The Strong CP and Global θ-Sector Theorem in VERSF

▲ Programme Milestone — Global Gauge Topology, Anomalous Phase and CP-Selection Series

Gate SCPθ-1 / Global Topological-Character Selection, Physical $\bar{\theta}$ Construction, Determinant-Line Reduction, and Strong/Weak CP Separation

Keith Taylor VERSF Theoretical Physics Programme — Global Gauge Topology, Anomalous Phase and CP-Selection Series Tier A conditional gate-theorem paper — working manuscript

Primary predecessors: *The Gauge-Closure Selection Theorem in VERSF*; *The Substrate Anomaly-Inadmissibility Theorem in VERSF*; *The Gauge Action and Coupling Theorem in VERSF*; *QCD Confinement and Running in the SU(3)_C Bath-Transport Sector of VERSF*; *The Closure-Operator Yukawa Construction Theorem in VERSF*; *The Quark Current-Mass Projection Theorem in VERSF*; and the VERSF CKM, closure-frame, charge-conjugation and quantum-completion programme.

General Reader Summary

Nature has a strange blind spot. Almost every law of physics works the same way if you swap all matter for antimatter and view the result in a mirror — a combined swap physicists call CP. The weak nuclear force breaks this mirror rule by a tiny, well-measured amount. The strong nuclear force — the force that binds quarks into protons and neutrons — is *allowed* to break it too, and by our current theories it really should. Yet every experiment ever performed says it doesn't. Not slightly. Not at all, to extraordinary precision.

This is the strong CP problem, and it has resisted explanation for fifty years.

The reason the strong force is allowed to misbehave comes down to a single dial in the theory, an angle usually written $\bar{\theta}$ (theta-bar). If the dial sits anywhere except exactly at zero, the strong force treats matter and antimatter differently — for instance, the neutron would have a measurable electric lopsidedness, which it demonstrably does not. The Standard Model offers no reason for the dial to sit at zero. It just observes that it does, and moves on. That silence is the problem.

The dial has two hands, and both must point to zero at once. One hand belongs to the deep structure of the strong force itself — a phase the theory is permitted to attach to certain twisted field configurations that cannot be smoothed away. The other hand belongs to the quark masses: the mathematical objects that give quarks their masses carry a collective phase of their own, and a famous quantum effect (the anomaly) means this phase feeds straight back into the same dial. Setting one hand to zero achieves nothing if the other still points somewhere else. Only the sum is physical.

This paper builds the VERSF answer to both hands, and states precisely what it does and does not prove.

For the first hand, the key idea is that in VERSF the weights of those twisted configurations are not free choices — they come from the underlying substrate, and the paper argues they must be genuine positive weights, like probabilities, not quantities that can secretly hide a phase. A phase with nothing in the ontology to carry it is forbidden bookkeeping. If that holds, the first hand is forced to zero. And pleasingly, mirror symmetry alone could never do this: it narrows the dial to two positions, zero and halfway round, and it takes positivity to eliminate the second one.

For the second hand, the paper proves a dramatic simplification. One might fear that removing the quark-mass phase requires policing dozens of independent numbers. It doesn't. The entire strong-CP content of the quark masses collapses, exactly, into the sign of a single quantity — a kind of oriented volume built from the way quarks attach to the substrate. If that one quantity comes out positive, the second hand reads zero. The paper also proves this could never happen by luck: for arbitrary attachments the quantity points in a completely random direction, so a positive result would be evidence of real structure. And it identifies an exact shortcut — if the substrate's matter–antimatter map lines up the left-handed and right-handed attachments, positivity follows automatically, no computation needed.

A striking symmetry emerges: both hands end the same way. In each case, symmetry alone narrows the answer to a final two-way choice, and in each case only positivity — the substrate behaving like a genuine measure — makes the last selection. The two halves of a fifty-year-old problem turn out to have the same shape.

Crucially, none of this throws out the physics we need to keep. The twisted configurations (instantons) survive. The quantum froth of strong-force topology survives. And the small, real CP violation of the weak force survives untouched — the paper proves explicitly that the strong dial can read zero while quark mixing stays complex.

The honest caveat: this paper does not claim the decisive calculations have been done. It proves that if two specific, well-defined conditions hold — positive substrate weights on the twisted sectors, and a positive orientation for that single quark-mass quantity — then the strong force must respect the matter–antimatter mirror. Both conditions are now stated sharply enough to be computed, and sharply enough to fail. That is the advance: a fifty-year mystery reduced to two checkable questions.

In one sentence:

If the substrate can neither attach an unearned phase to the global structure of the strong force nor hide one in the quark masses, then strong CP conservation stops being an unexplained coincidence and becomes a theorem — while everything we actually observe, from instantons to weak CP violation, survives intact.

Scope and Closure Box

Claim	SCPθ-1 status
Local colour algebra is $\mathfrak{su}(3)_C$	Inherited
Faithful Standard Model group is $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$	Inherited conditionally from GCS-1
Local colour Hessian or Z_3 determines θ	Rejected
Liftable SU(3) bundles carry integer Pontryagin number	Standard exact result
Relative phases on an additive topological lattice are unitary characters	Exact
Ordinary liftable colour character is $e^{i\theta\nu}$	Exact
CP alone selects $\theta = 0$ uniquely	Rejected — $\theta = \pi$ is also a CP fixed point
Positive primitive support on sector generators forces the trivial character	Exact conditional theorem
VERSF primitive measure is real and nonnegative	Open microscopic premise
No independent pseudoscalar sector ledger exists	Conditional VERSF selection premise
Full $\mathbb{Z}_6$-quotient bundle and line-operator lattice	Open
Quotient may modify individual θ-periodicities and permit discrete topological data	Inherited global-QFT fact; explicit VERSF audit open
Axial chiral rotations shift θ through the fermion-measure Jacobian	Standard quantum result
$\bar{\theta} = \theta + \arg\det(M_u M_d)$ is basis invariant	Exact
A massless quark makes $\bar{\theta}$ unphysical	Exact conditional escape; not assumed
Quark determinant phase reduces to a determinant-line overlap	Exact
Determinant-line overlap is convention-independent jointly with θ_{sub}	Exact
Coincident chiral embeddings force $\Delta_f \geq 0$	Exact theorem — structural route to Gate B
A real structure forces $\mathrm{Im}\,\Delta_q = 0$ but not the sign	Exact — residual $\mathbb{Z}_2$ orientation datum
$\Delta_q < 0$ ray is the determinant-line analogue of $\theta = \pi$	Exact structural correspondence

Claim	SCPθ-1 status
arg Δ_q is uniformly distributed for generic embeddings	Numerical structural result — Gate B cannot hold by accident
Exact identities (Theorems 7, 9, 11; Lemma 20.1) certified numerically	Done — Appendix K
Positivity of $\hat{C}_Y$ alone fixes arg det Y_f	Rejected
Positive combined quark determinant-line orientation gives arg det(M_u M_d) = 0	Exact conditional theorem
Vanishing determinant phase forces vanishing CKM phase	Rejected
Strong/weak CP separation	Exact existence theorem
$\theta_{sub} = 0$ and positive determinant line imply $\bar{\theta} = 0$	Exact conditional synthesis
CP-even vacuum at $\bar{\theta} = 0$	Conditional on no spontaneous CP breaking
Nonzero topological susceptibility is compatible with $\bar{\theta} = 0$	Exact
RG and thresholds preserve $\bar{\theta} = 0$ automatically	Not assumed; criterion derived
Numerical neutron EDM	Open; not calculated
Empirical confirmation	Not established

Abstract

The VERSF gauge programme derives the local colour connection from bath transport, the Yang–Mills coefficient from the reduced substrate response, and the faithful Standard Model gauge quotient from the common central kernel of the inherited matter and completion-interface representations. None of those local or representation-level results fixes the global QCD θ-sector. The strong-CP problem therefore remains a distinct gate.

This paper establishes the Strong CP and Global θ-Sector Theorem at conditional theorem-architecture level.

Let X be a compact oriented Euclidean four-manifold, or a regulated finite-volume geometry with admissible boundary conditions. On a liftable $SU(3)_C$ bundle P_3, define

$$\nu_3(P_3, A) = (1/8\pi^2) \int_X \mathrm{tr}(\mathscr{F}_3 \wedge \mathscr{F}_3) \in \mathbb{Z}.$$

Composition of global sectors is additive. Any multiplicative relative phase compatible with composition is therefore a character of the topological charge group. On the ordinary colour subsector,

$$\chi_\theta(\nu) = e^{i\theta\nu}.$$

Local gauge dynamics determines the sector weights Z_ν but does not determine χ_θ.

The first central result is the Primitive-Measure Trivial-Character Theorem. Let $\mathfrak{T}_G$ be the quotient-compatible topological charge lattice selected by the faithful global VERSF gauge lift. Suppose the primitive substrate sector weights W_s are real and nonnegative; the positive-support sectors generate $\mathfrak{T}_G$; orientation reversal maps $s \mapsto -s$ without changing the primitive weight; and no independently derived pseudoscalar record supplies a one-dimensional topological representation. If the physical primitive measure must remain real and nonnegative sector by sector, then every admissible character satisfies

$$\chi(s) = 1$$

on the generating support and hence on all of $\mathfrak{T}_G$. In particular,

$$\theta_{sub} = 0 \pmod{2\pi}$$

on the liftable colour generator. The theorem excludes $\theta = \pi$ because the alternating character changes the sign of odd-charge sector weights, given positive odd-sector support. Positivity, not CP invariance alone, performs the selection. The paper answers the natural objection directly: ordinary vectorlike Euclidean QCD at $\theta = 0$ also has a positive measure, but that positivity is a property of one point on the θ-circle, not a selection principle; nothing in ordinary QFT forbids appending the character. The VERSF claim is ontological — the sector weights are primitive measure data, not amplitudes, so a character deformation is not a free move.

The faithful quotient requires a separate global audit. For

$$G_{SM}^{faithful} = [SU(3)_C \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6,$$

not every admissible bundle need lift independently to the three covering factors. Non-liftable sectors can correlate colour, weak and hypercharge characteristic data and alter individual angle periodicities. SCPθ-1 therefore defines the full topological data as a character

$$\chi_{glob} \in \mathrm{Hom}(\mathfrak{T}_G, U(1))$$

and restricts the ordinary strong angle only after the quotient-compatible lattice has been derived. A local Lie-algebra Hessian cannot close this gate. (χ_{glob} denotes the global character throughout; χ_{top} is reserved for the topological susceptibility.)

The second central result concerns the quark determinant. Under the anomalous chiral change of variables

$$q_L \mapsto U_L\, q_L,\quad q_R \mapsto U_R\, q_R,$$

the quark mass matrix transforms as $M \mapsto U_L^\dagger M U_R$, while the fermionic measure shifts the colour angle by

$\theta_3 \mapsto \theta_3 + \arg\det U\_L - \arg\det U\_R$.

Consequently,

$$\bar{\theta} = \theta_3 + \arg\det(M\_u\, M\_d) \pmod{2\pi}$$

is invariant. Setting the primitive angle to zero is therefore insufficient.

Using the inherited VERSF construction

$Y\_f^c = \mathcal{E}\_fL^\dagger\, \hat{C}\_Y\, \mathcal{E}\_fR,\ \hat{C}\_Y \geq 0$,

the Determinant-Line Reduction Theorem gives, for three full-rank generations,

$\det Y\_f^c = \langle \Omega\_fL, (\Lambda^3\hat{C}\_Y)\, \Omega\_fR \rangle \equiv \Delta\_f$.

Hence

$\arg\det(M\_u\, M\_d) = \arg(\Delta\_u\, \Delta\_d)$.

All strong-CP information carried by the complete quark Yukawa matrices is compressed into one phase on the combined anomalous determinant line. A convention-independence lemma establishes that this compression is not an artefact of volume-element bookkeeping: rephasing the reference volume elements shifts $\arg \Delta\_q$ and θ_sub oppositely, so the physical content is exactly the invariant pair. If the VERSF orientation and charge-conjugation lift provides a real positive trivialisation of the combined line,

$\Delta\_u\, \Delta\_d > 0$,

then

$\arg\det(M\_u\, M\_d) = 0 \pmod{2\pi}$.

This requirement does not make the CKM matrix real. The determinant phase belongs to the anomalous U(1) volume orientation; weak CP violation belongs to the relative SU(3) flavour frame. The Strong/Weak CP Separation Theorem constructs full-rank positive-determinant Yukawa matrices with nonzero Jarlskog invariant.

Three structural results then constrain Gate B before any microscopic input exists. The Coincident-Embedding Positivity Theorem shows that if the charge-conjugation lift identifies the left and right quark determinant volumes, $\Delta\_f = \langle \Omega\_f, (\Lambda^3\hat{C}\_Y)\, \Omega\_f \rangle \geq 0$ holds exactly, because the exterior cube of a positive operator is positive; this converts Debt 15 from a blind evaluation into a structural criterion on the conjugation lift. The Reality–Orientation Decomposition shows that a compatible real structure forces $\mathrm{Im}\, \Delta\_q = 0$ exactly while leaving $\mathrm{sign}(\Delta\_q)$ as an unresolved $\mathbb{Z}_2$ orientation datum, so the trivialisation $J\_\mathcal{D}$ must supply orientation beyond reality; the negative ray is the exact determinant-line analogue of the alternating $\theta = \pi$ character, and both gates therefore terminate in a $\mathbb{Z}_2$ selection that positivity, not CP, must perform. A genericity audit (Appendix K) establishes numerically that arg Δ_q is uniformly distributed on the circle for Haar-generic complex embeddings: positivity of the combined determinant line is measure-zero generic, so a positive VERSF result would certify structure and a complex result unambiguously fails the gate. The same appendix certifies the paper's exact identities — the determinant-line reduction, the joint convention-independence under full $U(3)^4$ anomalous rotations, and the strong/weak separation — to machine precision, and records the audited exterior-power pipeline that will evaluate Δ_q the day the embeddings are frozen.

Combining the two selection results yields the main conditional theorem:

$\theta_{sub} = 0$ and $\Delta_u \Delta_d > 0 \Longrightarrow \bar{\theta} = 0$,

provided the anomalous Jacobian is included, no quark is silently removed, the CP-even vacuum branch is selected, and RG flow and threshold matching preserve the invariant.

At $\bar{\theta} = 0$, the vacuum energy is even and

$\partial E/\partial\bar{\theta}|_0 = 0$, $\chi_{top} = \partial^2 E/\partial\bar{\theta}^2 |_0 = \lim_{V\to\infty} \langle \nu^2\rangle_c / V \geq 0$.

Strong CP conservation therefore does not require the disappearance of instantons or topological susceptibility.

SCPθ-1 closes the exact character classification, the physical $\bar{\theta}$ construction, the determinant-line reduction and its convention-independence, the strong/weak CP separation, and the conditional implication from the two substrate selection conditions to $\bar{\theta} = 0$. It does not yet derive the primitive global measure, the full $\mathbb{Z}_6$-quotient topological lattice, the numerical closure-Yukawa embeddings, the positive combined determinant orientation, the complete quantum Jacobian from substrate primitives, or the radiative correction. Those are the ordered calculations now required.

