The VERSF Master Substrate Action and Constitutive-Descent Theorem

A Common Microscopic Parent for Scalar Response, Gauge Stiffness, Fermion Completion, Flavour Geometry, Colour Relaxation and Global Quantum Measure

▲ Programme Keystone — Unified Substrate Dynamics and Standard Model Constitutive Closure Series

Gate MASD-1 / Independent Link–Current–Record Variables, Universal Defect-Rate Functional, Polar Bond Compatibility, Measure-Fixed Action Normalisation, Derived Scalar Readout, Chiral Completion Kernel, Common Hessian Descent and Blind Finite-Regulator Evaluation

Keith Taylor VERSF Theoretical Physics Programme **Intercalary programme keystone: Six sector gates → MASD-1 → HMGBC-1 → HFB-1 → Renormalised Parameter Closure → Final Audit MASD-1 defines and tests the candidate machine; HMGBC-1 derives its operational metric, universal generation geometry and branch selector; HFB-1 executes the resulting physical branch. Tier A conditional master-action construction and candidate-execution theorem — July 2026**

Principal Claim

The six final-completion papers of the VERSF Standard Model programme each end at the same missing object. The scalar gate needs the momentum-dependent substrate Hessian; the Yukawa gate needs the readout projector, chiral embeddings and an independently derived left–right bilinear; the gauge gate needs the physical embedding, complete auxiliary response and absolute normalisation; the QCD gate needs an intrinsic colour–record intertwiner the wheel incidence geometry cannot supply; the global-measure gate needs an explicit finite-regulator rate functional. These are not five unrelated debts. They are five derivatives, restrictions or quotient constructions of one missing master action.

This paper defines the minimal common parent.

At finite regulator, the independent substrate variables are

$$\mathfrak{X} = (\rho,\ U,\ J,\ \Phi;\ \Psi,\ \Psi^{-}),$$

where ρ is the closure-normalised substrate amplitude, U the independent oriented link transport, J the independent transported record current, Φ the independent record/completion morphism and Ψ the complete CAR/Fock matter carrier. The Higgs interface is the scalar-compatible radial and orientation sector of ρ and Φ, not an appended field.

The bosonic action is the squared norm of one complete defect vector,

$$\mathbf{D} = (\mathcal{D}_C, \mathcal{D}_J, \mathcal{D}_T, \mathcal{D}_R, \mathcal{D}_B, \mathcal{D}_H),$$

in one canonical operational metric:

$$\Gamma_{bos}^{(n)} = \Gamma_V^{(n)} + \tfrac{1}{2} \langle \mathbf{D}, W_{prim} \mathbf{D} \rangle$$

The six defects are

$$\mathcal{D}_C = \text{vacuum-relative link holonomy},$$

$$\mathcal{D}_J = J - \kappa(\rho_t - U \rho_s),$$

$$\mathcal{D}_T = \partial_\sigma \rho + \partial_U J,$$

$$\mathcal{D}_R = \Phi_{jk}^{tr} \Phi_{ij}^{tr} - \Phi_{ik}^{tr},$$

$$\mathcal{D}_B = \Pi_j U_{ij} \Pi_i - \mathrm{Pol}(\Pi_j \Phi_{ij}^{tr} \Pi_i),$$

$$\mathcal{D}_H = X_H - (\varphi_H/\Lambda\star) P_H^R,$$

where X_H is the vacuum-relative positive polar stretch of the local left–right completion morphism Φ_H, φ_H is the full Higgs radial modulus and P_H^R is the representation-frozen right scalar-access projector (§47F), intertwined with its left partner P_H^L by the polar orientation V_H. Spatial record transport and local completion are distinct typed objects: composition constrains Φ^{tr}, mass generation lives on Φ_H.

The term $\mathcal{D}_B$ is the constitutive bridge. It compares the independent physical link with the polar transport selected by the independently varying record morphism on the occupied support, so endpoint-frame changes remain pure gauge while intrinsic support, bond and completion changes do not. Its mixed Hessian is the precise location at which a physical colour, weak or Abelian intertwiner may arise. The fermion kinetic term uses the same link U as the closure action; there is no independent matter-vertex normalisation.

Masses come from the positive polar stretch X_H of the local completion morphism (§31, §47F), which vanishes at the unbroken interface, and the sixth defect makes it track the Higgs modulus: on a defect-free branch $X_{(H,\star)} = (\varphi_{(H,\star)}/\Lambda\star)P_H^R$, so the scalar-response operator is $\mathcal{Y}\star = V_{(H,\star)}^\dagger P_H^R$ exactly on the minimal branch and the previous zero-readout obstruction on homogeneous vacua is removed. In general the relaxed scalar derivative determines the readout projectors as outputs,

$$P_Y^L = \mathrm{supp}(\mathcal{Y}\star \mathcal{Y}\star^\dagger), \quad P_Y^R = \mathrm{supp}(\mathcal{Y}\star^\dagger \mathcal{Y}\star)$$

related by the typed polar-orientation identity P_Y^L = V_(H,⋆)† P_Y^R V_(H,⋆) (§33), which reduces to literal equality P_Y^L = P_Y^R only after a common-carrier identification, in particular when V_(H,⋆) = I; their ranks are calculated rather than assumed.

Matter addresses are terminal-record subspaces, and the primitive left–right term is written in the raw representation-typed address maps $A\_fX^{(0)}$, frozen before any response is evaluated:

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⟦ Γ_LR⁽⁰⁾ = Λ⋆ ∑_(f=u,d,e,ν) [ Ψ¯_fL (A_fL⁽⁰⁾)† 𝒪_H A_fR⁽⁰⁾ Ψ_fR + h.c. ] ⟧
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No Yukawa matrix, derived projector or canonical embedding appears in this definition. The right-to-left completion operator 𝒪_H = V_H†X_H carries the polar orientation inside the bilinear, so the term is well typed and gauge invariant for distinct chiral representations; the embeddings 𝓔_fX and positive kinetic-address factors R_fX are calculated consequences. Two independent routes then return the Yukawa operators:

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Y_f^mixed = R_fL 𝓔_fL† 𝒴⋆ 𝓔_fR R_fR,

Y_f^Hess = R_fL 𝓔_fL† P_Y^L Ĥ_cl^red P_Y^R 𝓔_fR R_fR,
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the first from the relaxed derivative of the primitive bilinear, the second from the Schur-reduced bosonic Hessian. Their equality is not imposed. It is the calculable test

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⟦ P_Y^L 𝒴⋆ P_Y^R ≟ P_Y^L Ĥ_cl^red P_Y^R ⟧
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which, when it holds, implies CY3′ for every sector simultaneously. The master action can pass it, fail it or reveal a controlled residue.

The same master Hessian produces Z_H from the scalar two-point momentum coefficient; Z_3, Z_2, Z_q from the curvature/auxiliary Schur complement; Y_u, Y_d, Y_e, Y_ν from the mixed completion derivative; M_R and M_L^irr from symmetric neutral terminal-record compressions; and B_τ from constrained minimisation in the nonzero-triality sector. The global finite-regulator quantum measure is

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dμ_VERSF⁽ⁿ⁾ = Z_n⁻¹ e^(−Γ_master⁽ⁿ⁾) s_ferm⁽ⁿ⁾ dν_quotient⁽ⁿ⁾
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on the complete faithful-group orbit stack, and on the positive-pushforward branch the absolute action unit is fixed by the Radon–Nikodym rate functional of the substrate history measure — never fitted sector by sector.

Two branches of the master action have been executed on the canonical K = 7 wheel cell. The link–record branch (§47A) confirms that 𝒟_B generates a nonzero mixed Hessian, with exact reduced stiffness k_red(λ) = aλ + bcλ/(bλ + c), while exact gauge coboundaries remain null: the load-bearing new mechanism is operative, not merely typed. The factor-projected branch (§47B) evaluates that response through the frozen D_6 gauge isotypes, returning the exact wheel triple (K_3, K_2, K_q)^wheel = (24/5, 35/6, 3/2), reducing the entire remaining coupling debt to three projections π_A of one internal embedding, and returning the programme's first executed negative: the unit-isometric lift underweights primitive hypercharge, and the physical branch must satisfy $\text{ɲ}_q/\text{ɲ}_2 > 140$ or fire F26. The internal-norm section (§47C) then derives the exact Casimir-spectral operators whose three traces return the ɲ_A, the separability certificate that must pass before any ɲ_A is quoted, and the conditional target triples — mutually exclusive, with the colour conflict an exact factor of six — that the physical calculation must decide among. The response-package section (§47D) then makes supplying that matrix mechanical: two bare canonical fourteen-channel metrics are already explicit, and the physical matrix is the Schur-reduced pullback of the master Hessian along fourteen frozen tangent vectors, with the gauge embedding generated by generator differentiation rather than allocation.

Two further mechanisms have been derived and executed. The scalar-stretch branch (§47G) removes the zero-Yukawa obstruction: on the pre-registered rank-nine charged carrier it returns rank $P_Y = 9$, full-rank $Y_u = Y_d = Y_e = I_3$, exact mass–vertex linearity $M_f = (v_{cl}/\sqrt{2})Y_f$, a positive scalar–stretch Hessian and an exactly vanishing isolated CY3′ residue — masses now exist, though degenerate and unhierarchical on the unit branch. The corrected gauge embedding (§47H–§47I) lands in the tensor carrier $\mathscr{H}_{14} \otimes \mathrm{HS}(\mathscr{H}_{SM})$ rather than mapping twelve generators into fourteen channels: executed at shape 30926×12 with exact rank 12, exact factor orthogonality and isotropy, it derives the internal norms $(\text{ɲ}_3, \text{ɲ}_2, \text{ɲ}_q) = (6, 13/2, 378)$ — precisely the matter-census triple of §47C.6, now a calculated benchmark rather than a diagnostic — and therefore still fails the declared lower electroweak branch. The remaining physics is thereby localised in the nonfactorising parts of the one common Hessian: $D\mathcal{D}_B$, $D\mathcal{D}_H$, the cross blocks W_{BH}, W_{CH}, W_{RH}, and $H_{\eta\eta}^{-1}$ on the physical background (§47J). A first fully concrete candidate branch (CFB-1, §47K) has now been solved end to end through that entire chain — stationary solve, automatic differentiation, numerical quotient, complete Schur relaxation and direct nonfactorising gauge pullback — returning the candidate triple (4.8498, 5.8868, 1.5190), a definite electroweak failure, degenerate identity flavour, and no boundary certificate: the computation runs, and the simplest fully frozen branch does not produce the Standard Model. A second candidate branch (CFB-2, §47L) with a non-isotropic graph-resolvent completion law then returned hierarchical spectra in all four sectors, an exact 15° CKM plane, the first calculated Z_H and a full neutral spectrum — and still failed the electroweak branch after a $\sqrt{7002}$ local Abelian enhancement was screened away by the auxiliary relaxation, localising the remaining deficit in the auxiliary structure itself. A third branch (CFB-3, §47M) then derives the bond and record metric from the same fourth-moment rule — $(w_3, w_2, w_q) = (5/2, 21/8, 7003)$ — and becomes the first to pass the $Z_Y > Z_2$ ordering gate, with $G_{red} = \mathrm{diag}(6, 115/16, 7005/2)$, full three-angle CKM mixing and the first nonzero charged CP invariant $J_{CKM} = 0.0073$, at the cost of overshooting the Abelian magnitude. The metric restriction of CFB-3 rests on a Schur-forced representation-covariant operator (§47M.1); its generation graphs, however, are pre-registered only on an honor-system basis, not yet frozen under the §47E manifest before the solve, and this limitation is stated at first mention so no reader mistakes qualitative capacity for certified blindness.

This paper does not claim that the unique history-derived physical master action has been numerically evaluated. It does report three complete finite candidate-branch evaluations — CFB-1, CFB-2 and CFB-3 — each using additional declared constitutive assumptions not yet derived from the substrate history measure, and it claims something narrower but decisive:

The non-identifiability established by the sector gates is hereby converted into one explicit, common, finite-regulator forward calculation, whose gauge–record, factor-projection, scalar–stretch and tensor-embedding mechanisms have been executed, and whose final physical pass is the common nonfactorising Hessian evaluation of §§48–53. No sector-specific constitutive matrix remains admissible. The master action either returns the required Standard Model response operators, returns different operators, or fails one of its positivity, locality, rank, anomaly or continuum tests.

Making a single branch physical is not the work of this paper. The operational metric W_prim, the universal generation geometry and the branch selector that every candidate branch here freezes by hand are derived downstream in HMGBC-1; the selected physical branch is executed, and the boundary certificate issued, in HFB-1. MASD-1 defines and tests the machine; HMGBC-1 and HFB-1 make one branch physical.

General Reader Summary

The six final-completion papers showed what VERSF must calculate. They also showed why the calculation could not yet be completed.

The missing ingredient was not another formula for a particle mass or coupling. It was the underlying rule from which all those formulas must come.

A physical theory is not complete merely because it contains a symbol called a Hessian, a gauge response, a record map or a Yukawa operator. It must define one action — one rule assigning a cost and a quantum weight to every allowed microscopic configuration — and those objects must then appear as derivatives of that same rule.

This paper proposes that common rule.

The microscopic substrate is described by four independent bosonic objects.

The first is the local substrate amplitude. It says which closure directions are occupied and how strongly.

The second is the link transport. It says how an internal state at one event is compared with an internal state at another.

The third is the current. It says how recorded substrate content actually flows.

The fourth is the record/completion map. It says how one admissible record is completed, transported and composed into another.

A link derived only from endpoint states cannot generate a physical gauge stiffness: a link built by comparing two endpoint frames changes by a pure gauge transformation, and its variation cancels around every small loop. The link, current and record map must therefore be treated as independent physical variables, and this paper does so from the outset.

Independence does not mean they are unrelated. Their relation is imposed dynamically.

The action penalises six failures:

1. failure of link transport to close around a loop;
2. failure of the physical current to agree with local transported substrate flow;
3. failure of current conservation;
4. failure of record maps to compose;
5. failure of the physical link to agree, on the occupied record support, with the polar transport carried by the record map;
6. failure of the local completion stretch — the thing that gives matter its mass — to track the Higgs displacement.

The fifth is the bridge. It allows the record structure to influence physical gauge transport without turning the link into a mere endpoint convention. It is the possible source of the nonfactorising response that the gauge and QCD papers require.

All six failures are measured in one inherited operational metric. Colour, weak isospin and hypercharge are not given separate coefficients. Their different strengths must arise because they occupy different subspaces, deform different record profiles and relax through different physical auxiliary modes.

The same discipline extends to every dimensionful number in the machine. The vacuum strength, the stiffness of the potential, the flow conductances and the single scale that converts completion into mass are all derived — from the geometry of the regulator and from the machine's own rate functional — never calibrated against a known particle mass or coupling. And if the machine turns out to admit several distinct internal settings, all of them must be published; no setting may be quietly preferred because its output looks more like the Standard Model.

Matter uses the same links as the gauge curvature. There is therefore no independent freedom to rescale the matter current after the gauge action has been calculated.

Masses arise from the positive stretch of the record/completion map. The unitary part of that map transports orientation and gauge phase; the positive part measures how the record geometry is stretched by completion. The Higgs-compatible part of that stretch couples left-handed and right-handed matter.

This gives an independent microscopic fermion coupling — and it is written down in a deliberately blind order. The rule for how matter couples uses only raw address labels fixed in advance, before any response of the machine has been computed. The refined objects that eventually appear in the mass formulas are calculated afterward, from the machine's output.

Nothing answer-shaped is smuggled into the starting rule. In particular, the fermion coupling is not defined to equal the closure response. The two are separately calculated from the same master action and then compared. That comparison is the CY3′ test.

The paper also derives the scalar-readout space rather than choosing its size. The directions that respond to the physical Higgs radial displacement are calculated, and the support of that response becomes the Yukawa readout space. Even the question of whether the left-handed and right-handed readout spaces match is a calculated result, not an assumption. If the readout is too small, the flavour programme fails. If it has the required size but returns the wrong spectra or frames, the physical branch fails. No rank may be increased afterward because a result looks wrong.

The same action produces the Higgs residue, gauge couplings, Yukawa matrices, neutral mass blocks and colour-response Hessian. This is the core claim: the Standard Model sectors are different readings of one substrate response, not separately engineered mechanisms.

One part of the machine has now actually been switched on. The genuinely new component — the bond term that lets the record structure talk to the forces — was tested on the smallest honest arena the programme possesses, the seven-event wheel. The test asked the one question everything depends on: does this term respond to real physical twisting while staying perfectly silent under mere relabelling of reference frames? It does, exactly. Relabellings cost precisely nothing; every genuine loop deformation acquires a positive, exactly computable stiffness; and the term produces the cross-talk between links and records that two of the six papers proved was missing. The test used placeholder unit weights rather than the physical ones, so it returns no Standard Model number yet — but the mechanism the whole construction rests on is now demonstrated to work, not merely asserted. A second execution then sorted that response into the three force channels and computed the wheel's contribution to each force strength exactly. It also delivered the programme's first honest negative result: the simplest symmetric choice of internal weighting puts the hypercharge sector at badly the wrong strength relative to the weak sector. Because the machine has no adjustable dials, this failure converts into a single precise numerical demand on the one object still to be calculated — which is exactly how a falsifiable programme is supposed to behave. A third step then wrote down, exactly, the three missing numbers as three simple traces of one matrix the programme has yet to produce — and showed that the older competing branch choices now demand visibly different values of those traces, one pair disagreeing by exactly a factor of six. The coming calculation cannot satisfy them all; it must decide. Only the downstream step is mechanical: once the history-derived operational metric, the universal completion geometry and the physical branch are supplied, forming the Hessian and reading off the boundary is a matter of differentiation and export. Deriving those upstream objects — the metric, the generation geometry and the branch — is the remaining central task, and it is the work of the two papers that follow this one, not of this one. Two more components have since been switched on. The mass mechanism itself now works: the benchmark branch returns nonzero full-rank charged mass matrices, removing the previous zero-mass obstruction — though all masses come out equal on this simplest setting, so the interesting differences between particles are still to come. And the corrected map from the machine to the three forces was built in full, at its true size, reproducing the textbook matter content exactly — which confirms the machinery, and also confirms that the simplest separable version still puts the hypercharge sector at the wrong strength. Both results point at the same place: everything still missing lives in the genuinely entangled part of the machine, which only the full physical calculation can reach.

Three complete candidate machines have since been solved end to end. Each generates more of the qualitative structure of the Standard Model than the last: the first is degenerate, the second produces a hierarchy of masses and one plane of mixing, the third produces a full three-generation mixing pattern with the first trace of matter–antimatter asymmetry and, for the first time, the correct ordering of the electroweak forces. This is genuine progress, but it must be read honestly, and the paper reads it so: each machine reached its result by a hand-chosen internal rule, not by deriving that rule. An engine that can be made to produce hierarchy, mixing and asymmetry — and equally their absence, and equally the wrong values — by choosing its internal wiring is demonstrating capacity, not prediction. It becomes a prediction only once the deeper measure of the theory selects that wiring uniquely, and none of the three machines does that yet. The paper states this at full strength rather than burying it: at present the construction has relocated the Standard Model's free parameters into the choice of internal rule, not eliminated them.

There is a genuine step forward on that missing rule. The thing that would fix the machine's settings — instead of leaving them to be chosen — is the theory's deepest rulebook, the "measure" that weighs every possible microscopic history. This paper builds the mechanism that turns that rulebook into the machine's settings: it shows how the settings, the harmless relabellings that must cost nothing, the absolute scale, and the geometry behind the generations of particles all fall out of one weighting law. The missing ingredient that mechanism needs — the single rule for how one instant of the substrate refines into the next — is then identified, in a proposed form, not as a free choice but as the machine's own natural "heat flow" across its built-in notion of distance. Under that identification two of the rule's health conditions follow, and the derivatives the machine needs become ordinary geometric quantities rather than new free functions.

But this is a proposal, not a settled result, and it turns on one question it does not yet answer: is that built-in notion of distance itself forced by earlier results, or is it the new place where the freedom hides? If the distance is defined so as to make the construction work, then the distance itself is the unproven choice; if the distance is fixed independently, then the property the construction relies on has to be proved for the real, full-sized structure. Either way the same single question remains open. So the honest reading is that the mystery has been made smaller and pushed to one sharp place where it may finally be answerable — not that it has been answered.

Finally, the paper now contains its own finish line. It states, as a precise checklist, exactly which numbers must be produced — all three force strengths, the two Higgs parameters, the three charged mass matrices, the full neutrino block and the strong phase — under which frozen conditions, with which error ledgers, before the programme may declare the microscopic derivation complete. And it sets a gatekeeping rule: the paper that finally converts these raw numbers into the quantities experiments measure is forbidden from opening until that checklist is fully populated by real frozen outputs. Those outputs are produced by the two papers that come next — one that builds the machine's missing measure and settings, and one that runs the machine on the settings it selects — not by this paper. No benchmark, placeholder or experimentally borrowed value counts. Assembly is transport, not rescue.

The global measure paper then supplies the final interpretation. At finite regulator, the master action is the rate functional of the substrate history measure. Its quotient-compatible quantum weight is reconstructed on the complete faithful Standard Model bundle stack. If the proposed positive-pushforward ontology succeeds, its absolute normalisation is fixed by that measure, and the strong-phase consequence is sharper than reality: a positive measure excludes both generic complex phases and the signed $\bar{\theta} = \pi$ case, so positive pushforward implies $\bar{\theta} = 0$ or a global obstruction.

In plain terms:

VERSF has reached the point where it must stop proposing separate reasons for different Standard Model numbers and define one microscopic machine that produces all of them. This paper defines the first complete candidate for that machine, turns its first gear — which turns — builds the mechanism that would fix the machine's settings from the theory's deepest rulebook, shows it working on a small test case, and identifies the one missing rule as the machine's own natural heat flow rather than a free choice, proving its two health conditions. What now remains is not to build the rule but to check, on the full-sized structure, that its geometry is regular enough and that its single underlying ingredient is itself forced — a much smaller and more concrete task than the one this programme started with.

Headline Results and Conditionality

Object	Master-action result	Status
Primitive configuration	$\mathfrak{X} = (\rho, U, J, \Phi; \Psi, \bar{\Psi})$	defined
Link, current and record independence	imposed at configuration-space level	structural requirement
Universal potential	$\Gamma_V = \sum_v (\lambda_{sub}/4)(\rho_v^\dagger \rho_v - \rho_c^2)^2$	inherited
Closure defect	vacuum-relative logarithmic plaquette holonomy	explicit
Current constitutive defect	$J - \kappa(\rho_t - U \rho_s)$	explicit
Transport defect	$\partial_\sigma \rho + \partial_U J$	explicit
Record defect	$\Phi_{jk} \Phi_{ij} - \Phi_{ik}$	explicit
Bond compatibility defect	$U - \mathrm{Pol}(\Phi^{tr})$ on occupied support	explicit bridge

Object	Master-action result	Status
Scalar-stretch defect	$\mathcal{D}_H = X_H - (\varphi_H/\Lambda\star)P_H^R$ on the local completion morphism (target carrier)	explicit sixth defect, conditional on Premise MA9 (§47F)
Unified local carrier	fixed isometric substrate/record realisation on $\mathscr{H}_i$	closed modulo F21 (Theorem 8A); the realisation-freedom that F21 tests is the very thing assumed frozen
Absolute response metric	one operational metric; full induced block metric W_{XY}	conditional on Universal Defect-Rate Principle
Residual cross blocks	$W_{XY} = \iota_X^\dagger W_{prim} \iota_Y$, calculated overlaps, never inserted	closed structurally (Theorem 9A)
Primitive scale ledger	$(\ell_n, a_t, \rho_c, \lambda_{sub}, \kappa_e, \Lambda\star)$ all derived	conditional (Theorem 19A)
Independent factor coefficients	forbidden	closed structurally
Gauge response	common Hessian Schur complement	executable
Intrinsic auxiliary intertwiner	generated only by $\mathcal{D}_B$ and other non-coboundary variations	executable
Wheel link–record branch	$H_{uy} = -c I$ with exact gauge nullity and $k_{red}(\lambda) = a\lambda + bc\lambda/(b\lambda+c)$	executed pass (§47A)
Factor-projected wheel triple	$(K_3, K_2, K_q)^{wheel} = (24/5, 35/6, 3/2)$ on the D_6-isotypic lift	executed pass (§47B)
Residual coupling debt	three internal projections π_A of one fourteen-channel embedding	open, exactly specified
Abelian stiffness condition	$Z_Y > Z_2 \Leftrightarrow \pi_q/\pi_2 > 140$; unit lift gives 1, census gives ≈58	sharp open target (F26)
Internal projectors	$P_A = \mathcal{E}_A(\mathcal{E}_A^\dagger\mathcal{E}_A)^{-1}\mathcal{E}_A^\dagger$ from calculated embeddings; Casimir P_A^{Cas} as route-agreement check	derived construction (§47C)
Separability certificate	$\varepsilon_A^{sep} = 0$ required before quoting any π_A; direct Z_A fallback defined	closed as construction
Conditional target triples	(5/4, 39/28, 279) vs (5/24, 0.405, 205.2); colour conflict exactly ×6	computed, mutually exclusive
Bare fourteen-channel metrics	W_{14}^{stat} and Dirichlet Q_{14}^{Dir}, fully explicit in the frozen basis	supplied (§47D)
Response-package pipeline	H_{14}^{red} and E_{int} exported mechanically from background + tangents	closed as construction
Freeze-and-generation package	data+code hash manifest; certified-refusal generator; separable-benchmark and	supplied; 18/18 tests pass on the executed source (§47E, App. G)

Object	Master-action result	Status
	authoritative physical_direct modes; honest effective-norm status	
Scalar-stretch execution	rank-9 P_Y; $Y_u = Y_d = Y_e = I_3$ full rank; exact linearity; isolated CY3′ = 0	executed pass, unhierarchical (§47G)
Corrected gauge embedding	tensor carrier $\mathcal{V}$_int; 30926×12; rank 12; exact orthogonality/isotropy	executed pass (§47H–§47I)
Minimal tensor internal norms	(6, 13/2, 378) — the census triple, derived not inserted	executed; fails declared lower EW branch
Remaining physical locus	D$\mathcal{D}$_B, D$\mathcal{D}$_H, W_BH/W_CH/W_RH, H_$\eta\eta^{-1}$ on physical background	open (§47J)
Scalar kinetic residue	momentum coefficient of common reduced Hessian	executable
Completion-strain operator	positive logarithmic polar stretch	explicit
Scalar readout projectors P_Y^(L,R)	genuinely typed left/right supports of $\mathcal{D}$_(φ_H)$\mathcal{O}$_H, intertwined by V_H; = P_H^(L,R) exactly on the minimal branch	explicit and calculable
P_Y^(L,R) typed relation	P_Y^L = V_(H,⋆)† P_Y^R V_(H,⋆); literal equality only after common-carrier identification or when V_(H,⋆) = I	output; literal on executed V_H = I branch
Ranks of P_Y^(L,R)	outputs	rank 9 returned on executed charged branch (§47G)
Address-to-record map	canonical inclusion of terminal-record subspaces	conditional Address-as-Record Principle
Chiral embeddings	metric polar factors of projected address inclusions	explicit
Fermion bilinear	raw-address compression of completion strain; no derived projector, embedding or Yukawa inserted	explicit and primitive
Kinetic-address factors R_fX	polar radii of the canonicalised address maps; $R\_fX = I_3$ is a certificate	executable
Y_f^mixed	relaxed scalar derivative of the independent bilinear	executable
Y_f^Hess	compression of the Schur-reduced closure Hessian	executable
CY3′	equality of independently returned operators; universal operator form on P_Y^(L,R)	preserved and strengthened as test; never yet exercised nontrivially (F9)

Object	Master-action result	Status
Branch ledger β	every executed branch published with its manifest; finiteness NOT established (constitutive freedom is continuous)	branch-publication requirement; finiteness open, F29 (§57A)
Standard mass linearity	$M_f = (v_{cl}/\sqrt{2}) Y_f$	separate interface-linearity certificate
Neutral M_R	symmetric terminal-record completion compression	executable
M_L^{irr}	two-interface symmetric active compression	executable and order-suppressed only if returned
Gauge couplings	$g_s = Z_3^{(-1/2)}$, $g = Z_2^{(-1/2)}$, $g' = 6 Z_q^{(-1/2)}$	inherited conversion
Physical colour stiffness	no census identification; common Hessian output	open evaluation
Triality wall coefficient	constrained master-action minimum	explicit target
Global quantum measure	$e^{(-\Gamma_{master})} s_{ferm} dv_{quotient}$	finite-regulator construction
Strong phase	inherited combined-line calculation	downstream
Numerical Standard Model outputs	not returned in this paper	open
History-rate functional (W_{prim}, completion law)	consumer derived and demonstrated (Fisher route, K=7: $I_{\tau\tau} = 0.395\ldots$); operator proposed as $R_n = \zeta_n V_n e^{(-a_t L_n)}$ within an unbuilt HMGBC-1 package; HP1 proved, HP2 conditional, heat-flow identification itself a physical posit (§10.1.1) — not inherited, not established	**narrowed to finite-regulator evaluation of an explicit (proposed) construction — the load-bearing gate (F19/F20, §10.1, §10.1.1)**
Fisher-metric consumer of rate functional	$W_{prim} = D^2\mathcal{J}_n(0)$; null quotient, absolute unit and completion geometry all derived from one kernel	derived, demonstrated on K=7 (§10.1)
Refinement transfer admissibility	HP1 terminal-cone positivity, HP2 record-basis reversibility	HP1 proved; HP2 proved conditional on minimal-fibre submersion + basic measure; physical applicability + g_n uniqueness open (proposed HMGBC-1 package, §10.1.1)
Common six-defect zero locus genericity	numerical existence on toy cells only; 39 CFB-1 null modes counted but not analytically classified; differing early returns	open, untested (§20.1, F28)

Object	Master-action result	Status
	are a reproducibility item, not branch uncertainty	
Physical execution protocol	admission gate, six-defect Jacobian ledger, nonfactorising reduction, boundary certificate	fully specified (§§48–53)
Physical boundary certificate $\mathcal{P}\_{SM}^{VERSF}(\mu\star)$	$\{ g_s, g, g', \mu_H^2, \lambda, Y_u, Y_d, Y_e, \mathscr{B}_\nu, \bar{\theta} \}$	not yet issued (§53.3)
Parameter-Closure admission	Admission Rule 53.1: closed until the boundary table is populated	not yet admitted
Master-action definition gate	pass on stated premises	conditional
Bond-mechanism execution gate	gauge nullity, positivity, nonzero mixed block, exact Schur separation	passed on frozen unit branch
Fully concrete CFB-1 action	226-variable, 286-residual six-defect implementation	executed (§47K)
CFB-1 stationary solve	$\lVert R\star \rVert = 3.6\times10^{-13}$, common defect zero	numerical pass
CFB-1 full Hessian	226^2, 39 numerical-null and 187 positive modes	executed
CFB-1 direct reduced gauge matrix	diag(4.8498, 5.8868, 1.5190) up to $\sim10^{-15}$ cross terms	executed
CFB-1 electroweak test	$Z_Y = 0.0422 \ll Z_2$; $\sin^2\theta_W = 0.9929$	branch fail
CFB-1 flavour return	$Y_u = Y_d = Y_e = I_3$	full rank but physical fail
CFB-1 physical certificate	not issued	candidate branch only
Fully concrete CFB-2 branch	340-variable, 512-residual non-isotropic completion, K7×T4	executed (§47L)
CFB-2 stationary solve	exact defect zero, $\lVert \mathbf{D}\star \rVert = 0$	numerical pass
CFB-2 gauge return	diag(5.4194, 6.5753, 1.9998); $\gamma_q = \sqrt{7002}$ screened ~3500×	branch fail (EW); screening located in auxiliary structure
CFB-2 scalar residue	$Z_H = 2.6553$, $m^2_{H,curv} = 0.7532$	first calculated Z_H
CFB-2 flavour return	hierarchical σ in all four sectors; exact 15° 2–3 CKM plane	executed; not physical hierarchy
CFB-2 neutral return	full 6×6 spectrum, $M_R = 2Y_\nu$	executed candidate
CFB-2 physical certificate	not issued	candidate branch only

Object	Master-action result	Status
Fully concrete CFB-3 branch	388-variable, 605-residual fourth-moment metric topology	executed (§47M)
CFB-3 gauge return	diag(6, 115/16, 7005/2); $Z\_Y > Z_2$ — F26 ordering gate passes first time	executed; magnitude overshoots ($\sin^2\theta = 0.069$)
CFB-3 flavour return	four hierarchical sectors; all three CKM angles; J_CKM = 0.0073	first charged CP invariant
CFB-3 scalar and neutral	Z_H = 2.3489; six Takagi masses	executed candidate
CFB-3 pre-registration status	metric Schur-forced; generation graphs honor-system, not manifest-frozen	recorded as such (§47M.1, F17)
Candidate-branch "progression"	CFB-1/2/3 each add hand-set knobs (and CFB-2/3 also change arena, wheel → K7×T4); later branches reach more SM structure because they are more tuned, not because a derivation converges	executed; capacity shown, NOT accumulating evidence; prediction open (§47M.7)
CFB-3 physical certificate	not issued	candidate branch only
Physical master-action evaluation	defined as the frozen execution of §§48–53	boundary certificate not yet issued

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Abstract

The six final-completion papers of the VERSF Standard Model programme reduce scalar normalisation, fermion flavour, gauge strength, nonperturbative colour response and global quantum consistency to a common set of missing microscopic data. The microscopic Yukawa gate proves that J_C, J_R, P_Y^micro, the chiral embeddings and the dressed left–right bilinear are non-identifiable from the current corpus. The gauge-response gate supplies an explicit holonomy-response functional but leaves the physical embedding, action conversion and nonfactorising auxiliary response open. The QCD gate proves that the five apparent wheel modes are pure-gauge directions and do not generate the proposed five-unit colour screening; a genuine intrinsic record–link intertwiner is required. The global-measure gate supplies the quotient-stack, determinant-line and positive-pushforward architecture but requires an explicit finite-regulator action or rate functional.

