# The Substrate Anomaly-Inadmissibility Theorem in VERSF

**▲ Programme Milestone — Standard Model Gauge-Consistency Series**

## Deriving Gauge-Anomaly Cancellation as a Constraint of Persistent Record-Current Closure

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Successor to *The Electroweak Representation and Connection Closure in VERSF*.

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**Headline result — anomaly cancellation as substrate admissibility, not phenomenological repair.**

The predecessor paper derives the one-generation electroweak representation algebra

$\mathfrak{g}^{EW} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$,

acting on the chiral matter skeleton

$L_L, e_R, Q_L, u_R, d_R$,

with local connection form

$\mathcal{A}_\mu^{EW} = g\, W_\mu^i T_i + g'\, B_\mu Y$.

That paper interprets anomaly freedom as representation admissibility but leaves the stronger claim owed: why, in VERSF, anomalous chiral ledgers are forbidden by the substrate itself.

The present paper supplies the missing theorem.

**Central claim.** A chiral source ledger with a nonzero gauge, mixed, or gravitational anomaly cannot define a persistent record-current sector in VERSF. The reason is structural: a nonzero anomaly is a failure of local source accounting under admissible transport. It produces a residual in the ledger that cannot be absorbed into a committed record. In VERSF language:

anomalous ledger ⇒ non-closed record current ⇒ path-dependent commitment transport ⇒ substrate inadmissibility.

Anomaly cancellation is therefore not imported from Standard Model consistency. It is the condition under which a chiral gauge/source representation can be written into the substrate as a stable, refinement-persistent record.

**Scope of the claim.** The theorem is substrate-native in its admissibility interpretation, but it still inherits the standard local anomaly-descent structure at the record-layer QFT limit (premise SI-7). This paper derives why a nonzero anomaly is substrate-inadmissible, given that inherited structure. A later paper must derive the descent sequence internally from commitment complexes.

Applied to the inherited one-generation skeleton, the inadmissibility theorem forces the unique anomaly-safe ledger:

$L_L : (1, 2, -\frac{1}{2})$, $e_R : (1, 1, -1)$,

$Q_L : (3, 2, \frac{1}{6})$, $u_R : (3, 1, \frac{2}{3})$, $d_R : (3, 1, -\frac{1}{3})$.

The paper does not derive masses, Yukawas, CKM, PMNS, electroweak symmetry breaking, W/Z masses, the Weinberg angle, or full colour dynamics. It derives why anomalous charge ledgers cannot persist as physical substrate records.

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# General Reader Summary

The previous papers in this series built the matter table and the weak-force operating system. They showed how VERSF arrives at a one-generation structure with electron-like, neutrino-like, up-quark-like, and down-quark-like roles; why the weak force acts on left-handed pairs; why the right-handed particles are weak singlets; and why the electroweak operator has the form physicists write as

$SU(2)_L \times U(1)_Y$.

One important issue remained.

The charge table was checked against a famous set of quantum consistency conditions called anomaly cancellation. In ordinary particle physics, an anomaly is a kind of mathematical leak. A symmetry may look valid at the classical level, but once quantum effects are included, the bookkeeping no longer balances. If that happens to a gauge symmetry, the theory loses its consistent bookkeeping: the currents do not close, unphysical gauge choices can affect physical answers, and the theory cannot be trusted.

In the Standard Model, a remarkable thing happens. The charges of quarks and leptons cancel these leaks completely. The quark sector and the lepton sector balance each other exactly — the cancellation is arithmetic, not approximate. This is why the charge table feels so special. It is not merely a list of numbers; it is a ledger that balances.

The previous VERSF paper used the balancing condition to show that the quark charges are forced once the lepton charges are fixed. This paper asks the deeper question:

Why must the ledger balance at all?

The answer proposed here is simple in spirit.

VERSF treats reality as built from committed records. A physical structure is not permitted to be merely locally plausible; it must be able to persist as a stable record under refinement and transport. A charge ledger that leaks is not a stable record. If, after moving around a small closed loop of local transformations, the accounting does not return to itself, then the substrate cannot write that structure into permanent physical reality.

In everyday terms: an anomaly is a bank ledger that loses money every time it is audited from a different route. Such a ledger cannot be the bookkeeping system of the universe. VERSF says this is not a mathematical inconvenience. It is a substrate-level impossibility.

The paper's main claim is therefore:

anomaly cancellation is not a rule added after the fact; it is the condition that charge information can become a persistent physical record.

This upgrades the earlier result. The previous paper said:

If anomaly cancellation is required, the Standard Model charge table follows.

This paper says:

In VERSF, anomaly cancellation *is* required, because anomalous ledgers cannot be committed as stable substrate records.

That is the milestone. The Standard Model charge table is no longer merely anomaly-checked. It becomes anomaly-forced by the deeper rule that only closed, persistent, non-leaking source ledgers can exist.

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# Contents



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# Abstract

The preceding VERSF papers derive a one-generation chiral matter skeleton and an electroweak representation/connection closure. The matter-representation paper obtains the sectors

$L_L$, $e_R$, $Q_L$, $u_R$, $d_R$

and shows that, given lepton normalization and perturbative anomaly freedom, the quark hypercharges have a unique real completion:

$Y_Q = 1/6$, $Y_u = 2/3$, $Y_d = -1/3$.

The electroweak-representation paper then derives the representation algebra

$\mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$

and the local connection form

$\mathcal{A}\_\mu^{EW} = g\, W\_\mu^i\, T_i + g'\, B\_\mu\, Y.$

Both papers record the same outstanding debt: anomaly cancellation is used as an admissibility filter, but a fully VERSF-native derivation requires showing that anomalous chiral ledgers are substrate-inadmissible.

This paper supplies that derivation at the gauge-consistency layer. Let a chiral source ledger be a record-current representation assigning local source grades to spinorial matter sectors. In VERSF, such a ledger is admissible only if it is persistent under local commitment transport, refinement-stable under admissible coarse-graining, and compatible with bit-conservation/source-balance constraints. A gauge anomaly is precisely an obstruction to this persistence: it produces a non-vanishing residual in the divergence of the quantum source current — equivalently, a nontrivial local cohomology class in the descent sequence of the gauge/source representation. Such a residual cannot be absorbed into a boundary of the committed record. It is a true ledger leak.

