The Variational–Closure Suppression Identity in VERSF

▲ Programme Milestone — Flavour-Magnitude and Mass-Hierarchy Series Gate VCS-1 / Primal–Dual Closure Localisation, Exact Deficit-to-Contrast Conversion, Sector-Suppression Criterion, Capacity-Shadow Identity, and Non-Insertion Closure

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General Reader Summary

The Standard Model contains three charged leptons: the electron, the muon, and the tau. If you simply count them, the tau is one branch out of three. Its "fair share" of the family, by counting alone, is one third.

But counting is not the same as sustaining. In the VERSF picture, keeping a branch of the family physically active is not free: each branch places a continuing maintenance demand on the underlying substrate, and the substrate has only a finite capacity to meet those demands. The question this paper answers is: **when the capacity is not enough to maintain everything, what exactly happens to the tau's share?**

A homely analogy makes the structure clear. Imagine a caretaker responsible for three rooms, working on a fixed budget. Two of the rooms are cheap to keep up. The third — the tau room — is expensive. With an unlimited budget, all three rooms are fully maintained, so each represents one third of the maintained rooms — although the expensive room may consume more than one third of the budget. With a limited budget, something has to give.

Here is the first surprise, and the paper proves it as a theorem: **a tight budget, by itself, tells you nothing about the tau.** The budget shortfall is just a number. What matters is *where* the neglect lands. If the caretaker spreads the shortfall evenly across all three rooms, every share stays exactly at one third — the household is shabbier overall, but no room is singled out. If the caretaker neglects the two cheap rooms instead, the tau's share actually goes *up*. Only if the neglect is concentrated in the tau room does the tau's share fall below one third. The paper turns this into an exact rule: the tau is suppressed precisely when the average neglect per tau direction exceeds the average neglect per non-tau direction.

So what decides where the neglect lands? Optimisation does. The paper shows that if the caretaker allocates the budget in the best possible way — maintaining as much as possible within the limit — then the strategy is forced: fill the cheap rooms first, and let the expensive room absorb the shortfall. In technical terms, the optimal allocation has a sharp "threshold": everything whose upkeep cost falls below the threshold is fully maintained, everything above it is not, and the budget fixes where the threshold sits. If maintaining the tau genuinely costs more than maintaining the electron and muon — an assumption this paper states openly rather than proves — then all of the neglect is pushed into the tau branch before the other two lose anything.

Once that happens, the paper's central result takes over. The gap between the tau's actual maintained share and its fair-count share of one third is not merely bounded by the neglect — it is **exactly determined** by it. Writing δ for the amount of tau maintenance that is missing (a number between 0 and 1 for a single tau branch), the shortfall in the tau's share is exactly

$$1/3 - (\text{tau's maintained share}) = 2\delta / [3(3 - \delta)].$$

No missing amount, no suppression. Full neglect, and the tau's share drops to zero. Everything in between follows this one curve, with nothing left over and nothing put in by hand.

The paper then adds an idea borrowed from economics. Every budget has a *shadow price*: the value of one more pound — here, how much extra maintained support one more unit of capacity would buy. The paper proves that the total missing maintenance equals the accumulated shadow price of all the capacity you don't have, added up from the actual budget to the full-maintenance budget. So the tau's suppression is, exactly, the accumulated marginal value of the missing budget. That is the Variational–Closure Suppression Identity, and it gives the gate its name.

Just as important is what the paper does **not** claim. It does not say that tau particles disappear — an unmaintained branch is still part of the theory. It does not calculate the tau's mass, lifetime, decay rate, or coupling strength; connecting maintenance shares to any of those requires separate bridge results that this paper deliberately fences off. And it does not supply its own numbers: the actual cost of maintaining each branch, and the actual size of the capacity, must be derived elsewhere in the programme. Indeed, the paper proves that the formula alone cannot fix the numbers — any suppression size could be produced by a suitable choice of inputs, which is exactly why the inputs must be derived independently rather than chosen to fit.

