# The Substrate Anomaly-Descent Theorem in VERSF

**▲ Programme Milestone — Quantitative Normalisation Series Gate QN-3 / First-Principles Localization-Stiffness Descent**

*Deriving the canonical 8/3 localization stiffness from finite-substrate anomaly descent rather than assigning it as a convention representative*

Keith Taylor VERSF Theoretical Physics Programme — Quantitative Normalisation Series Tier A origin-stability gate paper, QN-3

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## General Reader Summary

The first normalisation paper, QN-1, fixed a serious notation problem.

QN-1 showed that VERSF must not use the symbol κ loosely. The number 8/3, the reciprocal 3/8, the channel-log quantity ln 14, the gap scale $\sqrt{(4/3)}\cdot\xi^{-1}$, and the gravitational-response coupling $\kappa_{eff}$ are not five values of one parameter. They are five different typed quantities, each with its own role, exponent level, and dimensional status.

But that discipline left one deeper question open:

**Why is the canonical localization stiffness 8/3 in the first place?**

This paper answers that question.

The answer is not that 8/3 is convenient. It is not fitted from a mass ratio. It is not secretly ln 14 in disguise. It is not a count of continuum dimensions imported from the emergent geometry.

The result comes from a finite-substrate mismatch.

In VERSF, a physical record is not born directly inside the emergent continuum. It descends from a deeper finite substrate into the physical support sector. For a substrate commitment to become a record at all, it must satisfy four primitive obligations: it must **occur**, it must be **distinguishable**, it must **continue**, and it must **return-normalise** consistently into the ledger of admissible records. Each obligation blocks one primitive way a record can fail: nothing happens, nothing identifiable happens, nothing lasts, or nothing connects back to the whole. But the physical support quotient into which records localize has only three independent support directions.

So the substrate carries a minimal, irreducible anomaly:

**Four primitive commitment obligations must descend into three physical support channels.**

The least-distorting, positive, isotropic descent spreads the four units of commitment charge evenly across the three physical support directions. Each channel therefore carries the one-sided descent load

$g\_AD = 4/3$.

A physical localization is not one-sided. A record must both descend into physical support **and** remain return-compatible with the substrate ledger. Neither branch can be privileged before physical time emerges, so the two branches carry equal load, and the total stiffness doubles:

$\kappa\star = 2\cdot g\_AD = 2\cdot(4/3) = 8/3$.

This paper therefore proves the core result:

**The canonical VERSF localization stiffness 8/3 is the two-sided stiffness generated when the four-unit commitment charge of the minimal record tetrad, forced down by the substrate anomaly, descends isotropically into the three-direction physical support quotient.**

This closes the origin debt left open by the κ-convention audit of QN-1.

In one sentence:

**VERSF's 8/3 localization stiffness is not a floating convention: it is the forced two-sided stiffness of the four-unit commitment charge that the substrate anomaly drives into three physical support channels.**

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# Scope Box — What QN-3 Closes and What It Does Not

| Claim | Status |
|---|---|
| Bare κ convention drift | Inherited closed from QN-1 |
| First-principles origin of $\kappa\star = 8/3$ | **Closed here**, conditional on the anomaly-descent premises AD-1 – AD-8 |
| One-sided coefficient $g\_\kappa = 4/3$ | **Derived here** as the isotropic descent load |
| Two-sided localization stiffness $2g\_\kappa = 8/3$ | **Derived here** |
| Reciprocal compliance $c\star = 3/8$ | Recovered as $1/\kappa\star$ |
| Gap scale $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$ | Recovered as the inverse-length branch-gap scale |
| ln 14 versus 8/3 bridge | Not closed here |
| Full mass-hierarchy derivation | Not closed here |

| Claim | Status |
|---|---|
| Gravitational κ_eff derivation | Not closed here |
| Empirical validation of VERSF | Outside scope |

This paper closes the **origin of the canonical localization stiffness** inside the VERSF normalisation chain. It does not claim to complete the Standard Model derivation programme or the gravitational-response programme.

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# Abstract

QN-1 established the convention discipline required for VERSF's quantitative programme: κ-like quantities must be typed by role, exponent level, dimensional status, and physical slot. It closed the κ-drift problem but left the first-principles origin of the canonical localization stiffness

$\kappa\star = 8/3$

as an explicit bridge debt.

This paper discharges that debt by deriving 8/3 from finite-substrate anomaly descent.

The central premise is that a terminal physical record is not merely a point in a pre-existing geometry. It is the stable physical-sector image of a finite-substrate commitment. A minimal record-making commitment must satisfy four primitive obligations — occurrence, distinction, continuation, and return-normalisation — blocking, respectively, the four primitive failure modes of recordhood: nullity, indistinguishability, evanescence, and detachment. These four obligations are necessary, mutually independent, and jointly sufficient; they are not continuum dimensions but record-formation requirements. The adjoint-removed physical support quotient into which such a commitment descends has three independent support directions. A finite substrate therefore carries a primitive anomaly: a rank-four commitment structure must be represented through a rank-three physical support quotient. The anomaly is the mismatch; the quantity that descends is the commitment charge.

Let the substrate commitment charge be

$\mathfrak{C} = 4$

and let the physical support rank be

$d_{phys} = 3$.

A positive, isotropic, least-distorting descent distributes $\mathfrak{C}$ across the three support directions with equal load. The unique admissible descent vector is

$a_1 = a_2 = a_3 = \mathfrak{C}/d\_phys = 4/3$,

and this uniqueness is robust: uniform descent is the minimiser of **every** strictly convex permutation-symmetric distortion functional, not merely the quadratic one. The number 4/3 is the one-sided anomaly-descent coefficient

$g\_AD = 4/3$.

Terminal record stability is bidirectional: the physical record must descend into support and also remain return-compatible with the substrate ledger. Positivity, adjoint removal, and pre-temporal reciprocity force the two branches to carry equal load. Hence the canonical two-sided amplitude-level localization stiffness is

$\kappa\_AD = 2\cdot g\_AD = 8/3$.

The theorem proves that, under the stated anomaly-descent premises, 8/3 is not assigned, fitted, or inferred from numerical coincidence. It is the unique two-sided stiffness compatible with finite commitment, rank-four minimal record obligation, rank-three physical support, isotropic descent, positivity, and bidirectional terminality, and the channel–exponent bridge assigns that load to the canonical radial slot. For the quadratic descent cost, the Lagrange-dual multiplier equals 8/3 identically; this is registered as an algebraic bookkeeping identity, not as independent corroboration, since the dual value is cost-dependent while the primal load is not.

The paper recovers the QN-1 relations

$g\_\kappa = 4/3$, $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$, $c\star = 3/8$, $\kappa\_prob = 16/3$,

now with an explicit origin rather than an inherited assignment.

QN-3 therefore closes the first-principles localization-stiffness descent gate: the canonical VERSF coefficient 8/3 is the two-sided residue of substrate anomaly descent from four primitive record obligations into three physical support channels.

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# Contents



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# 0. Inheritance, Firewalls, and Aim

## 0.1 Inherited results

QN-3 inherits the following results from the earlier quantitative-normalisation and physical-sector papers.

**I1 — Finite commitment scale.** VERSF uses a finite coherence/commitment scale

$\xi \approx 8.2 \times 10^{-5}$ m,

with associated minimal continuation time

$\tau_s = \xi/c \approx 0.28$ ps.

QN-3 does not rederive $\xi$.

**I2 — Typed κ-discipline.** The symbol $\kappa$ is not a free scalar. QN-1 established that $\kappa$-like objects must be typed by role, convention, exponent level, and dimensional status. A bare, untyped $\kappa$ is inadmissible anywhere in the quantitative programme.

**I3 — Canonical localization convention.** QN-1 fixed the canonical amplitude-level localization convention as

$\psi(u) = \psi(0)\cdot e^{(-\kappa\star\cdot u)}$, $u = \rho/\xi$,

with $\kappa\star = 8/3$ as the inherited canonical localization-stiffness representative.

**I4 — Gap-scale relation.** QN-1 registered the relation

$m\star^2 = (\kappa\star/2)\cdot\xi^{-2}$,

and therefore $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$ once $\kappa\star = 8/3$.

**I5 — Physical-sector positivity.** Quantitative VERSF gates operate in the adjoint-removed physical quotient. Negative-norm residue, ghost directions, and unphysical adjoint leakage are not permitted to contribute to physical localization stiffness.

**I6 — Record discipline.** Physical quantities must be tied to terminal record consequences or to structures required for such consequences. A coefficient is physical only if its role in record formation is fixed.

QN-3 accepts these inherited results and addresses the specific origin question left open by QN-1. In the gate sequence, QN-3 follows the Standard Model convention and normalisation audit (QN-2) but does not depend on it: the inheritance above runs through QN-1 and the physical-sector papers only.

# 0.2 The firewalls

This paper does **not** claim:

- that 8/3 is derived from ln 14;
- that 8/3 is fitted to Standard Model masses;
- that 8/3 is a continuum dimension count;
- that the substrate commitment tetrad is the coordinate structure of the emergent 4-manifold;
- that time is fundamental;

- that 4/3 is accepted merely because it is half of 8/3;
- that 3/8 is accepted merely because it is the reciprocal of 8/3;
- that all later mass, coupling, hierarchy, or gravitational-response gates are now closed.

It proves a narrower and stronger claim:

**Given the VERSF substrate-to-record descent premises, the canonical one-sided anomaly-descent load is uniquely 4/3, and the two-sided amplitude-level localization stiffness is uniquely 8/3.**

## 0.3 The aim

QN-1 answered:

Are VERSF's κ-like quantities coherent once typed?

QN-3 answers:

Why is the canonical localization stiffness $\kappa^\star$ equal to 8/3?

The claim is not convention assignment. It is origin descent.

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# 0′. Predictive-Content Ledger

| Object | Prior status | Status here |
|---|---|---|
| Bare κ | Disallowed by QN-1 | Inherited disallowed |
| $\kappa^\star = 8/3$ | Inherited canonical representative | **Derived** as anomaly-descent stiffness |
| $g_\kappa = 4/3$ | Inherited half-stiffness gap coefficient | **Derived** as one-sided descent load |
| $m^\star = \sqrt{(4/3)}\cdot\xi^{-1}$ | Inherited gap scale | Recovered from $g_{AD}$ |
| $c^\star = 3/8$ | Inherited reciprocal compliance | Recovered as $1/\kappa^\star$ |
| $\kappa_{prob} = 16/3$ | Inherited probability-level exponent | Recovered as $2\kappa^\star$ |
| Substrate anomaly | Implicit | Defined |
| Minimal record-commitment tetrad | Implicit | Defined |
| Failure-mode basis for $\mathfrak{C} = 4$ | Implicit | Defined (§4.2′), proved (Lemma 1) |

| Object | Prior status | Status here |
|---|---|---|
| Descent stiffness operator A_AD | Absent | Formulated: trace = charge, isotropic eigenvalue = load (§6.6) |
| Branch-load additivity | Assumed | Derived from log-attenuation composition (§8.3′) |
| Equal branch loads | Stated in AD-7 | Derived from branch-swap invariance (§8.2′) |
| Full-charge descent (AD-4) | Stated | Argued from obligation support and minimality (§3.6) |
| General descent law κ = b·r_sub/d_phys | Absent | Supplied with counterfactual table (§9.7) |
| Physical support quotient rank | Inherited physical-sector fact | Used as d_phys = 3 |
| Positive isotropic descent | Implicit | Stated as anomaly-descent law |
| Least-distorting descent | Implicit | Proved by convex minimisation; robustness proved in Appendix C |
| One-sided branch load | Inherited | Derived as 4/3 |
| Two-sided terminality | Inherited in closure language | Made explicit |
| Localization stiffness | Inherited | Derived as 2·(4/3) = 8/3 |
| Quadratic dual identity | Absent | Registered as algebraic bookkeeping (Appendix B) |
| Channel–exponent bridge | Implicit | Stated as AD-8, proved as Lemma 7 |
| Anomaly vs commitment-charge typing | Conflated | Separated (§3.1) |
| ln 14 bridge | Open | Remains open |
| Gravitational κ_eff | Open | Remains open |
| Full mass/coupling derivation | Outside scope | Remains outside scope |

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# 1. The Open Debt After QN-1

## 1.1 What QN-1 achieved

QN-1 closed the convention problem.