The present paper constructs the minimal common master action capable of supplying all these outputs without introducing sector-specific Standard Model parameters.

At regulator level n, the physical configuration is

```
𝔛_n = (ρ, U, J, Φ; Ψ, Ψ̄),
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on a closure-compatible oriented cellular complex. The link U, current J and record map Φ are independent variables. The complete leading local bosonic action is

$$\Gamma\_bos^{(n)} = \Gamma\_V^{(n)} + \tfrac{1}{2} \sum\_(X = C,J,T,R,B,H) \|\mathcal{D}\_X\|^2\_(W\_X),$$

with the six explicit defects

$$\mathcal{D}\_C(p) = (-i/|p|) \, \mathrm{Log}_0(H\_(p,\star)^{-1} H\_p),$$

$$\mathcal{D}\_J(e) = J\_e - \kappa\_e (\rho\_t(e) - U\_e \rho\_s(e)),$$

$$\mathcal{D}\_T = \partial\_\sigma \rho + \partial\_U J,$$

$$\mathcal{D}\_R(i,j,k) = \Phi\_jk \Phi\_ij - \Phi\_ik,$$

and

$$\mathcal{D}\_B(e) = \Pi\_t(e) U\_e \Pi\_s(e) - \mathrm{Pol}(\Pi\_t(e) \Phi\_e\hat{}tr \Pi\_s(e)),$$

and

$$\mathcal{D}\_H(i) = X\_(H,i) - (\varphi\_H/\Lambda\star) P\_H\hat{}R,$$

the scalar-to-completion-stretch defect on the local left–right completion morphism (§47F). Spatial record transport Φ^tr and the local completion map Φ_H are distinct typed objects; the mass-generating positive stretch lives on the latter, so a homogeneous stretch no longer conflicts with exact spatial composition.

The projectors Π_i are spectral support projectors of the local record Gram operators and are therefore calculated rather than inserted. The polar factor is the unique partial isometry in the polar decomposition on the rank-stable occupied support.

All variables act on one unified local operational carrier through fixed isometric substrate and record realisation maps, so every defect and bond comparison is well typed. The defect terms are priced by the complete induced residual metric W_XY = ι_X† W_prim ι_Y of one canonical operational metric: the diagonal blocks are the sector weights and the off-diagonal blocks are calculated overlaps, not inserted interactions. Their absolute coefficient is fixed, on the positive-pushforward branch, by identifying the master action with the finite-regulator Radon–Nikodym rate functional of the substrate history measure. The construction therefore contains no independently adjustable colour, weak, Abelian, scalar, record or completion coefficient at the declared order. The primitive dimensional tuple (ℓ_n, a_t, ρ_c, λ_sub, κ_e, Λ⋆) is likewise closed: each entry is derived from the regulator geometry, the history-rate functional or the completion-carrier residue, never from a Standard Model observable.

At an admissible background every defect vanishes. The bosonic second variation consequently has the exact Gauss–Newton form

$$\mathbb{G}\star = G\_V + D\mathbf{D}\star\dagger \, W\_prim \, D\mathbf{D}\star.$$

Second derivatives of the constitutive maps do not enter the quadratic Hessian. Gauge and null directions are removed, the complete same-order auxiliary carrier is relaxed and the physical kernel is the Schur complement.

The scalar two-point residue, the gauge kinetic response and the closure Hessian are different projections of $\mathbb{G}_\star^{\mathrm{red}}$. The physical gauge coefficients are the factorwise traces of its curvature block. The scalar coefficient is the p^2 derivative along the Higgs-normal embedding. The flavour mediator is the scalar-compatible compression of the canonical closure block.

The independently derived fermion completion operator comes from the positive polar stretch of the local completion morphism. If

$$\Phi_{(H,i)} = B_{(H,i)}\, V_{(H,i)}$$

is its polar decomposition, define the vacuum-relative strain

$$X_H = \log\left(B_{(H,\mathrm{sym})}^{-1/2}\, B_H\, B_{(H,\mathrm{sym})}^{-1/2} \right),$$

which the sixth defect ties to the Higgs radial modulus; on the minimal exact branch $\mathcal{Y}_\star = V_{(H,\star)}^\dagger P_H^R$, with $P_Y^R = P_H^R$ and $P_Y^L = P_H^L$.

The microscopic scalar-response operator is

$$\mathcal{Y}_\star = \Lambda_\star\, [\mathcal{D}_{(\varphi_H)}\, \mathcal{O}_H]_\star,$$

and the physical scalar-compatible readout projectors are its left and right supports,

$$P_Y^L = \mathrm{supp}(\mathcal{Y}_\star \mathcal{Y}_\star^\dagger), \quad P_Y^R = \mathrm{supp}(\mathcal{Y}_\star^\dagger \mathcal{Y}_\star),$$

related by the typed polar-orientation identity $P_Y^L = V_{(H,\star)}^\dagger P_Y^R V_{(H,\star)}$, which reduces to literal equality $P_Y^L = P_Y^R \equiv P_Y^{\mathrm{micro}}$ only after a common-carrier identification, in particular when $V_{(H,\star)} = I$.

The primitive fermion bilinear is

$$\Gamma_{LR}^{(0)} = \Lambda_\star \sum_f \bar{\Psi}_{fL}\, (A_{fL}^{(0)})^\dagger\, \mathcal{O}_H\, A_{fR}^{(0)}\, \Psi_{fR} + \mathrm{h.c.}, \quad \mathcal{O}_H = V_H^\dagger X_H,$$

with the raw address maps $A_{fX}^{(0)} = P_{fX}^{\mathrm{rep}} J_\Omega S_{fX}$ frozen by representation and interface typing (H or $\tilde{H}$) alone, before any response is evaluated.

The chiral embeddings are derived by identifying occupied matter addresses with terminal-record subspaces and polar-canonicalising their projected inclusions. No mass, angle, phase or coupling enters their construction.

The mixed Yukawa operator is

```
Y_f^mixed = K_fL^(-1/2) [ 𝓓_(φ_H) ( δ²Γ_master / δΨ⁻_fL δΨ_fR ) ]⋆
K_fR^(-1/2) = R_fL 𝓔_fL† 𝒴⋆ 𝓔_fR R_fR,
```

where the positive factors R_fX are the calculated kinetic-address radii of the canonicalised address maps.

The Hessian-compression operator is

```
Y_f^Hess = R_fL 𝓔_fL† P_Y^L Ĥ_cl^red P_Y^R 𝓔_fR R_fR.
```

Their difference is the physical CY3′ residue. Because X_H is defined from the independent completion polar stretch rather than from Ĥ_cl^red, the equality is not true by construction. This central internal consistency test has not yet been exercised in any regime where it could fail: every executed instance is identity-trivial with both sides equal to I, and the hierarchical branches return only the completion-route Yukawa, with Y_f^Hess uncomputed (F9, §55).

The same action defines the physical colour boundary and triality wall response. The colour coefficient is calculated from the curvature/auxiliary Schur complement, with no identification of the matter census index with primitive stiffness. The wall coefficient is the constrained minimum of the master action per unit area in a fixed nonzero-triality sector; no centre penalty is inserted.

The first quadratic branch of the action is executed on the canonical $K = 7$ wheel cell in one adjoint-generator channel. The bond-compatibility term returns a nonzero link–record mixed block $H_{uy} = -c\, I_{12}$ with exactly six null directions equal to the common gauge-frame modes, exact reduced stiffness $k_{red}(\lambda) = a\lambda + bc\lambda/(b\lambda + c)$ on the cycle spectrum $\{1, 2, 2, 4, 4, 5\}$, and full Hessian spectrum $\{0^6, 1, 2^8, 3, 4^4, 5, 6^2, 7\}$ on the frozen unit branch. The mechanism required by the gauge and QCD gates is therefore executable and operative. Projecting the branch through the frozen D_6-isotypic factor sectors returns exact factor-separated wheel responses $(K_3, K_2, K_q)^{wheel} = (24/5, 35/6, 3/2)$ with colour isotropy, reduces the remaining coupling calculation to three internal projections π_A of one fourteen-channel embedding, and fixes the sharp requirement $\pi_q/\pi_2 > 140$ for the physical electroweak branch — a condition the unit-isometric lift (ratio 1) and the matter-census diagnostic (ratio ≈ 58) both fail. The internal norms themselves are then constructed exactly: Casimir-spectral projectors $P_3 = C_3/3$ and $P_2 = C_2/2$, embedding-column projectors $P_A = \mathcal{E}_A(\mathcal{E}_A^\dagger\mathcal{E}_A)^{-1}\mathcal{E}_A^\dagger$ as the primary construction with Casimir projectors as route-agreement checks, direct Gram-trace formulas $\pi_A = (1/d_A)\mathrm{Tr}(\mathcal{E}_A^\dagger W_{int}^{red}\mathcal{E}_A)$, a separability certificate ε_A^{sep} with a direct nonfactorising fallback, and conditional target triples that convert the programme's remaining branch choices — including an exact factor-of-six colour conflict — into three calculable traces of one matrix. Supplying that matrix is reduced to a mechanical export: the bare stationary and Dirichlet fourteen-channel metrics are given explicitly, and the physical H_{14}^{red} is the auxiliary-Schur-reduced pullback $E_{14}^\dagger \mathbb{G}_\star E_{14}$ of the master Hessian along fourteen frozen tangent vectors, with the twelve gauge columns derived by generator differentiation and the gauge-active profile space returned as a three-dimensional calculation, and the whole pipeline is realised as a hash-verified freeze-and-generation package — the frozen object being data, basis, conventions and code together — whose generator refuses unfrozen, tampered, non-Hermitian, indefinite, negative-mode or rank-deficient inputs, emits effective internal norms only under the explicitly frozen wheel-factorisation assumption, and carries an authoritative physical-direct mode for the nonfactorising evaluation.

One fully concrete finite-regulator candidate branch has now also been executed end to end (CFB-1, §47K): a 226-variable, 286-residual six-defect action on the canonical wheel is solved to a common defect zero at $\|R\star\| \sim 10^{-13}$, automatically differentiated, spectrally quotiented (39 null, 187 positive modes) and Schur-reduced without any factorised assumption, returning the direct candidate gauge triple $(Z_3, Z_2, Z_q) = (4.8498, 5.8868, 1.5190)$ — an executed negative: $Z_Y \ll Z_2$ and identity flavour, published as a candidate-branch failure rather than repaired. A second branch (CFB-2, §47L; 340 variables, exact defect zero) adds a graph-resolvent non-isotropic completion law with hierarchical Yukawa spectra, an exact single-plane CKM rotation, the first calculated scalar residue $Z_H = 2.6553$ and a complete 6×6 neutral spectrum, while demonstrating that even a representation-derived fourth-moment Abelian amplitude of $\sqrt{7002}$ is screened to $Z_q \approx 2$: the electroweak deficit is auxiliary-structural, not local. A third branch (CFB-3, §47M; 388 variables) uses one common fourth-moment candidate metric built from representation fourth moments — through a Schur-forcing that keeps the per-factor restriction representation-covariant, so Firewall M5 is not violated by a hand-tuned coefficient — and passes the lower electroweak ordering for the first time — $Z_Y > Z_2$ with $(Z_3, Z_2, Z_q) = (6, 115/16, 7005/2)$ — while returning full three-angle CKM mixing, the first nonzero charged CP invariant, and six Takagi neutral masses; the Abelian magnitude overshoots — $\sin^2\theta = 0.069$ against ≈ 0.23, and J_{CKM} two orders too large — so the ordering is reached but the values are not, and the sign of the Abelian miss flips relative to CFB-1/CFB-2, the signature of a hand-set amplitude rather than a converging derivation. The large fourth-moment weights (7002/7003/7005) are asserted, not derived here, and HMGBC-1 replaces this candidate metric with the stronger proposed physical object $W_{prim} = \mathbb{I}_F[K_n]$. Its generation graphs are pre-registered on an honor-system basis only and are not yet frozen under the §47E manifest before the solve (§47M.1); this is recorded rather than glossed, because the capacity-versus-prediction reading of §47M.7 depends on the graph choice being genuinely blind.

The physical execution itself is now fully specified inside the paper: an evaluation admission gate (Admission Rule 48.1), the complete six-defect Jacobian and 6×6 cross-metric publication ledger, the nonfactorising Hessian reduction with its separability residues, the boundary set $\mathcal{P}_{SM}^{VERSF}(\mu\star) = \{g_s, g, g', \mu_H^2, \lambda, Y_u, Y_d, Y_e, \mathscr{B}_\nu, \theta\}$ with its publication table, and the Parameter-Closure Admission Rule forbidding the next paper from opening until that table is populated by frozen returns. The paper therefore supplies the first common forward action for the final-completion programme, together with its executed benchmark branches and its complete physical execution protocol.

It does not yet supply its physical numerical matrices — and one limitation must be stated at full strength rather than left among the falsifiers. Every executed branch freezes its operational metric and completion law by hand: the Radon–Nikodym history-rate functional that Premises MA1 and MA8 invoke has not been constructed, and the CFB progression, whatever its qualitative returns, computes no part of it. At the present stage MASD-1 has relocated the Standard Model's freedom, not eliminated it: the free parameters have been moved out of one-coefficient-per-sector and into the choice of operational metric and completion law. Whether that relocation is a reduction depends entirely on the history measure uniquely selecting those functions. On this front the status is now substantially advanced. A candidate Fisher-metric construction (§10.1) derives the entire *consumer* of the rate functional — W_{prim} as the relative-entropy curvature of the Doob-transformed history kernel, the gauge/coboundary null quotient as the kernel of statistical distinguishability, the absolute per-step normalisation, and the universal completion geometry — and demonstrates it end to end on the $K = 7$ cell ($I_{\tau\tau} = 0.395\ldots$). The refinement operator that this consumer requires is then supplied, in a *proposed* HMGBC-1 theorem package (§10.1.1) — HMGBC-1 not yet being a completed paper — not as a choice but as inherited heat transfer, $R_n = \zeta_n V_n e^{(-a_t L_n)}$, the heat semigroup of the operational Dirichlet form: its polar factor is that semigroup, terminal-cone positivity (HP1) is proved, record-basis reversibility (HP2) is proved conditional on the coarse maps being minimal-fibre submersions with basic measure, and the six defect-derivatives reduce by Duhamel to derivatives of the operational metric, measure and coarse map — regulator data, not free functions. F19 is thereby narrowed from "construct an undefined measure" to "evaluate an explicit construction and check its regularity": the residual debt is (a) numerical verification, across two regulator levels, that the actual coarse maps are minimal-fibre Riemannian submersions with basic measure, and (b) the uniqueness of the operational metric g_n. This is a genuine step down from "is there even an object" to "does the explicit object satisfy its regularity conditions and is its one input forced" — but it is not yet a passed gate, and the full construction, together with the branch selector, is the task of HMGBC-1; HFB-1 then executes the selected physical branch through §§48–53, populating the boundary table $\mathcal{P}_{SM}^{VERSF}(\mu\star)$ with frozen numerical returns carrying full uncertainty and branch ledgers. Its strict pass condition is that frozen physical execution — the complete six-defect Jacobian and cross-metric, the common nonfactorising Hessian, and the populated boundary table — which is in turn the admission condition of the Renormalised Parameter-Closure paper further downstream (Admission Rule 53.1).

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Frozen Symbol Glossary

The following objects recur throughout and are, in several cases, used in tables and result statements before their flowing-text definition. This glossary is frozen: the symbols below carry exactly these meanings everywhere in the paper.

Symbol	Meaning	Defined in
$\mathbb{G}\star$	the common bosonic Hessian (master-action second variation) at the frozen background	§21
P_{phys}	the physical quotient projector removing gauge and terminal-null directions	§22
G_{red}, $\hat{H}_{cl}^{red}$	the quotient-reduced physical Hessian on the retained carrier (the "reduced closure Hessian")	§22, §47D
Z_A, $\mathfrak{n}_A$	sector stiffness Z_A (Hessian curvature, *inverse* to coupling: $g_A = Z_A^{(-1/2)}$); effective internal norm $\mathfrak{n}_A = Z_A/K_A^{wheel}$	§26, §47B
K_A^{wheel}	frozen wheel coefficients $(K_3, K_2, K_q) = (24/5, 35/6, 3/2)$	§47B
$\tilde{E}_3$, $\tilde{E}_2$, $\tilde{E}_q$	direct gauge-generator embeddings on the retained carrier (physical_direct mode)	§47H, App. G
$\mathcal{K}_g$	authoritative gauge response $\mathcal{K}_g = \tilde{E}^\dagger G_{red}\tilde{E}$, no factorisation imposed	§47H, §50
$\mathcal{Y}\star$	the frozen Yukawa response operator on the completion sector (mixed-route Yukawa)	§37, §47D
Y_f^{mixed}, Y_f^{Hess}	the two independently generated Yukawa routes whose equality CY3′ tests	§37, App. E
$P_Y^{(L,R)}$	left/right Yukawa projectors, typed and intertwined by $P_Y^L = V_{(H,\star)}^\dagger P_Y^R V_{(H,\star)}$ on an aligned polar branch	§31A
$V_{(H,\star)}$	the aligned polar (left–right completion) intertwiner	§31
R_n	refinement transfer $R_n = \zeta_n V_n e^{(-a_t L_n)}$ (posited as operational heat flow)	§10.1.1
L_n, $\mathbb{T}_n$	operational Dirichlet-form generator and its heat semigroup $\mathbb{T}_n = e^{(-a_t L_n)}$	§10.1.1
W_{prim}	the proposed operational metric $W_{prim} = \mathbb{I}_F[K_n]$ (Fisher curvature of the Doob kernel) — an HMGBC-1 object	§10.1
R_f, R_{fX}	universal generation geometry $R_f = S_f^\dagger(I + L_\Omega)^{-1}S_f$ and its sector restriction — HMGBC-1 objects	§53.4
B_{phys}	the physical branch selector (argmin over the branch ledger) — an HMGBC-1 object	§53.4

Symbol	Meaning	Defined in
β	the six-component branch ledger (vacuum, support, polar, bundle, orientation, refinement)	§57A
$\mathcal{D}$_A	the six named primitive defects (A = 1…6), the arguments of the master action	§47F, MA2
$\mathcal{D}$_B	the intrinsic non-coboundary defect direction, flagged as most likely to break HP2	§10.1.1

Two conventions are worth restating here because they invert naive readings. *Stiffness is inverse to coupling*: a larger Z is a stiffer, hence weaker-coupled, sector, so "Abelian stiffness too small" and "hypercharge coupling too large" name the same fact (§26). And *"not executed" is a distinct certificate value from "fail"*: an untested equality (CY3′/F9) is neither passing nor failing evidence (§37, App. H).

Part I — The Missing Common Parent

1. Aim and Programme Position

The six final-completion papers close most of the theorem architecture of the VERSF Standard Model programme but expose one programme-level omission.

A parameter-closure paper cannot assemble physical Standard Model parameters until one action has generated them.

The scalar paper defines the microscopic two-point residue but does not possess the transport coefficients needed to evaluate it. The neutral paper freezes a carrier census and physical branch but awaits the actual neutral quadratic operator. The Yukawa paper proves exactly why the current bosonic functional cannot determine the physical Yukawa matrices. The gauge paper constructs one response functional but retains an underived response metric, embedding and auxiliary descent. The QCD paper proves that the proposed wheel screening does not arise from the wheel incidence complex. The global-measure paper defines the nonperturbative measure architecture but needs an explicit regulator-level rate functional and transfer dynamics.

The master-action paper is therefore not optional editorial consolidation. It is a missing derivational prerequisite. The dependency chain is

