ledger; it is a forward branch contingent on PH-ρ and PH-EMB-FIRE.

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20. Main theorem

Theorem (quark-sector closure ledger). Assume:

- the inherited role-odd quark generator Ω_q with a, b, c, ϕ;
- the minimal committed-quark C_3 Hessian return $G^B\_C_3 = \kappa\_C_3 I_6$;
- pre-readout equivariance and generation-blind role trace;
- role-isotype isotropy **PH-ISO**;
- shared closure profile **PH-SAME**;
- generation-depth readout **PH-RDO**, including 2↔3 survival and no direct 3↔1 insertion;
- gapped role-mixing block **PH-GAP**;
- phase legs **PH-ROOT**, **PH-HERM**, **PH-ROT**;
- the inherited symmetric split-BCH composition.

Then the quark-sector CKM ledger closes to the internal return

$\zeta\_C_3 = b/(\sqrt{3})e^{(5\pi i/6)}$, $\Delta_{13} = \frac{1}{2}\, a\, \zeta\_C_3$,

which, with a = 9/40, b = 81/2000, evaluates to

$\zeta\_C_3 = -0.0202500 + 0.0116913i$, $\Delta_{13} = -0.0022781 + 0.0013153i$, $\lvert\Delta_{13}\rvert = 0.0026306$.

The assembled unitary CKM output is then confronted empirically as

$\lvert V\_ub\rvert = 0.00356$, $\alpha = 84.5°$, $\beta = 22.5°$, $\gamma = 73.0°$, $\lvert J\rvert = 3.00 \times 10^{-5}$, graded separately: good α, β, J, low $|V_{ub}|$, high γ, with the CP agreement **Conditional** on independent phase provenance (PH-ROOT, PH-HERM, PH-ROT) and the $|J|$ magnitude additionally **Conditional** on independent amplitude provenance (PH-AMP-FIRE, with PH-SAME and PH-EMB-FIRE).

Proof. The minimal C_3 Hessian return gives $G^B\_C_3 = \kappa\_C_3 I_6$. The pre-readout block $\mathcal{K}$ therefore commutes with R_3, is circulant, and has equal branch weights. Under PH-ISO and PH-SAME the total role-even common-mode budget equals the inherited role-odd 2↔3 norm b^2; since the three branches are orthogonal and equal,

$$3|\zeta|^2 = b^2 \Rightarrow |\zeta| = b/(\sqrt{3}).$$

PH-RDO selects the 2↔3 survivor. PH-ROOT supplies the Hermitian half-transport target, PH-HERM selects the principal Hermitian root $e^{(i\pi/3)}$, and PH-ROT applies the $+i$ frame rotation, giving

$$\arg \zeta = \pi/2 + \pi/3 = 5\pi/6, \quad \zeta\_C_3 = b/(\sqrt{3})e^{(5\pi i/6)}.$$

Direct multiplication gives $[\Omega_0(\zeta), \Omega\_q]_{13} = -a\zeta$. The symmetric split-BCH convention contributes $-\frac{1}{2}[\Omega_0, \Omega\_q]$ to the logarithm, so the generated residue is $\Delta_{13} = \frac{1}{2} a\, \zeta\_C_3$. The numerical values follow by substitution; exponentiation gives the empirical CKM confrontation. ■

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21. Falsifiers and exposures

The programme can fail locally at named points.