The central theorem is:

$[\Omega\_anom] \neq 0 \Rightarrow$ no persistent VERSF record-current sector.

Gauge, mixed, gravitational, and cubic U(1) anomalies are therefore not merely conventional quantum-field-theoretic inconsistencies. In VERSF they are failures of substrate admissibility: a locally transported source ledger that does not close cannot be written as an irreversible, refinement-stable commitment record.

Applied to the inherited one-generation skeleton, the theorem upgrades the previous anomaly audit. The four conditions

$SU(3)^2U(1)$, $SU(2)^2U(1)$, $grav^2U(1)$, $U(1)^3$

are now substrate-admissibility conditions, not external tests. Their unique solution is

$L\_L : (1, 2, -½)$, $e\_R : (1, 1, -1)$,

$Q\_L : (3, 2, ⅙)$, $u\_R : (3, 1, ⅔)$, $d\_R : (3, 1, -⅓)$,

with the optional sterile $\nu\_R : (1, 1, 0)$ as the unique anomaly-neutral chiral singlet extension.

The result is conditional on the inherited chiral matter skeleton, the electroweak connection closure, and the standard local form of chiral anomaly descent. The theorem is substrate-native in its admissibility interpretation, but it inherits the local anomaly-descent structure at the record-layer QFT limit; deriving that descent sequence internally from commitment complexes is recorded as an open debt. It does not derive gauge kinetic terms, coupling constants, electroweak symmetry breaking, masses, generations, full colour dynamics, or renormalised Standard Model phenomenology. It derives why anomaly-free ledgers are the only ledgers admissible as persistent substrate records.

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# 0. Notation, Conventions, and Firewalls

## Notation

$\mathscr{H}_1$ — the one-particle spinorial source-carrier Hilbert space.

L_L, e_R, Q_L, u_R, d_R — the inherited one-generation chiral matter skeleton.

$\mathfrak{g}^{EW} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$ — the inherited electroweak representation algebra.

$\mathcal{A}_\mu^{EW} = g\, W_\mu^i\, T_i + g'\, B_\mu\, Y$ — the inherited electroweak connection form.

$J_A^\mu$ — the source current associated with a generator $T_A$ of a local source/gauge representation.

$\mathcal{A}_A$ — the local (consistent) anomaly functional: the obstruction to conserving the corresponding source current after quantisation of chiral matter (convention fixed in §3).

$\Omega_{anom}(\mathscr{R})$ — the anomaly ledger of a chiral source representation $\mathscr{R}$; $\Omega_{ledger}$ — the total anomaly ledger of the full matter table. $[\cdot]$ denotes the cohomology class.

$\mathfrak{A}_{source}$ — the set of substrate-admissible source representations.

Throughout this paper the hypercharge convention is

$Q = T_3 + Y.$

Some companion papers use $Q = T_3 + Y_{SM}/2$. The conventions are related by $Y_{SM} = 2Y$. All hypercharge values in this paper use the $Q = T_3 + Y$ convention.

## Representation Firewall

This paper inherits the matter skeleton and the electroweak representation/connection closure. It does not rederive L_L, e_R, Q_L, u_R, d_R; nor $\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$; nor $\mathcal{A}_\mu^{EW} = g\, W_\mu^i\, T_i + g'\, B_\mu\, Y$. Its purpose is narrower and deeper:

why must the inherited chiral ledger be anomaly-free?

### Mass Firewall

This paper does not compute fermion masses, Yukawa couplings, CKM, PMNS, Higgs couplings, generation hierarchy, or the Higgs vacuum expectation value. Mass belongs downstream. The present paper concerns the consistency of source ledgers before mass operators are attached.

### Electroweak-Breaking Firewall

This paper does not derive SU(2)_L × U(1)_Y → U(1)_EM, nor the photon, W/Z masses, the Weinberg angle, or closure-norm condensation. Those belong to the downstream electroweak symmetry-breaking module.

### Gauge-Dynamics Firewall

This paper does not derive gauge kinetic terms or field equations. It uses the inherited connection form and studies the admissibility of the chiral source ledger coupled to that connection.

### Anomaly-Scope Firewall

This paper addresses perturbative local gauge/source anomalies and the mixed gravitational-U(1) anomaly in the one-generation electroweak representation. It also records the global SU(2) even-doublet condition as an inherited audit line. It does not derive the full nonperturbative global anomaly classification of all possible VERSF gauge sectors.

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# 0′. Predictive-Content Ledger

| Object | Grade | Status |
|---|---|---|
| Fermionic Fock sector | Inherited | From Fock reconstruction |
| One-generation chiral matter skeleton | Inherited / Conditional | From matter-representation paper |
| Electroweak representation algebra | Inherited / Conditional | From electroweak representation closure |
| Local W/B connection form | Inherited / Conditional | From connection closure |
| Hypercharge convention $Q = T_3 + Y$ | Convention | Used throughout |
| Perturbative anomaly obstruction | Inherited | Standard QFT input via SI-7; interpreted substrate-theoretically |
| Record-current persistence | Inherited | VERSF source-admissibility requirement |

| Object | Grade | Status |
|---|---|---|
| Bit-conservation / source-balance condition | Inherited | Ledger must close |
| Anomaly as ledger leak | Candidate | New interpretation; central construction of this paper |
| Substrate anomaly-inadmissibility theorem | Conditional | Theorem given SI-1–SI-7 |
| Hypercharge rigidity as substrate admissibility | Conditional | Strengthens prior anomaly audit |
| Sterile ν_R : (1, 1, 0) uniqueness | Exact | Within the normalized ledger; see §10 scope note |
| Full substrate-native descent construction | Owed | Not completed (Debt 1) |
| Nonperturbative global anomaly classification | Owed | Partially addressed (§12) |
| Gauge kinetic dynamics | Firewalled | Not derived |
| Mass / Yukawa sector | Firewalled | Not addressed |
| EWSB / Higgs sector | Firewalled | Not addressed |
| Three generations | Owed | Not derived |

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# 1. Purpose and Claim Level

The preceding matter-representation and electroweak-representation papers make anomaly cancellation load-bearing.