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1. Notation, Conventions and Firewalls

1.1 Colour connection

The primitive colour connection is

$$\mathcal{A} = A_\mu^a T_a dx^\mu,$$

with Hermitian generators

$$[T_a, T_b] = i f_{ab}^c T_c, \quad \mathrm{tr}(T_a T_b) = \tfrac{1}{2} \delta_{ab}.$$

Its curvature is

$$\mathcal{F} = d\mathcal{A} + i \mathcal{A} \wedge \mathcal{A}.$$

No explicit coupling is included in the primitive connection convention. Canonical coupling information resides in the kinetic coefficient and field normalisation.

1.2 Pontryagin convention

On a liftable SU(3) bundle over a closed oriented Euclidean four-manifold,

$$\nu[A] = (1/8\pi^2) \int_X \mathrm{tr}(\mathcal{F} \wedge \mathcal{F}) = (1/32\pi^2) \int_X d^4x\, F_{\mu\nu}^a \tilde{F}^{a \mu\nu} \in \mathbb{Z},$$

with

$$\tilde{F}^{\mu\nu} = \tfrac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}.$$

For manifolds with boundary, noncompact geometries, quotient bundles or singular defects, the admissible boundary and lifting conditions must be stated before this integer formula is used.

1.3 Euclidean θ convention

The Euclidean sector sum is written

$$Z(\theta) = \sum_{\nu \in \mathbb{Z}} e^{i\theta\nu} Z_\nu,$$

equivalently

$S\_E[A; \theta] = S\_E[A; 0] - i\theta\, \nu[A]$.

All signs in the chiral-Jacobian section are chosen so that

$\bar{\theta} = \theta + \arg\det(M\_u M\_d)$

is invariant.

1.4 Local/global firewall

The local objects

$\mathfrak{su}(3)\_C$, $\mathscr{F}$, Z_3, g_s

do not determine: the global gauge group; the bundle classes; the line-operator spectrum; the topological charge lattice; the θ-periodicity; or any discrete topological character.

A local Hessian can calculate the coefficient of F^2. It cannot select the global weighting of disconnected gauge sectors.

1.5 Covering-group/faithful-quotient firewall

The inherited VERSF claim is

$G\_SM\text{^}faithful = [SU(3)\_C \times SU(2)\_L \times U(1)\_Y]/\mathbb{Z}_6$.

This paper inherits that result conditionally. It does not assume that every bundle of the quotient lifts separately to bundles of the three covering factors.

The ordinary integer colour charge is therefore first proved on the **liftable colour subsector**. The complete theorem uses a quotient-compatible topological lattice $\mathfrak{T}\_G$, which remains to be calculated.

1.6 Bare-θ/physical-θ̄ firewall

The primitive gauge phase θ_sub is not directly physical once massive quarks are present. The invariant is

$\bar{\theta} = \theta\_sub + \arg\det(M\_u M\_d)$.

A paper deriving only θ_sub = 0 has not solved strong CP.

1.7 CP/fixed-point firewall

CP sends

$\nu \mapsto -\nu$, $\theta \mapsto -\theta$.

For 2π-periodic θ, CP invariance permits

$\theta = 0$ or $\theta = \pi$.

Orientation pairing alone does not select zero. The exclusion of π requires an additional condition — here, primitive nonnegative sector weights with positive odd-sector support.

1.8 Strong/weak CP firewall

Strong CP is controlled by $\bar{\theta}$. Weak quark CP violation is controlled by relative flavour-frame invariants such as

$\mathcal{J}_q = \mathrm{Im}\,\mathrm{Tr}([H_{uL}, H_{dL}]^3)$.

The conditions

$\bar{\theta} = 0$ and $\mathcal{J}_q \neq 0$

are mathematically compatible.

1.9 Topology/CP firewall

The condition $\bar{\theta} = 0$ does not imply $\nu[A] = 0$ for every configuration and does not imply $\chi_{top} = 0$. It implies only that sectors $+\nu$ and $-\nu$ are not assigned a CP-odd relative character and that the vacuum expectation of the CP-odd charge vanishes on the CP-even branch.

1.10 Gauge-anomaly/axial-anomaly firewall

Gauge anomalies must cancel for consistency and substrate admissibility.

The singlet axial anomaly is different. It is a physical quantum statement that moves phase between the colour topological term and the quark determinant. It must be retained, not cancelled.

1.11 Positive-operator/determinant-phase firewall

The closure-readout operator

$\hat{C}_Y \geq 0$

has nonnegative eigenvalues. This does not imply that

$\det(\mathscr{E}_{fL}^\dagger \hat{C}_Y \mathscr{E}_{fR})$

is real or positive, because the left and right embedded determinant volumes can be complex and misaligned.

1.12 Field-redefinition/solution firewall

A chiral field redefinition can make either θ or arg det M vanish in a chosen convention. It cannot change their sum.

A basis convention is not a strong-CP solution.

1.13 Massless-quark firewall

If an exactly massless quark exists, an axial rotation can remove $\bar{\theta}$. SCPθ-1 does not use that route unless the VERSF mass programme independently produces an exact zero singular value.

1.14 Axion firewall

No axion, Peccei–Quinn field, additional pseudoscalar or hidden chiral sector is inserted. Such a completion may be studied separately, but it cannot be used to claim that minimal VERSF already solves strong CP.

1.15 Positivity-scope firewall

The Primitive-Measure Trivial-Character Theorem assumes a real nonnegative **primitive substrate sector measure**. Ordinary QCD at nonzero θ has a complex Euclidean weight. The theorem is therefore a proposed VERSF selection mechanism, not an identity of arbitrary quantum field theories. Section 11 states precisely what the mechanism adds beyond the familiar positivity of vectorlike QCD at θ = 0.

1.16 Notation firewall — characters, susceptibility, and the three θ symbols

χ_{glob} always denotes the global topological character in $\mathrm{Hom}(\mathfrak{T}_G, U(1))$; χ_{sub} denotes its substrate-selected value; χ_{top} always denotes the topological susceptibility. The three symbols are never interchanged.

The three θ symbols are likewise fixed: **θ** is the generic coordinate on the character circle of an abstract $\mathbb{Z}$ sector group; **θ_3** is that coordinate on the liftable colour generator, in whatever convention the surrounding equation declares; **θ_sub** is the substrate-selected primitive value of θ_3, i.e. the value picked out by the VERSF measure before the fermion determinant is combined. The invariant $\bar{\theta}$ is convention-free and is always formed as the sum of the colour angle and the mass-determinant phase in the same convention.

1.17 Numerical non-insertion firewall

No experimental bound on the neutron electric dipole moment, no measured quark mass, no CKM phase, no lattice topological susceptibility and no fitted instanton density may be used to choose:

- the global character;
- the determinant-line orientation;
- the vacuum branch;
- the VERSF embeddings;
- the radiative counterterm.

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2. Predictive-Content Ledger

Object	Grade	Status
Local SU(3)_C connection	Inherited	Bath-transport programme
Colour kinetic coefficient Z_3	Inherited conditional	Gauge-action programme
Asymptotic freedom and Λ_3 architecture	Inherited conditional	QCR-1
Faithful $\mathbb{Z}_6$ quotient	Inherited conditional	GCS-1
Full quotient bundle classification	Open	New global target
Liftable colour Pontryagin integer	Standard exact	Used here
Quotient-compatible topological lattice $\mathfrak{T}$_G	Defined target	Numerical/global derivation open
Character classification	Exact	Derived here
CP fixed points 0, π on ordinary colour circle	Exact	Derived here
Primitive measure positivity	VERSF premise/target	Open microscopically
No-Private-Character Principle	New selection principle	Conditional
Trivial-character consequence	Exact conditional theorem	Derived here
θ_sub = 0	Conditional VERSF output	Requires measure gate

Object	Grade	Status
Axial fermion-measure Jacobian	Standard quantum result	Inherited once quantum gate closes
Physical $\bar{\theta}$	Exact invariant	Derived
Closure-Yukawa form	Inherited conditional	CMSY-1
Determinant-line reduction	Exact	Derived here
Joint convention-independence of (θ_{sub}, arg Δ_q)	Exact	Derived here
Positive combined determinant orientation	VERSF target	Open
Strong/weak CP independence	Exact	Derived here
CP-even vacuum branch	Conditional	Positivity/no-spontaneous-CP gate
Topological susceptibility at $\bar{\theta} = 0$	Exact definition	Numerical value open
RG stability condition	Exact identity	Derived here
Threshold matching of $\bar{\theta}$	Exact structural relation	Derived
Numerical $\bar{\theta}$	Open	No number inserted
Numerical EDM	Open	Downstream
Empirical confirmation	Not established	Blind comparison outstanding

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3. Inherited VERSF Infrastructure

3.1 Local colour gauge origin

The colour connection is inherited from continuous bath transport on a threefold indistinguishable carrier. The local internal algebra is $\mathfrak{su}(3)_C$.

3.2 Gauge action and running

The colour kinetic coefficient is inherited as a reduced substrate response,

$g_s(k) = Z_3(k)^{-1/2}$,

with universal one-loop running once the background-field quantum gate is supplied.

The coefficient of F^2 and the coefficient of $F \tilde{F}$ are different kinds of data. The first is a local positive response coefficient; the second is a global topological character.

3.3 Confinement and centre structure

QCR-1 separates local adjoint response from global centre-sensitive data. The same distinction is load-bearing here. Triality concerns the $\mathbb{Z}_3$ centre and line operators; the strong θ-sector concerns bundle topology and sector characters.

Neither is derivable from the eight-dimensional local Lie algebra alone.

3.4 Faithful global quotient

The Gauge-Closure Selection paper identifies, conditional on the inherited representation census,

$$G_{SM}^{faithful} = [SU(3)_C \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6,$$

with central congruence

$$2t + 3d + q \equiv 0 \pmod 6.$$

That paper explicitly leaves the bundle, line-operator, monopole and θ-periodicity consequences open. SCPθ-1 takes up that debt.

3.5 Anomaly inadmissibility

The substrate anomaly paper interprets a nonzero gauge-anomaly class as a persistent source-ledger leak. SCPθ-1 inherits the distinction between:

- a forbidden gauge anomaly, which destroys local gauge consistency;
- the physical singlet axial anomaly, which relates the fermion determinant phase to the colour topological term.

3.6 Closure-operator Yukawas

The canonical quark Yukawa operators are inherited in the form

$$Y_u^c = \mathcal{E}_{uL}^\dagger \hat{C}_Y \mathcal{E}_{uR}, \quad Y_d^c = \mathcal{E}_{dL}^\dagger \hat{C}_Y \mathcal{E}_{dR},$$

with

$$\hat{C}_Y = P_Y K_{cl}^{-1/2} H_{cl} K_{cl}^{-1/2} P_Y \geq 0.$$

The eigenvalues of the common operator are real and nonnegative. Complex phases can enter through the spectral projectors, chiral embeddings, interface contractions and transported kinetic structure.

3.7 Canonical CP equivalence

Canonical normalisation is an invertible field redefinition. It can redistribute phase between kinetic and interaction operators but cannot create or remove a genuine CP obstruction. SCPθ-1 therefore performs its determinant audit on canonically normalised mass operators without mistaking normalisation phases for new physics.

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4. Purpose and Claim Levels

SCPθ-1 answers seven questions.

1. What global object carries the strong θ-angle?
2. Why does local colour dynamics fail to determine it?
3. What VERSF property could select the trivial topological character?
4. How does the axial anomaly combine the colour phase with the quark determinant?
5. How can the determinant phase be calculated from the closure-Yukawa construction?
6. Can strong CP vanish while CKM CP remains nonzero?
7. What must be checked for vacuum and radiative stability?

The paper distinguishes four claim levels.

Level I — Exact global and algebraic identities

- Pontryagin integrality on liftable SU(3) bundles;
- character classification;
- CP fixed points;
- basis invariance of $\bar{\theta}$;
- determinant-line reduction and its joint convention-independence;
- strong/weak CP independence;
- RG and threshold identities.

Level II — Standard quantum-field inputs

- anomalous fermion-measure Jacobian;
- instanton-sector decomposition;
- transfer of phase under chiral rotations;
- positivity arguments for vectorlike QCD under declared conditions.

Level III — Conditional VERSF selection theorems

- primitive positive measure selects the trivial character;
- combined closure-derived determinant line is positively oriented;

- no unearned pseudoscalar sector ledger exists;
- the selected vacuum remains CP even.

Level IV — Open calculations

- full quotient bundle lattice;
- primitive substrate measure;
- instanton-sector support;
- numerical closure Hessian and embeddings;
- combined determinant overlap;
- complete quantum running and observable matching.

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5. Named Premises

SCP-1 — Local colour inheritance. The local colour connection takes values in $\mathfrak{su}(3)\_C$.

SCP-2 — Faithful global-group inheritance. The physical gauge group is the VERSF-derived faithful quotient

$G\_SM\text{\^{}}faithful = [SU(3)\_C \times SU(2)\_L \times U(1)\_Y]/\mathbb{Z}_6$.

SCP-3 — Global bundle lift. VERSF supplies a functor from admissible substrate transport histories to principal G_SM^faithful-bundles, transition data or an equivalent global gauge object, preserving composition and orientation reversal.

SCP-4 — Topological-sector completeness. The lift identifies the full additive lattice $\mathfrak{T}\_G$ of topological charges and discrete topological data relevant to the path integral.

SCP-5 — Liftable colour normalisation. On the subsector admitting an SU(3)_C lift, the colour generator is normalised by

$\nu_3 = (8\pi^2)\text{\^{}}(-1) \int \mathrm{tr}(\mathscr{F}_3 \wedge \mathscr{F}_3) \in \mathbb{Z}$.

SCP-6 — Primitive sector measure. Before any topological character is attached, VERSF derives sector weights W_s from the same microscopic closure measure that derives the local field action.

SCP-7 — Primitive nonnegativity. For every admissible sector in the support,

$W\_s \in \mathbb{R}\_\{\geq 0\}$.

This is a VERSF premise to be derived, not a property assumed of arbitrary θ-deformed QCD.

SCP-8 — Generating support. The sectors with $W_s > 0$ generate $\mathfrak{T}_G$, or at least generate the colour topological subgroup whose character is being selected. In particular, when the π-exclusion corollary is invoked, at least one odd-charge sector has $W_s > 0$.

SCP-9 — Orientation pairing. There is an exact substrate involution

$\mathscr{R} : s \mapsto -s$

with

$W_{-s} = W_s$.

SCP-10 — No private topological character. A one-dimensional global phase representation is admissible only if supplied by an independently derived physical pseudoscalar record, boundary datum or dynamical field. No such primitive exists in minimal VERSF.

SCP-11 — Sectorwise primitive admissibility. The primitive physical measure remains real and nonnegative on every supported sector; destructive phase weighting cannot be appended as unearned bookkeeping. Sectorwise means before orientation pairing: positivity of the paired sum alone is strictly weaker and does not select the character (Appendix C).

SCP-12 — Quantum fermion gate. The VERSF fermionic path-integral measure reproduces the standard singlet axial Jacobian with the inherited colour representation content.

SCP-13 — Closure-Yukawa inheritance. The canonical quark Yukawa operators have the CMSY-1 compression form, with independently derived $\hat{C}_Y$ and chiral embeddings.

SCP-14 — Full-rank massive branch. The main determinant theorem is applied on the branch where Y_u and Y_d have nonzero determinant. Exact zero modes are handled separately.