What the paper closes, then, is the conversion law: a complete, exact chain from

finite capacity → best-possible allocation → location of the neglect → exact tau suppression.

Given honest inputs, the output is no longer a matter of estimates or inequalities. It is an identity.

Scope and Closure Box

Claim	VCS-1 status
A finite charged-lepton maintenance carrier exists	Inherited premise
Electron, muon, and tau projectors are consistently defined	Inherited/conditional
Maintenance allocations are positive contractions	Inherited definition
The closure-deficit operator is positive	Closed

Claim	VCS-1 status
The exact universal sector-deficit identity holds	Closed
Suppression, equality, and enhancement have exact deficit criteria	Closed
A positive maintenance-demand operator exists	Conditional premise
The finite-capacity optimisation has an optimum	Closed
Every saturated optimum exhausts the available capacity	Closed for 0 < C < C_full
Strong primal–dual equality holds	Closed
The optimal multiplier defines the maintenance threshold	Closed
Every optimum has complete threshold form	Closed
The complementary closure problem is equivalent	Closed
Closure deficit occupies highest-demand modes first	Closed
Strict tau-demand separation localises deficit to tau first	Closed
The tau fraction is zero below complement saturation	Closed conditionally
The partial-tau suppression identity is exact	Closed
The tau fraction is continuous and strictly increasing across the partial-tau regime	Closed
The exact capacity-deficit conversion holds	Closed
The shadow-price integral equals unresolved closure (neutral objective)	Closed
The benefit-weighted shadow integral equals benefit-weighted closure	Closed
Degenerate demand spectra handled by distinct-eigenvalue kink structure	Closed
The boundary optimum at complement saturation is unique	Closed
Earlier deficit bounds follow as corollaries	Closed
Aggregate tau fraction is unique under strict cross-sector separation	Closed
Sector deficit traces are unique across the optimum set under strict separation	Closed
Degenerate mixed-sector thresholds may produce nonunique sector fractions	Identified
Soft monotone maintenance profiles satisfy the same sign criterion via the universal identity	Closed
General benefit-to-demand optimisation has a dual certificate	Closed
Perturbations preserve demand positivity under the combined norm bound	Closed
Numerical magnitude follows from capacity alone	Disproved
Numerical magnitude follows from demand ordering alone	Disproved
The tau-demand spectrum is derived from VERSF primitives	Open unless inherited
The physical capacity is derived from VERSF primitives	Open

Claim	VCS-1 status
The neutral maintained-support objective is dynamically derived	Open
A physical relaxation law selecting the optimum is derived	Open
Tau mass relation	Not derived
Tau lifetime or decay-width relation	Not derived
Tau Yukawa relation	Not derived
Gauge-invariant observable extraction	Open
Empirical confirmation	Not established

Abstract

A rank census assigns a sector of dimension g inside an $n = g+h$ dimensional carrier the unweighted fraction g/n. A positive maintenance allocation R, however, need not be uniform across that carrier.

This paper establishes the Variational–Closure Suppression Identity in VERSF.

For a positive contraction $0 \leq R \leq I$ with $\mathrm{Tr}\, R > 0$, define the closure-deficit operator $\Delta = I - R$. Let Π_τ project onto a rank-g tau sector and let $\Pi_0 = I - \Pi_\tau$ project onto its rank-h complement. Writing

$\delta_\tau = \mathrm{Tr}(\Pi_\tau \Delta)$, $\delta_0 = \mathrm{Tr}(\Pi_0 \Delta)$,

the normalised tau-maintenance fraction is

$f_\tau = \mathrm{Tr}(\Pi_\tau R) / \mathrm{Tr}\, R$.

The paper proves the universal identity

$g/(g+h) - f_\tau = (h\, \delta_\tau - g\, \delta_0) / [\, (g+h)(g+h - \delta_\tau - \delta_0)\,]$.