The matter-representation paper shows:

lepton normalization + anomaly freedom ⇒ $Y_Q = \frac{1}{6}$, $Y_u = \frac{2}{3}$, $Y_d = -\frac{1}{3}$.

The electroweak-representation paper strengthens the interpretation: anomaly freedom is not phenomenological fitting but representation admissibility. That paper nevertheless leaves the stronger result owed:

derive anomaly inadmissibility from substrate structure itself.

This paper discharges that debt. The target chain is:

local chiral source ledger → anomaly functional → record-current nonclosure → substrate inadmissibility → anomaly-free Standard Model ledger.

## Central Claim

A chiral weak/source ledger is substrate-admissible in VERSF only if its total anomaly class vanishes:

$[\Omega_{ledger}] = 0$.

If $[\Omega_{ledger}] \neq 0$, the ledger cannot define a persistent record-current sector. It fails the substrate's commitment-closure condition.

Applied to the inherited one-generation electroweak skeleton, this condition forces the anomaly-safe hypercharge ledger already found in the prior papers — but now as a substrate necessity rather than an imported test.

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# 2. Inherited Background

The paper inherits four structural results.

## 2.1 The chiral matter skeleton

The one-generation matter skeleton is:

$L_L = (\nu_L, e_L),\ e_R,$

$Q_L = (u_L, d_L),\ u_R,\ d_R.$

The hypercharge ledger is:

$L_L : (1, 2, -\tfrac{1}{2}),\ e_R : (1, 1, -1),$

$Q_L : (3, 2, \tfrac{1}{6}),\ u_R : (3, 1, \tfrac{2}{3}),\ d_R : (3, 1, -\tfrac{1}{3}).$

The optional sterile neutral sector is $\nu_R : (1, 1, 0)$. This paper does not rederive the skeleton.

## 2.2 The electroweak representation and connection

The inherited representation algebra is

$\mathfrak{g}^{EW} = \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y,$

with inherited local connection form

$\mathcal{A}_\mu^{EW} = g\, W_\mu^i\, T_i + g'\, B_\mu\, Y.$

This paper does not derive the connection form or its dynamics. It studies whether the chiral source ledger coupled to this connection is admissible.

### 2.3 Persistent record currents

VERSF source sectors require persistent record-current closure. A physical source current must not merely be formally definable; it must survive local transport and refinement as a committed record. The core requirement is

$$\partial_\mu J_A^\mu = 0,$$

or its covariant generalisation

$$D_\mu J_A^\mu = 0,$$

for every generator whose local transport is part of the admissible source representation. If a current fails this condition because of a non-removable local obstruction, the ledger is not physically admissible.

### 2.4 Bit conservation and source balance

VERSF's bit-conservation/source-balance discipline forbids ledger leakage. A committed source grade must balance under local refinement and closed transport. The governing principle:

a source grade that changes under a closed local audit is not a source grade of a persistent record.

Anomaly freedom is therefore expected, before any calculation, to be a substrate admissibility condition. The remainder of the paper makes this expectation a theorem.

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## 3. What an Anomaly Means in VERSF Language

In ordinary field theory, a gauge anomaly means that a symmetry of the classical action fails after quantising chiral fermions. For a local gauge/source transformation with parameter $\alpha^A(x)$, the quantum effective action transforms as

$$\delta_\alpha \Gamma = \int d^4x\, \alpha^A(x)\, \mathcal{A}_A(x),$$

where $\mathcal{A}_A(x)$ is the anomaly functional. Equivalently, the associated current fails to conserve:

$$D_\mu J_A^\mu = \mathcal{A}_A.$$

For a gauge symmetry this is fatal: the source current is no longer consistently preserved.

### Convention — consistent anomaly

Throughout this paper, $\mathcal{A}$_A denotes the *consistent* anomaly: the functional obtained from the variation of the quantum effective action, satisfying the Wess–Zumino consistency conditions and therefore defining a 1-cocycle on the local gauge/source group. The covariant anomaly differs from it by the Bardeen–Zumino polynomial and does not carry the same cocycle structure. All cohomological statements in this paper — non-exactness (SI-3), counterterm invariance (Lemma 1), and the descent sequence (§13) — are statements about the consistent form.

### VERSF translation

In VERSF, $\mathcal{A}$_A is a ledger leak.

The classical ledger asserts D_μ J_A^μ = 0. The anomaly asserts D_μ J_A^μ = $\mathcal{A}$_A ≠ 0.

This means local commitment transport around the same source structure depends on the path taken through the gauge/source configuration. The substrate cannot commit such a structure as a refinement-stable record, because the answer changes under admissible local comparison.

Thus $\mathcal{A}$_A ≠ 0 is not merely a quantum correction. It is a failure of record closure.

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# 4. Substrate Admissibility Premises

The theorem rests on seven named premises. Each is stated with its falsifier, so the conditional structure of the result is explicit.

### SI-1 — Persistent source-current requirement

A physical gauge/source representation in VERSF must define a persistent record-current sector.

**Statement.** D_μ J_A^μ = 0 must hold for every local generator T_A of the source representation after all admissible quantum/record corrections are included.

**Falsifier.** A physical source representation with nonzero unavoidable current divergence.

### SI-2 — Local-transport path independence

Transporting a committed source ledger around an infinitesimal closed comparison loop must return the same ledger.

**Statement.** The source ledger must be closed under local comparison.

**Falsifier.** A physical ledger whose value depends on the local audit path.

## SI-3 — Anomaly as non-exact ledger residue

A gauge/source anomaly is represented by a local cohomology class [Ω_anom]. If the class is nonzero, the anomaly cannot be removed by counterterm redefinition, relabelling, or boundary adjustment.

**Falsifier.** A nonzero anomaly class that is nevertheless absorbed into admissible substrate bookkeeping.

## SI-4 — Bit-conservation / source-balance constraint

A committed source grade cannot leak under refinement. All local gain/loss terms must either be balanced by opposite source terms or vanish.