```
⟦ Six sector gates ⟶ MASD-1 ⟶ HMGBC-1 ⟶ HFB-1 ⟶ Renormalised Parameter
Closure ⟶ Final Derivation Audit ⟧
```

MASD-1 does not renumber the sector papers. It is an intercalary programme keystone, required because the six gates independently identify the same missing parent. It defines and tests the candidate machine but does not by itself make any single branch physical. Two downstream papers do that work: HMGBC-1 derives the operational metric W_prim, the universal generation geometry and the physical branch selector that MASD-1 leaves as candidate premises; HFB-1 then executes the selected physical branch through the evaluation protocol of §§48–53 and issues the boundary certificate. The Renormalised Parameter Closure paper consumes that certificate. MASD-1 is therefore upstream of HMGBC-1 and HFB-1, not directly upstream of Parameter Closure.

2. Why the Sector Actions Cannot Remain Separate

Suppose one writes Γ_scalar, Γ_gauge, Γ_Yukawa, Γ_colour, and chooses each independently. Even if every functional is individually invariant and positive, the result does not derive one Standard Model. It defines several theories whose normalisations, backgrounds and microscopic carriers may be mutually unrelated.

A common master action is required for five reasons. First, the scalar vacuum must be the same vacuum used by the fermion and gauge sectors. Second, the link carrying matter must be the same link whose curvature defines the gauge kinetic term. Third, the auxiliary modes that relax a scalar or gauge response must be part of one common fluctuation census — a mode cannot be retained in one paper and omitted in another because it alters an answer. Fourth, the fermion bilinear and closure Hessian must be independently generated by one parent if CY3′ is to be a test. Fifth, the global quantum measure must weight precisely the same configurations whose Hessians and vertices are being calculated.

3. Claim Grades and Scope

This paper separates four grades.

Grade M1 — Exact master-action mathematics. Gauge covariance, positivity of squared defects, Gauss–Newton reduction, Schur complements, the polar differential, spectral-support projectors and the extraction formulas.

Grade M2 — Conditional constitutive theorem. The claim that the leading local rate functional is the universal squared norm of the six declared defects and that physical matter addresses are terminal-record subspaces.

Grade M3 — Numerical finite-regulator evaluation. Requires actual background links, records, currents, metrics and projectors.

Grade M4 — Physical Standard Model comparison. Occurs only after common-scheme matching, threshold transport and blind publication.

The present paper closes M1, defines M2 and specifies M3. It does not claim M4.

4. Binding Type Conventions

The substrate amplitude is $\rho_i \in \mathscr{H}_{(sub,i)}$. The physical link is $U_{ij} : \mathscr{H}_{(sub,i)} \to \mathscr{H}_{(sub,j)}$, with $U_{ji} = U_{ij}^{-1}$. The current is target-based: $J_{ij} \in \mathscr{H}_{(sub,j)}$.

The record variable carries two typed components. Spatial record transport is $\Phi_{ij}^{tr} : \mathscr{H}_{(rec,i)} \to \mathscr{H}_{(rec,j)}$, and the local scalar-compatible left–right completion morphism is $\Phi_{(H,i)} : \mathscr{H}_{(L,i)}^{rec} \to \mathscr{H}_{(R,i)}^{rec}$. These are not the same object: a homogeneous nonzero mass stretch on Φ^{tr} would conflict with exact spatial record composition, so the completion stretch must live on the local morphism. Where Φ appears without a superscript in common sums it denotes the pair $(\Phi^{tr}, \{\Phi_{(H,i)}\})$.

The positive closure Hessian is not a transport generator. The Standard Model connection is a tangent direction of the physical link carrier, not a coefficient already inserted into U.

The scalar radial field ϱ is the canonically normalised fluctuation about the broken vacuum; the symmetric interface is at $\varrho = -v_{cl}$. The full canonically normalised Higgs radial modulus is written φ_H, with $\varphi_H = 0$ at the symmetric interface and $\varphi_{(H,\star)} = v_{cl}/\sqrt{2}$ on the broken branch. Derivatives that fix mass–vertex linearity are taken with respect to φ_H, which removes any radial-normalisation ambiguity.

All Hessians are real physical second variations. Complex Hermitian notation is used only on complex-linear blocks. The bosonic residual terms carry the prefactor ½, and every Hessian formula in this paper is stated in that convention. The convention is frozen here once and may not drift between sectors.

5. Master Premises and Firewalls

Premise MA1 — Universal defect-rate principle. At leading local finite-regulator order, the negative logarithm of the absolute substrate history density is the squared norm of the complete primitive defect vector in the canonical operational metric, plus the inherited substrate potential.

Premise MA2 — Primitive completeness. At the declared order, the primitive bosonic consistency failures are exhausted by closure, current realisation, transport conservation, record composition, bond compatibility and scalar-to-completion stretch (§47F). The five-defect form of this ledger omitted the typed distinction between spatial record transport and the local completion morphism, and its enlargement to six is exactly the event this premise's falsifier contemplates; the ledger is re-frozen here at six. A future additional independent defect enlarges the master action and triggers F1. This is not a hypothetical: the ledger has already been revised once, from five defects to six (the typed split of spatial transport from local completion, §47F), so "exactly six" is a conjecture with a track record of one revision, and F1 is correspondingly a live falsifier, not a formality.

Premise MA3 — Independent variables. (ρ, U, J, Φ) are independently varied. Their physical relations are returned by the stationary equations rather than imposed through endpoint parametrisations.

Premise MA4 — Address-as-record principle. A physical matter address is a terminally distinguishable record subspace. The address carrier is the direct sum of those subspaces, and its map to the record carrier is canonical inclusion.

Premise MA5 — Shared-link matter transport. The same U appears in curvature, record transport and fermionic covariant transport.

Premise MA6 — Completion-strain coupling. The leading left–right matter completion is generated by the positive polar stretch X_H of the local scalar-compatible completion morphism Φ_H, coupled to the Higgs radial modulus through the sixth primitive defect $\mathcal{D}_H$ (§47F). The stretch lives on the local morphism, not on spatial record transport.

Premise MA7 — Complete auxiliary census. Every same-order mode that can respond to scalar displacement, curvature or completion is retained until the common Schur reduction.

Premise MA8 — Measure/action matching. On the positive-pushforward branch, the absolute normalisation of the master action is fixed by the finite-regulator history measure. It may not be re-normalised factor by factor.

Premise MA9 — Linear scalar–stretch constitutive law. At leading declared order, the local completion strain tracks the Higgs radial modulus linearly and isotropically on the frozen scalar-access projector: $\mathcal{D}_H = X_H - (\varphi_H/\Lambda\star)P_H^R$ on the target carrier, so that $X_{(H,\star)} = (\varphi_{(H,\star)}/\Lambda\star)P_H^R$ at stationarity. This is a constitutive premise on the same footing as MA1, not a derived result: the substrate history-rate functional may return this law, or a nonlinear or non-isotropic refinement of it, which would enlarge $\mathcal{D}_H$ as a declared-order revision rather than a fit.

Premise MA9′ — Generation-resolved scalar–stretch law. On a generation-resolved candidate branch, the sixth defect takes the form $\mathcal{D}_{(H,f)} = X_{(H,f)} - (\varphi_H/\Lambda\star)\, R_f$, where the positive completion operators R_f must be derived from the common address and completion geometry. The isotropic MA9 branch is the special case $R_f = P_{(H,f)}^R$. Until the R_f are derived upstream, MA9′ remains a candidate premise, and any branch executing sector-dependent R_f is a test of MA9′, not of MA9. Sector-resolved R_f choices are also the natural locus of F17, and their pre-registration must be made checkable through the §47E freeze manifest rather than asserted (§47M.1).

Firewall M1 — No Hessian-defined bilinear. The fermion bilinear may not be defined as an integral of the desired Hessian compression. Such a definition would force CY3′ by construction.

Firewall M2 — No endpoint-only gauge link. A link constructed only as $E_j E_i^{-1}$ is a frame coboundary and cannot supply physical curvature stiffness.

Firewall M3 — No matter-census stiffness. Representation traces may test matter response and beta-function counting. They do not define primitive gauge stiffness.

Firewall M4 — No selected readout rank. The ranks of the readout projectors $P_Y^{(L,R)}$ are calculated from the scalar response. They are not chosen from the required number of fermion masses.

Firewall M5 — No fitted defect weight. No relative coefficient may be selected using a mass, coupling, mixing angle, QCD scale or string tension.

Part II — Primitive Configuration and Symmetry

6. The Finite-Regulator Substrate Complex

Let $\mathfrak{S}_n = (V_n, E_n, F_n, C_n)$ be an oriented finite cellular regulator with vertices, edges, elementary faces and four-cells. The regulator includes the closure-compatible wheel and diamond subcomplexes used in the sector evaluations. Each edge $e : i \to j$ has geometric primal and dual weights $|e|$, $|e★|$; each face p has $|p|$, $|p★|$. These determine the discrete exterior-calculus weights. They are not coupling constants.

7. Independent Microscopic Variables

The complete bosonic field space is $\mathcal{X}_n = \mathcal{X}_\rho \times \mathcal{X}_U \times \mathcal{X}_J \times \mathcal{X}_\Phi$. The variables are independent because they answer different physical questions. ρ records occupied substrate content. U compares internal states between events. J carries transported record content. Φ realises record completion and composition, through its two typed components: spatial transport Φ^{tr} and the local left–right completion morphisms Φ_H (§4, §47F).

A density does not determine a connection. A connection does not determine the transported current. A current does not determine the complete record morphism.

8. Gauge and Record Transformations

For local internal transformations G_i,

$$\rho_i \mapsto G_i\, \rho_i, \quad U_{ij} \mapsto G_j\, U_{ij}\, G_i^{-1}, \quad J_{ij} \mapsto G_j\, J_{ij}, \quad \Phi_{ij} \mapsto G_j\, \Phi_{ij}\, G_i^{-1}.$$

The matter fields transform in their inherited faithful Standard Model representations. All master defects transform covariantly, and their norms are gauge invariant. The covariance proofs are collected in Appendix A.

8A. The Unified Local Operational Carrier

The substrate and record carriers perform different physical roles, but every expression in the master action must act on a mathematically common local carrier before its terms may be added, compared or differentiated.

For each regulator vertex i, let $\mathscr{H}_i$ be the complete finite local operational carrier. The substrate-amplitude carrier and the terminal-record carrier are distinguished isometrically embedded subcarriers:

$$r_i^{sub} : \mathcal{H}_{(sub,i)} \hookrightarrow \mathcal{H}_i, \quad r_i^{rec} : \mathcal{H}_{(rec,i)} \hookrightarrow \mathcal{H}_i, \quad (r_i^{sub})^\dagger r_i^{sub} = I_{(sub,i)}, \quad (r_i^{rec})^\dagger r_i^{rec} = I_{(rec,i)}.$$

The embeddings are inherited from the operational-realisation map and are fixed before the master action is evaluated. They are not selected from Standard Model outputs. The physical substrate amplitude is represented on $\mathscr{H}_i$ by $\tilde{\rho}_i = r_i^{sub} \rho_i$, and the record/completion morphism by $\tilde{\Phi}_{ij} = r_j^{rec} \Phi_{ij} (r_i^{rec})^\dagger$. The physical link is an operator on the complete operational carrier, $U_{ij} : \mathscr{H}_i \longrightarrow \mathscr{H}_j$, and the current takes values in the complete target carrier: $J_{ij} \in \mathscr{H}_j$.

All subsequent occurrences of ρ_i, Φ_{ij}, U_{ij} and J_{ij} in common operator sums are understood in these realised forms. The tildes may be suppressed only after this typing has been stated.

Theorem 8A — Common-Carrier Realisation Theorem

Given the fixed isometric realisation maps r_i^{sub} and r_i^{rec}: (1) the Gram operator G_i is a positive operator on one well-defined carrier $\mathscr{H}_i$; (2) the support projector Π_i acts on the same carrier as U_{ij}; (3) the supported record map $M_{ij} = \Pi_j \tilde{\Phi}_{ij} \Pi_i$ has the same source and target supports as $\Pi_j U_{ij} \Pi_i$; (4) the bond residual $\mathcal{D}_B(i,j) = \Pi_j U_{ij} \Pi_i - \mathrm{Pol}(\Pi_j \tilde{\Phi}_{ij} \Pi_i)$ is therefore well typed.

The relative position of the substrate and record subcarriers inside $\mathscr{H}_i$ is a physical invariant of the operational realisation. If it remains continuously adjustable after the operational metric and exact symmetries are fixed, the master action has not closed its carrier geometry and F21 fires.

Strengthened Gram operator. The local Gram operator is

$$G_i = \tilde{\rho}_i \tilde{\rho}_i^\dagger + \sum_{(e : t(e)=i)} \tilde{\Phi}_e \tilde{\Phi}_e^\dagger + \sum_{(e : s(e)=i)} \tilde{\Phi}_e^\dagger \tilde{\Phi}_e + G_i^{term}$$

where $G_i^{term} = \sum_{(\omega\in\Omega_i)} w_\omega P(\omega,i)^{rec}$, $w_\omega > 0$, is the terminal-record support contribution inherited from the occupied-address ledger. The term G_i^{term} ensures that a terminally occupied matter address remains visible to the support projector even when the corresponding classical record amplitude vanishes at the vacuum. Its weights are fixed uniformly by the operational measure and may not be chosen separately by particle species.

The occupied support is $\Pi_i = \mathbf{1}_{[g_{occ}, \infty)}(G_i)$, where $g_{occ} > 0$ lies inside a regulator-stable spectral gap. The value of g_{occ} is fixed by the gap, not selected to produce a desired rank.

9. The Full Canonical Residual Metric

The claim that all sectors descend from one operational metric is strongest when implemented before the residual carrier is artificially decomposed.

Let $\mathscr{R}_{prim}$ be the complete primitive residual carrier, and let $\iota_X : \mathscr{R}_X \longrightarrow \mathscr{R}_{prim}$, $X \in \{C, J, T, R, B, H\}$, be the fixed typed embeddings of the six defect carriers. The geometric primal and dual weights of §6 are carried by these embeddings; they introduce no coupling constants. Define the complete defect map $\mathbf{D} = \sum_X \iota_X \mathcal{D}_X \in \mathscr{R}_{prim}$. Let $W_{prim} > 0$ be the canonical operational response metric on $\mathscr{R}_{prim}$, induced by the stationary measure π of the finite substrate transport operator through

$$W_{op} = \mathrm{diag}(\pi) \otimes I_{orientation} \otimes I_{internal}.$$

The induced residual blocks are $W_{XY} = \iota_X^\dagger W_{prim} \iota_Y$. The diagonal terms are $W_{XX} = W_X$. The off-diagonal terms are not additional couplings; they are fixed overlap matrices produced by one common metric. The complete bosonic defect action is therefore

$$[\![\Gamma_{defect}^{(n)} = \tfrac{1}{2} \sum_{(X,Y)} \langle \mathcal{D}_X, W_{XY} \mathcal{D}_Y \rangle]\!] \quad \text{equivalently} \quad \Gamma_{defect}^{(n)} = \tfrac{1}{2} \langle \mathbf{D}, W_{prim} \mathbf{D} \rangle.$$

Theorem 9A — Metric-Induced Cross-Defect Theorem

Every same-order cross-defect term allowed by the primitive operational geometry is already contained in W_{XY}. Consequently: (1) no cross-defect coefficient may be introduced independently; (2) a vanishing cross block is a calculated orthogonality result; (3) a nonzero cross block is a substrate response, not an inserted interaction; (4) positivity of W_{prim} guarantees positivity of the full defect action even when individual off-diagonal blocks are nonzero.

The block-diagonal form $W_{XY} = 0$, $X \neq Y$, is the orthogonal-defect branch. It may be used only after the corresponding operational inner products have been evaluated and shown to vanish. This full-metric form strengthens the physical gauge and flavour calculation because the required mixing may arise both through the common derivatives of $\mathcal{D}_B$ and through operationally fixed overlap between defect carriers. Neither mechanism introduces a free Standard Model parameter.

If the typed embeddings introduce unspent continuous scale freedom, MA1 is not closed and F2 fires; if the substrate/record realisation of §8A retains such freedom, F21 fires.

10. The Universal Defect-Rate Principle

At leading local regulator order, define

$$[\![\Gamma\_bos^{(n)} = \Gamma\_V^{(n)} + \tfrac{1}{2} \langle \mathbf{D}, W\_prim\, \mathbf{D} \rangle]\!]$$

The Universal Defect-Rate Principle states that this functional is the quadratic part of the negative logarithm of the absolute substrate history density around the admissible branch. More precisely, let $\mu\_hist^{(n)}$ be the regulated substrate history measure and $\mu_0^{(n)}$ the canonical reference volume induced by the operational metric. Then, on the physical support,

$$-\log(d\mu\_hist^{(n)} / d\mu_0^{(n)}) = \Gamma\_bos^{(n)} + \Gamma\_ferm^{(n)} + \Gamma\_top^{(n)} + O(\|\mathbf{D}\|^3),$$

and the quadratic response metric is consequently

$$[\![W\_prim = D^2[-\log(d\mu\_hist^{(n)} / d\mu_0^{(n)})]\_(\mathbf{D}=0)]\!]$$

on the primitive residual carrier. This equation is the direct evaluation address for MA1 and MA8. It fixes both the relative residual geometry and the absolute action conversion. Its advantage is not that it guarantees the desired Standard Model outputs; its advantage is that it removes the freedom to change the action after those outputs are seen. It is also the single object no executed branch of this paper has yet constructed: every CFB branch freezes W_prim by hand, and the derivational status of the whole construction turns on whether this functional is built and shown to return a specific metric (F19). The status of that gate is sharpened — though not closed — by the candidate construction of §10.1.

10.1 Candidate Fisher-Metric Route to W_prim, and the Narrowed Form of F19

A candidate construction of the history-rate functional exists and is developed in full in HMGBC-1. It is summarised here because it changes the *form* of F19: the gate is no longer "construct an undefined measure," but a single sharply stated admissibility question about one operator. The construction derives the entire consumer of the rate functional and demonstrates it end to end on the K = 7 cell; it does not by itself close F19, because the one VERSF-specific input it requires is still unbuilt.

Inputs. The operational (Born) measure is the unique quadratic form $\mu\_op(x) = \langle \Omega, P\_x\, \Omega \rangle$ on an exhaustive orthogonal terminal-record partition $\{P\_x\}$; the refinement operator has the canonical polar factorisation $R\_n = U\_n |R\_n|$ with positive irreversible factor $\mathbb{T}\_n = |R\_n|$; and the six typed defect coordinates $\mathbf{D} = (\mathcal{D}\_C, \mathcal{D}\_J, \mathcal{D}\_T, \mathcal{D}\_R, \mathcal{D}\_B, \mathcal{D}\_H)$ parametrise the transfer, $\mathbb{T}\_n(\mathbf{D})$.

Two premises the construction needs, on the same footing as MA1. Operator positivity is not enough to define classical transition probabilities, and Hilbert-space self-adjointness is not enough to give reversibility in the record basis. Two premises are therefore named:

Premise HP1 — Terminal-cone positivity. There exists a terminal-record cone $\mathcal{C}_n$ with $\mathbb{T}_n \mathcal{C}_n \subseteq \mathcal{C}_n$, on which $\mathbb{T}_n$ is irreducible (positivity-improving), so that Perron–Frobenius applies in the record basis and yields a strictly positive Perron vector r_n and eigenvalue λ_n.

Premise HP2 — Record-basis reversibility. $\mathbb{T}_n$ is self-adjoint in the terminal-record inner product, not merely as a Hilbert-space operator, so that the induced kernel satisfies detailed balance. Both HP1 and HP2 are conditions *on the unbuilt* R_n; neither can be checked until R_n is supplied, and $\mathcal{D}_B$ — an intrinsic non-coboundary response by design — is exactly the direction most likely to endanger HP2.

The consumer, fully derived. Given HP1 and HP2, the Doob-transformed kernel

$$K_n(y|x) = t_n(x,y)\, r_n(y) \,/\, (\lambda_n\, r_n(x))$$

is stochastic ($\sum_y K_n(y|x) = 1$), has stationary law $\pi_n(x) = r_n(x)^2/\sum_z r_n(z)^2$, and satisfies detailed balance $\pi_n(x)K_n(y|x) = \pi_n(y)K_n(x|y)$. The relative-entropy rate of the defect-deformed law against the defect-free law,

$$\mathcal{I}_n(\mathbf{D}) = \sum_{(x,y)} \pi_{(n,0)}(x)\, K_{(n,0)}(y|x)\, \log[\, K_{(n,0)}(y|x) \,/\, K_n(y|x;\, \mathbf{D})\,]$$

satisfies $\mathcal{J}_n(0) = 0$, $\partial\mathcal{J}_n(0) = 0$, and returns the operational metric as its Fisher information,

$$(W_{prim}^{(n)})_{ab} = \partial_a \partial_b\, \mathcal{I}_n(0) = \sum_{(x,y)} \pi_{(n,0)}(x)\, K_{(n,0)}(y|x)\, S_a(x,y)\, S_b(x,y)$$

with the score

$$S_a(x,y) = t_a(x,y)/t(x,y) + r_a(y)/r(y) - r_a(x)/r(x) - \lambda_a/\lambda$$

$$\lambda_a = r^\dagger\, \mathbb{T}_a\, r, \qquad r_a = (\lambda I - \mathbb{T})^+\, (\mathbb{T}_a - \lambda_a I)\, r$$

($^+$ the reduced Moore–Penrose inverse on $r\perp$). This derives every diagonal and cross block — W_{CC} … W_{HH} and W_{BH}, W_{CH}, W_{RH}, … — from one probability law, with no inserted cross-coefficient, and it does so mechanically once $\mathbb{T}_0$ and its six defect-derivatives $\partial_{(\mathcal{D}_A)}\mathbb{T}$ are known.

Four MASD-1 posits become consequences. (i) Positivity of W_{prim} is automatic (a Fisher metric is PSD, $W_{prim} \succeq 0$). (ii) The gauge/record-coboundary null directions of §22 are automatic: a pure terminal-frame change $\mathbb{T} \mapsto G^{-1}\mathbb{T}G$ leaves the stochastic kernel unchanged up to relabelling, so $S_a = 0$ and $W_{prim}\, v_{gauge} = 0$ — the null quotient is the kernel of statistical distinguishability of the history law. This does **not** imply that gauge and frame coboundaries exhaust the kernel: a positive-semidefinite Fisher matrix may carry additional *statistically invisible* directions — defect variations that leave the one-step history law unchanged for reasons beyond gauge or frame — and any such direction sits in ker W_prim without being a gauge mode. The physical rank of W_prim must therefore be *calculated*, not assumed equal to the physical dimension minus the gauge count. This is the history-measure face of the unclassified 39 null modes of §20.1 and is coupled to F28: an unexplained W_prim null direction is either a genuine redundancy to be quotiented or a signal that the reduced response is not rigid along the zero locus. (iii) The absolute normalisation of §9 is fixed per transfer step: $-\log(d\mu\_\mathbf{D}^{(N)}/d\mu\_0^{(N)}) \approx (N/2)\langle \mathbf{D}, W\_prim\ \mathbf{D}\rangle$, with unit 1/a_t and no independent colour/weak/Abelian/scalar/completion conversion. (iv) The generation geometry descends from the same kernel: detailed balance gives symmetric conductances c_xy = π_n(x)K_n(y|x), a universal record Laplacian Δ_Ω = B_Ω† C B_Ω, the completion resolvent R_Ω = (W_H + Δ_Ω)$^{-1}$ W_H, and the sector operators R_f = S_fL† V_(H,⋆)† R_Ω S_fR — one universal geometry, not four chosen graphs.

Executed demonstration (K = 7). On the wheel, the published stationary measure π = (1/31)(7,4,4,4,4,4,4) and reversible Dirichlet form $Q_7 = \Pi(I - T_0)$ give the symmetric generator $L_7 = \Pi^{(-1/2)} Q_7 \Pi^{(-1/2)}$ with spectrum {0, ½, ½, 1, 1, 31/28, 5/4}. The one-step transfer $\mathbb{T}_7 = e^{(-L_7)}$ has strictly positive spectrum, Perron vector r = √π, Perron eigenvalue 1, and Doob kernel $K_7 = e^{(-(I-T_0))}$ satisfying $K_7\mathbb{1} = \mathbb{1}$, $\pi K_7 = \pi$ and detailed balance to numerical precision. The one-step Fisher curvature along the refinement-time deformation $K\_\tau = e^{(-\tau(I-T_0))}$ is

```
⟦ I_ττ = ∑_(x,y) π(x) K₁(y|x) [∂_τ log K_τ(y|x)]²_(τ=1) = 0.3954250563 ⟧
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reproduced by a direct second finite difference of the KL rate. This is a **plumbing demonstration, not an admissibility one**: it shows that VERSF's own operational and Dirichlet data generate a nonzero, absolutely normalised history-rate curvature, i.e. that the pipeline runs and normalises. It deliberately runs along τ, a single refinement-time direction, not along the six defects, and along a generator (symmetric PSD by construction) for which HP1 and HP2 *cannot* fail. It therefore certifies the machinery in the one place the hard premises are safe and says nothing about the directions where they are at risk — in particular nothing about $\mathcal{D}$_B, the intrinsic non-coboundary direction most likely to break minimal fibres and hence HP2. No admissibility conclusion may be drawn from it.

What this closes, and what it does not. Closed: the consumer of the rate functional — the map from a parametrised admissible transfer to W_prim, to the null quotient, to the absolute normalisation, and to the completion geometry — is fully derived and demonstrated. Not closed: the one VERSF-specific input, the explicit microscopic refinement operator

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⟦ (ρ, U, J, Φ) ↦ R_n(ρ, U, J, Φ) ⟧
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and its six defect-derivatives, together with proofs that it satisfies HP1 and HP2. F19 is therefore **narrowed, not passed**: its content is now exactly this single operator and its admissibility, and everything downstream has explicit formulas the moment it is supplied.

Honest reading — relocation one level deeper, and where it now stands. This is the same move as §47M.7, one level down: MASD-1 relocated the Standard Model's freedom into (W_prim, completion law); the Fisher-metric route relocates (W_prim, completion law) into the single operator R_n and its six derivatives. A Fisher metric built from a *chosen* $\mathbb{T}_n(\mathbf{D})$ would still be a chosen metric in information-geometric dress. F19 passes only when R_n is *derived* from the substrate axioms (finite distinguishability, irreversible commitment, the Void), not selected. The capacity-versus-prediction question is thus inherited by R_n — and §10.1.1 supplies R_n, within a *proposed* HMGBC-1 theorem package, as a construction forced by data already in the corpus, with HP1 proved and HP2 proved conditionally on geometric premises whose physical validity is open. What remains open descends accordingly, and is stated there. The package is proposed, not established: it changes the form of the F19 debt without yet paying it.

10.1.1 The Refinement Operator as Inherited Heat Transfer (proposed HMGBC-1 theorem package, included conditionally)

Grade and status of this subsection. HMGBC-1 does not yet exist as a completed paper. The construction below is its *proposed* theorem package, reproduced here at grade "proposed/conditional": it is not inherited as an established result, and it is not part of the unconditional Grade M1 mathematics of MASD-1. Its algebraic core (Theorem 10.1.1) and HP1 (Theorem 10.1.2) are genuine theorems once the operational Dirichlet form is fixed; HP2 (Theorem 10.1.3) is a theorem only *conditional on* geometric premises whose validity for the actual MASD coarse maps is open. Nothing here may be cited elsewhere in the paper as a closed result, and the F19 status lines that reference it are written accordingly.

The conjecture that the refinement transfer is the semigroup generated by the defect action can, within this proposed package, be discharged by identifying that semigroup with data already fixed upstream. The construction is summarised because it changes the *form* of F19 from "supply a missing operator" to "evaluate an explicit one and check its regularity" — a change that holds even though the package is proposed rather than established.

The construction. The positive factor of refinement is not free: it is the heat transfer of the inherited operational Dirichlet form. Let L_n be the Laplace–Beltrami operator of the operational distinguishability metric g_n on the record manifold, with operational measure μ_n, and let V_n be the isometric pullback of the coarse-graining map $q_n : M_{(n+1)} \to M_n$. Then

$$⟦\ R_n = \zeta_n V_n e^{(-a_t L_n)}\ ⟧$$

with ζ_n a positive normalisation. Nothing in this expression is chosen: g_n, μ_n and q_n are prior VERSF data, a_t is the frozen Euclidean transfer step (§19A), and the exponential is fixed once the generator is.

Theorem 10.1.1 (polar form). Since V_n is an isometry, $V_n^\dagger V_n = I$, so

$$⟦\ R_n^\dagger R_n = \zeta_n^2 e^{(-a_t L_n)} V_n^\dagger V_n e^{(-a_t L_n)} = \zeta_n^2 e^{(-2 a_t L_n)}\ ⟧$$

hence the positive factor is $|R_n| = \zeta_n e^{-a_t L_n} = \zeta_n \mathbb{T}_n$ and the polar isometry is exactly V_n. Within the proposed package the object MASD-1 called $\mathbb{T}_n$ is therefore the operational heat semigroup, and Identification 10.2 (below, demoted to a corollary of this package) becomes a consequence rather than a free posit — conditionally on the package's premises, not as an established MASD-1 result.

Theorem 10.1.2 (HP1, terminal-cone positivity). The heat semigroup $\mathbb{T}_n = e^{-a_t L_n}$ of a Dirichlet form is Markov: $\mathbb{T}_n f \geq 0$ for $f \geq 0$ and $\mathbb{T}_n \mathbb{1} = \mathbb{1}$. Order-preservation of the isometric pullback then gives $R_n \mathcal{C}_n \subseteq \mathcal{C}_n$ on the terminal-record cone, and the strictly positive heat kernel of an irreducible (connected-support) Dirichlet form makes $\mathbb{T}_n$ positivity-improving, so Perron–Frobenius applies in the record basis and yields a strictly positive Perron vector r_n with a spectral gap. This discharges HP1 as a theorem *once the operational Dirichlet form is fixed* — it does not depend on the coarse-map geometry of Theorem 10.1.3: operator positivity was never sufficient, and the Markov/Dirichlet structure is exactly what supplies cone positivity and irreducibility.

Theorem 10.1.3 (HP2, strong lumpability by submersion). If q_n is a Riemannian submersion with minimal fibres and the operational measure is basic (product) across refinement, $d\mu_{n+1} = d\mu_n \, d\nu_{F_n}$ with base-independent fibre measure, then the Laplacian of a basic function is basic, $L_{n+1} V_n = V_n L_n$, and functional calculus lifts this to $\mathbb{T}_{n+1} V_n = V_n \mathbb{T}_n$ — which is exactly strong lumpability of the induced record kernel. Detailed balance in the record basis (HP2) follows, and $\pi_n = r_n^2/\|r\|^2$ is the reversible stationary law of §10.1. This discharges HP2 as a theorem *conditional on* the three geometric properties named — minimal-fibre Riemannian submersion and basic measure. Its physical applicability is therefore open: whether the actual MASD coarse maps of §2 satisfy those properties on the real regulator is exactly the numerical question of item (1) below, and until it is settled HP2 holds for the model maps, not yet for the physical ones.

The six derivatives are regulator data. By Duhamel,

$$\partial_{\mathcal{D}_A} \mathbb{T}_n = -\int_0^{a_t} e^{-(a_t-s)L_n} \left(\partial_{\mathcal{D}_A} L_n\right) e^{-s L_n} \, ds$$

so every defect-derivative of the transfer reduces to $\partial_{\mathcal{D}_A} L_n$ — that is, to derivatives of the operational Dirichlet form's metric, measure and coarse map (∂g_n, $\partial\mu_n$, ∂q_n). These are finite-regulator geometric quantities, not free functions. No derivative of an unspecified R_n survives anywhere in the Fisher metric.

Branch free energy uses the raw weight, and the two forms coincide. The branch selector of §53.4 is the least-free-energy branch. Two forms appear in the paper and must be reconciled: §53.4 writes $f_\beta = -(1/a_t) \log \lambda_{0,\beta}$, while the construction here gives $f_\beta = -(1/2a_t) \log(Z_{n+1,\beta}/Z_{n,\beta})$. These are the same object, not a factor-of-two ambiguity. The positive factor is $|R_n| = \zeta_n e^{-a_t L_n}$; since $e^{-a_t L_n}$ has Perron eigenvalue 1, the Perron eigenvalue of $|R_n|$ is

$$\lambda_{0,\beta} = \zeta_\beta = \sqrt{Z_{n+1,\beta} / Z_{n,\beta}}$$

Substituting into the §53.4 form gives $-(1/a_t) \log \lambda_{(0,\beta)} = -(1/a_t) \log \sqrt{Z_{(n+1,\beta)}/Z_{(n,\beta)}} = -(1/2a_t) \log(Z_{(n+1,\beta)}/Z_{(n,\beta)})$, which is exactly the construction form. The factor of two is absorbed by the square root in ζ_β; the two expressions are proved equal, and §53.4 is to be read with $\lambda_{(0,\beta)} = \zeta_\beta$. The comparison is computed from the *unnormalised* Perron weight ζ_n: HP1, HP2 and W_prim survive if the operational measures are normalised, but the branch selector does not, so $Z_{(n,\beta)}$ must be published unnormalised. This is a real constraint on the corpus, recorded as such.

The one physical posit, stated plainly (re Theorem 10.1.1). Theorem 10.1.1 — that the polar factor of R_n is $e^{-a_t L_n}$ — is a *trivial* algebraic consequence of the definitional form $R_n = \zeta_n V_n e^{-a_t L_n}$; it proves nothing about physics. The physical content is entirely in the *choice of that form*: the identification of the microscopic refinement transfer with the operational heat flow. Demoting Identification 10.2 to a corollary is therefore a presentational move, not a mathematical one — the hypothesis "refinement = heat flow of the operational metric" remains a physical posit; it has merely been relocated into the definition of R_n. This subsection does not derive that posit from the substrate axioms. It shows that *if* the posit holds, HP1 follows unconditionally and HP2 follows conditionally, and the six derivatives become geometric data. No referee should read Theorem 10.1.1 as converting the posit into a theorem.

The single gating question, and why the two "conditionals" are one. The residual F19 debt is often written as two items — coarse-map regularity and g_n uniqueness — but they are two faces of one circularity, and it is cleaner to state it as a single gate:

Is the operational distinguishability metric g_n forced by prior VERSF results, independently of the refinement construction? If g_n is *defined* by horizontal lift through the coarse map (§9), then the minimal-fibre submersion property is tautological but g_n is a derived object whose identity with the operational metric the rest of the programme uses is then unestablished. If g_n is given *independently*, then the submersion/minimal-fibre/basic-measure regularity of the actual maps (the ordered-holonomy link map, the Karcher-mean completion map — with $\mathcal{D}_B$ the direction most likely to break minimal fibres) becomes a set of theorems to prove, verifiable only numerically across at least two regulator levels. Either horn leaves F19 open: the first hides the freedom in g_n, the second hides it in the unproven regularity. F19 is genuinely closed as a construction only when this one question is answered — g_n shown forced *and* the actual maps shown regular under that forced g_n.

Two information-geometric faces of the same gate. The Fisher-metric route of §10.1 carries the identical circularity in a different guise, and it should be read as part of this one gate rather than as separate caveats: (a) the operational Born reference state must *be* the Perron state of the refinement transfer, or $\langle r_n, P_x r_n\rangle = \pi_n(x)$ is a definition dressed as content; and (b) repeated coarse-graining must stay Markov (strong lumpability of the record fibres) — which is exactly HP2, i.e. exactly the minimal-fibre/basic-measure regularity above. Both reduce to "is g_n forced, and are the maps regular under it." They are not independent debts.

What is and is not closed. Closed, *conditional on the posit and on the single gate above*: R_n is an explicit construction from inherited data rather than a per-sector choice; its polar factor is the operational heat semigroup; HP1 is a theorem; HP2 is a theorem given the geometric premises;

and the six defect-derivatives are regulator data. Not closed: the physical posit itself, and the single gating question. The correct summary is that F19's *form* has changed — from "construct an undefined measure" to "answer one metric-uniqueness-and-regularity question and then evaluate" — while its *substance* remains open.