It showed that VERSF contains several κ-like quantities —

$8/3$, $3/8$, $\ln 14$, $\sqrt{(4/3)}\cdot\xi^{-1}$, $\kappa_{eff}$

— but that these are not rival values of one parameter. Instead:

- 8/3 is the canonical amplitude-level localization stiffness;
- 3/8 is reciprocal compliance;
- ln 14 is a channel-log weight;
- $\sqrt{(4/3)}\cdot\xi^{-1}$ is an inverse-length gap scale;
- $\kappa_{eff}$ is a dimensional response coefficient.

That result was necessary. Without it, later quantitative VERSF derivations would be vulnerable to the objection that κ changes value between calculations.

## 1.2 What QN-1 left open

QN-1 deliberately did not derive 8/3 from first principles. It treated $\kappa\star = 8/3$ as the inherited canonical representative and registered the deeper question as a bridge debt:

**Why 8/3?**

QN-3 is the bridge theorem that answers this.

## 1.3 The QN-3 strategy

QN-3 derives 8/3 from four ingredients:

1. a minimal finite-substrate record commitment has four primitive obligations;
2. the physical support quotient has three independent support directions;
3. the commitment charge held under the unresolved anomaly must descend positively and isotropically into those three channels;
4. a terminal record requires two equal branches: descent into physical support and return-compatibility with the substrate ledger.

The result is forced:

one-sided load = 4/3, two-sided stiffness = $2\cdot(4/3) = 8/3$.

This is the whole logic of QN-3. Everything else in the paper is the disciplined statement of premises, proofs of the intermediate lemmas, a variational reformulation with its dual identity correctly typed, and the falsifiers that keep the result honest.

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# 2. Named Premises of the Anomaly-Descent Theorem

The theorem rests on eight named premises. Each carries an explicit falsifier. The theorem is exactly as strong as this premise set; nothing else is smuggled in.

**AD-1 (Finite descent).** Every terminal physical record is the physical-sector image of a finite-substrate commitment; no record originates natively inside the emergent continuum. *Falsifier:* exhibit a terminal record whose formation requires no substrate commitment.

**AD-2 (Record-obligation minimality).** The minimal record-forming commitment carries exactly four primitive obligations because a terminal record must block exactly four independent failure modes: nullity, indistinguishability, evanescence, and detachment. These are blocked, respectively, by occurrence, distinction, continuation, and return-normalisation. The four obligations are necessary, mutually independent, and jointly sufficient for minimal terminal recordhood. *Falsifier:* show that a terminal record survives the removal of one obligation, that one obligation is derivable from the other three, or that a fifth independent primitive failure mode must be blocked before terminal recordhood is possible.

**AD-3 (Support rank three).** The adjoint-removed physical support quotient has exactly three independent support directions: d_phys = 3. *Falsifier:* exhibit a fourth independent physical support direction surviving adjoint removal, or show that localized records require fewer than three.

**AD-4 (Charge conservation).** Descent preserves the total commitment charge: the full commitment load $\mathfrak{C} = 4$ descends; none is discarded, none is created. *Falsifier:* exhibit a lawful record-forming descent that dissipates or amplifies commitment load.

**AD-5 (Positivity).** Descended loads are non-negative in every physical support channel; no channel may carry negative load, since negative load is adjoint residue and is excluded from the physical quotient. *Falsifier:* exhibit a physical localization sustained by negative support load.

**AD-6 (Pre-record isotropy).** Prior to record realization, no unit direction in the physical support quotient is distinguished from any other: the isotropy is continuous, not merely discrete. Invariance under the permutation group $S_3$ acting on the three support channels is the restriction of this continuous statement to the channel basis, and is the form consumed by the load-vector argument (Lemma 2); the operator formulation (§6.6) consumes the full continuous statement. The descent is least-distorting: it adds no structure absent from the anomaly. The invariance is required of the realized descent, not merely of the legislating law: no admissible realization carries directional information prior to record formation. *Falsifier:* exhibit a primitive (pre-record) anisotropy in the descent law or in any admissible realized descent — discrete (a privileged channel) or continuous (a privileged direction in the support quotient, including the channel diagonal).

**AD-7 (Bidirectional terminality).** Terminal record stability requires two branches — descent into physical support and return-normalisation into the substrate ledger — and, prior to emergent physical time, neither branch may carry a privileged primitive stiffness; the branches are equal. *Falsifier:* exhibit a terminal record that is one-sided (never return-normalised), or a lawful pre-temporal asymmetry between the branches.

**AD-8 (Channel–exponent bridge).** The two-sided load carried by a support direction is a rate of amplitude log-attenuation per unit normalized displacement along that direction. The radial variable $u = \rho/\xi$ of I3 is a one-dimensional path coordinate along a normalized ray, not a coordinate sum over the three support channels. With isotropy (AD-6) making the per-direction rate common to every ray, the exponent of the canonical radial profile of I3 equals the per-direction two-sided load — for every ray, and never an aggregate of loads across channels. *Falsifier:* exhibit an admissible localization profile in which the radial amplitude exponent differs from the per-direction two-sided load — for example, one in which the exponent sums loads over directions.

Every quantitative step below cites the premises it consumes. The reader can therefore audit exactly which failure would break which step.

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# 3. Substrate Anomaly: Definition and Physical Meaning

## 3.1 What "anomaly" means here

In this paper, an anomaly is not a quantum-field-theoretic gauge anomaly, not a particle-sector anomaly, and not an observational irregularity.

A **substrate anomaly** is the irreducible mismatch between:

1. the number of primitive obligations required for a finite-substrate commitment to become a terminal record; and
2. the number of independent physical support directions available to represent that commitment in the physical quotient.

In symbols, the primitive anomaly arises because

$r_{sub} = 4$, $d_{phys} = 3$.

The substrate must satisfy four record-making obligations, but physical localization is supported by three independent support directions. That mismatch cannot be ignored. It forces a descent into the physical sector — and what descends, once typed correctly, is the commitment charge, appearing there as a localization stiffness.

Terminology is fixed here once. The **anomaly** is the mismatch itself: the fact that a rank-four obligation structure must be represented through a rank-three support quotient. The anomaly is *why* descent must occur. The **commitment charge** $\mathfrak{C} = 4$ is the full obligation load that descends. The anomaly is not the descended quantity, and in particular the descent load is not the rank deficit $4 - 3 = 1$: the deficit measures the mismatch; the charge measures what the mismatch forces downward (§5.4). No later section uses "anomaly" for the charge or "charge" for the mismatch.

## 3.2 Why anomaly descent is required

A finite-substrate commitment cannot remain indefinitely unresolved. If it did, it would not become a physical record (AD-1).

A physical record requires positive support, finite localization, no negative-norm leakage, stable continuation, and return-compatibility with the record ledger. The commitment charge held under the unresolved anomaly therefore has only three conceivable fates:

1. **it descends positively into the physical support quotient;**
2. it remains unresolved, and no terminal record forms;
3. it leaks into unphysical directions, violating physical-sector positivity (I5, AD-5).

Only the first is admissible in the physical sector. Descent is therefore not an option added to the framework; it is the sole channel through which records exist at all.

## 3.3 Definition — substrate commitment charge

Let $\mathcal{K}$ denote the minimal finite-substrate commitment module required for terminal record formation, with primitive rank

$\mathrm{rank}(\mathcal{K}) = 4.$

Define the substrate commitment charge as the primitive rank of the commitment:

$\mathfrak{C} = \mathrm{rank}(\mathcal{K}) = 4.$

The charge is a count of obligations and is stated as a count; no ambient algebra is invoked to dress it as a trace. It is not a tunable number: it counts the primitive obligations of the minimal commitment (AD-2) and nothing else.

## 3.4 Definition — physical support quotient

Let $\mathcal{P}$ denote the adjoint-removed physical support quotient, with support rank

$\mathrm{rank}(\mathcal{P}) = d_{phys} = 3.$

A localized physical record distributes support over three independent physical directions (AD-3).

## 3.5 Definition — anomaly descent

An **anomaly descent** is a positive map from substrate commitment load into the physical support quotient,

$D : \mathcal{K} \longrightarrow \mathcal{P}$,

represented at the level of support loads by a vector

$a = (a_1, a_2, a_3),\ a_i \geq 0,\ a_1 + a_2 + a_3 = \mathfrak{C} = 4.$

The $a_i$ are the descended charge loads in the three physical support directions. Non-negativity is AD-5; the sum constraint is AD-4.

## 3.6 Why the full charge descends

AD-4 can be argued rather than merely stated. Suppose less than the full charge descends: $\Sigma_i\, a_i < \mathfrak{C}$. Then at least one primitive record obligation has no physical support image. The resulting object may be a partial substrate commitment, but it is not a terminal physical record: an obligation without support is a failure mode left unblocked (AD-2; Lemma 1).

Suppose more than the full charge descends: $\Sigma_i\, a_i > \mathfrak{C}$. Then the physical quotient carries load supplied by no obligation of the minimal commitment tetrad. That is additional primitive structure with no source — a violation of minimality — and positivity (AD-5) forbids repairing the excess by negative load elsewhere.

A lawful minimal descent therefore satisfies exactly

$\Sigma_i\, a_i = \mathfrak{C} = 4$:

the descent neither discards an obligation nor creates a new one.

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# 4. The Minimal Record-Commitment Tetrad

## 4.1 Why the substrate commitment has rank four

A terminal record is not merely an event. It is a finite, distinguishable, continuable, ledger-compatible physical consequence. A minimal record-forming commitment must therefore satisfy four primitive obligations:

1. **Occurrence (E):** something is committed rather than nothing.
2. **Distinction (D):** the commitment is distinguishable from its complement or non-occurrence.

3. **Continuation (C):** the commitment persists over a finite support interval rather than vanishing as an uncommitted fluctuation.
4. **Return-normalisation (R):** the commitment remains compatible with the global record ledger, so that it is not a detached local fragment with no admissible normalisation.

These four obligations are necessary, mutually independent, and jointly sufficient. Each blocks exactly one primitive failure mode of recordhood; the correspondence is tabulated in §4.2′, and minimality is proved in Lemma 1.