**Falsifier.** A stable physical record whose source grade is not balanced.

## SI-5 — Refinement stability and cohomological functoriality

Admissible source ledgers must remain stable under substrate refinement.

**Statement.** Admissible refinement maps act on ledger cohomology and preserve nontriviality of classes: a refinement carries a nonzero anomaly class to a nonzero class. A defect that persists at the class level under refinement blocks record commitment.

**Remark.** The functoriality clause is a named assumption of this paper, not a consequence of the anomaly being cohomological — the construction of the substrate complex on which refinement acts, and of the induced map on its cohomology, is precisely Debt 1.

**Falsifier.** An admissible refinement map that trivialises a nonzero anomaly class, or a nonzero anomaly residue that remains physically admissible under refinement.

## SI-6 — No hidden anomaly reservoir

The one-generation matter skeleton contains no additional hidden chiral sectors capable of absorbing anomalies, except the optional sterile (1, 1, 0) sector, which contributes zero.

**Falsifier.** An unavoidable hidden chiral sector that cancels anomalies without appearing in the matter skeleton.

## SI-7 — Standard local anomaly descent applies to the record-layer chiral fields

At the record-layer field-theoretic limit, chiral fermion source currents carry the standard local anomaly structure.

**Falsifier.** VERSF chiral fields evade the usual local anomaly descent despite carrying ordinary chiral gauge/source coupling.

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# 5. The Ledger-Closure Principle

## Definition — Source ledger

A source ledger is an assignment of source grades to chiral matter sectors together with the currents those grades generate. For a generator $T_A$, the ledger current is

$J_A^\mu = :\bar{\psi} \gamma^\mu T_A \psi:$,

with left/right projections included where appropriate. The ledger is *closed* if

$D_\mu J_A^\mu = 0$

after quantum corrections.

## Principle — Ledger Closure

A source ledger is substrate-admissible only if its total non-exact anomaly class vanishes:

$[\Omega_{ledger}] = 0$.

## Interpretation

A vanishing anomaly class means the ledger can be globally and locally balanced. A nonzero anomaly class means there is no refinement-stable way to preserve the source account. This is the VERSF-native form of anomaly cancellation.

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# 6. The Substrate Anomaly-Inadmissibility Theorem

## Theorem 1 — Substrate Anomaly-Inadmissibility

Let $\mathscr{R}$ be a chiral source representation carried by persistent spinorial source-carrier fields in VERSF. Suppose $\mathscr{R}$ has nonzero anomaly class:

$[\Omega_{anom}(\mathscr{R})] \neq 0$.

Then $\mathscr{R}$ cannot define an admissible persistent record-current sector. Equivalently:

$[\Omega_{anom}] \neq 0 \Rightarrow \mathscr{R} \notin \mathfrak{A}_{source}$.

## Proof

The proof proceeds in five steps, each keyed to a named premise.

**Step 1 (currents).** A chiral source representation $\mathscr{R}$ defines local source currents $J_A^\mu$. By SI-1, persistence of the corresponding record-current sector requires $D_\mu J_A^\mu = 0$ for every local generator $T_A$.

**Step 2 (leak).** If $\mathscr{R}$ has a nonzero anomaly class, then by SI-7 the quantum record-layer current satisfies

$$D_\mu J_A^\mu = \mathcal{A}_A,$$

with $\mathcal{A}_A$ representing a non-exact local cohomology class. By SI-3 this residual cannot be eliminated by a counterterm, boundary relabelling, or gauge-choice convention. It is not bookkeeping noise; it is a genuine ledger residue.

**Step 3 (path dependence via the Wess–Zumino cocycle).** By SI-2, a persistent record ledger must be path-independent under local comparison. The consistent anomaly satisfies the Wess–Zumino consistency conditions and is therefore a 1-cocycle on the local gauge/source group; nontriviality of its class means the cocycle is not a coboundary. Composing admissible local transformations around a closed loop in the space of gauge/source configurations therefore accumulates a residual that no relabelling removes: the audit of the source ledger depends on the path taken around the loop. This is precisely the violation of SI-2. The source account is not closed.

**Step 4 (balance violation).** By SI-4, source grades cannot leak under committed record formation. Since $\mathcal{A}_A \neq 0$ is an unbalanced source term, the ledger violates bit-conservation/source-balance.

**Step 5 (refinement persistence of the defect).** By SI-5, admissible refinement acts class-preservingly on ledger cohomology. A nonzero anomaly class therefore persists under refinement — not because "cohomological" abstractly implies persistence, but because SI-5 asserts the required functoriality explicitly. Refinement cannot wash the defect away.

The representation therefore cannot define a persistent record-current sector. It is substrate-inadmissible. ■

## Reading

In ordinary QFT language, gauge anomalies are inconsistent because they violate gauge invariance and current conservation. In VERSF language, the deeper reason is that they prevent the source ledger from being written as a stable committed record.

An anomaly is not merely a bad equation. It is a record that cannot close.

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# 7. Corollary: Anomaly Freedom Is Not a Phenomenological Filter

### Corollary 1.1 — Anomaly freedom is substrate admissibility

For a VERSF chiral gauge/source sector, anomaly cancellation is a necessary condition of physical existence:

$[\Omega_{ledger}] = 0$

is not imposed because the Standard Model happens to work. It follows from the requirement that the source ledger be persistently commit-able.

### Proof

Immediate from Theorem 1. If $[\Omega_{ledger}] \neq 0$, the ledger is inadmissible; any physical ledger therefore satisfies $[\Omega_{ledger}] = 0$. ■

### Programme consequence

The prior hypercharge rigidity theorem is upgraded.

Earlier: *if anomaly freedom is imposed, the hypercharge table is forced.*

Now: *because anomaly freedom is substrate admissibility, the hypercharge table is substrate-forced within the inherited skeleton.*

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# 8. The Four Anomaly Channels as Four Ledger-Closure Tests

For the inherited one-generation electroweak skeleton, there are four local perturbative anomaly channels:

$SU(3)^2U(1)$, $SU(2)^2U(1)$, $grav^2U(1)$, $U(1)^3$.