SCP-15 — Combined determinant-line orientation. The substrate orientation, charge-conjugation and chiral attachment maps provide a real positive trivialisation of the combined anomalous quark determinant line, in the canonical volume-element frame defined in Section 21.

SCP-16 — No spontaneous CP breaking at the selected point. The physical vacuum at $\bar{\theta} = 0$ lies on a CP-even branch.

SCP-17 — Quantum stability. The complete beta functions and threshold maps preserve $\bar{\theta} = 0$, or produce a calculable correction that is included before any empirical comparison.

SCP-18 — Numerical non-insertion. No strong-CP observable is used to choose the global character, determinant orientation, branch or counterterm.

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6. Local Colour Dynamics Does Not Determine θ

The local Euclidean colour action begins

$S_{loc} = (Z_3/2) \int_X \mathrm{tr}(\mathscr{F} \wedge \star\mathscr{F}) + \cdots.$

The topological term is

$S_\theta = -i\theta\, (1/8\pi^2) \int_X \mathrm{tr}(\mathscr{F} \wedge \mathscr{F}).$

The first term depends on the local metric and contributes to the field equations. The second is locally a total derivative and distinguishes global sectors through its integral.

Proposition 6.1 — Local-Response Insufficiency

No calculation of the local quadratic colour Hessian

$\mathcal{K}_3 = E_3^\dagger\, \mathscr{H}^{red} E_3$

can by itself determine the topological character.

Proof

The local Hessian is a bilinear form on infinitesimal adjoint curvature variations. It determines the coefficient multiplying the metric contraction $F_{\mu\nu} F^{\mu\nu}$. The θ-term is controlled by the orientation-sensitive density $F_{\mu\nu} \tilde{F}^{\mu\nu}$, whose integral depends on the global bundle class and is unchanged by local continuous deformations within a sector. Two theories can possess the same local Hessian and different global sector characters. Therefore local response data are insufficient. ■

Consequence

The colour-holonomy response calculation and the strong-CP calculation are sequential but distinct:

$E_3^\dagger\, \mathscr{H}^{red} E_3 \longrightarrow Z_3$

does not imply θ_{sub}.

---

7. Liftable Colour Bundles and Pontryagin Sectors

Let $P_3 \to X$ be a principal SU(3) bundle.

Theorem 1 — Liftable Colour Pontryagin Census

For a smooth connection A on P_3,

$\nu_3(P_3) = (1/8\pi^2) \int_X \mathrm{tr}(\mathscr{F}_3 \wedge \mathscr{F}_3)$

is the second Chern number of the associated fundamental bundle and therefore satisfies

$\nu_3(P_3) \in \mathbb{Z}$.

It is constant under continuous deformations that preserve the bundle and boundary conditions.

Proof

Chern–Weil theory identifies the de Rham class $(1/8\pi^2)\, \mathrm{tr}(\mathscr{F}_3 \wedge \mathscr{F}_3)$ with $c_2(P_3)$ in the stated generator normalisation. Integration over the fundamental four-cycle gives an integer characteristic number. Continuous connection variations change the density by an exact form and leave the integral unchanged on a closed manifold or under fixed admissible boundary data. ■

Corollary 7.1 — Additive Colour Charge

Under disjoint composition or gluing with compatible boundary data,

$\nu_3[A_1 \# A_2] = \nu_3[A_1] + \nu_3[A_2]$.

The liftable colour sectors therefore carry an additive $\mathbb{Z}$ label.

Boundary qualification

On $\mathbb{R}^4$, finite-action conditions compactify the asymptotic data to the relevant homotopy class. On a manifold with boundary, the Chern–Simons boundary term must be controlled. On the faithful quotient, the individual colour number need not remain integral in every non-liftable bundle. Those cases belong to Section 9.

---

8. Topological Phases as Characters

Let $\mathfrak{T}$ be an additive abelian topological charge group. A sector weighting compatible with composition is a map

$\chi : \mathfrak{T} \to U(1)$

satisfying

$\chi(s_1 + s_2) = \chi(s_1)\, \chi(s_2)$.

Theorem 2 — Topological-Character Classification

Every composition-compatible relative phase weighting is a unitary character,

$\chi \in \mathrm{Hom}(\mathfrak{T}, U(1))$.

For $\mathfrak{T} = \mathbb{Z}$,

$\chi_\theta(\nu) = e^{i\theta\nu}$, $\theta \in \mathbb{R}/2\pi\mathbb{Z}$.

Proof

Composition compatibility is exactly the homomorphism condition from the additive sector group to the multiplicative unit circle. Every homomorphism from $\mathbb{Z}$ is determined by $\chi(1) \in U(1)$. Writing $\chi(1) = e^{i\theta}$ gives $\chi(\nu) = e^{i\theta\nu}$, with θ defined modulo 2π. ■

Interpretation

The θ-angle is not another local coupling extracted from the colour Hessian. It is the coordinate of a one-dimensional representation of the global sector group.

The strong-CP question can therefore be written sharply:

Which character of the colour topological group does VERSF select?

---

9. Faithful-Group Quotient and the Global Charge Lattice

The faithful group is inherited as

$G = [SU(3)_C \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6$.

A G-bundle need not lift globally to an independent bundle for each covering factor. Let

$w_6(P) \in H^2(X, \mathbb{Z}_6)$

denote schematically the obstruction to such a lift. The exact characteristic-class realisation must be derived for the VERSF global carrier.

Definition 9.1 — Quotient-Compatible Topological Lattice

Let $\mathfrak{T}_G(X)$ be the additive group or lattice generated by the allowed characteristic data of all VERSF-admissible G-bundles on X, including:

- liftable integer instanton numbers;
- correlated fractional covering-factor charges in non-liftable sectors;
- magnetic or line-operator data required by the quotient;
- admissible discrete topological terms.

The complete global topological weighting is

$\chi_{glob} \in \mathrm{Hom}(\mathfrak{T}_G, U(1))$.

Theorem 3 — Liftable-Subsector Restriction

If $w_6(P) = 0$, the G-bundle lifts to the covering product and the colour characteristic number is integral:

$\nu_3 \in \mathbb{Z}$.

The restriction of any full character χ_{glob} to the liftable colour generator has the form

$\chi_{glob} |_{\mathbb{Z}_3^{top}} = e^{i\theta_3\nu_3}$,

where $\mathbb{Z}_3^{top}$ denotes the additive $\mathbb{Z}$ of liftable colour topological charge — the instanton-number subgroup, not the $\mathbb{Z}_3$ centre of the gauge group.

Proof

A lift supplies an ordinary SU(3) bundle, so Theorem 1 applies. The restriction of a character to the resulting additive $\mathbb{Z}$ subgroup is classified by Theorem 2. ■

Known global-QFT imports and their limits

The general phenomenon is established in the external literature: on quotient gauge groups the θ-angle periodicities can extend, correlate across factors, and mix with discrete topological terms and line-operator choices (Aharony–Seiberg–Tachikawa; Tong for the Standard Model quotients). SCPθ-1 imports the *existence* of these phenomena as a warning, not their specific realisation. The VERSF audit must derive which of the four global forms of the Standard Model gauge group the substrate actually selects, and compute the resulting joint periodicity lattice, rather than assuming the covering-product answer.

Global-quotient firewall

The ordinary statement $\theta_3 \sim \theta_3 + 2\pi$ is secure on the isolated liftable integer generator. The periodicity of a coordinate called "the colour angle" on the full quotient lattice may be correlated with shifts of electroweak angles or discrete characters.

SCPθ-1 therefore refuses two shortcuts:

1. it does not infer the full global θ-torus from the local algebra;
2. it does not claim to have calculated the quotient periodicities before $\mathfrak{T}$_G is derived.

Global selection target

The substrate selection theorem should ultimately act on the whole lattice:

$\chi\_sub(s) = 1 \;\forall\; s \in \mathfrak{T}\_G$,

not only on the liftable colour line. The ordinary strong result is the restriction of that stronger statement.

---

10. CP Action and the 0/π Fixed-Point Theorem

Orientation reversal sends

$\mathscr{F} \wedge \mathscr{F} \mapsto -\mathscr{F} \wedge \mathscr{F}$,

hence $\nu \mapsto -\nu$. For a character,

$\chi\_\theta(-\nu) = \chi\_\theta(\nu)^* = e^\wedge(-i\theta\nu)$.

Theorem 4 — CP Fixed Points on the Liftable Colour Circle

For a 2π-periodic colour angle, the topological character is CP invariant iff

$\theta \equiv -\theta \pmod{2\pi}$.

Therefore

$\theta = 0$ or $\theta = \pi \pmod{2\pi}$.

Proof

CP invariance requires $e^{i\theta\nu} = e^{-i\theta\nu}$ for every $\nu \in \mathbb{Z}$. Setting $\nu = 1$ gives $e^{2i\theta} = 1$, so $\theta = 0$ or π modulo 2π. ■

Quotient caveat

The 2π periodicity assumed here is the liftable-subsector statement. On the full quotient lattice $\mathfrak{T}_G$, angle identifications may be joint (Section 9), and the CP fixed-point set of the complete character group must be recomputed after $\mathfrak{T}_G$ is derived (F26). Theorem 4 is exact where stated and provisional beyond it.

Consequence

A VERSF proof based only on orientation symmetry would not solve the selection problem. It would reduce a continuous parameter to two discrete branches.

The additional question is:

Why the trivial character rather than the alternating character?

---

11. Primitive Sector Measure and Positivity

Let the primitive VERSF sector sum be

$Z_{sub} = \sum_{s \in \mathfrak{T}_G} W_s$,

where W_s is obtained by integrating or summing the microscopic closure weight over histories in sector s, before any independent topological character is appended.

The proposed minimal-VERSF properties are:

$W_s \geq 0$, $W_{-s} = W_s$.

The first is stronger than CP. It says that the primitive closure measure is an actual nonnegative measure rather than a conventionally chosen complex amplitude on the sector set.

11.1 What this adds beyond ordinary QCD positivity

A referee should immediately object: Euclidean vectorlike QCD at $\theta = 0$ already has a real nonnegative measure — the quark determinant is positive for positive masses, and the gauge weight is real. Why is VERSF positivity anything more than a restatement of the $\theta = 0$ point?

The answer is that ordinary positivity is a *property* of one point on the θ-circle, not a *selection principle*. In ordinary QFT, the θ-deformed theories

$Z(\theta) = \sum_\nu e^{i\theta\nu} Z_\nu$

are all equally consistent; nothing in the formalism prefers the positive-measure member of the family. The angle is an input.

The VERSF claim is ontological, not formal. The sector weights W_s are proposed to be primitive measure data of the substrate — the same closure measure that generates the local field action — rather than amplitudes on which characters may be freely appended. If the primitive measure is the physical object, then a character deformation is not a change of description; it is the insertion of phase information that no substrate structure carries. That insertion is what the No-Private-Character Principle forbids.

The distinction is falsifiable: F7 states that if the actual minimal substrate measure is intrinsically sign-indefinite or complex before any character is attached, the selection mechanism fails and the θ-angle reverts to an input.

Emergence-layer placement. One further specification is required, because VERSF elsewhere *derives* complex amplitudes and Hilbert-space interference from the substrate. The positivity claim of SCP-7 is a claim about the layer **below** that emergence: W_s must be computed from the primitive closure measure before the reconstruction of complex amplitude structure, not read off from the emergent amplitude layer, where weights are already complex for the same reason Minkowski-signature QCD is complex-weighted at every θ. If the sector decomposition is only available after amplitudes exist, positivity is unjustified and F7 fires. The natural comparison object is then Euclidean: an atemporal primitive measure has no phase generated by time evolution, and its field-theoretic image is a real-weighted sum over completed configurations — which is precisely the Euclidean sector sum, not the Minkowski one. Making this placement precise is part of Debt 5: the derivation of W_s must exhibit the layer at which the topological decomposition is performed and show that it precedes amplitude emergence. Signature-dependence of "the physical measure is positive" is thereby acknowledged rather than hidden: the claim is positivity of the primitive atemporal measure, with the Euclidean weighting as its derived image.

11.2 Orientation reversal versus commitment irreversibility

SCP-9 requires an exact involution $\mathscr{R}$ with $W_{-s} = W_s$. This premise deserves scrutiny beyond bookkeeping, because VERSF's most distinctive ontological commitment is a **directed** primitive process — commitment events are irreversible — while SCP-9 asserts an exact reversal symmetry of the measure that process generates. These are not automatically compatible, and the paper must type the claim rather than assume it.

What $\mathscr{R}$ acts on. $\mathscr{R}$ is an involution on completed configuration sectors: it reverses the orientation class of the recorded field configuration, sending the sector label s to −s. It does **not** act on the direction of commitment. The commitment ordering is the process by which a configuration becomes recorded; the orientation class is a property of what is recorded. SCP-9 is therefore the claim that the primitive weight of a completed sector depends only on the recorded configuration class and is blind to the orientation of that class — not the (false) claim that substrate dynamics can be run backwards.

Why this is nontrivial. Nothing in the typing guarantees the claim. If the directedness of commitment leaves an orientation-sensitive imprint on the completed measure — if histories realising ν and −ν are reached with systematically different primitive weight because of the arrow in the process — then $W_\nu \neq W_{-\nu}$, the substrate sources CP violation at the measure level, and the selection theorem fails at its foundation rather than at its algebra. This is the precise point where the theorem's premises meet the programme's foundations, and it is the Gate A twin of the Gate B $\mathbb{Z}_2$ analysis in Proposition 21.2: there, reality had to be earned before orientation; here, orientation-blindness of a directed process must be earned before pairing.

Sharpened falsifier. F11 is accordingly strengthened below: it fails not only if the involution is misconstructed, but if commitment directedness is shown to bias the completed sector measure between orientation classes. Debt 8 must construct $\mathscr{R}$ on actual substrate histories and prove the blindness, not merely define the map on labels.

Definition 11.1 — Primitive Character Deformation

A character deformation is

$W_s \mapsto W_s^\chi = \chi(s)\, W_s.$

A nontrivial character introduces phase information that is absent from the magnitude W_s.

Lemma 11.1 — Support-Generator Lemma

Let $S_+ \subset \mathfrak{T}_G$ be the set of sectors with $W_s > 0$. If S_+ generates $\mathfrak{T}_G$, then a character is determined by its values on S_+.

Proof

Every element of $\mathfrak{T}_G$ is a finite integer combination of supported generators. The homomorphism property determines the character on every such combination. ■

Lemma 11.2 — Sectorwise versus Paired Admissibility

Sectorwise admissibility (SCP-11) requires $\chi(s) W_s \in \mathbb{R}_{\geq 0}$ for each supported s. The strictly weaker paired condition — nonnegativity of $W_s \chi(s) + W_{-s} \chi(-s) = 2 W_s \cos \varphi_s$ — admits every character with $\cos \varphi_s \geq 0$ on the support and therefore does not select. The selection theorem rests on the sectorwise form, and the sectorwise form is the one licensed by the ontic reading of W_s: a primitive measure is a measure sector by sector, not only in orientation-averaged combinations. (Proof detail in Appendix C.)

---

12. The No-Private-Character Principle

The Gauge-Closure Selection paper excludes private ledgers on indistinguishable local sectors: a dynamically load-bearing distinction cannot be preserved if no admissible record carries it.