# 4.2 Definition — commitment tetrad

The four obligations form the **minimal commitment tetrad**

$\mathcal{K} = \langle E, D, C, R \rangle$, $\mathrm{rank}(\mathcal{K}) = 4$.

This is not a continuum tetrad. It is a record-formation tetrad, defined prior to any emergent geometric structure.

# 4.2′ The Four Primitive Failure Modes

The commitment tetrad is not a descriptive checklist. It is a minimal failure-prevention basis. A candidate record can fail in exactly four primitive ways:

| Failure mode | Missing obligation | Result |
|---|---|---|
| Nullity | Occurrence (E) | Nothing is committed |
| Indistinguishability | Distinction (D) | No identifiable record exists |
| Evanescence | Continuation (C) | The commitment does not persist into record form |
| Detachment | Return-normalisation (R) | The local consequence cannot enter the global record ledger |

A terminal record exists only when all four failure modes are blocked. This is why the numerator in the anomaly-descent load is four. It is not a continuum dimension count, not a channel count, and not a fitted integer. It is the primitive rank of terminal recordhood.

# 4.3 Lemma 1 — Record-obligation minimality

**Lemma 1.** A minimal terminal record requires exactly four primitive substrate obligations: occurrence, distinction, continuation, and return-normalisation. The four are necessary, mutually independent, and jointly sufficient.

**Proof.** *Necessity.* A terminal record must occur: if occurrence is absent, there is no committed consequence and therefore no record (nullity). It must be distinguishable: if distinction is absent, the alleged record cannot be separated from non-record, background, or complement — present, perhaps, in an undifferentiated sense, but not identifiable as a record (indistinguishability). It must continue over finite support: if continuation is absent, the commitment is evanescent and never enters record form (evanescence). It must be return-normalisable: if return-normalisation is absent, the consequence may be locally present, distinguishable, and persistent, yet it remains a detached fragment with no consistent embedding in the wider ledger of possible records (detachment). All four obligations are therefore necessary.

*Independence.* No obligation is derivable from the conjunction of the other three. Occurrence is not derivable from distinction, continuation, and return-normalisation: a ledger may contain an admissible, distinguishable slot with continuation rules, but if nothing is committed into that slot, no record occurs — specification is not commitment. Distinction is not derivable from occurrence, continuation, and return-normalisation: a commitment may occur and persist in a ledger-compatible way, but if it cannot be separated from background or complement, it is not identifiable as a record. Continuation is not derivable from occurrence, distinction, and return-normalisation: a commitment may occur, be identifiable, and be of an admissible type, but if it vanishes without finite support it never becomes a terminal record. Return-normalisation is not derivable from occurrence, distinction, and continuation: a local consequence may occur, be identifiable, and persist, yet admit no consistent embedding in the global ledger — the detached fragment. The four obligations are therefore mutually independent.

*Sufficiency.* If occurrence holds, there is a committed consequence rather than nullity. If distinction holds, that consequence is identifiable against background or complement. If continuation holds, it persists over finite support and is not merely evanescent. If return-normalisation holds, it is admissible in the global record ledger and is not a detached fragment. A non-null, identifiable, finitely supported, ledger-admissible consequence satisfies the definition of a minimal terminal record. No fifth primitive obligation is required.

Since the four obligations are necessary, mutually independent, and jointly sufficient, the minimal record-forming commitment has exactly four primitive obligations:

$r\_sub = \mathrm{rank}(\mathcal{K}) = 4$. ■

## 4.4 Corollary — Why the commitment charge is four

The substrate commitment charge is four because terminal recordhood has four independent primitive failure modes. A minimal commitment must block nullity, indistinguishability, evanescence, and detachment, and by Lemma 1 these blocks are individually necessary, mutually independent, and jointly sufficient. The charge counts them:

$\mathfrak{C} = \mathrm{rank}(\mathcal{K}) = 4$.

Thus $\mathfrak{C} = 4$ is not a choice of numerical convenience. It is the primitive rank of terminal record formation — the sole source of the numerator in the 4/3 descent coefficient. No continuum dimension count, channel census, or empirical input enters here.

## 4.5 Non-circularity of the tetrad

A critic may say the paper defines "record" so that four obligations are required. The tetrad is not obtained that way. It is obtained by analysing the primitive ways in which recordhood can fail.

The four obligations correspond to four mutually independent negations: no occurrence, no distinction, no continuation, no ledger admissibility. Each negation destroys terminal recordhood in a different way (§4.2′), and once all four negations are blocked, minimal recordhood is achieved (Lemma 1, sufficiency). The tetrad is therefore a failure-completion basis, not a stipulative definition.

This matters for the anomaly-descent theorem because the value 4 entering

g_AD = 4/3

is not a free count. It is the number of independent primitive failures that must be prevented for a finite substrate commitment to become a terminal physical record.

## 4.6 The fifth-obligation challenge

The exactness claim r_sub = 4 is falsifiable, and the challenge can be made precise. A proposed fifth primitive obligation is admissible only if it supplies a fifth independent failure mode of terminal recordhood. It is not enough to name another useful feature. The proposal must pass three tests:

1. **Necessity:** without the proposed obligation, terminal recordhood fails.
2. **Independence:** the proposed obligation is not reducible to occurrence, distinction, continuation, or return-normalisation, singly or jointly.
3. **Sufficiency disruption:** the four-obligation tetrad no longer suffices once the fifth failure mode is exposed.

If no such fifth failure mode is supplied, the minimal rank remains r_sub = 4. This turns the exactness of the tetrad from a stipulation into a standing test.

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# 5. The Physical Support Quotient

## 5.1 The physical support sector

A physical record localizes into the adjoint-removed physical quotient. This quotient has three independent support directions. They are not introduced as arbitrary coordinates. They are the minimal physical support directions that remain once unphysical adjoint residue has been removed and localized terminal records are still supportable.

Thus

$d\_phys = 3$.

## 5.2 Why the quotient is rank three

A physical record must have local support, and local support must allow independent separation along mutually independent directions. Fewer than three directions cannot carry the full localization structure exhibited by observed spatial records. More than three would introduce a support direction with no surviving physical carrier in the adjoint-removed quotient — precisely the kind of unsupported direction that adjoint removal excludes.

QN-3 therefore adopts, as an inherited physical-sector structure (AD-3):

$\mathrm{rank}(\mathcal{P}) = 3$.

## 5.3 Structural Fact — Physical support rank

**Structural Fact (inherited).** In the VERSF physical-sector quotient used by the quantitative normalisation programme, the local physical support rank is three: $d\_phys = 3$.

This is not proved here, and it is deliberately not dressed as a lemma. It is the adjoint-removed physical-sector structure inherited through I5 and stated as premise AD-3. Restating an inherited premise in declarative voice and appending "■" would invite a circularity charge; QN-3 instead consumes the fact openly, and falsifier F3 keeps it permanently exposed.

## 5.4 The rank mismatch — and why the load is 4/3, not 1

Combining Lemma 1 and the Structural Fact of §5.3 gives the primitive substrate-to-physical mismatch — the anomaly proper of §3.1:

$\mathrm{rank}(\mathcal{K}) - \mathrm{rank}(\mathcal{P}) = 4 - 3 = 1$.

But the anomaly is not the descended quantity, and the descended load is **not** the rank difference. Descent must carry the full commitment charge (AD-4): all four obligations must be represented through the three available channels, not merely the "excess" one. The relevant quantity is therefore the load ratio

$\mathfrak{C}/d\_phys = 4/3$,

not the rank deficit 1. This distinction matters: a rank-deficit reading would make the stiffness independent of the tetrad structure, whereas the charge-conservation reading (AD-4) makes every obligation count. It is the second reading that record discipline (I6) demands, since each obligation is individually required for terminality. This is the typing fixed in §3.1: the anomaly (the deficit) is why descent occurs; the charge (four units) is what descends.

# 6. Positive Isotropic Descent

## 6.1 Why descent must be positive

Physical-sector localization cannot use negative descended load to cancel positive load. Negative load in a support channel would correspond to adjoint residue, ghost contribution, or non-physical subtraction from the record-support sector — all excluded by I5 and AD-5.

Therefore the descent vector satisfies

$a_i \geq 0$ for every i.

This is not optional. It is physical-sector positivity applied at the level of descent.

## 6.2 Why descent must be isotropic

Before a physical record localizes into a particular realized configuration, the substrate anomaly has no preferred physical support axis. A descent rule assigning unequal primitive loads to the three support directions would install a preferred physical axis at the level of the substrate itself — an anisotropy with no source (AD-6).

Therefore the descent law is invariant under permutations of the three physical support directions. The symmetry group is

$S_3$.

Discrete channel-permutation invariance is the shadow, in the channel basis, of the continuous content of AD-6: no unit direction in the support quotient is distinguished. The discrete form suffices for the load-vector argument below; the continuous form is what the operator formulation of §6.6 consumes.

Pre-record isotropy is not final-state isotropy. It does not assert that every realized physical record is spatially isotropic. It asserts only that the primitive descent law contains no preferred support channel before a specific record is realized. Once a record localizes, boundary conditions, interactions, measurement couplings, or environmental structure may select directions; that later anisotropy belongs to record realization, not to the primitive substrate-to-support descent. AD-6 forbids built-in anisotropy at the descent level, and forbids nothing about anisotropic physical configurations after localization.

## 6.3 Why descent must be least-distorting

The descent must not add structure that was not present in the anomaly. The anomaly carries total load 4 and no directional information; the physical quotient offers three channels and privileges none. The admissible descent is therefore the least-distorting positive distribution of total load 4 across three channels.

Stated variationally: among all vectors $a \in \mathbb{R}^3$ with $a_i \geq 0$ and $a_1 + a_2 + a_3 = 4$, the descent selects the vector minimizing a strictly convex, permutation-symmetric distortion functional. The quadratic

$$\mathcal{D}(a) = a_1^2 + a_2^2 + a_3^2$$

is the minimal such cost — and, crucially, the result below does not depend on the quadratic choice (Appendix C).

A loophole must be closed here. $S_3$-invariance of the descent **law** does not by itself force $S_3$-invariance of the descent **outcome**. A concave cost on the simplex is minimised at a vertex, e.g. (4, 0, 0); and "select one channel uniformly at random" is a perfectly $S_3$-invariant law whose every realization is maximally anisotropic. AD-6 therefore demands more than symmetric legislation: it demands that the *realized* descent carry no directional information prior to record formation, since an anisotropic realization would constitute directional structure with no source, however symmetric the law that produced it. Strict convexity is the formalization of that demand — it is exactly the condition under which the symmetric law has a unique, and therefore symmetric, realization. Falsifier F7 accordingly targets the pair ⟨permutation symmetry, strict convexity⟩, not the quadratic representative.

# 6.4 Lemma 2 — Unique positive isotropic descent

**Lemma 2.** The unique positive, isotropic, least-distorting descent of a substrate commitment charge $\mathfrak{C} = 4$ into three physical support directions is

$$a_1 = a_2 = a_3 = 4/3.$$

**Proof.** *Symmetry route.* The descent vector satisfies $a_i \geq 0$ and $a_1 + a_2 + a_3 = 4$. Isotropy (AD-6) requires the descent law to be invariant under all permutations of the three channels. The only vector in the constraint simplex fixed by the full permutation group $S_3$ has equal components, $a_1 = a_2 = a_3 = a$; the constraint then gives $3a = 4$, so $a = 4/3$.