In VERSF these are four distinct ways a source ledger can fail to close.

### 8.1 $SU(3)^2U(1)$

Tests whether the colour-supported source ledger is compatible with the hypercharge grade. A nonzero result means *colour transport leaks hypercharge* — quark-like $C_3$-supported records would be incompatible with a persistent hypercharge ledger.

### 8.2 $SU(2)^2U(1)$

Tests whether weak unresolved-branch transport is compatible with the hypercharge grade. A nonzero result means *weak branch transport leaks hypercharge* — the weak doublet ledger would be inadmissible.

### 8.3 $\mathrm{grav}^2U(1)$

Tests whether the total hypercharge source ledger is compatible with universal record transport through the emergent gravitational/background geometry. A nonzero result means *the total source grade is not globally balanced* — a violation of the universal source-balance discipline.

Unlike the other three channels, this one presupposes the record-layer geometric structure: it is evaluated at the emergent-geometry record layer, where the 4-manifold through which universal transport is defined already exists (per SI-7). Below the layer at which geometric structure emerges, the gravitational channel has no independent meaning; its VERSF content is the global balance of the total source grade under universal record transport.

### 8.4 $U(1)^3$

Tests whether the abelian source grade is internally self-consistent. A nonzero result means *hypercharge sources leak into themselves under local quantum transport* — the $U(1)_Y$ ledger would be internally inadmissible.

### Reading

The four anomaly tests are not arbitrary field-theory rituals. They are the four independent audit channels of a chiral electroweak source ledger, and by Theorem 1 each is a substrate-admissibility condition.

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# 9. Hypercharge Rigidity from Substrate Inadmissibility

The inherited one-generation content, written entirely in terms of left-handed Weyl sectors, is:

$Q_L : (3, 2, Y_Q)$,

$u_L^c : (\bar{3}, 1, -Y_u)$,

$d_L^c : (\bar{3}, 1, -Y_d)$,

$L_L : (1, 2, Y_L)$,

$e_L^c : (1, 1, -Y_e)$.

Lepton normalization fixes

$Y_L = -\frac{1}{2}, Y_e = -1$.

The unknowns are $Y_Q$, $Y_u$, $Y_d$. By Theorem 1, the anomaly ledger must close; the following are therefore substrate-admissibility conditions, not external checks.

### 9.1 $SU(2)^2U(1)$

The weak doublet ledger gives

$3Y_Q + Y_L = 0$.

Substituting $Y_L = -\frac{1}{2}$:

$3Y_Q - \frac{1}{2} = 0 \Rightarrow Y_Q = \frac{1}{6}$.

### 9.2 $SU(3)^2U(1)$

The colour ledger gives

$2Y_Q - Y_u - Y_d = 0$.

With $Y_Q = \frac{1}{6}$:

$Y_u + Y_d = \frac{1}{3}$.

### 9.3 $grav^2U(1)$

The total hypercharge ledger gives

$6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$.

Substituting $Y_Q = \frac{1}{6}, Y_L = -\frac{1}{2}, Y_e = -1$:

$1 - 3(Y_u + Y_d) - 1 + 1 = 0 \Rightarrow Y_u + Y_d = \frac{1}{3}$.

This reproduces the $SU(3)^2U(1)$ condition from a physically distinct ledger-closure channel. It is algebraically redundant after the first two conditions but physically nontrivial: the same source table passes an independent global balance audit.

### 9.4 $U(1)^3$

The cubic hypercharge ledger gives

$$6Y\_Q^3 - 3Y\_u^3 - 3Y\_d^3 + 2Y\_L^3 - Y\_e^3 = 0.$$

Substituting $Y\_Q = \frac{1}{6}$, $Y\_L = -\frac{1}{2}$, $Y\_e = -1$:

$$1/36 - 3(Y\_u^3 + Y\_d^3) - \frac{1}{4} + 1 = 0 \Rightarrow Y\_u^3 + Y\_d^3 = 7/27.$$

Let

$$s = Y\_u + Y\_d = \frac{1}{3},\ p = Y\_u\, Y\_d.$$

Using the identity $Y\_u^3 + Y\_d^3 = s^3 - 3ps$:

$$1/27 - p = 7/27 \Rightarrow p = -2/9.$$

Thus Y_u and Y_d are the roots of

$$t^2 - \frac{1}{3}\, t - 2/9 = 0.$$

The discriminant is

$$\Delta = 1/9 + 8/9 = 1,$$

so

$$t = (\frac{1}{3} \pm 1)/2 \Rightarrow \{Y\_u, Y\_d\} = \{\frac{2}{3}, -\frac{1}{3}\}.$$

The quadratic returns an unordered pair; the anomaly conditions alone do not distribute the two roots between the up-type and down-type sectors. The assignment of $Y\_u = \frac{2}{3}$ to u_R and $Y\_d = -\frac{1}{3}$ to d_R is fixed by the inherited weak-doublet ordering Q_L = (u_L, d_L), where $T_3(u\_L) = +\frac{1}{2}$ and $T_3(d\_L) = -\frac{1}{2}$: via $Q = T_3 + Y$ with $Y\_Q = \frac{1}{6}$, the doublet components carry electric charges $\frac{2}{3}$ and $-\frac{1}{3}$ respectively, and each right-handed singlet must match the electric charge of its doublet partner, since Q = Y for weak singlets.

This pairing invokes no mass operator. That u_R is the right-handed completion of the up-type spinorial sector — and d_R of the down-type — is inherited at the representation level from the Fock-reconstruction and matter-representation papers, which fix which right-handed singlet completes which doublet component before any Yukawa or Dirac pairing is defined. The mass firewall (§0) is therefore intact: the root distribution uses inherited representation data, not mass data. Therefore:

$$Y\_Q = \frac{1}{6},\ Y\_u = \frac{2}{3},\ Y\_d = -\frac{1}{3}.$$

## Theorem 2 — Hypercharge ledger from substrate inadmissibility

Given the inherited one-generation skeleton, lepton normalization, and no extra anomaly-carrying chiral exotics (SI-6), the substrate anomaly-inadmissibility theorem forces the unique real quark hypercharge ledger

$Y_Q = \frac{1}{6}$, $Y_u = \frac{2}{3}$, $Y_d = -\frac{1}{3}$.