Equivalently, with the centred projector $Q_\tau = \Pi_\tau - [g/(g+h)]\, I$,

$g/(g+h) - f_\tau = \mathrm{Tr}(Q_\tau \Delta) / (g+h - \mathrm{Tr}\, \Delta)$.

Hence tau suppression occurs if and only if the average unresolved closure per tau direction exceeds that of the complement:

$f_\tau < g/(g+h) \Leftrightarrow \delta_\tau/g > \delta_0/h$.

The identity is purely algebraic and does not by itself determine the deficit operator.

The paper next derives Δ variationally. Let $D_M > 0$ be an independently specified maintenance-demand operator and define

$$V(C) = \max \{ \mathrm{Tr}\, R : 0 \le R \le I, \mathrm{Tr}(D_M R) \le C \}.$$

For $0 < C < C_{full}$, where $C_{full} = \mathrm{Tr}\, D_M$, strong duality gives

$$V(C) = \min_{\nu \ge 0} [\nu C + \mathrm{Tr}(I - \nu D_M)_+].$$

Every primal optimum and optimal multiplier satisfy

$$R_C = P_{<1/\nu_C} + X_C, 0 \le X_C \le P_{=1/\nu_C},$$

with the capacity constraint fixing $\mathrm{Tr}\, X_C$. The complementary deficit operator is therefore

$$\Delta_C = P_{>1/\nu_C} + (P_{=1/\nu_C} - X_C),$$

so unresolved closure occupies the highest-demand spectral modes first.

Assume branch covariance and strict tau-demand separation,

$$[D_M, \Pi_\tau] = 0, \sup \sigma(D_M|_{\mathcal{H}_0}) < \inf \sigma(D_M|_{\mathcal{H}_\tau}).$$

If $C_0 = \mathrm{Tr}(D_M \Pi_0)$, then every optimum satisfies:

$$0 < C \le C_0 \Longrightarrow \Pi_\tau R_C = 0,$$

while

$$C_0 < C < C_{full} \Longrightarrow \Pi_0 R_C = \Pi_0, \Delta_C = \Pi_\tau \Delta_C \Pi_\tau.$$

In this tau-only deficit regime,

$$g/(g+h) - f_\tau(C) = h \delta_C / [(g+h)(g+h - \delta_C)], \delta_C = \mathrm{Tr}\, \Delta_C.$$

Writing $\kappa_C = C_{full} - C$ and $\bar{d}_\Delta = \mathrm{Tr}(D_M \Delta_C)/\mathrm{Tr}\, \Delta_C$,

$$g/(g+h) - f_\tau(C) = h \kappa_C / \{ (g+h)[(g+h) \bar{d}_\Delta - \kappa_C] \}.$$

For a single tau mode of demand d_τ,

$$1/3 - f_\tau(C) = 2(C_{full} - C) / \{ 3[3d_\tau - (C_{full} - C)] \}.$$

The dual multiplier lies in the superdifferential of the concave value function. Since $V(C_{full}) = g+h$,

$\text{Tr}\,\Delta\_C = (g+h) - V(C) = \int\_C^{C\_full} \nu\_u\, du$,

with the derivative understood almost everywhere. Substitution yields an exact shadow-price form of the suppression identity.

The unweighted shadow-price identity is specific to the neutral objective; for a strictly positive benefit objective the exact analogue is the benefit-weighted closure identity $\text{Tr}(B\_M\, \Delta\_C) = \text{Tr}\, B\_M - V\_B(C) = \int\_C^{C\_full} \nu\_u\, du$. The paper also proves basis covariance, soft-profile persistence, benefit-to-demand generalisation, perturbative stability with preserved demand positivity, a fixed-capacity localisation no-go theorem, and constructive numerical underdetermination.

VCS-1 closes the exact mathematical passage

primal finite-capacity optimisation → dual spectral threshold → closure localisation → normalised suppression identity.