*Variational route.* Minimizing $\mathcal{D}(a) = a_1^2 + a_2^2 + a_3^2$ subject to $\Sigma_i\, a_i = 4$ gives, by Lagrange stationarity, $2a_i - \lambda = 0$ for every i, hence all $a_i$ equal, hence $a_i = 4/3$. Uniqueness follows from strict convexity of $\mathcal{D}$ on the compact constraint simplex. The non-negativity constraints are inactive at the minimiser, so the interior stationary point is the global minimum.

*Robustness.* By Appendix C, the uniform vector minimises **every** strictly convex permutation-symmetric functional on the simplex, so the conclusion is independent of the quadratic choice. ■

# 6.5 Interpretation

The value 4/3 is not inserted. It is the unique load per physical support direction when four primitive substrate obligations descend evenly into three physical support channels. Define

$g_{AD} = 4/3$.

This is the **one-sided anomaly-descent coefficient**.

## 6.6 The descent load as a stiffness operator

The descent vector $a = (4/3, 4/3, 4/3)$ is a ledger shorthand. The invariant object is a positive stiffness operator on the physical support quotient,

$A_{AD} : \mathcal{P} \to \mathcal{P}$,

whose diagonal entries in a channel basis are the descended loads. Two facts fix it completely.

First, charge conservation (AD-4) fixes its trace:

$\mathrm{Tr}_{\mathcal{P}}(A_{AD}) = \mathfrak{C} = 4$.

(The trace here is a genuine operator trace on the three-dimensional support quotient — unlike a decorated count, there is an actual operator to trace.)

Second, isotropy must be consumed in its full continuous form (AD-6). Discrete $S_3$-invariance alone is not enough at the operator level: the general operator commuting with all channel permutations is

$A = \alpha \cdot \mathbb{1}_{\mathcal{P}} + \beta \cdot (J - \mathbb{1}_{\mathcal{P}})$,

where J is the all-ones matrix. Trace conservation gives $3\alpha = \mathfrak{C} = 4$, so the diagonal load is 4/3 for any $\beta$ — but for $\beta \neq 0$ the per-direction stiffness $n^T \cdot A \cdot n$ is direction-dependent: the eigenvalues are $\alpha + 2\beta$ along the channel diagonal $(1, 1, 1)/\sqrt{3}$ and $\alpha - \beta$ on its orthogonal complement, so a nonzero $\beta$ installs the channel diagonal as a distinguished axis. That is directional structure with no source — exactly what the continuous clause of AD-6 forbids: no unit direction in the support quotient is distinguished prior to record formation. Hence $\beta = 0$, every unit vector is an eigenvector with the common eigenvalue, and

$A_{AD} = g_{AD} \cdot \mathbb{1}_{\mathcal{P}}$.

Taking the trace,

$3 \cdot g_{AD} = 4 \Longrightarrow g_{AD} = 4/3$.

This resolves cleanly a question the vector notation leaves open: why the one-sided coefficient is 4/3 rather than the total 4. The conserved commitment charge is the **trace** of the descent operator. The scalar stiffness seen by a normalized support displacement is its **isotropic eigenvalue**. For any unit support direction $n \in \mathcal{P}$,

$$n^T \cdot A\_AD \cdot n = n^T \cdot (4/3) \cdot \mathbb{1}\_\mathcal{P} \cdot n = 4/3.$$

Every physical support direction sees the same one-sided descent stiffness 4/3. No direction sees the total charge, because the total charge is not a per-direction quantity; it is the channel sum.

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# 7. The One-Sided 4/3 Descent Coefficient

## 7.1 Definition

The one-sided anomaly-descent coefficient is

$$g\_AD = \mathfrak{C}/d\_phys = 4/3.$$

## 7.2 Physical meaning

The coefficient 4/3 measures the branch load carried by each physical support direction after the commitment charge of the minimal tetrad descends into the physical quotient. It is "one-sided" because it describes one branch of record formation: descent from substrate commitment into physical support. Equivalently (§6.6), g_AD is the isotropic eigenvalue of the descent stiffness operator, whose trace is the conserved charge $\mathfrak{C} = 4$.

## 7.3 Lemma 3 — One-sided branch coefficient

**Lemma 3.** Under premises AD-1 – AD-6, the one-sided branch coefficient is

$$g\_AD = 4/3.$$

**Proof.** By Lemma 1 (via AD-2), the substrate commitment charge is $\mathfrak{C} = 4$. By AD-3 (Structural Fact, §5.3), the physical support rank is d_phys = 3. By AD-4 the full charge descends, by AD-5 the loads are non-negative, and by Lemma 2 (via AD-6) the descent is uniform. Hence the load per channel — the one-sided branch coefficient — is

$$g\_AD = \mathfrak{C}/d\_phys = 4/3. \blacksquare$$

## 7.4 Relation to the gap coefficient

QN-1 treated $g\_\kappa = 4/3$ as the quadratic half-stiffness gap coefficient, defined downstream of $\kappa\star$ by $g\_\kappa = \kappa\star/2$. QN-3 reverses the arrow of explanation:

$g\_\kappa = g\_AD$.

The gap coefficient is **primary**: it is the one-sided descent load, and κ⋆ is its two-sided completion. Consequently

$$m\star^2 = g\_AD \cdot \xi^{-2} \Longrightarrow m\star = \sqrt{(g\_AD)} \cdot \xi^{-1} = \sqrt{(4/3)} \cdot \xi^{-1}.$$

The gap coefficient is therefore not a loose half of 8/3. It is the quantity from which 8/3 follows.

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# 8. Bidirectional Terminality and the Two-Sided 8/3 Stiffness

## 8.1 Why one-sided descent is not enough

A one-sided descent produces local physical support, but a terminal record requires more. A terminal record must also remain compatible with the substrate ledger from which it descended; otherwise it is a detached physical fragment, not a record (obligation R of the tetrad, now acting at the level of branches).

Record formation therefore has two branches:

1. **descent branch:** substrate commitment descends into physical support;
2. **return branch:** physical support remains normalisable back into the substrate record ledger.

This is not ordinary time reversal. It is pre-temporal record reciprocity: both branches are constitutive of terminality, and both are prior to emergent physical time.

## 8.2 Equality of the two branches

At the substrate level there is no prior physical time direction that could privilege descent over return. If the two branches carried different primitive stiffnesses, the local record algebra would contain a built-in pre-physical arrow — an unlicensed asymmetry inserted before time emerges. Positivity (AD-5) independently forbids the alternative escape of cancelling one branch against the other with negative load.

Therefore the descent and return branches carry equal load (AD-7):

$$g\_down = g\_return = g\_AD = 4/3.$$

## 8.2′ The branch-swap involution

The equality of the two branches can be stated as an invariance. Let $\sigma\_br$ denote the pre-temporal branch-swap involution that exchanges the two terminality branches:

$\sigma\_br$ : descent $\leftrightarrow$ return.

(The symbol $\sigma\_br$ is used rather than any variant of R, to avoid collision with obligation R.) This is not ordinary time reversal. It is the statement that, before physical time emerges, the substrate ledger contains no primitive orientation distinguishing descent into support from return-normalisation into the ledger.

Let the terminal branch action carry the two positive branch requirements B_down and B_return with loads g_down and g_return:

$S\_term = g\_down \cdot B\_down + g\_return \cdot B\_return.$

Then

$\sigma\_br(S\_term) = g\_down \cdot B\_return + g\_return \cdot B\_down,$

and pre-temporal orientation-freedom requires $\sigma\_br(S\_term) = S\_term$, that is,

$(g\_down - g\_return) \cdot (B\_down - B\_return) = 0.$

The two branch requirements are distinct: descent into physical support and return into the substrate ledger are different transport channels. Their distinctness is established independently by the predicate/channel argument of Lemma 5 (§8.5 — a forward reference; that argument does not rely on the present section), so $B\_down \neq B\_return$, and therefore

$g\_down = g\_return.$

If the two coefficients differed, the substrate would carry a built-in branch orientation before physical time exists: an additional primitive structure licensed nowhere in AD-1 – AD-8. Equal branch load is therefore not a modelling convenience; it is forced by pre-temporal reciprocity. This also sharpens the internal structure of AD-7: given two distinct branches and no primitive pre-temporal orientation, equality is derived rather than separately assumed, and AD-7's falsifier accordingly targets the orientation claim.

## 8.3 Definition — two-sided localization stiffness

The two-sided amplitude-level localization stiffness is the total branch load required for terminal record stability:

$\kappa\_AD = g\_down + g\_return = 2 \cdot g\_AD = 2 \cdot (4/3) = 8/3.$

## 8.3′ Branch composition and log-attenuation

The two branch loads add — rather than multiply, average, or combine in quadrature — because localization is measured by log-attenuation, the QN-1 invariant.

The exponential branch form below is not an ansatz: it is the QN-1 log-attenuation convention (I3) applied per branch. Both branches attenuate the same support profile, so both share the normalized variable u.

Let the descent branch contribute the amplitude attenuation

$\psi(u) \mapsto e^{(-g\_down \cdot u)} \cdot \psi(0)$,

and let the return-normalisation branch contribute

$\psi(u) \mapsto e^{(-g\_return \cdot u)} \cdot \psi(0)$.

A terminal record must satisfy both branch requirements, so the attenuation factors compose multiplicatively:

$e^{(-g\_down \cdot u)} \cdot e^{(-g\_return \cdot u)} = e^{(-(g\_down + g\_return) \cdot u)}$.

The invariant log-attenuation is therefore additive in the branch loads:

$L(u) = -\ln( \psi(u)/\psi(0) ) = (g\_down + g\_return) \cdot u$.

Hence the two-sided amplitude-level localization stiffness is the branch sum,

$\kappa\_AD = g\_down + g\_return$,

and with the equal branches of §8.2′,

$\kappa\_AD = 2 \cdot g\_AD = 8/3$.

The doubling rule is thus not an arithmetic convention: it is multiplicative composition of attenuations, read through QN-1's logarithmic invariant.

# 8.4 Lemma 4 — Bidirectional terminality

**Lemma 4.** A terminal VERSF record requires two equal anomaly-descent branches: descent into physical support and return-normalisation into the substrate ledger.

**Proof.** A physical record must be locally supported, otherwise it has no physical presence; this requires the descent branch. A physical record must be ledger-compatible, otherwise it cannot be normalised as a terminal record in the VERSF record algebra; this requires the return branch. Both branches are prior to emergent physical time, so no branch may be assigned a preferred primitive stiffness without inserting an unlicensed pre-temporal asymmetry (AD-7). Physical-sector positivity forbids assigning negative load to either branch (AD-5). Hence both branches are positive and equal; formally, invariance of the terminal branch action under the branch-swap involution $\sigma_{br}$ forces the same conclusion (§8.2′):

$g_{down} = g_{return}$. ■

## 8.5 Lemma 5 — No double counting of return-normalisation

The sharpest available objection to the factor of 2 is that obligation R already sits inside the charge $\mathfrak{C} = 4$, so counting a return branch counts return twice. This lemma discharges that objection.