### Proof

By Theorem 1 the four anomaly channels are substrate-admissibility conditions. §9.1 fixes $Y_Q$ linearly; §9.2 fixes the sum $Y_u + Y_d$; §9.4 fixes the product $Y_u Y_d$, and the resulting quadratic has discriminant 1, hence exactly two real roots, distributed by the doublet $T_3$ structure. §9.3 provides an independent consistency audit. The ledger above is the unique real solution. ■

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# 10. Sterile-Neutrino Uniqueness

A candidate right-handed neutral singlet has representation

$\nu_R : (1, 1, Y_\nu)$.

It contributes nothing to $SU(3)^2U(1)$ or $SU(2)^2U(1)$, being a colour and weak singlet. It contributes $Y_\nu$ to the gravitational-U(1) anomaly and $Y_\nu^3$ to the cubic $U(1)^3$ anomaly. (In the left-handed Weyl language of §9, the conjugate $\nu_L^c$ contributes $-Y_\nu$ and $-Y_\nu^3$; the vanishing condition is sign-independent.) Both vanish iff

$Y_\nu = 0$.

Therefore $\nu_R : (1, 1, 0)$ is the unique anomaly-neutral chiral singlet extension.

### Corollary 2.1 — Sterile singlet rigidity

The optional sterile sector is not arbitrary. If a chiral singlet is added without disturbing the anomaly ledger, it must carry $Y = 0$ — it is necessarily sterile with respect to the electroweak source ledger. Sterility is thus a ledger-forced property, not a model-building choice.

### Scope note — the B−L flat direction

Uniqueness here is conditional on the upstream lepton normalization $Y_L = -\frac{1}{2}$, $Y_e = -1$, taken as fixed. If instead all five hypercharges were re-solved jointly with $\nu_R$ present and $Y_\nu$ free, the anomaly system admits a one-parameter family of solutions along the B−L direction, and uniqueness would fail. That flat direction is excluded upstream, by the normalization firewall of the matter-representation paper — not by the anomaly ledger of this paper. Corollary 2.1 asserts rigidity of the singlet extension of the already-normalized ledger, nothing stronger.

## 11. Relation to $C_3$ Support and Lepton–Quark Locking

The anomaly-inadmissibility theorem also explains why the lepton and quark sectors must appear together.

The weak anomaly condition

$3Y\_Q + Y\_L = 0$

says that the three colour-supported copies of the quark doublet balance the single lepton doublet. In VERSF terms:

$C_3$-support + single lepton support = closed weak/source ledger.

Without the lepton branch, the quark ledger leaks. Without the quark branch, the lepton ledger leaks. The one-generation matter table is therefore not an arbitrary union of sectors. It is the minimal source-balanced ledger.

### Theorem 3 — Lepton–quark co-admissibility

Given the inherited $C_3$-supported quark branch and single-support lepton branch, neither branch alone defines an anomaly-admissible electroweak source ledger — perturbatively for nonzero charges, and globally regardless of charges. The pair is co-admissible precisely when the hypercharges take the Standard-Model values.

### Proof

The obstruction is two-channel.

*Perturbative channel.* If L_L is removed, the $SU(2)^2U(1)$ condition becomes 3Y_Q = 0, inconsistent with nonzero quark charges. If Q_L is removed, the same condition becomes Y_L = 0, inconsistent with the charged lepton branch.

*Global channel.* The lepton-only ledger carries one weak doublet; the quark-only ledger carries three (the colour copies of Q_L). Both counts are odd, so each branch alone fails the SU(2) even-doublet condition of §12 regardless of its hypercharge assignment — including the trivially charged case that evades the perturbative channel.

Together the branches carry 3 + 1 = 4 doublets, satisfy 3Y_Q + Y_L = 0, and the remaining anomaly conditions force Y_u and Y_d as in Theorem 2. ■

### Reading

The lepton sector and quark sector are not merely both present. They are ledger partners, and the locking is two-channel: perturbative (hypercharge balance) and global (doublet parity). Each makes the other admissible, and neither channel alone can be evaded by tuning charges.

---

# 12. Global SU(2) Even-Doublet Condition

The perturbative anomaly conditions are not the whole story. There is also a global SU(2) consistency condition: the number of left-handed SU(2) doublets of Weyl fermions must be even.

The inherited one-generation skeleton has three coloured copies of $Q_L$ and one lepton doublet $L_L$:

$$N_{doublets} = 3 + 1 = 4,$$

which is even.

### VERSF reading

An odd number of weak doublets would mean the global topology of weak-branch transport fails to return consistently under a large SU(2) transformation. In VERSF terms, the weak-branch record would acquire a global sign obstruction under admissible large transport. The even-doublet condition is therefore a *global* record-closure condition — the nonperturbative counterpart of the local ledger-closure tests of §8, and a second, independent instance of the lepton–quark locking of §11: the colour triplication of $Q_L$ is precisely what makes the doublet count even once $L_L$ is included.

### Status note

This paper records the global SU(2) audit and its VERSF interpretation. A full classification of all possible nonperturbative global anomaly obstructions remains owed.

---

# 13. The Anomaly Polynomial as a Ledger Curvature

The local anomaly information can be packaged into an anomaly polynomial. In a four-dimensional chiral gauge theory, local anomalies are encoded by a six-form $I_6$. Schematically,

$$I_6 = \mathrm{Tr}_{\mathscr{R}}(F^3) + \mathrm{Tr}_{\mathscr{R}}(F)\, p_1(R),$$

where F is the gauge/source curvature and $p_1(R)$ carries the gravitational contribution (relative coefficients suppressed; the expression is schematic and should not be audited literally). The descent procedure gives

$I_6 = dI_5$, $\delta I_5 = dI_4^{(1)}$,

and the anomaly is represented by $\int I_4^{(1)}$.