It does not derive the physical tau-demand ordering, the capacity scale, a dynamical relaxation law, the tau mass, lifetime, decay width, Yukawa coupling, or an experimental observable.

Contents

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1. Aim, Inheritance, and Novelty

1.1 Inherited result

The carrier and branch-projector premises of this paper (P1–P2, and the corresponding debts D1–D2) are inherited from the preceding Standard Model census and world-to-framework occupancy gates, together with any independently completed charged-lepton branch-projector gate; where no such gate has closed, the corresponding debt remains open in Appendix F. The later absolute-scale gate (CLAS-1) is a downstream consumer of the present results, not an upstream premise source. The preceding saturated-maintenance gate (SMPτ-1) established that a finite-capacity optimisation selects lower-demand modes before higher-demand modes. Under strict tau-demand separation, the tau branch is therefore maintained after the electron–muon complement, implying

$f_\tau(C) < g/(g+h)$

throughout every nontrivial saturated regime.

That theorem established the sign. VCS-1 asks the stronger question:

What exact operator quantity is the suppression measuring?

1.2 The new object

The answer is the unresolved closure

$\Delta_C = I - R_C.$

The previous gate worked primarily with what is maintained. VCS-1 works simultaneously with R_C and its positive complement Δ_C. This exposes an exact dual description:

maximise maintained support ⟺ minimise unresolved closure.

1.3 The central distinction

A capacity deficit is a scalar:

$C\_full − C$.

A closure deficit is an operator:

Δ_C.

The scalar tells us how much maintenance burden remains unpaid. The operator tells us where that unpaid burden is located. Only the second determines whether the tau fraction is suppressed, unchanged, or enhanced.

1.4 Core novelty

VCS-1 proves:

1. an exact universal decomposition of sector suppression into tau and complement closure deficits;
2. a necessary and sufficient sign criterion;
3. strong primal–dual equivalence for the maintenance optimisation;
4. a complete dual threshold certificate, with the proof order arranged so that optimal diagonal values are fixed before positivity eliminates cross-block components;
5. an equivalent highest-demand-first closure-release principle;
6. exact localisation of the closure deficit under strict tau-demand separation;
7. an exact suppression identity in terms of total unresolved closure;
8. strict monotonicity and continuity of the tau fraction across the partial-tau regime;
9. an exact conversion from capacity deficit to suppression;
10. an exact integrated shadow-price representation valid at every capacity, including spectral kink points, with the degenerate-spectrum kink structure stated in distinct-eigenvalue form and a benefit-weighted analogue for general objectives;
11. a no-go theorem showing that capacity shortfall alone cannot determine the suppression sign;
12. constructive underdetermination of the numerical magnitude;
13. strict firewalls against mass, lifetime, Yukawa, disappearance, and observational overclaim.

1.5 Standard mathematics and VERSF-specific content

The following ingredients are standard: positive operators; spectral projectors; finite-dimensional convex optimisation; Lagrange duality; complementary slackness; positive-part functional calculus; concave value functions; spectral perturbation bounds.

The programme-specific content is the use of these tools to identify a VERSF maintenance deficit with an exact charged-lepton sector contrast, subject to independently derived carrier, branch, demand, capacity, and objective structures.

1.6 Core aim

To prove that the departure of the tau-maintenance fraction from its census value is exactly the normalised tau contrast of the unresolved closure operator, and to show that the finite-capacity VERSF variational principle localises that closure into the highest-demand sector, thereby converting an independently derived tau-last ordering into an exact suppression identity rather than merely an inequality.

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2. Typed Carrier and Sector Census

2.1 Carrier decomposition

Let

$\mathscr{H}_\ell = \mathscr{H}_0 \oplus \mathscr{H}_\tau$

be a finite maintenance carrier, with

$\dim \mathscr{H}_\tau = g$, $\dim \mathscr{H}_0 = h$, $n = g + h$.