**Lemma 5.** Obligation R contributes exactly once, to the commitment charge; the terminality factor 2 contributes exactly once, as a count of transport branches. The two contributions are of different type, and neither duplicates the other.

**Proof.** Obligation-level R is a **predicate on the commitment**: the commitment must be ledger-compatible. It is one of the four conditions a commitment must satisfy to be record-forming at all, and it therefore contributes one unit to $\mathfrak{C} = 4$ (Lemma 1).

Branch-level return is a **channel of the descent map**: the direction along which realized load is carried back into the substrate ledger. It is not an obligation; it is one of the two transport branches along which the whole charge must be sustained.

The distinction is exhibited by what each branch carries. The return branch does not carry obligation R alone; it carries the full four-obligation charge, exactly as the descent branch does. A record that is merely ledger-compatible in specification (R satisfied) but whose realized support carries no return load is a detached fragment — compatible in principle, unreconciled in fact. Conversely, a return branch carrying only R-load would return an occurrence-free, distinction-free residue, which is not a normalisable record. So each branch transports (E, D, C, R) in full; neither branch is "the R branch."

If R were the doubling, the return branch would carry one unit against the descent branch's four, and the stiffness would be some hybrid such as $(4 + 1)/3 = 5/3$. It is not: both branches carry $\mathfrak{C}/d_{phys} = 4/3$, and the total is $8/3$.

Operationally, the two counts break under different falsifiers, which is the test that they are distinct: removing obligation R from the tetrad changes $\mathfrak{C}$ from 4 to 3 and destroys the *reason* a return branch exists (AD-2 breaks, F1/F2); removing the return branch changes the branch count from 2 to 1 and destroys the *realization* of R (AD-7 breaks, F8/F9). A single double-counted quantity could not fail in two independent ways. R therefore motivates AD-7 without being absorbed into it: the premises are related, as premises about the same record structure must be, but they are not the same premise. Under AD-1 – AD-8,

$\kappa\star = 2 \cdot (\mathfrak{C}/d_{phys}) = 2 \cdot (4/3) = 8/3$,

with each factor counted once. ■

## 8.6 Lemma 6 — Two-sided stiffness

**Lemma 6.** The two-sided localization stiffness generated by substrate anomaly descent is

$\kappa_{AD} = 8/3$.

**Proof.** By Lemma 3, $g_{AD} = 4/3$. By Lemma 4, terminality requires two equal branches, and branch loads add under log-attenuation composition (§8.3′). Therefore

$\kappa_{AD} = g_{down} + g_{return} = 2\cdot g_{AD} = 2\cdot(4/3) = 8/3$. ■

## 8.7 Interpretation — the anatomy of 8/3

The number 8/3 is not a convention choice. It is the two-sided cost of turning a finite substrate commitment into a terminal physical record, and every factor in it is structural:

$\kappa\star = (2 \times 4)/3$.

- The **4** is the minimal commitment tetrad (occurrence, distinction, continuation, return-normalisation).
- The **2** is bidirectional terminality (descent plus return).
- The **3** is the physical support rank.

Nothing in the numerator or denominator is empirical, fitted, or approximate.

## 8.8 Lemma 7 — Channel-to-exponent bridge

One step remains before the derived quantity may be called κ⋆. Lemma 6 delivers a per-direction two-sided load; the canonical convention I3 defines κ⋆ as the exponent of a radial profile. Identifying the two silently would be exactly the unstated slot-assignment that QN-1's typing discipline exists to forbid. The identification is therefore stated as premise AD-8 and proved as a bridge lemma.

**Lemma 7.** Under AD-6 and AD-8, the canonical amplitude-level radial exponent equals the two-sided per-direction load:

$\kappa\star = \kappa_{AD} = 8/3$.

**Proof.** By Lemma 6, each physical support direction carries the two-sided load $\kappa_{AD} = 8/3$. By AD-8, the two-sided load carried by a direction is a rate of amplitude log-attenuation per unit normalized displacement along that direction — never an aggregate of loads across channels. By isotropy (AD-6, Lemma 2), that rate is common to every direction, so the decay rate along any ray — in particular along the radial variable $u = \rho/\xi$ of the canonical convention I3 — is the single-direction load. Hence the coefficient in $\psi(u) = \psi(0)\cdot e^{(-\kappa\star\cdot u)}$ is $\kappa\star = \kappa_{AD} = 8/3$. ■

The operator formulation of §6.6 makes the bridge concrete. The radial variable $u = \rho/\xi$ is a one-dimensional support-distance coordinate along a normalized ray $n \in \mathcal{P}$. It is not the coordinate sum $u_1 + u_2 + u_3$, and it does not represent simultaneous independent traversal of the three support channels. A radial displacement selects a unit direction, and the two-sided descent operator

$$K\_AD = 2\cdot A\_AD = (8/3)\cdot \mathbb{1}\_\mathcal{P}$$

assigns that direction the stiffness

$$n^T\cdot K\_AD\cdot n = 2\cdot(4/3) = 8/3.$$

The canonical radial exponent therefore reads the isotropic eigenvalue of the two-sided support operator, never its trace. Read this way, AD-8 does the minimal work left to a premise: it fixes that the radial convention of I3 probes the operator as a one-dimensional path must — one direction at a time, eigenvalue rather than trace — and F16 prices exactly the denial of that reading.

Two typing clauses complete the assignment. First, the descent derivation fixes the **value** of the exponent; the QN-1 convention I3 fixes its **level** — amplitude rather than probability. Neither is chosen to make the other land: the value is forced by AD-1 – AD-8, and the level is a QN-1 convention fixed before this paper existed. The probability-level exponent then follows by the QN-1 rule $\kappa\_prob = 2\kappa\star$. Second, AD-8 is a substantive, falsifiable premise (F16), not a notational identification: deny it, and the derivation still yields a per-direction load of 8/3, but the claim that this load is $\kappa\star$ lapses.

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# 9. The Main Theorem

## 9.1 Statement

**Theorem 1 — Substrate Anomaly-Descent Theorem.** Let VERSF physical record formation satisfy premises AD-1 – AD-8:

1. a terminal record descends from a finite-substrate commitment (AD-1);
2. the minimal substrate commitment has four primitive obligations — occurrence, distinction, continuation, and return-normalisation (AD-2);
3. the adjoint-removed physical support quotient has three independent support directions (AD-3);
4. descent conserves the total commitment charge (AD-4);
5. descended loads are non-negative in every channel (AD-5);
6. the descent is isotropic and least-distorting (AD-6);
7. terminal record stability is bidirectional, with equal descent and return-normalisation branches (AD-7);

8. the amplitude-level exponent along any support direction equals the two-sided load carried by that direction (AD-8);

and let the canonical localization convention be the amplitude-level two-sided support convention

$\psi(u) = \psi(0)\cdot e^{(-\kappa\star\cdot u)}$, $u = \rho/\xi$.

Then the canonical localization stiffness is uniquely

**$\kappa\star = 8/3$.**

Moreover, the one-sided gap coefficient is uniquely

**$g_\kappa = 4/3$,**

and the corresponding inverse-length gap scale is

**$m\star = \sqrt{(4/3)}\cdot\xi^{-1}$.**

## 9.2 Proof

By AD-2 and Lemma 1, the finite substrate commitment carries four primitive obligations, so its commitment charge is $\mathfrak{C} = 4$. By AD-3 (Structural Fact, §5.3), the commitment charge must descend into three independent support directions, $d_{phys} = 3$. By AD-4, AD-5, AD-6 and Lemma 2, the descent is the unique positive, isotropic, least-distorting distribution, giving the one-sided branch coefficient

$g_{AD} = \mathfrak{C}/d_{phys} = 4/3$

(Lemma 3). By AD-7 and Lemma 4, terminal record stability requires two equal branches — descent into physical support and return-normalisation into the substrate ledger. Therefore, by Lemma 6, each support direction carries the two-sided load

$\kappa_{AD} = 2\cdot g_{AD} = 8/3$,

with no double counting of obligation R (Lemma 5). By AD-8 and Lemma 7, this per-direction two-sided load is the exponent of the canonical radial convention, so the derived quantity is $\kappa\star$ itself:

$\kappa\star = 8/3$.

The one-sided coefficient is $g_\kappa = g_{AD} = 4/3$, and the inverse-length gap scale is

$m\star = \sqrt{(g_\kappa)}\cdot\xi^{-1} = \sqrt{(4/3)}\cdot\xi^{-1}$.

Uniqueness at each step is inherited from Lemma 2 (unique descent vector) and Lemma 4 (forced branch equality); the slot assignment is Lemma 7 (AD-8). ■

# 9.3 Corollary 1 — 8/3 is not fitted

The coefficient 8/3 is determined entirely by structural integers:

$\kappa\star = (2\cdot r\_sub)/d\_phys = (2\cdot 4)/3$.

No empirical mass, coupling, channel count, or hierarchy value enters the derivation at any point.

# 9.4 Corollary 2 — 4/3 is primary, not decorative

The coefficient 4/3 is not obtained by halving 8/3. It is the primary one-sided anomaly-descent load, and the canonical localization stiffness is its two-sided completion, $\kappa\star = 2g\_\kappa$. This reverses the explanatory direction of the inherited formulation: the gap coefficient now has an origin, and the stiffness inherits it.

# 9.5 Corollary 3 — 3/8 is compliance against a derived stiffness

Since $\kappa\star = 8/3$, the reciprocal compliance coefficient is

$c\star = 1/\kappa\star = 3/8$.

The compliance coefficient thereby inherits its origin from the anomaly-descent derivation of $\kappa\star$; it is not an independent constant.

# 9.6 Remark — The quadratic dual identity

For the quadratic distortion functional, the Lagrange multiplier enforcing charge conservation is

$\lambda = 2\cdot(\mathfrak{C}/d\_phys) = 8/3$.

This equality is algebraic, not corroborative: quadratic stationarity gives $a_i = \lambda/2$, so $\lambda = 2\cdot(\mathfrak{C}/d)$ identically, for any charge and any rank. The dual reproduces the two-sided value by construction, and it carries no independent-confirmation weight. For other admissible costs the primal minimiser is unchanged (Appendix C) but the multiplier is not — for $\Phi(a) = \Sigma_i a_i^4$ it is 256/27. The dual value is cost-dependent bookkeeping; what is cost-independent, and what the theorem uses, is the primal load 4/3 (Proposition C.1). The factor of 2 in $\kappa\star$ is supplied by bidirectional terminality (AD-7, Lemma 4), not by duality.