The global analogue is:

Principle NPC — No-Private-Character Principle

A topological phase character is substrate-admissible only if it is carried by an independently derived physical structure, such as:

- a primitive pseudoscalar record;
- an oriented boundary datum;
- a dynamical axion-like field;
- a discrete topological sector supplied by the global bundle lift;
- or another explicitly typed source of one-dimensional holonomy.

A character that changes quantum interference while corresponding to no substrate record is private global bookkeeping and is excluded.

Reading

A nontrivial θ-character is not inert. It changes the relative amplitude of globally distinct histories and therefore changes observables. If no microscopic object carries that phase, the substrate would be paying a physical interference cost to preserve information that nothing in its ontology records.

Status

NPC is a VERSF selection principle, not a theorem of ordinary QCD. Its value is that it makes the remaining foundational burden explicit:

Derive the primitive sector measure and prove that minimal VERSF contains no independent pseudoscalar character carrier.

---

13. The Primitive-Measure Trivial-Character Theorem

Theorem 5 — Primitive-Measure Trivial-Character Theorem

Assume SCP-3–SCP-11. Let

$\chi \in \mathrm{Hom}(\mathfrak{T}_G, U(1))$

be an admissible character deformation of the primitive sector measure. If

$W_s > 0 \Longrightarrow \chi(s)\, W_s \in \mathbb{R}_{\geq 0}$,

and the positive-support sectors generate $\mathfrak{T}_G$, then

$\chi(s) = 1 \ \forall\, s \in \mathfrak{T}_G$.

Proof

For every supported sector s, W_s is strictly positive and real. The product $\chi(s)\, W_s$ is required to be real and nonnegative. Since $|\chi(s)| = 1$, the only allowed value is $\chi(s) = 1$. By the Support-Generator Lemma and the homomorphism property, χ is unity on the group generated by the support, which is all of $\mathfrak{T}_G$. ■

Remark on the weight of the theorem

The deductive step is short by design. The entire content of the result lives in the premises — sectorwise admissibility (SCP-11), generating support (SCP-8), and the absence of a private carrier (SCP-10). SCPθ-1 treats this as a feature: the theorem converts a vague hope ("some symmetry sets θ to zero") into three sharply falsifiable microscopic conditions, each with its own debt and falsifier. A short proof over exactly stated premises is worth more than a long proof over hidden ones.

Corollary 13.1 — Strong Primitive Character

On the liftable colour generator,

$\chi\_sub(\nu) = 1$,

so

$\theta\_sub = 0 \pmod{2\pi}$.

Corollary 13.2 — Exclusion of the π Branch

Suppose in addition that at least one odd-charge sector has positive primitive support (the odd-support clause of SCP-8). At $\theta = \pi$,

$\chi\_\pi(\nu) = (-1)^\nu$,

so that odd sector's weight becomes negative. Therefore the alternating character violates sectorwise primitive nonnegativity and is excluded.

Without positive odd-sector support the corollary does not apply, and the π branch must be excluded by other means or acknowledged as open (F10).

What the theorem really proves

It does not prove that ordinary QCD forbids a θ-term.

It proves:

positive generating primitive measure + no independent phase carrier ⟹ trivial topological character.

The physical work is to derive the hypotheses from VERSF.

---

14. Large-Gauge Single-Valuedness

A θ-vacuum may also be described as a one-dimensional representation of the group of large gauge transformations acting on wavefunctionals.

Let $\mathcal{G}/\mathcal{G}_0$ denote the disconnected components of the gauge-transformation group. In the liftable colour sector,

$\pi_0(\mathcal{G}) \cong \mathbb{Z}$.

A θ-state transforms as

$\Psi_\theta[A^{g_n}] = e^{in\theta} \Psi_\theta[A]$.

Theorem 6 — Character/Flat-Line Equivalence

The following data are equivalent:

1. a character $e^{i\theta\nu}$ weighting topological sectors;
2. a one-dimensional representation $e^{in\theta}$ of large gauge transformations;
3. a flat U(1) line over the gauge-orbit configuration space with holonomy $e^{i\theta}$.

Proof sketch

The winding of a large gauge transformation changes the Chern–Simons functional by an integer and maps between equivalent descriptions of adjacent topological histories. The sector phase, transformation law of the wavefunctional and flat-line holonomy are three realisations of the same character of the integer component group. ■

Corollary 14.1 — Single-Valued Persistent Record

If the VERSF physical state is required to be a single-valued scalar record on the true gauge-orbit space, rather than a section of an independently supplied nontrivial flat line, then only the trivial large-gauge character is admissible.

Firewall

This corollary is not independent of the No-Private-Character Principle. It is its canonical-state formulation. A nontrivial flat line is allowed only if VERSF derives the line as physical structure.

---

15. Instantons and Sector Support

The trivial-character theorem requires support on generators of the topological lattice. Merely classifying sectors does not prove that the primitive dynamics assigns them nonzero weight.

Definition 15.1 — Sector Support Functional

Let

$$W_s = \int_{\mathcal{C}_s} d\mu_{sub}\, e^{-\mathcal{S}_{sub}}$$

schematically denote the primitive weight of the configuration sector $\mathcal{C}_s$.

Required calculation

VERSF must establish one of:

1. $W_{\pm 1} > 0$ on the liftable colour line; or
2. positive support on another set generating the same topological subgroup.

Proposition 15.1 — Vanishing-Support Limitation

If all sectors with nonzero colour charge have exactly zero primitive weight, then θ is dynamically irrelevant but not selected by Theorem 5.

Proof

The partition function and all supported observables are independent of the character on zero-weight sectors. No experiment in that theory can distinguish θ, but the character is not mathematically fixed by positivity. ■

This limitation is falsifier F22 in premise form: a mechanism that empties the nonzero-topology sectors evades the selection problem rather than solving it.

Programme reading

A strong VERSF solution should not destroy the topological sectors to evade the question. It should show:

$W_{\pm 1} > 0$ and $\chi_{sub} = 1$.

That preserves nontrivial QCD topology while selecting CP.

---

16. The Anomalous Chiral Jacobian

Let the quark mass terms be

$\mathscr{L}_m = -\bar{u}_L M_u u_R - \bar{d}_L M_d d_R + \text{h.c.}$

Consider general unitary chiral changes of variables,

$u_L \mapsto U_{uL} u_L$, $u_R \mapsto U_{uR} u_R$,

and similarly for d. The mass matrices transform as

$M_u \mapsto U_{uL}^\dagger M_u U_{uR}$, $M_d \mapsto U_{dL}^\dagger M_d U_{dR}$.

The anomalous fermion measure shifts the colour angle.

Theorem 7 — Anomalous Phase-Transfer Theorem

Under the combined quark change of variables,

$\theta_3 \mapsto \theta_3 + \arg\det U_{uL} - \arg\det U_{uR} + \arg\det U_{dL} - \arg\det U_{dR}$.

Meanwhile,

$\arg\det(M_u M_d) \mapsto \arg\det(M_u M_d) - \arg\det U_{uL} + \arg\det U_{uR} - \arg\det U_{dL} + \arg\det U_{dR}$.

Therefore their sum is invariant.

Proof

The mass transformation law gives the determinant-phase shift directly. The fermion-measure shift is the integrated singlet axial anomaly, with the same representation normalisation and opposite total phase. Adding the two transformed quantities cancels every basis-dependent determinant phase. ■

VERSF significance

The substrate anomaly-inadmissibility theorem concerned gauge anomalies. Here the anomalous Jacobian must survive. If the VERSF quantum measure failed to reproduce it, the theory would not match QCD's chiral topology and the strong-CP analysis would be invalid.

---

17. The Physical $\bar{\theta}$ Invariant

Theorem 8 — Physical Strong-Phase Theorem

For nonzero quark determinant,

$$\bar{\theta} = \theta_3 + \arg\det(M_u M_d) \pmod{2\pi}$$

is invariant under all nonsingular unitary changes of quark flavour basis, including anomalous axial rephasings.

Proof

Immediate from Theorem 7. ■

Corollary 17.1 — Bare-Zero Insufficiency

The condition $\theta_3 = 0$ does not imply $\bar{\theta} = 0$ unless arg det(M_u M_d) = 0.

Corollary 17.2 — Real-Mass Convention

One may choose a convention in which the quark determinant is real and positive. In that convention the complete physical phase appears in the colour term:

$\theta_3^{\text{mass-real}} = \bar{\theta}$.

Alternatively, one may choose $\theta_3 = 0$, in which case the complete phase resides in the mass determinant. Neither convention changes the physics.

---

18. Massless-Quark and Axion Firewalls

Proposition 18.1 — Exact Massless-Quark Escape

If one quark has an exactly vanishing mass eigenvalue, the corresponding axial phase is not fixed by a mass term. An anomalous axial rotation can then remove θ_3, making $\bar{\theta}$ unobservable.

SCPθ-1 position

The VERSF mass programme currently targets nonzero light-quark current masses. The massless route is therefore not used as the primary solution.

If a future blind closure-Yukawa calculation produces

det M_u = 0 or det M_d = 0,

the strong-CP theorem must be rewritten around the exact chiral zero mode.

Proposition 18.2 — Axion Route Is a Different Theory

A dynamical pseudoscalar a(x) with coupling

$(\theta + a/f_a)\, \mathscr{F}\tilde{F}$

can relax the effective angle. This introduces a new physical character carrier and lies outside minimal SCPθ-1.

The No-Private-Character Principle does not exclude a phase carried by a genuinely derived field. It excludes only an unearned phase with no carrier.

---

19. Closure-Operator Yukawa Inheritance

At the matching scale μ⋆,

$M_f^c = (v_{cl}/\sqrt{2})\, Y_f^c,\ f \in \{u, d\},$

with $v_{cl} > 0$ in the chosen vacuum convention. Hence

$\arg\det(M_u M_d) = \arg\det(Y_u^c Y_d^c).$

The inherited operators are

$Y_f^c = \mathcal{E}_{fL}^\dagger\, \hat{C}_Y\, \mathcal{E}_{fR},$

where

$\hat{C}_Y = \sum_\alpha \lambda_\alpha \Pi_\alpha,\ \lambda_\alpha \geq 0.$

Key observation

The eigenvalues λ_α carry no phase. Every determinant phase arises from the relative oriented volumes selected by:

- the left embedding;
- the right embedding;
- the spectral projectors;
- the interface contraction;
- the kinetic transport included in the canonical embeddings.

Strong CP therefore has an exact location in the VERSF Yukawa architecture.

---

20. The Determinant-Line Reduction Theorem

Let the canonical left and right quark carriers each have dimension three. Choose unit volume elements

$\omega_{fL} \in \det \mathscr{H}_{fL}^c$, $\omega_{fR} \in \det \mathscr{H}_{fR}^c$.

Define their embedded determinant vectors,

$\Omega_{fL} = (\Lambda^3 \mathscr{E}_{fL})\, \omega_{fL}$, $\Omega_{fR} = (\Lambda^3 \mathscr{E}_{fR})\, \omega_{fR}$.

These live in $\Lambda^3 \mathscr{H}_{cl}^c$.

Theorem 9 — Determinant-Line Reduction

For

$Y_f^c = \mathscr{E}_{fL}^\dagger\, \hat{C}_Y\, \mathscr{E}_{fR}$,

the determinant is

$$\det Y_f^c = \langle \Omega_{fL}, (\Lambda^3 \hat{C}_Y)\, \Omega_{fR} \rangle \equiv \Delta_f.$$

Proof

Exterior powers preserve composition:

$\Lambda^3 Y_f^c = (\Lambda^3 \mathscr{E}_{fL}^\dagger)(\Lambda^3 \hat{C}_Y)(\Lambda^3 \mathscr{E}_{fR})$.

Since the determinant spaces of the three-dimensional fermion carriers are one-dimensional, $\Lambda^3 Y_f^c$ acts by multiplication by $\det Y_f^c$. Evaluating between the unit determinant vectors gives the displayed overlap. ■

Lemma 20.1 — Joint Convention-Independence

The individual overlap Δ_f depends on the phase choice of the unit volume elements: rephasing $\omega_{fL} \mapsto e^{i\alpha_{fL}} \omega_{fL}$ and $\omega_{fR} \mapsto e^{i\alpha_{fR}} \omega_{fR}$ sends

$\Delta_f \mapsto e^{i(\alpha_{fR} - \alpha_{fL})}\, \Delta_f$.

But such a rephasing is precisely an anomalous determinant rotation of the corresponding chiral quark fields, which by Theorem 7 simultaneously shifts

$\theta_3 \mapsto \theta_3 + (\alpha_{fL} - \alpha_{fR})$.

Therefore the pair

$(\theta_3, \arg(\Delta\_u\ \Delta\_d))$

transforms so that $\theta_3 + \arg(\Delta\_u\ \Delta\_d) = \bar{\theta}$ is convention-independent. The determinant-line reduction is a reduction of the invariant, not a statement dependent on volume-element bookkeeping.

Consequence

Any claim of the form "$\Delta\_u\ \Delta\_d > 0$" is meaningful only relative to a *canonically supplied* volume-element frame in which θ_sub has been independently defined. Section 21 makes that frame explicit; SCP-15 refers to it. A frame chosen after the fact to make Δ_q positive is falsifier F15 in action.

Corollary 20.2 — Combined Strong Determinant

arg det(M_u M_d) = arg(Δ_u Δ_d).

Corollary 20.3 — Spectral Expansion

Let Ĉ_Y = ∑_α λ_α Π_α with **distinct** eigenvalues λ_α and eigenspaces V_α of arbitrary multiplicity. The antisymmetric cube decomposes over occupation patterns, not ordered triples:

$\Lambda^3\mathcal{H}\_cl = \bigoplus\_\{(k)\}\ \bigotimes\_\alpha\ \Lambda\text{^}\{k\_\alpha\}\ V\_\alpha$,

where (k) = (k_α) runs over all patterns with ∑_α k_α = 3 and 0 ≤ k_α ≤ dim V_α, and each summand is understood as canonically embedded in $\Lambda^3\mathcal{H}$_cl by the wedge product. On the summand labelled (k), the operator acts by the scalar ∏_α λ_α^{k_α}:

$\Lambda^3\hat{C}\_Y = \sum\_\{(k)\}$ (*∏α λ_α^{k_α}) Π^(3)*{(k)},

with Π^(3)_{(k)} the orthogonal projector onto ⊗_α Λ^{k_α} V_α. Therefore

Δ_f = ∑_{(k)} (*∏α λ_α^{k_α}) ⟨Ω_fL, Π^(3)*{(k)} Ω_fR⟩.

For a nondegenerate spectrum every admissible pattern has k_α ∈ {0,1} and the sum reduces to strictly ordered distinct triples with coefficient λ_α λ_β λ_γ; for degenerate spectra the repeated-eigenvalue sectors $\Lambda^2(V\_\alpha) \otimes V\_\beta$ and $\Lambda^3(V\_\alpha)$, with coefficients $\lambda\_\alpha^2\ \lambda\_\beta$ and λ_α^3, are genuine additional summands. All coefficients are nonnegative in every case. The phase is entirely in the determinant-volume pairings. This partition form is the same object used in the proof of Proposition 21.1 (restriction of Ĉ_Y^⊗3 to the antisymmetric subspace) and computed in Appendix K.1 (third compound matrix); the three presentations agree.

Importance

The strong-CP mass problem is not a nine-entry phase problem per quark sector. It is one combined determinant-line phase:

$\varphi_{det} = \arg(\Delta_u \Delta_d)$.

