The Substrate Cohomology, Global Gauge Measure, and Positive-Pushforward Strong-Phase No-Go Theorem in VERSF

Finite-Regulator Positive-Pushforward No-Go Theorem and Pass Programme

▲ Programme Milestone — Global Quantum Measure, Faithful-Quotient Topology and Strong-Phase Completion Series

Gate SGMC-1R / Operational-to-History Measure Extension, Faithful-Group Bundle Stack, Determinant-Line Descent, Gribov-Free Global Quantisation, Triality-Compatible Reflection-Positive Reconstruction, Conditional Quark-Line Real Trivialisation, and Positive-Pushforward Strong-Phase No-Go Theorem

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Paper 6 of the Final Eight Standard Model Completion Papers
Tier A global non-perturbative completion theorem — July 2026

General Reader Summary

The earlier Quantum Gauge Completion paper explained how the VERSF Standard Model is quantised in ordinary perturbative calculations. It supplied gauge fixing, ghosts, BRST and BV symmetry, perturbative anomaly cancellation, renormalisation and the local physical-state rulebook. That was essential, but it addressed the theory near one regular gauge background. A complete quantum gauge theory must also work across every topological bundle, every large gauge transformation and every part of the gauge-orbit space.

This paper tackles that global problem.

Its first move is to separate three different notions of probability. VERSF already has a quadratic probability measure on finite operational alternatives. But a probability rule for a finite state is not automatically a probability measure on an entire substrate history. And a history measure is not automatically a functional measure on continuum gauge and matter fields. The paper therefore inserts the missing middle layer. Finite operational measures must agree under every

admissible refinement. If they form a consistent and tight projective family, they define one regular history measure. The continuum measure is then the pushforward of that history measure—not a separately chosen path-integral weighting.

The second move is global gauge topology. The faithful Standard Model gauge group is the product of colour, weak isospin and hypercharge divided by a common six-element centre. This quotient changes which bundles and line operators exist. A complete measure must include quotient bundles that cannot be lifted to three independent product-group bundles. Those sectors are genuine configurations of the physical group.

The third move is to stop pretending that one gauge condition can define the theory everywhere. Non-Abelian gauge fixing can intersect the same orbit repeatedly. Paper 6 therefore defines the nonperturbative measure directly on gauge orbits, retaining bundle sectors and stabilisers. Familiar Faddeev–Popov and BRST expressions remain useful, but only as local coordinates on this global object.

The fourth move concerns fermions. A chiral determinant is not generally an ordinary function of the gauge field. It is a section of a complex line bundle. Perturbative anomaly cancellation says the curvature of the total Standard Model determinant line vanishes. A flat line may still have nontrivial global holonomy. Paper 6 therefore requires a faithful-group bordism and eta-invariant audit before the chiral measure is declared globally consistent.

The flavour papers of the completion series sharpen the strong-phase calculation. The physical strong phase is

$$\bar{\theta} = \theta_3 + \arg\det(M_uM_d).$$

The microscopic Yukawa paper provides a conditional Hermitian representative in which the up- and down-quark determinants are both positive. On that branch,

$$\arg\det(M_uM_d) = 0.$$

This is real progress: it is the first explicit conditional trivialisation of the quark determinant factor. It does not settle the physical answer, because the representative's right-handed frames and orientation have not yet been returned by the frozen substrate. The strict physical branch still has to calculate them.

The quark-mixing papers also make an important distinction unavoidable. CKM CP violation and strong CP are not the same phase. CKM CP violation comes from the relative orientation of the left-handed up and down flavour frames. The determinant phase comes from the full left–right mass maps. A theory may therefore have a nonzero Jarlskog invariant while $\det(M_uM_d)$ is positive. The conditional VERSF flavour branch does exactly that. Paper 6 promotes this from a warning into a theorem.

The strong-interaction papers sharpen the relation to confinement. Paper 5 already identifies the centre-surface operator, triality sectors, the colour stiffness and the wall-response coefficient required for a nonzero triality gap. Paper 6 supplies the global measure, reflection-positive transfer matrix and physical Hilbert-space reconstruction in which those sectors must live. It also tests a candidate source for the wall coefficient: the real part of the substrate-to-continuum Jacobian may generate a positive Euclidean penalty for gradients of the gauge-action density. A correct one-dimensional Cauchy–Schwarz argument gives a positive wall cost when nonzero triality enforces distinct core and exterior action densities across a uniformly bounded wall.

That local wall cost is not yet confinement. A shortcut entropic area-law argument fails in three places: fixed total flux does not by itself give an area-extensive field-strength bound; a positive density of flux configurations does not imply exponential area suppression; and the required positivity must be Euclidean, not Lorentzian. Paper 6 keeps the valid wall mechanism and rejects the invalid area-law shortcut. The physical route remains: derive triality-sector coercivity, establish longitudinal persistence, reconstruct a positive transfer matrix, and then prove the potential or Wilson-loop theorem.

The physical Hilbert space is reconstructed only after reflection positivity. Paper 6 then compares that Osterwalder–Schrader space with the terminal-record GNS space obtained from adjoint removal. In the adjoint-only colour restriction, the transfer matrix must commute with the centre-surface operator so that the same triality sectors appear in both reconstructions.

The final strong-phase verdict remains a trichotomy:

$\bar{\theta} = 0$, $\bar{\theta} \neq 0$, or a global obstruction.

A conditional branch with a real quark determinant exists, so on that branch the entire question reduces to the substrate topological character. The strict result still requires the physical quark determinant phase and the substrate sector character to be calculated independently.

The sharpest single result is a fork. If the continuum quantum object really is an ordinary positive probability measure pushed forward from committed substrate records, then a nonzero $\bar{\theta}$ is not merely unlikely — it is impossible; the only alternatives are $\bar{\theta} = 0$ or a global measure obstruction. But that "if" is itself a strong foundational premise. Quantum theory keeps complex amplitudes before commitment and positive probabilities after it, and the strong phase lives exactly at that interface. The paper therefore states the fork as a no-go theorem and keeps the physical pass on the ledger until four named calculations — history decoherence, the transfer-operator derivation, the loop-generator census and the topology-support witness — are executed.

In plain terms:

Paper 6 builds the global quantum stage, identifies the exact line on which strong CP lives, proves that a positive pushforward measure cannot carry a nonzero strong phase, supplies the first conditional real quark-line branch, proves why weak CP cannot decide strong CP, and gives a corrected candidate mechanism by which the global measure may contribute to confinement — without pretending that the remaining physical calculations, or the foundational positive-measure premise itself, have already been established.

Principal Claim

Principal claim. The paper takes the positive-pushforward claim literally and derives its consequence at finite regulator. If the complete continuum gauge object is an ordinary positive pushforward probability measure over the supported topological sectors of the faithful quotient, then a residual nonzero strong phase is impossible. The result is a structural fork:

$$[\![\text{ ordinary positive global pushforward } \Longrightarrow \bar{\theta}\_\text{VERSF} = 0 \pmod{2\pi}]\!]$$

or the positive-pushforward description of the continuum quantum object is globally obstructed. This is a **Positive-Pushforward No-Go Theorem for a Nonzero Strong Phase**, proved conditionally in Part IXA. The mathematical distance between premise and conclusion is short and the paper says so exactly: within this architecture the premise is logically equivalent to the fork, up to the obstruction alternative — P-POS $\Longrightarrow$ ($\bar{\theta} = 0$ or obstruction) and $\bar{\theta} \neq 0 \Longrightarrow \neg$P-POS — so the theorem's value lies in the foundational commitment it isolates (Premise P-POS) and in the combined-line closure of the θ_3-to-quark-mass rotation loophole, not in the positive-measure-implies-no-phase implication, which is the standard complex-action fact underlying the Euclidean sign problem.

The physical strong-phase pass is not yet claimed. Four executed substrate calculations separate the no-go theorem from a strict pass: terminal-history decoherence (P-H1), microscopic derivation of the positive Euclidean transfer operator (P-H2), completeness of the determinant-line loop-generator census (P-H3), and a unit-topology support witness with regulator-removal stability (P-H4). The fork itself additionally rests on the foundational premise that the uncommitted continuum object is an ordinary positive measure rather than a complex line-valued amplitude functional (Premise P-POS): positive Born probability for committed terminal records does not by itself imply a positive pointwise measure over uncommitted gauge histories. Part IXA states the construction, the no-go proof, the named premises and the remaining hinges. Silence is not an admissible outcome: the completed evaluation must return zero, a named obstruction, or a demonstrated failure of P-POS.

The Quantum Gauge Completion paper closes the local perturbative rulebook. It does not construct the global quantum theory.

The present paper supplies the strongest global completion assemblable from the frozen VERSF stack, including the flavour, colour and confinement papers of the completion series. Its claim has seven parts.

First, the word "measure" is split into three typed objects that must not be conflated:

$$\mu\_\text{op}^{(\alpha)} \longrightarrow \mu\_\text{hist} \longrightarrow \mu\_\text{cont} = \mathfrak{Q}\_{*}\mu\_\text{hist}.$$

The finite Born/operational measure is not itself a history measure, and a history measure is not itself the continuum gauge-field functional measure. For a directed family of finite admissible substrate quotients, projective consistency, operational-indistinguishability

invariance and tightness give a unique regular history measure. This is the exact bridge required before the continuum measure can be called substrate-derived.

Second, the continuum target is the complete faithful-group configuration stack

$$\mathfrak{C}_{G_{SM}}(X) = \coprod_{[P] \in \mathrm{Bun}_{G_{SM}^{faithful}}(X)} \left[\mathcal{A}(P) \times \Gamma(\mathcal{F}_P) \times \Gamma(\mathcal{S}_P) / \mathcal{G}(P) \right],$$

$$G_{SM}^{faithful} = (SU(3)_C \times SU(2)_L \times U(1)_Y)/\mathbb{Z}_6.$$

Every quotient bundle is represented locally by lifted transition functions whose triple-overlap defect is a six-cocycle

$$\tilde{g}_{ij}\tilde{g}_{jk}\tilde{g}_{ki} = \zeta^{n_{ijk}}, \quad [n_{ijk}] = \nu_6(P) \in H^2(X, \mathbb{Z}_6),$$

and the inherited matter fields descend because

$$2t+3d+q \equiv 0 \pmod{6}.$$

Non-liftable $\nu_6(P)\neq 0$ sectors are part of the physical global census. Restricting the measure to the covering product or to one trivial bundle is not a global Standard Model construction.

Third, the measure is defined directly on the gauge-orbit stack. A globally unique gauge slice is neither assumed nor required. Faddeev–Popov, BRST and BV appear only as local coordinate descriptions on regular orbit charts. This avoids substituting a signed global copy count for the physical measure and keeps the Neuberger 0/0 mechanism outside the fundamental definition.

Fourth, the fermionic sector is promoted from a positive CAR/Fock carrier to a family of chiral Dirac operators over every faithful-group bundle. Its determinant is a section of a line bundle, not generally a complex-valued function. Local anomaly cancellation makes the combined line flat; the global eta-invariant/bordism character decides whether it is actually trivialisable.

Fifth, the microscopic Yukawa evaluation paper supplies the first explicit conditional strong-phase branch. On its declared Hermitian corpus representative

$$\det Y_d^C>0, \quad \det Y_u^C>0,$$

so

$$\arg\det(M_u^C M_d^C) = 0 \pmod{2\pi}.$$

This does not yet prove the physical quark determinant phase. The right-handed frames, canonical orientation and dressed mixed bilinear remain substrate outputs. It does,

however, establish a genuine conditional real trivialisation of the quark determinant factor and reduces that branch to

$$e^{i\bar{\theta}_C} = \chi_{sub}(1).$$

Sixth, the paper proves a Weak-CP/Strong-Phase Independence Theorem. A nonzero Jarlskog invariant measures relative left-frame misalignment, while arg det (M_uM_d) measures the anomalous axial orientation of the full left–right mass maps. VERSF may therefore return CKM CP violation and a positive real quark determinant simultaneously. The conditional quark-sector branch is an explicit example.

Seventh, Paper 5's triality architecture is linked to the global measure and physical Hilbert space. If the reconstructed transfer matrix commutes with the correctly embedded centre-surface operator, the adjoint-only physical Hilbert space decomposes into triality sectors. A candidate local wall coefficient may arise from the real part of the substrate pushforward Jacobian through the Euclidean gradient functional

$$S_{EG} = c_6^{sub}/\Lambda_{sub}^2 \int_X |d\varphi_E|^2/(\varphi_E+\varepsilon), \quad \varphi_E = \tfrac{1}{2}\, \mathrm{tr}\, F_{\mu\nu}F_{\mu\nu}.$$

The paper derives the correct conditional wall-cost inequality for this branch. It does not inherit the earlier area-law argument: one fixed centre flux does not produce an area-extensive F^2 bound, a positive probability of flux configurations does not imply an area law, and Lorentzian tr F^2 is not positive. The physical area law still requires triality-sector coercivity, longitudinal slicing, reflection positivity and a transfer-matrix theorem.

The invariant strong phase lives on

$$\mathcal{L}_{\bar{\theta}} = \mathcal{L}_{\theta_3} \otimes \mathcal{L}_{(\det M_u)} \otimes \mathcal{L}_{(\det M_d)},$$

$$e^{i\bar{\theta}} = e^{i\theta_3} \det(M_uM_d)/|\det(M_uM_d)|.$$

The frozen substrate must return exactly one of three outcomes:

$$[\![\text{ positive real trivialisation} \Rightarrow \bar{\theta} = 0]\!],$$

$$[\![\text{ non-real unitary trivialisation} \Rightarrow \bar{\theta} \neq 0]\!],$$

$$[\![\text{ no global trivialisation} \Rightarrow \text{global anomaly or measure obstruction}]\!].$$

The paper does not choose the attractive branch by wording. It contains one conditional real quark-line branch, but the strict physical answer still requires the substrate-selected topological character and the physical blind determinant phases.

For Paper 5, this construction supplies the global measure, reflection-positive reconstruction, triality-compatible Hilbert architecture and a candidate measure-generated contribution to B_τ. It does not by itself establish the global coercive gap. The $N_f=3\to 0$ matching bridge D9 remains separate.

Headline Results and Conditionality

Object	Result in this paper	Status
Finite operational measure	$\mu_{op}^{(\alpha)}=\|\psi_\alpha\|^2$ on each admissible finite quotient	inherited
Operational-to-history bridge	projective-extension theorem for μ_{hist}	exact given SG1–SG3
Continuum measure	$\mu_{cont}=\mathfrak{Q}_*\mu_{hist}$	exact definition; physical evaluation open
Pushforward Jacobian	$\mathcal{J}_{(sub\to cont)}$ retained with real, nonlocal and line-phase parts	formula fixed; functional open
Candidate measure-generated wall term	$S_{EG}=(c_6^{sub}/\Lambda_{sub}^2)\int$	$d\varphi_E$
Conditional entropic wall cost	$B_\tau^{EG}>0$ under uniform core/exterior contrast and wall thickness	exact inequality given premises
Entropic area-law shortcut	rejected	fixed total flux/probability arguments insufficient
Faithful global group	$[SU(3)\times SU(2)\times U(1)]/\mathbb{Z}_6$	inherited/derived upstream
Quotient bundle class	$\nu_6(P)\in H^2(X,\mathbb{Z}_6)$	derived here
Complete configuration space	faithful-group bundle stack over every [P]	derived here
Global gauge fixing	not used; measure defined on orbit stack	construction theorem
Local BRST/BV	recovered on regular quotient charts	inherited local theorem
Fermion carrier	positive CAR/Fock carrier with local-field bridge	inherited conditionally
Chiral determinant	section of $\mathscr{L}_{ferm}$	exact typing
Local anomaly	curvature of total determinant line vanishes for inherited census	inherited from QGC
Global anomaly	bordism/eta-invariant character $\mathfrak{a}_{SM}$	exact audit target; evaluation open

Object	Result in this paper	Status
Chiral measure	global section from a local spectral regulator	conditional construction
Reflection positivity	explicit OS criterion and transfer-matrix reconstruction	conditional on SG9
OS/GNS identification	unitary equivalence on the common physical algebra	exact given SG10
Triality-compatible reconstruction	$[T, \hat{U}_\omega(\Sigma)]=0 \Rightarrow \mathcal{H}=\bigoplus_\tau \mathcal{H}_\tau$ in adjoint-only theory	conditional theorem
Physical mass magnitudes	singular values of canonical M_u, M_d	inherited
Physical determinant phase	$\arg\det(M_u M_d)$	strict output still open
Conditional MCHY branch	$\det Y_u^C>0$, $\det Y_d^C>0 \Rightarrow \arg\det(M_u^C M_d^C)=0$	conditional reference/corpus result
Weak-CP/strong-phase independence	Jarlskog invariant and determinant phase are independent invariants	exact theorem
Strong-phase line	$\mathcal{L}_{\bar\theta}=\mathcal{L}_{\theta_3}\otimes\mathcal{L}_{(\det M_u)}\otimes\mathcal{L}_{(\det M_d)}$	derived
Conditional branch reduction	$e^{i\bar\theta_C}=\chi_{sub}(1)$	exact on MCHY representative branch
Strong-phase verdict	positive-real, nonzero-phase, or obstruction	exact trichotomy
Terminal-history measure	$\mu_{sub}^{(a, R)}(h)=D(h, h)$ from class operators	exact given decoherence premise P-DEC (P-H1)
Euclidean transfer identification	$\mathbb{T} =$	R
Reflection symmetry of positive factor	$\Theta\mathbb{T}\Theta^{-1}=\mathbb{T}$, Θ	$\Omega\rangle=$
Record-subalgebra reflection positivity	$\omega((\Theta F)^*F) \geq 0$ on terminal-record algebra	conditional lemma under P-TR and P-REF; commutative algebra, nearly classical; full-algebra OS open
Positive continuum representability	continuum object is an ordinary positive pushforward	foundational Premise P-POS; derivation owed
Positive-pushforward no-go theorem	positive pushforward $\Longrightarrow \bar\theta=0$, else obstruction	conditional theorem (Theorem IXA-7)
Physical strong-phase pass	requires P-H1–P-H4 executed and P-POS established	not yet established

Object	Result in this paper	Status
Numerical physical $\bar{\theta}$	requires physical determinant phase and substrate character	open
Paper 5 Debt D8	measure, triality-compatible reconstruction and candidate local B_τ branch supplied	structural progress; coercive pass open
Paper 5 Debt D9	$N_f=3\to0$ matching	not discharged here

Abstract

The Quantum Gauge Completion of the VERSF Standard Model establishes the local perturbative quantum architecture of the inherited gauge and matter system: local orbit disintegration, Faddeev–Popov determinants, ghost typing, BRST and BV closure, perturbative anomaly cancellation, algebraic renormalisation and local physical cohomology. It explicitly leaves open the global quantum measure, the faithful-quotient bundle sum, determinant-line topology, Gribov control, the nonperturbative chiral measure, reflection positivity, the nonperturbative physical Hilbert space and the invariant strong phase. The present paper constructs the strongest global completion architecture supported by the frozen corpus and incorporates the flavour and confinement returns of the completion series.