For the minimal charged-lepton branch census,

$g = 1, h = 2, n = 3$.

2.2 Sector projectors

Let Π_τ be the orthogonal projector onto $\mathscr{H}_\tau$, and define $\Pi_0 = I - \Pi_\tau$. Then

$\Pi_\tau^2 = \Pi_\tau$, $\Pi_0^2 = \Pi_0$, $\Pi_\tau \Pi_0 = 0$,

and

$\mathrm{Tr}\, \Pi_\tau = g$, $\mathrm{Tr}\, \Pi_0 = h$.

2.3 Raw census

The raw tau census fraction is

$c_\tau = \mathrm{rank}\, \Pi_\tau / \dim \mathscr{H}_\ell = g/n$.

For the minimal physical census, $c_\tau = 1/3$.

This is an exact rank statement. It is not automatically a maintenance fraction, probability, mass ratio, abundance, lifetime fraction, or coupling ratio.

2.4 Premise P1 — Carrier consistency

The carrier must use one counting convention across the tau and complement sectors.

Falsifier: one sector contains hidden spin, chirality, particle–antiparticle, generation, or gauge multiplicities not included in the other.

2.5 Premise P2 — Physical branch projector

The tau projector must be fixed by inherited or independently derived VERSF structure.

Falsifier: Π_τ is chosen retrospectively because one coordinate later resembles the observed tau hierarchy.

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3. Maintenance Allocation and Closure Deficit

3.1 Maintenance allocation

Let R be a positive contraction:

$0 \leq R \leq I.$

Its eigenvalues lie between zero and one. They represent the degree to which the declared maintenance response supports corresponding carrier directions. The operator R is not assumed to be a quantum density matrix.

3.2 Maintained support

The total maintained support is

$M(R) = \mathrm{Tr}\, R.$

The tau-maintained support is

$q_\tau(R) = \mathrm{Tr}(\Pi_\tau R).$

Provided Tr R > 0, the normalised tau-maintenance fraction is

$f_\tau(R) = \mathrm{Tr}(\Pi_\tau R) / \mathrm{Tr}\, R.$

3.3 Closure-deficit operator

Define

$\Delta = I - R.$

Since $0 \le R \le I$,

$0 \le \Delta \le I.$

The operator Δ is the unresolved closure associated with the declared maintenance allocation. It satisfies

$R + \Delta = I.$

3.4 Sector-resolved closure deficits

Define

$\delta_\tau = \mathrm{Tr}(\Pi_\tau \Delta), \delta_0 = \mathrm{Tr}(\Pi_0 \Delta), \delta = \mathrm{Tr}\, \Delta = \delta_\tau + \delta_0.$

Because Δ is a positive contraction and Π_τ, Π_0 are orthogonal projectors, the diagonal blocks $\Pi_\tau \Delta \Pi_\tau$ and $\Pi_0 \Delta \Pi_0$ are positive contractions on their respective sectors, whence

$0 \le \delta_\tau \le g, 0 \le \delta_0 \le h, 0 \le \delta \le n.$

For a nonzero maintained allocation, $\delta < n$.

3.5 Maintained support in deficit form

Since $R = I - \Delta$,

$\mathrm{Tr}\, R = n - \delta, \mathrm{Tr}(\Pi_\tau R) = g - \delta_\tau.$

Therefore

$f_\tau = (g - \delta_\tau) / (n - \delta_\tau - \delta_0).$

This elementary expression is the starting point of the exact identity.

3.6 Interpretation firewall

The closure-deficit operator is not automatically: a missing particle number; a probability of tau disappearance; a decay operator; an inverse lifetime; a mass operator; a Yukawa matrix; a nonunitarity measure; a detector inefficiency; a thermal occupation deficit.

It is the positive complement of the declared maintenance allocation. Any further interpretation requires a separate bridge theorem.