# 9.7 The general anomaly-descent law and counterfactuals

The QN-3 result is the special case of a general descent law, and stating the general law makes the dependency structure fully explicit. Let r_sub be the primitive substrate obligation rank, d_phys the physical support rank, and b the number of terminality branches. Under positive, isotropic, charge-conserving descent, the one-branch load is

$$g = r_{sub}/d_{phys},$$

and the total localization stiffness is

$$\kappa = b\cdot g = b\cdot r_{sub}/d_{phys}.$$

VERSF terminal record formation has (b, r_sub, d_phys) = (2, 4, 3), hence

$$\kappa\star = (2\cdot 4)/3 = 8/3.$$

The value 8/3 enjoys no protection if the premise values change:

| **Changed premise triple (b, r_sub, d_phys)** | **Resulting κ** |
|---|---|
| (2, 3, 3) — three obligations | 2 |
| (2, 5, 3) — five obligations | 10/3 |
| (2, 4, 4) — four support directions | 2 |
| (1, 4, 3) — one-sided terminality | 4/3 |
| unequal branches | not fixed by the law |

The law is a ratio and is therefore not injective in its arguments: the rows (2, 3, 3) and (2, 4, 4) both give $\kappa = 2$. Output degeneracy under distinct counterfactuals is an expected property of a ratio law and carries no significance; the falsifiers act on the premise values, not on the quotient.

Each counterfactual row is the quantitative face of a falsifier already in §13: F1/F2 move the numerator, F3 moves the denominator, F8/F9 move the branch factor. A fitted constant would survive such changes; a derived one cannot.

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# 10. Recovery of the QN-1 κ-Ledger

## 10.1 Canonical stiffness

QN-1 assigned $\kappa\star = 8/3$ as the canonical amplitude-level localization stiffness. QN-3 derives it:

$$\kappa\star = 2\cdot(4/3) = 8/3.$$

## 10.2 Probability exponent

QN-1 established that the probability-density exponent for the same amplitude profile is $\kappa_{prob} = 2\kappa\star$. Therefore

$\kappa_{prob} = 2\cdot(8/3) = 16/3$.

Two independent factors of 2 now sit in the ledger and must not be conflated. The first, $g_{AD} \to \kappa\star$, is terminality: descent plus return (AD-7). The second, $\kappa\star \to \kappa_{prob}$, is the amplitude-to-probability squaring of the profile (QN-1 convention). That $16/3 = 4\cdot(4/3)$ is a composition of these two distinct doublings; it is not a direct map from the four obligations to the probability exponent, and no such map is claimed.

## 10.3 Gap coefficient

QN-1 used $g_\kappa = \kappa\star/2$. QN-3 explains why this halving is lawful: the half is the one-sided branch,

$g_\kappa = g_{AD} = 4/3 = (1/2)\cdot(8/3)$.

## 10.4 Gap scale

QN-1 used $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$. QN-3 recovers it directly from the one-sided descent coefficient:

$m\star = \sqrt{(g_{AD})}\cdot\xi^{-1} = \sqrt{(4/3)}\cdot\xi^{-1}$.

## 10.5 Compliance coefficient

QN-1 typed $c\star = 3/8$ as reciprocal compliance. QN-3 supplies the stiffness origin:

$c\star = 1/\kappa\star = 1/(8/3) = 3/8$.

## 10.6 Channel-log separation remains intact

QN-3 does not identify ln 14 with 8/3. The derivation here uses $2 \times 4/3$, not ln 14. The QN-1 separation therefore stands:

$\lambda_{14} = \ln 14$

remains a channel-log weight, not the origin of $\kappa\star$. A future bridge may relate the 14-channel census to anomaly descent, but that relation is neither required nor claimed here.

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# 11. Why This Is Not Numerology

## 11.1 The obvious criticism

A critic might say:

You wanted 8/3, so you invented a 4, a 3, and a factor of 2.

This criticism would be valid if those numbers were free. They are not.

- The **4** is the rank of the minimal record-commitment tetrad — a count of record obligations proved necessary, mutually independent, and jointly sufficient (AD-2, Lemma 1), each with its own falsifier.
- The **3** is the rank of the adjoint-removed physical support quotient — an inherited physical-sector structure, not a choice made in this paper (AD-3).
- The **2** is bidirectional terminality — descent plus return, forced by the definition of a terminal record and by the absence of a pre-temporal arrow (AD-7).

Thus $8/3 = (2 \times 4)/3$ is not a decorative arithmetic identity. It is a descent stiffness whose every factor is independently anchored and independently breakable.

## 11.2 The single standard

QN-1 insisted that numerical closeness is not a mechanism. QN-3 applies the same standard to itself. The derivation of 8/3 does not rely on closeness to another number. It does not use $\ln 14 \approx 8/3$. It does not use mass ratios. It does not select a rational value for convenience. It supplies what the QN-1 bridge standard demands:

1. a dimensional setting (dimensionless amplitude-level stiffness in units of $u = \rho/\xi$);
2. a domain of validity (the adjoint-removed physical quotient under AD-1 – AD-8);
3. a mechanism (charge-conserving positive isotropic descent, doubled by terminality);
4. an exact result (8/3, with no error term).

## 11.3 The role of 4/3

The appearance of 4/3 is decisive. In weaker versions of the programme, one could only say $4/3 = (8/3)/2$ — true, but explanatorily empty. QN-3 strengthens the chain by deriving 4/3 **first**, as the one-sided anomaly-descent load, and then obtaining $\kappa\star = 2g\_AD$. The half-stiffness is not a consequence of notation. It is the primary quantity.

## 11.4 Why ln 14 is not used

The value $\ln 14 \approx 2.639057$ is close to $8/3 \approx 2.666667$. QN-3 makes no use of that closeness. This matters: were 8/3 justified by proximity to ln 14, the mismatch

$$8/3 - \ln 14 \approx 0.027609$$

would survive as a hidden approximation inside every downstream exponent. QN-3 avoids that entirely. The descent derivation gives exactly 8/3, and the ln 14 relation remains a separate, honestly open bridge question.

## 11.5 Robustness of the descent rule

A subtler numerological worry is that the quadratic distortion functional in Lemma 2 was chosen to produce the uniform answer. Appendix C removes this worry: on the constraint simplex, the uniform vector (4/3, 4/3, 4/3) is the unique minimiser of every strictly convex permutation-symmetric functional. The result depends on symmetry and convexity — that is, on AD-6 — not on the specific cost.

## 11.6 The falsifiability of the result

QN-3 is not unfalsifiable. It has sharp dependencies. If the minimal commitment rank is not four, the result changes. If the physical support rank is not three, the result changes. If descent is anisotropic, or lossy, or one-sided, or branch-asymmetric, the result changes. The coefficient 8/3 is not protected by notation. It stands or falls with premises AD-1 – AD-8, and §13 lists the breakage modes explicitly. That is what makes the theorem meaningful.

## 11.7 Why the four is not a hidden fit

The strongest available accusation against QN-3 is that the integer four has been selected because it leads to 8/3. The record-obligation theorem blocks that accusation.

The four is fixed before the descent calculation begins. It is the number of primitive failure modes that must be prevented for terminal recordhood: nullity, indistinguishability, evanescence, and detachment (§4.2′, Lemma 1). Only after this failure basis is established does the descent calculation begin, distributing the already-fixed charge $\mathfrak{C} = 4$ across the already-fixed physical support rank $d_{phys} = 3$. The result

$$g_{AD} = 4/3$$

therefore follows from record minimality and physical support rank, not from target-fitting. The corresponding falsifier is exact: exhibit a fifth independent primitive failure mode, or reduce the four to three, and the numerator — and with it 8/3 — changes.

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# 12. Relation to ln 14, Gap Scales, and Later Gates

## 12.1 Relation to ln 14

The paired-channel census gives $2K = 14$ and hence the channel-log weight $\lambda_{14} = \ln 14$. QN-3 does not derive $\lambda_{14}$ and does not equate it with $\kappa\star$. Any later connection must show how the channel census couples to the anomaly-descent structure; that remains a separate bridge theorem with its own burden of mechanism.

## 12.2 Relation to the gap scale

QN-3 strengthens the gap-scale chain. In the inherited formulation, $m\star^2 = (\kappa\star/2)\cdot\xi^{-2}$; in the descent formulation,

$m\star^2 = g\_AD\cdot\xi^{-2}$, $g\_AD = 4/3$, $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$.

The inverse-length gap is the physical imprint of one-sided anomaly descent.

## 12.3 Relation to mass-hierarchy gates

Mass-hierarchy and Yukawa gates are exponent-sensitive: mass ratios, suppression cascades, and localization overlaps depend strongly on exponent normalisation. QN-3 supplies the first-principles source for the coefficient those gates consume. The inheritance clause for later papers is:

The canonical amplitude-level localization stiffness is $\kappa\star = 8/3$, derived by QN-3 as the two-sided anomaly-descent stiffness $2\cdot(4/3)$ from the minimal four-obligation substrate commitment into the three-direction physical support quotient.

## 12.4 Relation to gravitational response

QN-3 does not derive $\kappa\_eff$. A gravitational-response coupling is dimensional and belongs to a source–response equation; the anomaly-descent stiffness is dimensionless and belongs to localization attenuation. A future gravitational bridge must still supply nondimensionalisation, a domain of validity, a mechanism, and error control or exact equality. QN-3 does not bypass that requirement.

## 12.5 Relation to the Standard Model derivation programme

QN-3 advances the Standard Model derivation programme indirectly but importantly. It derives no particle mass, mixing, or coupling. What it does is stabilise the exponent foundation on which those derivations stand. Before QN-3, a critic could ask: "You use 8/3 — where does it come from?" After QN-3, the answer is: it is the two-sided localization stiffness generated by finite-substrate anomaly descent, with named premises and explicit falsifiers.

# 13. Falsification Conditions

| # | Failure | Consequence |
|---|---|---|
| F1 | Minimal record commitment does not require four primitive obligations | $\mathfrak{C} = 4$ fails; AD-2 breaks |
| F2 | The four obligations are not independent | Commitment tetrad rank reduces; AD-2 breaks |
| F3 | The physical support quotient is not rank three | Denominator 3 fails; AD-3 breaks |
| F4 | Descent need not preserve total commitment charge | Load conservation fails; AD-4 breaks |
| F5 | Physical-sector descent permits negative load | Positivity fails; AD-5 breaks |
| F6 | Descent is anisotropic before record realization | Isotropic 4/3 result fails; AD-6 breaks |
| F7 | Least-distorting descent is not the correct physical rule | Variational derivation fails; AD-6 breaks (but see Appendix C for the weakened requirement actually needed) |
| F8 | Terminal record formation is one-sided only | Factor of 2 fails; AD-7 breaks |
| F9 | Descent and return branches may carry unequal primitive stiffness | $\kappa\star = 2g_{AD}$ fails; AD-7 breaks |
| F10 | 4/3 is used downstream without anomaly-descent origin | QN-3 inheritance is being ignored |
| F11 | 8/3 is treated as ln 14 | QN-1/QN-3 separation fails |
| F12 | 3/8 is used as stiffness rather than reciprocal compliance | QN-1 typing fails |
| F13 | $m\star$ is treated as dimensionless | Gap-scale typing fails |
| F14 | A later hierarchy gate changes $\kappa\star$ without changing the anomaly-descent premises | Parameter drift reappears |
| F15 | An observable requires a different canonical stiffness under the same premises | QN-3 is falsified or incomplete |
| F16 | The radial amplitude exponent does not equal the per-direction two-sided load | Channel-to-exponent bridge fails; AD-8 breaks |

These falsifiers are deliberate. A genuine derivation must be breakable, and it must say where.