The first result is an Operational-to-History Measure Extension Theorem. Let $\{X_\alpha, r_{\beta\alpha}\}$ be the directed family of finite operational substrate quotients, with universal operational measures μ_α. If the marginals are projectively consistent,

$$(r_{\beta\alpha})_*\mu_\beta = \mu_\alpha,$$

operationally invariant and tight, then the inverse-limit history space

$$X_{hist} = \varprojlim_\alpha X_\alpha$$

carries a unique regular measure μ_{hist} with those marginals. This theorem supplies the missing bridge between finite Born/operational probability and a complete substrate history measure without identifying the two objects.

The second result constructs the continuum target as the faithful-group configuration stack

$$\mathfrak{C}_{(G_{SM})}(X) = \coprod_{([P] \in \mathrm{Bun}_{(G_{SM}^{faithful})}(X))} \left[\mathcal{A}(P) \times \Gamma(\mathcal{F}_P) \times \Gamma(\mathcal{S}_P) / \mathcal{G}(P) \right],$$

with

$$G_{SM}^{faithful} = [SU(3)_C \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6.$$

Local lifts of a quotient bundle define a cocycle class

$\nu_6(P) \in H^2(X, \mathbb{Z}_6)$,

and the complete measure must include non-liftable sectors. The continuum measure is the pushforward

$\mu_{cont} = \mathfrak{Q}_*\mu_{hist}$,

with the full substrate Jacobian retained. The global definition is made directly on the gauge-orbit stack; local Faddeev–Popov/BRST/BV descriptions are recovered only on regular charts.

The third result constructs the global chiral-measure target. The positive CAR/Fock reconstruction supplies the algebraic fermion carrier, while the chiral matter and electroweak representation papers supply the faithful one-generation representation family. A regulated family of chiral Dirac operators therefore defines a determinant/Pfaffian line over the complete configuration stack. Local anomaly cancellation makes the total line flat. Global consistency additionally requires triviality of the faithful-group bordism/eta-invariant character. A global chiral measure is a local, integrable, gauge-equivariant section of this line; it is not obtained by choosing a phase independently in each gauge chart.

The fourth result strengthens the flavour input to the strong-phase problem. The physical invariant is

$\bar{\theta} = \theta_3+\arg\det(M_uM_d) \pmod{2\pi}$.

The microscopic Yukawa paper requires the blind publication of the physical determinant phases and supplies a conditional Hermitian corpus representative with

$\det Y_d^C>0,\ \det Y_u^C>0$.

Consequently,

$\arg\det(M_u^CM_d^C) = 0$

on that declared branch, and the combined phase reduces to

$e^{i\bar{\theta}_C} = \chi_{sub}(1)$.

This is a conditional real trivialisation of the quark determinant factor, not the strict physical answer: the canonical right frames, dressed mixed bilinear and anomaly-compatible orientation remain substrate outputs.

The fifth result is a Weak-CP/Strong-Phase Independence Theorem. CKM CP violation is measured by invariants of the relative left-handed Hermitian pair

$H_u^L = M_uM_u^\dagger,\ H_d^L = M_dM_d^\dagger$, whereas arg det $(M_u M_d)$ depends on the full left–right maps. Right-unitary phase rotations can change the determinant phase without changing either left Hermitian operator, and matrices with positive determinants can carry a nonzero Jarlskog invariant. The conditional VERSF quark branch exhibits precisely this structure. CKM agreement therefore neither solves nor obstructs strong CP.

The sixth result gives the reflection-positive reconstruction chain. If the complete regulated measure satisfies Osterwalder–Schrader positivity, covariance, regularity and clustering, it reconstructs a positive Hilbert space $\mathscr{H}_{OS}$, transfer matrix and Hamiltonian. If the Euclidean and terminal-record states agree on the common physical algebra, $\mathscr{H}_{OS}$ is canonically isometric to the adjoint-removal GNS space. In an adjoint-only colour restriction, compatibility with Paper 5 additionally requires

$$[T, \hat{U}_\omega(\Sigma)] = 0,$$

giving the triality-sector decomposition of the physical Hilbert space. With fundamental quarks, this is replaced by the corresponding static-source/screening architecture rather than an exact one-form superselection rule.

The seventh result evaluates a candidate measure-generated contribution to the triality wall coefficient. If the real part of the pushforward Jacobian contains the Euclidean local term

$$S_{EG} = c_6^{sub}/\Lambda_{sub}^2 \int_X |d\varphi_E|^2/(\varphi_E+\varepsilon), \quad \varphi_E = \tfrac{1}{2}\, \mathrm{tr}\, F_{\mu\nu}F_{\mu\nu},$$

then a correct normal-line Cauchy–Schwarz estimate gives a positive wall cost when nonzero triality enforces distinct core and exterior values across a uniformly bounded wall. This supplies a candidate local contribution B_τ^{EG}. It does not prove a global triality gap or area law. The earlier fixed-flux patch argument has the wrong area scaling, a positive probability of flux configurations does not imply exponential area suppression, and Lorentzian positivity cannot be used. The physical confinement theorem still requires global sector coercivity, longitudinal slicing, reflection positivity and regulator-uniform convergence.

The combined strong-phase line is

$$\mathcal{L}_{\bar\theta} = \mathcal{L}_{\theta_3} \otimes \mathcal{L}_{(\det M_u)} \otimes \mathcal{L}_{(\det M_d)}.$$

Its substrate section returns exactly one of three statuses: a positive real trivialisation with $\bar\theta=0$; a non-real unitary trivialisation with a calculated nonzero $\bar\theta$; or a global anomaly/measure obstruction. No measured strong-CP bound, electric-dipole result, lattice topological weight or observed quark phase may select the branch.

The paper therefore closes the global architecture, supplies the first conditional real quark-line branch, and sharpens Paper 5's D8 interface. A strict numerical pass remains dependent on the physical projective marginals, pushforward Jacobian, faithful bundle census, chiral regulator, reflection-positive decomposition, substrate topological character and blind physical determinant phases. Silence is not an admissible final result: the evaluation must return zero, a nonzero phase or an obstruction.

Finite-regulator no-go theorem and pass programme. Part IXA constructs the regulated terminal-history measure from class operators under a named decoherence premise, states the positive-transfer identification as a premise requiring microscopic derivation, performs the lifting-obstruction census and a conditional determinant-line audit on a periodic cellular regulator, and proves the Positive-Pushforward No-Go Theorem: under Premise P-POS, an ordinary positive quotient-compatible pushforward forces

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⟦ θ⁻_VERSF = 0 (mod 2π) ⟧
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or fails to exist, in which case the return is a named global measure obstruction. The verdict of this paper is two-line:

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⟦ finite-regulator positive-pushforward no-go theorem — established
conditionally ⟧

⟦ physical strong-phase pass — not yet established ⟧
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The pass requires executing hinges P-H1–P-H4 and independently establishing P-POS. The implication inside the no-go theorem is, stripped of VERSF framing, the standard complex-action statement that $\theta \neq 0$ Euclidean gauge theory admits no positive-measure representation; the content claimed as new is the foundational premise P-POS, its logical equivalence to the conclusion up to the obstruction alternative stated openly, and the combined-line closure that prevents hiding the phase by rotating θ_3 into the quark mass matrices.

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Part I — The Global Quantum Problem

1. Aim and Place Among the Final Eight

This paper is Paper 6 of the final eight-paper VERSF Standard Model completion programme.

Papers 1–5 address the scalar vacuum, neutral-fermion branch, microscopic Yukawa operators, gauge-response coefficients and the strong-interaction scale/triality architecture. The Quantum Gauge Completion paper supplies the local perturbative quantum rulebook. The present paper has a different task:

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local quantum field theory ⟶ one globally defined nonperturbative quantum
theory.
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The distinction is not editorial. It separates four statements that are frequently conflated:

1. the local gauge algebra closes;
2. perturbative gauge anomalies cancel;
3. the gauge-fixed theory has BRST cohomology and perturbative unitarity;
4. the complete theory possesses a globally defined measure, a nonperturbative chiral determinant, a positive physical Hilbert space and a consistent sum over every faithful-group bundle.

The first three do not prove the fourth.

The paper has two principal outputs.

Global-measure output. Construct the exact mathematical chain by which the substrate's finite operational probability data can define a complete history measure and push forward to the faithful Standard Model configuration stack.

Strong-phase output. Construct and evaluate, in the precise conditional sense developed below, the invariant combined topological/quark determinant line. The paper must return a positive real trivialisation, a nonzero phase, or an obstruction once the frozen input slots are supplied.

A definition-only paper would not pass. A paper repeating local BRST closure would not pass. A paper setting $\bar{\theta}=0$ because zero is phenomenologically attractive would fail the programme's non-insertion discipline.

2. Why Local Quantum Gauge Completion Is Not Global Completion

The local perturbative gauge construction operates in a regular orbit neighbourhood. One assumes that the gauge action is locally free after exact stabilisers and zero modes are separated, chooses a valid local slice, inserts the Faddeev–Popov determinant, introduces ghosts and obtains a nilpotent BRST differential. The BV master action then packages the local gauge redundancy in a coordinate-independent perturbative form.

None of this determines:

- which principal G_SM^faithful-bundles occur;
- whether a non-liftable quotient bundle was omitted;
- whether the chiral determinant line has nontrivial flat holonomy;
- whether one global gauge slice exists;
- whether a nonperturbative regulator admits a consistent chiral phase;
- whether the Euclidean measure is reflection positive;
- whether the resulting nonperturbative state space is positive and complete;
- which phase the substrate assigns to the combined strong line.

The local/global separation can be written compactly. Let $\mathfrak{g}$ be the Lie algebra and G the faithful global group. Local BRST data depend on $\mathfrak{g}$. Bundle sums, line operators, large gauge transformations, torsion sectors and determinant-line holonomy depend on G.

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𝔤_SM = 𝔰𝔲(3) ⊕ 𝔰𝔲(2) ⊕ 𝔲(1)
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does not by itself determine

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G_SM^faithful = (SU(3) × SU(2) × U(1))/ℤ₆.
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The same local connection one-form can therefore belong to globally inequivalent quantum theories.

Local/Global Firewall. No result stated only in one regular gauge patch may be used as a substitute for the global bundle, determinant-line, Gribov, chiral-measure or reflection-positivity calculation.

3. Inherited Frozen Stack

The paper consumes the following frozen or conditionally frozen VERSF structures.

3.1 Gauge structure

The inherited faithful group is

$$G_{SM}^{faithful} = (SU(3)_C \times SU(2)_L \times U(1)_Y)/\mathbb{Z}_6,$$

with generator

$$\zeta = (\omega I_3, -I_2, e^{i\pi/3}), \quad \omega = e^{2\pi i/3}.$$

Every inherited field with colour triality t, weak duality d and integer hypercharge q=6Y satisfies

$$2t+3d+q \equiv 0 \pmod 6.$$

3.2 Local quantum gauge architecture

The Quantum Gauge Completion paper supplies:

- local Faddeev–Popov orbit disintegration;
- the typed ghost complex;
- BRST nilpotency and BV closure;
- local Standard Model anomaly cancellation;
- the axial measure Jacobian;
- perturbative renormalisation and physical cohomology;
- the invariant strong-phase formula;
- explicit global debts.

3.3 Operational measure

The operational-probability programme supplies finite-dimensional or finite-resolution operational carriers $\mathscr{H}_\alpha$ with positive quadratic measure

$$\mu_\alpha(\psi) = \|\psi\|_\alpha^2$$

and invariance under operationally structure-preserving maps. The present paper does not identify this directly with a field-functional measure.

3.4 Physical quotient and positivity

The Adjoint-Removal theorem supplies a physical local algebra

$$\mathfrak{A}\_P(\mathcal{O}) = \mathfrak{A}\_K(\mathcal{O})/\mathfrak{J}\_AR(\mathcal{O}),$$

positive terminal-record states and a GNS representation free of negative-norm physical vectors. Microcausality descends once the kinematic antecedent is supplied.

3.5 Fermion carrier

The matter programme supplies a positive one-particle Hilbert space, antisymmetric Fock space, full CAR algebra, cyclic free vacuum and an emergent relativistic Dirac-field construction, all at the grades declared in those papers. These supply the carrier on which the chiral Dirac family must be built.

3.6 Yukawa and mass matrices

Paper 3 supplies the exact address of the physical matrices

$$M\_f(\mu\_\star) = v\_cl/\sqrt{2}Y\_f(\mu\_\star), \quad f = u,\ d,\ e,$$

and requires publication of determinant and phase invariants. The blind physical matrices are an input slot to the final strong-phase evaluation and are not replaced here by illustrative grids or observed masses.

3.7 Paper 5 global strong-sector architecture

Paper 5 supplies the centre operators, triality projectors, local wall-response target, and the exact statement of Debt D8: a quotient-compatible global colour measure, reflection-positive transfer matrix, physical Hamiltonian, sector coercivity and continuum convergence.

4. Claim Grades, Premises and Firewalls

The paper uses four grades.

1. **Exact structural result.** Follows from stated mathematical objects and premises.
2. **Conditional global theorem.** Follows uniquely once named substrate returns exist.
3. **Reference completion.** An explicit mathematically consistent construction demonstrating non-emptiness, not a substrate selection theorem.
4. **Open numerical object.** Formula and data address are fixed, but the substrate return is absent.

The main premises are:

SG1 — Directed finite-resolution system. The substrate admits a directed family of finite admissible operational quotients X_α.

SG2 — Projective consistency. For every refinement $\beta \geq \alpha$, the coarse map $r_{\beta\alpha}:X_\beta \to X_\alpha$ satisfies

$$(r_{\beta\alpha})_*\mu_\beta = \mu_\alpha.$$

SG3 — Tightness and regularity. The family is tight in the topology used for the history limit, and all spaces are standard Borel or compact Hausdorff at regulator level.

SG4 — Measurable substrate-to-continuum map. There is a measurable coarse-graining map

$$\mathfrak{Q}:X_{hist} \to \mathfrak{C}_{(G_{SM})}(X)$$

compatible with locality, field typing and gauge equivalence.

SG5 — Gauge-equivariance. Substrate transformations representing continuum gauge changes map to the same point of the quotient stack.

SG6 — Complete faithful bundle census. Every admissible $G_{SM}^{faithful}$-bundle is included, including non-liftable $\nu_6 \neq 0$ sectors.

SG7 — Local spectral fermion regulator. On every regulated bundle sector there exists a gauge-covariant local chiral Dirac family with the correct continuum index and CAR/Fock limit.

SG8 — Global anomaly integrability. The tensor product of all chiral determinant/Pfaffian and counterterm lines has trivial anomaly character on the complete faithful-group bordism generators.

SG9 — Reflection positivity. The complete regulated measure, including the fermionic line section, satisfies the declared reflection-positive decomposition.

SG10 — Record/Euclidean compatibility. The Euclidean expectation functional and the terminal-record state agree on their common physical observable algebra.

SG11 — Nonzero physical quark determinants. $\det M_u \det M_d \neq 0$ on the frozen physical branch. The massless branch is separately typed.

SG12 — Non-insertion. No observed strong phase, neutron electric dipole bound, quark mass phase or measured topological susceptibility selects any premise, section or branch.

SG13 — Triality-compatible reconstruction. In the adjoint-only colour restriction, the regulated measure and transfer matrix represent the correctly embedded centre-surface operator and satisfy

$$[T_{(a, R)}, \hat{U}_\omega(\Sigma)] = 0.$$

In full QCD, the corresponding static-source and screening sectors are represented without falsely imposing an exact one-form superselection rule.

4.1 Measure firewall

```
μ_op ≠ μ_hist ≠ μ_cont
```

as types. Equality may be proven only after specifying the relevant identification or pushforward.

4.2 Gauge-fixing firewall

A local gauge slice may be used to calculate. It may not define the global theory.

4.3 Positivity firewall

GNS positivity of a record state and perturbative BRST unitarity do not, separately or jointly, prove Osterwalder–Schrader reflection positivity of the complete Euclidean gauge measure.

4.4 Strong-phase firewall

Neither θ_3 nor arg det (M_uM_d) is separately physical. Only the combined line and its invariant holonomy may be reported.

4.5 Empirical firewall

The strong-phase branch is frozen before any comparison with the empirical bound on strong CP.

Part II — From Operational Probability to a Substrate History Measure

5. The Three-Measure Distinction

The finite operational measure, the history measure and the continuum functional measure answer different questions.

5.1 Finite operational measure

For an admissible finite-resolution carrier X_α, μ_α assigns weight to operational alternatives visible at that resolution. In a Hilbert representation this is the quadratic measure inherited from the operational programme.

5.2 Substrate history measure

A history is a compatible choice of operational state at every resolution. The history measure assigns weight to cylinder sets specifying finitely many such choices. It must reproduce every μ_α as a marginal.

5.3 Continuum gauge measure

The continuum measure assigns weight to gauge, matter and scalar configurations, bundle by bundle, after coarse-graining. It is obtained from the history measure by pushforward and generally includes a nontrivial Jacobian and a determinant-line phase.

The logical chain is therefore:

```
finite operational probability → consistent history probability → continuum field measure.
```

Skipping the middle arrow is a type error.

6. Finite Operational Quotients and Refinement Maps

Let $\mathfrak{R}$ be the directed set of admissible finite substrate resolutions. For each $\alpha \in \mathfrak{R}$, let

$$X_\alpha$$

be the set of operationally distinct substrate configurations at resolution α, already quotiented by distinctions that no admissible record at that resolution can witness.

For $\beta \succeq \alpha$, refinement gives a measurable coarse map

$$r_{\beta\alpha}: X_\beta \to X_\alpha.$$

These maps obey

$$r_{\alpha\alpha} = \mathrm{id}, \quad r_{\gamma\alpha} = r_{\beta\alpha} \circ r_{\gamma\beta} \quad (\gamma \succeq \beta \succeq \alpha).$$

The operational measure programme supplies a probability measure μ_α on each X_α. The first genuine global test is not the form of any one μ_α. It is whether the family is compatible:

$$(r_{\beta\alpha})_* \mu_\beta = \mu_\alpha.$$

This condition says that refining and then forgetting the extra distinctions gives the same probabilities as never introducing those distinctions.

It is the history-level form of refinement stability and operational commensurability.

7. The Operational-to-History Measure Extension Theorem

Define the inverse-limit history space

$$X_{hist} = \varprojlim_{(\alpha\in\mathfrak{R})} X_\alpha = \{ (x_\alpha)_\alpha : r_{\beta\alpha}(x_\beta) = x_\alpha \}.$$

Let $\pi_\alpha : X_{hist} \to X_\alpha$ be the canonical projections.

Theorem 1 — Operational-to-History Measure Extension

Assume SG1–SG3. Then there exists a unique regular probability measure μ_{hist} on the cylinder sigma-algebra of X_{hist}, extending to the declared Radon completion, such that

$$(\pi_\alpha)_* \mu_{hist} = \mu_\alpha \text{ for every } \alpha.$$

Proof

For every finite family of resolutions, directedness supplies a common refinement γ. The corresponding joint distribution is defined by pulling the cylinder specification to X_γ and evaluating with μ_γ. Projective consistency makes this definition independent of the chosen common refinement. The family of cylinder distributions is therefore finitely consistent.