Code	Failure condition	Consequence
F1	full su(8) support contains extra C_3 branch copies	minimal cell not exhaustive
F2	C_3-invariant Cartan leakage couples to the role-even branch	democratic branch not clean
F3	P_C, P_R, P_q, or role trace is generation-biased before readout	pre-readout democracy fails
F4	role metric is anisotropic	norm transfer b^2 fails
F5	role-even common mode and role-odd 2↔3 transport use different closure profiles	$|\zeta|$ budget fails
F6	P_Y rescales the 2↔3 survivor	amplitude changes
F7	P_Y weights 1↔2 at leading order	Cabibbo protection fails
F8	P_Y weights 3↔1 at leading order	residue inserted, not generated
F9	Ω_mix not gapped	clean role-even reading fails
F10	PH-ROOT not derived independently	phase target exposed

Code	Failure condition	Consequence
F11	PH-HERM not derived independently	root selection exposed
F12	PH-ROT not derived independently	CP sign / magnitude exposed
F13	diagonal Cartan term not removable by allowed rephasing	rephasing audit fails
F14	higher-order corrections move $\|V_{ub}\|$ further from data	empirical tension worsens
F15	γ remains high under improved inputs	quark-sector shape tension persists
F16	phase legs were selected by CKM data contact	CP agreement circular, not predictive
F17	base amplitudes a, b, c were fixed with reference to $\|V_{us}\|$, $\|V_{cb}\|$, or J	$\|J\|$-magnitude success laundered, not independent
F18	weak-frame orientation is independent of the Ω_q conventions	PH-ROT's only known proof route is circular

The burden has changed. A critic no longer needs to say vaguely that the CKM correction was assumed — the critic can point to a specific failure condition. That is what sector-level progress looks like.

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22. What this paper does not claim

This paper does **not** claim:

- that the full su(8) projected Hessian has been exhaustively decomposed;
- that the direct-target 3↔1 readout suppression has been structurally forced;
- that $5\pi/6$ is independently predictive unless the phase firewall is secured;
- that a and b are new predictions of this paper;
- that the CKM sector is perfectly matched;
- that the full Standard Model has been derived.

It claims instead:

The quark mixing sector has been reduced to a finite closure ledger with a returned minimal-cell C_3 curvature, an exact commutator-generated residue, a full empirical confrontation, and a short list of named remaining exposures.

That is the correct programme-level statement.

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23. Programme consequence

The quark-sector claim has sharpened in stages:

- *broad* — perhaps the projected Hessian explains the CKM residue;
- *conditional* — if the projected Hessian returns this branch, the residue follows;
- *minimal-cell* — inside the committed-quark C_3 cell, the projected Hessian returns the branch;
- *closure ledger* — the CKM sector is now a finite, auditable ledger of inherited entries, returned curvature, exact residue generation, empirical comparison, and named exposures.

That is not final closure; it is a meaningful sector-level milestone. The next papers are no longer exploratory but targeted:

1. **C_3 embedding-exhaustion paper** — prove PH-EMB in the full su(8) projected support.
2. **Readout-selection paper** — prove structural suppression of the direct 3↔1 branch.
3. **Phase-firewall paper** — derive PH-ROOT, PH-HERM, PH-ROT without CKM data contact.
4. **Residual-γ paper** — determine whether the high-γ tension is a higher-order / input issue or a genuine falsification signal.

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24. Conclusion

The VERSF quark-flavour programme has reached a sharper stage. The CKM sector is not yet closed in the final microscopic sense, but it is no longer an unbounded target — it is a finite closure ledger.

The inherited generator supplies $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\phi = 2\pi/3$. The minimal committed-quark C_3 Hessian return supplies a democratic role-even branch; under role-isotype norm transfer and shared closure profile, $|\zeta| = b/\sqrt{3}$; under the phase firewall, $\arg \zeta = 5\pi/6$. Thus

$\zeta\_C_3 = b/(\sqrt{3})e^{(5\pi i/6)}$, $\Delta_{13} = ½\ a\ \zeta\_C_3$,

with

$\zeta\_C_3 = -0.0202500 + 0.0116913i$, $\Delta_{13} = -0.0022781 + 0.0013153i$, $|\Delta_{13}| = 0.0026306$.