This is the exact quantity VERSF must calculate.

---

21. The Combined Quark Determinant-Line Selection Theorem

Define the combined anomalous determinant line

$\mathcal{D}_q = \det \mathscr{H}_{uL}^{c*} \otimes \det \mathscr{H}_{uR}^c \otimes \det \mathscr{H}_{dL}^{c*} \otimes \det \mathscr{H}_{dR}^c$.

The product $\det(Y_u Y_d)$ is a section of this one-dimensional complex line before a chiral orientation convention is fixed.

Premise DLT — Positive Determinant-Line Trivialisation

The VERSF orientation, charge-conjugation, completion and occupancy maps provide a canonical real structure

$J_{\mathcal{D}} : \mathcal{D}_q \to \mathcal{D}_q$

and an orientation ray

$\mathcal{D}_q^+ \subset \mathrm{Fix}(J_{\mathcal{D}})$

such that the closure-Yukawa section lies in the positive ray.

Equivalently, in the frame trivialised by $J_{\mathcal{D}}$,

$\Delta_u \Delta_d > 0$.

This is a load-bearing derivational premise, not a convention. Lemma 20.1 shows why a basis rotation cannot establish it: anomalous determinant rotations of the volume elements simultaneously shift θ. DLT is the claim that the *same* substrate structure that defines θ_{sub} also fixes the volume-element frame, and that in that jointly defined frame the section is positive.

Theorem 10 — Combined Determinant-Line Selection

Under SCP-13–SCP-15 and the full-rank premise,

$\arg\det(M_u M_d) = 0 \pmod{2\pi}$.

Proof

The positive real completion scale contributes no phase. By Theorem 9,

$\det(M_u M_d) = (v_{cl}/\sqrt{2})^6 \Delta_u \Delta_d$.

The prefactor is positive. By DLT, $\Delta_u \Delta_d > 0$ in the canonical frame. Therefore the determinant has zero phase modulo 2π. ■

Why positivity of $\hat{C}_Y$ is insufficient

A positive operator yields a nonnegative quadratic expectation when the same vector occurs on both sides. Here the left and right determinant vectors differ:

$\langle \Omega_L, (\Lambda^3\hat{C}_Y)\, \Omega_R \rangle$.

That cross-pairing can be complex. DLT is therefore a real physical bridge that the existing Yukawa theorem does not yet close.

Proposition 21.1 — Coincident-Embedding Positivity

Suppose the substrate charge-conjugation and completion structure identifies the left and right determinant vectors of a quark sector:

$\Omega_{fL} = \Omega_{fR} \equiv \Omega_f$.

Then

$\Delta_f = \langle \Omega_f, (\Lambda^3\hat{C}_Y)\, \Omega_f \rangle \geq 0$,

with strict positivity iff $\Omega_f \notin \ker(\Lambda^3\hat{C}_Y)$.

Proof

$\hat{C}_Y \geq 0$ implies $\hat{C}_Y^{\otimes 3} \geq 0$ on $\mathscr{H}_{cl}^{\otimes 3}$. The antisymmetriser commutes with $\hat{C}_Y^{\otimes 3}$, so the restriction $\Lambda^3\hat{C}_Y$ to the antisymmetric subspace is positive. The overlap is then a quadratic form of a positive operator, hence real and nonnegative; it vanishes iff Ω_f lies in the kernel. ■

Consequence

This is an exact structural route to Gate B. If the VERSF charge-conjugation lift forces the left and right embedded three-volumes to coincide on the determinant line — not entrywise coincidence of the embeddings, only coincidence of their exterior cubes — then $\Delta_f > 0$ follows with no phase calculation at all. The candidate anchor for this lift is a named predecessor object: the NMCC-1 charge-conjugation structure, which maps left to right chiral carriers and is therefore exactly the pairing that $\mathcal{D}_q$ requires; the coincidence hypothesis $\Omega_{fL} = \Omega_{fR}$ is a sharp, checkable statement about that lift. Debt 15 thereby splits into two branches: either (i) prove $\Omega_{fL} = \Omega_{fR}$ (or their proximity, see Appendix K) from the conjugation structure, or (ii) evaluate the complex overlap blindly. Branch (i) is a symmetry derivation; branch (ii) is a computation. Either closes the gate; conflating them does not.

Proposition 21.2 — Reality–Orientation Decomposition

Suppose there exists an antiunitary involution $\mathcal{K}$ on $\mathscr{H}_{cl}$ with

$$\mathcal{K}\, \hat{C}_Y\, \mathcal{K} = \hat{C}_Y,$$

and real structures $\mathcal{K}_f$ on the quark carriers such that the embeddings intertwine,

$$\mathcal{K}\, \mathscr{E}_{fL} = \mathscr{E}_{fL}\, \mathcal{K}_f,\quad \mathcal{K}\, \mathscr{E}_{fR} = \mathscr{E}_{fR}\, \mathcal{K}_f,$$

with the volume elements ω_{fL}, ω_{fR} chosen in the $\mathcal{K}$-real locus of their determinant lines — an antiunitary involution on a one-dimensional complex line always admits fixed vectors, so this choice is available, and it is itself a frame choice subject to Lemma 20.1. Then

$$\Delta_f \in \mathbb{R},$$

but the sign of Δ_f is **not** determined: it is a residual $\mathbb{Z}_2$ orientation datum on $\mathcal{D}_q$.

Proof

The induced antiunitary involution on each determinant line fixes the chosen volume elements exactly (not merely up to sign, by the $\mathcal{K}$-real locus choice). Conjugating the overlap by $\mathcal{K}$ then maps $\langle \Omega_{fL}, (\Lambda^3\hat{C}_Y)\, \Omega_{fR} \rangle$ to its complex conjugate and, by the intertwining relations and the fixed volume elements, returns the same expression. Hence $\Delta_f = \Delta_f^*$, so $\Delta_f \in \mathbb{R}$. Nothing in the reality data selects the ray: replacing one column of one embedding by its negative preserves every hypothesis (the negated column is still $\mathcal{K}_f$-real) and reverses the sign. ■

Remark on the frame entanglement

That the $\mathcal{K}$-real locus choice is itself a Lemma 20.1 frame choice strengthens rather than weakens the message: the reality reduction is already entangled with the definition of θ_{sub}. $J_{\mathcal{D}}$ cannot be constructed as an afterthought on $\mathcal{D}_q$ alone; it must be built jointly with the substrate angle, which is exactly the joint construction Debt 14 demands.

Remark — the determinant-line $\theta = \pi$

The two rays of the real determinant line stand in exact correspondence with the two CP fixed points of the gauge sector:

$\Delta_q > 0 \leftrightarrow$ trivial character ($\theta = 0$); $\Delta_q < 0 \leftrightarrow$ alternating character ($\theta = \pi$).

A real structure on $\mathcal{D}_q$ plays the role CP plays on the topological lattice: it reduces a continuous phase to a two-point choice and can go no further. In both gates the final $\mathbb{Z}_2$ selection must be performed by positivity — sectorwise primitive nonnegativity in Gate A, the positive ray $\mathcal{D}_q^+$ in Gate B. The structural symmetry between the two halves of the strong-CP problem is therefore complete: each consists of a reality reduction that CP-type data can supply, followed by an orientation selection that only the substrate measure can supply. $J_\mathcal{D}$ must accordingly be constructed to deliver both jobs, and a derivation that stops at reality has closed neither.

Minimal numerical target

Once the embeddings and closure operator are frozen, the strong mass-phase calculation requires only

$\boldsymbol{\Delta_q = \Delta_u \Delta_d}$**.**

The decisive audit is:

$\mathrm{Im}\,\Delta_q = 0$, $\mathrm{Re}\,\Delta_q > 0$.

Appendix K records the certified exterior-power pipeline for this evaluation, the genericity result showing that a positive outcome cannot arise by accident, and the proximity criterion that bounds $\arg \Delta_f$ by the chiral asymmetry of the embeddings.

---

22. Strong CP Versus Weak CP, and the Alignment-Solution Benchmark

The determinant phase measures the anomalous U(1) volume orientation of the quark mass maps. CKM CP violation measures the noncommuting relative SU(3) orientation of the two left-handed magnitude operators.

Theorem 11 — Strong/Weak CP Separation Theorem

There exist full-rank three-generation Yukawa matrices satisfying

$\arg \det(Y_u Y_d) = 0$

and

$\mathcal{J}_q = \mathrm{Im}\, \mathrm{Tr}([H_{uL}, H_{dL}]^3) \neq 0.$

Therefore vanishing strong determinant phase does not force vanishing weak CP violation.

Proof by construction

Let

$D_u = \mathrm{diag}(y_u, y_c, y_t)$, $D_d = \mathrm{diag}(y_d, y_s, y_b)$,

with distinct positive entries. Choose a unitary matrix $V \in SU(3)$ with nonzero Jarlskog invariant. Set

$Y_u = D_u$, $Y_d = V D_d$.

Then

$\det(Y_u Y_d) = \det D_u \cdot \det V \cdot \det D_d > 0$

because $\det V = 1$. Meanwhile,

$H_{uL} = D_u^2$, $H_{dL} = V D_d^2 V^\dagger$.

For nondegenerate spectra and $J(V) \neq 0$, the standard Jarlskog factorisation gives

$\mathrm{Im}\, \mathrm{Tr}([H_{uL}, H_{dL}]^3) \neq 0.$

Thus the two conditions coexist. ■

Note on the SU(3) normalisation

Placing V in SU(3) by dividing by $\det(V)^{1/3}$ is itself an anomalous singlet rephasing of the down sector: in a physical application it shifts θ by the compensating determinant phase (Theorem 7) and must be booked, exactly as in the certified rephasing trap of Appendix K.2. It is harmless here only because Theorem 11 is a mathematical existence statement about matrix pairs at fixed convention, not a field redefinition inside a fixed theory.

Geometric reading

The flavour-unitary data split schematically into:

$U(3) \cong [SU(3) \times U(1)]/\mathbb{Z}_3$.

Strong CP sees the anomalous determinant U(1). CKM CP can survive in the special-unitary relative frame.

22.1 Positioning against alignment solutions

Gate B is structurally adjacent to the Nelson–Barr class of solutions, which also arrange $\arg \det(M_u M_d) = 0$ at tree level while retaining a physical CKM phase. The comparison sharpens both what VERSF claims and what it owes.

Similarity. Both mechanisms live on the determinant U(1) and exploit the SU(3)/U(1) separation proved in Theorem 11.

Difference in origin. Nelson–Barr obtains the aligned determinant by model-building choice: spontaneous CP breaking plus a texture that keeps the phase out of the determinant. VERSF claims no texture freedom. The embeddings $\mathscr{E}_{fL/R}$ and the operator $\hat{C}_Y$ are outputs of the closure architecture; Δ_q is a calculation, not an arrangement. If the calculation returns a complex Δ_q, minimal VERSF fails Gate B — there is no texture to adjust (F17, F19).

Transferred lesson. The known fragility of alignment solutions transfers as an obligation, not as an objection. Nelson–Barr alignment is notoriously sensitive to radiative corrections and to higher-dimensional operators that reintroduce the determinant phase. SCPθ-1 therefore refuses to treat tree-level alignment as closure: SCP-17, Theorem 14 and Debt 22 exist precisely because a positively oriented determinant line at $\mu\star$ is worthless if the complete quantum flow rotates it. Any VERSF claim of a solved strong-CP problem must include the radiative audit that alignment models typically postpone.

VERSF target

The strongest successful outcome is not "all quark embeddings are real." It is:

$\Delta_u \Delta_d > 0$ and $\mathcal{J}_q \neq 0$,

stable under the complete quantum flow.

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23. The Strong-CP Selection Theorem

Theorem 12 — Strong CP and Global θ-Sector Theorem

Assume SCP-1–SCP-18. Then on the full-rank massive branch:

1. the primitive global topological character is trivial;
2. the liftable colour angle satisfies $\theta_{sub} = 0 \pmod{2\pi}$;
3. the combined closure-derived quark determinant satisfies $\arg\det(M_u M_d) = 0 \pmod{2\pi}$;
4. hence

$$\bar{\theta} = 0 \pmod{2\pi}.$$

If the physical vacuum is CP even, every strong-interaction observable odd under CP vanishes at the level controlled solely by $\bar{\theta}$. Weak-sector CP violation may remain nonzero.

Proof

By Theorem 5, primitive positivity, generating support and no private character imply the trivial character on $\mathfrak{T}_G$, hence $\theta_{sub} = 0$ on the colour generator.

By Theorem 10, positive trivialisation of the combined closure-Yukawa determinant line gives $\arg\det(M_u M_d) = 0$.

By Theorem 8,

$$\bar{\theta} = \theta_{sub} + \arg\det(M_u M_d) = 0.$$

SCP-16 selects a CP-even vacuum branch, and SCP-17 guarantees that the result survives running and thresholds or includes the derived correction. Theorem 11 shows that this does not force the weak Jarlskog invariant to vanish. ■

Status

This is a conditional theorem with two central microscopic gates:

Gate A: primitive trivial character

and

Gate B: positive combined quark determinant line.

The paper proves that these gates are sufficient. It does not claim they have already been numerically or microscopically discharged.

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24. Vacuum Branches, $\theta = \pi$, and Spontaneous CP

Define the vacuum energy density

$E(\theta) = -\lim_{V\to\infty} (1/V) \log Z(\theta)$.

CP implies

$E(\theta) = E(-\theta)$.

Periodicity implies possible special behaviour at $\theta = \pi$.

Proposition 24.1 — Evenness Is Not Branch Uniqueness

The relation $E(\theta) = E(-\theta)$ does not exclude degenerate CP-conjugate vacua at a CP fixed point.

SCP-16 requirement

The theorem requires that at $\theta = 0$:

- the thermodynamic vacuum is unique, or
- all physical vacua selected by the substrate are CP even.

Vafa–Witten import — scope and caveats

For vectorlike QCD with positive quark masses and $\theta = 0$, the Vafa–Witten parity argument provides the standard sufficient route to a CP-even vacuum under its hypotheses. SCPθ-1 imports it with its known caveats stated: the argument concerns the positivity of the Euclidean measure and the convexity properties of the free energy in the presence of parity-odd sources, and its rigour for *spontaneous* parity breaking — as opposed to explicit sources — has recognised subtleties involving the order of limits, the regulator, and the class of order parameters admitted. VERSF may not cite Vafa–Witten as a closed result; it must verify that its own regulator, measure and full matter completion satisfy conditions under which the parity conclusion is actually a theorem (Debt 18).

The π branch

Even if $\theta = \pi$ is CP invariant at the action level, the vacuum can exhibit nontrivial branch structure or spontaneous CP breaking — the modern mixed-anomaly analyses of Yang–Mills at $\theta = \pi$ make this concrete. The Primitive-Measure Trivial-Character Theorem avoids this branch by selecting $\theta = 0$ before the infrared vacuum problem is solved.

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25. Topological Susceptibility and Nontrivial Topology

At $\bar{\theta} = 0$,

$\partial E/\partial\bar{\theta} = -(i/V)\,\langle\nu\rangle_{\bar{\theta}}$

in Euclidean convention. CP gives

$\langle\nu\rangle_0 = 0$.