The standard projective-limit extension theorem supplies a unique measure on the cylinder sigma-algebra. Tightness and regularity supply the declared Radon extension and exclude probability loss into a noncompact refinement boundary. Uniqueness follows because cylinder sets generate the sigma-algebra. ■

Remark — Exactness requirement

The projective-limit machinery requires exact marginal consistency: $(r_{\beta\alpha})_* \mu_\beta = \mu_\alpha$ as an identity, not an approximation. If the finite marginals are only approximately consistent — as they are whenever they arise from approximately decoherent history families — the theorem as stated does not apply, and no measure with the stated marginals need exist. An approximate variant with explicit error control (marginal defects summable along the directed family, with the limit measure unique up to a controlled total-variation error) would be required; it is not proved here. This exactness requirement propagates forward to Premise P-DEC of Part IXA and is part of hinge P-H1.

Corollary 1.1 — No independent history weighting

Once the finite operational measures and refinement maps are fixed, no additional history-level probability functional may be inserted without changing at least one finite operational marginal.

Corollary 1.2 — Operationally invisible distinctions remain null

If two raw histories have the same image in every X_α, they define the same point of X_{hist}. No history measure can assign them different physical weights.

8. Uniqueness, Null Distinctions and Commensurability

The extension theorem turns the operational-indistinguishability discipline into a global statement.

Let T be a transformation of the history space whose projections T_α preserve every finite operational measure:

$$(T_\alpha)_*\mu_\alpha = \mu_\alpha.$$

If T commutes with the projective maps, then

$$T_*\mu_{hist} = \mu_{hist}.$$

This is the history-level form of the Universal Measure Principle: probability depends on substrate content, not on a presentation of that content.

The theorem also fixes the proper relation to the adjoint-removal quotient. Operationally null completion residue must already be absent from the finite quotients or lie in the kernel of every π_α. It cannot reappear as an independently weighted continuum field.

9. What the Extension Theorem Does Not Calculate

Theorem 1 does not establish SG1–SG3 for the physical substrate. In particular, it does not calculate:

- the actual set of finite operational quotients;
- the full refinement maps;
- tightness of the family;
- the continuum coarse-graining map;
- the substrate-to-continuum Jacobian;

- the determinant-line phase.

The theorem closes the mathematical question:

What must be shown for finite VERSF operational probability to define one complete history measure?

It does not claim the substrate has passed those tests until their data are published.

Part III — Continuum Pushforward and Faithful-Group Configuration Space

10. The Substrate-to-Continuum Map

Let X be the regulated or limiting emergent Euclidean four-manifold. The continuum target is the stack of faithful-group gauge, matter and scalar configurations.

The substrate-to-continuum map is

$\mathfrak{Q}:X\_hist \to \mathfrak{C}\_(G\_SM)(X).$

It assigns to a compatible substrate history:

- a principal G_SM^faithful-bundle P→ X;
- a connection $A \in \mathcal{A}(P)$;
- chiral matter sections in the associated bundles;
- scalar/interface sections;
- the retained source and record data required by the effective action.

The map must be measurable, local or controlled quasi-local at the declared scale, and compatible with the field typing of the gauge and matter programmes.

11. Pushforward Measure and Jacobian

The continuum measure is defined by

$\mu\_cont(E) = \mu\_hist(\mathfrak{Q}^{-1}(E))$

for measurable subsets E of the configuration stack. Symbolically,

$$[\![\mu_{cont} = \mathfrak{Q}_* \mu_{hist}.]\!]$$

In a local coordinate chart this may be represented as

$$d\mu_{cont}^{abs} = \mathcal{J}_{(sub \to cont)}[A, \Phi]\, e^{(-S_{bos}[A, \Phi])}\, |\mathrm{PfD}[A, \Phi]|\, d\nu_{chart}.$$

The equality is a local density representation, not the global definition.

The logarithm of the Jacobian is an effective functional:

$$-\log \mathcal{J}_{(sub \to cont)} = \Delta S_{local} + \Delta S_{nonlocal} + i\Delta\varphi_{line}.$$

The local real part contributes to coupling and operator matching. The nonlocal part tests the validity of the effective local expansion. The phase belongs to the determinant/counterterm line and may not be discarded by declaring the Jacobian positive.

Jacobian Firewall. Setting $\mathcal{J}_{(sub \to cont)}=1$ is a branch assumption, not a derivation.

11.1 Candidate measure-generated entropic-gradient term

The real local part of the pushforward Jacobian may contain the Euclidean gauge-invariant operator

$$S_{EG}[A] = c_6^{sub}/\Lambda_{sub}^2 \int_X |d\varphi_E|^2/(\varphi_E+\varepsilon),\quad \varphi_E = \tfrac{1}{2} \mathrm{tr}\,(F_{\mu\nu}F_{\mu\nu}) \geq 0.$$

The coefficient c_6^{sub} is not set to O(1). It is an output of the substrate coarse-graining/Jacobian calculation. The term is written in Euclidean signature because neither tr $F_{\mu\nu}F^{\mu\nu}$ nor $\partial_\mu\varphi\partial^\mu\varphi$ is positive in Lorentzian signature.

This operator is a candidate local source of the wall-response coefficient required by Paper 5. It is not by itself an area-law or confinement theorem. Its physical use requires:

1. a regulator realisation compatible with reflection positivity;
2. a nonzero-triality condition that enforces distinct core and exterior values of φ_E;
3. a uniform upper bound on the wall thickness;
4. global sector coercivity and longitudinal persistence.

The exact conditional wall inequality is proved in §46B.

12. Gauge-Basicity of the Absolute Measure

Let $\mathcal{G}(P)$ act on the fields in bundle sector P. The absolute measure is gauge-basic if

$$|\mu_{cont}|(gE) = |\mu_{cont}|(E)$$

for every gauge transformation g.

Theorem 2 — Gauge-Basic Pushforward

Assume SG4–SG5 and that the history measure is invariant under substrate transformations mapped by $\mathfrak{Q}$ to continuum gauge transformations. Then the absolute pushforward measure descends uniquely to the quotient stack.

Proof

If E and gE differ only by a gauge transformation, gauge-equivariance gives

$$\mathfrak{Q}^{-1}(gE) = \tilde{g}\mathfrak{Q}^{-1}(E)$$

for the corresponding substrate transformation $\tilde{g}$. Invariance of μ_hist therefore gives equal weights. The pushforward is constant on gauge orbits and defines a measure on the quotient. Stabiliser information is retained by the stack rather than erased by a naive orbit-space quotient. ■

The theorem applies to the absolute density. The phase of a chiral determinant is a section of a line and requires the separate descent of Part IV.

13. The Faithful Standard Model Group

The common central element

$$\zeta = (\omega I_3, -I_2, e^{i\pi/3})$$

acts trivially on the inherited Standard Model matter and scalar ledger. Its powers form the full common kernel. Therefore

$$G\_SM^{faithful} = \tilde{G}\_SM/\langle\zeta\rangle, \quad \tilde{G}\_SM = SU(3)\_C \times SU(2)\_L \times U(1)\_Y.$$

The quotient leaves the local Lie algebra unchanged but changes global topology, admissible bundles and line operators.

A field of centre charges (t, d, q) descends precisely when

$$2t+3d+q \equiv 0 \pmod 6.$$

This relation is the field-by-field compatibility condition used below.

14. The Six-Cocycle and Quotient-Bundle Census

Let {U_i} be a good cover of X, with faithful-group transition functions

$$g_{ij}: U_i \cap U_j \to G_{SM}^{faithful}.$$

Choose local lifts

$$\tilde{g}_{ij}: U_i \cap U_j \to \tilde{G}_{SM}.$$

On a triple overlap, the lifts need not satisfy the covering-product cocycle condition. Instead,

$$\tilde{g}_{ij}\tilde{g}_{jk}\tilde{g}_{ki} = \zeta^{n_{ijk}}, \quad n_{ijk} \in \mathbb{Z}_6.$$

The quadruple-overlap identity makes n a Cech two-cocycle. A change of lifts changes n by a coboundary. Therefore

$$[\![\ \nu_6(P) = [n] \in H^2(X, \mathbb{Z}_6) \ ]\!]$$

is an invariant of the quotient bundle.

Its reductions

$$\nu_3(P) = \nu_6(P) \bmod 3, \quad \nu_2(P) = \nu_6(P) \bmod 2$$

record the correlated colour- and weak-centre obstructions. The hypercharge transition data are tied to the same six-cocycle by the quotient, rather than forming an independently selectable line bundle.

Theorem 3 — Faithful-Bundle Descent

For an inherited representation with centre charges (t, d, q), the associated local transition functions obey the ordinary cocycle condition on triple overlaps if and only if

$$2t+3d+q \equiv 0 \pmod 6.$$

Proof

The triple-overlap defect acts in the representation by

$$\rho(\zeta^{n_{ijk}}) = \exp\left[(i\pi\, n_{ijk})/3(2t+3d+q) \right].$$

This equals unity for every n_{ijk} exactly when the congruence holds. ■

Complete census requirement

The bundle sum is

$$\sum_{[P] \in Bun_{(G_{SM}^{faithful})}(X)}.$$

It may be organised by $\nu_6(P)$, instanton numbers, torsion classes, spin or spin-G structures and boundary data. It may not be replaced by the subset $\nu_6=0$ unless the substrate proves all other sectors inadmissible.

15. The Complete Configuration Stack

For each principal bundle P, let:

- $\mathcal{A}(P)$ be the affine space of connections;
- $\mathcal{F}_P$ be the direct sum of inherited chiral matter bundles;
- $\mathcal{S}_P$ be the scalar/interface bundle;
- $\mathcal{G}(P)$ be the gauge group.

The global configuration object is

$$[\![\mathfrak{C}_{(G_{SM})}(X) = \coprod_{[P]} [\mathcal{A}(P) \times \Gamma(\mathcal{F}_P) \times \Gamma(\mathcal{S}_P) / \mathcal{G}(P)].]\!]$$

Square brackets denote the quotient stack. This retains isotropy groups at reducible connections and avoids treating singular orbits as ordinary manifold points.

The complete partition functional is schematically

$$Z = \sum_{[P]} \int_{(\mathfrak{C}_P)} s_{ferm}\, e^{(-S_{bos})}\, d\mu_{cont}^{abs},$$

where s_ferm is a section of the determinant/Pfaffian line, not an ordinary global scalar until Part IV closes.

16. Line Operators, Triality and Quotient Compatibility

The faithful quotient correlates colour triality, weak centre charge and hypercharge. Consequently the globally admissible Wilson/'t Hooft line spectrum is not the direct product of independently chosen colour, weak and Abelian lines.

A line operator carrying centre data (t, d, q) is genuine only when its endpoint or dressing makes the total quotient charge trivial. The same congruence

$$2t+3d+q \equiv 0 \pmod 6$$

governs local fields and admissible global line dressings.

This matters for Paper 5. The colour-centre operator used to label triality sectors must be embedded into the complete faithful-group line algebra. In the adjoint-only restriction the $\mathbb{Z}_3$ one-form symmetry is exact. With fundamental Standard Model quarks it is explicitly broken and line operators may end on matter. The global measure must represent both statements without identifying them.

Part IV — Determinant Lines and the Nonperturbative Chiral Measure

17. The Regulated Dirac Family

For each bundle sector P and regulated gauge/scalar background (A, Φ), the inherited fermion carrier must supply a chiral Dirac family

$$D\_(\rho, A, \Phi) : \mathcal{H}\_(\rho, L) \to \mathcal{H}\_(\rho, R)$$

for every irreducible chiral representation ρ in the frozen matter census.

At finite regulator (a, R), the family must be:

1. gauge covariant;
2. local in the regulator metric;
3. compatible with the faithful quotient;
4. continuous on admissible gauge backgrounds;
5. index-correct in the continuum limit;
6. compatible with the positive CAR/Fock carrier;
7. stable under the regulator-removal scheme.

A useful reference realisation is a Ginsparg–Wilson/overlap-type family satisfying

$$\gamma_5 D + D\gamma_5 = a D\gamma_5 D,$$

with

$$\hat{\gamma}_5 = \gamma_5(1-aD), \quad \hat{P}_- = (1-\hat{\gamma}_5)/2.$$

This is an admissible mathematical implementation of the chiral measure gate. The strict VERSF route must derive or universality-match the required regulated kernel from the substrate-to-continuum and CAR constructions rather than merely import a lattice operator by name.

18. The Chiral Determinant/Pfaffian Line

For a family of chiral Fredholm operators, the determinant is not naturally a number. It is an element of the line

$$\mathrm{Det}\, D_\rho = \Lambda^{top} \ker D_\rho^* \otimes \Lambda^{top} \mathrm{coker}\, D_\rho.$$

The total fermion line is

$$\mathcal{L}_{ferm} = \otimes_{(\rho \in \mathcal{R}_{chiral})} \mathrm{Det} D_\rho,$$

with Pfaffian replacements where a real or pseudoreal Majorana structure requires them.

The regulated fermion measure is a section

$$s_{ferm} \in \Gamma(\mathcal{L}_{ferm}).$$

A choice of local chiral basis gives a local representative. On chart overlaps, basis changes multiply that representative by a transition phase. A globally defined chiral measure exists exactly when these local representatives glue to a global section with the required gauge covariance.

19. Local Anomaly Curvature and Its Cancellation

The determinant line carries a natural connection. Its curvature is the transgression of the local anomaly polynomial. Schematically,

$$i/2\pi F_{(\mathcal{L}_{ferm})} = \int_X I_6(F, R)_{(2)}.$$

The inherited Standard Model census makes the local gauge-anomaly polynomial vanish generation by generation. Therefore the total determinant line is flat after the allowed local counterterm lines are included:

$$F_{(\mathcal{L}_{total})} = 0.$$

Flat does not mean trivial. A flat line can have nontrivial holonomy around noncontractible loops in the configuration stack.

Flat/Trivial Firewall. Local anomaly cancellation proves zero curvature. Global anomaly freedom additionally requires trivial holonomy.

20. Global Holonomy, Eta Invariants and Bordism

Let γ be a closed loop in the faithful configuration stack. It defines a five-dimensional mapping torus Y_γ. The determinant-line holonomy is represented by the exponentiated eta invariant of the associated five-dimensional Dirac operator:

```
Hol_γ(ℒ_total) = exp(2π i(η(D_(Y_γ))+h)/2)
```

in the declared convention, with the local index contribution included consistently.

The global anomaly is therefore a character

```
α_SM: Ω₅^(Spin-G_SM^faithful) ⟶ U(1).
```

The precise tangential structure is ordinary spin where the quotient bundle admits it and the appropriate spin-G refinement where the gauge quotient participates in lifting the tangent structure.

Global anomaly freedom is

```
⟦ α_SM = 1 on every bordism generator. ⟧
```

21. The Faithful-Group Anomaly Audit

The inherited local census establishes:

- all perturbative gauge anomalies cancel;
- the ordinary SU(2) Witten sign anomaly is absent because there are twelve left-handed weak doublets across three generations;
- gauge-singlet right-handed neutrinos do not change the internal gauge anomaly.

The remaining faithful-group audit must evaluate:

1. non-liftable ν_6 bundle sectors;
2. torsion bordism generators;
3. spin versus spin-G sectors;
4. mixed gauge–gravitational global phases;
5. the optional neutral-fermion completion;
6. the full determinant-line holonomy.

Theorem 4 — Global-Anomaly Closure Criterion

Assume the local anomaly polynomial vanishes. Then a global gauge-equivariant chiral measure exists on the complete faithful configuration stack if and only if the total determinant/counterterm line has trivial bordism holonomy character.

Proof

Zero local anomaly makes the total line flat. A flat complex line admits a global gauge-equivariant trivialisation exactly when its holonomy representation of the fundamental group — equivalently its anomaly-theoretic bordism character on the mapping-torus cycles generated by large gauge transformations and quotient topology — is trivial. ■

The theorem reduces the global anomaly problem to a finite generator audit once the relevant bordism group is computed. This paper records external computations only as corroboration; a VERSF-native closure requires the substrate regulator and fermion census to reproduce the trivial character directly.

22. A Substrate Spectral Chiral-Measure Construction

Assume SG7–SG8. Let $\{v_j(A)\}$ be a local orthonormal basis of the regulated left-handed subspace $\hat{P}_-(A)\mathscr{H}$. The local measure connection is

$$\mathcal{A}_\eta(A) = i\sum_j \langle v_j(A), \delta_\eta v_j(A)\rangle.$$

Its curvature is

$$\mathcal{F}_{\eta\zeta} = i\mathrm{Tr}\left(\hat{P}_-[\delta_\eta \hat{P}_-, \delta_\zeta \hat{P}_-] \right),$$

which reproduces the regulated local anomaly.

When the total anomaly curvature and global holonomy vanish, the connection is globally integrable. Choose a base configuration A_0 and a positive reference orientation of the total fermion line there. Parallel transport with the integrable connection gives a global section

$$s_{ferm}^{glob}(A).$$

Theorem 5 — Nonperturbative Chiral-Measure Construction

Under SG7–SG8, the regulated Standard Model chiral measure exists globally, is local, respects the faithful quotient, includes all topological sectors and has a regulator-independent continuum section, up to one overall constant phase per connected physical component.

The residual constant phases are not automatically free parameters. They are fixed by the substrate history measure and become the topological sector character analysed in Part VII.

23. Locality, Integrability and Continuum Limit

The chiral measure must pass three separate tests.

23.1 Locality

The variation of the measure phase under a local variation of the gauge background must be an exponentially or regulator-locally supported current at finite regulator, with a controlled continuum limit.

23.2 Integrability

For every closed loop γ in configuration space,

$$\exp(i\oint_\gamma \mathcal{A}) = \mathrm{Hol}_\gamma(\mathcal{L}_{total}).$$

The right-hand side must be unity for an anomaly-free theory, except for the separately retained physical topological/strong-phase line.

23.3 Continuum limit

If $s_{(a, R)}$ is the finite-regulator section, one requires compatible convergence on cylinder observables and local correlation functions:

$$s_{(a, R)}\, d\mu_{(a, R)} \Rightarrow s\, d\mu_{cont}$$

as $a \to 0$, $R \to \infty$, in a topology strong enough to preserve gauge invariance, reflection positivity and the determinant-line holonomy.

Part V — Gribov-Free Global Quantisation

24. Why One Global Gauge Slice Cannot Be the Definition

A gauge condition such as

$$\partial^\mu A_\mu = 0$$

can intersect one non-Abelian gauge orbit more than once. These intersections are Gribov copies. At the Gribov horizon the Faddeev–Popov operator develops zero modes and the local determinant vanishes. Therefore a local gauge slice is not a global coordinate system on configuration space.

This does not invalidate perturbative gauge fixing. It fixes its scope.

The global theory should not be defined by the statement:

choose one representative from every gauge orbit.

It should be defined by:

integrate gauge-invariant data over the quotient object, using local slices only where they are valid coordinates.

25. Orbit-Stack Integration at Finite Regulator

Let $K_{(a, R)}$ be a finite regulator complex with oriented links ℓ and vertices v. For each faithful bundle sector, the link-configuration space is locally represented by

$$\mathcal{U}_{(a, R)}(P) = \prod_\ell G_{SM}^{faithful},$$

with the bundle twisting encoded in transition data and boundary conditions. The regulator gauge group is

$$\mathcal{G}_{(a, R)}(P) = \prod_v G_{SM}^{faithful}.$$

The product Haar measure $d\nu_H(U)$ is gauge invariant. The substrate pushforward modifies it by the gauge-basic absolute density and the determinant-line section.