When the full unitary CKM matrix is assembled, the CP sector becomes realistic: β and J land close to measured values, and α is good. But the empirical ledger is not perfect — $|V\_ub|$ is low and γ remains high — and the CP agreement is not yet fully independent, because it rests heavily on the phase 5π/6. The phase firewall is therefore the programme's most exposed success, not its safest one. The returned amplitude also sits at the least favourable corner of its own allowed band — ρ = 1 is the worst-fitting in-band value (2.47σ, §12) — so the γ tension is not a small residual to be polished but a signal that the full projection must move ρ off the minimal-cell value.

A candidate full-projection refinement ($\rho = \sqrt{3}/2$, §18) would reduce the γ tension to $\approx 1.7°$ at the cost of a growing α tension, and sits just past the in-band empirical optimum near $\rho \approx 0.90$. It is recorded as a conditional forward branch — not a ledger revision — pending an independent projection return of PH-ρ under PH-EMB-FIRE.

The result rests on a named stack of premises — PH-ISO, PH-SAME, PH-RDO, PH-GAP, PH-ROOT, PH-HERM, PH-ROT, PH-EMB, PH-EMB-FIRE, PH-AMP-FIRE — each carrying an explicit falsifier. That is a strength of bookkeeping, not of proof: the honest reading is that the quark-mixing sector has been *organised*, not *closed*. The controlling claim therefore stays deliberately modest, and no stronger wording elsewhere in the paper should be read as displacing it:

Final status. Finite CKM-sector closure ledger achieved; full microscopic closure still owed.

That is a genuine advance. It gives the Standard Model programme a concrete quark-sector milestone and converts the remaining work into a short list of named, local, falsifiable problems.

Appendix A. Numerical ledger

Quantity	Exact expression	Value
a	9/40	0.225
b	81/2000	0.0405
c	243/100000	0.00243
ϕ	$2\pi/3$	120°
$\lvert\zeta_{C_3}\rvert$	$27\sqrt{3}/2000$	0.0233827
arg ζ_{C_3}	$5\pi/6$	150°
ζ_{C_3}	$-81/4000 + i\cdot 27\sqrt{3}/4000$	$-0.0202500 + 0.0116913\, i$
Δ_{13}	$-729/320000 + i\cdot 243\sqrt{3}/320000$	$-0.0022781 + 0.0013153\, i$
$\lvert\Delta_{13}\rvert$ (lead)	—	0.0026306
$\lvert\Delta_{13}\rvert$ (conv)	—	0.0026332
$\lvert\Delta_{12}\rvert/a$ (lead)	$\sqrt{3}\, cb/(6a)$	1.26×10^{-4}
$\lvert\Delta_{12}\rvert/a$ (conv)	—	2.11×10^{-4}
$\lvert V_{ub}\rvert$ (full)	—	0.00356
α (full)	unitarity triangle	84.5°
β (full)	unitarity triangle	22.5°
γ (full)	unitarity triangle	73.0°
$\lvert J\rvert$ (full)	—	3.00×10^{-5}

Audit: the assembled unitary satisfies $\lVert V^\dagger V - I\rVert \approx 2.5 \times 10^{-16}$; $\alpha + \beta + \gamma = 180°$ exactly.

Appendix B. Reference empirical benchmarks

The comparison in this draft uses the following 2025 PDG benchmark values:

$|V_{us}| = 0.22431 \pm 0.00085$, $|V_{cb}| = (41.1 \pm 1.2)\times10^{-3}$, $|V_{ub}| = (3.82 \pm 0.20)\times10^{-3}$, $\alpha = 84.1 (+4.5/-3.8)^\circ$, $\gamma = 65.7 \pm 3.0^\circ$, $\sin 2\beta = 0.709 \pm 0.011$, $J = (3.12 +0.13/-0.12)\times10^{-5}$.

In the final manuscript these should be supplemented by date-stamped references to the selected comparison standard: PDG direct averages, PDG global fit, CKMfitter, UTfit, or a deliberately conservative tree-level-only benchmark. The choice of benchmark materially affects the size of the γ tension and should be fixed before, not after, the comparison.