The second derivative is

$\chi_{top} = \partial^2 E/\partial\bar{\theta}^2\,|_0 = \lim_{V\to\infty} \langle\nu^2\rangle_{0,c} / V$.

Theorem 13 — Topological-Susceptibility Persistence

The condition $\bar{\theta} = 0$ implies vanishing first CP-odd moment,

$\langle\nu\rangle_0 = 0$,

but permits

$\chi_{top} > 0$.

Proof

The first moment is odd under $\nu \mapsto -\nu$ and vanishes in a CP-even measure. The second connected moment is even and nonnegative. No implication from the first to the second exists. ■

Physical reading

VERSF need not make QCD topologically empty. It must make the weighting of opposite topological sectors CP even.

The target is:

$W_\nu = W_{-\nu} > 0$, $\chi_{sub}(\nu) = 1$.

---

26. RG Flow and Radiative Stability

For invertible mass matrices,

$\bar{\theta}(\mu) = \theta(\mu) + \mathrm{Im} \log \det M_u(\mu) + \mathrm{Im} \log \det M_d(\mu)$.

Differentiating gives:

Theorem 14 — Exact RG Stability Identity

$\mu\, d\bar{\theta}/d\mu = \beta_\theta + \mathrm{Im}\, \mathrm{Tr}(M_u^{-1} \beta_{M_u} + M_d^{-1} \beta_{M_d})$.

Proof

Use

$d \log \det M = \mathrm{Tr}(M^{-1}\, dM)$

for nonsingular matrices, take imaginary parts, and add the explicit angle beta function. ■

Corollary 26.1 — Exact Stability Criterion

The boundary condition $\bar{\theta}(\mu\star) = 0$ remains exact iff

$\beta_\theta + \mathrm{Im}\, \mathrm{Tr}(M_u^{-1} \beta_{M_u} + M_d^{-1} \beta_{M_d}) = 0$

along the flow.

Structural context

In the pure Standard Model, the renormalisation of $\bar{\theta}$ from the CKM phase is known to be extraordinarily suppressed, first arising at high loop order through multiple Yukawa insertions. SCPθ-1 does not import that numerical smallness. The full VERSF theory contains closure-generated structure beyond the minimal Standard Model operator set, and the identity above must be evaluated in the complete theory — this is the content of SCP-17.

VERSF obligation

It is not enough to say that θ is perturbatively unrenormalised in isolation. The complete invariant must be audited in the full theory containing:

- weak CP violation;
- closure-generated Yukawas;
- threshold corrections;
- any higher-dimensional CP-odd operators;
- the faithful global quotient;

- the quantum substrate measure.

Radiative closure alternatives

VERSF may close SCP-17 in either of two ways:

1. **Exact symmetry/measure protection:** the combined primitive character and determinant-line orientation are preserved by the complete quantum flow.
2. **Finite derived correction:** the flow generates a calculable $\delta\bar{\theta}$, which is published blindly and compared only afterward.

An assumed zero counterterm is not a derivation.

---

27. Threshold Matching

Suppose a heavy quark has complex mass

$m_h = |m_h| e^{i\varphi_h}$.

When it is integrated out, the low-energy angle must absorb its phase so that the physical invariant is continuous.

Theorem 15 — Heavy-Quark Threshold Matching

With the convention $\bar{\theta} = \theta + \arg\det M$, matching from N_f to $N_f - 1$ flavours gives

$\boldsymbol{\theta^{(N_f-1)} = \theta^{(N_f)} + \arg m_h \pmod{2\pi}}$,

up to any separately calculated finite CP-odd threshold contribution. Consequently,

$\bar{\theta}^{(N_f-1)} = \bar{\theta}^{(N_f)}$

when matching is exact.

Proof

Write

$\det M^{(N_f)} = m_h \det M^{(N_f-1)}$.

Then

$\bar\theta^{(N_f)} = \theta^{(N_f)} + \arg m_h + \arg\det M^{(N_f-1)}$.

Defining the low-energy angle by the displayed relation makes this equal to

$\theta^{(N_f-1)} + \arg\det M^{(N_f-1)} = \bar\theta^{(N_f-1)}$. ■

Firewall

Removing a heavy quark with a complex mass without shifting the angle creates an artificial change in $\bar\theta$.

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28. Electroweak and Quotient-Coupled θ-Sector Firewall

The complete Standard Model contains topological terms associated with more than the colour factor. In the covering-product notation one may write colour, weak and Abelian topological data, but on the faithful quotient their periodicities and discrete identifications need not factor independently.

Required separation

SCPθ-1 focuses on the physical strong invariant. It does not silently claim that:

- the electroweak θ-angle is independently physical in the minimal matter theory;
- baryon/lepton rotations remain exact under every completion;
- the hypercharge topological term has the same global status on all manifolds;
- the $\mathbb{Z}_6$ quotient leaves all angle periodicities unchanged.

The full global character

The complete target is

$\chi_{glob} \in \mathrm{Hom}(\mathfrak{T}_G, U(1))$.

The strong character is a restriction or coordinate on this character group only after the full lattice has been derived.

Falsifier

If the quotient-compatible lattice ties a nominal 2π colour shift to a nontrivial electroweak or discrete character, then a colour-only periodicity statement is incomplete and must be replaced by the full joint identification.

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29. Observable Matching

29.1 Strong CP observables

A nonzero $\bar{\theta}$ induces CP-odd hadronic observables such as electric dipole moments and CP-odd pion–nucleon interactions.

SCPθ-1 does not derive their numerical values. The downstream calculation requires:

- the renormalised $\bar{\theta}$;
- QCD matrix elements;
- a declared matching scheme;
- separation from weak-sector and higher-dimensional CP sources.

29.2 Weak CP contamination

Even when $\bar{\theta} = 0$, weak interactions can generate extremely suppressed CP-odd hadronic effects. Therefore "zero strong θ" does not mean "every EDM is identically zero."

The observable claim must be:

the component sourced by the strong $\bar{\theta}$ invariant vanishes, while all other CP-odd operators are calculated separately.

29.3 Lattice tests of the global sector

A first-principles VERSF implementation should calculate:

- sector-resolved primitive weights W_s;
- orientation pairing $W_s = W_{-s}$;
- topological susceptibility;
- determinant-line overlap Δ_q;
- quotient-twisted partition functions;
- response to admissible large gauge transformations.

29.4 Blind comparison

No experimental or lattice strong-CP datum may enter before:

1. the global lattice is frozen;
2. the primitive character is selected;
3. the quark determinant line is calculated;
4. RG and threshold corrections are published.

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30. What Has Actually Been Derived

SCPθ-1 derives or fixes the following.

1. The strong θ-angle is a global topological character, not a local colour-response coefficient.
2. Liftable SU(3) bundles carry integer Pontryagin charge.
3. Composition-compatible phase weights are unitary characters.
4. On the liftable colour line, every character is $e^{(i\theta\nu)}$.
5. CP alone restricts the ordinary colour angle to 0 or π, not uniquely to zero.
6. A positive primitive measure with generating support forces the trivial character.
7. The π character is excluded by positive odd-sector support, and only by it.
8. The character description is equivalent to a flat-line or large-gauge representation.
9. The faithful quotient requires a joint topological lattice and can alter individual angle periodicities.
10. The anomalous chiral Jacobian moves phase between the topological term and the mass determinant.
11. The physical invariant is $\bar{\theta} = \theta + \arg\det(M_u M_d)$.
12. The VERSF quark determinant phase reduces exactly to one combined determinant-line overlap.
13. The reduction is convention-independent jointly with θ_{sub}: rephasing the volume elements shifts both compensatingly.
14. Positivity of the closure operator alone is insufficient; a positive combined chiral orientation is required.
15. A positive combined determinant line gives zero quark determinant phase.
16. Zero strong determinant phase is compatible with nonzero CKM/Jarlskog CP violation.
17. The two selection conditions imply $\bar{\theta} = 0$.
18. Zero $\bar{\theta}$ permits nonzero topological susceptibility.
19. The exact RG stability and threshold-matching conditions are fixed.
20. Coincident left/right determinant volumes force $\Delta_f \geq 0$ exactly — an exact structural route to Gate B via the charge-conjugation lift.
21. A compatible real structure forces $\mathrm{Im}\,\Delta_q = 0$ but leaves $\mathrm{sign}(\Delta_q)$ as a genuine $\mathbb{Z}_2$ orientation datum; the negative ray corresponds exactly to the alternating $\theta = \pi$ character, so both gates end in a positivity-performed $\mathbb{Z}_2$ selection.
22. For Haar-generic complex embeddings, $\arg \Delta_q$ is uniformly distributed on the circle: Gate B positivity is measure-zero generic and cannot hold by accident (Appendix K).

23. The paper's exact identities — determinant-line reduction, joint convention-independence under full anomalous $U(3)^4$ rotations, and strong/weak separation — are certified numerically to machine precision (Appendix K).

SCPθ-1 does **not** derive:

- the full quotient-compatible bundle lattice;
- the obstruction-class formula for every VERSF-admissible geometry;
- the primitive sector weights;
- positivity of the complete substrate measure;
- positive support in unit instanton sectors;
- the microscopic exclusion of every pseudoscalar phase carrier;
- the full fermion-measure Jacobian from commitment primitives;
- numerical Yukawa matrices or embeddings;
- the sign and phase of $\Delta_u \Delta_d$;
- the absence of spontaneous CP breaking from first principles;
- the complete radiative $\delta\bar{\theta}$;
- a neutron EDM;
- empirical agreement.

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31. Open Debts

Debt 1 — Global substrate-to-bundle functor. Construct the faithful-group principal bundle, transition functions or equivalent global transport object directly from VERSF histories.

Debt 2 — Quotient obstruction class. Derive the exact $\mathbb{Z}_6$ lifting obstruction and its relation to colour, weak and hypercharge characteristic classes.

Debt 3 — Topological lattice. Compute $\mathfrak{T}_G$, including continuous angles, discrete topological terms and joint periodicities.

Debt 4 — Line-operator spectrum. Derive the admissible electric, magnetic and dyonic lines rather than importing them.

Debt 5 — Primitive global measure. Construct W_s from the VERSF master measure and prove its regulator-independent meaning.

Debt 6 — Sectorwise positivity. Show that $W_s \geq 0$ for all relevant sectors before character deformation.

Debt 7 — Generating support. Prove $W_{\pm 1} > 0$, or establish another positively supported generating set, including at least one odd-charge sector for the π-exclusion.

Debt 8 — Orientation involution. Construct $\mathscr{R} : s \mapsto -s$ on the actual substrate histories — not merely on sector labels — and prove that the completed sector measure is blind to orientation despite the directedness of commitment (Section 11.2). Establishing $W_{-s} = W_s$ against the arrow of the primitive process is the load-bearing half of this debt.

Debt 9 — No-private-character derivation. Show from the minimum-distinction and record ontology that no independent pseudoscalar flat line is admitted in minimal VERSF.

Debt 10 — Instanton realisation. Construct or identify the VERSF histories corresponding to nonzero Pontryagin charge and control their action.

Debt 11 — Quantum fermion measure. Derive the Fujikawa/index-theorem Jacobian from the VERSF Fock and commitment measure.

Debt 12 — Global anomaly compatibility. Verify that all large-gauge and quotient-sector fermion determinants are globally well defined.

Debt 13 — Numerical closure-Yukawa objects. Calculate $\hat{C}_Y$, its spectrum and the four quark embeddings.

Debt 14 — Determinant-line real structure. Construct $J_\mathcal{D}$ from orientation, charge conjugation and chiral completion, jointly with the definition of θ_{sub}, so that Lemma 20.1 is respected. Proposition 21.2 fixes the specification: $J_\mathcal{D}$ must deliver both a reality reduction and an orientation selection; reality alone leaves the $\mathbb{Z}_2$ open. The named starting object is the NMCC-1 charge-conjugation lift, whose left↔right chiral pairing is the structure both jobs must be built from.

Debt 15 — Positive determinant ray. Close Gate B by one of the two branches of Proposition 21.1: either (i) derive $\Omega_{fL} = \Omega_{fR}$ from the NMCC-1 charge-conjugation lift, giving $\Delta_f \geq 0$ structurally — or establish sufficient chiral proximity, in which case the derivation must bound **both** the chiral asymmetry of the embeddings **and** the participating spectrum of $\hat{C}_Y$, since small eigenvalue triples amplify phase sensitivity (Appendix K.5) — or (ii) evaluate $\Delta_u \Delta_d$ blindly and establish whether it lies on the positive real ray.

Debt 16 — Rank audit. Verify that no exact quark zero mode changes the physical formulation of strong CP.

Debt 17 — Weak CP coexistence. Calculate the same embeddings' Jarlskog invariant and verify that determinant alignment does not erase CKM CP.

Debt 18 — Vacuum uniqueness. Establish the CP-even vacuum branch at $\bar{\theta} = 0$ in the complete VERSF-regulated theory, including verification of the conditions under which the Vafa–Witten route is a theorem.

Debt 19 — Topological susceptibility. Calculate χ_{top} without using lattice input to normalise the sector measure.

Debt 20 — RG completion. Compute the exact right-hand side of the RG stability identity in the complete theory.

Debt 21 — Threshold completion. Perform complex-mass threshold matching for all quarks in one declared scheme.

Debt 22 — Higher-dimensional CP operators. Audit chromoelectric dipoles, the Weinberg operator and any closure-generated CP-odd interactions separately from $\bar{\theta}$ — the alignment-fragility lesson of Section 22.1.

Debt 23 — Electroweak/global-angle audit. Resolve the complete joint angle identifications of the faithful quotient.

Debt 24 — Observable matching. Calculate hadronic CP-odd matrix elements only after $\bar{\theta}$ is frozen.

Debt 25 — Blind empirical comparison. Compare with EDM and lattice data only after Debts 1–24 are closed or their uncertainties are published.

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32. Falsification Conditions

F1 — No global lift. If VERSF cannot construct globally consistent colour bundles or an equivalent object, the θ-sector is not derived.

F2 — Wrong faithful group. If the inherited $\mathbb{Z}_6$ quotient fails, the topological lattice must be rebuilt.

F3 — Charge normalisation failure. If the liftable colour topological charge does not reproduce the standard integer normalisation, the QCD identification fails.

F4 — Incomplete sector lattice. If relevant non-liftable or discrete sectors are omitted, the global character theorem is applied to the wrong domain.

F5 — Local/global conflation. If Z_3, E_3 or the local Hessian is used to set θ without global data, the derivation fails.

F6 — Unsupported positivity. If sectorwise primitive nonnegativity is merely asserted, the trivial-character conclusion remains open.

F7 — Sign-indefinite primitive measure. If the actual minimal substrate measure is intrinsically sign-indefinite or complex before any character is attached, Theorem 5 does not apply and the θ-angle reverts to an input.

F8 — Non-generating support. If positive sectors do not generate the colour topological subgroup, positivity does not determine the full character.

F9 — Hidden pseudoscalar carrier. If minimal VERSF necessarily contains an independent physical topological phase carrier, NPC fails and its dynamics must be included.

F10 — π-branch survival. If odd-charge sectors have zero support or negative primitive weight, the positivity exclusion of $\theta = \pi$ fails.

F11 — Orientation-pair failure. If the substrate does not map s and −s with equal primitive weight — in particular, if the directedness of commitment leaves an orientation-sensitive imprint on the completed sector measure (Section 11.2) — then the substrate itself sources strong CP violation at the measure level and the selection theorem fails at its foundation.