For a gauge-invariant observable F, define

$$\int_{[\mathcal{U}/\mathcal{G}]} F d\bar{\mu} := 1/\mathrm{Vol}\,\mathcal{G} \int_{\mathcal{U}} F(U)\, d\mu(U)$$

when the action is free. For reducible configurations, the quotient stack retains the stabiliser weight; schematically the orbit contribution carries 1/|Stab(U)| in the finite case or the corresponding group-volume factor.

This is not gauge fixing. It is orbit integration.

Theorem 6 — Gribov-Free Regulator Measure

For compact regulator gauge group and a gauge-basic absolute density, the expectation of every gauge-invariant cylinder observable is well defined on the quotient stack without choosing a global gauge slice.

Proof

Gauge invariance makes the integrand constant on each orbit. Compactness supplies finite Haar volume at finite regulator. Quotient-stack integration divides the orbit volume while retaining stabiliser data. No representative is selected, so multiple intersections of any later gauge slice do not alter the definition. ■

26. Recovery of Local Faddeev–Popov and BRST Charts

On a regular orbit neighbourhood $\mathcal{O}$ where a gauge condition G(A)=0 defines a valid local section, orbit disintegration gives

```
∫_([𝒪/𝒢])Fdμ̄ = ∫_(G(A) = 0) F(A) det M_FP[A] dμ_slice(A).
```

The determinant is represented by the local ghost integral. BRST and BV encode independence of the chosen local section and the overlap consistency of regular charts.

Thus:

```
orbit-stack measure ⟹ local FP/BRST/BV descriptions.
```

The reverse implication does not hold globally.

27. The Neuberger Firewall

A naive BRST-exact gauge-fixing partition function on a compact lattice can sum Gribov copies with alternating signs and produce a zero numerator and zero denominator. The orbit-stack definition does not insert this signed copy count into the fundamental measure.

Local BRST charts may still be used for perturbation theory. They are glued as local descriptions of a measure defined independently of global gauge fixing.

Neuberger Firewall. No nonperturbative partition function is defined as a globally BRST-exact signed sum over gauge copies.

28. The Global Gauge-Measure Theorem

Theorem 7 — Global Gauge-Measure Construction

Assume SG1–SG8. Then:

1. the finite operational measures extend uniquely to μ_hist;
2. the measurable map $\mathfrak{Q}$ pushes μ_hist to the complete faithful-group configuration stack;
3. the absolute pushforward measure is gauge-basic;
4. all faithful quotient bundle sectors are included;
5. the fermionic phase is a global section of the anomaly-free determinant/counterterm line;
6. gauge-invariant expectation values are defined without a global gauge slice;
7. local FP/BRST/BV formulae are recovered on regular orbit charts.

The theorem is unique up to the substrate-selected topological sector character and normalisation fixed by the finite marginals.

Proof

Items 1–3 follow from Theorems 1–2. Item 4 follows from the six-cocycle census and SG6. Items 5 follows from Theorems 4–5. Item 6 follows from Theorem 6. Item 7 follows from local orbit disintegration and the inherited Quantum Gauge Completion theorem. ■

The remaining questions are positivity, physical reconstruction and the value of the residual sector character. These are Parts VI and VII.

Part VI — Reflection Positivity and Physical Hilbert-Space Reconstruction

29. The Euclidean Reflection Structure

Let the regulated Euclidean 4-manifold possess a time-reflection involution

```
Θ:(τ, x) ↦ (-τ, x)
```

acting anti-linearly on complex functionals and with the standard field-dependent action on gauge, scalar and fermion variables.

Let $\mathfrak{A}_+$ be the algebra of gauge-invariant physical observables supported at positive Euclidean time.

For $F \in \mathfrak{A}_+$, define

$$\langle F, F \rangle_{OS} = \int (\Theta F) F \, d\mu_{full}.$$

Reflection positivity is

$$[\![\langle F, F \rangle_{OS} \geq 0 \text{ for every} F \in \mathfrak{A}_+.]\!]$$

The full measure includes the bosonic density, chiral determinant section and topological sector weights. Positivity must therefore be tested after the combined line is assembled, not only for the absolute bosonic density.

30. Reflection-Positive Decomposition

A sufficient regulator-level condition is a decomposition

$$S_{(a, R)} = S_+ + \Theta S_+ + \sum_k c_k B_k \Theta B_k, \quad c_k \geq 0,$$

with the fermion kernel admitting the corresponding reflection-positive bilinear decomposition and the sector character transforming compatibly under reflection.

For a positive class function plaquette action, the link integral can be expanded in positive representation coefficients across the reflection plane. Scalar nearest-neighbour terms admit the same form when the kinetic metric is positive. Fermions require the regulator-specific reflection-positivity theorem for the chosen chiral kernel.

Premise RP-SG

The substrate pushforward at finite regulator admits such a decomposition on the physical quotient, uniformly in the regulator sequence.

This premise is stronger than positivity of each finite operational marginal. It is the exact extra condition needed to reconstruct Lorentzian dynamics.

31. Osterwalder–Schrader Reconstruction

Let

$\mathcal{N}\_OS = \{F \in \mathfrak{A}_+ : \langle F, F\rangle\_OS = 0\}.$

The physical pre-Hilbert space is

$\mathcal{H}\_OS^0 = \mathfrak{A}_+/\mathcal{N}\_OS,$

with completion

$⟦ \mathcal{H}\_OS = \mathcal{H}\_OS^0. ⟧$

Euclidean time translations by a_t define a positive contraction T_(a, R) on this space. Under time-translation invariance and the semigroup property,

$T\_(a, R) = e^{\wedge}(-a\_t\ H\_(a, R)),\ H\_(a, R) \geq 0.$

The vacuum is the class of the constant functional where it is cyclic and unique under the clustering premise.

Theorem 8 — Reflection-Positive Hilbert Reconstruction

Assume SG9 together with Euclidean covariance, regularity, clustering and compatible regulator convergence. Then the global VERSF measure reconstructs a positive physical Hilbert space, vacuum, transfer matrix and non-negative Hamiltonian.

No negative-norm physical state survives the OS quotient.

32. Transfer Matrix and Physical Hamiltonian

At finite regulator,

$H\_(a, R)^{\wedge}phys = -1/a\_tlog\ T\_(a, R)$

is defined on the support of the positive transfer matrix.

The continuum Hamiltonian is obtained through Mosco or norm-resolvent convergence of the regulated quadratic forms:

$H\_(a, R)^{\wedge}phys \longrightarrow H^{\wedge}phys.$

This convergence is the point at which regulator-uniform sector gaps, triality projectors and vacuum uniqueness become physical continuum statements.

33. Terminal-Record GNS Comparison and Triality Compatibility

The Adjoint-Removal programme constructs a positive terminal-record state ω_{rec} on the physical observable algebra $\mathfrak{A}_P$, with GNS space

$$\mathcal{H}_{GNS} = \mathfrak{A}_P/\mathcal{N}_{rec}.$$

The Euclidean measure defines a state ω_E on the corresponding time-zero physical algebra.

Theorem 9 — OS/GNS Identification

Assume SG10. If

$$\omega_E(A) = \omega_{rec}(A)$$

for every element of a dense common physical *-subalgebra and the two null ideals agree there, then

$$[A]_{OS} \mapsto [A]_{GNS}$$

extends uniquely to a unitary isomorphism

$$\mathcal{H}_{OS} \simeq \mathcal{H}_{GNS}.$$

Proof

Equality of the states gives equality of inner products on the common dense algebra. Equality of null ideals makes the map well defined and isometric. Density and completion give a unitary extension. ■

Theorem 9A — Triality-Compatible Reconstruction

Assume SG13 in the adjoint-only colour restriction. Let $\hat{U}_\omega(\Sigma)$ be the correctly embedded centre-surface operator with spectrum $\{1, \omega, \omega^2\}$. If

$$[T_{(a, R)}, \hat{U}_\omega(\Sigma)] = 0$$

at every regulator and the commutation survives the OS/GNS identification and continuum limit, then

$$\mathcal{H}_{phys}^{(N_f = 0)} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2,$$

where

$\hat{U}_\omega(\Sigma)|_{(\mathcal{H}_\tau)} = \omega^\tau.$

The transfer matrix and Hamiltonian preserve each $\mathcal{H}_\tau$.

With dynamical fundamental quarks, $\mathbb{Z}_3$ one-form symmetry is explicitly broken. The theorem is then replaced by a static-source/screening decomposition: the global measure must still represent the centre surface action and its endpoints on matter, but it may not claim exact triality superselection of the vacuum Hilbert space.

Proof

At finite regulator, a normal unitary commuting with the positive transfer matrix has spectral projectors commuting with the transfer matrix. These projectors descend through the OS quotient and, by SG10, through the GNS identification. Regulator convergence preserves the decomposition when the projectors and forms converge compatibly. ■

This is the exact bridge between the measure-based, terminal-record and Paper 5 triality-sector notions of the physical state space.

34. Microcausality and the Continuum Limit

Reflection-positive reconstruction supplies spectral positivity and a Lorentzian Hilbert space after analytic continuation. Full microcausality additionally requires the continuum Schwinger functions to satisfy the locality and analyticity conditions needed for spacelike graded commutation.

The Adjoint-Removal theorem then gives the descent:

$[A, B]_g \in \mathfrak{J}_{AR} \Longrightarrow [[A], [B]]_g = 0.$

Paper 6 therefore splits microcausality into two auditable steps:

1. derive locality of the reconstructed continuum fields modulo terminally null residue;
2. use the adjoint-removal quotient to obtain exact physical microcausality.

Neither step may be replaced by perturbative cutting identities.

Part VII — The Combined Strong-Phase Line

35. Why θ_3 Is Not Separately Physical

The colour topological term is

$$S_{\theta_3} = i\theta_3 Q_3[A], \quad Q_3[A] = 1/32\pi^2 \int_X F^a_{\mu\nu}\tilde{F}^{a\mu\nu}.$$

A chiral rephasing of a quark field changes the fermion measure by the axial Jacobian and shifts θ_3. The same transformation changes the phase of the quark mass determinant. Therefore neither contribution is invariant alone.

For nonzero quark determinants, the invariant phase is

$$\llbracket\ \bar{\theta} = \theta_3 + \arg\det(M_u M_d) \pmod{2\pi}.\ \rrbracket$$

36. Quark Determinant Phases and Canonical Matrices

The strong-phase input must be the physical, canonically normalised, blind quark mass maps returned by the microscopic Yukawa programme:

$$M_u = v_{cl}/\sqrt{2}Y_u, \quad M_d = v_{cl}/\sqrt{2}Y_d.$$

The positive singular values determine current-mass magnitudes:

$$m_{(u, i)} = \sigma_i(M_u), \quad m_{(d, i)} = \sigma_i(M_d).$$

They determine

$$|\det(M_u M_d)| = \prod_i m_{(u, i)} \prod_j m_{(d, j)},$$

but they do not determine

$$\arg\det(M_u M_d).$$

This is the mass-magnitude/strong-phase firewall:

$$\llbracket\ \text{mass-spectrum closure} \neq \text{determinant-line phase closure.}\ \rrbracket$$

The strict Paper 3 output package must therefore publish

$$\det M_u, \quad \det M_d, \quad \varphi_u = \arg\det M_u, \quad \varphi_d = \arg\det M_d,$$

together with:

- the canonical left and right kinetic metrics;
- the substrate-derived chiral embeddings;
- the anomaly-compatible orientation;
- stability under precision refinement;
- invariance under allowed frame changes.

For a biunitary factorisation

$$U_{uL}^\dagger M_u U_{uR} = D_u,$$

$$\arg\det M_u = \arg\det U_{uL} - \arg\det U_{uR} + \arg\det D_u.$$

A chiral frame change shifts the fermion measure and θ_3. The separate determinant phase is therefore not an observable outside the combined line, but it remains a required factor in that line.

The present corpus contains three different grades that must not be merged:

1. **Physical blind matrices:** still owed from the complete dressed Hessian, projector and chiral embeddings.
2. **Blind baseline matrices:** explicit but structurally insufficient; they are not the physical flavour return.
3. **Conditional corpus representatives:** useful reference branches assembled under inherited premises; they may supply conditional determinant-phase statements but do not replace the physical output.

36A. Weak-CP/Strong-Phase Independence Theorem

Define the left Hermitian mass operators

$$H_u^L = M_u M_u^\dagger, \quad H_d^L = M_d M_d^\dagger.$$

The weak CP invariant may be represented by

$$\mathcal{J}_{weak} = \mathrm{Im}\,\mathrm{Tr}\left([H_u^L, H_d^L]^3\right),$$

or equivalently by the Jarlskog invariant after division by the nondegenerate mass-difference factors.

Theorem 9B — Weak-CP/Strong-Phase Independence

For nondegenerate full-rank three-generation mass maps:

1. $\mathcal{J}$_weak is determined by the relative left-handed frames and mass singular values.
2. arg det (M_uM_d) is not determined by H_u^L, H_d^L.
3. A nonzero $\mathcal{J}$_weak is compatible with

   arg det (M_uM_d) = 0.

4. A change in arg det (M_uM_d) need not change $\mathcal{J}$_weak.

Proof

Right multiplication

$$M_u \mapsto M_u V_R$$

by any unitary V_R leaves

$$M_uM_u^\dagger$$

unchanged, hence leaves every weak invariant constructed from H_u^L, H_d^L unchanged. But

$$\det (M_uV_R) = \det M_u \det V_R,$$

so the determinant phase changes when det V_R≠1. Conversely, choose positive diagonal singular-value matrices D_u, D_d, left frames with a nonzero CKM/Jarlskog phase and right frames whose determinants make det M_u, det M_d positive. Then $\mathcal{J}$_weak≠0 while arg det (M_uM_d)=0. ■

The anomalous chiral transformation accompanying a right-frame phase shifts θ_3, so the combined $\bar{\theta}$ remains the physical invariant. The theorem says that weak CP data cannot determine the strong phase and a real quark determinant cannot erase CKM CP violation.

36B. Conditional MCHY Real-Trivialisation Branch

The MCHY-1C Hermitian corpus branch supplies declared positive-Hermitian charged-sector representatives with nonzero positive determinants. In the published numerical branch,

$$\det Y_d^C \simeq 4.0624 \times 10^{-10} > 0,$$

$$\det Y_u^C \simeq 1.3330 \times 10^{-7} > 0.$$

With the positive vacuum magnitude v_cl>0,

$$\det (M_u^CM_d^C) = (v_{cl}/\sqrt{2})^6 \det (Y_u^CY_d^C) > 0.$$

Therefore

$$\arg \det (M_u^C M_d^C) = 0 \pmod{2\pi}.$$

Proposition 9C — Conditional Quark-Line Real Trivialisation

On the declared MCHY-1C Hermitian representative branch, the quark determinant factor admits a positive real trivialisation:

$$\det (M_u^C M_d^C)/|\det (M_u^C M_d^C)| = 1.$$

Consequently,

$$e^{i\bar{\theta}_C} = \chi_{\mathrm{sub}}(1).$$

This is a real conditional calculation, not a physical strong-CP solution. It remains conditional because:

- the Hermitian left/right representative convention is not yet returned by the microscopic dressed functional;
- the physical right-handed frames remain open;
- the canonical anomaly-compatible orientation is not yet fixed;
- every CY3′ residue remains unevaluated;
- the full physical matrices are not yet the blind substrate outputs.

The strict theorem must therefore publish both:

$$\varphi_q^{phys} = \arg \det (M_u M_d),$$

and the diagnostic reference value

$$\varphi_q^C = 0.$$

Agreement would promote the branch. Disagreement would identify the missing right-frame or dressed-line phase.

37. The Combined Invariant Line

Let $\mathscr{L}_{\theta_3}$ be the unitary line carrying the topological sector character, and let $\mathscr{L}_{(\det M_u)}$, $\mathscr{L}_{(\det M_d)}$ carry the quark determinant phases. Define

$$\mathcal{L}_{\bar{\theta}} = \mathcal{L}_{\theta_3} \otimes \mathcal{L}_{(\det M_u)} \otimes \mathcal{L}_{(\det M_d)}.$$

A substrate-defined sector character is a multiplicative map

$\chi\_sub:\mathcal{T}_3 \to U(1),$

where $\mathcal{T}_3$ is the additive group of admissible colour topological sectors after quotient and boundary conditions are included. On the integer instanton subgroup,

$\chi\_sub(Q) = e^{(iQ\theta_3^{sub})}$

when the character is continuous in the declared sense.

The combined substrate section is

$s\_\bar{\theta} = s\_\theta_3^{sub} \otimes (\det M\_u)/(|\det M\_u|) \otimes (\det M\_d)/(|\det M\_d|).$

For a unit topological generator,

$$e^{(i\bar{\theta}\_sub)} = \chi\_sub(1) \det (M\_uM\_d)/|\det (M\_uM\_d)|.$$

The strict physical return uses the blind matrices. On the conditional MCHY representative,

$$e^{(i\bar{\theta}\_C)} = \chi\_sub(1).$$

Paper 6 must publish the strict physical value and may publish the conditional branch alongside it as a diagnostic. The branch may not replace the strict result.

38. The Strong-Phase Trichotomy Theorem

Theorem 10 — Strong-Phase Trichotomy

Assume SG6–SG8 and SG11. The substrate-induced combined line $\mathscr{L}\_\bar{\theta}$ has exactly one of the following statuses.

Branch A — Positive real trivialisation

There exists a nowhere-vanishing gauge-equivariant section s with

$s>0$

in the substrate-selected real orientation on every connected physical component. Then

$$e^{i\bar{\theta}} = 1, \quad \bar{\theta} = 0 \pmod{2\pi}.$$

Branch B — Non-real unitary trivialisation

The line is globally trivial, but the substrate section has nontrivial unitary holonomy

$\mathrm{Hol}(\mathcal{L}\_\bar{\theta}) = e^{i\vartheta}, \vartheta \not\equiv 0 \pmod{2\pi}$.

Then

$$[\![\bar{\theta} = \vartheta.]\!]$$

This is a prediction, not an inconsistency. It becomes an empirical falsifier only after the result is transported to a strong-CP observable without target-data adjustment.

Branch C — Obstruction

The combined line has no globally consistent gauge-equivariant trivialisation. Equivalently, its anomaly/holonomy class is nontrivial in a way not removable by an allowed local counterterm. Then the putative global Standard Model measure is obstructed.

Proof

A complex line over each connected component is either nontrivialisable, or trivialisable. In the trivialisable case, the substrate-selected nonzero section has a well-defined unitary phase relative to the positive reference orientation. That phase is either unity or not. These cases are mutually exclusive and exhaustive. Gauge and chiral basis invariance follows because the three factors transform oppositely and only their tensor product is used. ■

The theorem is deliberately a trichotomy, not a zero-phase theorem. Any stronger conclusion requires a substrate selector.

39. The Positive-Real Branch

A sufficient certificate for Branch A is:

1. the total anomaly character is trivial;
2. the substrate history measure gives a reflection-compatible real structure on $\mathscr{L}\_\bar{\theta}$;
3. the reference section at the trivial connection is positive;
4. parallel transport in the combined flat line preserves that real orientation;
5. the physical quark determinant product is positive in the same orientation.

Under these conditions the section cannot acquire a phase around any physical loop and

$\bar{\theta} = 0$.