Corollary 10.2 (former Identification 10.2). Under the posit, the refinement transfer is the Euclidean semigroup $\mathbb{T}_n(\mathbf{D}) = e^{(-a_t L_n(\mathbf{D}))}$ generated by the operational Dirichlet form, and its six defect-derivatives reduce by Duhamel to the $\partial_{(\mathcal{D}_A)} L_n$ that MASD-1 already supplies as the $D\mathcal{D}_A$. This is the content of Theorems 10.1.1–10.1.3 given the posit; it is not a derivation of the posit.

Coupling to F28. The score formula uses $r_a = (\lambda I - \mathbb{T})^+(\mathbb{T}_a - \lambda_a I)\, r$, whose reduced resolvent diverges when the leading eigenvalue is near-degenerate — the same near-degeneracy that would signal a non-isolated zero locus. The Fisher metric is therefore well-conditioned exactly when F28 is satisfied and ill-conditioned exactly when it is not, so a proof of a Perron gap (HP1 irreducibility) services both F19 and F28 at once.

The two joins are the single gate. Theorem-style statements in this route reduce to two identifications — that the operational Born reference state *is* the Perron state (so $\langle r_n, P_x r_n\rangle = \pi_n(x)$ is content, not definition), and that repeated coarse-graining stays Markov (strong lumpability). These are not extra caveats: §10.1.1 shows they are the two information-geometric faces of the single gating question (is g_n forced, and are the actual maps regular under it), and the F19 status line is written against that one gate, not against a list of independent conditions.

Part III — The Bosonic Master Action

11. The Substrate Potential

The amplitude potential is $\Gamma_V^{(n)} = \sum_{(i\in V_n)} v_i\, (\lambda_{sub}/4)\, (\rho_i^\dagger\rho_i - \rho_c^2)^2$. Its physical second variation supplies one radial stiffness per occupied event, $H_V = 2\,\lambda_{sub}\, \rho_c^2\, P_{rad}$. It supplies no tangential generation or flavour hierarchy.

12. Vacuum-Relative Closure Defect

For an oriented face p, $H_p[U] = \overleftarrow{\prod}_{(e\in\partial p)} U_e^{\sigma(p,e)}$. *With frozen background holonomy* $\mathcal{H}(p,\star)$,

$$[\![\ \mathcal{D}_C(p) = (-i/|p|)\ \mathrm{Log}_0(\ H_{(p,\star)}^{-1}\ H_p\ )\ ]\!]$$

The logarithm branch is frozen at the identity before evaluation. For an infinitesimal Standard Model connection insertion $U_e \mapsto U_{(e,\star)} \exp[i \mathcal{E}_{SM}(a_e)]$, the residual tends to the embedded continuum curvature.

13. Constitutive Current Defect

Define the minimal transported substrate difference $\Delta_e \rho = \rho_{t(e)} - U_e \rho_{s(e)}$. The current is not identified with this difference. Its failure to agree with the minimal transported flow is

$$[\![\mathcal{D}_J(e) = J_e - \kappa_e \Delta_e \rho]\!]$$

The coefficients κ_e are not free sector couplings. They are the edge transport conductances returned by the stationary operational metric and geometric edge weights.

14. Transport-Conservation Defect

Let ∂_U denote the covariant incidence operator. For a vertex i, $(\partial U J)i = \sum(e : t(e)=i) J_e - \sum(e : s(e)=i) U_e^{-1} J_e$. Define

$$[\![\mathcal{D}_T(i) = \partial_\sigma \rho_i + (\partial_U J)_i]\!]$$

This is the finite-regulator continuity defect.

15. Record-Composition Defect

For every composable triple $i \to j \to k$,

$$[\![\mathcal{D}_R(i,j,k) = \Phi_{jk}^{tr} \Phi_{ij}^{tr} - \Phi_{ik}^{tr}]\!]$$

This acts on the spatial transport sector only; the local completion morphisms Φ_H are constrained by $\mathcal{D}_H$ (§47F), not by composition. This is independent of link closure. A connection may be flat while the record maps fail to compose, and vice versa.

16. Polar Bond-Compatibility Defect

16.1 Occupied support

The positive local Gram operator is the strengthened form of §8A, assembled on the unified local carrier $\mathscr{H}_i$:

$$G_i = \tilde{\rho}_i \tilde{\rho}_i^\dagger + \sum_{(e : t(e)=i)} \tilde{\Phi}_e \tilde{\Phi}_e^\dagger + \sum_{(e : s(e)=i)} \tilde{\Phi}_e^\dagger \tilde{\Phi}_e + G_i^{term},$$

with the terminal-record contribution G_i^{term} keeping every occupied matter address visible to the support even where the classical record amplitude vanishes at the vacuum. Let Π_i be its spectral projector onto the nonzero rank-stable occupied band, $\Pi_i = \mathbf{1}_{[g_{occ}, \infty)}(G_i)$, with g_{occ} fixed inside the certified gap. At numerical finite precision, the zero/nonzero split must be certified by a spectral gap that survives regulator refinement. It may not be set by flavour data. Failure of a stable gap fires F18.

16.2 Polar transport

Define the supported record map $M_{ij} = \Pi_j \tilde{\Phi}_{ij} \Pi_i$. On a rank-stable branch, $M_{ij} = B_{ij} V_{ij}$, where $B_{ij} = (M_{ij} M_{ij}^\dagger)^{1/2}$ is positive on im Π_j, and $V_{ij} = B_{ij}^{-1} M_{ij}$ is the unique polar partial isometry from im Π_i to im Π_j.

16.3 Compatibility

The bond defect is

$$\mathcal{D}_B(i,j) = \Pi_j U_{ij} \Pi_i - V_{ij}$$

Both terms transform as $G_j (\cdot) G_i^{-1}$, so the defect is gauge covariant. This term does not eliminate U as an independent variable. It gives the action a reason to align the physical link with the independently generated record transport. Its linearisation contains a pure endpoint-frame coboundary, which cancels around a face, and an intrinsic polar response from changes in the support and positive bond stretch, which need not cancel. The latter is the candidate physical intertwiner missing from the factorised gauge and colour calculations. The polar differential and its coboundary/intrinsic split are derived in Appendix B.

17. The Complete Bosonic Action

Using the full induced residual metric, the bosonic master action is

$$\Gamma_{bos}^{(n)} = \Gamma_V^{(n)} + \tfrac{1}{2} \sum_{X,Y \in \{C,J,T,R,B,H\}} \langle \mathcal{D}_X, W_{XY} \mathcal{D}_Y \rangle$$

In expanded notation, $\Gamma_{bos}^{(n)} = \Gamma_V^{(n)} + \tfrac{1}{2} \sum_X \|\mathcal{D}_X\|^2(W_X) + \mathrm{Re} \sum_{X<Y} \langle \mathcal{D}_X, W_{XY} \mathcal{D}_Y \rangle$. Every diagonal and off-diagonal block is fixed by $W_{XY} = \iota_X^\dagger W_{prim} \iota_Y$, with the geometric primal and dual weights of §6 carried inside the typed embeddings. No term in this action is independently normalised by colour, weak isospin, hypercharge, generation, chirality or particle species. At a common defect zero, the second variation is

$$\mathbb{G}^\star = G_V + D\mathbf{D}^{\star\dagger} W_{prim} D\mathbf{D}^\star$$

This expression includes every cross response generated by the common operational geometry.

18. Locality, Positivity and Reversal

Every term is nonnegative in Euclidean signature. Every term is local on one vertex, edge or elementary face, except for the spectral support and polar operations on the finite local record block. Under edge reversal, $U_{ji} = U_{ij}^{-1}$, $\Phi_{ji} = \Phi_{ij}^{-1}$ where defined, and the corresponding residual norm is unchanged.

19. Limited Uniqueness at Declared Order

The action is not claimed to be the unique possible all-order substrate functional. It is the minimal functional generated by: the inherited primitive variables; the six declared defects; the inherited operational metric; gauge covariance; positivity; reversal symmetry; leading quadratic defect pricing; and no inserted factor coefficients. A competitor at the same declared order must either (1) add a new independent defect, (2) use a different operational metric, (3) introduce cross-defect terms, (4) add higher powers of the residuals, or (5) violate one of the stated symmetries. Each alternative is visible and falsifiable.

19A. Primitive Dimensional Ledger

A common master action cannot be numerically closed unless every dimensionful primitive is independently assigned and derived. Let the frozen primitive scale tuple be $\mathfrak{S}_{scale} = (\ell_n, a_t, \rho_c, \lambda_{sub}, \{\kappa_e\}, \Lambda^\star)$. Its entries have distinct roles.

Primitive	Role	Required derivation
ℓ_n	spatial refinement unit	regulator geometry
a_t	Euclidean transfer step	refinement/transfer theorem
ρ_c	substrate vacuum norm	stationary minimum of the absolute rate functional
λ_{sub}	radial fourth derivative	fourth radial variation of the rate functional
κ_e	current-conductance map	current block of W_{prim} and edge geometry
$\Lambda^\star$	completion-to-four-dimensional mass conversion	substrate-to-continuum dimensional descent

No member of this tuple may be fixed by a Standard Model mass, coupling or confinement observable.

Theorem 19A — Primitive Scale Closure

Given the evaluated history-rate functional and the frozen regulator geometry:

$$\rho_c = \mathrm{argmin}_\rho\ \Gamma_{bos}[\rho, U\star, J\star, \Phi\star],$$

$$\lambda_{sub}^{bare} = (1/6)\ \partial^4\Gamma_{bos}/\partial\varrho^4\ |\star$$

in the declared canonical radial convention — the bare substrate quartic, to be distinguished from the physical relaxed quartic $\lambda(\mu\star)$ of §51.1, which includes the auxiliary Schur correction of the full vertex tower and is the object entering the boundary set —

$$\kappa_e = \mathrm{argmin}_\kappa\ \| J_e - \kappa\ (\rho_t - U_e\ \rho_s)\ \|^2_{(W_{prim})},$$

evaluated before matter or gauge data are inspected — noting that on a homogeneous branch with $\Delta_e\rho\star = \rho_t - U_e\rho_s = 0$ this minimisation is indeterminate, in which case κ_e is extracted instead from the quadratic current-response block of W_prim or from a certified nonzero perturbation of the background — and

$$\Lambda\star = \ell_{comp}^{-1}\ Z_{comp}^{(-1/2)},$$

where ℓ_{comp} and Z_{comp} are respectively the correlation length and canonical residue of the completion-strain mode returned by the common Hessian. Thus $\Lambda\star$ is not an independent mass inserted into the left–right bilinear; it is the canonical mass conversion of the completion carrier. A failure to return any member of $\mathfrak{S}_{scale}$ leaves the corresponding dimensional prediction open and fires F20.

Part IV — Common Hessian and Physical Reduction

20. The Admissible Vacuum

An admissible background $\mathfrak{X}\star = (\rho\star, U\star, J\star, \Phi\star)$ satisfies simultaneously $\rho\star^\dagger\rho\star = \rho_c^2$ on the occupied support, $\mathcal{D}_C(U\star) = 0$ on evaluated faces, $\mathcal{D}_J = 0$, $\mathcal{D}_T = 0$, $\mathcal{D}_R = 0$, $\mathcal{D}_B = 0$ on the occupied support, and $\mathcal{D}_H = 0$. At such a background every defect vanishes and every residual norm is stationary. The scalar sector may still carry the physical broken vacuum through the inherited potential and the completion-strain coupling.

20.1 Existence and Genericity of the Common Zero Locus

The reduction of §21 presupposes that a common background annihilating all six defects simultaneously exists and is, in the relevant sense, isolated. Neither is automatic. The six defects are heterogeneous — holonomy, current realisation, continuity, record composition, bond polar compatibility and completion stretch — and there is no a priori guarantee that their zero sets intersect, still less that the intersection is a smooth stratified manifold of the expected codimension whose only null directions are gauge.

This paper does not prove genericity. It establishes existence numerically on the finite toy cells actually executed, and records the genericity question as open (F28).

What is established. On the canonical $K = 7$ wheel cell, the CFB-1, CFB-2 and CFB-3 branches each return a background at which the assembled residual vector reaches machine zero: $\|R\star\| \sim 10^{-13}$ for CFB-1, and exact defect zeros for CFB-2 and CFB-3. A common zero therefore exists on these cells. This is evidence for existence on the executed cells, not a proof of it in general.

What is not established. The null space of the Gauss–Newton Hessian at these points has not been analytically classified. The candidate executions return, for CFB-1, 39 numerical-null modes; their full decomposition into gauge, frame, record-coboundary and completion directions has not yet been published. It is therefore not established that every null direction is an identified gauge, frame or controlled completion direction; that decomposition is an open item, not a closed result. The claim of "no unexplained null direction" is downgraded to "no unexplained null direction has yet been catalogued, and the catalogue is not complete."

On differing early CFB-1 returns. Earlier development implementations produced differing CFB-1 gauge returns. Because those executions were not shown to share one complete source-and-data manifest — the same residual source, variable ordering, stationary point, retained directions, quotient, auxiliary basis and reduction code — the difference cannot be interpreted as physical branch uncertainty or as evidence of a non-isolated zero locus. It instead demonstrates the necessity of source-level reproduction and exact manifest equality. No structural conclusion about the zero locus may be drawn from those differing numbers, and they are not cited here as evidence for or against genericity.

The genericity falsifier. The genericity question is nonetheless real and open on its own terms. If, on refinement under one frozen manifest, the common six-defect zero locus is found to carry continuous physical moduli that change the reduced gauge, scalar or flavour response — so that the boundary certificate of §53 is not a function of the frozen inputs alone but depends on an unfixed choice of point within the zero locus — then the master action has not closed its background, and F28 fires. Its resolution requires either a proof that the physical reduced response is constant along the zero locus (a gauge-invariance-of-response statement) or a derived principle selecting a unique point, and neither is in hand. This must be tested by manifest-equal refinement, not by comparing unmatched development runs.

21. Universal Gauss–Newton Hessian

Theorem 21.1 — Gauss–Newton Reduction

Let $\mathbf{D}(\mathfrak{X})$ be the assembled defect map and $\Gamma\_\mathrm{defect} = \frac{1}{2}\langle \mathbf{D}, W\_\mathrm{prim}\, \mathbf{D}\rangle$. At a common zero $\mathbf{D}(\mathfrak{X}\star) = 0$, the physical second variation is exactly

$$[\![\mathbb{G}^\star = G_V + D\mathbf{D}^{\star\dagger}\ W_{prim}\ D\mathbf{D}^\star]\!]$$

Proof. The general second variation of ½⟨**D**, W_prim **D**⟩ is D**D**† W_prim D**D** + ⟨**D**, W_prim D^2**D**⟩. At a common zero the second term vanishes because **D**($\mathfrak{X}^\star$) = 0. The potential contributes G_V. ■

The consequence is structural: no second derivative of any constitutive map — link, current, record, bond or completion — enters the quadratic physical Hessian. Only first-order sensitivities D$\mathcal{D}$_X appear, priced by one common W_prim. This is what makes the Hessian's physical blocks calculable without inserting response constants.

22. Gauge and Null Quotient

The kernel of $\mathbb{G}^\star$ contains the exact zero modes: infinitesimal gauge transformations, endpoint frame changes and, where present, exact defect-preserving flat directions. Let $\mathcal{N}$ be that kernel. The physical operator is defined on the orthogonal complement, $\mathbb{G}^{\star\,phys} = \mathbb{G}^\star |_{(\mathcal{N}^\perp)}$. No physical response may be read from $\mathcal{N}$. The complete removal of frame directions is what leaves the intrinsic bond response, if any, in the physical block. In the executed branches, the numerically null directions counted so far are consistent with identified gauge, frame and flat completion directions, but their complete analytic classification has not been published (§20.1): for CFB-1, 39 null modes are counted and not yet fully decomposed into gauge, frame, record-coboundary and completion directions. The quotient is therefore performed against the numerically identified null space, with the classification recorded as an open item.

23. Complete Auxiliary Census

Let the physical modes be split into slow physical channels φ (scalar-normal, gauge-tangent, completion-strain, closure) and fast auxiliary channels η (support-tangent, radial, record-transport, current-response). The census must be complete: every same-order mode that couples to a physical channel is retained until the Schur step, by Premise MA7. Omitting an auxiliary mode is equivalent to inserting a constraint and fires F7.

24. Exact Schur Reduction

Write the quotiented Hessian in block form,

$$\mathbb{G}^{\star\,phys} = [\ [\ H_{\varphi\varphi},\ H_{\varphi\eta}\ ],\ [\ H_{\eta\varphi},\ H_{\eta\eta}\ ]\ ].$$

Theorem 24.1 — Physical Schur Complement

If $H_{\eta\eta} \succ 0$ on the auxiliary block, the exact physical operator on the slow channels is

$$[\![\hat{H}_{red} = H_{\varphi\varphi} - H_{\varphi\eta}\ H_{\eta\eta}^{-1}\ H_{\eta\varphi}]\!]$$

This is not perturbative. It is the exact reduction of the quadratic form after the complete auxiliary block is relaxed. Every physical extraction in Part VI is a projection of this one operator. If $H_{\eta\eta}$ is not positive on the retained census, the background is not a stable physical vacuum and F8 fires.

25. Scalar Two-Point Descent

Let n_H be the canonical Higgs-normal embedding into the slow scalar channel, carrying the operational-metric normalisation. The scalar two-point response is the momentum-resolved compression

$$[\![\Pi_H^{scalar}(p) = n_H^\dagger \hat{H}_{red}(p)\, n_H]\!]$$

Its expansion $\Pi_H^{scalar}(p) = m_{(H,curv)}^2 + Z_H p^2 + O(p^4)$ defines the curvature mass and canonical residue. The physical scalar mass parameter is $\mu_H^2 = m_{(H,curv)}^2/Z_H$, and the canonically normalised Higgs field is $\varphi_H^{can} = Z_H^{(1/2)} \varphi_H$. No scalar normalisation is inserted; both coefficients are compressions of the one common Hessian.

26. Gauge-Response Descent

Let $\{T_A\}$ be the frozen physical embedding of the Standard Model generators into the gauge-tangent slow channel, factorised by simple factor and hypercharge. The gauge response is

$$[\![(Z_g)_{(AB)} = T_A^\dagger \hat{H}_{red}\, T_B]\!]$$

On the physical factor decomposition this is block-diagonal with entries Z_3, Z_2, Z_q. The physical couplings are

$$[\![g_s = Z_3^{(-1/2)}, \quad g = Z_2^{(-1/2)}, \quad g' = 6\, Z_q^{(-1/2)}]\!]$$

in the inherited normalisation of the gauge-sector gate. **Sign convention (fixed once, here).** Throughout this paper "stiffness" denotes the Hessian curvature Z, which is *inverse* to the coupling: a larger Z is a stiffer response and hence a *weaker* coupling, since $g = Z^{(-1/2)}$. Statements elsewhere that an "Abelian force is too weak" refer to the Abelian *stiffness* Z_Y being too small; in coupling language the identical fact reads "the hypercharge coupling g′ is too large," because with $Z_Y = Z_q/36$ one has $g' = 6\, Z_q^{(-1/2)} = Z_Y^{(-1/2)}$, so small $Z_Y \Leftrightarrow$ large g′. Both phrasings describe the same failure — the Abelian sector sits at the wrong point relative to the weak sector — and the reader should hold the stiffness reading as primary. The factor 6 in $g' = 6\, Z_q^{(-1/2)}$ is the primitive hypercharge normalisation $q = 6Y$ (frozen in the manifest conventions, Appendix G.4): the primitive Abelian generator is normalised to integer charges, so the physical hypercharge coupling carries the 6 that converts q back to Y. The embedding $\{T_A\}$ is not a free choice: §47H derives it by differentiating the frozen faithful representation, landing in the tensor carrier, so no allocation of channels to factors is performed by hand.

27. Closure-Hessian Descent

Let P_cl be the projector onto the closure-compatible slow block. The reduced closure operator is

$$\hat{H}_{cl}^{red} = P_{cl}\, \hat{H}_{red}\, P_{cl}$$

This is the object whose scalar-compatible compression supplies the Hessian route to the Yukawa operators, and whose curvature block supplies the colour response. It is a projection of the same $\hat{H}_{red}$, not a separately defined operator.

28. Colour-Response and Wall Descent

The physical colour stiffness is the colour-factor curvature entry Z_3 of §26, with no identification of the matter census index with primitive stiffness (Firewall M3). The triality-wall coefficient is the constrained master-action minimum per unit area in a fixed nonzero-triality sector,

$$B_\tau = \min_{\mathfrak{X} \in \mathcal{T}_\tau} \left(\Gamma_{master}[\mathfrak{X}] / A \right)$$

where $\mathcal{T}_\tau$ is the class of configurations carrying a unit of nonzero triality across a fixed wall of area A, with the vacuum value subtracted. No centre-symmetry penalty is inserted by hand; the wall tension is whatever the master action assigns to the constrained sector.

Part V — Fermions and the Completion Kernel

29. Shared-Link CAR/Fock Transport

The matter carrier is the CAR/Fock space over the occupied faithful Standard Model representation, transported by the same physical link U used in the closure action (Premise MA5). The primitive fermion kinetic term is

$$\Gamma_{kin}^{(0)} = \sum_{(e\,:\,i\to j)} \bar{\Psi}_j\, \gamma^e \left(\Psi_j - U_e \Psi_i \right) + \text{h.c.},$$

with U in its inherited representation on the matter carrier. There is no separate matter-vertex normalisation: any rescaling of the matter transport would be a rescaling of the same U that fixes the gauge curvature, and is therefore forbidden by Firewall M2's companion condition.

30. Terminal Records as Primitive Matter Addresses

By Premise MA4, a physical matter address is a terminally distinguishable record subspace. Let Ω_i be the finite occupied-address ledger at vertex i and $P_{(\omega,i)}^{rec}$ the corresponding terminal-record support projector. The address carrier at i is $\mathcal{A}_i = \bigoplus_{(\omega\in\Omega_i)} \mathrm{im}\, P_{(\omega,i)}^{rec}$, and the address-to-record map is canonical inclusion $\mathcal{A}_i \hookrightarrow \mathcal{H}_{(rec,i)}$. No refined embedding, angle or phase is present at this stage; only the raw terminal supports.

31. Polar Completion Strain

Let $\Phi_{(H,i)}$ be the local scalar-compatible left–right completion morphism, with polar decomposition $\Phi_{(H,i)} = B_{(H,i)} V_{(H,i)}$, $B_{(H,i)} \geq 0$, $V_{(H,i)}$ a partial isometry. Let $B_{(H,sym)}$ be the frozen symmetric-interface positive part. The vacuum-relative completion strain is

$$⟦\ X_H = \log\left(B_{(H,sym)}^{(-1/2)}\, B_H\, B_{(H,sym)}^{(-1/2)} \right)\ ⟧$$

It is Hermitian, vanishes at the symmetric interface, and measures how far the completion geometry is stretched relative to the symmetric reference. Its differential is derived in Appendix B.

31A. The Typed Left–Right Completion Operator

Define the completion operator

$$⟦\ \mathcal{O}_H = V_H^\dagger X_H\ ⟧$$

carrying the polar orientation V_H inside the bilinear. This object is well typed as a map from the right chiral record carrier to the left chiral record carrier, so a bilinear $\bar{\Psi}_L\, \mathcal{O}_H\, \Psi_R$ is gauge invariant even when left and right sit in distinct Standard Model representations. The separation of the unitary orientation V_H from the positive stretch X_H is what allows mass generation (through X_H) to be distinguished from gauge/orientation transport (through V_H).

32. The Primitive Typed Left–Right Fermion Action

The raw representation-typed address maps are frozen before any response is evaluated,

$$\llbracket\ A_{fX}^{(0)} = P_{fX}^{rep}\ J_{\Omega}\ S_{fX}\ \rrbracket$$

where P_{fX}^{rep} is the frozen representation projector for sector f and chirality X, J_Ω the address-ledger inclusion and S_{fX} the interface-typing selector (H or $\tilde{H}$). The primitive left–right fermion action is

$$\llbracket\ \Gamma_{LR}^{(0)} = \Lambda\star \sum_{(f=u,d,e,\nu)}\ [\ \bar{\Psi}_{fL}\ (A_{fL}^{(0)})^\dagger\ \mathcal{O}_H\ A_{fR}^{(0)}\ \Psi_{fR} + h.c.\ ]\ \rrbracket$$

No Yukawa matrix, derived projector, canonical embedding or mixing angle appears in this definition (Firewall M1). Only raw address labels and the completion operator are present.

33. The Typed Scalar-Readout Operator and Chiral Supports

The microscopic scalar-response operator is the relaxed derivative of the completion operator with respect to the Higgs radial modulus,

$$\llbracket\ \mathcal{Y}\star = \Lambda\star\ [\ \mathcal{D}_{(\varphi_H)}\ \mathcal{O}_H\ ]\star\ \rrbracket$$

Its left and right supports are the physical scalar-compatible readout projectors,

$$\llbracket\ P_Y^L = \mathrm{supp}(\ \mathcal{Y}\star\ \mathcal{Y}\star^\dagger\ ), \quad P_Y^R = \mathrm{supp}(\ \mathcal{Y}\star^\dagger\ \mathcal{Y}\star\ )\ \rrbracket$$

Their ranks are outputs, not inputs (Firewall M4). The typed general relation between them is $P_Y^L = V_{(H,\star)}^\dagger P_Y^R V_{(H,\star)}$, carrying the polar orientation $V_{(H,\star)}$; this reduces to literal equality $P_Y^L = P_Y^R \equiv P_Y^{micro}$ only after a common-carrier identification of the left and right chiral record carriers, in particular when $V_{(H,\star)} = I$. On the minimal exact branch, $X_{(H,\star)} = (\varphi_{(H,\star)}/\Lambda\star)\, P_H^R$, so $\mathcal{Y}\star = V_{(H,\star)}^\dagger P_H^R$, with $P_Y^R = P_H^R$ and $P_Y^L = V_{(H,\star)}^\dagger P_H^R V_{(H,\star)} = P_H^L$. Statements of "$P_Y^L = P_Y^R$" elsewhere in this paper are shorthand for this typed identity on the common-carrier or $V_H = I$ branch.

34. Derived Canonical Address Maps and Embeddings

The raw address maps are canonicalised by polar decomposition on their projected supports. Write $P_Y^X A_{fX}^{(0)} = \mathcal{E}_{fX} R_{fX}$, where $\mathcal{E}_{fX}$ is the partial isometry (the derived chiral embedding) and $R_{fX} = (\ (A_{fX}^{(0)})^\dagger P_Y^X A_{fX}^{(0)}\ )^{(1/2)}$ is the positive kinetic-address radius. The condition $R_{fX} = I_3$ is a certificate that the raw address was already canonically normalised on the readout support; departures are calculated, not inserted.

34A. Neutral Symmetric Completion

The neutral sector uses symmetric terminal-record compressions. The heavy Majorana block is

$$M_R = \Lambda_\star \left(A_{\nu R}^{(0)} \right)^T \mathcal{C}\, \mathcal{O}_H^{sym} A_{\nu R}^{(0)}$$

with $\mathcal{C}$ the charge-conjugation intertwiner and $\mathcal{O}_H^{sym}$ the symmetric completion operator; the irreducible active block M_L^{irr} is the two-interface symmetric active compression, order-suppressed only if the calculation returns it so. Neither block is inserted; both are compressions of the same completion operator.

35. The Exact Mixed-Variation Yukawa Operator

The mixed Yukawa operator is the relaxed scalar derivative of the primitive bilinear, compressed onto the readout supports and dressed by the kinetic-address radii,

$$Y_f^{mixed} = R_{fL}\, \mathcal{E}_{fL}^\dagger\, \mathcal{Y}_\star\, \mathcal{E}_{fR}\, R_{fR}$$

where $\mathcal{Y}_\star = P_Y^L\, \mathcal{Y}_\star\, P_Y^R$. This is one of two independent routes to the physical Yukawa operator.

36. Mass and Vertex Linearity Certificate

At the broken vacuum $\varphi_{(H,\star)} = v_{cl}/\sqrt{2}$, the mass operator is the value of the completion bilinear, and the Higgs vertex is its φ_H derivative. Because both come from the same completion operator, they satisfy the interface-linearity certificate

$$M_f = (v_{cl}/\sqrt{2})\, Y_f$$

exactly, provided the completion strain is linear in φ_H on the readout support (Premise MA9). Any departure from this linearity is a calculated higher-order completion effect and is published as such.

37. CY3′ as an Independent Master-Action Equality

The second, independent route to the Yukawa operator is the compression of the Schur-reduced closure Hessian,

$$Y_f^{Hess} = R_{fL}\, \mathcal{E}_{fL}^\dagger\, P_Y^L\, \hat{H}_{cl}^{red}\, P_Y^R\, \mathcal{E}_{fR}\, R_{fR}$$

Because Y_f^{mixed} comes from the completion polar stretch and Y_f^{Hess} comes from the bosonic closure Hessian, and because the completion strain X_H is defined independently of $\hat{H}_{cl}^{red}$, the two are not equal by construction. Their equality is the calculable test

$$P_Y^L \mathcal{Y}\star P_Y^R \stackrel{?}{=} P_Y^L \hat{H}_{cl}^{red} P_Y^R$$

which, when satisfied, implies CY3′ simultaneously for every sector. The master action may pass it, fail it, or reveal a controlled residue $\Delta_f = Y_f^{mixed} - Y_f^{Hess}$.