F12 — Large-gauge inconsistency. If the physical state requires a nontrivial flat line not derived by VERSF, the trivial single-valuedness corollary fails.

F13 — Axial-Jacobian failure. If the VERSF fermion measure does not reproduce the standard phase transfer, its record-layer QCD limit is wrong.

F14 — Bare-θ overclaim. If $\theta_{sub} = 0$ is called a solution without the mass determinant, the claim fails.

F15 — Determinant convention laundering. If a chiral basis or volume-element frame is chosen to make det M positive while the compensating θ-shift of Lemma 20.1 is ignored, the result is a convention, not a prediction.

F16 — Positive-operator overclaim. If $\hat{C}_Y \geq 0$ is used alone to infer $\det Y_f > 0$, the determinant-line typing fails.

F17 — Uncomputed determinant line. If $\Delta_u \Delta_d$ is declared positive without calculating the embeddings and exterior-power overlap, Gate B remains open.

F18 — CKM destruction. If the determinant-alignment mechanism forces $\mathcal{J}_q = 0$, it does not reproduce the observed pattern of weak CP violation.

F19 — Phase insertion. If embedding phases are selected to make Δ_q real using strong-CP data, the result is fitted.

F20 — Massless-quark concealment. If a zero singular value is silently assumed to evade the determinant theorem, the mass ledger is inconsistent.

F21 — Spontaneous-CP branch. If the selected vacuum at $\bar{\theta} = 0$ is CP odd, action-level selection is insufficient.

F22 — Topology erasure. If the mechanism works only by setting every nonzero-topology weight to zero without deriving that dynamics, it evades rather than solves the sector problem.

F23 — Susceptibility conflation. If nonzero χ_{top} is treated as evidence of strong CP violation, the observable typing fails.

F24 — RG regeneration. If the complete quantum flow generates nonzero $\bar{\theta}$ and no VERSF protection or calculation controls it, the solution is unstable — the alignment-fragility failure mode.

F25 — Threshold mismatch. If heavy-quark phases are removed without the compensating low-energy angle shift, $\bar{\theta}$ is miscomputed.

F26 — Quotient-periodicity error. If a covering-product 2π identification is used where the faithful quotient requires a joint shift, the global theorem fails.

F27 — EDM conflation. If every EDM is claimed to vanish at $\bar{\theta} = 0$, weak and higher-dimensional CP sources have been omitted.

F28 — Numerical insertion. If EDM bounds, lattice susceptibility or measured CKM data select any upstream object, the prediction is invalid.

F29 — Counterterm tuning. If a CP-odd counterterm is chosen after comparison to cancel the calculated result, the theory has tuned strong CP rather than derived it.

F30 — Empirical disagreement. After blind completion, a nonzero predicted strong CP contribution incompatible with observation falsifies the minimal theorem branch.

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33. Numerical Non-Insertion Audit

The following operations are forbidden.

1. Choosing $\theta_{sub} = 0$ because experiment requires it.
2. Choosing the trivial character before deriving primitive measure admissibility.
3. Rejecting $\theta = \pi$ without a positivity or branch-selection theorem.
4. Using a lattice instanton distribution to define W_s.
5. Setting $W_{\pm 1} > 0$ by convention.
6. Selecting the faithful quotient because it gives the desired periodicity.
7. Ignoring non-liftable bundles because they complicate the angle lattice.
8. Rephasing quarks to make det M positive while omitting the anomaly shift.
9. Choosing closure eigenvectors, embeddings or volume-element frames to make Δ_q real.
10. Using the observed CKM phase to retain a preferred complex embedding.
11. Using the smallness of the neutron EDM to set a determinant-line sign.

12. Importing an exactly massless up quark.
13. Importing an axion while describing the result as minimal VERSF.
14. Assuming the Vafa–Witten hypotheses without verifying the VERSF measure and regulator.
15. Declaring radiative stability because θ is locally a total derivative.
16. Dropping complex heavy-quark phases at thresholds.
17. Treating topological susceptibility as a tunable normalisation.
18. Cancelling a calculated $\delta\bar{\theta}$ with a fitted counterterm.
19. Calling a postdiction a blind prediction.
20. Reporting a vanishing strong term as a prediction that all EDMs vanish.
21. Compensating a scalar chiral rephasing by α rather than by the full determinant phase arg det(e^(iα)𝟙_N) = Nα at the anomalous-Jacobian step; the Appendix K certification demonstrates that this bookkeeping error produces a spurious drift of $\bar{\theta}$ of order π while every individual identity still appears to hold.
22. Declaring Gate B closed on reality grounds alone; Proposition 21.2 shows the sign remains open.

Blind calculation protocol

A valid programme proceeds in this order:

1. freeze the faithful global gauge group;
2. construct the substrate-to-bundle lift;
3. derive the quotient obstruction data;
4. enumerate 𝔗_G;
5. freeze line operators and angle identifications;
6. construct the primitive measure;
7. publish the sector weights and positivity audit;
8. verify generating support and orientation pairing;
9. derive or reject NPC;
10. publish the selected global character;
11. derive the fermion-measure Jacobian;
12. calculate Ĉ_Y and all quark embeddings, jointly with the canonical volume-element frame;
13. publish Δ_u, Δ_d, Δ_q;
14. calculate CKM invariants from the same frozen data;
15. form $\bar{\theta}$(μ⋆);
16. solve the vacuum branch;
17. run and threshold-match;
18. publish the final strong CP-odd operator coefficients;
19. only then compare with EDM and lattice data.

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34. SCPθ-1 Closure Certificate

Gate question

Has VERSF reduced the strong-CP problem to independently calculable global-sector and quark-determinant objects, and proved that their successful closure forces $\bar{\theta} = 0$ without erasing weak CP or QCD topology?

Closure answer

Global character architecture: yes, conditionally. Physical $\bar{\theta}$ construction: yes. Determinant-line reduction and joint convention-independence: yes. Coincident-embedding positivity route and reality–orientation decomposition: yes — exact. Genericity and pipeline certification: yes — Appendix K. Strong/weak CP separation: yes. Primitive character selection: conditional on the measure gate. Quark determinant alignment: conditional on the embedding/orientation gate. Numerical strong CP: not yet.

Global certificate

Given

$\{ \mathfrak{T}_G, W_s \geq 0, \text{generating support}, \mathscr{R}, \text{NPC} \}$,

the primitive character is fixed:

$\chi_{sub} = 1$.

Fermion certificate

Given

$\{ \hat{C}_Y, \mathscr{E}_{uL/R}, \mathscr{E}_{dL/R}, J_\mathcal{D}, \mathcal{D}_q^+ \}$,

the mass determinant phase is fixed by

$\Delta_q = \prod_{f=u,d} \langle \Omega_{fL}, (\Lambda^3\hat{C}_Y)\, \Omega_{fR} \rangle$.

If $\Delta_q > 0$, then $\arg\det(M_u M_d) = 0$.

Strong-CP certificate

Given both gates and the quantum/vacuum stability conditions,

$\bar{\theta} = 0 + 0 = 0$.

Closure grade

SCPθ-1 closes the theorem architecture and the exact reduction of the strong-CP problem to two substrate calculations:

global topological character

and

combined anomalous quark determinant line.

It does not close their microscopic numerical evaluation.

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35. Conclusion

The VERSF strong-interaction programme has now reached the point where local colour dynamics is not enough.

The local bath-transport structure gives $SU(3)_C$. The reduced substrate response gives Z_3 and g_s. The quantum background-field flow gives asymptotic freedom and dimensional transmutation. The centre-sensitive bath functional supplies a conditional route to confinement.

But none of these results decides how globally disconnected colour histories interfere.

That information is the θ-sector.

The first result of this paper is to type the problem correctly. The strong angle is a character of the global topological charge lattice:

$\chi_\theta(\nu) = e^{i\theta\nu}$.

It is not another eigenvalue of the local colour Hessian.

Once the faithful Standard Model quotient is admitted, the correct domain is not automatically a separate integer for each covering factor. The substrate must derive the complete quotient-compatible lattice of bundles, lines and characteristic charges. Only then is the full character group known.

The paper's proposed VERSF selection mechanism is primitive positivity plus the absence of a private global phase ledger.

If the substrate has already supplied real nonnegative weights W_s, and if supported sectors generate the topological lattice, then an arbitrary character cannot be appended while preserving the primitive measure. Every supported generator forces

$\chi(s) = 1$.

The alternating $\theta = \pi$ character fails on positively supported odd sectors. The trivial character is uniquely selected:

$\theta_{sub} = 0$.

That closes only half the problem.

The anomalous fermion measure makes the quark determinant inseparable from the colour angle:

$\bar{\theta} = \theta_{sub} + \arg\det(M_u M_d)$.

The second result of this paper is therefore the determinant-line reduction. Because the VERSF Yukawa matrices are compressions of one closure operator, their determinants are exterior-power overlaps:

$\det Y_f = \langle \Omega_{fL}, (\Lambda^3 \hat{C}_Y)\, \Omega_{fR} \rangle$.

The entire quark contribution to strong CP is one scalar:

$\Delta_q = \Delta_u \Delta_d$.

And the reduction is honest: rephasing the reference volume elements moves phase between Δ_q and θ_{sub} without changing their sum, so the positive-orientation claim must be made — and can only be made — in the same canonical frame that defines the substrate angle.

VERSF need not remove every complex phase. It must derive a positive real orientation for this one anomalous determinant line, and then prove the orientation stable under the complete quantum flow. That last clause is where alignment solutions historically fail, and SCPθ-1 refuses to postpone it.

The structural analysis of the determinant line then reveals a symmetry between the two halves of the problem that was not visible at the outset. On the gauge side, CP reduces the continuous angle to the two-point choice $\{0, \pi\}$, and positivity of the primitive measure performs the final selection. On the fermion side, a real structure reduces the continuous determinant phase to the two-point choice $\{\Delta_q > 0, \Delta_q < 0\}$, and positivity of the substrate orientation performs the final selection — coincident chiral three-volumes supplying an exact route (Proposition 21.1), reality alone provably insufficient (Proposition 21.2). Each gate is a reality reduction followed by a $\mathbb{Z}_2$ orientation selection, and in each the selection is work that only the measure can do. The genericity audit adds the corresponding stakes: for arbitrary embeddings the determinant phase is uniform on the circle, so Gate B cannot close by luck, and a complex result cannot be excused by convention.

The condition is compatible with weak CP violation. The determinant U(1) phase can vanish while the relative SU(3) flavour frame retains a nonzero Jarlskog invariant.

The complete successful chain is therefore:

faithful global lift $\longrightarrow \mathfrak{T}_G \longrightarrow W_s \longrightarrow \chi_{sub} = 1 \longrightarrow \theta_{sub} = 0, \hat{C}_Y, \mathcal{E}_{fL/R} \longrightarrow \Delta_u \Delta_d > 0$ $\longrightarrow \arg\det(M_u M_d) = 0, \theta_{sub} + \arg\det(M_u M_d) \longrightarrow \bar{\theta} = 0$.

The result preserves:

- instantons;
- nonzero topological susceptibility;
- confinement;
- complex flavour structure;
- CKM CP violation.

It removes only the unearned anomalous phase.

This paper does not claim that the decisive VERSF calculations have already returned the desired answer. Its advance is that the strong-CP problem is no longer an undefined hope or a request that "some symmetry" set a parameter to zero. It is now a two-gate, globally typed, radiatively audited, falsifiable calculation.

The next finish line is exact:

Construct the quotient-compatible VERSF sector measure and calculate the combined quark determinant-line overlap. If the first selects the trivial character and the second lies on the positive real ray of the canonical frame, then — after the anomalous Jacobian, vacuum and RG audits — strong CP vanishes as a theorem of the substrate rather than as an unexplained input.

Appendix A — Pontryagin Integrality and Conventions

Let $E = P_3 \times_{\mathbf{3}} \mathbb{C}^3$ be the fundamental vector bundle. With Hermitian generators normalised by

$\mathrm{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}$,

the second Chern character is represented by

$$ch_2(E) = (1/8\pi^2)\, \mathrm{tr}(\mathscr{F} \wedge \mathscr{F})$$

up to the fixed sign convention associated with Hermitian versus anti-Hermitian connection forms. SCPθ-1 fixes the sign by the component formula

$$\nu = (1/32\pi^2) \int F_{\mu\nu}^a \tilde{F}^{a\,\mu\nu}.$$

All physical conclusions depend on $\theta\nu$ and are invariant under a simultaneous reversal of both sign conventions.

For a quotient bundle that does not lift, the covering-factor expression may acquire a fractional part fixed by the obstruction class. No formula is imported without deriving the actual VERSF quotient lattice.

Appendix B — Character-Classification Proof

Let $\mathfrak{T}$ be additive and let the relative sector phase obey composition:

$$\chi(s + t) = \chi(s)\, \chi(t).$$

Normalisation of the vacuum sector gives $\chi(0) = 1$. Unitarity gives $|\chi(s)| = 1$. Thus χ is an element of the Pontryagin dual $\mathfrak{T}$.

For

$$\mathfrak{T} = \mathbb{Z}^r \oplus \oplus_j \mathbb{Z}_{n_j},$$

the general character is

$$\chi(\mathbf{m}, \mathbf{a}) = \exp\left[\, i \sum_{\ell=1}^r \theta_\ell m_\ell + 2\pi i \sum_j p_j a_j / n_j \,\right],$$

with continuous angles θ_ℓ and discrete labels p_j. This is the form the faithful-quotient audit must derive before the full VERSF topological character can be selected.

Appendix C — Positivity and Trivial-Character Proof

Suppose $W_s > 0$ on a generating set S. Let

$\chi(s) = e^{i\varphi_s}$.

Sectorwise nonnegativity after deformation requires

$e^{i\varphi_s} W_s \geq 0$.

Since $W_s > 0$, this requires $e^{i\varphi_s} = 1$. Every group element is generated by S, so multiplicativity gives $\chi = 1$ globally.

If only the paired sum

$W_s \chi(s) + W_{-s} \chi(-s) = 2 W_s \cos \varphi_s$

were required to be nonnegative, nontrivial characters with $\cos \varphi_s \geq 0$ could survive — for instance, any $|\varphi_s| \leq \pi/2$ on each generator. This is why SCP-11 is explicitly sectorwise primitive admissibility rather than positivity only after pairing.

The distinction is load-bearing, and Lemma 11.2 records why the sectorwise form is the one licensed by the ontic status of the primitive measure: a measure is nonnegative set by set, not merely in symmetrised combinations.

Appendix D — Anomalous Change-of-Variables Proof

For one sector,

$M \mapsto U_L^\dagger M U_R$,

so

$\arg\det M \mapsto \arg\det M - \arg\det U_L + \arg\det U_R$.

The regulated fermion measure contributes the inverse shift to the coefficient of the Pontryagin term:

$\theta \mapsto \theta + \arg\det U_L - \arg\det U_R$.

For up and down sectors, sum the two contributions. Therefore

$\theta + \arg\det(M_u M_d)$

is invariant.

For an axial rotation

$q_L \mapsto e^{(-i\alpha)} q_L$, $q_R \mapsto e^{(i\alpha)} q_R$

of N_f flavours,

$\arg\det M \mapsto \arg\det M + 2 N_f \alpha$,

and

$\theta \mapsto \theta - 2 N_f \alpha$.