Corollary 10.1 — Positive-Measure Strong-Phase Trivialisation

If the substrate sector character is trivial,

$\chi\_sub(1) = 1,$

and the blind physical matrices satisfy

$\det (M\_uM\_d)>0,$

in the anomaly-compatible canonical orientation, then Branch A holds.

This is an exact corollary. The two antecedents are calculations, not assumptions to be selected after comparison with experiment.

Corollary 10.2 — MCHY Reference-Branch Reduction

On the MCHY-1C Hermitian representative,

$\det (M\_u^CM\_d^C)>0.$

Therefore Branch A on that representative is equivalent to one remaining condition:

$$[\![\chi\_sub(1) = 1.]\!]$$

A nontrivial χ_sub(1) would instead give Branch B even though the quark determinant is positive. This makes the conditional branch maximally informative: it isolates the topological sector character as the sole remaining phase on that branch.

40. The Nonzero-Holonomy Branch

If

$\chi\_sub(1) = e^(i\theta_3^sub)$

and

$\det (M\_uM\_d)/|\det (M\_uM\_d)| = e^(i\varphi\_q),$

then

$$[\![\theta^-\_sub = Arg(e^(i(\theta_3^sub+\varphi\_q))).]\!]$$

The phase is not fitted. Its uncertainty is the certified numerical uncertainty of the two independent inputs.

A nonzero result must be carried forward even if it is phenomenologically uncomfortable. The later audit, not this paper, decides whether the theory survives comparison.

41. The Obstruction Branch

Branch C can arise through:

- a nontrivial faithful-group bordism anomaly;
- a determinant line with nontrivial gauge holonomy not cancelled by the full census;
- failure of the substrate chiral regulator to define a local integrable measure;
- an omitted quotient bundle sector needed for descent;
- incompatibility of the fermion line with reflection positivity;
- zeros of the determinant that prevent the claimed nonzero section on the chosen branch.

An obstruction is a result. It says the frozen VERSF Standard Model completion is globally inconsistent or incomplete.

42. Massless-Quark Firewall

If

$$\det(M_u M_d) = 0,$$

then $\arg\det(M_u M_d)$ is undefined. A massless chiral direction may remove the topological angle by an exact rephasing. The correct branch is then not the three-way non-degenerate trichotomy above but the massless-quark branch:

$$\bar{\theta} \text{ is unphysical along the exact massless direction.}$$

Paper 3's full-rank charged-sector certificate is expected to exclude this branch, but Paper 6 may not assume that certificate before it is returned.

43. RG and Threshold Stability

The combined phase obeys the identity inherited from Quantum Gauge Completion:

$$\mu \, d\bar{\theta}/d\mu = \beta_\theta + \mathrm{ImTr}\left(M_u^{-1}\beta_{(M_u)} + M_d^{-1}\beta_{(M_d)} \right).$$

In an anomaly-consistent scheme the separate terms may shift under chiral conventions while the combined line remains invariant.

When a heavy quark with complex mass m_h is integrated out, the effective topological phase shifts by the phase transferred from the determinant, so that

$$\bar{\theta}^{(n_f-1)} = \bar{\theta}^{(n_f)} \pmod{2\pi}$$

when matching is performed on the combined line.

Theorem 11 — Strong-Phase Transport Invariance

If the RG flow and threshold maps preserve the anomaly-compatible determinant-line connection, the value of $e^{i\bar{\theta}}$ is independent of renormalisation scale and of the order in which heavy complex masses are integrated out.

The later parameter-closure paper must transport this invariant, not transport θ_3 and the quark phases as unrelated numbers.

44. Blind Numerical Evaluation Protocol

The strict physical branch and the conditional MCHY diagnostic branch must be kept separate throughout.

44.1 Freeze inputs

Publish before empirical comparison:

- regulator family and complete bundle census;
- finite operational quotients, marginals and coarse maps;
- substrate history measure or cylinder specification;
- $\mathfrak{Q}$ and $\mathcal{J}$_(sub→ cont);
- the sector partition functions and determinant-line orientation;
- physical blind M_u, M_d;
- conditional representatives M_u^C, M_d^C, explicitly labelled;
- centre-surface operator and triality projectors;
- reflection-positive decomposition.

44.2 Calculate the substrate topological character

For a primitive unit sector, define

```
χ_sub(1) = lim_(a → 0, R → ∞) (Z_(Q = 1)^sub/|Z_(Q = 1)^sub|)/(Z_(Q =
0)^sub/|Z_(Q = 0)^sub|),
```

only after determinant-line and quotient-bundle consistency are imposed.

If the topology contains torsion or fractional faithful-quotient sectors, evaluate the complete character on a generator set rather than infer it from the integer instanton subgroup alone.

44.3 Calculate the strict physical quark determinant phase

Use a certified complex factorisation of the physical canonical matrices:

$\varphi\_q^{phys} = \mathrm{Argdet}\ M\_u + \mathrm{Argdet}\ M\_d.$

Track the phase continuously under precision refinement. Reject branch jumps caused by pivoting or SVD conventions. Verify invariance of the combined phase under allowed chiral frame changes.

44.4 Evaluate the conditional diagnostic branch

Confirm independently that

$$\varphi_q^C = \arg\det(M_u^C M_d^C) = 0$$

for the frozen MCHY representative. Then calculate

$$\bar{\theta}_C = \mathrm{Arg}\chi_{sub}(1).$$

The diagnostic branch tests whether the physical phase debt lies entirely in the substrate topological character or whether the strict right-frame/dressed-line calculation adds a phase absent from the representative.

44.5 Combine the strict physical inputs

$$\bar{\theta}_{phys} = \mathrm{Arg}\left[\chi_{sub}(1)\det(M_u M_d)/|\det(M_u M_d)|\right].$$

44.6 Triality and positivity audit

At each regulator:

$$[T_{(a,R)}, \hat{U}_\omega(\Sigma)] = 0$$

must hold in the adjoint-only restriction, and the OS norm must be nonnegative on each sector. The projectors and forms must converge compatibly.

44.7 Publish one of three physical verdicts

- $\bar{\theta}_{phys}=0$ with certified exact or interval proof;
- $\bar{\theta}_{phys}\neq 0$ with a certified interval excluding zero;
- obstruction, with the failed line, bordism, locality or positivity certificate identified.

The conditional MCHY branch is published as a separate diagnostic result. No fourth physical verdict is admissible.

Part VIII — Relation to Paper 5 and Global QCD

45. What This Paper Supplies to Debt D8

Paper 5 defines D8 as the quotient-compatible global colour measure, reflection-positive transfer matrix, physical transverse Hamiltonian, global sector coercivity, centre commutation and continuum convergence.

The present paper supplies:

1. a global faithful-stack measure architecture;
2. direct orbit integration without a global gauge slice;
3. the reflection-positive OS reconstruction criterion;
4. the transfer matrix and Hamiltonian definition;
5. the OS/GNS identification;
6. the centre-surface/triality compatibility theorem;
7. regulator convergence conditions;
8. a candidate measure-generated local contribution B_τ^{EG} from the pushforward Jacobian.

Restricting the global measure to an adjoint-only colour theory gives the measure/Hilbert component of the strict Paper 5 triality theorem, provided the restriction is separately matched, reflection positive and compatible with the centre operator.

46. What Remains for the Triality Gap

Paper 6 does not by itself calculate or prove:

- the physical colour stiffness Z_3;
- the complete local wall coefficient B_τ;
- the centre-sensitive global lift and primitive flux normalisation;
- the global coercive inequality in every nonzero-triality sector;
- the longitudinal slicing constant;
- a regulator-uniform sector gap;
- a finite-slope upper bound or subadditivity theorem;
- existence of the physical string-tension limit.

The measure cannot manufacture these facts. It can generate candidate local operators and provide the positive Hilbert space in which a proven form bound becomes a physical gap.

The correct division of labour is:

Paper 5: colour response, wall coefficient, coercivity and slicing

Paper 6: global measure, centre-compatible positivity and Hamiltonian reconstruction.

46A. Candidate Entropic-Gradient Realisation of the Wall Coefficient

The entropic-confinement manuscript identifies a useful candidate local operator but does not supply a valid area-law theorem. The useful component is retained in corrected form.

Assume the real local part of the substrate Jacobian contains

$$S_{EG} = c_6^{sub}/\Lambda_{sub}^2 \int_X |d\varphi_E|^2/(\varphi_E+\varepsilon),\quad \varphi_E = \tfrac{1}{2}\, \mathrm{tr}\, F_{\mu\nu}F_{\mu\nu}.$$

The coefficient must be derived from $\mathfrak{Q}$ and μ_{hist}. External gluon condensates, fitted flux-tube enhancements and an O(1) choice of c_6 may be used only for downstream phenomenological comparison, never to define the substrate coefficient.

The operator can contribute to the local triality wall cost if nonzero triality forces a transition between a core action density and an exterior action density. The existence of that transition is not supplied by the gradient term; it belongs to the global centre/triality sector architecture.

46B. Conditional Entropic Wall-Cost Lemma and Area-Law Firewall

Lemma 12A — Conditional Entropic Wall Cost

Let $s\in[0, \delta]$ be a normal coordinate through a Euclidean wall. Assume

$$\varphi_E(0) \geq \varphi_+,\quad \varphi_E(\delta) \leq \varphi_-,\quad \varphi_+>\varphi_-,\quad \delta \leq \delta_{max}.$$

Then

$$\int_0^\delta (\partial_s\varphi_E)^2/(\varphi_E+\varepsilon)\,ds \geq 4/\delta\ \left(\sqrt{(\varphi_++\varepsilon)} - \sqrt{(\varphi_-+\varepsilon)} \right)^2.$$

Hence a wall family of area A_Γ satisfies

$$S_{EG} \geq B_\tau^{EG} A_\Gamma,$$

with

$$B_\tau^{EG} = \frac{4c_6^{sub}}{\Lambda_{sub}^2 \delta_{max}} \left(\sqrt{\varphi_+ + \varepsilon} - \sqrt{\varphi_- + \varepsilon} \right)^2 > 0.$$

Proof

By Cauchy–Schwarz,

$$\int_0^\delta \frac{(\varphi')^2}{\varphi+\varepsilon} ds \int_0^\delta ds \geq \left(\int_0^\delta \frac{|\varphi'|}{\sqrt{\varphi+\varepsilon}} ds \right)^2.$$

The second integral is bounded below by the total monotone variation,

$$2\left| \sqrt{\varphi(0)+\varepsilon} - \sqrt{\varphi(\delta)+\varepsilon} \right|,$$

giving the result. Integration over the wall family yields the area bound. ■

The lemma supplies only a local coefficient. Four earlier shortcuts are rejected.

Fixed-total-flux firewall

One total centre flux spread over $n \sim A/\ell^2$ patches gives, at best,

$$\sum_{j=1}^n \Phi_j^2/\ell^2 \geq \Phi^2/A,$$

not a term proportional to A. Extensivity requires an independently proved statement that every longitudinal slice or elementary wall cell carries a fixed irreducible triality burden.

Probability/area-law firewall

A mixture

$$(1-p)+pe^{-\sigma A}$$

approaches $1-p$, not zero, as $A\to\infty$. Moreover,

$$1-p+pe^{-x} \geq e^{-px}$$

by convexity. A positive probability of flux configurations therefore does not imply an area law. One needs a transfer-matrix linear-potential theorem or an independently derived large-deviation/centre-vortex characteristic function with an area-extensive rate.

Signature firewall

The positive functional is Euclidean. No confinement inequality follows from treating Lorentzian

$$\operatorname{tr} F_{\mu\nu}F^{\mu\nu}$$

or $\partial_\mu\varphi\partial^\mu\varphi$ as pointwise positive.

Reflection-positivity firewall

A continuum higher-derivative term may fail OS positivity. The substrate regulator must realise S_{EG} through a reflection-positive local interaction or show that it is the continuum limit of one. Until that is proved, B_τ^{EG} is a candidate Euclidean wall coefficient, not a physical Hamiltonian term.

If Paper 5 supplies global triality coercivity and longitudinal slicing, and Paper 6 supplies reflection positivity and convergence, the local coefficient can enter the strict gap theorem. Without those steps, no area law is claimed.

47. Why D9 Remains Separate

D9 is the finite matching bridge from the physical $N_f=3$ VBF theory to an adjoint-only $N_f=0$ theory and its independent scale Λ_0.

A global measure can be defined for both theories without proving that one is obtained from the other by the required finite matching. Therefore Paper 6 does not silently discharge D9.

A future D9 calculation must integrate out or deform away the three fundamental flavours in a controlled path, track the determinant and topological lines, preserve reflection positivity, and derive the relation between Λ_3 and Λ_0. Until then, an adjoint-only triality gap cannot be expressed as a physical pure-Yang–Mills tension in terms of Λ_3.

Part IXA — Finite-Regulator Global Evaluation and Strong-Phase No-Go Theorem

47A. Purpose and Standard of Proof

The preceding parts construct the global objects and state the conditions under which they exist. This part performs the stronger task required by the programme brief to the extent the frozen stack currently permits: it replaces the abstract assumptions by one explicit finite-regulator construction, derives the strongest strong-phase statement that construction supports — a conditional no-go theorem — and reduces the remaining distance to a physical pass to four named executable calculations and one foundational premise.

The route chosen is the **positive-pushforward route**. It takes literally the VERSF claim that physical probability is an ordinary positive measure on operationally distinct substrate histories and that the continuum gauge measure is its pushforward. This choice has a sharp consequence. A genuine pushforward of a positive measure cannot retain an uncancelled complex sector phase. Therefore the combined topological and quark determinant line must admit a positive real trivialisation, or the proposed pushforward does not exist as a physical probability measure.

One caution governs the whole part. Positive Born probability for completed terminal records coexists, throughout the VERSF quantum programme, with complex amplitudes before commitment; interference and the Born rule depend on that distinction. Positive probabilities for records therefore do not by themselves imply a positive pointwise measure over uncommitted continuum gauge histories. The statement that the continuum quantum object is an ordinary positive measure is recorded below as a named foundational premise, **Premise P-POS — Positive continuum representability**, not as a corollary of the operational Born measure. Where a construction depends on it, that dependence is stated.

The finite-regulator construction is organised as seven returns:

a. an explicit positive substrate history measure;

b. an explicit gauge-basic substrate-to-field readout;

c. a complete quotient-bundle census on a declared regulator;

d. a finite-dimensional determinant-line and global-anomaly audit;

e. a positive transfer operator and reflection-positive reconstruction;

f. nonzero support in the unit colour-topology sector;

g. the combined strong-phase verdict.

No measured strong-CP information is used in any step.

47B. Finite Operational History Measure from Terminal-Record Projectors

Fix a finite, reflection-symmetric cellular regulator

```
K_(a, R)
```

with spatial extent R, Euclidean time spacing a_t, and finite operational resolution a. Let

```
ℋ_(a, R)^op
```

be the finite operational Hilbert carrier supplied by the finite-packing and operational-Hilbert programme at this regulator.

Let

$\{P_h : h \in \mathfrak{H}_{(a, R)}\}$

be the maximal terminal-record partition of the identity, where each P_h projects onto one complete operationally distinguishable regulated history class. Thus

$P_h P_{h'} = \delta_{hh'} P_h, \quad \sum_{h \in \mathfrak{H}_{(a, R)}} P_h = 1.$

The orthogonality is the Hilbert representation of closure incompatibility. The exhaustiveness is the finite-regulator form of the claim that every terminally distinguishable history belongs to exactly one complete record class.

Let $|\Omega_{(a, R)}\rangle$ be the normalised frozen substrate state returned by the admissible-background construction,

$\langle\Omega_{(a, R)}|\Omega_{(a, R)}\rangle = 1.$

Define

$$\mu_{sub}^{(a, R)}(h) := \langle\Omega_{(a, R)}|P_h|\Omega_{(a, R)}\rangle.$$

This is not an independently chosen Gibbs weight. It is the universal operational measure applied to the complete terminal-record partition.

That definition, however, is not yet a probability measure over full time-extended quantum histories. A regulated history is in general represented not by a single projector but by a time-ordered class operator

$C_h = P_{(h_n)}(t_n) \cdots P_{(h_2)}(t_2)\ P_{(h_1)}(t_1),$

and the interference between two histories is measured by the decoherence functional

$D(h, h') = \langle\Omega_{(a, R)}|\ C_{(h')}^\dagger\ C_h\ |\Omega_{(a, R)}\rangle.$

Time-slice projectors need not commute, and products of projectors at different times are not themselves an orthogonal projector partition. Classical additivity of the coarse-grained probabilities holds only when the history family decoheres:

$D(h, h') = 0 \quad (h \neq h').$

The single-partition definition above is therefore the fully committed limit, in which every history class is represented by one terminal projector and decoherence is automatic. For the general time-extended family the following premise is required.

Premise P-DEC — Exact terminal-history decoherence. The substrate commitment dynamics establishes exact decoherence of the complete regulated terminal-history family: $D(h, h') = 0$ for $h \neq h'$, identically and not merely to good approximation. Irreversible terminal commitment is the mechanism expected to enforce this, but it is a substrate calculation, not an axiom, and it is carried as hinge P-H1.

Exactness warning. The exactness in P-DEC is load-bearing, not cosmetic. Theorem 1 is a Kolmogorov/projective extension theorem and requires exactly consistent marginals; the extension machinery is unforgiving of approximation. Exact additivity of the history measure holds only when $D(h, h') = 0$ exactly, whereas generic physical decoherence — including the medium and strong grades of the decoherent-histories literature — is approximate, with exponentially small but nonzero off-diagonal remainders. Exact decoherence is plausible only in a strict commitment/superselection limit, in which terminally committed records are represented in superselected sectors with no residual overlap, and it may fail for time-extended families outside that limit. Discharging P-H1 therefore requires one of two routes: prove exact decoherence of the terminal family in the commitment superselection limit, or reformulate the extension theorem for approximately consistent marginals with explicit, regulator-uniform error control. Neither route has been executed; until one is, the chain from operational probability to history measure carries an exactness assumption that the decoherence formalism does not generically supply.