# 14. QN-3 Closure Certificate

| QN-3 condition | Result |
|---|---|
| QN-1 κ-ledger inherited | Closed |
| Open origin debt identified | Closed |
| Named premise set AD-1 – AD-8 stated with falsifiers | Closed |
| Substrate anomaly defined | Closed |
| Minimal commitment tetrad defined | Closed |
| Four primitive record-failure modes identified | Closed (§4.2′) |
| Necessity of the four obligations proved | Closed (Lemma 1) |
| Independence of the four obligations proved | Closed (Lemma 1) |
| Sufficiency of the four obligations proved | Closed (Lemma 1) |
| Exactness of r_sub = 4 established | Closed, conditional on the terminal-record definition |
| Non-circularity of the tetrad count addressed | Closed (§4.5) |
| Rank-four substrate obligation justified | Closed, conditional on record-obligation minimality: necessity, independence, and joint sufficiency of the four terminal-record obligations (AD-2, Lemma 1) |
| Rank-three physical support quotient stated | Closed, inherited physical-sector structure (AD-3) |
| Positive descent requirement stated | Closed |
| Isotropic descent requirement stated | Closed |
| Least-distorting descent rule supplied | Closed |
| Cost-independence of uniform descent proved | Closed (Appendix C) |
| Uniform descent load derived | Closed |
| One-sided coefficient 4/3 derived | Closed |
| Bidirectional terminality stated | Closed |
| Equal-branch condition stated | Closed |
| Two-sided stiffness 8/3 derived | Closed |
| Quadratic dual identity registered with correct (algebraic) weight | Closed (Appendix B) |
| Return double-counting objection discharged | Closed (Lemma 5) |
| Channel-to-exponent bridge stated and proved | Closed (AD-8, Lemma 7) |
| Law-invariance / outcome-invariance loophole closed | Closed (§6.3) |
| Descent load formulated as positive support operator | Closed (§6.6) |

| QN-3 condition | Result |
|---|---|
| Trace / eigenvalue distinction stated | Closed (§6.6) |
| One-sided 4/3 identified as isotropic eigenvalue, not total charge | Closed (§6.6) |
| Continuous vs discrete isotropy distinguished; channel-diagonal loophole closed | Closed (AD-6, §6.6) |
| Branch-load addition derived from log-attenuation composition | Closed (§8.3′) |
| AD-8 stated as a ray-rate premise, not a channel-aggregate | Closed (AD-8, Lemma 7) |
| Radial variable typed as one-dimensional ray coordinate; exponent reads eigenvalue of two-sided operator, not trace | Closed (§8.8) |
| Equal-branch condition derived from branch-swap invariance | Closed (§8.2′) |
| Full-charge descent argued (no under- or over-descent) | Closed (§3.6) |
| Fifth-obligation challenge posed as standing test | Closed (§4.6) |
| Pre-record vs final-state isotropy distinguished | Closed (§6.2) |
| General law κ = b·r_sub/d_phys with counterfactual table | Closed (§9.7) |
| Quadratic representative derived as second-order normal form (shape forced within the nondegenerate-second-variation subclass; scale by canonical unit normalization) | Closed (Appendix B) |
| Anomaly / commitment-charge terminology separated | Closed (§3.1) |
| Gap scale $m\star = \sqrt{(4/3)}\cdot\xi^{-1}$ recovered | Closed |
| Compliance coefficient 3/8 recovered | Closed |
| Probability exponent 16/3 recovered | Closed |
| ln 14 separation preserved | Closed |
| Gravitational κ_eff kept separate | Closed |
| Full mass/coupling derivation | Outside scope |
| Empirical validation | Outside scope |

**QN-3 closure grade:** closed as a substrate anomaly-descent origin theorem, conditional on the VERSF premises that terminal records require a four-obligation substrate commitment (AD-2), that physical support has rank three (AD-3), that anomaly descent is charge-conserving, positive, and isotropic (AD-4 – AD-6), that terminality is bidirectional with equal branches (AD-7), and that the amplitude exponent along any direction is that direction's two-sided load (AD-8).

The strongest supportable closure claim is therefore: QN-3 closes the first-principles origin of $\kappa\star = 8/3$ within VERSF by deriving it from record-obligation minimality, charge-conserving isotropic descent, bidirectional terminality, and the channel–exponent bridge. The result remains conditional on AD-1 – AD-8, with the numerator 4, the denominator 3, and the doubling factor 2 each independently anchored and independently falsifiable.

In summary: the numerator 4 is established by record-obligation minimality (Lemma 1); the denominator 3 is the inherited physical support rank (AD-3); the one-sided coefficient 4/3 is the isotropic eigenvalue of the descent operator whose trace is the conserved charge (§6.6); and the doubling factor 2 is forced by pre-temporal branch reciprocity and realized as log-attenuation composition (§8.2′, §8.3′).

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# 15. Conclusion

QN-1 solved the κ-drift problem. It showed that VERSF's κ-like quantities must be typed and that 8/3, 3/8, ln 14, $m\star$, and $\kappa_{eff}$ are not interchangeable. But it left a deeper question open: why is the canonical localization stiffness 8/3?

QN-3 answers that question.

The result comes from finite-substrate anomaly descent. A terminal record requires four primitive substrate obligations — occurrence, distinction, continuation, and return-normalisation. The physical support quotient has three independent support directions. Positive, isotropic, charge-conserving descent therefore distributes four units of commitment charge across three support channels, giving the one-sided branch coefficient

$g_{AD} = 4/3$.

A terminal record is not one-sided. It must descend into physical support and remain return-compatible with the substrate ledger, and the two branches are equal by pre-temporal reciprocity and positivity. Therefore the canonical amplitude-level localization stiffness is

$\kappa\star = 2\cdot g_{AD} = 2\cdot(4/3) = 8/3$.

This derives the coefficient that QN-1 had to inherit. It also explains why the gap coefficient is 4/3, why the inverse-length scale is $\sqrt{(4/3)}\cdot\xi^{-1}$, why the reciprocal compliance coefficient is 3/8, and why the probability-level exponent is 16/3. The uniform-descent step is proved robust against the choice of distortion functional, the return branch is shown not to double-count obligation R, and the quadratic dual identity is registered with its correct, purely algebraic weight.

The theorem does not use ln 14. It does not fit a mass. It does not rely on numerical proximity. It derives the canonical stiffness from structural descent, under eight named premises, each with an explicit falsifier.

In one sentence:

**The canonical VERSF localization stiffness 8/3 is the two-sided anomaly-descent residue of a four-obligation finite substrate commitment distributed across the three-direction physical support quotient.**

---

# Appendix A — Minimal Algebraic Skeleton

Let the minimal substrate commitment module be

$\mathcal{K} = \langle E, D, C, R \rangle$, $\mathrm{rank}(\mathcal{K}) = 4$,

and the physical support quotient be

$\mathcal{P}$, $\mathrm{rank}(\mathcal{P}) = 3$.

Define the substrate commitment charge

$\mathfrak{C} = \mathrm{rank}(\mathcal{K}) = 4$.

Let the descent vector be

$a = (a_1, a_2, a_3)$, $a_i \geq 0$, $a_1 + a_2 + a_3 = 4$.

Isotropy ($S_3$-invariance) forces

$a_1 = a_2 = a_3 = 4/3$,

so the one-sided coefficient is

$g\_AD = 4/3$.

Terminality requires two equal branches,

$g\_down = g\_return = g\_AD$,

hence

$\kappa\star = g\_down + g\_return = 2 \cdot g\_AD = 8/3$.

The derived ledger follows:

$m\star = \sqrt{(g\_AD)}\cdot\xi^{-1} = \sqrt{(4/3)}\cdot\xi^{-1}$, $c\star = 1/\kappa\star = 3/8$, $\kappa\_prob = 2\cdot\kappa\star = 16/3$.

By AD-8, the per-direction two-sided load occupies the canonical radial slot:

$\psi(u) = \psi(0)\cdot e^{(-\kappa\star\cdot u)}$, $u = \rho/\xi$, $\kappa\star = 8/3$.

---

# Appendix B — Variational Derivation and the Quadratic Dual Identity

Appendix C shows that the uniform descent vector is independent of the particular strictly convex symmetric distortion functional. This appendix has a narrower purpose: it asks what the dual stiffness is when the descent cost is represented by the canonical quadratic form.

The distinction is stated up front. The uniform load $a_i = 4/3$ is symmetry-robust. The multiplier value $\lambda = 8/3$ is the dual stiffness of the quadratic representative, not of every admissible convex cost. The quadratic is nevertheless the correct representative for the *stiffness* question, and this can be argued rather than asserted. Stiffness is a second-variation object: the Hessian-level cost of displacing the support profile away from the least-distorting descent. Take any admissible cost $\Phi$ — strictly convex and permutation-symmetric — with nondegenerate second variation at the optimum, and expand it to second order about the uniform minimiser $a\bigstar$. The Hessian of a symmetric $\Phi$ at the symmetric point is itself $S_3$-invariant, hence of the form $h\cdot\mathbb{1} + k\cdot(J - \mathbb{1})$; restricted to the admissible variation space — charge-conserving displacements with $\Sigma_i\, \delta a_i = 0$ — the all-ones component annihilates every variation, and the restricted Hessian is $(h - k)\cdot\mathbb{1}$, a strictly positive multiple of the identity by the nondegeneracy assumption. Strict convexity alone guarantees only $h - k \geq 0$: the admissible cost $\Phi(a) = \Sigma_i\, (a_i - 4/3)^4$ is strictly convex and permutation-symmetric yet has identically vanishing Hessian at $a\bigstar$, and such degenerate members define no second-variation stiffness at all. Every admissible cost with nondegenerate second variation therefore has the same second-order normal form on the constraint surface: the quadratic, up to positive scale. The functional $\mathcal{D}(a) = \Sigma_i\, a_i^2$ is that normal form under the canonical unit normalization; within the nondegenerate subclass the shape is forced, the scale is the canonical convention, and the multiplier quoted below is the dual stiffness under that normalization. The quadratic is therefore not chosen to force uniformity — uniformity is already forced by symmetry (Proposition C.1) — and it is not chosen to force 8/3.

The isotropic descent result can be expressed as a constrained minimisation. Let $a_i$ be the descended charge load in physical support direction i. The admissible set is the simplex

$\mathcal{A} = \{ a \in \mathbb{R}^3 : a_i \geq 0,\ a_1 + a_2 + a_3 = 4 \}$.

Select the least-distorting positive descent by minimising

$\mathcal{D}(a) = a_1^2 + a_2^2 + a_3^2$.

The Lagrangian is

$\mathcal{L}(a, \lambda) = \Sigma_i a_i^2 - \lambda\cdot(\Sigma_i a_i - 4)$.

Stationarity gives

$\partial\mathcal{L}/\partial a_i = 2a_i - \lambda = 0 \Longrightarrow a_i = \lambda/2$ for every i.

Applying the constraint,

$3\cdot(\lambda/2) = 4 \Longrightarrow \lambda = 8/3$, $a_i = 4/3$.

The primal descent load is $g_{AD} = 4/3$, and the multiplier enforcing charge conservation is $\lambda = 8/3$ — numerically the two-sided stiffness. This coincidence must be typed correctly, and its correct type is algebraic, not corroborative.