This test is the central internal-consistency claim of the completion sector, and its present evidential status must be stated plainly. It has not yet been exercised in any regime where it could fail. Every executed instance is identity-trivial: on CFB-1 and the scalar-stretch branch of §47G both sides equal I on the readout support, so the equality holds vacuously. On the hierarchical branches CFB-2 and CFB-3, only the completion-route operator Y_f^{mixed} is computed; Y_f^{Hess} — the Schur-reduced closure-Hessian compression — is not evaluated, so the two routes are never compared where they could differ. Until Y_f^{Hess} is computed on a branch with nondegenerate spectra and compared against Y_f^{mixed}, CY3′ remains an untested equality: the completion sector's central internal-consistency claim currently has **zero discriminating evidence** — not weak evidence, none — because the only cases run are those in which the equality cannot fail. This is recorded as F9 (§56) and as an unexecuted item in the remaining ledger (§55).

Part VI — Recovery of the Six Sector Gates

38. Scalar-Sector Recovery

The scalar gate required the momentum-dependent substrate Hessian. It is supplied by §25: $\Pi_H^{scalar}(p) = n_H^\dagger \hat{H}_{red}(p) n_H$, with $(m_{(H,curv)}^2, Z_H)$ its low-momentum coefficients and $\mu_H^2 = m_{(H,curv)}^2/Z_H$. No scalar residue is inserted.

39. Neutral-Branch Recovery

The neutral gate required the neutral quadratic operator and carrier census. They are supplied by §34A: M_R from the symmetric right compression and M_L^{irr} from the two-interface symmetric active compression, both compressions of the same completion operator on the frozen neutral census.

40. Yukawa-Sector Recovery

The Yukawa gate proved non-identifiability of J_C, J_R, P_Y^micro, the chiral embeddings and the dressed bilinear. All five are supplied: J_C, J_R as the closure and record currents; P_Y^micro as the calculated readout support of §33; the embeddings $\mathcal{E}$_fX as the polar factors of §34; and the dressed bilinear as Γ_LR$^{(0)}$ of §32. The two Yukawa routes of §35 and §37 and their equality test replace the previously missing dressing.

41. Gauge-Sector Recovery

The gauge gate retained an underived response metric, embedding and auxiliary descent. All three are supplied: the response metric as the compression of Ĥ_red (§26); the embedding as the differentiated faithful representation landing in the tensor carrier (§47H); and the auxiliary descent as the common Schur reduction (§24). The physical couplings follow by the inherited conversion.

42. QCD-Sector Recovery

The QCD gate proved that the five wheel modes are pure-gauge and cannot supply the proposed screening. The master action supplies the required intrinsic intertwiner through $\mathcal{D}$_B, whose mixed Hessian block is nonzero while exact coboundaries remain null (§47A). The physical colour stiffness is the Z_3 curvature entry, with no census identification (Firewall M3).

43. Global-Measure Recovery

The global-measure gate required an explicit finite-regulator rate functional. The master action supplies the **candidate density architecture and evaluation address** for the global measure — the form Γ_master$^{(n)}$ = Γ_bos$^{(n)}$ + Γ_ferm$^{(n)}$ + Γ_top$^{(n)}$ and the finite-regulator quantum measure

$$[\![\ d\mu\_VERSF^{(n)} = Z\_n^{-1}\ e\hat{}(-\Gamma\_master^{(n)})\ s\_ferm^{(n)}\ d\nu\_quotient^{(n)}\ ]\!]$$

on the complete faithful-group orbit stack, with the fermionic Pfaffian sign s_ferm$^{(n)}$ and the quotient volume dν_quotient$^{(n)}$. This is the architecture and the address, not the evaluated measure: the actual measure is returned only after W_prim, the history kernel and the determinant-line data are physically derived. On the positive-pushforward branch the absolute normalisation is fixed by the history measure (MA8), and the strong-phase consequence is stronger than reality: a positive pushforward excludes both generic complex phases and the signed $\bar{\theta} = \pi$ case, so **positive pushforward ⟹ $\bar{\theta}$ = 0 or a global obstruction** (§52.7). The construction of the rate functional itself is the gate F19, now narrowed by §10.1 and §10.1.1: its consumer (W_prim as Fisher curvature of the Doob kernel, plus the null quotient and per-step normalisation) is derived and demonstrated, and the refinement operator is supplied in a proposed HMGBC-1 package with HP1 proved and HP2 conditional; the residual debt is the finite-regulator verification of that package and the uniqueness of g_n.

44. Cross-Sector Consistency Tests

Because one action generates all sectors, the following equalities are tests, not definitions: (1) the vacuum used by the scalar residue equals the vacuum transporting fermions; (2) the link fixing gauge curvature equals the link transporting matter; (3) the auxiliary census relaxing the scalar equals the one relaxing the gauge and completion; (4) Y_f^{mixed} equals Y_f^{Hess} (CY3′); (5) the measure weights the same configurations whose Hessians are computed. Each may fail.

44A. Master Ward–Response Identities

Gauge invariance of Γ_{master} implies exact discrete Ward identities relating the current-response block, the bond block and the gauge-tangent compression. Schematically, $\sum_A T_A^\dagger (\partial \mathbb{G}^\star / \partial \theta_A) = 0$ on the physical block, so the gauge-response matrix Z_g is transverse and the derived couplings are frame independent. These identities are consequences of Appendix A covariance and provide an internal check on any numerical Hessian: a computed $\mathbb{G}^\star$ that violates transversality beyond the certified tolerance fires F5.

Part VII — Executable Programme and Physical Evaluation

45. Frozen Finite-Cell Background

The executable programme uses the canonical $K = 7$ wheel cell as the smallest closure-compatible arena carrying nontrivial holonomy, record composition and terminal addresses. The background links, records, currents and metrics are frozen before evaluation and recorded in the manifest of Appendix G. No background entry may be adjusted after a response is read.

46. Required Input Matrices

To evaluate the common Hessian, the following frozen objects are required: the background link $U^\star$ on every edge; the background record maps $\Phi^\star$; the background current $J^\star$; the operational metric W_{prim}; the support projectors Π_i; the Higgs-normal embedding n_H; the generator embedding $\{T_A\}$; and the address maps $A_{fX}^{(0)}$. Each is fixed by the regulator geometry and the frozen realisation maps, and each is listed with its hash in the freeze manifest.

47. Common Hessian Computation

The common Hessian is assembled as $\mathbb{G}\star = G\_V + D\mathbf{D}\star\dagger\ W\_prim\ D\mathbf{D}\star$, quotiented by the null space $\mathcal{N}$, and Schur-reduced on the auxiliary census to $\hat{H}\_red$. The following subsections execute successive branches of this computation, from the first isolated bond-response test to fully concrete finite-regulator candidate branches.

47A. First Executed Master-Action Branch — Canonical Wheel Link–Record Response

The bond-compatibility mechanism is tested in isolation on the wheel in one adjoint-generator channel. The frozen background sets $\mathcal{D}\_B = 0$, so the branch tests only the second variation. The mixed link–record block returns

$$⟦\ H\_uy = -c\ I_{12}\ ⟧$$

with exactly six null directions equal to the common gauge-frame modes, and the full Hessian spectrum on the frozen unit branch is $\{0^6, 1, 2^8, 3, 4^4, 5, 6^2, 7\}$. The exact reduced stiffness along the cycle spectrum $\{1, 2, 2, 4, 4, 5\}$ is

$$⟦\ k\_red(\lambda) = a\lambda + bc\lambda/(b\lambda + c)\ ⟧$$

Result. The mixed block is nonzero (the intrinsic intertwiner exists), while exact gauge coboundaries remain null (the frame directions cost nothing). This is the load-bearing pass: the mechanism the gauge and QCD gates require is operative, not merely typed. The branch uses placeholder unit weights and therefore returns no Standard Model coupling; it certifies mechanism, not magnitude.

47B. Factor-Projected Wheel Response and First Gauge-Coupling Return

Projecting the branch through the frozen D_6-isotypic factor sectors returns exact factor-separated wheel responses

$$⟦\ (K_3,\ K_2,\ K\_q)\text{^}wheel = (24/5,\ 35/6,\ 3/2)\ ⟧$$

with exact colour isotropy. This reduces the entire remaining coupling calculation to three internal projections π_A of one fourteen-channel embedding, and returns the programme's first executed negative. Converting to the unit-isometric lift, the Abelian-to-weak stiffness ratio is

$$⟦\ Z\_Y > Z_2 \iff \pi\_q/\pi_2 > 140\ ⟧$$

The unit-isometric lift gives $\pi\_q/\pi_2 = 1$ and the matter-census diagnostic gives ≈ 58, both far below 140. The physical branch must therefore satisfy $\pi\_q/\pi_2 > 140$ or fire F26. This is a sharp, falsifiable numerical target extracted with no free parameter.

47C. Exact Internal-Norm Operators, Factorisation Certificate and Conditional Target Triples

The three internal projections are constructed exactly. The Casimir-spectral projectors are $P_3 = C_3/3$ and $P_2 = C_2/2$ on the internal carrier; the embedding-column projectors are

$$[\![P_A = \mathcal{E}_A (\mathcal{E}_A^\dagger \mathcal{E}_A)^{-1} \mathcal{E}_A^\dagger]\!]$$

taken as the primary construction, with the Casimir projectors serving as an independent route-agreement check. The internal norms are the Gram traces

$$[\![\pi_A = (1/d_A) \mathrm{Tr}(\mathcal{E}_A^\dagger W_{int}^{red} \mathcal{E}_A)]\!]$$

Before any π_A is quoted, the separability certificate $\varepsilon_A^{sep} = \| [P_A, W_{int}^{red}] \| / \| W_{int}^{red} \|$ must vanish to tolerance; if it does not, the factorised trace is not admissible and the direct nonfactorising Z_A of §47D must be used instead.

Census benchmark and conditional targets. The matter-census triple on the 47-dimensional carrier is $(\pi_3, \pi_2, \pi_q) = (6, 13/2, 378)$, with $\pi_q = 378 = \sum q^2$ and ratio $756/13 \approx 58.15$. This is a calculated diagnostic benchmark, not a stiffness identification (Firewall M3). The conditional target triples that the physical calculation must decide among are mutually exclusive; the colour entry of the two leading candidate resolutions differs by exactly a factor of six, so the coming calculation cannot satisfy both and must select. The candidate resolutions recorded are (5/4, 39/28, 279) and (5/24, 0.405, 205.2).

47D. Generating the Response Package from the Master Action

Supplying the physical fourteen-channel matrix is reduced to a mechanical export. Two bare canonical fourteen-channel metrics are given explicitly in the frozen basis: the stationary metric W_{14}^{stat} and the Dirichlet metric Q_{14}^{Dir}. The physical reduced matrix is the auxiliary-Schur-reduced pullback of the master Hessian along fourteen frozen tangent vectors,

$$[\![H_{14}^{red} = E_{14}^\dagger \mathbb{G}^\star E_{14} - (\text{auxiliary Schur correction})]\!]$$

where $E_{14} = [e_1 \ldots e_{14}]$ are the frozen tangent vectors. The twelve gauge columns are derived by generator differentiation, $E_A = \partial_{(\theta_A)} \mathfrak{X}^\star$, not by allocation, and the gauge-active profile space is returned as a three-dimensional calculation. The internal embedding matrix E_int and its reduced metric W_{int}^{red} are exported by the same pipeline, so the π_A of §47C become traces of a produced object rather than inserted numbers.

47E. The Frozen Branch Package and Enforced Generation Protocol

The freeze package makes the frozen object be data, basis, conventions and code together, bound by one hash manifest. The generator refuses to run on unfrozen, tampered, non-Hermitian, indefinite, negative-mode or rank-deficient inputs, emitting a certified refusal rather than a number. It has two modes: a separable-benchmark mode, which emits effective internal norms only under the explicitly declared wheel-factorisation assumption and labels them as such; and an authoritative physical_direct mode, which performs the nonfactorising evaluation and is the one whose output may enter a boundary certificate. The complete manifest schema, the freeze and generation source, and the eighteen-test acceptance log are reproduced verbatim in Appendix G; all eighteen tests pass, and the log was produced by exactly the source reproduced there. The freeze manifest is also the locus at which a candidate branch's constitutive graph choice must be registered and hashed before the solve; a branch that hashes only its outputs, not its graph choice, is pre-registered on honor-system only, and §47M.1 records exactly where the current branches stand on this. What this package certifies is plumbing, not physics: several of its acceptance tests (T4, T14, T17) are planted-value recovery checks — a known answer is injected and the pipeline must return it — which validate the machinery, not any Standard Model claim, and the package does not contain the CFB stationary solvers (§55).

47F. The Scalar-to-Completion-Stretch Defect

The sixth defect ties the completion stretch to the Higgs modulus. On the target carrier,

$$\llbracket\ \mathcal{D}_H(i) = X_{(H,i)} - (\varphi_H/\Lambda\star)\ P_H^R\ \rrbracket$$

where P_H^R is the representation-frozen right scalar-access projector and P_H^L its left partner, intertwined by the polar orientation V_H. At stationarity on the minimal branch, $X_{(H,\star)} = (\varphi_{(H,\star)}/\Lambda\star)\ P_H^R$, so the previous zero-readout obstruction on homogeneous vacua — where a completion strain defined on spatial transport would have had to vanish to preserve composition — is removed, because the stretch now lives on the local completion morphism (§4, §31). This defect is conditional on Premise MA9 (isotropic linear law) or, on generation-resolved branches, MA9′; a nonlinear or non-isotropic history-derived law would enlarge $\mathcal{D}_H$ as a declared-order revision, not a fit.

47G. First Executed Scalar-Stretch Branch

The scalar-stretch mechanism is executed on the pre-registered rank-nine charged carrier. It returns

$$\llbracket\ \operatorname{rank} P_Y = 9, \quad Y_u = Y_d = Y_e = I_3\ \rrbracket$$

with exact mass–vertex linearity $M_f = (v_{cl}/\sqrt{2})\, Y_f$, a positive scalar–stretch Hessian, and an exactly vanishing isolated CY3′ residue. Masses now exist — the previous zero-Yukawa obstruction is removed — but they are degenerate and unhierarchical on the unit branch: all three charged sectors return the identity, so no generation splitting or mixing appears. This is a genuine pass of the existence question and a null result on the structure question. The CY3′ residue vanishes here only because both routes return I on the readout support, so the test is not exercised nontrivially (§37, F9).

47H. Corrected Gauge-to-Response Embedding

The corrected embedding lands in the tensor carrier

$$\mathcal{V}_{int} = \mathcal{H}_{14} \otimes \mathrm{HS}(\mathcal{H}_{SM})$$

rather than mapping twelve generators into fourteen channels. The generator embedding is derived by differentiating the frozen faithful representation, $T_A = \partial_{\theta_A} R(\theta)|_0$, embedded into $\mathrm{HS}(\mathscr{H}_{SM})$ and paired with the fourteen-channel profile. This resolves the earlier dimension mismatch and removes any hand allocation of channels to factors.

47I. Executed Tensor-Embedding Branch

The tensor embedding is executed at shape 30926×12 with exact rank 12, exact factor orthogonality and exact isotropy. It derives the internal norms

$$(\mathfrak{n}_3, \mathfrak{n}_2, \mathfrak{n}_q) = (6, 13/2, 378)$$

precisely the matter-census triple of §47C — now a calculated benchmark from the embedding rather than a diagnostic count. Because $\mathfrak{n}_q/\mathfrak{n}_2 = 378/(13/2) = 756/13 \approx 58.15 < 140$, this separable tensor branch still fails the declared lower electroweak branch (F26). The failure is now localised: the required Abelian enhancement cannot come from the separable embedding and must come from the nonfactorising part of the common Hessian.

47J. Revised Gate Verdict

The executed branches localise all remaining physics in the nonfactorising parts of one common Hessian: the bond differential $D\mathcal{D}_B$, the completion differential $D\mathcal{D}_H$, the cross blocks W_{BH}, W_{CH}, W_{RH}, and the inverse auxiliary block $H_{\eta\eta}^{-1}$ on the physical background. Every separable, factorised or unit-isometric shortcut has now been executed and has either passed as mechanism or failed as magnitude. What remains is not a search for a new term; it is the evaluation of the entangled blocks of the action already written down. This is the content of §§48–53.

47K. First Fully Concrete Finite-Regulator Execution — CFB-1

The first fully concrete candidate branch fixes every constitutive object explicitly and solves the entire chain end to end. It is a 226-variable, 286-residual six-defect action on the canonical wheel, with an isotropic unit completion law and a stationary operational metric.

Solve. The stationary solve reaches a common defect zero at $\|R\star\| = 3.6\times10^{-13}$. The full 226×226 Hessian has 39 numerically null modes and 187 positive modes.

Reduction. Spectral quotient of the 39 null modes, complete auxiliary Schur relaxation and direct nonfactorising gauge pullback return the reduced gauge triple

$$(Z_3, Z_2, Z_q) = (4.849843136, 5.886806950, 1.518955532)$$

i.e. diag(4.8498, 5.8868, 1.5190) up to $\sim10^{-15}$ cross terms.

Electroweak test. The derived couplings are $1/\sqrt{Z_3} = 0.4541$, $1/\sqrt{Z_2} = 0.4122$ and $6/\sqrt{Z_q} = 4.868$, and the Abelian stiffness is $Z_Y = Z_q/36 = 1.518955532/36 = 0.042193$. Hence

$$\sin^2\hat{\theta}_W = Z_2/(Z_2 + Z_Y) = 5.886806950/(5.886806950 + 0.042193) = 0.99288$$

Since $Z_Y = 0.0422 \ll Z_2$, the Abelian stiffness is far too small — equivalently the hypercharge coupling $g' = Z_Y^{-1/2}$ is far too large (§26 convention) — and the branch fails the electroweak ordering.

Flavour and certificate. The flavour return is degenerate, $Y_u = Y_d = Y_e = I_3$, and no boundary certificate is issued. CFB-1 is published as a candidate-branch failure: the computation runs end to end, and the simplest fully frozen branch does not produce the Standard Model. Earlier development implementations of this branch returned differing gauge triples; because those runs were not shown to share one complete source-and-data manifest, that difference is not evidence of physical branch uncertainty or of a non-isolated zero locus, only of the need for source-level reproduction under exact manifest equality (§20.1). The triple reported here is the one produced by the frozen manifest of this execution.

47L. Second Concrete Execution — CFB-2: Non-Isotropic Completion

The second branch adds a graph-resolvent non-isotropic completion law on the K7×T4 arena, with 340 variables and 512 residuals. It reaches an exact defect zero, $\|\mathbf{D}\star\| = 0$.

Gauge return. The reduced triple is diag(5.4194, 6.5753, 1.9998), with a local Abelian enhancement $\gamma\_q = \sqrt{7002}$ arising from the representation fourth-moment amplitude. That enhancement is screened by the auxiliary relaxation by a factor of ~3500, so the physical Z_q returns to ≈ 2 and the branch still fails the electroweak ordering. The decisive lesson is structural: even a representation-derived fourth-moment Abelian amplitude of √7002 is screened away by the auxiliary block, so the electroweak deficit is auxiliary-structural, not a matter of local amplitude.

Scalar and flavour. The branch returns the first calculated scalar residue Z_H = 2.6553 with m^2_(H,curv) = 0.7532, hierarchical singular-value spectra σ in all four sectors, and an exact 15° two-three CKM plane (a single-plane rotation). This is genuine hierarchy and mixing — but only the completion-route Yukawa Y_f^mixed is computed; Y_f^Hess is not, so CY3′ is not exercised.

Neutral. The neutral sector returns a full 6×6 spectrum with M_R = 2Y_ν. No boundary certificate is issued; CFB-2 is a candidate branch.

47M. Third Concrete Execution — CFB-3: Fourth-Moment Metric and Full Mixing

The third branch derives the bond and record metric itself from the same representation fourth-moment rule, rather than only the completion amplitudes. It is a 388-variable, 605-residual action with a fourth-moment metric topology.

Metric. The derived metric weights are $(w_3, w_2, w\_q) = (5/2, 21/8, 7003)$, and the reduced gauge block is

```
⟦ G_red = diag(6, 115/16, 7005/2) ⟧
```

so $Z_3 = 6$, $Z_2 = 115/16 = 7.1875$ and Z_q = 7005/2 = 3502.5. The Abelian stiffness is now Z_Y = Z_q/36 = 97.29, so

```
⟦ Z_Y > Z₂ ⟧
```

for the first time in the programme, and the lower electroweak ordering gate F26 passes. The weak mixing returns $\sin^2\theta = Z_2/(Z_2 + Z\_Y) = 7.1875/(7.1875 + 97.29) = 0.0688$, i.e. ≈ 0.069.

This ordering pass must be read with three honesties held together. First, the qualitative ordering $Z\_Y > Z_2$ is correct but the *magnitude* is not close: $\sin^2\theta = 0.069$ against the physical $\sin^2\theta\_W \approx 0.23$ is off by more than a factor of three, and the charged CP invariant J_CKM = 0.0073 overshoots the physical $J \approx 3\times10^{-5}$ by more than two orders of magnitude. The miss is a large quantitative failure, not a residual normalisation. Second, "only the normalisation remains" would therefore be a misreading: what is demonstrated is capacity to reach the correct *ordering*, not proximity to the correct *values*, and this phrase must not travel outside the capacity-versus-prediction framing of §47M.7. Third, the large weights 7002/7003/7005 that drive the Abelian entry are representation fourth moments *asserted* into the metric; their specific values are not derived from the substrate axioms in this paper. Compared to CFB-1 and CFB-2, whose Abelian stiffness was too small, CFB-3 overshoots — Z_Y too large, g′ too small — so across the three branches the sign of the miss flips with the choice of amplitude, which is the signature of a hand-tuned knob, not of a converging derivation. The deficit that remains is therefore not merely the history normalisation of the metric; it is the derivation of the metric itself, which is deferred to HMGBC-1.

Flavour and CP. The branch returns four hierarchical sectors and, for the first time, all three CKM mixing angles together with a nonzero charged CP invariant

```
⟦ J_CKM = 0.00727984 ⟧
```

Scalar and neutral. The scalar residue is Z_H = 2.3489, and the neutral sector returns six Takagi masses. No boundary certificate is issued; CFB-3 is a candidate branch.

47M.1 Pre-Registration Status of CFB-3

The metric restriction of CFB-3 is not a set of three hand-tuned numbers: the weights (w_3, w_2, w_q) are the three isotypic restrictions of one common fourth-moment candidate operator W_prim$^{(4)}$ = I + $\mathbb{M}_4$, and the Schur-forcing that makes each per-factor restriction scalar is representation-covariant, so no coefficient is fitted to an observable and Firewall M5 is not violated on that count. This is the firmer part of the branch, but it must not be overstated. W_prim$^{(4)}$ is a candidate, not the physical metric: the operator $\mathbb{M}_4$ is not explicitly defined on the complete primitive residual carrier — its domain, its off-diagonal blocks, its action on the inert directions, its positivity and its covariance relative to the six defect carriers are not all supplied here. CFB-3 should therefore be read as *using one common fourth-moment candidate metric*, not as *deriving the physical metric*. The physical object is proposed downstream in HMGBC-1 as W_prim = 𝕀_F[K_n], where the metric is the information form of a derived transfer kernel rather than an assumed fourth-moment topology.

The completion graphs are a different matter, and it must be stated squarely. CFB-2 and CFB-3 hash their outputs; they do not demonstrate that the constitutive graph choice was itself frozen and hashed before the solve. The pre-registration claim for the generation-resolving graphs is therefore honor-system, and it is recorded here as such. This limitation is load-bearing, not cosmetic: the capacity-versus-prediction argument of §47M.7 depends on the generation graphs being genuinely blind, and an honor-system pre-registration is weaker than a manifest-frozen one. Future branches must freeze their constitutive graph choices under the §47E manifest before the solve, so that the pre-registration is certified rather than asserted. Both F17 and Premise MA9′ carry this cross-reference, so the three places a skeptic would look — the branch execution, the falsifier and the constitutive premise — agree on the current status.

47M.7 Capacity Versus Prediction

The CFB-1 → CFB-2 → CFB-3 sequence generates, in order, degeneracy, then hierarchy plus one mixing plane, then full three-generation mixing with CP violation and correct electroweak ordering. It is tempting to score this as accumulating success. It must be scored the opposite way, and it is not even a clean sequence. Each later branch reaches more Standard Model structure precisely because it carries more hand-set structure — a non-isotropic completion law (CFB-2), then an asserted fourth-moment metric (CFB-3) — and CFB-2 and CFB-3 also change the arena from the bare wheel to K7×T4, with different variable counts and residual definitions. The three are therefore not comparable points on one convergence curve; they are three differently-tuned machines on two different cells. "The later ones look more like the Standard Model" is thus a statement about how many knobs were turned, not about evidence accumulating for the construction.

An engine that can be driven to produce hierarchy, mixing and CP violation — and, by other choices of its internal completion graph and metric, equally to produce degeneracy, wrong orderings and vanishing CP — is demonstrating capacity, not prediction. Each qualitative Standard Model feature appeared because a constitutive graph or metric rule was chosen to make it appear. The construction has, at this stage, proved that the master action has the expressive capacity to contain the Standard Model's flavour and gauge structure; it has simultaneously proved that this capacity alone predicts nothing, because the same capacity contains the non-Standard-Model alternatives equally.

The flexibility is therefore a liability to be discharged, not an achievement to be celebrated. A fundamental theory earns a prediction only when a principle upstream of the branch choice selects one constitutive rule uniquely. The only candidate for that principle in this programme is the history-rate functional of MA1/MA8 (§10, F19), which no executed branch has constructed — and which §10.1.1 now supplies only as a *proposed* HMGBC-1 heat-transfer package, conditional on the single g_n-uniqueness-and-regularity gate, not as a derived-and-verified object. Until that gate is settled and the functional shown to return a specific W_prim and a specific completion law, every CFB result is a demonstration of what the action can be made to do, not of what it does.

Stated at full strength: at the present stage MASD-1 has relocated the Standard Model's free parameters, not removed them. It has moved them out of "one coefficient per sector" and into "choice of operational metric and completion graph." Whether this relocation is a genuine reduction depends entirely on whether the history measure selects those functions uniquely. That is an open question, and it is the question on which the entire derivational claim of the paper turns.

47M.8 Next-Step Ledger

The concrete next steps that would move any CFB branch from capacity toward prediction are: (1) close F19 by verifying, numerically across two regulator levels, that the actual coarse maps are minimal-fibre Riemannian submersions with basic measure, and settling the uniqueness of the operational metric g_n — the refinement operator R_n = ζ_n V_n e^(−a_t L_n) and the theorems HP1/HP2 are already supplied by §10.1.1, so the residual F19 debt is this verification, not a construction; (2) derive the completion graphs R_f of Premise MA9′ from the common address geometry rather than choosing them, and freeze them under the §47E manifest before the solve, discharging the honor-system status of §47M.1; (3) compute Y_f^{Hess} on a hierarchical branch and compare it against Y_f^{mixed}, exercising CY3′ nontrivially and discharging F9; (4) resolve the CFB-1 reduced-triple instability of §20.1, either by proving response-constancy along the zero locus or by a derived selection principle, discharging F28. None of these four is closed. All are concrete and executable.

48. Frozen Physical Background and Evaluation Admission Gate

Admission Rule 48.1 — Physical Evaluation Admission

A branch may enter the physical boundary-certificate protocol of §§49–53 only if it satisfies, before any output is read: (1) every constitutive object — link, current, record, completion, metric, graph — is frozen and hashed under the §47E manifest, including the generation graph choice; (2) the six-defect residual reaches a certified common zero with a stable spectral gap (§16.1, F18); (2b) every null direction removed in the §22 quotient is analytically classified as gauge, endpoint-frame, record-coboundary, or a derived flat completion direction, with a published catalogue exhausting the numerical null count — an uncatalogued null mode blocks admission, because the reduced operator $\hat{H}_{red}$ is defined by the quotient and an unidentified physical flat direction would alter the Schur complement and hence every reported response; this clause is not satisfied by any branch in this paper; (3) the auxiliary block $H_{\eta\eta}$ is certified positive on the complete census (§24, F8); (4) no object in the branch was fixed using a Standard Model observable (Firewall M5). A branch failing any clause is a candidate branch and may report results only under the candidate label, as CFB-1, CFB-2 and CFB-3 do. This rule is a protocol admitting a computation, not a proved theorem about the substrate.

49. Full Six-Defect Differential and Operational Cross-Metric

The physical evaluation requires the complete Jacobian $D\mathbf{D}^\star = (D\mathcal{D}_C, D\mathcal{D}_J, D\mathcal{D}_T, D\mathcal{D}_R, D\mathcal{D}_B, D\mathcal{D}_H)$ at the frozen background, and the full 6×6 induced cross-metric $W_{XY} = \iota_X^\dagger W_{prim} \iota_Y$ with every off-diagonal block published, not assumed zero. The bond and completion differentials $D\mathcal{D}_B$ (Appendix B) and $D\mathcal{D}_H$ are the nonfactorising blocks localised in §47J. The publication ledger lists each block, its rank, its norm and its separability residue.

50. Common Physical Hessian and Nonfactorising Reduction

50.1 Assembly

The physical Hessian is $\mathbb{G}\star = G_V + D\mathbf{D}\star^\dagger W_{prim} D\mathbf{D}\star$, assembled with the full cross-metric of §49.

50.2 Quotient and Schur

The null space $\mathcal{N}$ is removed (§22) and the complete auxiliary census is Schur-relaxed (§24) to $\hat{H}_{red}$, with the cross blocks W_{BH}, W_{CH}, W_{RH} and the inverse auxiliary block $H_{\eta\eta}^{-1}$ carried exactly.

50.3 Separability residues

For each gauge factor A, the separability residue $\varepsilon_A^{sep} = \|[P_A, W_{int}^{red}]\|/\|W_{int}^{red}\|$ is published. Only if it vanishes to tolerance may the factorised trace π_A be quoted; otherwise the direct nonfactorising Z_A is used (§47C, §47D).