Theorem IXA-1 — Consistent Terminal-History Measure

Let $\{C_h : h \in \mathcal{H}_{(a, R)}\}$ be an exhaustive family of regulated terminal-history class operators satisfying

$$\sum_{h \in \mathfrak{H}_{(a, R)}} C_h = 1,$$

and let Premise P-DEC hold. Then

$$\mu_{sub}^{(a, R)}(h) := D(h, h) = \|C_h \Omega_{(a, R)}\|^2$$

is a positive, normalised, additive history measure on $\mathcal{H}_{(a, R)}$. For a coarse history $\bar{h}$ with class operator

$$C_{\bar{h}} = \sum_{h \in \bar{h}} C_h,$$

additivity holds exactly:

$$\mu_{sub}^{(a, R)}(\bar{h}) = \sum_{h \in \bar{h}} \mu_{sub}^{(a, R)}(h).$$

In the fully committed limit $C_h = P_h$, the class operators reduce to one orthogonal exhaustive projector partition, P-DEC is automatic, and $\mu_{sub}^{(a, R)}(h) = \langle \Omega_{(a, R)} | P_h | \Omega_{(a, R)} \rangle$. If $c: \mathcal{H}_{(a', R')} \to \mathcal{H}_{(a, R)}$ is an admissible coarse-graining and the coarse class operators satisfy

$$C_{\bar{h}} = \sum_{h : c(h) = \bar{h}} C_h,$$

then

$$c_*\mu_{sub}^{(a', R')} = \mu_{sub}^{(a, R)}.$$

Proof

Positivity: $\mu_{sub}^{(a, R)}(h) = D(h, h) = \|C_h \Omega_{(a, R)}\|^2 \geq 0$. Normalisation and additivity: expanding the exhaustiveness relation,

$$1 = \|\sum_h C_h \Omega\|^2 = \sum_h D(h, h) + \sum_{(h \neq h')} D(h, h') = \sum_h \mu_{sub}^{(a, R)}(h),$$

where the cross terms vanish by P-DEC. For a coarse history $\bar{h}$,

$$\mu_{sub}^{(a, R)}(\bar{h}) = \|\sum_{(h \in \bar{h})} C_h \Omega\|^2 = \sum_{(h \in \bar{h})} \|C_h \Omega\|^2 = \sum_{(h \in \bar{h})} \mu_{sub}^{(a, R)}(h),$$

again by P-DEC applied within $\bar{h}$. The refinement pushforward relation is the same computation applied to the coarse-graining map c. ■

Consequence

SG1–SG3 are no longer independent measure axioms at finite regulator. They reduce to two concrete substrate tests:

1. the complete terminal-history class-operator family decoheres exactly: $D(h, h') = 0$ for $h \neq h'$ (Premise P-DEC, hinge P-H1) — or the extension theorem is reformulated with controlled error for approximate decoherence;
2. refinement coarse-graining acts by summing the fine class operators belonging to one coarse history.

If either test fails, the operational probability programme does not extend to a history measure. If both pass, projective consistency and additivity are automatic. The previously hidden assumption — that time-extended terminal records form one orthogonal projector partition — is thereby converted into a concrete substrate calculation: prove that irreversible terminal commitment decoheres the relevant history family.

47C. Positive Euclidean Transfer from Refinement Polar Decomposition

The unified refinement-Hilbert construction gives every finite-regulator refinement map a polar decomposition

$$R_{(a, R)} = U_{(a, R)} T_{(a, R)}, \quad T_{(a, R)} := |R_{(a, R)}| \geq 0.$$

The unitary factor carries reversible pre-commitment transport. The positive factor carries commitment-compatible contraction.

The polar decomposition exists unconditionally. What does not follow from it is the identification of physical Euclidean evolution with the positive factor. The physical Euclidean step could a priori be R itself, $U|R|$, a dilation of R, or a separately derived transfer kernel — and the discarded unitary factor is exactly where phase and orientation information may live. In a strong-phase paper this is the dangerous step: selecting $|R|$ because it yields reflection positivity would assume the object the section claims to derive. The identification is therefore stated as a premise.

Premise P-TR — Positive-transfer identification. The physical Euclidean one-step operator at regulator (a, R) is the positive polar factor of the physical refinement map. Discharging P-TR requires one of the following, derived from the microscopic substrate update rather than selected for its consequences:

1. $R_{Euclidean} = |R|$ from reflection/Wick continuation of the substrate commitment dynamics; or
2. $R_{Euclidean} = S^{-1} T S$ for a positive self-adjoint T and an explicit similarity S; or
3. an independent construction of the Euclidean transfer kernel with reflection positivity proved directly.

P-TR is carried as hinge P-H2.

Premise P-REF — Reflection symmetry of the positive factor. The positive polar factor and the frozen state are reflection symmetric: $\Theta\mathbb{T}_{(a, R)}\Theta^{-1} = \mathbb{T}_{(a, R)}$ and $\Theta|\Omega_{(a, R)}\rangle = |\Omega_{(a, R)}\rangle$. This is stated as its own premise rather than assumed inline: nothing in the polar decomposition itself guarantees that the positive factor of the substrate refinement map commutes with the reflection, and the property must be derived from the reflection behaviour of the microscopic update alongside P-TR.

Under P-TR and P-REF, define the normalised one-step transfer operator

$$\mathbb{T}_{(a, R)} := \frac{T_{(a, R)}}{\|T_{(a, R)}\|}, \quad 0 \le \mathbb{T}_{(a, R)} \le 1.$$

By Premise P-REF, the regulator and frozen state are reflection symmetric:

$$\Theta\mathbb{T}_{(a, R)}\Theta^{-1} = \mathbb{T}_{(a, R)}, \quad \Theta|\Omega_{(a, R)}\rangle = |\Omega_{(a, R)}\rangle.$$

For the commutative algebra generated by terminal-record projectors on Euclidean time slices, define the cylinder functional by the transfer prescription

$$\omega_{(a, R)}(F_0, F_1, \ldots, F_n) = \langle\Omega_{(a, R)}|\, F_0\mathbb{T}\, F_1\mathbb{T}\cdots\, \mathbb{T}\, F_n\, |\Omega_{(a, R)}\rangle.$$

The corresponding diagonal cylinder probabilities are the terminal-record probabilities of Theorem IXA-1. Off-diagonal entries supply the pre-commitment quantum correlations reconstructed after quotienting by the OS null ideal.

Theorem IXA-2 — Regulated Record-Subalgebra Reflection Positivity

Assume Premises P-TR and P-REF. Let F be any polynomial in terminal-record projectors supported on the positive-time half of the regulator. Then

$$\omega_{(a, R)}((\Theta F)^* F) \geq 0.$$

Proof

Write the positive-time insertion represented by F as an operator $\hat{F}_+$ acting after the reflection plane. Reflection symmetry and self-adjointness of $\mathbb{T}$ give

$$\omega_{(a, R)}((\Theta F)^* F) = \langle \Omega | \hat{F}_+^* \mathbb{T}^m \hat{F}_+ | \Omega \rangle = \| \mathbb{T}^{(m/2)} \hat{F}_+ | \Omega \rangle \|^2 \geq 0.$$

The same proof applies after quotienting by gauge-equivalent record descriptions because the observables are gauge basic. ■

Scope boundary

Theorem IXA-2 is a record-subalgebra reflection-positivity lemma, conditional on P-TR and P-REF, and its scope should not be oversold. The terminal-record subalgebra is commutative, and reflection positivity on a positive-spectrum abelian algebra is nearly automatic once a positive self-adjoint transfer operator is granted — the lemma is closer to classical probability than to constructive field theory. Essentially all of the quantum-field-theoretic weight lies outside it: full Standard Model Osterwalder–Schrader positivity requires the same inequality on the complete physical algebra containing gauge holonomies, scalar observables, chiral fermion bilinears, topological-sector operators, the determinant-line section and the centre-surface operators. It has not been shown here that all of these observables are represented by admissible F, that reflection acts correctly on each of them, or that the fermionic and topological factors preserve the displayed norm-square form. That extension carries essentially all the difficulty of the reconstruction and remains untouched as debt D-SG8. Throughout this paper, "reflection-positive reconstruction" in the Part IXA branch means record-subalgebra reflection positivity in this sense.

Corollary IXA-2.1 — Physical Hamiltonian

The OS reconstruction gives

$$\mathcal{H}_{(a, R)}^{OS}, \quad \mathbb{T}_{(a, R)} = e^{(-a_t H_{(a, R)})}, \quad H_{(a, R)} \geq 0.$$

Thus, on the record subalgebra and under P-TR and P-REF, reflection positivity is not an extra sign assumption: it is inherited from the positive factor of the substrate refinement map. The corresponding statement for the full physical algebra is not inherited and remains D-SG8.

Firewall

Premise P-TR is a substantive VERSF statement, not a convenience. The statement $\mathbb{T} = |R|/\|R\|$ must be derived from the microscopic substrate update, not selected because it yields reflection positivity: the unitary factor removed by the polar decomposition may contain precisely the global phase information whose value this paper evaluates. Until P-TR is discharged by one of the three routes above, Theorem IXA-2 remains conditional on the main object it would otherwise appear to derive, and reflection positivity for the full theory stays on the open ledger.

47D. Canonical Substrate-to-Field Readout and Gauge Basicity

For every regulated history class h, define the field readout

```
𝔔_(a, R)(h) = [ {U_e(h)}, {Φ_v(h)}, {Ψ_(f, v)(h)} ]_(G_SM^faithful),
```

where:

- $U_e(h) \in G_{SM}^{faithful}$ is the inherited transport holonomy on oriented edge e;
- $\Phi_v(h)$ is the completion-interface scalar readout at vertex v;
- $\Psi_{(f, v)}(h)$ is the regulated chiral matter carrier readout in the representation slot f;
- square brackets denote the faithful-group orbit, including the correlated $\mathbb{Z}_6$ identification.

The map is canonical in the following sense. Two regulated histories have the same image exactly when every faithful-group gauge-invariant cylinder observable constructed from link holonomies, scalar contractions and matter bilinears has the same value on them. Equivalently, $\mathfrak{Q}_{(a, R)}$ is the joint Gelfand-spectrum map of the regulated physical cylinder algebra.

Define

```
⟦ μ_cont^(a, R) := (𝔔_(a, R))_* μ_sub^(a, R). ⟧
```

Theorem IXA-3 — Positive Gauge-Basic Pushforward

The measure $\mu_{cont}^{(a, R)}$ is a positive normalised Radon measure on the finite-regulator faithful-group orbit space. It is gauge basic and includes every bundle sector represented in the image of $\mathfrak{Q}_{(a, R)}$.

Proof

A measurable pushforward of a positive probability measure is a positive probability measure. Gauge-related link, scalar and matter representatives define the same orbit by construction, so the pushforward depends only on the orbit class. The finite regulator makes the orbit space compact modulo finite stabiliser strata, hence the measure is Radon. ■

No-independent-Jacobian consequence

When the pushforward is rewritten in local field coordinates, the substrate-to-continuum Jacobian is the Radon–Nikodym derivative of this already-positive measure relative to the chosen local reference volume. Its absolute value and any induced local operators are calculable coordinate data. They are not an extra probability law.

47E. Faithful-Quotient Bundle Census on the Periodic Test Regulator

To test genuinely non-liftable sectors, take the canonical periodic regulator

$$X_{(a, R)} = T^4_{(a, R)}.$$

Choose local product-group lifts of every quotient link variable to

$$SU(3) \times SU(2) \times U(1).$$

Around each oriented plaquette f, the product of lifted links may close only up to the diagonal central generator. Write

$$\prod_{(e \in \partial f)} \tilde{U}_e = \zeta_6^{(z_f)} \tilde{U}_f^{curv}, \quad z_f \in \mathbb{Z}_6.$$

The three-cell consistency condition gives

$$\delta z = 0,$$

and a change of lifts changes z by a coboundary. Therefore

$$[z] = \nu_6(P) \in H^2(T^4, \mathbb{Z}_6).$$

Since

$$H^2(T^4, \mathbb{Z}_6) \cong (\mathbb{Z}_6)^6,$$

the regulator contains

$$6^6$$

distinct discrete lifting-obstruction classes before the continuous instanton and Abelian flux data are added.

The complete finite-regulator census is the tuple

$\mathfrak{B}$_(a, R) = (ν_6; c_1^Y; k_2; k_3; w_2(TX); boundary/spin data),

subject to the descent congruences imposed by the common $\mathbb{Z}_6$ centre. The discrete cochain algorithm above is finite and exhaustive: every quotient bundle on the regulator is represented by one cocycle class and the associated integer/torsion characteristic data.

Theorem IXA-4 — Finite Lifting-Obstruction Census

On the periodic regulator, the $(\mathbb{Z}_6)^6$ calculation exhausts the $\mathbb{Z}_6$ lifting-obstruction sectors: the quotient-bundle sum must run over every $[z]\in(\mathbb{Z}_6)^6$, and restricting it to [z]=0 omits physical faithful-group bundles.

Census boundary

The lifting-obstruction classification is not the complete faithful-group bundle census. The complete census additionally requires the correlated continuous and torsion characteristic data compatible with each [z]: the hypercharge class c_1^Y, the instanton classes c_2^(2) and c_2^(3) with their quotient congruences, torsion refinements, spin or spin-G structures, and the possible fractional instanton data induced by ν_6. The finite algorithm of this subsection is complete for the obstruction sectors; the compatibility relations for the remaining characteristic data are carried in debt D-SG5 and must be produced before the census as a whole is called complete.

47F. Finite Determinant Line and Global-Anomaly Audit

At finite regulator, every one-particle fermion space is finite dimensional. Choose a gauge-covariant chiral lattice operator satisfying the Ginsparg–Wilson relation and possessing the correct continuum index. Let

P^_(f, L)(U), P_(f, R)

be its regulated chiral projectors in representation slot f.

Define the determinant line at background U by

$\mathcal{L}$_f(U) = detimP^_(f, L)(U) ⊗ (detimP_(f, R))^*.

The complete fermion line is

$\mathcal{L}$_ferm = ⊗_f$\mathcal{L}$_f.

For a closed loop γ in the faithful-group orbit space, choose orthonormal chiral frames along the loop. The finite holonomy is the determinant of the ordered product of adjacent overlap matrices:

```
Hol_γ(ℒ_ferm) = det∏_j [ ⟨ v_i(U_j), v_k(U_(j+1))⟩ ].
```

This formula is directly executable and contains no continuum regularisation ambiguity.

The four-dimensional faithful-group anomaly audit has three generator classes:

1. **contractible gauge loops**, detected by local anomaly curvature;
2. **the nontrivial $\pi_4(SU(2))\cong\mathbb{Z}_2$ loop**, detected by the Witten sign;
3. **faithful-quotient lift loops**, in which product-group lifts differ by the common $\mathbb{Z}_6$ generator.

For the inherited Standard Model representation census:

- the perturbative cubic and mixed anomaly polynomial vanishes;
- each generation contains four left-handed fundamental weak doublets, three coloured and one leptonic, so the Witten sign is $(-1)^4=+1$;
- every matter slot obeys the faithful descent congruence, so the diagonal $\mathbb{Z}_6$ generator acts trivially on the physical representation and a pure change of product-group lift contributes no determinant holonomy.

Theorem IXA-5 — Conditional Regulated Determinant-Line Trivialisation

Assumption (generator completeness, hinge P-H3). The three generator classes above — perturbative anomaly loops, the SU(2) Witten loop, and diagonal $\mathbb{Z}_6$ changes of product-group lift — generate the fundamental loop data of the regulated faithful-group orbit stack. Under this assumption,

```
Hol_γ(ℒ_ferm) = 1
```

for every closed loop γ, and $\mathscr{L}$_ferm admits a global unitary section at finite regulator.

Proof

Contractible loops have trivial holonomy because the total local anomaly curvature vanishes. The unique four-dimensional weak global-anomaly generator has sign +1 because the number of fundamental weak doublets is even. Quotient-lift loops act on the matter fields through the common centre, which is the identity on every faithful matter representation, so no holonomy arises from that pointwise action; spectral-flow and eta-invariant contributions along such loops are bordism data, are not excluded by the triviality of the pointwise action, and are covered only by the generator-completeness assumption. The holonomy is multiplicative under loop concatenation, so it is trivial on the loop group generated by these classes. ■

Audit boundary

The generator-completeness assumption is the remaining mathematical hinge of this subsection, and it is exactly the global calculation the programme brief requires. A faithful quotient can carry torsion and mixed spin-G bordism data not reducible to the three intuitive loop classes. The required output remains the evaluated character

$$\alpha\_SM : \Omega_5\hat{}(\text{Spin-G})(B\ G\_SM\hat{}\text{faithful}) \longrightarrow U(1)$$

on an actual generator set of the bordism group, not on the expected generators alone. Until that group and its character are computed, Theorem IXA-5 is a conditional criterion, not a determinant-line trivialisation result. The paper therefore publishes the finite overlap determinant on every generator of the computed cellular orbit graph as the definitive check, and carries generator completeness as hinge P-H3.

47G. Positive-Pushforward Strong-Phase Selection

Let $\mathscr{L}\_\bar{\theta}$ be the combined line

$$\mathcal{L}\_\theta^{-} = \mathcal{L}\_\theta_3 \otimes \mathcal{L}\_(\det\ M\_u) \otimes \mathcal{L}\_(\det\ M\_d).$$

After the anomaly line has been globally trivialised, decompose the regulated field space into integer colour-topology sectors $\mathfrak{C}\_\nu$. Relative to a positive absolute measure $d\lambda\_\nu$, write the most general residual constant phase as

$$d\mu\_(\text{cont},\ \nu)\hat{}(a,\ R) = \rho\_\nu(A,\ \Phi,\ \Psi)\ e\hat{}i\nu\theta^{-}\ d\lambda\_\nu,\ \rho\_\nu \geq 0.$$

The combined phase is

$$e\hat{}i\theta^{-} = \chi\_\text{sub}(1)\ \det\ (M\_uM\_d)/|\det\ (M\_uM\_d)|.$$

Lemma IXA-6 — No Complex Phase in a Positive Pushforward

If $\mu\_\text{cont}\hat{}(a, R)$ is the pushforward of the positive substrate probability measure of Theorem IXA-1, then

$$e\hat{}i\nu\theta^{-} = 1$$

for every topology sector ν on which ρ_ν is nonzero on a set of positive measure.

Proof

A pushforward of a positive measure is a positive real measure. Therefore its Radon–Nikodym derivative relative to any positive reference measure is real and nonnegative almost everywhere. Since ρ_ν≥0 and the remaining multiplier is a constant unit complex number, positivity forces that multiplier to equal +1 on every supported sector. ■

Remark — Positivity, not reality, is the operative constraint

The lemma excludes $\bar{\theta} = \pi$ as well as generic phases, and it is positivity that does the work. At $\bar{\theta} = \pi$ the sector multiplier is e^(iνπ) = (-1)^ν, which is real on every sector but signed on odd ν. A merely real measure would tolerate it; the requirement that the pushforward be a positive measure forces the multiplier to +1 and excludes the signed alternative. The no-go therefore covers the entire circle of candidate phases, including the reality-preserving point $\bar{\theta} = \pi$.

Unit-sector support (hinge P-H4)

The periodic faithful-group regulator admits an embedded unit-instanton colour configuration, and nonzero quark masses lift the corresponding fermion zero modes so that the complete massive determinant is nonzero. Two statements are nonetheless still owed before the ν=1 sector may be called supported:

```
𝔔⁻¹(unit-topology neighbourhood) ≠ ∅,

μ_sub(𝔔⁻¹(unit-topology neighbourhood)) > 0.
```

Existence of a continuum field configuration does not prove that the substrate coarse-graining map reaches it, and an abstract Born measure does not imply full support on every admissible configuration. Both statements are direct finite calculations: exhibit a specific substrate history h_1 with $\mathfrak{Q}_3(\mathfrak{Q}(h_1)) = 1$ and $\mu\_sub(h_1) > 0$, or a positive-measure cylinder neighbourhood with that property. Until performed, unit-sector support is carried as hinge P-H4 and every theorem below that invokes it is conditional on P-H4.

Theorem IXA-7 — Positive-Pushforward No-Go Theorem for a Nonzero Strong Phase

Assume:

1. Theorems IXA-1–IXA-5, with their premises P-DEC, P-TR, P-REF and the generator-completeness assumption;
2. nonzero physical quark determinants;
3. positive support in the ν=1 colour-topology sector (hinge P-H4);
4. **Premise P-POS** — the continuum physical object is the ordinary positive pushforward measure (𝔔_(a, R))_*μ_sub^(a, R), rather than a complex line-valued amplitude functional.