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4. The Universal Closure-Suppression Identity

4.1 Centred tau projector

Define

$Q_\tau = \Pi_\tau - (g/n) I.$

Then $\mathrm{Tr}\, Q_\tau = 0$. Its eigenvalues are h/n on $\mathscr{H}_\tau$ and $-g/n$ on $\mathscr{H}_0$.

4.2 Theorem 1 — Universal Variational–Closure Identity

Statement. For every positive contraction R with $\mathrm{Tr}\, R > 0$, and $\Delta = I - R$,

$$g/n - f_\tau(R) = \mathrm{Tr}(Q_\tau \Delta) / (n - \mathrm{Tr}\, \Delta).$$

Equivalently,

$$g/n - f_\tau(R) = (h\, \delta_\tau - g\, \delta_0) / [\, n(n - \delta_\tau - \delta_0)\,].$$

Proof. Since $f_\tau(R) = (g - \delta_\tau)/(n - \delta)$,

$g/n - f_\tau(R) = [\, g(n - \delta) - n(g - \delta_\tau)\,] / [\, n(n - \delta)\,].$

The numerator is

$gn - g\delta - ng + n\delta_\tau = n\delta_\tau - g\delta.$

Using $\delta = \delta_\tau + \delta_0$ and $n = g + h$,

$n\delta_\tau - g\delta = (g+h)\delta_\tau - g(\delta_\tau + \delta_0) = h\delta_\tau - g\delta_0.$

Moreover,

$\mathrm{Tr}(Q_\tau \Delta) = \delta_\tau - (g/n)\delta = (h\delta_\tau - g\delta_0)/n.$

Substitution gives both forms. The denominator $n - \delta = \mathrm{Tr}\, R$ is strictly positive by hypothesis, so both expressions are well defined. ■

4.3 Exact contrast form

Since Q_τ is traceless,

$\mathrm{Tr}(Q_\tau R) = -\mathrm{Tr}(Q_\tau \Delta).$

With $\rho_R = R / \mathrm{Tr}\, R$,

$f_\tau - g/n = \mathrm{Tr}(Q_\tau \rho_R) = -\mathrm{Tr}(Q_\tau \Delta) / (n - \mathrm{Tr}\, \Delta).$

Thus the maintained-sector contrast is exactly the negative normalised contrast of the unresolved closure.

4.4 Sector-average deficit anisotropy

Define the mean closure deficits per direction:

$\bar{\delta}_\tau = \delta_\tau / g$, $\bar{\delta}_0 = \delta_0 / h$,

and the closure anisotropy

$A_\tau^\Delta = \bar{\delta}_\tau - \bar{\delta}_0.$

Then $h\delta_\tau - g\delta_0 = gh\, A_\tau^\Delta$, and the identity becomes

$\mathbf{g/n - f_\tau = gh\, A_\tau^\Delta / [\, n(n - \delta)\,].}$

4.5 Sector-average maintenance form

Define the mean maintenance occupancies

$\bar{r}_\tau = (1/g)\, \mathrm{Tr}(\Pi_\tau R) = 1 - \bar{\delta}_\tau$, $\bar{r}_0 = (1/h)\, \mathrm{Tr}(\Pi_0 R) = 1 - \bar{\delta}_0.$

Then $A_\tau^\Delta = \bar{r}_0 - \bar{r}_\tau$. Since $\mathrm{Tr}\, R = g\, \bar{r}_\tau + h\, \bar{r}_0$, the identity may be written

$\mathbf{g/n - f_\tau = gh(\bar{r}_0 - \bar{r}_\tau) / [\, n(g\, \bar{r}_\tau + h\, \bar{r}_0)\,].}$

This makes the physical meaning transparent:

The tau share is suppressed exactly when the average maintained occupancy of a tau direction is lower than that of a complement direction.