50.4 Branch-uncertainty ledger

Before any branch-uncertainty entry is meaningful, the null-mode catalogue of Admission Rule 48.1(2b) must be complete: an unclassified null mode is not a source of quotable uncertainty but a source of undefinedness in $\hat{H}_{red}$, and error bars on an undefined operator are not admissible. The 39 CFB-1 null modes are uncatalogued (§20.1); consequently the CFB gauge triples of §§47K–47M are reported as branch-diagnostic returns of a not-yet-physical reduction, and no physical weight may be placed on their specific values pending that catalogue.

Every reduced response is published with a branch-uncertainty entry recording its variation across certified refinements of one and the same frozen manifest — same residual source, variable ordering, quotient, auxiliary basis and reduction code. Variation across unmatched development implementations is not a branch-uncertainty entry and may not be quoted as one; it is a reproducibility failure, addressed by source-level reproduction, not by an error bar. A nonzero branch-uncertainty that does not shrink under manifest-equal refinement is evidence for F28 and must be reported alongside the central value, never suppressed.

51. Physical Scalar and Gauge Boundary Return

51.1 Scalar

The scalar boundary returns $(Z_H, m^2_{(H,curv)})$ from §25, hence $\mu_H^2 = m^2_{(H,curv)}/Z_H$ and the physical relaxed quartic $\lambda(\mu\star)$, which includes the auxiliary Schur correction of the full vertex tower and is distinct from the bare substrate quartic λ_{sub}^{bare} of §19A.

51.2 Gauge

The gauge boundary returns (Z_3, Z_2, Z_q) from §26 after the separability check of §50.3, hence $g_s = Z_3^{(-1/2)}$, $g = Z_2^{(-1/2)}$, $g' = 6\, Z_q^{(-1/2)}$, each with its branch-uncertainty entry.

52. Physical Flavour, Neutral, Colour and Strong-Phase Return

The flavour boundary returns Y_u, Y_d, Y_e from the mixed route (§35) and, once computed, Y_f^{Hess} from the Hessian route (§37), with the CY3′ residue Δ_f published. The neutral boundary returns M_R and M_L^{irr} (§34A). The colour boundary returns Z_3 (already in §51.2) and the triality-wall coefficient B_τ (§28). The strong-phase boundary returns $\bar{\theta}$ from the inherited combined-line calculation on the positive-pushforward branch. Each entry carries its branch-uncertainty and separability ledger.

53. MASD-1 Physical Boundary Certificate

53.1 The boundary set

The physical boundary certificate is the frozen table

$$[\![\ \mathcal{P}\_SM^{VERSF}(\mu\star) = \{\ g_s,\ g,\ g',\ \mu_H^2,\ \lambda,\ Y_u,\ Y_d,\ Y_e,\ \mathcal{B}_\nu,\ \theta^- \ \}\ ]\!]$$

where $\mathcal{B}_\nu = (M_R, M_L^{irr})$ is the neutral block. The certificate is issued only when every entry is a frozen numerical return of an admitted branch (Admission Rule 48.1), with full uncertainty and branch ledgers.

53.2 Strict ledger

Each entry is published with: its central value; its branch-uncertainty across manifest-equal refinements of one frozen background; its separability residue where applicable; the hash of the frozen background that produced it; and its falsifier status. No entry may be a benchmark, placeholder or experimentally borrowed value.

53.3 Certificate status

No boundary certificate has been issued. CFB-1, CFB-2 and CFB-3 are all candidate branches under §48.1, but for branch-specific reasons, and the graph objection does not apply uniformly. CFB-1 is nonphysical because it uses a declared benchmark metric and an isotropic completion law, is reduced through a spectral quotient rather than a derived one, and has no history provenance; it does not use a generation-resolved graph in the CFB-2/3 sense, so its inadmissibility is a metric/provenance matter, not a graph-freeze matter. CFB-2 and CFB-3 additionally use generation-resolving completion graphs that are pre-registered on honor-system only, not frozen and hashed under the §47E manifest before the solve (§47M.1), and their operational metrics are candidate objects rather than history-derived. The boundary table is therefore empty of frozen physical returns, and the issuance of a certificate awaits the history-derived metric and branch of HMGBC-1 and the frozen execution of HFB-1.

53.3A CFB-1 candidate ledger

$(Z_3, Z_2, Z_q) = (4.8498, 5.8868, 1.5190)$; $\sin^2\theta_W = 0.99288$ (fail); $Y_u = Y_d = Y_e = I_3$ (degenerate); Z_H not returned; differing earlier development returns are a reproducibility item, not a branch-uncertainty (§20.1); reduction rests on the uncatalogued null space of §20.1 — triple is diagnostic, not a physical response; reason nonphysical: declared benchmark metric, isotropic completion law, spectral quotient and missing history provenance — not a generation-graph objection; certificate: not issued.

53.3B CFB-2 candidate ledger

$(Z_3, Z_2, Z_q) = (5.4194, 6.5753, 1.9998)$ after $\sqrt{7002}$ screening (fail, auxiliary-structural); $Z_H = 2.6553$; hierarchical σ four sectors; 15° 2–3 CKM plane; $M_R = 2Y_\nu$; Y_f^{Hess} not computed (F9); reduction rests on the uncatalogued null space of §20.1 — triple is diagnostic, not a physical response; certificate: not issued.

53.3C CFB-3 candidate ledger

$(Z_3, Z_2, Z_q) = (6, 115/16, 7005/2)$; $Z_Y > Z_2$ (F26 ordering passes), $\sin^2\theta = 0.069$ (magnitude overshoot); three CKM angles; $J_{CKM} = 0.00727984$; $Z_H = 2.3489$; six Takagi neutral masses; generation graphs honor-system, not manifest-frozen (§47M.1, F17); Y_f^{Hess} not computed (F9); reduction rests on the uncatalogued null space of §20.1 — triple is diagnostic, not a physical response; certificate: not issued.

Admission Rule 53.1 — Parameter-Closure Admission

The Renormalised Standard Model Parameter-Closure paper may not open until $\mathcal{P}_{SM}^{VERSF}(\mu\star)$ is fully populated by frozen returns of a single admitted branch. On the corrected programme order that admitted branch is produced not by MASD-1 but by its two successors: HMGBC-1 derives the history operational metric, the universal generation geometry and the branch selector, and HFB-1 executes the selected physical branch through §§48–53 and issues the certificate. Parameter Closure consumes that certificate. No benchmark, placeholder or experimentally borrowed value counts toward this condition. Assembly of physical parameters is transport of frozen microscopic returns, not rescue by fitted inputs. This is a protocol on the programme's workflow, not a theorem about the substrate.

53.4. History-Derived Boundary Handoff

The final programme order is

$$[\![\text{Six sector gates} \to \text{MASD-1} \to \text{HMGBC-1} \to \text{HFB-1} \to \text{Renormalised Parameter Closure} \to \text{Final Derivation Audit}]\!]$$

MASD-1 defines and tests the common machine. HMGBC-1 derives its history metric, universal generation geometry and physical branch. HFB-1 executes that derived machine and issues the microscopic boundary certificate. The Renormalised paper transports the certificate; it does not complete it.

The candidate branches CFB-1, CFB-2 and CFB-3 demonstrate that the master-action architecture is executable and expressive. They do not determine the physical microscopic boundary, because their operational metrics, completion laws or branch choices are declared candidate inputs. The physical handoff begins only after HMGBC-1 returns, at finite regulator, the three history-derived objects

$$[\![W\_\text{prim}^{(n)} = \mathbb{I}\_F[\mathbb{K}\_n]]\!]$$

the information form of an explicit positive substrate transfer kernel $\mathbb{K}\_n$;

$$[\![R\_\Omega^{(n)} = (I + \mathcal{L}\_\Omega^{(n)})^{-1}]\!]$$

from one universal terminal-record completion geometry; and

$$[\![\mathfrak{B}\_\text{phys}^{(n)} = \text{argmin}\_(\beta \in \mathfrak{B}\_\text{adm}^{(n)})\ f\_\beta^{(n)}, \quad f\_\beta^{(n)} = -(1/a\_t) \log \lambda\_(0,\beta)^{(n)} = -(1/2a\_t) \log(Z\_(n+1,\beta)/Z\_(n,\beta))]\!]$$

the branch of least Perron free-energy density of the same transfer law. The two written forms are equal because the Perron eigenvalue of the raw positive factor is $\lambda\_(0,\beta) = \zeta\_\beta = \sqrt{(Z\_(n+1,\beta)/Z\_(n,\beta))}$ (§10.1.1); the factor of two is absorbed by that square root, and there is no normalisation ambiguity. The sector completion responses must then be compressions of the one universal completion geometry,

$$[\![R\_f^{(n)} = S\_fL^\dagger\ V\_(H,\star)^\dagger\ R\_\Omega^{(n)}\ S\_fR, \quad f = u, d, e, \nu]\!]$$

with no independent sector graph, magnetic flux, factor weight or completion matrix admissible at this stage. This is the object that replaces, and is stronger than, the fourth-moment candidate metric of CFB-3 (§47M.1): the metric is the information form of a derived transfer kernel, and the generation geometry is one universal resolvent, not a hand-chosen family. HFB-1 uses these returns to solve the common master action, construct the exact physical quotient, perform the complete auxiliary Schur reduction, and issue the first history-derived microscopic boundary package.

53.5. The HFB-1 Completion Certificate

The complete HFB-1 handoff package is

```
⟦ 𝔥_HFB1⁽ⁿ⁾ = { 𝕂_n, π_n, W_prim⁽ⁿ⁾, Δ_Ω⁽ⁿ⁾, β_phys⁽ⁿ⁾, f_β⁽ⁿ⁾,
                𝔖_scale⁽ⁿ⁾, P_phys⁽ⁿ⁾, G_red⁽ⁿ⁾, Ẽ₃⁽ⁿ⁾, Ẽ₂⁽ⁿ⁾, Ẽ_q⁽ⁿ⁾,
                Z_H⁽ⁿ⁾, v_cl⁽ⁿ⁾, B_τ⁽ⁿ⁾, Ĥ_cl^(red,n),
                P_Y^(L,n), P_Y^(R,n), R_fX⁽ⁿ⁾, ℰ_fX⁽ⁿ⁾,
                𝒫_SM^VERSF(μ⋆), Σ_uncertainty⁽ⁿ⁾, 𝒞_certificates⁽ⁿ⁾, 𝔥_source⁽ⁿ⁾
} ⟧
```

where π_n is the stationary history state; β_phys$^{(n)}$ is the data-independently selected branch; 𝔖_scale$^{(n)}$ is the complete primitive scale ledger (§19A); Σ_uncertainty$^{(n)}$ is the numerical, history, auxiliary, regulator, branch and premise ledger; 𝒞_certificates$^{(n)}$ contains the Ward, CY3′, support, polar, quotient, isotropy, wall and determinant-line certificates; and 𝔥_source$^{(n)}$ contains the immutable hashes of the data, conventions and source code. The package must be published at no fewer than two regulator levels, n and n+1, together with the blocking map connecting them.

53.5.1 Mandatory boundary table

The HFB-1 certificate must contain, with no blank or benchmark-only row:

Boundary object	Numerical return	Scheme and scale	Microscopic provenance	Certificates
g_s	required	(μ⋆, 𝔖⋆)	history-derived direct $\mathcal{K}_3$	positivity, isotropy, Ward
g	required	(μ⋆, 𝔖⋆)	history-derived direct $\mathcal{K}_2$	positivity, isotropy, Ward
g′	required	(μ⋆, 𝔖⋆)	history-derived direct 𝒦_q	positivity, Abelian normalisation
μ_H^2	required	(μ⋆, 𝔖⋆)	scalar two-point kernel	momentum and regulator convergence
λ	required	(μ⋆, 𝔖⋆)	relaxed fourth scalar vertex	auxiliary and scheme ledger
Y_u	full matrix	common canonical frame	mixed and Hessian routes	CY3′ and rank
Y_d	full matrix	common canonical frame	mixed and Hessian routes	CY3′ and rank
Y_e	full matrix	common canonical frame	mixed and Hessian routes	CY3′ and rank
ℬ_ν	full block	common canonical frame	common neutral completion	Takagi, active/sterile, PMNS

Boundary object	Numerical return	Scheme and scale	Microscopic provenance	Certificates
θ̄	required	determinant-line convention	gauge, Yukawa and line contributions	branch-stable combined phase

The accompanying non-boundary certificate must also return Z_H, v_cl, B_τ, P_Y^(L,R), R_fX, 𝓔_fX and Ĥ_cl^red.

53.5.2 Nontrivial CY3′ requirement

The hierarchical physical branch must publish both routes,

```
⟦ Y_f^mixed = R_fL 𝓔_fL† 𝒴⋆ 𝓔_fR R_fR ⟧
```

```
⟦ Y_f^Hess = R_fL 𝓔_fL† P_Y^L Ĥ_cl^red P_Y^R 𝓔_fR R_fR ⟧
```

the universal residue

```
⟦ Δ_univ^CY3′ = P_Y^L 𝒴⋆ P_Y^R − P_Y^L Ĥ_cl^red P_Y^R ⟧
```

and the normalised residue

```
⟦ ε_CY3′ = ‖Δ_univ^CY3′‖ / max( ‖P_Y^L 𝒴⋆ P_Y^R‖, ‖P_Y^L Ĥ_cl^red P_Y^R‖ ) ⟧
```

A hierarchical completion-route result without the corresponding Hessian route does not satisfy this gate. This is exactly the gate that no branch in this paper has passed (F9): every executed instance is identity-trivial or computes Y_f^mixed alone.

The nontrivial CY3′ comparison must be executed on the physically selected hierarchical branch β_phys returned by HMGBC-1 (§53.4), not on an arbitrary hierarchical candidate. A pass of Y_f^mixed = Y_f^Hess on some hand-chosen hierarchical branch would test the machinery on a non-physical wiring and would not license the physical certificate; the equality is discriminating only where the branch is itself derived. Until β_phys exists and both routes are computed on it, CY3′ carries zero discriminating evidence (§37) and its certificate value is NOT_EXECUTED (Appendix H), which by Admission Rule 53.2(7) blocks the Renormalised paper regardless of any other returns.

53.5.3 Full strong-phase requirement (typed)

The physical strong phase is a single invariant, not a sum of three freely chosen numbers, and the decomposition below is a typing of one object, not an assertion of three independent physical phases. It must be published with each term's precise meaning and with an explicit non-double-counting certificate.

The invariant. The physical, basis-independent strong phase is

$$[\![\ \bar\theta = \theta_{top} + \arg\det(\mathbb{M}_u\,\mathbb{M}_d) \pmod{2\pi}\ ]\!]$$

where θ_{top} is the coefficient of the topological density in the substrate measure (the analogue of the QCD vacuum angle, returned by the determinant-line/positive-pushforward structure of §43, §52) and $\mathbb{M}_f = (v_{cl}/\sqrt{2})Y_f$ are the physical mass operators. This two-term invariant is the only physical content; any further term must be shown to be zero or to be a rewriting of these two.

What the determinant-line construction is. The determinant line is the framework in which $\arg\det(\mathbb{M}_u\,\mathbb{M}_d)$ is globally well defined across the faithful-group orbit stack — it fixes the reference section against which the determinant phase is measured. It is therefore not an additive third phase. The earlier three-term writing $\bar\theta = \theta_{gauge} + \arg\det(Y_u\,Y_d) + \theta_{line}$ is admissible only under the identification $\theta_{gauge} \equiv \theta_{top}$, $\arg\det(Y_u\,Y_d) \equiv \arg\det(\mathbb{M}_u\,\mathbb{M}_d)$ (the $(v_{cl}/\sqrt{2})$ factors are real and carry no phase), and $\theta_{line} \equiv$ the global-section correction that makes arg det single-valued on the stack — in which θ_{line} is not a residual holonomy to be added but the bookkeeping that makes the determinant term itself well defined. On a trivial determinant line (no global obstruction) $\theta_{line} = 0$ by construction, and the invariant reduces to the standard two terms.

The three admissible fates of θ_{line}, and the certificate. θ_{line} must be classified into exactly one of:

1. **Zero** — the determinant line is trivial (globally trivialisable section); then $\bar\theta = \theta_{top} + \arg\det(\mathbb{M}_u\,\mathbb{M}_d)$ with no third term. This is the generic expected case.
2. **A global anomaly obstruction** — the determinant line is nontrivial and admits no global section; then θ_{line} is not a number to add but a statement that arg det is not globally single-valued, and $\bar\theta$ is defined only relative to a declared section, with the obstruction class published. This is the "global obstruction" alternative already named in F24.
3. **A genuine residual physical holonomy** — distinct from both θ_{top} and the determinant phase. This case is forbidden unless accompanied by a proof that the holonomy is not already contained in θ_{top} or in $\arg\det(\mathbb{M}_u\,\mathbb{M}_d)$. Absent that proof, case 3 is treated as a double-counting error, not a physical phase.

Non-double-counting certificate (mandatory). The strong-phase row is populated only when the branch publishes: (i) θ_{top} computed from the topological sector of the measure alone, with the Yukawa sector set to a real reference; (ii) $\arg\det(\mathbb{M}_u\,\mathbb{M}_d)$ computed from the Yukawa determinants alone, on the fixed determinant-line section; (iii) an explicit demonstration that θ_{line} falls into fate 1 or fate 2 above — i.e. that no phase information is counted in both (i) and (ii). A returned $\bar\theta$ that requires a nonzero fate-3 θ_{line} without the accompanying independence proof fails the certificate and does not populate the boundary table.

Consistency with F24. Positive pushforward requires the physical invariant $\bar\theta \in \{0, \text{global obstruction}\}$ (§43, F24); this is a condition on the two-term invariant above, and it is what excludes both generic complex phases and the signed $\bar\theta = \pi$ case. The typing here is what makes F24 checkable: without separating θ_{top}, the determinant phase and the section bookkeeping, "$\bar\theta = 0$" cannot be verified without risking that a hidden phase has merely moved between terms.

53.5.4 Triality requirement

The wall coefficient must be evaluated on a replicated family,

$$B_\tau = \liminf_{A_\Gamma \to \infty} \left[\inf_{\mathcal{C}_\tau(\Gamma)} \Gamma_{master} - \inf_{\mathcal{C}_0} \Gamma_{master} \right] / A_\Gamma$$

with at least three wall areas and two regulator levels published.

53.6. Renormalised-Paper Input Contract

The Renormalised Standard Model Parameter-Closure paper is a transport theorem. Its input is exactly one completed package $\mathfrak{h}_{HFB1}^{pass}$. Its permitted operations are the native-substrate-to-declared-continuum scheme bridge, the $\mu\star \to \mu$ coupled renormalisation-group running, threshold matching, electroweak symmetry breaking, running masses and pole extraction, composed as

$$\mathcal{O}_{physical} = \mathcal{P}_{pole} \circ \mathcal{E}_{EWSB} \circ \mathcal{M}_{threshold} \circ \mathcal{U}_{RG} \circ \mathcal{S}_{bridge} [\mathfrak{h}_{HFB1}^{pass}]$$

where $\mathcal{S}_{bridge}$ converts the frozen substrate scheme to the declared renormalisation scheme; $\mathcal{U}_{RG}$ performs the coupled running; $\mathcal{M}_{threshold}$ performs decoupling and matching; $\mathcal{E}_{EWSB}$ constructs the broken-phase mass and mixing objects; and $\mathcal{P}_{pole}$ extracts physical poles and observables. The paper is forbidden to: (1) change W_{prim}; (2) change Δ_Ω or any sector selector; (3) choose another branch; (4) rescale a gauge embedding; (5) insert a missing Yukawa entry; (6) set v, λ, a threshold or a matching scale from experiment; (7) remove a failed Ward, CY3′, wall or determinant-line certificate; (8) select a renormalisation prescription because it improves agreement. A failed microscopic return remains a failed return after transport, and a discrepancy between the transported VERSF return and experiment is a physical result of the programme, not permission to modify the input.

Admission Rule 53.2 — HFB-1 Completion Rule

The Renormalised Standard Model Parameter-Closure paper may begin numerical evaluation only if: (1) an explicit transfer kernel $\mathbb{K}_n$ has been published; (2) W_{prim} has been calculated from its information geometry; (3) all sector completion responses descend from one Δ_Ω; (4) the physical branch has been selected by the transfer law; (5) the exact null quotient has been published; (6) the complete HFB-1 boundary table contains no blank or benchmark-only row; (7) CY3′ has been evaluated nontrivially on the hierarchical branch; (8) $\bar{\theta}$ is published as the two-term physical invariant $\theta_{top} + \arg\det(\mathbb{M}_u \mathbb{M}_d)$, with the determinant-line section declared, θ_{line} classified into fate 1 or 2 of §53.5.3, and the non-double-counting certificate passed — a fate-3 residual without an independence proof fails this condition; (9) B_τ has been returned from a large-area family; (10) at least two regulator levels have passed the declared convergence criteria; (11) all numerical inputs and source code are covered by one immutable handoff manifest. Failure of any condition leaves the next paper architecturally writable but numerically closed. Like Admission Rule 53.1, this is a programme-workflow protocol, not a theorem about the substrate.

Part VIII — Closure

54. What This Paper Defines

This paper defines: one common finite-regulator master action $\Gamma\_master^{(n)}$ over independent primitive variables (ρ, U, J, Φ; Ψ, Ψ̄); six explicit primitive defects with one induced operational cross-metric; a universal Gauss–Newton reduction to one common physical Hessian; the scalar, gauge, closure, colour and wall projections of that Hessian; an independent completion-strain fermion bilinear with derived readout projectors, chiral embeddings and two independent Yukawa routes joined by CY3′; a finite-regulator quantum measure on the faithful-group orbit stack; a complete physical execution protocol with an admission gate and a boundary certificate; and three fully executed concrete candidate branches. It proves the exact mathematics (Grade M1) and defines the conditional constitutive theorem (Grade M2).

55. What Remains to Be Calculated

The remaining work falls into three honestly distinct categories, and they are not interchangeable.

Category A — Implemented but not physically validated. The bond mechanism (§47A), the factor projection (§47B), the internal-norm operators (§47C), the response-package pipeline (§47D–§47E), the scalar-stretch existence branch (§47G), and the tensor embedding (§47I) are all executed and pass their stated internal tests. They are validated as mechanism and as construction, not as physics: none returns a Standard Model number, and each uses a frozen unit or benchmark input in place of the physical one. "Closed as construction" for these items means executed as written, not physically confirmed.

Category B — Defined but underived. The history-rate functional W_prim and the completion law (MA1, MA8, §10) are defined by their evaluation address; their entire consumer is constructed and demonstrated (the Fisher-metric route of §10.1); and the refinement operator itself is proposed as inherited heat transfer, R_n = ζ_n V_n e^(−a_t L_n), in a proposed HMGBC-1 theorem package, with HP1 proved, HP2 proved conditional on geometric premises whose physical validity is open, and the six defect-derivatives reduced to operational-Dirichlet-form data (§10.1.1). The executed CFB branches still freeze W_prim by hand because they predate this construction, not because it is missing. The residual gap is no longer a missing object: it is the numerical verification that the actual coarse maps satisfy the minimal-fibre/submersion/basic-measure regularity, and the uniqueness of the operational metric g_n (F19, F20). The primitive scale tuple 𝔖_scale (§19A) is derived only conditionally on that functional. The generation graphs R_f (MA9′) are chosen, not derived (§47M.1). Until Category B is discharged, the framework is a structured reparametrisation, and its physical outputs are candidate, not predicted (§47M.7).

Category C — Defined but never executed. The CY3′ physical test (§37) has never been exercised in a regime where it could fail: Y_f^Hess is uncomputed on every hierarchical branch, so the equality Y_f^mixed = Y_f^Hess is untested — its status is *not executed*, which is distinct from *failing* (F9). The nonfactorising physical Hessian evaluation of §§48–53 has not been run on any admitted branch; no boundary certificate exists (§53.3). The genericity of the common six-defect zero locus (§20.1) is neither proved nor executed to resolution; it is open and must be tested by manifest-equal refinement, and the differing early CFB-1 returns are a reproducibility item, not evidence about it (F28). The full analytic classification of the 39 CFB-1 null modes is also outstanding.

No item may be moved from Category B or C to Category A by relabelling. Discharging B and C is the entire remaining physical content of the programme.

Reproducibility status of the candidate branches. A separate honesty point, orthogonal to the three categories: the freeze-and-generation package of Appendix G covers the *response-generation* pipeline (the fourteen-channel pullback, the Schur reduction, the certificate guards), but it does **not** contain the complete CFB-1, CFB-2 and CFB-3 stationary solvers, their residual-map definitions, their automatic-differentiation graphs, or their frozen background data. Until those complete branch packages are released under immutable manifests on the same footing as the response package, the CFB numerical results reported in §§47K–47M and §53.3A–C are *reported executions*, not yet *independently reproducible calculations*. This does not weaken their status as executed candidate-branch failures — the point of each is a negative or a candidate return, not a physical certificate — but it does mean that reproduction currently requires trusting the reported runs rather than re-deriving them from published source. Releasing the CFB solvers under immutable manifests is a required deliverable before any CFB return could be promoted beyond "reported execution."

56. Falsification Conditions

- **F1** — A new independent primitive bosonic defect at the declared order enlarges the master action beyond six terms. **Status: live.** The ledger already grew from five to six (§47F, MA2); "exactly six" is a conjecture that has been falsified once and revised, so a seventh defect is a real possibility, not an excluded one.
- **F2** — The typed defect embeddings carry unspent continuous scale freedom, so MA1 is not closed.
- **F3** — A required cross-defect response cannot be produced by W_prim and must be inserted.
- **F4** — The bond mixed Hessian vanishes, so no intrinsic intertwiner exists (contradicted on the wheel, §47A).
- **F5** — The computed $\mathbb{G}^\star$ violates gauge transversality beyond tolerance (§44A).
- **F6** — A frame coboundary acquires nonzero stiffness, so a null direction is mispriced.

- **F7** — An auxiliary mode is omitted from the census, effectively inserting a constraint.
- **F8** — The auxiliary block $H_{\eta\eta}$ fails positivity on the retained census, so the background is not a stable vacuum.
- **F9** — CY3′ returns a calculated nonzero residue beyond tolerance on an admitted branch, so the two independently generated Yukawa routes disagree. This is the *failure* condition, and it is distinct from the *test's execution status*. **CY3′ physical test status: not executed** — Y_f^{Hess} has not been computed on a nondegenerate branch, so $Y_f^{mixed} = Y_f^{Hess}$ has never been exposed to genuine failure; every executed instance is identity-trivial or computes only the completion route (§37, §55C). "Not executed" is not "failing": F9 fires only on a calculated disagreement, which cannot occur until the test is run. The honest status is that the completion sector's central consistency claim has zero discriminating evidence to date.
- **F10** — The two Yukawa routes disagree beyond the published residue on an admitted branch.
- **F11** — The readout rank is selected to match a required number of masses rather than calculated (Firewall M4).
- **F12** — A defect weight is fixed using a mass, coupling, angle, QCD scale or wall tension (Firewall M5).
- **F13** — The fermion bilinear is defined through the desired Hessian compression (Firewall M1).
- **F14** — The gauge link is constructed as an endpoint-only coboundary (Firewall M2).
- **F15** — The colour stiffness is identified with the matter census index (Firewall M3).
- **F16** — Mass–vertex linearity $M_f = (v_{cl}/\sqrt{2})Y_f$ fails beyond the published completion nonlinearity.
- **F17** — A generation-resolved completion graph R_f is chosen to produce a desired spectrum and its pre-registration is not certifiable under the freeze manifest. **Locus:** the honor-system status of the CFB-2/CFB-3 graphs (§47M.1); the graphs are pre-registered by assertion, not by manifest-freeze, so this falsifier's premise is currently only partly guarded.
- **F18** — No stable spectral gap separates occupied and empty record supports, so Π_i is not well defined.
- **F19** — The history-rate functional W_{prim} of §10 cannot be constructed, or does not return a specific metric, so MA1/MA8 have no evaluated content. **This is the load-bearing gate:** every executed branch freezes W_{prim} by hand, so the derivational claim of the whole paper is conditional on this falsifier not firing. **Status: narrowed to finite-regulator evaluation of an explicit construction (§10.1, §10.1.1).** The Fisher-metric route derives the entire consumer of the rate functional and demonstrates it on the $K = 7$ cell ($I_{\tau\tau} = 0.395\ldots$); HMGBC-1 then proposes the refinement operator as inherited heat transfer — a posited identification, not a derivation — $R_n = \zeta_n V_n e^{(-a_t L_n)}$, shows its polar factor is the operational heat semigroup, and discharges terminal-cone positivity (HP1) and record-basis reversibility (HP2) as theorems, with the six defect-derivatives reduced by Duhamel to derivatives of the operational Dirichlet form. F19 no longer awaits a missing construction; the residual debt is (a) numerical verification that the actual coarse maps are minimal-fibre Riemannian submersions with basic measure across two regulator levels, and (b) uniqueness of the operational metric g_n. Named as THE gate in the abstract, the verdict and the conclusion.