### VERSF reading

The anomaly polynomial is the curvature of the source ledger. If its cohomology class is nonzero, the ledger has irreducible curvature: it cannot be flattened by any local counterterm.

$[I_6] \neq 0 \Rightarrow$ the source ledger has non-removable curvature.

A non-removable ledger curvature cannot define a persistent flat record of source conservation. It violates admissible closure.

### Theorem 4 — Anomaly polynomial closure criterion

A chiral source ledger is perturbatively substrate-admissible only if its total anomaly polynomial has vanishing cohomology class:

$[I_6]\_\text{ledger} = 0$.

### Proof

The anomaly polynomial represents the cohomology class of the local anomaly via descent. If the class is nonzero, Theorem 1 applies and the ledger is inadmissible. If the class vanishes, no perturbative local anomaly obstructs record-current closure. ■

### Status note

The criterion is perturbative and local. It must be supplemented by global anomaly audits such as the SU(2) even-doublet condition of §12.

---

# 14. Why Counterterms Cannot Save a Nonzero Class

A possible objection: anomalies can sometimes be shifted between currents by adding local counterterms. This is true, and it does not weaken the theorem. Local counterterms can move anomaly contributions between ledger lines. They cannot remove a nonzero total cohomology class.

In VERSF terms: a counterterm can relabel which page of the ledger shows the leak; it cannot make a leaking ledger balance.

### Lemma 1 — Counterterm relocation does not restore admissibility

If $[\Omega_{ledger}] \neq 0$, no local counterterm can make the ledger substrate-admissible.

### Proof

A local counterterm changes the anomaly representative by an exact term and leaves the cohomology class invariant. Substrate admissibility requires the class to vanish (Theorem 1); a nonzero class therefore remains inadmissible under any counterterm choice. ■

### Remark

This is why SI-3 is stated cohomologically. The substrate does not audit representatives; it audits classes. Only class-level closure is record-closure.

---

# 15. Why This Is Not Circular

The paper does not assume the Standard Model charge ledger and then celebrate that it cancels anomalies. The logical order is:

1. The VERSF matter-representation structure supplies the skeleton: $L_L$, $e_R$, $Q_L$, $u_R$, $d_R$.
2. Lepton single-support normalization fixes $Y_L = -½$, $Y_e = -1$.
3. Substrate anomaly-inadmissibility requires $[\Omega_{ledger}] = 0$.
4. The four ledger-closure channels force $Y_Q = ⅙$, $Y_u = ⅔$, $Y_d = -⅓$.

The charge ledger is not inserted. It is the unique closure of the inherited skeleton under substrate source-balance.

### What remains conditional

The derivation is conditional on the inherited skeleton and the local anomaly-descent structure (SI-7). If the skeleton changes, or if hidden chiral exotics are required (contra SI-6), the ledger can change. Both are recorded as falsifiers in §20.

---

# 16. What Has Actually Been Derived

This paper derives, conditionally on SI-1–SI-7:

1. the anomaly as a VERSF ledger leak;
2. a nonzero anomaly class as record-current nonclosure;
3. substrate inadmissibility of anomalous chiral source ledgers (Theorem 1);
4. anomaly freedom as a substrate admissibility condition (Corollary 1.1);
5. hypercharge rigidity as substrate-forced rather than externally imposed (Theorem 2);
6. lepton–quark co-admissibility (Theorem 3);
7. sterile $\nu_R$ : (1, 1, 0) uniqueness (Corollary 2.1);
8. anomaly-polynomial closure as the perturbative ledger criterion (Theorem 4);
9. counterterm-insensitivity of the inadmissibility result (Lemma 1).

This paper does not derive:

1. the matter skeleton itself;
2. the electroweak representation algebra;
3. the local W/B connection form;
4. gauge kinetic terms;
5. coupling constants;
6. electroweak symmetry breaking;
7. photon / W / Z mass structure;
8. fermion masses;
9. Yukawa couplings;
10. CKM / PMNS;
11. three-generation replication;
12. the full nonperturbative anomaly classification.

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# 17. Relation to the Previous Electroweak Paper

The predecessor paper answered: *what electroweak operator algebra acts on the matter skeleton?* It derived

$\mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$

and the local connection form

$\mathcal{A}_\mu^{EW} = g\, W_\mu^i\, T_i + g'\, B_\mu\, Y,$

and interpreted anomaly freedom as representation admissibility while leaving the full substrate proof owed.

The present paper answers: *why are anomalous electroweak ledgers impossible in VERSF?*

The programme chain becomes:

fermion algebra → chiral matter skeleton → electroweak representation/connection → substrate anomaly-inadmissibility.

This strengthens every upstream charge result: each hypercharge value in the series now rests on a substrate necessity rather than a consistency check.

---

# 18. Relation to the Downstream EWSB Paper

The closure-norm condensation paper requires a stable electroweak source representation before the vacuum condenses. If the chiral ledger were anomalous, the closure-norm condensate would couple to an unstable source system: the gauge transport modes would not define a persistent record-current sector.

This paper supplies the missing precondition:

the electroweak source ledger is anomaly-admissible.

Only then can the downstream EWSB module act on it. The sequence is:

representation closure → anomaly-admissible ledger → closure-norm condensation → broken electroweak vacuum.

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# 19. Open Debts

### Debt 1 — Full substrate cohomology construction

This paper identifies anomaly classes with ledger cohomology classes. A deeper paper should construct the relevant substrate cohomology directly from commitment complexes and derive the descent sequence internally, rather than importing the record-layer anomaly formalism through SI-7.

### Debt 2 — Nonperturbative global anomaly classification

The SU(2) even-doublet condition is recorded. A complete VERSF classification of all possible global anomaly obstructions remains owed.

### Debt 3 — Hidden-sector exclusion theorem

The no-extra-chiral-exotics condition (SI-6) is inherited. A stronger theorem should show that no hidden anomaly-carrying sectors are forced by the substrate.

### Debt 4 — Continuous colour lift

The SU(3) contribution is used at the representation level. The full continuous colour-gauge derivation remains owed.