Appendix C. Algebraic transcription checks

Two factored intermediate forms are prone to a factor-of-3 transcription slip in which the radical migrates from the denominator to the numerator. Both are recorded here so the correct forms in §§7–8 can be verified at a glance.

C.1 — Curvature imaginary part. From $\zeta_{C_3} = (b/\sqrt{3})\cdot e^{5\pi i/6}$ with $e^{5\pi i/6} = -\sqrt{3}/2 + i/2$, the imaginary part is

$b/(\sqrt{3})\cdot\frac{1}{2} = b/(2\sqrt{3}) = \sqrt{3}\, b/6$, not $\sqrt{3}\, b/2$.

The two differ by a factor of 3; only $\sqrt{3}\, b/6 = 0.0116913$ reproduces the stated value.

C.2 — Residue imaginary part. From $\Delta_{13} = \frac{1}{2} a\, \zeta_{C_3}$, the imaginary part is

$a/2\cdot\sqrt{3}\, b/6 = \sqrt{3}\, a\, b/12 = a\, b/(4\sqrt{3})$, not $\sqrt{3}\, a\, b/4$.

Only $\sqrt{3}\, ab/12 = 0.0013153$ reproduces the stated value.

C.3 — Cabibbo back-reaction. The leading estimate $|\Delta_{12}|_{lead} = \frac{1}{2} c\, |\zeta_{C_3}| = cb/(2\sqrt{3}) = \sqrt{3}\, cb/6$, giving $|\Delta_{12}|_{lead}/a = \sqrt{3}\, cb/(6a) = 1.26 \times 10^{-4}$.

All exact rational-radical forms ($\zeta_{C_3} = -81/4000 + i\cdot27\sqrt{3}/4000$; $\Delta_{13} = -729/320000 + i\cdot243\sqrt{3}/320000$), the commutator identity $[\Omega_0, \Omega_q]_{13} = -a\zeta$, the assembled-unitary residual $\|V^\dagger V - I\| \approx 2.5 \times 10^{-16}$, and the exact angle sum $\alpha + \beta + \gamma = 180^\circ$ are numerically reproducible from the inputs $a = 9/40$, $b = 81/2000$, $c = 243/100000$, $\phi = 2\pi/3$.

Appendix D. Candidate full-projection numerical ledger

Quantity	Exact expression	Value
ρ	$\sqrt{3}/2$	0.866025
$\lvert\zeta_{full}\rvert$	b/2	0.0202500
ζ_{full}	$-81\sqrt{3}/8000 + i\cdot 81/8000$	$-0.0175370 + 0.0101250\,i$
$\Delta_{13}^{(full)}$	$-729\sqrt{3}/640000 + i\cdot 729/640000$	$-0.0019729 + 0.0011391\,i$
$\lvert\Delta_{13}^{(full)}\rvert$	ab/4	0.0022781
α	unitarity triangle	90.4°
β	unitarity triangle	22.2°
γ	unitarity triangle	67.4°
$\lvert J\rvert$	—	2.84×10^{-5}
$\lvert V_{ub}\rvert$	—	0.00350
Cartan ratio r	$\lambda_{inv}/\lambda_{reg}$ at $\rho = \sqrt{3}/2$	1.22

Audit: with $\rho = \sqrt{3}/2$, $|\zeta| = b/2$, $\arg \zeta = 5\pi/6$, the assembled unitary $V = \exp(-\Omega_0 + \Omega_q/2)\cdot\exp(+\Omega_0 + \Omega_q/2)$ reproduces the triangle above. The σ-optimum over ρ against the direct benchmark is $\rho \approx 0.904$ at 1.75σ, versus the candidate's 1.88σ and the closed-ledger 2.47σ. The isotypic band anchors are $\rho = 1$ (retention 1) and $\rho = \sqrt{4/6} = 0.816$ (isotropic Cartan share 2/6).