- **F20** — Any member of the primitive scale tuple $\mathfrak{S}_{\text{scale}}$ cannot be returned, leaving the corresponding dimensional prediction open. Depends on F19.
- **F21** — The substrate/record realisation of §8A retains continuous freedom after metric and symmetry are fixed, so the carrier geometry is not closed.
- **F22** — The neutral block M_L^{irr} is not order-suppressed when the calculation returns it, contradicting the required active hierarchy.
- **F23** — The measure fails quotient-compatibility on the faithful-group stack, so $d\mu_{\text{VERSF}}$ is ill defined.
- **F24** — On the positive-pushforward branch the strong phase is not $\bar{\theta} = 0$ (or a demonstrated global obstruction). Positive pushforward is stronger than reality: it excludes both generic complex phases and the signed $\bar{\theta} = \pi$ case, so a returned $\bar{\theta} \notin \{0,$ global obstruction$\}$ fails the branch.
- **F25** — The fermionic Pfaffian sign s_{ferm} is not well defined or not continuous under refinement.
- **F26** — The physical electroweak branch fails $\eta_q/\eta_2 > 140$, so the Abelian stiffness Z_Y is too small relative to Z_2 (equivalently g' too large; §26 convention). **Status:** the separable/unit/census/tensor branches all fire this (ratios 1, ≈58); CFB-1 and CFB-2 fire it; CFB-3 passes the ordering but overshoots the magnitude — its Z_Y is too *large*, i.e. g' too small — so the miss changes sign, not just size (§47B, §47I, §47K–§47M).
- **F27** — The triality-wall coefficient B_τ is negative or zero, so no confining wall tension exists.
- **F28** — The common six-defect zero locus carries continuous physical moduli that change the reduced response under manifest-equal refinement, so the boundary certificate is not a function of the frozen inputs alone. **Status: open, untested, and the likeliest single point of failure for the boundary certificate.** It must be tested by refining one and the same frozen manifest, not by comparing unmatched development runs; the differing early CFB-1 returns are a reproducibility item and are not evidence for or against this falsifier (§20.1, §50.4). The full analytic classification of the 39 CFB-1 null modes (§20.1) is a *hard prerequisite*, not a formality: until every null direction is shown to be gauge, frame or a catalogued flat completion mode, the reduction of §24 rests on an unverified premise, and a hidden physical modulus would make the §53 boundary certificate a function of an unfixed choice rather than of the frozen inputs. Of all open falsifiers this is the one most likely to break the reduction, because it attacks the premise the whole descent presupposes.
- **F29** — The admissible branch set is not finite and isolated: the history-derived metric, completion law and admissibility rules fail to collapse the continuous constitutive freedom (positive-metric cone, completion families, holonomy and background moduli) to a finite set, so branchwise determinacy cannot be claimed. **Status: open, and open in two separable senses.** (a) *Per-branch:* whether each admissible transfer has an isolated dominant eigen-branch (coupled to the Perron gap of §10.1.1). (b) *Cross-branch:* whether all branches embed in one common history operator with one reference measure, so that their Perron data are comparable at all (§57A.1). Resolution of (a) does not resolve (b). The Perron selection of HMGBC-1 must discharge both; a green (a) with an open (b) is selection among incommensurable models, which is no selection.

57. Non-Insertion Protocol

No measured Standard Model boundary value is numerically inserted into the displayed candidate inputs: no coupling, mass, mixing angle, CP phase, QCD scale, wall tension or scalar residue appears as an entered number. Every such quantity is defined as a projection, compression or trace of one common Hessian, or as a return of the history measure. The readout ranks are calculated; the embeddings are polar factors; the scale tuple is (conditionally) derived from the rate functional.

This must not be overstated into "no free parameter anywhere." The candidate branches nevertheless retain constitutive freedom in their operational metrics and completion graphs, and those objects — not any entered number — determine the spectra, mixing and CP structure. The candidate branches therefore demonstrate capacity rather than parameter-free prediction: the freedom has been relocated from entered coefficients into the choice of metric and graph (§47M.7), and it is removed only when the history measure selects those objects uniquely (HMGBC-1). The freeze package of §47E and Appendix G enforces the *no-entered-value* discipline mechanically — its generator refuses inputs that would violate it — but a green freeze suite certifies that discipline, not parameter-free prediction.

57A. Branch Resolution and Determinacy

Branch-Publication Requirement 57A

Finiteness of the admissible branch set is **not** established here, and the earlier "finite branch ledger theorem" is withdrawn. At the declared order the defect ledger is fixed at six (MA2), but the remaining constitutive freedom is continuous, not finite: positive operational metrics form a continuous cone; completion operators form continuous families; graph weights and background holonomies vary continuously; and stationary backgrounds may form continuous moduli. Pre-registration constrains *when* a choice is fixed; it does not make a continuous choice space finite. No product-of-finitely-many-choices argument is therefore available, and none is claimed.

What is required instead is a publication discipline:

Every executed branch must be published in full with its frozen manifest, and the programme may claim finite branchwise determinacy only after the history-derived metric, completion law and admissibility rules are shown to return a finite, isolated branch set. Until that is shown, finiteness is an open condition, and its failure — a positive-dimensional or continuous admissible branch family that the measure does not collapse — is a falsifier (F29: the measure fails to select a finite isolated branch set).

57A.1 The common-history-space precondition on Perron selection

Branch selection by least Perron free-energy density (§53.4) is only well posed if the competing branches are components of one history operator, not separately constructed models. A Perron eigenvalue is an invariant of a single irreducible transfer operator; comparing the Perron data of two independently defined operators — as CFB-1, CFB-2 and CFB-3 currently are, with different arenas, metrics and completion laws — compares differently normalised objects and carries no selection content. The apparent selector $\mathrm{argmin}_\beta f_\beta$ is therefore not yet a physical selector.

The precondition that makes it one is a single theorem, and it is hereby made an explicit HMGBC-1 deliverable, not an assumed property:

Common-Branch-Measure Obligation. There exists one master substrate history operator $\mathbb{T}_{hist}$ on one common history space $\mathscr{H}_{hist}$, of which every admissible branch β is a component (an invariant subspace, sector, or boundary condition), such that: (i) all branch weights $Z_{(n,\beta)}$ and normalisations descend from one common reference measure on $\mathscr{H}_{hist}$; (ii) the branch free energies f_β are the block Perron exponents of that one operator; and (iii) the physical branch is the unique dominant component. Only under (i)–(iii) is $\mathrm{argmin}_\beta f_\beta$ a comparison of eigen-branches of one operator rather than of separate models.

Until this obligation is discharged, F29 remains fully open even if every individual branch exhibits its own Perron gap: a per-branch gap establishes internal stationarity, not cross-branch selection. The two are distinct, and the paper does not conflate them. A per-branch gap is necessary for selection to be well conditioned — it is the F19/F28 coupling of §10.1.1 — but it is not sufficient for selection to occur.

The correct place to settle this is the Perron selection of HMGBC-1: if the history transfer kernel has a unique dominant (Perron) eigen-branch, the admissible physical branch is isolated, and finiteness is earned rather than assumed. Selection among branches is the work of the history measure (F19); this requirement only ensures that no branch is silently preferred and that finiteness is treated as a claim to be proved downstream, not a theorem asserted here.

58. Fenced Lemmas and the Conditional Master-Action Determination

A sharp distinction must be drawn between what this paper proves and what it conditionally asserts, because the two have been run together in earlier drafts.

Genuinely proved (Grade M1 mathematics). The following are proved mathematics, independent of any constitutive premise: the gauge covariance of all six primitive defects (Appendix A); the Gauss–Newton reduction $\mathbb{G}\star = G_V + D\mathbf{D}\star^\dagger W_{prim} D\mathbf{D}\star$ at a common zero (Theorem 21.1); the exact Schur complement $\hat{H}_{red}$ (Theorem 24.1); the polar, logarithmic and support differentials and their coboundary/intrinsic split (Appendix B); the metric-induced cross-defect structure (Theorem 9A); the common-carrier realisation typing (Theorem 8A); and the tensor-embedding rank and orthogonality (§47H–§47I). These stand regardless of whether the constitutive premises hold. Finiteness of the branch ledger is NOT among the proved results: it was claimed as a theorem in an earlier draft and is withdrawn (§57A), because the admissible constitutive freedom is continuous; finiteness is an open condition to be earned by the Perron selection of HMGBC-1.

Not theorems — protocols and conditionals. The following are labelled correctly as what they are, not as proved theorems. Admission Rule 48.1 and Admission Rule 53.1 are protocols governing when a computation may be admitted and when the next paper may open; they are workflow rules, not statements about the substrate. The Universal Defect-Rate Principle (MA1) and the Measure/Action Matching (MA8) are constitutive premises with an evaluation address, not derived results.

Conditional Master-Action Determination (Conditional Claim 58). *If* the history-rate functional of §10 exists and returns a specific W_prim and completion law (i.e. F19 does not fire), *and if* the six-defect zero locus is response-rigid (F28 does not fire), *then* the master action determines the boundary set 𝒫_SM^VERSF(μ⋆) as a function of the frozen regulator geometry alone, with no inserted Standard Model parameter. This is a conditional, and both antecedents are currently open. It is stated as a conditional determination, not proved as a determination theorem, because its load-bearing hypotheses are exactly the unconstructed objects of Category B and the unresolved locus of Category C.

58A. Strengthened Constitutive Descent

Subject to the same two antecedents, the descent is strengthened by the following internal checks, each of which is itself either proved (M1) or executable: the Ward–response transversality of §44A; the separability certificate of §47C/§50.3; the CY3′ equality of §37; and the branch-uncertainty rigidity of §50.4. Passing all four on an admitted branch would upgrade Conditional Claim 58 from conditional to executed for that branch. None has yet been passed on an admitted branch, because no branch is yet admitted.

59. Closure Certificate

Caveat header. "Closed as construction" below means proved as Grade M1 mathematics or executed as written in code, not physically validated, and every item is subject to the relocated-freedom caveat of §47M.7: an executed construction that freezes W_prim or a completion graph by hand demonstrates capacity, not prediction. This list may not be quoted out of context as a claim that the Standard Model has been derived.

Closed as construction: the six-defect master action (§17); the common-carrier typing (§8A); the metric-induced cross-defect structure (§9); the Gauss–Newton and Schur reductions (§§21, 24); the completion-strain bilinear and its two Yukawa routes (§§32, 35, 37); the branch-publication requirement (§57A) — note: branch finiteness is NOT closed, only the publication discipline is; the bond-mechanism execution (§47A); the factor projection and its sharp F26 target (§47B); the internal-norm operators and separability certificate (§47C); the response-package pipeline and freeze protocol (§47D–§47E); the scalar-stretch existence branch (§47G); the tensor embedding (§47I); the three concrete candidate executions CFB-1/2/3 (§47K–§47M); and the physical execution protocol §§48–53.

Open: the history-rate functional and completion law (F19, F20, Category B); CY3′ nontrivial exercise (F9, Category C); the nonfactorising physical Hessian evaluation and boundary certificate (§53.3, Category C); the zero-locus genericity (F28, Category C); and the generation-graph derivation and manifest-freeze (F17, §47M.1).

Verdict. MASD-1 supplies the first common finite-regulator forward action for the six-gate final-completion programme, proves its exact reduction mathematics, executes every separable and benchmark mechanism it rests on, and runs three complete concrete candidate branches whose combined qualitative output covers hierarchy, mixing, CP violation and correct electroweak ordering. It does not derive the Standard Model, and it does not by itself make any branch physical: that is the work of its two successors, HMGBC-1 (which derives the operational metric, the universal generation geometry and the branch selector) and HFB-1 (which executes the selected branch and issues the certificate). The single object on which the derivational claim turns — the history-rate functional that would make W_prim and the completion law outputs rather than inputs (F19) — is not evaluated here, but its status is now substantially advanced rather than open: the Fisher-metric route derives and demonstrates the entire consumer of that functional (§10.1), and HMGBC-1 proposes the refinement operator as inherited heat transfer — a posited identification, not a derivation — $R_n = \zeta_n V_n e^{(-a_t L_n)}$, proving terminal-cone positivity and record-basis reversibility as theorems and reducing the six derivatives to operational-Dirichlet-form data (§10.1.1). The word "supplies" must be read at grade: HMGBC-1 is not yet a completed paper, the heat-flow identification $R_n = \zeta_n V_n e^{(-a_t L_n)}$ is a physical posit and not a theorem (§10.1.1), and neither the identification nor the metric g_n has been established. "Relocated, not removed" is therefore the operative verb throughout: MASD-1 has moved the freedom to a single named place, not discharged it. F19 is thereby moved from "is there even an object" to "does the explicit object satisfy its regularity conditions on the real regulator, and is its one input — the operational metric g_n — forced." Until that verification is done and g_n shown to be inherited rather than free, the paper has relocated the Standard Model's free parameters, now up to g_n and the coarse-map regularity, rather than eliminated them (§47M.7, §10.1.1). The construction is honest about this at every level: the proved lemmas are fenced from the conditional determination, and the withdrawn finite-branch theorem is now an open condition (§57A, §58); the CY3′ physical test is flagged as not executed — not failing — in three places (§37, §55, F9); the genericity gap has a section and a falsifier, with its 39 null modes counted but not yet classified and the differing early CFB-1 returns treated as a reproducibility item rather than as evidence (§20.1, F28); the flexibility of the CFB progression is scored as a liability, not a triumph (§47M.7); the CFB-3 metric is labelled a candidate, not the physical object (§47M.1); and the load-bearing gate is named as such in the abstract, here and in the conclusion. On those terms, and only those, the paper is complete as a construction and a protocol.

60. Conclusion

The six final-completion papers did not leave six separate holes. They left one: a common microscopic action from which every Standard Model sector must descend as a derivative, restriction or quotient of the same object. This paper writes that action down, at finite regulator, over independent link, current, record and completion variables, with six defects priced by one operational metric and no inserted sector coefficient. It reduces the action exactly to one common Hessian, projects that Hessian into every sector, and supplies an independent completion-strain route to the fermion masses joined to the Hessian route by a genuine internal test.

It then turns the first gears. The bond mechanism works; the factor projection returns exact wheel responses and a sharp falsifiable target; the scalar stretch produces masses; the tensor embedding reproduces the matter census; and three concrete branches run end to end, each generating more of the Standard Model's qualitative structure than the last.

None of this derives the Standard Model, and the paper says so without hedging. The engine has been shown to have the capacity to contain the Standard Model's flavour, gauge and CP structure, and — by the same freedom of internal wiring — the capacity to contain its alternatives. Capacity is not prediction. The prediction would begin only when the history-rate functional of the theory selects one wiring uniquely, and the status of that functional has now advanced markedly: its entire consumer is built and demonstrated, and the refinement operator it needs is supplied — not chosen — as the heat transfer of the inherited operational Dirichlet form, $R_n = \zeta_n V_n e^{(-a_t L_n)}$, with its two admissibility conditions proved as theorems (§10.1.1). What remains is no longer to build an object but to verify, on the real regulator, that the coarse maps have the required geometry and that the one surviving input — the operational metric — is itself forced by prior results. Until that is done, the free parameters have been relocated, not removed — now one level deeper still, up to that metric and that geometry.

MASD-1 is therefore not the immediate numerical input to Renormalised Parameter Closure. Its immediate consumer is HMGBC-1, which must turn the operational metric, completion geometry and branch label from declared candidate structures into returns of one substrate history law; HFB-1 then performs the first admitted physical execution. The handoff to the Renormalised paper is now exact: it is one immutable object, $\mathfrak{h}\_{HFB1}^{pass}$ (§53.5, Appendix H), containing the complete microscopic boundary, scale and scheme labels, branch provenance, uncertainty ledger and structural certificates. Once that object exists, no further microscopic choice is permitted; renormalisation becomes transport rather than rescue (§53.6, Admission Rule 53.2). At the close of this paper that object does not yet exist — its boundary fields are empty and its CY3′ certificate is not executed — which is the precise, machine-checkable statement of what remains.

What the paper does deliver is a finish line and a gatekeeper: a precise checklist of the frozen numerical returns required before the microscopic derivation may be declared complete, an admission rule forbidding the Renormalised Parameter-Closure paper from opening until that checklist is populated by real frozen outputs of a single admitted branch, and a falsifier set in which the one gate that decides everything is named as such. Those frozen returns are produced not here but in the two papers that follow: HMGBC-1 builds the one measure — the operational metric, the universal generation geometry and the branch selector — that would turn the master action's settings into a prediction, and HFB-1 executes the branch it selects. VERSF has reached the point where it must stop proposing separate reasons for different Standard Model numbers and instead build that measure. This paper defines the machine, demonstrates that its first gear turns, states plainly the one thing still missing, and hands that thing to HMGBC-1.

Appendix A — Gauge Covariance of the Primitive Defects

Under a local internal transformation $G = \{G_i\}$, the primitive variables transform as $\rho_i \mapsto G_i \rho_i$, $U_{ij} \mapsto G_j U_{ij} G_i^{-1}$, $J_{ij} \mapsto G_j J_{ij}$, $\Phi_{ij} \mapsto G_j \Phi_{ij} G_i^{-1}$. Each defect is covariant term by term.

Closure. $H_p \mapsto G_{b(p)} H_p G_{b(p)}^{-1}$ at the base event $b(p)$, so the vacuum-relative logarithm transforms by conjugation and $\|\mathcal{D}_C\|$ is invariant.

Current. $J_e - \kappa_e(\rho_t - U_e \rho_s) \mapsto G_j[J_e - \kappa_e(\rho_t - U_e \rho_s)]$, covariant by left action, so $\|\mathcal{D}_J\|$ is invariant.

Transport. The covariant incidence $(\partial_U J)_i \mapsto G_i(\partial_U J)_i$, and $\partial_\sigma \rho_i \mapsto G_i \partial_\sigma \rho_i$, so $\mathcal{D}_T \mapsto G_i \mathcal{D}_T$.

Record. Each factor transforms as $G_k(\cdot)G_i^{-1}$ on the composable triple, so $\Phi_{jk} \Phi_{ij} - \Phi_{ik} \mapsto G_k(\Phi_{jk} \Phi_{ij} - \Phi_{ik})G_i^{-1}$.

Bond. The Gram operator transforms as $G_i (\cdot) G_i^\dagger$, so the support projector obeys $\Pi_i \mapsto G_i \Pi_i G_i^{-1}$ (for unitary G_i), the supported map $M_{ij} \mapsto G_j M_{ij} G_i^{-1}$, and by uniqueness of the polar decomposition the polar isometry obeys $V_{ij} \mapsto G_j V_{ij} G_i^{-1}$. Both terms of $\mathcal{D}_B$ therefore transform as $G_j(\cdot)G_i^{-1}$.

Scalar-stretch. Under G_i the completion morphism obeys $\Phi_{(H,i)} \mapsto G_{(L,i)} \Phi_{(H,i)} G_{(R,i)}^\dagger$; the polar positive part transforms by G_R-conjugation, $\mathcal{D}_H : X_H \mapsto G_R X_H G_R^\dagger$ and $\mathcal{O}_H \mapsto G_L \mathcal{O}_H G_R^\dagger$, and $P_H^R \mapsto G_R P_H^R G_R^\dagger$, so $\mathcal{D}_H$ is covariant.

Consequently Γ_{bos} is invariant term by term, and every off-diagonal block of W_{XY}, being an inner product of two covariant residuals in the invariant operational metric, is invariant. ■

Appendix B — Differential of the Polar Factor

Let $M = B V$ be the polar decomposition on the rank-stable support, $B = (MM^\dagger)^{1/2} > 0$, V a partial isometry. The differential of the positive factor solves the Sylvester (Lyapunov) equation

$$\llbracket\ \delta B \cdot B + B \cdot \delta B = \delta M \cdot M^\dagger + M \cdot \delta M^\dagger\ \rrbracket$$

which has a unique solution for $B \succ 0$ on the support. The isometry differential is

$$\llbracket\ \delta V = B^{-1} (\delta M - \delta B \cdot V)\ \rrbracket$$

For the bond defect, the intrinsic content is isolated by the coboundary/intrinsic split of the record variation. Writing an infinitesimal gauge/frame change as $\delta M = i(\theta_j M - M \theta_i)$, the induced $\delta V = i(\theta_j V - V \theta_i)$ is a pure coboundary and cancels in $\mathcal{D}_B$ against the identical variation of $\Pi_j U_{ij} \Pi_i$; only the intrinsic part of δM — the part not of coboundary form — survives in the mixed Hessian. This is the mechanism by which frame directions stay null while support and bond-stretch changes cost action (§16.3, §47A).

The support projector differential is

$$\llbracket\ \delta\Pi = \Pi^\perp\ \delta G \cdot S_G + S_G \cdot \delta G \cdot \Pi^\perp\ \rrbracket$$

with S_G the reduced resolvent (Moore–Penrose inverse of $G - g_{occ}$ on the support), giving the bound $\|\delta\Pi\| \le (2/g_\star)\|\delta G\|$ where $g_\star$ is the certified spectral gap (§16.1). The logarithmic strain differential is $\delta X_H = \int_0^\infty (B+s)^{-1}\, \delta B\, (B+s)^{-1}\, ds$ in the symmetric-reference frame. ■

Appendix C — Master Hessian Block Ledger

The master Hessian on the slow/fast channels is published as a labelled block table. Each block records its source defects, its rank and its norm.

Block	Contributing defects	Publication requirement
$(\delta\rho, \delta\rho)$	V, T, J, B	radial + tangential rank
$(\delta\rho, \delta U)$	J, B	coboundary content certified null
$(\delta U, \delta U)$	C, B	gauge-transverse rank (F5)
$(\delta U, \delta\Phi)$	B	mixed bond block; nonzero required (F4)
$(\delta\Phi, \delta\Phi)$	R, B, H	record + completion rank
$(\delta J, \delta J)$	J, T	current-response positivity
$(\delta\Phi_H, \delta\Phi_H)$	H	scalar-stretch positivity (§47G)
$(\delta U, \delta\Phi_H)$	B, H	W_{BH} cross block
$(\delta\rho, \delta\Phi_H)$	H	W_{CH} cross block
$(\delta\Phi, \delta\Phi_H)$	R, H	W_{RH} cross block

The direct bond block (δU, $\delta\Phi$) must be nonzero; a vanishing mixed bond block fires F4 (contradicted on the wheel, §47A). Every off-diagonal block is an evaluated overlap in W_prim, not an inserted coupling (Theorem 9A).

Appendix D — Address and Embedding Algorithm

The chiral embeddings and kinetic-address radii are produced by a fixed eleven-step algorithm, executed after the background is frozen and hashed.

1. Assemble the local Gram operator G_i (§8A) including the terminal-record contribution, and certify the occupied-support spectral gap (F18).
2. Form the polar decomposition of every supported map and certify polar rank stability (F18).
3. Freeze and hash the raw address maps $A_{fX}^{(0)}$ before any response is evaluated (Firewall M1).
4. Compute the completion operator $\mathcal{O}_H = V_H^\dagger X_H$ and the scalar response $\mathcal{Y}_\star = \Lambda_\star[\mathcal{D}_{(\varphi_H)}\mathcal{O}_H]_\star$ (F24 sign/phase audit).
5. Compute the readout projectors P_Y^L, P_Y^R as the supports of $\mathcal{Y}_\star\mathcal{Y}_\star^\dagger$ and $\mathcal{Y}_\star^\dagger\mathcal{Y}_\star$ (Firewall M4); certify their coincidence.
6. Project the raw address maps onto the readout supports.
7. Polar-decompose each projected inclusion into $\mathcal{E}_{fX} R_{fX}$; certify rank 3 (F7/F8) per sector.
8. Certify $R_{fX} = I_3$ where it holds (F23), else record the calculated departure.
9. Form the primitive bilinear compression $B_{fX} = \mathcal{E}_{fX} R_{fX}$.
10. Certify commutant uniqueness of the embedding and the shared-doublet condition across up/down and neutrino/charged-lepton sectors.
11. Hash every produced object before any spectrum is inspected.

No step consults a Standard Model observable; the rank-3 certificates and the $R_{fX} = I_3$ certificate are outputs.

Appendix E — CY3′ Evaluation Package

A CY3′ evaluation is a publication of ten items, in this fixed order, on a single admitted branch.

1. The reduced closure Hessian $\hat{H}_{cl}^{red}$ (§27).
2. The completion polar pair (B_H, V_H) and the strain X_H.
3. The readout supports P_Y^L, P_Y^R and their ranks.
4. The kinetic-address factors R_{fL}, R_{fR} per sector.
5. The universal residue operator on $P_Y^L \hat{H}_{cl}^{red} P_Y^R$.
6. The four completion/Hessian operator pairs (Y_f^{mixed}, Y_f^{Hess}) for $f = u, d, e, \nu$.
7. The interface-linearity residues $M_f - (v_{cl}/\sqrt{2})Y_f$ (F16).

8. The singular values and determinant phases of each Y_f.
9. The regulator dependence of the residue under one refinement step.
10. The pass/residue/premise/architecture outcome statement.

No residue may be absorbed by a field redefinition, a rank change or a relabelling. If $\Delta_f = Y_f^{mixed} - Y_f^{Hess}$ exceeds the published tolerance, the outcome is a fail (F10), not a redefinition. A CY3′ package has not yet been published on any branch, because Y_f^{Hess} is uncomputed on every hierarchical branch (F9).

Appendix F — Dependency Map

Predecessor gates (inputs to MASD-1). The six sector-gate series — the electroweak/Higgs completion series (HR), the neutral-fermion carrier census (NFCC), the Yukawa completion (MCHY), the gauge-response gate (GRCE), the QCD running/confinement gate (QCR) and the strong-CP/global-measure gate (SGMC) — each identify a component of the missing common parent and hand it to MASD-1. They precede MASD-1; each is a consumer of a *projection* of the common Hessian only in the sense that MASD-1 supplies the object each gate had left open.

Successor papers (consumers of MASD-1's construction). MASD-1 → **HMGBC-1 → HFB-1** → Renormalised Parameter Closure → Final Audit. HMGBC-1 derives the operational metric $W_{prim} = \mathbb{I}_F[K_n]$, the universal generation geometry and the physical branch selector (its Perron selection is the correct place to settle branch finiteness, §57A); HFB-1 executes the selected physical branch through §§48–53 and issues the boundary certificate; Renormalised Parameter Closure consumes that certificate; the Final Audit checks the whole chain. MASD-1 is upstream of HMGBC-1 and HFB-1, not directly upstream of Parameter Closure.

Additional inputs (consumed by MASD-1). The faithful-group $\mathbb{Z}_6$ quotient structure; a candidate operational metric of the stationary substrate transport operator (the physical metric being derived only downstream in HMGBC-1); the CAR/Fock matter carrier; and the $K = 7$ closure architecture.

External mathematical tools. Polar decomposition; the Sylvester/Lyapunov equation; spectral (resolvent) perturbation theory; discrete exterior calculus on cellular complexes; Gauss–Newton and Schur-complement reduction; Perron–Frobenius theory for the stationary measure; the Kolmogorov extension theorem for the history measure; Ginsparg–Wilson/overlap constructions for the chiral matter carrier; Osterwalder–Schrader positivity for the reflection structure; and Mosco convergence for the continuum limit of the quadratic forms.

Appendix G — The Frozen Branch Package: Manifest, Tools and Source

This appendix reproduces the complete freeze-and-generation package of §47E, so that the paper is self-contained: a reader holding only this document can reconstruct the package byte-for-byte, freeze a branch and generate the fourteen-channel response. The distributed package and this appendix are generated from the same source; the executed test log below was produced with exactly this code.

G.1 Enforced ordering and the frozen object

The package implements the §57 protocol as software, and the frozen object is data + basis + conventions + code. The order is fixed: export the input arrays; run the freeze tool, which refuses unresolved branch labels and publishes a SHA-256 manifest covering every numerical input and every code, basis and convention file; then run the generator, which verifies every one of those hashes before computing anything. The generator refuses to run without a manifest, refuses if any frozen input or code file was modified after hashing, and refuses unmanifested input files. An explicitly unfrozen benchmark run is possible only through a dedicated flag, prints a warning, and labels every output with `frozen_run: false`. Output directories are excluded from the freeze hash.

Certificates enforced inside the generation, at the tolerances frozen in the manifest: the input Hessian must be the quotient-reduced physical parent (supplied directly, or computed inside the generator from an explicit quotient projector); Hermiticity is certified before any hermitisation, with residuals published for the parent Hessian, H_{14}^bare, H_{14}^red and every factor block; the auxiliary block must be positive definite; H_{14}^red must have no negative physical mode; the gauge embedding must have rank twelve at the frozen tolerance; each coupling's imaginary residual is audited; and effective internal norms π_A = Z_A/K_A^wheel are emitted only in the separable-benchmark mode when every isotropy and cross-factor certificate passes, under the status `defined_under_frozen_wheel_factorisation` — those certificates do not prove physical factorisation — otherwise `not_defined_nonseparable` with only the direct Z_A retained. The `physical_direct` mode (inputs G_red + $\tilde{E}_3$, $\tilde{E}_2$, Ẽ_q) computes the authoritative pullback $\mathcal{K}$_g = Ẽ†G_redẼ with no factorisation imposed and no effective norms.

G.2 The frozen channel basis (basis14.csv)

index	label	orientation	channel
0	h_plus	plus	h
1–6	b1_plus … b6_plus	plus	b1 … b6
7	h_minus	minus	h
8–13	b1_minus … b6_minus	minus	b1 … b6

Once frozen, rows and columns may never be silently reordered. Gauge multiplicity is not carried by these fourteen profile labels; it lives in the operator factor of the tensor carrier $\mathcal{V}$_int = $\mathscr{H}_{14} \otimes$ HS($\mathscr{H}$_SM) of §47H.

G.3 Input arrays and shapes

With N the dimension of the physical (quotient-reduced) tangent carrier and M the number of auxiliary directions:

File	Shape	Contents
inputs/physical_parent_hessian.npy	(N, N)	G_phys = P_phys $\mathbb{G}^\star$ P_phys, the quotient-reduced parent of §22
inputs/full_hessian.npy + inputs/P_phys.npy	(N_full, N_full), (N_full, N)	alternative: the generator applies G_phys = P_phys†G_full P_phys itself
inputs/E14.npy	(N, 14)	the fourteen master tangent vectors Ξ_m
inputs/Eeta.npy	(N, M)	complete auxiliary tangent basis (optional if M = 0)
inputs/E3.npy	(D, 8)	colour derivative embedding; D = 14 (channel) or D = 14·k (tensor, §47H, metric applied as H_{14}^red ⊗ I_HS)
inputs/E2.npy	(D, 3)	weak derivative embedding
inputs/Eq.npy	(D, 1)	primitive-hypercharge derivative vector
inputs/G_red.npy + Etilde3/2/q.npy	(M, M), (M, 8/3/1)	physical_direct mode: reduced physical Hessian and direct gauge embeddings on the full retained carrier
inputs/rho_star.npy, U_star.npy, J_star.npy, Phi_star.npy, W_prim.npy	—	the frozen background and common residual metric

The companion file `canonical_bare_matrices.npz` supplies π_7, W_{14}^stat, the exact Q_7 of §47D.2, and Q_{14}^Dir in both orientation-summed and orientation-averaged conventions, as pipeline benchmarks.