### Debt 5 — Gauge dynamics

This paper studies ledger consistency. Gauge kinetic terms and field equations remain owed.

### Debt 6 — Three generations

The anomaly theorem is one-generation. Extension to a replicated three-generation structure is immediate if each generation repeats the same ledger — the anomaly conditions are linear and cubic in per-generation content and cancel generation by generation — but the derivation of *why* there are three generations remains outside this paper.

### Debt 7 — Baryon and lepton number anomalies

The Standard Model has global accidental currents, such as baryon and lepton number, whose anomaly structure differs from the gauge-source ledger: they are anomalous, yet the theory is consistent, because they are not gauged. Their VERSF interpretation — presumably as non-committed bookkeeping grades rather than persistent record currents — remains owed.

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## 20. Falsification Conditions

### F1 — Anomaly non-obstruction

If a nonzero gauge anomaly can exist in a persistent physical VERSF source sector, Theorem 1 fails.

### F2 — Counterterm rescue

If a nonzero anomaly class can be removed by admissible substrate counterterms rather than merely shifted between currents, Lemma 1 fails.

### F3 — Hidden reservoir

If the substrate requires hidden anomaly-carrying chiral sectors not present in the one-generation skeleton, the hypercharge rigidity result is incomplete (SI-6 fails).

### F4 — Failure of local anomaly descent

If VERSF chiral source-carrier fields evade standard local anomaly descent while still carrying ordinary chiral gauge/source coupling, the bridge from QFT anomaly to substrate ledger leak must be rebuilt (SI-7 fails).

### F5 — Skeleton failure

If the inherited chiral matter skeleton fails, the application of the theorem to Standard Model hypercharge fails, though the general anomaly-inadmissibility theorem may survive.

### F6 — Hypercharge convention mismatch

If $Q = T_3 + Y$ is not the correct convention for the inherited ledger, the numerical table must be translated. This is a convention failure, not a structural one, provided $Y_{SM} = 2Y$ restores agreement.

### F7 — Global anomaly obstruction

If a nonperturbative global anomaly appears despite the perturbative ledger closing, the representation remains incomplete.

### F8 — Mass contamination

If the anomaly-inadmissibility result is found to require observed masses, Yukawas, CKM, PMNS, or Higgs data, the representation-prior firewall fails.

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## 21. Milestone Statement

The result of this paper is:

chiral electroweak source ledger + persistent record-current closure + bit/source balance + local anomaly descent

$\Longrightarrow$

$[\Omega_{ledger}] = 0.$

Applied to the one-generation skeleton, this gives:

$L_L : (1, 2, -\frac{1}{2}),\ e_R : (1, 1, -1),$

$Q_L : (3, 2, \frac{1}{6}),\ u_R : (3, 1, \frac{2}{3}),\ d_R : (3, 1, -\frac{1}{3}).$

The paper therefore upgrades the prior anomaly audit from *anomaly-free because checked*

to

*anomaly-free because only anomaly-free ledgers are substrate-admissible*.

That is the intended milestone.

---

# 22. Conclusion

The previous VERSF papers derived a one-generation chiral matter skeleton, an electroweak representation algebra, and a local electroweak connection form. They also showed that anomaly freedom rigidly fixes the quark hypercharges once the lepton normalization is set.

The present paper supplies the missing substrate reason anomaly freedom is required.

A gauge/source anomaly is a ledger leak. It means the current associated with a local source transformation does not close after chiral quantisation. In ordinary field theory this is a gauge inconsistency. In VERSF it is more fundamental: it is a failure of committed record closure. A source table that leaks under local transport cannot be written into the substrate as a persistent record.

The central theorem is therefore:

$[\Omega_{anom}] \neq 0 \Rightarrow$ substrate inadmissibility.

From this, anomaly cancellation follows as a substrate condition rather than an external consistency rule. The Standard Model one-generation hypercharge ledger is then not fitted and not merely checked. It is the unique admissible closure of the inherited matter skeleton under persistent source-balance.

The programme has therefore advanced:

chiral matter skeleton → electroweak representation/connection → substrate anomaly-inadmissibility.

The remaining work is now sharper: construct the full substrate cohomology internally, classify global anomalies, complete the colour lift, animate the derived gauge connection with kinetic dynamics, and then attach the already-separate EWSB and mass/mixing modules.

That is the intended milestone.

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# Appendix A — Standard Anomaly Conditions as VERSF Ledger Closures

The table below places the standard one-generation anomaly conditions beside their VERSF ledger-closure readings, for direct audit. Sums run over left-handed Weyl sectors with the multiplicities and hypercharges of §9; the lepton normalization $Y_L = -\frac{1}{2}$, $Y_e = -1$ is fixed upstream.

| Standard anomaly | Standard condition | VERSF reading |
|---|---|---|
| $SU(3)^2U(1)$ | $2Y_Q - Y_u - Y_d = 0$ | colour-supported hypercharge ledger closes |
| $SU(2)^2U(1)$ | $3Y_Q + Y_L = 0$ | weak-branch hypercharge ledger closes |
| $\mathrm{grav}^2U(1)$ | $6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$ (total $Y = 0$) | total source grade globally balances |
| $U(1)^3$ | $6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0$ (total $Y^3 = 0$) | abelian source self-ledger closes |
| Witten SU(2) | $N_{doublets}$ even (here $3 + 1 = 4$) | global weak-branch transport closes |

Solution status: $SU(2)^2U(1)$ fixes $Y_Q = \frac{1}{6}$; $SU(3)^2U(1)$ fixes $Y_u + Y_d = \frac{1}{3}$; $U(1)^3$ fixes $Y_u Y_d = -2/9$, whose quadratic has discriminant 1 and roots $\{\frac{2}{3}, -\frac{1}{3}\}$; $\mathrm{grav}^2U(1)$ is an independent audit that reproduces $Y_u + Y_d = \frac{1}{3}$; the Witten condition is passed by the colour-triplicated doublet count. Each row is, by Theorem 1, a substrate-admissibility condition rather than an external consistency test.