Then a residual nonzero strong phase is impossible: either

⟦ $\theta^- = 0 \pmod{2\pi}$ ⟧

holds, or no ordinary positive quotient-compatible pushforward measure exists on the claimed configuration space and the return is a named global measure obstruction. On the positive branch, equivalently,

⟦ $\chi_{sub}(1) = \left(\det(M_uM_d)/|\det(M_uM_d)| \right)^{-1}.$ ⟧

Proof

Lemma IXA-6 applied to the supported unit sector gives

$e^{i\theta^-} = 1.$

The second identity is the definition of the combined invariant line. ■

What this theorem is and is not

Theorem IXA-7 is a no-go theorem, not the physical strong-phase evaluation. It establishes the fork: if the complete continuum gauge object is an ordinary positive pushforward probability measure over the supported topological sectors, then $\bar{\theta} \neq 0$ cannot occur; the alternatives are $\bar{\theta} = 0$ or the non-existence of that positive representation. It does not establish which branch of the fork the physical substrate occupies, because Premise P-POS is not derived from the operational Born measure alone. Positive probabilities for committed records coexist with complex amplitudes before commitment, and the strong phase lives precisely at that interface: the manuscript keeps the amplitude level and the committed-record level typed apart, and P-POS asserts a relation between them that must be independently established.

Three honesty declarations calibrate the theorem's distance.

Exact equivalence up to the obstruction alternative. The mathematical distance between premise and conclusion is short, and the paper states this precisely rather than hedging it. P-POS $\Longrightarrow$ ($\bar{\theta} = 0$ or global obstruction), and contrapositively $\bar{\theta} \neq 0 \Longrightarrow \neg$P-POS. The premise and the conclusion are therefore logically equivalent up to the obstruction alternative: within this architecture, asserting P-POS just is asserting the fork. The genuinely new content is the ontological commitment P-POS itself, together with the combined-line closure below, not the positive-measure-implies-no-phase implication.

Relation to the complex-action problem. Stripped of VERSF framing, Lemma IXA-6 is the standard statement underlying the Euclidean sign problem: gauge theory at $\theta \neq 0$ has a complex action and admits no ordinary positive-measure representation. That fact is well known in lattice field theory and is not claimed as new here. What VERSF adds is the assertion, via P-POS, that the physical continuum object is such a positive measure — a foundational commitment standard Euclidean field theory does not make and lattice practice deliberately circumvents.

What the combined line genuinely closes. The construction $\mathscr{L}_{\bar{\theta}} = \mathscr{L}_{\theta_3} \otimes \mathscr{L}_{(\det M_u)} \otimes \mathscr{L}_{(\det M_d)}$ closes the obvious evasion: the phase cannot be hidden by rotating θ_3 into the quark mass matrices, because a complex fermion determinant spoils positivity of the measure exactly as the gauge θ-term does. The no-go applies to the invariant combination, not to a convention-dependent decomposition, and this loophole-closure is the theorem's real strength.

Calling $\bar{\theta} = 0$ a returned VERSF prediction before P-DEC, P-TR, P-H3, P-H4 and P-POS are discharged would overstate what has been achieved.

Interpretation

The theorem does not require the topological factor and the quark determinant factor to be separately real. It requires their product to be the positive real section selected by the substrate probability measure. Thus a nontrivial physical quark determinant phase is permitted only if the substrate topological character cancels it exactly.

On the conditional MCHY Hermitian representative,

$$\arg \det (M_u^C M_d^C) = 0,$$

so the theorem reduces to

$$\chi_{sub}(1) = 1.$$

The strict theorem is stronger: it returns $\bar{\theta}=0$ even if the blind physical determinant phase is nonzero.

47H. Strong-Phase Obstruction Alternative

The positive-pushforward theorem creates a clean falsification fork.

If the global anomaly audit fails, if the combined line has nontrivial holonomy, or if a supported topology sector necessarily carries a non-real Radon–Nikodym phase, then no ordinary positive measure $\mu_{cont}=\mathfrak{Q}_*\mu_{sub}$ exists on the claimed configuration space.

That is not a nonzero-$\bar{\theta}$ branch within the positive-measure architecture. It is the obstruction branch:

⟦ no positive quotient-compatible continuum pushforward exists. ⟧

The physical alternatives are therefore reduced from three to two by the positive-probability ontology:

⟦ $\bar{\theta} = 0$ ⟧ or ⟦ global measure obstruction. ⟧

A genuine nonzero strong phase would require abandoning Premise P-POS — the identification of the continuum quantum object with an ordinary positive pushforward measure — and replacing it by a complex line-valued amplitude functional. That is a coherent alternative, not an absurdity: it is the standard Euclidean architecture at nonzero θ, and it is the direction in which the framework's own pre-commitment amplitude level points. Adopting it would be a different VERSF architecture and must be stated as such. The reduction from three physical branches to two therefore holds under P-POS and only under P-POS.

47I. Regulator Removal

Let (a_n, R_n) be a refinement sequence with

```
a_n → 0, R_n → ∞.
```

Assume:

- tightness of the positive measures μ_cont^(a_n, R_n);
- convergence of all gauge-invariant cylinder correlators;
- strong-resolvent convergence of H_(a_n, R_n) on a common dense core;
- stability of the determinant-line trivialisation and of unit-sector support;
- no probability loss into regulator-boundary or singular-orbit strata.

Then every weak limit is a positive gauge-basic continuum measure. The equality

```
e^iθ̄ = 1
```

is discrete and regulator independent, so it survives the limit.

Theorem IXA-8 — Conditional Continuum Positive-Measure Strong-Phase Theorem

Under the finite-regulator construction (with premises P-DEC, P-TR, P-REF, P-POS and hinges P-H3, P-H4) and the convergence conditions above, the VERSF continuum Standard Model has a quotient-compatible positive physical measure, a reflection-positive Hilbert reconstruction and

```
⟦ θ̄_VERSF = 0 (mod 2π). ⟧
```

If any convergence condition or global-line audit fails, the output is a named global obstruction rather than an unevaluated phase.

47J. Pass-Programme Ledger

Required return	Attempted return in this part	Grade
Explicit regulated substrate measure	$\mu\_sub^{(a, R)}(h)=D(h, h)=\lVert C\_h\Omega\rVert^2$	exact given terminal-history decoherence (P-DEC / P-H1)
Projective consistency	coarse class operators are sums of fine class operators	exact given P-DEC
Explicit continuum map	joint holonomy/scalar/matter orbit readout $\mathfrak{Q}\_(a, R)$	constructive; microscopic readout maps inherited
Gauge-basicity	orbit-valued map by construction	exact
Faithful quotient census	$H^2(T^4, \mathbb{Z}_6)=(\mathbb{Z}_6)^6$ lifting-obstruction sectors	exact for obstruction sectors; characteristic-data compatibility open (D-SG5)
Global Gribov treatment	orbit measure, no global slice	exact
Determinant-line audit	local curvature, Witten sign and quotient-lift loops	conditional on generator completeness (P-H3); Ω_5^(Spin-G) evaluation open
Reflection positivity	record-subalgebra lemma from positive polar factor	conditional on P-TR and P-REF (P-H2); abelian algebra only; full-algebra OS positivity open (D-SG8)
Physical Hilbert space	OS reconstruction from $\mathbb{T}$	exact under preceding identification
Strong phase	no-go fork: θ=0 or obstruction	conditional theorem under P-POS, P-DEC, P-TR, P-H3, P-H4; physical pass open
Alternative output	global measure obstruction	exact failure branch

Honest status

This part establishes the Positive-Pushforward No-Go Theorem in conditional form. It does not return the physical strong phase. Four substrate calculations remain load-bearing, and each has a definite finite form:

1. **P-H1 — Consistent-history decoherence.** Calculate $D(h, h') = \langle\Omega|C\_(h')^\dagger C\_h|\Omega\rangle$ for the complete terminal-history classes and prove exact off-diagonal vanishing in the commitment superselection limit, or supply an error-controlled approximate extension theorem. This creates the actual positive history measure; exactness is load-bearing because the projective extension is unforgiving of approximation.
2. **P-H2 — Euclidean transfer derivation.** Derive the transfer operator from the substrate dynamics and prove it positive; $\mathbb{T} = |R|/\lVert R\rVert$ must be returned as a theorem, not installed as a definition.
3. **P-H3 — Cellular bordism/holonomy computation.** Construct the complete loop-generator complex for the faithful quotient and evaluate $Hol\_(\gamma\_j)(\mathscr{L}\_ferm)$ for every

independent γ_j; publishing the three expected loop types is insufficient without a completeness proof.

4. **P-H4 — Topological support witness.** Exhibit a specific substrate history h_1 with $\mathfrak{Q}_3(\mathfrak{Q}(h_1)) = 1$ and $\mu\_sub(h_1) > 0$, or a positive-measure cylinder neighbourhood with that property.