G.4 The freeze manifest schema (freeze_manifest.template.json)

```
{
  "schema_version": "1.0",
  "branch_label": {
    "vacuum_id": "FILL_ME",
    "support_branch": "FILL_ME",
    "polar_branch": "FILL_ME",
    "bundle_sector": "FILL_ME",
    "fermion_orientation": "FILL_ME",
    "refinement_branch": "FILL_ME"
```

```
  },
  "numeric_convention": {
    "dtype": "complex128",
    "endianness": "little",
    "hessian_prefactor": "1/2 residual norm convention",
    "generator_normalization": {
      "SU3": "tr(T_a T_b)=1/2 delta_ab",
      "SU2": "tr(t_i t_j)=1/2 delta_ij",
      "U1": "q=6Y"
    },
    "tolerances": {
      "symmetry": 1e-10,
      "rank": 1e-10,
      "positivity": 1e-10,
      "support_gap": 1e-08,
      "isotropy": 1e-08,
      "cross_factor": 1e-08,
      "imaginary": 1e-10
    }
  },
  "required_files": {
    "basis14": "basis14.csv",
    "E14": "inputs/E14.npy",
    "Eeta": "inputs/Eeta.npy (optional if no auxiliaries)",
    "E3": "inputs/E3.npy",
    "E2": "inputs/E2.npy",
    "Eq": "inputs/Eq.npy",
    "background": {
      "rho_star": "inputs/rho_star.npy",
      "U_star": "inputs/U_star.npy",
      "J_star": "inputs/J_star.npy",
      "Phi_star": "inputs/Phi_star.npy",
      "W_prim": "inputs/W_prim.npy"
    },
    "physical_parent_hessian": "inputs/physical_parent_hessian.npy (or
full_hessian.npy + P_phys.npy)"
  },
  "sha256": {}
}
```

The `branch_label` fields realise, one-to-one, the six components of the branch ledger β of Theorem 57A (§57A); the matching-scale and scheme prescription referenced by F26 is declared here. The freeze tool refuses a manifest with any unresolved label: publishing the completed manifest is the act that freezes the branch, and changing any input or code file afterwards creates a different branch requiring a new manifest.

G.5 The freeze tool (freeze_inputs.py)

```
from __future__ import annotations
import argparse, hashlib, json
from pathlib import Path

def sha256(path: Path) -> str:
    h = hashlib.sha256()
```

```
    with path.open("rb") as f:
        for chunk in iter(lambda: f.read(1024 * 1024), b""):
            h.update(chunk)
    return h.hexdigest()

def main() -> None:
    p = argparse.ArgumentParser()
    p.add_argument("--root", type=Path, default=Path("."))
    p.add_argument("--manifest", type=Path, required=True)
    p.add_argument("--output", type=Path,
default=Path("freeze_manifest.json"))
    args = p.parse_args()

    root = args.root.resolve()
    data = json.loads(args.manifest.read_text())

    # A branch may not be frozen with unresolved labels: the manifest must
    # identify a physical branch, not merely hash a directory.
    for key, value in data.get("branch_label", {}).items():
        if not value or value == "FILL_ME":
            raise ValueError(f"Unresolved branch label: {key}. "
                             "Fill every branch_label field before
freezing.")

    excluded_dirs = {"outputs", "outputs_bench", "__pycache__"}
    hashes = {}
    for path in sorted(root.rglob("*")):
        if not path.is_file():
            continue
        if path.resolve() in {args.output.resolve()}:
            continue
        rel_parts = path.relative_to(root).parts
        if rel_parts and rel_parts[0] in excluded_dirs:
            continue
        rel = path.relative_to(root).as_posix()
        hashes[rel] = sha256(path)

    data["sha256"] = hashes
    args.output.write_text(json.dumps(data, indent=2) + "\n")
    print(f"Wrote {args.output} with {len(hashes)} file hashes "
          "(data + basis + conventions + code).")

if __name__ == "__main__":
    main()
```

G.6 The generator (generate_h14.py)

```
from __future__ import annotations
import argparse, hashlib, json, sys
from pathlib import Path
import numpy as np

# The frozen evaluation map is data + basis + conventions + code.
CODE_FILES = ("freeze_inputs.py", "generate_h14.py", "basis14.csv",
              "freeze_manifest.template.json", "canonical_bare_matrices.npz")
```

```
# Executed wheel coefficients of Section 47B (frozen unit branch of Section
47A).
K_WHEEL = {"SU3": 24.0/5.0, "SU2": 35.0/6.0, "U1": 3.0/2.0}
DEFAULT_TOL = {"symmetry": 1e-10, "rank": 1e-10, "positivity": 1e-10,
               "support_gap": 1e-08, "isotropy": 1e-08,
               "cross_factor": 1e-08, "imaginary": 1e-10}

def sha256(path: Path) -> str:
    h = hashlib.sha256()
    with path.open("rb") as f:
        for chunk in iter(lambda: f.read(1024 * 1024), b""):
            h.update(chunk)
    return h.hexdigest()

def herm_residual(a: np.ndarray) -> float:
    return float(np.linalg.norm(a - a.conj().T) / max(np.linalg.norm(a), 1e-
30))

def hermitize(a: np.ndarray) -> np.ndarray:
    return 0.5 * (a + a.conj().T)

def load(path: Path) -> np.ndarray:
    if not path.exists():
        raise FileNotFoundError(path)
    return np.asarray(np.load(path))

def check_columns(name, a, rows, cols):
    if a.shape != (rows, cols):
        raise ValueError(f"{name} must have shape {(rows, cols)}, got
{a.shape}")

def load_tolerances(root: Path, manifest_path: Path) -> dict:
    src = manifest_path if manifest_path.exists() else root /
"freeze_manifest.template.json"
    tol = dict(DEFAULT_TOL)
    try:
        data = json.loads(src.read_text())
        tol.update(data.get("numeric_convention", {}).get("tolerances", {}))
    except Exception:
        pass
    return tol

def verify_freeze(root: Path, manifest_path: Path) -> None:
    if not manifest_path.exists():
        raise ValueError(
            f"Freeze manifest {manifest_path} not found. Run freeze_inputs.py
"
            "before generating, or pass --unfrozen for an explicitly unfrozen
benchmark run.")
    data = json.loads(manifest_path.read_text())
    hashes = data.get("sha256", {})
    mismatched, missing = [], []
    # 1) Every hashed input and code file must exist and match.
    for rel, expected in hashes.items():
        if not (rel.startswith("inputs/") or rel in CODE_FILES):
            continue
```

```
        path = root / rel
        if not path.exists():
            missing.append(rel)
        elif sha256(path) != expected:
            mismatched.append(rel)
    # 2) The evaluation code itself must be in the manifest at all.
    for rel in CODE_FILES:
        if (root / rel).exists() and rel not in hashes:
            mismatched.append(rel + " (code not frozen in manifest)")
    # 3) No unmanifested numerical inputs may be present.
    for path in sorted((root / "inputs").glob("*.npy")):
        rel = path.relative_to(root).as_posix()
        if rel not in hashes:
            mismatched.append(rel + " (not in manifest)")
    if mismatched or missing:
        raise ValueError(
            "Frozen verification FAILED (data + code). "
            f"Mismatched or unmanifested: {mismatched}; missing: {missing}. "
            "Inputs or code were changed after freezing; re-freeze under a
new branch manifest.")
    print("Frozen verification passed: all inputs AND code match the
manifest.")

def refuse_or_warn(frozen: bool, msg: str) -> None:
    if frozen:
        raise ValueError(msg)
    print("WARNING (unfrozen run): " + msg, file=sys.stderr)

def run_physical_direct(inp: Path, out: Path, tol: dict, frozen: bool) ->
None:
    G = load(inp/"G_red.npy")
    if G.ndim != 2 or G.shape[0] != G.shape[1]:
        raise ValueError("G_red must be square")
    herm = {"G_red": herm_residual(G)}
    if herm["G_red"] > tol["symmetry"]:
        refuse_or_warn(frozen, f"G_red is not Hermitian within tolerance: "
                       f"{herm['G_red']:.3e} > {tol['symmetry']:.1e}.")
    G = hermitize(G)
    M = G.shape[0]
    E3 = load(inp/"Etilde3.npy"); E2 = load(inp/"Etilde2.npy"); Eq =
load(inp/"Etildeq.npy")
    for name, E, k in (("Etilde3", E3, 8), ("Etilde2", E2, 3), ("Etildeq",
Eq, 1)):
        if E.shape != (M, k):
            raise ValueError(f"{name} must have shape {(M, k)}, got
{E.shape}")
    Eint = np.concatenate([E3, E2, Eq], axis=1)
    sv = np.linalg.svd(Eint, compute_uv=False)
    rank_eint = int(np.sum(sv > tol["rank"] * sv[0]))
    if rank_eint != 12:
        raise ValueError(f"Etilde rank must be 12 at frozen rank tolerance,
got {rank_eint}")
    lam_min = float(np.linalg.eigvalsh(G).min())
    if lam_min < -tol["positivity"]:
        refuse_or_warn(frozen, f"G_red has a negative physical mode: "
                       f"lambda_min={lam_min:.6e}. Section 22 applies.")
    im_audit: dict = {}
```

```
    def block(E, name):
        K = E.conj().T @ G @ E
        herm[name] = herm_residual(K)
        return hermitize(K)
    K3, K2, Kq = block(E3, "K3"), block(E2, "K2"), block(Eq, "Kq")
    def audited_Z(val, name):
        eps_im = abs(np.imag(val)) / max(abs(val), 1e-30)
        im_audit[name] = float(eps_im)
        if eps_im > tol["imaginary"]:
            refuse_or_warn(frozen, f"{name} imaginary residual {eps_im:.3e}
above tolerance.")
        return float(np.real(val))
    Z3 = audited_Z(np.trace(K3)/8.0, "Z3")
    Z2 = audited_Z(np.trace(K2)/3.0, "Z2")
    Zq = audited_Z(Kq[0, 0], "Zq")
    def rel(a, b): return float(np.linalg.norm(a)/max(np.linalg.norm(b), 1e-
30))
    iso3 = rel(K3 - Z3*np.eye(8), K3); iso2 = rel(K2 - Z2*np.eye(3), K2)
    scale = max(float(np.linalg.norm(G)), 1e-30)
    cross = {"SU3_SU2": float(np.linalg.norm(E3.conj().T @ G @ E2))/scale,
             "SU3_U1": float(np.linalg.norm(E3.conj().T @ G @ Eq))/scale,
             "SU2_U1": float(np.linalg.norm(E2.conj().T @ G @ Eq))/scale}
    failed = []
    if iso3 > tol["isotropy"]: failed.append(f"isotropy_SU3={iso3:.3e}")
    if iso2 > tol["isotropy"]: failed.append(f"isotropy_SU2={iso2:.3e}")
    for k, v in cross.items():
        if v > tol["cross_factor"]: failed.append(f"cross_{k}={v:.3e}")
    diagnostics = {
        "frozen_run": frozen,
        "response_mode": "physical_direct",
        "authoritative": True,
        "N_retained": M, "rank_Eint": rank_eint,
        "min_eig_G_red": lam_min,
        "hermiticity_residuals": herm, "imaginary_residuals": im_audit,
        "Z3": Z3, "Z2": Z2, "Zq": Zq,
        "isotropy_residual_SU3": iso3, "isotropy_residual_SU2": iso2,
        "cross_factor_residuals": cross,
        "failed_certificates": failed,
        "note": ("Direct pullback K_g = Etilde^dag G_red Etilde on the full
retained "
                 "carrier; no wheel (x) internal factorisation imposed.
Cross-factor "
                 "blocks above tolerance are failed Ward/equivariance
certificates, "
                 "never gauge-kinetic mixing. No effective internal norms are
emitted "
                 "in this mode; the direct K_g is authoritative."),
        "tolerances_used": tol,
    }
    np.save(out/"Kg_K3.npy", K3); np.save(out/"Kg_K2.npy", K2);
np.save(out/"Kg_Kq.npy", Kq)
    (out/"diagnostics.json").write_text(json.dumps(diagnostics,
indent=2)+"\n")
    print(json.dumps(diagnostics, indent=2))

def main() -> None:
    p = argparse.ArgumentParser()
```

```
    p.add_argument("--root", type=Path, default=Path("."))
    p.add_argument("--output", type=Path, default=Path("outputs"))
    p.add_argument("--manifest", type=Path, default=None)
    p.add_argument("--unfrozen", action="store_true",
                   help="skip freeze verification; outputs labelled
frozen_run=false")
    args = p.parse_args()

    root = args.root
    manifest_path = args.manifest or (root / "freeze_manifest.json")
    frozen = not args.unfrozen
    if frozen:
        verify_freeze(root, manifest_path)
    else:
        print("WARNING: UNFROZEN RUN - outputs are benchmark/diagnostic only
"
              "and must not be quoted as frozen branch results.",
file=sys.stderr)

    tol = load_tolerances(root, manifest_path)
    inp = root / "inputs"
    out = args.output
    out.mkdir(parents=True, exist_ok=True)

    # --- Physical-direct route: pull the gauge response straight from the
    # reduced physical Hessian on the full retained carrier, with no
    # factorised (wheel (x) internal) assumption. This is the authoritative
    # physical mode of Sections 48-53; the channel/tensor route below is the
    # separable benchmark pipeline. Supplying both input sets is ambiguous
    # and refused.
    direct_files = [inp/"G_red.npy", inp/"Etilde3.npy", inp/"Etilde2.npy",
inp/"Etildeq.npy"]
    bench_files = [inp/"physical_parent_hessian.npy", inp/"full_hessian.npy"]
    has_direct = all(p.exists() for p in direct_files)
    if has_direct and any(p.exists() for p in bench_files):
        raise ValueError("Both physical-direct (G_red + Etilde*) and
separable-benchmark "
                         "(parent Hessian) inputs are present; remove one
input set.")
    if has_direct:
        run_physical_direct(inp, out, tol, frozen)
        return

    # --- Physical parent Hessian: the quotient must be explicit ---
    pph = inp / "physical_parent_hessian.npy"
    fh, pp = inp / "full_hessian.npy", inp / "P_phys.npy"
    if pph.exists():
        G = load(pph)
    elif fh.exists() and pp.exists():
        Gfull, P = load(fh), load(pp)
        G = P.conj().T @ Gfull @ P
        print("Applied physical quotient: G_phys = P_phys† G_full P_phys.")
    elif fh.exists():
        refuse_or_warn(frozen,
            "full_hessian.npy supplied without P_phys.npy. Supply "
            "physical_parent_hessian.npy (already quotient-reduced) or add
P_phys.npy; "
```

```
            "gauge/terminal-null directions must not enter the fourteen-
channel matrix.")
        G = load(fh)
        print("WARNING: treating full_hessian.npy as already quotient-
reduced.", file=sys.stderr)
    else:
        raise FileNotFoundError("physical_parent_hessian.npy (or
full_hessian.npy + P_phys.npy)")

    if G.ndim != 2 or G.shape[0] != G.shape[1]:
        raise ValueError("Hessian must be square")
    N = G.shape[0]

    # --- Hermiticity is certified before it is repaired ---
    herm = {"G": herm_residual(G)}
    if herm["G"] > tol["symmetry"]:
        refuse_or_warn(frozen,
            f"Supplied Hessian is not Hermitian within tolerance: residual "
            f"{herm['G']:.3e} > {tol['symmetry']:.1e}. This indicates an AD,
"
            "coordinate or export error; hermitisation would hide it.")
    G = hermitize(G)

    E14 = load(inp / "E14.npy"); check_columns("E14", E14, N, 14)
    Eeta_path = inp / "Eeta.npy"
    Eeta = load(Eeta_path) if Eeta_path.exists() else None
    if Eeta is not None and Eeta.shape[0] != N:
        raise ValueError("Eeta row count must equal Hessian dimension")

    E3 = load(inp/"E3.npy"); E2 = load(inp/"E2.npy"); Eq = load(inp/"Eq.npy")
    # Two admissible response carriers (Section 47H):
    #  - channel mode: D = 14, gauge embeddings land directly in the channel
carrier;
    #  - tensor mode:  D = 14*k for integer k, embeddings land in H14 (x)
HS(H_SM),
    #    and the response metric acts as H14_red (x) I_HS, applied by
reshape.
    D = E3.shape[0]
    if D == 14:
        mode = "channel"
    elif D % 14 == 0 and D > 14:
        mode = "tensor"
    else:
        raise ValueError(f"gauge embedding row dimension {D} must be 14
(channel mode) "
                         "or a multiple of 14 (tensor mode, Section 47H)")
    check_columns("E3", E3, D, 8); check_columns("E2", E2, D, 3);
check_columns("Eq", Eq, D, 1)
    Eint = np.concatenate([E3, E2, Eq], axis=1)
    sv = np.linalg.svd(Eint, compute_uv=False)
    rank_eint = int(np.sum(sv > tol["rank"] * sv[0]))
    if rank_eint != 12:
        raise ValueError(f"Eint rank must be 12 at frozen rank tolerance "
                         f"{tol['rank']:.1e}, got {rank_eint}")

    # --- Pullback and Schur reduction ---
    H14 = E14.conj().T @ G @ E14
```

```
    herm["H14_bare"] = herm_residual(H14)
    H14 = hermitize(H14)

    if Eeta is None or Eeta.shape[1] == 0:
        Hred, H14eta, Hetaeta = H14.copy(), None, None
    else:
        H14eta = E14.conj().T @ G @ Eeta
        Hetaeta = hermitize(Eeta.conj().T @ G @ Eeta)
        try:
            L = np.linalg.cholesky(Hetaeta)
        except np.linalg.LinAlgError as exc:
            eig = np.linalg.eigvalsh(Hetaeta)
            raise ValueError("Hetaeta is not positive definite; "
                             f"minimum eigenvalue={eig.min():.6e}") from exc
        x = np.linalg.solve(L.conj().T, np.linalg.solve(L, H14eta.conj().T))
        Hred = H14 - H14eta @ x
        herm["H14_red"] = herm_residual(Hred)
        Hred = hermitize(Hred)

    # --- Positivity of the reduced physical response ---
    lam_min = float(np.linalg.eigvalsh(Hred).min())
    if lam_min < -tol["positivity"]:
        refuse_or_warn(frozen,
            f"H14_red has a negative physical mode: lambda_min={lam_min:.6e}
< "
            f"-{tol['positivity']:.1e}. Section 22: an instability may not be
"
            "reinterpreted as a small coupling.")

    def apply_response(E):
        if mode == "channel":
            return Hred @ E
        # tensor mode: (Hred (x) I) E column-by-column via reshape
        k = D // 14
        out_cols = []
        for j in range(E.shape[1]):
            V = E[:, j].reshape(14, k)
            out_cols.append((Hred @ V).ravel())
        return np.column_stack(out_cols)

    im_audit: dict = {}
    def block(E, name):
        K = E.conj().T @ apply_response(E)
        herm[name] = herm_residual(K)
        return hermitize(K)
    K3, K2, Kq = block(E3, "K3"), block(E2, "K2"), block(Eq, "Kq")

    def audited_Z(val: complex, name: str) -> float:
        eps_im = abs(np.imag(val)) / max(abs(val), 1e-30)
        im_audit[name] = float(eps_im)
        if eps_im > tol["imaginary"]:
            refuse_or_warn(frozen,
                f"{name} has a significant imaginary part:
eps_Im={eps_im:.3e} > "
                f"{tol['imaginary']:.1e}. A complex coupling indicates a
typing, "
                "determinant-line, background or Hessian problem.")
```

```
        return float(np.real(val))
    Z3 = audited_Z(np.trace(K3) / 8.0, "Z3")
    Z2 = audited_Z(np.trace(K2) / 3.0, "Z2")
    Zq = audited_Z(Kq[0, 0], "Zq")

    # --- Certificates: isotropy, cross-factor, then (conditionally) internal
norms ---
    def rel(a, b): return float(np.linalg.norm(a) / max(np.linalg.norm(b),
1e-30))
    iso3 = rel(K3 - Z3*np.eye(8), K3)
    iso2 = rel(K2 - Z2*np.eye(3), K2)
    scale = max(float(np.linalg.norm(Hred)), 1e-30)
    cross = {"SU3_SU2": float(np.linalg.norm(E3.conj().T @
apply_response(E2)))/scale,
             "SU3_U1": float(np.linalg.norm(E3.conj().T @
apply_response(Eq)))/scale,
             "SU2_U1": float(np.linalg.norm(E2.conj().T @
apply_response(Eq)))/scale}
    failed = []
    if iso3 > tol["isotropy"]: failed.append(f"isotropy_SU3={iso3:.3e}")
    if iso2 > tol["isotropy"]: failed.append(f"isotropy_SU2={iso2:.3e}")
    for k, v in cross.items():
        if v > tol["cross_factor"]: failed.append(f"cross_{k}={v:.3e}")

    # The isotropy and cross-factor certificates do NOT prove that the
    # physical response factorises as (wheel) (x) (internal). They certify
the
    # benchmark's internal consistency only. The division by the frozen wheel
    # coefficients therefore defines EFFECTIVE norms under the frozen wheel
    # factorisation assumption; the physical_direct K_g is authoritative.
    if failed:
        norms_status = "not_defined_nonseparable"
        norms = None
    else:
        norms_status = "defined_under_frozen_wheel_factorisation"
        norms = {"n3": Z3 / K_WHEEL["SU3"], "n2": Z2 / K_WHEEL["SU2"],
                 "nq": Zq / K_WHEEL["U1"]}

    diagnostics = {
        "frozen_run": frozen,
        "embedding_mode": mode, "response_dim": D,
        "N_full": N, "N_aux": 0 if Eeta is None else int(Eeta.shape[1]),
        "rank_Eint": rank_eint,
        "min_eig_H14_red": lam_min,
        "hermiticity_residuals": herm,
        "imaginary_residuals": im_audit,
        "Z3": Z3, "Z2": Z2, "Zq": Zq,
        "isotropy_residual_SU3": iso3, "isotropy_residual_SU2": iso2,
        "cross_factor_residuals": cross,
        "failed_certificates": failed,
        "response_mode": f"separable_benchmark:{mode}",
        "authoritative": False,
        "effective_internal_norms_status": norms_status,
        "effective_internal_norms": norms,
        "wheel_coefficients_frozen": K_WHEEL,
        "tolerances_used": tol,
    }
```

```
    np.save(out/"H14_bare.npy", H14); np.save(out/"H14_red.npy", Hred)
    np.save(out/"K3.npy", K3); np.save(out/"K2.npy", K2);
np.save(out/"Kq.npy", Kq)
    if H14eta is not None:
        np.save(out/"H14_eta.npy", H14eta); np.save(out/"Hetaeta.npy",
Hetaeta)
    (out/"diagnostics.json").write_text(json.dumps(diagnostics,
indent=2)+"\n")
    print(json.dumps(diagnostics, indent=2))

if __name__ == "__main__":
    main()
```

G.7 Commands and the executed test log

Freeze, then generate:

```
python freeze_inputs.py --root . --manifest my_branch_manifest.json --output
freeze_manifest.json
python generate_h14.py  --root . --output outputs
```

The generator writes `H14_bare`, `H14_red`, `K3`, `K2`, `Kq`, the auxiliary blocks when present, and a diagnostics record containing the frozen-run label, carrier and auxiliary dimensions, rank of E_int, the minimum eigenvalue of H_{14}^red, all Hermiticity and imaginary residuals, Z_3, Z_2, Z_q, both isotropy residuals, all three cross-factor responses, the failed-certificate list, the `response_mode` and `authoritative` flags, and — in the separable-benchmark mode only — `effective_internal_norms_status` with the conditional effective π_A. In `physical_direct` mode the outputs are the $\mathcal{K}$_g factor blocks and diagnostics only.

VERSF H14 Pack — Executed Test Log. This log records the verification suite executed on exactly the source contained in this package (and reproduced verbatim above). All eighteen tests pass. The generator has two response modes: `separable_benchmark` (channel D = 14 or tensor D = 14k) applies the frozen wheel ⊗ internal factorisation and emits effective internal norms only when its isotropy and cross-factor certificates pass, under the status `defined_under_frozen_wheel_factorisation` — these certificates do NOT prove physical factorisation; `physical_direct` pulls $\mathcal{K}$_g = Ẽ†G_redẼ straight from the reduced physical Hessian on the full retained carrier with no factorisation imposed, is authoritative, and emits no effective norms. Supplying both input sets is refused as ambiguous.

#	Test	Result	Detail
T1	Freeze tool refuses placeholder branch labels	PASS	Unresolved branch label
T2	Generation refuses without a freeze manifest	PASS	manifest-not-found refusal
T3	Freeze hashes data + basis + conventions + code; excludes outputs	PASS	both tools, basis and template hashed

#	Test	Result	Detail
T4	Channel benchmark: planted Z recovered to 1e-9; effective norms to 1e-12; honest status	PASS	separable_benchmark:channel, authoritative: false, status defined_under_frozen_wheel_factorisation
T5	Code tampering after freeze detected	PASS	modified generate_h14.py refused
T6	Data tampering after freeze detected	PASS	single-entry 1e-9 change refused
T7	Hermiticity certified before repair	PASS	refused frozen; warned and labelled -- unfrozen
T8	Indefinite auxiliary block refused	PASS	minimum eigenvalue reported
T9	Negative physical mode refused at manifest positivity tolerance	PASS	§22 cited
T10	Rank enforced at manifest rank tolerance	PASS	rank-11 embedding refused
T11	Explicit quotient path G_phys = P_phys†G_full P_phys matches direct path	PASS	Z3 identical to 1e-9
T12	Nonseparable benchmark: effective norms withheld with named certificate	PASS	not_defined_nonseparable, cross_SU3_SU2 listed
T13	Unfrozen Dirichlet benchmark runs, loudly labelled	PASS	frozen_run: false
T14	Full §47I tensor benchmark, shape 30926×12, through separable_benchmark:tensor; honest status	PASS	traces (6, 13/2, 378) to 1e-12
T15	Tensor reshape metric matches explicit H_{14}^red ⊗ I_HS kron	PASS	Z and K3 block to 1e-9/1e-10 at D = 168
T16	Invalid embedding row dimension refused with mode diagnosis	PASS	channel/tensor rule enforced
T17	physical_direct: $\mathcal{K}$_g = Ẽ†G_redẼ on a 40-dim retained carrier; planted (Z_3, Z_2, Z_q) recovered to 1e-12; authoritative: true; no effective norms	PASS	direct mode is the physical route of §§48–53
T18	Dual-input ambiguity refused; nonfactorising G_red in direct mode names the failed cross-factor certificate rather than emitting norms	PASS	failed Ward/equivariance certificate published

Construction of T4/T12/T17/T18: reduced responses built to be exactly scalar on the factor blocks (recovery tests) or perturbed by a small colour–weak cross term (withholding tests), lifted through the inverse Schur formula where auxiliaries are present so the generator's reduction must recover the target exactly.

This appendix is the executed package, not a specification of it: the source above is what produced the log. It does not, and cannot, construct the history-rate functional (F19); a green suite certifies the plumbing and the anti-fitting guardrails, not the Standard Model.

Appendix H — Machine-Readable Handoff Schema

The HFB-1 boundary package is issued in one machine-readable object. Its status may equal `PASS` only when every boundary field is populated and every mandatory certificate has been executed; `NOT_EXECUTED` is a distinct certificate value from `FAIL` and is the current state of `CY3_prime` (F9).


```
{
  "schema": "VERSF-HFB1-HANDOFF-1.0",
  "status": "PASS | FAIL | INCOMPLETE",
  "branch": {
    "branch_id": "",
    "perron_eigenvalue": null,
    "free_energy_density": null,
    "selection_gap": null
  },
  "matching": {
    "mu_star": null,
    "native_scheme": "",
    "target_bridge_scheme": "",
    "primitive_scale_ledger": {}
  },
  "history": {
    "transfer_kernel_hash": "",
    "stationary_state_hash": "",
    "W_prim_hash": "",
    "Delta_Omega_hash": ""
  },
  "boundary": {
    "g_s": null,
    "g": null,
    "g_prime": null,
    "mu_H_squared": null,
    "lambda": null,
    "Y_u": null,
    "Y_d": null,
    "Y_e": null,
    "B_nu": null,
    "theta_bar": null
  },
```

```
  "supporting_returns": {
    "Z_H": null,
    "v_cl": null,
    "B_tau": null,
    "H_cl_red_hash": "",
    "P_Y_L_hash": "",
    "P_Y_R_hash": ""
  },
  "certificates": {
    "ward": "PASS | FAIL",
    "CY3_prime": "PASS | FAIL | NOT_EXECUTED",
    "mass_linearity": "PASS | FAIL",
    "null_quotient": "PASS | FAIL",
    "reflection_positivity": "PASS | FAIL",
    "triality_wall": "PASS | FAIL",
    "determinant_line": "PASS | FAIL"
  },
  "uncertainty": {
    "numerical": {},
    "history": {},
    "auxiliary": {},
    "regulator": {},
    "branch": {},
    "premise": {}
  },
  "regulator_levels": [],
  "source_manifest_hash": ""
}
```


This schema is the exact object the Renormalised Parameter-Closure paper consumes (§53.6). It carries no MASD-1 numerical return: at the close of this paper every `boundary` field is `null`, `CY3_prime` is `NOT_EXECUTED`, and `status` is `INCOMPLETE`, because the history-derived metric, generation geometry and branch of HMGBC-1 and the frozen execution of HFB-1 have not been performed.

End of MASD-1.