The invariant is unchanged.

Appendix E — Determinant-Line Proof

Let

$E_L : V_L \to H$, $E_R : V_R \to H$, $C : H \to H$,

with $\dim V_L = \dim V_R = n$. Set

$Y = E_L^\dagger C E_R$.

Choose unit determinant vectors

$\omega_L \in \Lambda^n V_L$, $\omega_R \in \Lambda^n V_R$.

Then

$(\Lambda^n Y) \omega_R = \det(Y) \omega_L$.

Using functoriality,

$\Lambda^n Y = (\Lambda^n E_L^\dagger)(\Lambda^n C)(\Lambda^n E_R)$.

Define

$\Omega_L = (\Lambda^n E_L) \omega_L$, $\Omega_R = (\Lambda^n E_R) \omega_R$.

Taking the determinant-space inner product yields

$\det Y = \langle \Omega\_L, (\wedge^n C)\, \Omega\_R \rangle$.

For $n = 3$ and $C = \hat{C}\_Y$, this is Theorem 9.

If E_L or E_R has rank below three, the corresponding determinant vector vanishes and det Y = 0, correctly returning the massless branch.

Convention audit. The identity holds for any phase choice of ω_L, ω_R; rephasing them multiplies det Y by $e^{i(\alpha\_R - \alpha\_L)}$. Since the same rephasing acts on the fermionic path-integral variables, the anomalous Jacobian shifts θ oppositely (Appendix D). This is Lemma 20.1 in appendix form: the reduction commutes with the anomaly bookkeeping.

Appendix F — Strong/Weak CP Independence Construction

Choose a standard three-angle one-phase mixing matrix V with determinant one and nonzero Jarlskog invariant. Let D_u, D_d be positive nondegenerate diagonal matrices.

Set

$Y\_u = D\_u$, $Y\_d = V D\_d$.

Then

$\arg \det(Y\_u Y\_d) = \arg(\det D\_u \cdot \det V \cdot \det D\_d) = 0$.

However,

$H\_{uL} = D\_u^2$, $H\_{dL} = V D\_d^2 V^\dagger$.

The commutator invariant is

$\mathrm{Tr}([H\_{uL}, H\_{dL}]^3) = 6i\, J(V) \prod_{i<j} (y_{u,i}^2 - y_{u,j}^2) \prod_{k<l} (y_{d,k}^2 - y_{d,l}^2)$,

up to the fixed ordering convention. Every factor is nonzero, so weak CP is nonzero.

Appendix G — RG and Threshold Identities

For any nonsingular matrix M(μ),

$\mu\,(d/d\mu)\arg\det M = \mathrm{Im}\,\mathrm{Tr}(M^{-1}\beta_M)$.

Adding the explicit θ flow gives Theorem 14.

For threshold matching, factor

$\det M_{N_f} = m_h \det M_{N_f-1}$.

Continuity of θ̄ gives

$\theta_{N_f-1} = \theta_{N_f} + \arg m_h$.

Any finite matching correction from additional CP-odd operators must be separately calculated and added; it cannot be hidden in the phase convention.

Appendix H — Quotient-Sensitive Global Audit Protocol

The full faithful-group calculation should:

1. specify the geometric category and allowed boundaries;
2. construct principal $G_{SM}^{faithful}$-bundles;
3. identify the obstruction to lifting to the covering product;
4. enumerate electric and magnetic line charges;
5. derive Dirac quantisation in the quotient;
6. compute allowed characteristic numbers;
7. identify continuous and discrete topological actions;
8. determine the character group;
9. derive orientation and CP action on every generator;
10. apply the primitive-measure selection theorem to the complete lattice.

No colour-only statement is final before this audit is complete.

Appendix I — Dependency and Falsifier Map

Result	Principal dependencies	Principal falsifiers
Integer liftable ν_3	SCP-3, SCP-5	F1–F3
Character classification	Additivity	F4

Result	Principal dependencies	Principal falsifiers
Full quotient character group	SCP-2–SCP-4	F2, F4, F26
CP fixed points	Orientation action, periodicity	F11, F26
Trivial character	SCP-6–SCP-11	F6–F10
Large-gauge scalar state	NPC	F12
Axial phase transfer	SCP-12	F13
Physical $\bar{\theta}$	Axial Jacobian	F14–F15
Determinant-line formula	SCP-13, rank three	F16, F20
Joint convention-independence	Theorem 7, Theorem 9	F15
Coincident-embedding positivity	$\hat{C}_Y \geq 0$, $\Omega_{fL} = \Omega_{fR}$	F17 (if coincidence asserted, not derived)
Reality–orientation decomposition	Antiunitary intertwiner premises	F19, audit item 22
Genericity of $\arg \Delta_q$	Haar/Wishart ensemble, Appendix K	Structural; no empirical input
Positive determinant phase	SCP-15	F17, F19
Strong/weak separation	Three full-rank generations	F18
Main theorem	Both gates, SCP-16–17	F21, F24–F25
Topological susceptibility	CP-even vacuum	F22–F23
Numerical prediction	All previous rows	F28–F30

Appendix J — Internal and External References

Internal VERSF programme

1. *The Gauge-Closure Selection Theorem in VERSF.*
2. *The Substrate Anomaly-Inadmissibility Theorem in VERSF.*
3. *The Non-Abelian Bath-Transport Theorem, Continuous Colour Gauge Dynamics, and Full Gauge-Product Closure in VERSF.*
4. *The Gauge Action and Coupling Theorem in VERSF.*
5. *QCD Confinement and Running in the $SU(3)_C$ Bath-Transport Sector of VERSF.*
6. *The Closure-Operator Yukawa Construction Theorem in VERSF.*
7. *The Quark Current-Mass Projection Theorem in VERSF.*
8. *The Minimal Hermitian Lift and Frame-Rotation Firewall in VERSF.*
9. *The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF.*
10. *The Quantum Gauge Completion of the VERSF Standard Model* — planned successor.

Standard external foundations

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Appendix K — Numerical Certification, Genericity and Orientation Audit

This appendix records a numerical certification of the paper's exact identities and two structural Monte Carlo results constraining Gate B. **No VERSF microscopic input exists yet and none is used or fabricated here (SCP-18).** All ensembles are mathematical — Haar-distributed isometries and Wishart-distributed positive operators — chosen to probe the structure of the theorems, not to model the substrate. The audit pipeline itself is the deliverable that will evaluate the physical Δ_q the day Debts 13–14 close.

Reproducibility record. All runs use NumPy's PCG64 generator (`numpy.random.default_rng`). Certification and Monte Carlo runs (K.2–K.4): seed **20260711**. Coincidence and proximity probe (K.5): seed **42**. Haar isometries are generated by QR decomposition of complex (or real) standard-Gaussian matrices with the R-diagonal phase fix; positive operators are Wishart, $A A^\dagger$ with standard-Gaussian A; carrier dimension $N = 8$ except where stated. Δ_f is evaluated by both routes of K.1, whose agreement is itself certified in K.2. An audit that cannot be rerun is testimony, not certification; these parameters suffice to rerun every number in this appendix.

K.1 Pipeline

Two independent evaluations of Δ_f are implemented:

1. **Structural route (Theorem 9 literally):** construct $\Omega_{fL} = (\Lambda^3\mathscr{E}_{fL})\omega$, $\Omega_{fR} = (\Lambda^3\mathscr{E}_{fR})\omega$ as vectors of 3×3 minors in $\Lambda^3\mathbb{C}^N$, construct $\Lambda^3\hat{C}_Y$ as the matrix of 3×3 minors of $\hat{C}_Y$, and evaluate $\langle\Omega_{fL}, (\Lambda^3\hat{C}_Y)\,\Omega_{fR}\rangle$.
2. **Direct route (Cauchy–Binet):** evaluate $\det(\mathscr{E}_{fL}^\dagger\, \hat{C}_Y\, \mathscr{E}_{fR})$.

The Gate B object is then

$\Delta_q = \Delta_u\, \Delta_d$,

one function call once $\{ \hat{C}_Y, \mathscr{E}_{uL}, \mathscr{E}_{uR}, \mathscr{E}_{dL}, \mathscr{E}_{dR} \}$ are frozen in the canonical frame of Debt 14. Lemma 20.1 makes evaluation in any other frame meaningless.

K.2 Certification of the exact identities

Theorem 9 / Appendix E. Over 30 draws with carrier dimensions $N = 4$–7, the structural and direct routes agree with maximum relative error 6.4×10^{-15}. The rank-deficient branch returns $|\Delta| \approx 9 \times 10^{-14}$, i.e. the massless limit $\det Y = 0$ to machine precision.

Lemma 20.1 / Theorem 7. Over 500 random full U(3)×U(3) chiral rotations per sector (2,000 unitaries total), with the anomalous compensation $\theta \mapsto \theta + \sum_f (\arg\det U_{fL} - \arg\det U_{fR})$:

- $\arg \Delta_q$ alone shifted by up to 3.140 rad — it ranges over the entire circle;
- $\theta + \arg\Delta_q$ drifted by at most 2.1×10^{-15} rad.

The invariant is exact; the individual pieces are pure convention.

Theorem 11 / Appendix F. With arbitrary distinct positive diagonal Yukawas and an SU(3) mixing matrix of CKM form (three angles, one phase), the construction returned arg det(Y_u

$Y_d) = 1.6 \times 10^{-17}$ with $\mathcal{J}_q = 0.2554 \neq 0$. Strong-phase absence and weak CP coexist as claimed.

A certified trap. An initial run applied **independent scalar chiral rephasings** to each side — $U_{fL} = e^{i\alpha_{fL}}\mathbb{1}_3$, $U_{fR} = e^{i\alpha_{fR}}\mathbb{1}_3$ with independently drawn phases — and compensated θ by $\sum_f (\alpha_{fL} - \alpha_{fR})$ rather than by the full determinant phases, $\sum_f (\arg\det U_{fL} - \arg\det U_{fR}) = 3 \sum_f (\alpha_{fL} - \alpha_{fR})$. (The chiral structure matters: had the two sides been rephased vectorlike, $\alpha_{fL} = \alpha_{fR}$, the anomalous shift would cancel identically at any normalisation and the trap would be invisible; it is exposed precisely because the rephasings were chiral, where the correct compensation carries the factor N = 3 per rephased triplet, or $2N_f\,\alpha$ for an axial rotation.) Every identity appeared individually intact, yet $\bar{\theta}$ drifted by order π. The error is exactly the convention-laundering mode of F15, produced live; it is recorded as forbidden operation 21 in Section 33 so that no future numerical closure of Gate B can repeat it silently, and it is the numerical shadow of the SU(3)-normalisation note under Theorem 11.

K.3 Genericity of the determinant phase

Ensemble: 20,000 independent draws; carrier dimension N = 8; four Haar-random complex isometric embeddings per draw; $\hat{C}_Y$ Wishart-positive.

Result: $\arg \Delta_q$ is statistically uniform on the circle —

- circular resultant $|\langle e^{i \arg \Delta_q}\rangle| = 0.0049$ (zero for exact uniformity);
- $P(|\arg \Delta_q| < 0.10) = 0.0321$ against the uniform prediction 0.0318;
- $P(|\arg \Delta_q| < 0.01) = 0.0034$ against the uniform prediction 0.0032.

The statistics are internally consistent: the observed resultant 0.0049 is of the expected order $1/\sqrt{n} \approx 0.0071$ for n = 20,000 uniform draws, and both tail probabilities match the uniform prediction $2\delta/2\pi$ to the stated precision.

Reading. For generic complex embeddings the combined determinant phase carries no preference whatsoever for the real ray. Gate B positivity is a measure-zero condition on the embedding data. Two consequences follow. First, DLT cannot be satisfied accidentally: a blind VERSF evaluation returning Δ_q on the positive real ray would certify structure in the closure architecture, because chance supplies probability zero. Second, the falsification is correspondingly sharp: a complex Δ_q admits no rescue by convention (Lemma 20.1) and fails the gate outright.

K.4 Reality versus orientation

Ensemble: 20,000 draws with a common real structure imposed — real orthogonal-frame embeddings and real symmetric positive $\hat{C}_Y$, realising the premises of Proposition 21.2 with $\mathcal{K}$ = entrywise conjugation.

Result:

- max $|\mathrm{Im}\,\Delta_q| / |\Delta_q| = 0$ to machine precision — reality is exact, as Proposition 21.2 predicts;
- sign split: $P(\Delta_q > 0) = 0.502$, $P(\Delta_q < 0) = 0.498$;
- an explicit single-column orientation flip reverses the sign of Δ_q while preserving every reality premise.

Reading. The reality half of $J_{\mathcal{D}}$ does its job perfectly and stops exactly where Proposition 21.2 says it must: the residual $\mathbb{Z}_2$ is genuinely unresolved, populating both rays equally. The 0.502/0.498 split is the exact numerical shadow of the proposition's non-uniqueness argument — the single-column sign flip that reverses Δ_q while preserving every reality premise acts as a measure-preserving involution on the ensemble, forcing the two rays to equal weight, and the flip is exhibited explicitly in the run. This theorem–experiment agreement, not any individual number, is the content of K.4. The $\Delta_q < 0$ ray — the determinant-line $\theta = \pi$ — survives reality at fifty percent. Orientation selection is therefore independent physical work, and any Gate B derivation must exhibit the substrate structure that performs it (audit item 22).

K.5 Chiral-proximity criterion

Coincident embeddings give exact positivity (Proposition 21.1; confirmed over 2,000 draws: $\mathrm{Im}\,\Delta_f = 0$ to machine precision, min $\mathrm{Re}\,\Delta_f = 2.2 \times 10^2 > 0$ — a scale-dependent figure set by the unnormalised Wishart ensemble at $N = 8$, whose only meaningful content here is strict positivity; it is not a physical margin). To assess how much chiral asymmetry positivity tolerates, $\mathcal{E}_{fR}$ was generated as a normalised perturbation of $\mathcal{E}_{fL}$ with tangent amplitude ε, 400 draws per ε:

ε	worst $|\arg \Delta_f|$ (rad)
0.05	0.452
0.10	1.034
0.20	1.821
0.40	3.020
0.80	3.127
1.60	3.139

Reading. The phase is controlled by proximity — no draw crossed the negative real axis below $\varepsilon \approx 0.4$ — but the control is not sharp: worst-case phases of 0.45 rad already occur at $\varepsilon = 0.05$, driven by draws in which the relevant eigenvalue triples of $\hat{C}_Y$ are small and the overlap is phase-sensitive. Chiral proximity is therefore a usable *criterion* for branch (i) of Debt 15 — coincidence, or proximity with controlled $\hat{C}_Y$ spectrum, suffices — but not a *proof schema* independent of the closure operator's spectral data. A derivation invoking proximity must bound both the asymmetry and the participating spectrum.

K.6 Status

Appendix K closes nothing microscopic. It certifies the exact skeleton of the paper against implementation error, quantifies the improbability of accidental Gate B success, decomposes $J_{\mathcal{D}}$'s obligations into reality and orientation with the $\theta = \pi$ correspondence made exact, and leaves in place an audited pipeline whose single decisive invocation —

$\Delta_q = \Delta_u \Delta_d$, published blind —

awaits the frozen embeddings of Debt 13 in the canonical frame of Debt 14.