Beyond P-H1–P-H4, the fork itself rests on Premise P-POS, which is a foundational claim about the relation between pre-commitment amplitudes and committed-record probabilities and must be independently established. The verdict of this part is therefore:

```
⟦ finite-regulator positive-pushforward no-go theorem — established
conditionally ⟧
```

```
⟦ physical strong-phase pass — not yet established ⟧
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Part IX — Closure, Falsification and Audit

48. The Substrate Cohomology, Global Gauge Measure, and Positive-Pushforward Strong-Phase No-Go Theorem

Theorem 12 — SGMC-1R Conditional Global Completion Theorem

Assume SG1–SG13. Then the VERSF Standard Model possesses a globally defined nonperturbative quantum completion with the following properties.

1. **History measure.** The finite operational measures extend uniquely to a regular substrate history measure μ_hist.
2. **Continuum pushforward.** μ_hist pushes forward through $\mathfrak{Q}$ to the complete faithful-group configuration stack, with the substrate Jacobian retained.
3. **Gauge-basicity.** The absolute measure descends to gauge-orbit space and includes every admissible faithful quotient bundle.
4. **Global chiral measure.** The total chiral determinant/Pfaffian line has a local, gauge-equivariant, faithful-quotient-compatible global section.
5. **Gribov control by substitution.** The global theory is defined on the quotient stack without a globally unique gauge slice; local FP/BRST/BV charts are recovered perturbatively.
6. **Positive physical reconstruction.** Reflection positivity reconstructs a positive Hilbert space, transfer matrix and Hamiltonian.
7. **Record compatibility.** The OS Hilbert space is canonically equivalent to the terminal-record GNS space on the common observable algebra.

8. **Triality compatibility.** In the adjoint-only colour restriction, the centre-surface operator commutes with the transfer matrix and the physical Hilbert space decomposes into triality sectors. In full QCD, the corresponding screening architecture is represented without a false exact superselection claim.
9. **Weak/strong CP separation.** The weak Jarlskog invariant and quark determinant phase are independent typed outputs.
10. **Conditional quark-line branch.** The MCHY Hermitian representative has

    $$\arg\det(M_u^C M_d^C) = 0,$$

    so its strong phase is entirely the substrate topological character.

11. **Strong-phase closure.** The strict combined line returns exactly one branch of the trichotomy:

    $$e^{i\bar{\theta}} = \chi_{sub}(1)\,\det(M_u M_d)/|\det(M_u M_d)|.$$

12. **Paper 5 interface.** The global measure/Hilbert component of D8 is supplied. A Jacobian-generated entropic-gradient term may contribute a calculable local B_τ, but global coercivity, slicing and D9 remain separate.

Proof

Assemble Theorems 1–11, Theorem 9A, Theorem 9B, Proposition 9C and the supporting lemmas of Parts II–VIII. ■

Theorem IXA-9 — SGMC-1P Conditional No-Go and Pass-Programme Theorem

Assume Premises P-DEC, P-TR, P-REF and P-POS, hinges P-H3 and P-H4, and the convergence conditions of Theorem IXA-8. Then the VERSF Standard Model possesses a positive faithful-quotient continuum measure, a reflection-positive physical Hilbert space on the record subalgebra, a globally trivial combined anomaly line and the fork

$$\llbracket\ \bar{\theta}_{VERSF} = 0 \pmod{2\pi}\ \rrbracket$$

or a named global measure obstruction. It does not return an unevaluated phase.

The hinge-independent content of this theorem is the no-go direction: it requires none of P-H1–P-H4, but it remains conditional on P-POS — under that premise alone, no branch with $\bar{\theta} \neq 0$ exists inside the positive-pushforward architecture. The pass direction — the assertion that the physical substrate in fact satisfies P-DEC, P-TR, P-REF, P-POS and P-H1–P-H4 and therefore returns $\bar{\theta} = 0$ — is a programme of four executed calculations plus one foundational derivation, and it is not claimed here.

Proof

Theorem IXA-1 (under P-DEC) constructs the positive regulated history measure; Theorems IXA-3–IXA-5 (under generator completeness) construct the faithful quotient and determinant-line section; Theorem IXA-2 (under P-TR) gives record-subalgebra reflection positivity; Theorem IXA-7 (under P-POS and P-H4) forces the combined phase to unity on the supported unit topology sector; Theorem IXA-8 preserves the result under regulator removal. ■

49. Strict Theorem Versus Reference Completion

Two routes must remain distinct.

49.1 Strict substrate route

The strict route evaluates the actual projective marginals, history measure, pushforward Jacobian, faithful bundle sectors, chiral Dirac family, bordism character, reflection-positive decomposition and strong-phase section from the frozen VERSF substrate.

Only this route can pass Paper 6 numerically.

49.2 Reference completion

A mathematically explicit reference model may be assembled from:

- compact faithful-group link variables with product Haar measure;
- a positive plaquette/scalar action;
- a local overlap/Ginsparg–Wilson chiral regulator;
- the anomaly-free Standard Model census;
- trivial topological sector character;
- a positive real base orientation of the combined determinant line.

This reference model demonstrates that the declared global architecture is nonempty. A second reference branch is supplied by the MCHY Hermitian corpus representative, whose quark determinant factor is positive. On either reference branch,

$$\theta^{-} = 0$$

only if the topological sector character is also trivial.

Neither reference construction proves that the VERSF substrate selects the reference measure, regulator, right-handed frames, sector character or positive branch.

50. Open-Debt Ledger

50.0 Decisive Hinge Ledger for the Positive-Pushforward Pass

The broad SG1–SG13 architecture debts below remain useful as a programme audit. For the finite-regulator pass programme, they collapse to four load-bearing calculations and one foundational premise:

P-H1 — Consistent-history decoherence (exact or controlled). Calculate the decoherence functional $D(h, h')=\langle\Omega|C_{h'}^\dagger C_h|\Omega\rangle$ for the complete regulated terminal-history class-operator family and prove $D(h, h')=0$ for $h \neq h'$ exactly, in the commitment superselection limit; or reformulate the projective extension theorem for approximately consistent marginals with explicit regulator-uniform error control. Either route discharges Premise P-DEC and creates the actual positive additive history measure of Theorem IXA-1. Approximate decoherence without error-controlled extension discharges nothing: the Kolmogorov machinery is unforgiving of approximation.

P-H2 — Euclidean transfer derivation. Derive the physical Euclidean one-step operator from the microscopic substrate update and prove it positive, by reflection/Wick continuation, by explicit similarity to a positive self-adjoint operator, or by independent construction of a reflection-positive transfer kernel; derive its reflection symmetry $\Theta\mathbb{T}\Theta^{-1}=\mathbb{T}$ in the same step. This discharges Premises P-TR and P-REF; $\mathbb{T} = |R|/\|R\|$ must arrive as a theorem, not a definition.

P-H3 — Global-loop completeness. Construct the complete loop-generator complex of the cellular faithful-quotient orbit stack, evaluate $\mathrm{Hol}_{\gamma_j}(\mathscr{L}_{ferm})$ on every independent generator γ_j, and evaluate the bordism character $\mathfrak{a}_{SM}$ on an actual generator set of $\Omega_5^{\mathrm{Spin}\text{-}G}(B\,G_{SM}^{faithful})$. Verifying only the local, Witten and $\mathbb{Z}_6$-lift loop classes is insufficient without a completeness proof.

P-H4 — Unit-topology support and convergence. Exhibit a specific substrate history h_1 with $\mathfrak{Q}_3(\mathfrak{Q}(h_1))=1$ and $\mu_{sub}(h_1)>0$ (or a positive-measure cylinder neighbourhood with that property), and prove that its support, the positive measure and the determinant section survive regulator removal.

Premise P-POS — Positive continuum representability. Establish that the uncommitted continuum quantum object is an ordinary positive pushforward measure rather than a complex line-valued amplitude functional. This is a foundational derivation about the amplitude/record interface, not a finite check, and the no-go fork of Theorem IXA-7 holds under it and only under it.

Executing P-H1–P-H4 and establishing P-POS promotes Theorem IXA-9 from a conditional no-go theorem to a strict substrate pass.

D-SG1 — Finite operational projective system. Publish the physical X_α, maps $r_{\beta\alpha}$ and marginals.

D-SG2 — Tightness. Prove no measure escapes into unresolved refinement directions.

D-SG3 — Coarse-graining map. Construct $\mathfrak{Q}$ on complete histories.

D-SG4 — Pushforward Jacobian. Evaluate $\mathcal{J}_{(\text{sub}\to\text{cont})}$, including real local operators, nonlocal residue and line phase.

D-SG5 — Faithful bundle census. Compute admissible ν_6, spin/spin-G, torsion and boundary sectors.

D-SG6 — Substrate chiral regulator. Derive or universality-match the local index-correct Dirac family.

D-SG7 — VERSF-native bordism audit. Evaluate $\mathfrak{a}_{SM}$ on all physical generators.

D-SG8 — Reflection positivity. Prove RP for the complete measure and combined line.

D-SG9 — OS/GNS compatibility. Match Euclidean and terminal-record states.

D-SG10 — Strong-sector character. Evaluate χ_{sub} on a complete topological generator set.

D-SG11 — Physical quark phases. Publish the blind physical arg det M_u and arg det M_d.

D-SG11A — Right-frame and orientation audit. Determine whether the physical substrate selects the MCHY Hermitian representative or adds an axial phase through the right frames/dressed bilinear.

D-SG12 — Triality-compatible restriction. Derive the adjoint-only restriction and prove centre commutation through regulator removal.

D-SG13 — Entropic-gradient coefficient. Calculate c_6^{sub} from the pushforward Jacobian without external condensate or string-tension input.

D-SG14 — Wall boundary data. Derive φ_+, φ_- and δ_{max} from the nonzero-triality sector.

D-SG15 — Global coercivity and slicing. Promote the local wall coefficient to a regulator-uniform sector gap.

D-SG16 — Continuum convergence. Preserve gauge invariance, line holonomy, positivity, centre action and clustering through regulator removal.

51. Falsification Conditions

F0 — Decoherence failure. If the complete regulated terminal-history family has persistently nonzero off-diagonal decoherence-functional entries, $D(h, h') \neq 0$ for $h \neq h'$ at every admissible coarse-graining, and no error-controlled approximate extension is available, then no positive additive substrate history measure exists and the positive-pushforward architecture fails at its first step. Approximate decoherence without a reformulated extension theorem also fails this condition: exact additivity is what the projective machinery consumes.

F1 — Marginal inconsistency. If some refinement obeys

$$(r_{\beta\alpha})_*\mu_\beta \neq \mu_\alpha,$$

no single substrate history measure exists with the claimed marginals.

F2 — Tightness failure. If probability escapes to unresolved refinement directions, the history measure is not a regular physical completion.

F3 — Nonmeasurable or nonlocal coarse map. If $\mathfrak{Q}$ cannot be defined with the declared locality and field typing, the continuum theory is not a substrate pushforward.

F4 — Gauge-basicity failure. If gauge-equivalent continuum configurations receive different absolute weights, the measure is not physical.

F5 — Omitted quotient sectors. If a non-liftable admissible faithful bundle is excluded, the global census is incomplete.

F6 — Global anomaly. If $\mathfrak{a}_{SM}\neq 1$ on any physical bordism generator, the chiral Standard Model measure is obstructed.

F7 — Chiral locality failure. If the regulated chiral measure is nonlocal or has no correct continuum index, the nonperturbative fermion construction fails.

F8 — Gribov dependence. If a gauge-invariant observable depends on which local gauge slice or copy is used, the orbit-stack construction or its implementation is wrong.

F9 — Neuberger zero. If the fundamental partition function is defined only as a globally BRST-exact signed copy sum and collapses to 0/0, the global construction fails its own firewall.

F10 — Reflection-positivity failure. If some positive-time physical functional has negative OS norm, no positive Lorentzian Hilbert space follows from the measure.

F11 — OS/GNS mismatch. If the two physical state constructions assign different expectations to a common terminal observable, at least one identification is incomplete.

F12 — Quark determinant instability. If $\arg\det(M_u M_d)$ changes under certified numerical precision or canonical-frame gauge, Paper 3 has not supplied a physical phase.

F13 — Strong-line obstruction. If the combined line cannot be trivialised, the claimed global Standard Model completion is obstructed. This is a valid Branch C result, not a silent failure.

F14 — Nonzero strong phase. A certified $\bar{\theta}\neq 0$ is a valid Branch B prediction. It falsifies VERSF only if later physical transport and blind empirical comparison exclude it.

F15 — Target-data dependence. If any measure phase, regulator orientation or branch is chosen using the observed strong-CP bound, the derivation fails.

F16 — Paper 5 mismatch. If the global measure does not commute with the correctly embedded centre operator or fails to support Paper 5's sector decomposition, D8 is not discharged.

F17 — Weak/strong CP conflation. If a CKM/Jarlskog phase is used as the quark determinant phase, or a positive determinant is used to erase weak CP violation, the strong-phase calculation is mistyped.

F18 — Reference/physical branch conflation. If arg det (M_u^CM_d^C)=0 is reported as the physical result before the right-frame and dressed-line audit, the derivation fails.

F19 — Entropic coefficient insertion. If c_6^sub, φ_± or δ_max are chosen from the observed string tension, the wall branch is postdiction.

F20 — Fixed-flux area-scaling error. If one global centre flux is used to infer an area-extensive F^2 cost without a per-slice or per-cell triality theorem, the confinement proof fails.

F21 — Flux-probability area-law error. If a nonzero probability of flux configurations is used as an upper exponential bound on the Wilson loop, the area-law inference fails.

F22 — Lorentzian positivity error. If Lorentzian F^2 or $(\partial\varphi)^2$ is treated as a positive Euclidean action density, the entropic branch is invalid.

F23 — Entropic RP failure. If the regulator realisation of S_EG violates reflection positivity, it cannot be promoted to the physical Hamiltonian.

52. Numerical Non-Insertion Audit

The following may not be used upstream:

- the empirical upper bound on $|\bar{\theta}|$;
- neutron or atomic electric dipole data;
- an observed quark mass to select a determinant phase;
- lattice-QCD topological-sector weights used as substrate input;
- a measured topological susceptibility used to normalise χ_sub;
- an assumed positive determinant orientation selected because it gives zero.

The following may be used as mathematical or consistency inputs without numerical insertion:

- the inherited matter representation census;
- the faithful $\mathbb{Z}_6$ quotient;
- standard projective-limit, determinant-line, bordism and OS mathematics;
- external anomaly calculations as downstream corroboration, clearly labelled;
- a reference regulator used to demonstrate existence, clearly separated from substrate selection.

53. SGMC-1R Closure Certificate

Closure item	Status
Local perturbative gauge architecture	inherited closed
Three-measure distinction	closed
Operational-to-history extension theorem	closed conditionally on SG1–SG3
Physical projective marginals	open calculation
History measure uniqueness	closed once marginals pass
Substrate-to-continuum map	exact target; open construction
Pushforward Jacobian	typed; open evaluation
Candidate S_EG operator	admissible Jacobian branch; coefficient open
Conditional entropic wall inequality	proved given sector boundary data
Entropic area-law theorem	not claimed
Faithful global group	inherited closed
Six-cocycle quotient-bundle architecture	closed
Complete physical quotient-bundle census	open evaluation
Gauge-basic absolute measure	closed conditionally on SG4–SG5
Global orbit-stack definition	closed
Global gauge slice required	no
Local FP/BRST/BV recovery	inherited/closed on regular charts
Neuberger firewall	closed structurally
Positive CAR/Fock carrier	inherited conditionally
Fermion determinant-line typing	closed
Local anomaly cancellation	inherited closed
Full faithful-group bordism audit	exact target; evaluation open
Nonperturbative chiral measure	conditional construction supplied
Reflection-positive reconstruction	exact theorem conditional on SG9
Physical transfer matrix/Hamiltonian	conditional on RP and convergence
OS/GNS comparison	exact theorem conditional on SG10

Closure item	Status
Triality-compatible OS/GNS sector decomposition	exact theorem conditional on SG13
Mass singular values	magnitude inputs only
Weak-CP/strong-phase independence	closed
Conditional MCHY quark determinant phase	0 on declared Hermitian representative
Physical arg det $(M_u M_d)$	owed by blind microscopic evaluation
Combined strong-phase line	closed
Strong-phase trichotomy	closed
Conditional branch $e^{i\bar{\theta}_C}=\chi_{sub}(1)$	closed
Substrate sector character χ_{sub}	open evaluation
Positive-pushforward strong phase	conditional no-go fork under P-POS: $\bar{\theta}=0$ or obstruction; physical pass open
Paper 5 D8 measure/Hilbert architecture	supplied
Candidate local B_τ^{EG}	conditional branch supplied
Paper 5 global coercive pass	open
Paper 5 D9	outside present closure
Measured strong-CP or string-tension data used upstream	no

SGMC-1P verdict.

`⟦ finite-regulator positive-pushforward no-go theorem — established conditionally ⟧`

`⟦ physical strong-phase pass — not yet established ⟧`

The positive-pushforward branch reduces the alternatives to $\bar{\theta}=0$ or a named global measure obstruction rather than leaving the phase unevaluated, but this fork holds conditionally on Premise P-POS and on hinges P-H1–P-H4. A strict substrate-native pass is claimed only after the history-decoherence, transfer-operator, bordism-generator and topology-support calculations are executed and P-POS is independently established. Until then the result is a conditional no-go theorem and a precisely specified pass programme, not a completed microscopic evaluation.

54. Conclusion

The local and global quantum-gauge problems are not the same problem. The local problem is the construction of perturbative gauge theory in a regular orbit patch. The global problem is the existence of one quotient-compatible physical measure over every faithful-group bundle and large-gauge sector, together with a positive Hilbert reconstruction and a globally meaningful chiral determinant line.

The architecture developed in Parts I–VIII closes the type system of that problem:

$$\mu\_op^{(\alpha)} \longrightarrow \mu\_hist \longrightarrow \mu\_cont = \mathfrak{Q}\_*\mu\_hist,$$

$$\mathfrak{C}\_(G\_SM)(X) = \coprod\_[P] \; [\; \mathcal{A}(P) \times \Gamma(\mathcal{F}\_P) \times \Gamma(\mathcal{S}\_P) \;/\mathcal{G}(P) \;],$$

$$\mathcal{L}\_\theta^{-} = \mathcal{L}\_\theta_3 \otimes \mathcal{L}\_(\det M\_u) \otimes \mathcal{L}\_(\det M\_d).$$

The finite-regulator construction of Part IXA goes further. It makes four concrete identifications, each carried with its named premise.

First, terminally distinguishable regulated histories are represented by an exhaustive class-operator family whose diagonal decoherence-functional values supply the substrate probabilities, conditional on terminal-history decoherence (Premise P-DEC):

$$\mu\_sub^{(a,\ R)}(h) = D(h,\ h) = \|C\_h\ \Omega\_(a,\ R)\|^2.$$

Decoherence makes additivity and refinement consistency automatic whenever a coarse class operator is the sum of its fine class operators; proving the decoherence itself is hinge P-H1.

Second, the Euclidean transfer operator is identified with the positive polar factor of the physical refinement map, as Premise P-TR:

$$R\_(a,\ R) = U\_(a,\ R)|R\_(a,\ R)|,\quad \mathbb{T}\_(a,\ R) = |R\_(a,\ R)|/\|R\_(a,\ R)\|.$$

Under P-TR this gives record-subalgebra reflection positivity directly and reconstructs a positive Hamiltonian; deriving the identification from the microscopic update, rather than installing it, is hinge P-H2.

Third, the continuum field measure is defined as the faithful-orbit pushforward of the positive terminal-history measure. Gauge fixing becomes a local coordinate device, not the definition of the theory. The periodic test regulator supplies an explicit non-liftable quotient-bundle census through

$$H^2(T^4,\ \mathbb{Z}_6) \cong (\mathbb{Z}_6)^6.$$

Fourth, the finite determinant-line audit reduces to local anomaly curvature, the Witten SU(2) loop and faithful-quotient lift loops. The inherited chiral census cancels the local anomalies, contains an even number of weak doublets, and is trivial under the diagonal $\mathbb{Z}_6$ generator. Conditional on completeness of this generator set, the regulated determinant line admits a global section.

These identifications turn the strong-phase question into a structural theorem. A genuine pushforward of a positive substrate probability measure is a positive real measure. It cannot carry a residual constant complex phase on a supported topological sector. Since the unit colour-topology sector is supported and the physical quark determinants are nonzero,

$⟦ \bar{\theta}\_VERSF = 0 \pmod{2\pi}. ⟧$

The theorem constrains only the invariant combination. It does not require

$\theta_3 = 0$

and

$\arg\det(M_u M_d) = 0$

separately. A nonzero blind quark determinant phase is permitted if the substrate topological character cancels it exactly. On the conditional MCHY Hermitian representative, the quark determinant is already positive, so the same result reduces to a trivial substrate topological character.

This also sharpens the obstruction branch. If a faithful quotient sector has unavoidable nontrivial determinant-line holonomy, if the transfer operator is not positive, or if the continuum object necessarily has a non-real Radon–Nikodym phase, then the VERSF continuum object is not an ordinary positive pushforward measure. The output is a global measure obstruction, not an unevaluated value of $\bar{\theta}$.

The positive-pushforward result is therefore a real theoretical commitment. Standard Euclidean gauge theory at nonzero θ uses a complex weight. The VERSF claim that the physical continuum measure is the pushforward of positive operational probability excludes that architecture unless the total phase cancels. Paper 6 consequently predicts exact strong-phase triviality or fails at the global-measure level.

This result does not erase the remaining implementation work. A strict publication-grade pass still requires explicit verification that:

1. the terminal-record projector partition is exhaustive at the chosen substrate regulator;
2. the physical Euclidean step is the positive polar factor of refinement;
3. the finite orbit-loop generator list is complete for the faithful quotient;
4. a unit-topology configuration has nonzero substrate-measure support;
5. the positive measures, determinant section and transfer operators survive regulator removal.

Those are the decisive hinges. They are narrower than the SG1–SG13 architecture ledger and each has a direct finite computation.

For Paper 5, the same construction supplies the global positive measure, transfer matrix and triality-compatible Hilbert stage required by D8. The candidate entropic-gradient term may contribute a local wall coefficient, but it does not replace the global coercivity, longitudinal slicing or D9 calculations.

The Paper 6 verdict is therefore the structural fork

⟦ ordinary positive quotient-compatible pushforward ⟹ θ̄ = 0 ⟧

⟦ failure of positive global trivialisation ⟹ global measure obstruction ⟧

together with the honest two-line status

⟦ finite-regulator positive-pushforward no-go theorem — established conditionally ⟧

⟦ physical strong-phase pass — not yet established ⟧

Within the positive-probability premise P-POS, a third physical branch with nonzero θ̄ is not available: it would require replacing the continuum probability measure by a complex line-valued amplitude functional and thereby changing the framework's foundational measure claim. P-POS itself, however, is a statement about the interface between pre-commitment amplitudes and committed-record probabilities and is owed an independent derivation. The paper's positive achievements stand without it: the structural fork, the reduction of the measure problem to the four finite hinges P-H1–P-H4, the conditional Hermitian quark branch with arg det (M_u^C M_d^C) = 0 kept correctly separate from the physical determinant phase, and the weak-CP/strong-phase independence theorem.

Appendix A — Projective-Limit Measure Proof

Let $\mathfrak{R}$ be directed and $(X_\alpha, \mathscr{B}_\alpha)$ standard Borel. For $\beta \succeq \alpha$, let $r_{\beta\alpha}$ be measurable, compositional and surjective on the physical support. Let μ_α be projectively consistent probability measures.

For a cylinder set

$C = \bigcap_{(j = 1)}^{n} \pi_{(\alpha_j)}^{-1}(E_j),$

choose $\gamma \succeq \alpha_j$ for every j and define

$\mu\_cyl(C) = \mu\_\gamma(\bigcap\_j \ r\_(\gamma\alpha\_j)^{-1}(E\_j)).$

If γ' is another common refinement, choose $\delta \succeq \gamma, \gamma'$. Projective consistency shows the two values agree after pullback to X_δ. Thus μ_cyl is well defined. Finite additivity follows from the finite measures. The standard extension theorem yields a probability measure on the generated sigma-algebra. Tightness gives the desired regular extension and compatibility with the inverse-limit topology.

The proof also shows why projective consistency is non-negotiable: one failed marginal equality gives two incompatible values for the same cylinder event.

Appendix B — Faithful-Group Six-Cocycle

Let

$1 \to \mathbb{Z}_6 \to G\tilde{\ }\_SM \to G\_SM\text{^}faithful \to 1$

be the central extension. A principal faithful-group bundle has local transition functions g_ij. Local lifts g̃_ij exist on double overlaps. Their triple products lie in the kernel, giving $n\_ijk \in \mathbb{Z}_6$. The Cech cocycle identity follows because the kernel is central. A new lift

$g\tilde{\ }\_ij' = \zeta\text{^}(m\_ij) g\tilde{\ }\_ij$

changes

$n\_ijk' = n\_ijk+m\_ij+m\_jk+m\_ki,$

so [n] is well defined in $H^2(X, \mathbb{Z}_6)$.

The bundle lifts to the covering product if and only if $\nu_6(P)=0$. The complete faithful theory includes both liftable and non-liftable sectors.

Appendix C — Determinant-Line and Bordism Audit

The audit package should publish:

1. the tangential structure used in each quotient sector;
2. a generator presentation of Ω_5^(Spin-G)(BG);
3. the five-dimensional Dirac operators for the inherited chiral census;
4. the exponentiated eta invariant on every generator;
5. the counterterm-line contribution;
6. the total character ɑ_SM;
7. the dependence on optional sterile neutrino branches;

8. the regulator-independence certificate.

A local anomaly polynomial table is not this audit.

Appendix D — Reflection-Positivity Proof Skeleton

At finite regulator split the fields into positive-time, negative-time and reflection-plane variables. Show:

1. the gauge/scalar action admits a positive character or square decomposition across the plane;
2. the fermion kernel has the required reflection adjointness;
3. the determinant-line section obeys

   $\Theta s(\Phi) = \bar{s}(\Theta\Phi)$;

4. topological sector weights pair under reflection without producing a negative OS quadratic form;
5. integration over reflection-plane variables preserves positivity;
6. the physical quotient commutes with reflection;
7. the inequality is uniform under regulator refinement.

Only after these seven rows pass should T and H be called physical.

Appendix E — Strong-Phase Evaluation Algorithm

Input A: canonical physical matrices M_u, M_d.
Input B: substrate topological sector character χ_{sub}.
Input C: determinant-line orientation and anomaly certificate.

1. Verify $\sigma_{min}(M_u), \sigma_{min}(M_d)>0$.
2. Compute log determinants using a phase-stable complex factorisation.
3. Compute $\varphi_q=\mathrm{Im}[\log\det M_u+\log\det M_d]$ with branch continuity.
4. Evaluate $\theta_3^{sub}=\mathrm{Arg}\chi_{sub}(1)$.
5. Return

   $\bar{\theta} = \mathrm{Arg}\, e^{i(\theta_3^{sub}+\varphi_q)}$.

6. Repeat under canonical-frame gauge transformations and threshold transport.
7. Certify one of:

- interval contains only 0 modulo 2π;
- interval excludes 0;
- line/phase cannot be defined globally.

Appendix F — Dependency Map and Internal References

Primary internal predecessors:

1. **The Quantum Gauge Completion of the VERSF Standard Model** — local FP/BRST/BV, anomaly and renormalisation architecture; global debts; combined strong line.
2. **The Gauge Closure Selection Theorem in VERSF** — faithful $\mathbb{Z}_6$ quotient and field congruence.
3. **Operational Distinguishability Geometry as the Primitive Probability Object** — operational quotient and measure invariance.
4. **The Universal Measure Principle** — cross-sector commensurability and universal measure.
5. **Probability as Admissible Measure on the VERSF Operational Hilbert Geometry** — quadratic finite operational measure.
6. **Unified Refinement Hilbert Geometry in VERSF** — complex carrier, unitary transport and positive contraction.
7. **The Adjoint-Removal Positivity and Microcausality Theorem in VERSF** — physical quotient and GNS positivity.
8. **Fock-Space Construction and the Full CAR Algebra in VERSF** — positive one-particle space, Fock space and CAR.
9. **From Substrate CAR Algebra to Relativistic Fermion Fields in VERSF** — emergent Dirac field and positivity–covariance bridge.
10. **The Microscopic Closure-Hessian and Full Yukawa Evaluation Theorem in VERSF** — physical quark matrices, determinant-phase publication requirement and the conditional positive-determinant corpus branch.
11. **The Quark-Sector Closure Ledger / Projected C_3 Hessian papers** — nonzero weak CP/Jarlskog branch and phase-provenance firewall.
12. **The Non-Perturbative QCD Scale and Triality-Gap Completion Theorem in VERSF** — D8 global measure interface, centre/triality architecture and D9 firewall.
13. **Entropic Confinement and Effective Quark Masses in the VERSF Framework** — candidate gradient-wall mechanism; area-law argument not inherited.
14. **The Final Eight Papers of the VERSF Standard Model Programme** — Paper 6 brief and closure test.

Selected external mathematical frameworks used as imported tools, not VERSF outputs:

- projective/Kolmogorov measure extension;
- principal-bundle obstruction theory for central quotients;

- determinant lines and families index theory;
- Dai–Freed eta-invariant anomaly formulation;
- Ginsparg–Wilson/overlap chiral regularisation;
- Gribov and Neuberger obstruction analysis;
- Osterwalder–Schrader reflection-positive reconstruction;
- GNS representation theory.