For the quadratic cost, stationarity gives $a_i = \lambda/2$ for every i, hence

$\lambda = 2\cdot(\mathfrak{C}/d_{phys})$

identically — for any charge and any rank. The quadratic dual reproduces the two-sided value by construction; what it adds is the canonical linear-response normalisation of the constraint (see the preamble above), not an independent derivation of 8/3.

The multiplier has a clean envelope reading. For the quadratic representative, the optimised descent cost is

$\mathcal{D}^\star(\mathfrak{C}) = 3\cdot(\mathfrak{C}/3)^2 = \mathfrak{C}^2/3$,

so the marginal optimised cost per unit of conserved charge is

$d\mathcal{D}^\star/d\mathfrak{C} = 2\mathfrak{C}/3$,

and at the physical charge,

$\lambda = (d\mathcal{D}^\star/d\mathfrak{C})|_{\mathfrak{C}=4} = 8/3$.

Within the quadratic representative, the canonical stiffness therefore appears in two complementary ways: as the sum of the descent and return branch loads, and as the marginal optimised descent cost of charge conservation. The complementarity is representative-specific — it certifies that the quadratic normalisation is internally coherent; it does not independently rederive 8/3.

The dependence on the cost makes this explicit. For a different strictly convex symmetric cost the primal minimiser is unchanged (Proposition C.1) but the multiplier is not. With $\Phi(a) = \Sigma_i a_i^4$, stationarity gives $4a_i^3 = \lambda$; the minimiser is still (4/3, 4/3, 4/3), and

$$\lambda = 4\cdot(4/3)^3 = 256/27 \neq 8/3.$$

The correct division of labour is therefore: Appendix C establishes that the primal load 4/3 is cost-independent and is the quantity the theorem consumes; this appendix establishes that the quadratic dual equals 2·(4/3) as a bookkeeping identity. The factor of 2 in $\kappa\star$ is supplied by bidirectional terminality (AD-7, Lemma 4), not by duality, and no reading of the dual multiplier as independent corroboration of 8/3 is admissible.

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# Appendix C — Robustness of Uniform Descent

Lemma 2 used the quadratic distortion functional. This appendix shows the conclusion does not depend on that choice.

**Proposition C.1.** Let $\Phi : \mathbb{R}^3 \to \mathbb{R}$ be any permutation-symmetric, strictly convex functional, and let

$$\mathcal{A} = \{ a \in \mathbb{R}^3 : a_i \geq 0, \Sigma_i a_i = 4 \}.$$

Then the unique minimiser of $\Phi$ on $\mathcal{A}$ is the uniform vector $a\bigstar = (4/3, 4/3, 4/3)$.

**Proof.** Let $a \in \mathcal{A}$ be any point and let $\sigma$ range over the six permutations in $S_3$. Each permuted vector $\sigma(a)$ lies in $\mathcal{A}$, and by symmetry $\Phi(\sigma(a)) = \Phi(a)$. The average

$$\bar{a} = (1/6)\cdot\Sigma\_\sigma\ \sigma(a) = (4/3, 4/3, 4/3) = a\bigstar$$

also lies in $\mathcal{A}$ by convexity of the simplex. By strict convexity of $\Phi$,

$$\Phi(a\bigstar) = \Phi(\bar{a}) \leq (1/6)\cdot\Sigma\_\sigma\ \Phi(\sigma(a)) = \Phi(a),$$

with equality only if all $\sigma(a)$ coincide — that is, only if $a = a\bigstar$. Hence $a\bigstar$ is the unique minimiser. ■

**Consequence.** The uniform descent load 4/3 is forced by symmetry (AD-6) and strict convexity alone. The quadratic in Lemma 2 and Appendix B is a representative, not an assumption doing hidden work. Falsifier F7 should therefore be read precisely: what must fail for the result to fail is not the quadratic choice, but the pair ⟨permutation symmetry, strict convexity⟩ of the physical descent cost. The dual multiplier, by contrast, is cost-dependent (Appendix B): across the admissible cost class the primal minimiser is invariant and the multiplier is not, so only the primal load carries theorem-level weight.

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# Appendix D — Referee Objections and Replies

## Objection 1 — You have just assumed the number four.

**Reply.** The number four is not a numerical target. It is the rank of the minimal record-commitment tetrad: occurrence, distinction, continuation, return-normalisation. The theorem is conditional on those obligations being primitive and independent (AD-2), and Lemma 1 proves their necessity, mutual independence, and joint sufficiency explicitly. If a critic can reduce the tetrad without destroying terminal record formation, QN-3 fails at F1/F2. That is an explicit falsifier, not a protected assumption.

## Objection 2 — You have shown four obligations are necessary, but not that they are sufficient.

**Reply.** Lemma 1 proves sufficiency explicitly. Occurrence blocks nullity; distinction blocks indistinguishability; continuation blocks evanescence; return-normalisation blocks detachment from the global ledger. Once all four primitive failure modes are blocked, the candidate consequence is non-null, identifiable, finitely supported, and ledger-admissible — precisely minimal terminal recordhood. A fifth primitive obligation would be required only if there were a fifth independent primitive failure mode not reducible to these four, and AD-2's falsifier makes exactly that an explicit break condition.

## Objection 3 — Is the commitment tetrad just the emergent 4-manifold in disguise?

**Reply.** No. The tetrad is not three support directions plus time. It is a pre-physical record-formation structure, defined before any emergent geometric or temporal structure exists. Time remains emergent in VERSF. The four obligations are E, D, C, R — not four coordinates of the emergent continuum. The numerical coincidence with the continuum dimension is exactly that: a coincidence of counts with entirely different origins, and the derivation never uses the continuum count.

## Objection 4 — Why divide by three rather than subtract three?

**Reply.** Because descent conserves the commitment charge (AD-4). All four obligations must be represented through the three available physical support channels; the load is the full charge distributed, 4/3 per channel, not the rank deficit $4 - 3 = 1$. The subtraction reading would make the stiffness insensitive to the tetrad structure, contradicting record discipline (I6), under which every obligation is individually required for terminality.

## Objection 5 — If each of the three channels carries 4/3, why is the one-sided coefficient not the total, 4?

**Reply.** Because the conserved total is the trace of the descent operator, while the coefficient entering a normalized support-direction localization law is its isotropic eigenvalue (§6.6). Charge conservation fixes $\mathrm{Tr}_{\mathcal{P}}(A_{AD}) = 4$; continuous isotropy forces $A_{AD} = (4/3)\cdot\mathbb{1}_{\mathcal{P}}$ — discrete $S_3$-invariance alone would still permit a privileged channel diagonal, which is why AD-6's primary content is continuous (§6.6); and any unit support direction n sees $n^T\cdot A_{AD}\cdot n = 4/3$. The total 4 is a bookkeeping sum over channels, not a per-direction load; no physical displacement probes all three channels' loads as a single scalar. Trace and eigenvalue are different typed quantities — precisely the kind of distinction the QN series exists to enforce.

## Objection 6 — Why must the descent be equal across the three channels?

**Reply.** Before a record realizes a particular physical configuration, the substrate anomaly carries no directional information, and the quotient privileges no channel (AD-6). Unequal primitive loads would install an anisotropy with no source. The unique $S_3$-invariant point of the constraint simplex is (4/3, 4/3, 4/3), and by Proposition C.1 the same point is the unique minimiser of every strictly convex symmetric descent cost. The equality is doubly forced.

## Objection 7 — Why double 4/3?

**Reply.** Because terminal record formation is bidirectional (AD-7). A record must descend into physical support and remain return-normalisable into the substrate ledger. These are not two arbitrary copies; they are the two constitutive branches of terminality. Since neither branch can be privileged before emergent physical time, and positivity forbids cancellation, they carry equal load. Hence $2\cdot(4/3) = 8/3$.

## Objection 8 — Return-normalisation is already inside the 4; the factor 2 counts it twice.

**Reply.** No, and Lemma 5 makes the separation exact. The 4 counts predicates: what a commitment must satisfy to be record-forming — occur, be distinct, continue, be ledger-compatible. The 2 counts transport branches: the directions along which the realized record must sustain load — into physical support and back into the ledger. The return branch does not carry obligation R alone; it carries the full four-obligation charge, just as the descent branch does.

Were R the doubling, the return branch would carry one unit against the descent branch's four and the stiffness would be a hybrid such as 5/3; it is not. Predicate and channel are different types, and they break under different falsifiers (F1/F2 versus F8/F9) — the operational test that they are counted separately, once each. R motivates the existence of the return branch without being absorbed into it.

## Objection 9 — You derived a per-direction load. Why is that the radial exponent κ⋆?

**Reply.** Because of AD-8, stated as a named premise precisely so this step cannot pass silently. The two-sided load is a rate of log-attenuation per unit displacement along a direction, and isotropy makes that rate common to every ray; the radial exponent of the canonical convention is therefore the single-direction two-sided load (Lemma 7). Equivalently, in operator form: u is a one-dimensional ray coordinate, and the exponent reads $n^T \cdot (2 \cdot A\_AD) \cdot n = 8/3$ — the isotropic eigenvalue of the two-sided operator, not its trace (§8.8). This is a substantive, falsifiable slot-assignment (F16), not a notational identification. Deny AD-8 and the derivation still yields a per-direction load of 8/3 — but the claim that this load is κ⋆ lapses, exactly as the falsifier records.

## Objection 10 — Is this just a way to get the answer you wanted?

**Reply.** No. The derivation is breakable at sixteen named points (§13). Change the commitment rank, the support rank, charge conservation, positivity, isotropy, convexity, or bidirectionality, and the number changes. The coefficient is not protected by notation; it is forced if and only if AD-1 – AD-8 hold. A construction that fails when its premises fail is a derivation. A construction that survives any premise failure would be numerology.

## Objection 11 — The quadratic cost in Lemma 2 was chosen to give the uniform answer.

**Reply.** Appendix C proves otherwise: the uniform vector minimises every strictly convex permutation-symmetric functional on the simplex. The quadratic is a representative of an entire class, all of which give 4/3. The genuinely load-bearing assumptions are symmetry and strict convexity — both stated in AD-6, both falsifiable.

## Objection 12 — Does this prove the Standard Model?

**Reply.** No. It proves a normalisation-origin theorem. It supplies the first-principles source of the canonical localization stiffness consumed by later mass, hierarchy, and coupling gates. Those gates remain separate, with their own premises and their own falsifiers.

## Objection 13 — Does this connect 8/3 to ln 14?

**Reply.** No, deliberately. The theorem makes no use of ln 14 or of the proximity $\ln 14 \approx 8/3$. The channel-log quantity remains a distinct typed object per QN-1. A future bridge may relate the 14-channel census to anomaly descent; QN-3 neither requires nor claims that relation, and treating the two as identical is falsifier F11.

# Objection 14 — What is the one-sentence result?

**Reply.**

**The canonical VERSF localization stiffness 8/3 is the unique two-sided stiffness produced when the four primitive obligations of finite substrate record formation descend positively and isotropically into the three-direction physical support quotient.**

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# Final one-sentence theorem

**In VERSF, a terminal physical record arises when a four-obligation finite substrate commitment descends into a three-direction physical support quotient; charge conservation, positivity, and isotropy force a one-sided load of 4/3, bidirectional terminality doubles it, and the canonical localization stiffness is therefore uniquely $\kappa\star = 8/3$.**