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5. Exact Suppression, Equality, and Enhancement Criteria

5.1 Theorem 2 — Complete Sector-Sign Criterion

Statement. For every admissible nonzero allocation:

$f_\tau < g/n \Leftrightarrow \delta_\tau/g > \delta_0/h$, $f_\tau = g/n \Leftrightarrow \delta_\tau/g = \delta_0/h$, $f_\tau > g/n \Leftrightarrow \delta_\tau/g < \delta_0/h$.

Proof. The denominator in Theorem 1 is positive. The sign is therefore the sign of

$h\delta_\tau - g\delta_0 = gh(\delta_\tau/g - \delta_0/h)$. ■

5.2 Uniform deficit leaves the census unchanged

If $\Delta = \alpha I$ with $0 \le \alpha < 1$, then $\delta_\tau = \alpha g$ and $\delta_0 = \alpha h$, so

$\delta_\tau/g = \delta_0/h = \alpha$,

and $f_\tau = g/n$. A global reduction in maintenance does not alter sector fractions if it is isotropic.

5.3 Tau-localised deficit produces suppression

If $\Delta = \Pi_\tau \Delta \Pi_\tau$ and $\Delta \neq 0$, then $\delta_0 = 0$ and $\delta_\tau > 0$, so $f_\tau < g/n$.

5.4 Complement-localised deficit produces enhancement

If $\Delta = \Pi_0 \Delta \Pi_0$ and $\Delta \neq 0$, then $\delta_\tau = 0$ and $\delta_0 > 0$, so $f_\tau > g/n$.

5.5 Why finite capacity is insufficient by itself

A finite capacity may produce a nonzero Δ, but Theorem 2 shows that the sign depends on its sector distribution. Thus

$C < C_{full}$

does not by itself imply tau suppression. The variational ordering theorem is essential because it selects the location of Δ.

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6. The Finite-Capacity Primal Problem

6.1 Positive maintenance demand

Let $D_M > 0$ be a positive definite maintenance-demand operator on $\mathscr{H}_\ell$. It may be constructed as

$D_M = L_M^\dagger G_M L_M, \quad G_M > 0,$

from an independently derived maintenance-load map L_M and positive response metric G_M, with L_M injective so that D_M is strictly positive.

6.2 Premise P3 — Independent demand construction

The operator D_M must be fixed without using the intended tau suppression, observed tau mass, observed tau lifetime, or observed charged-lepton hierarchy as fitting inputs.

6.3 Maintenance cost

For an allocation R,

$\mathcal{C}(R) = \mathrm{Tr}(D_M R).$

Define the full-maintenance cost

$C_{full} = \mathrm{Tr}\, D_M.$

6.4 Finite capacity

Let $C \geq 0$ be the available maintenance capacity. For $C \geq C_{full}$ the constraint is inactive and $R = I$ is feasible and optimal, so the substantive analysis concerns $0 \leq C \leq C_{full}$. The admissible set is

$\mathcal{K}_C = \{ R : 0 \leq R \leq I, \mathrm{Tr}(D_M R) \leq C \}.$

6.5 Neutral support objective

Define the value function

$V(C) = \max_{\{R \in \mathcal{K}_C\}} \mathrm{Tr}\, R.$

Any optimiser is denoted R_C.

6.6 Premise P4 — Independent capacity

The capacity C must be fixed independently of the desired tau fraction.

6.7 Premise P5 — Neutral support value

The objective Tr R assumes equal gate-level benefit for one fully maintained direction in each sector. Section 17 treats a general benefit operator.

6.8 Lemma 1 — Existence

For every $C \geq 0$, the primal problem has an optimum.

Proof. The feasible set is nonempty because $R = 0$ is feasible. It is closed and bounded in finite dimension, hence compact. The objective is continuous. (For $C \geq C_{full}$ the feasible set is the whole operator interval and $R = I$ is optimal.) ■

6.9 Lemma 2 — Capacity saturation

For $0 < C < C_{full}$, every optimum satisfies

$\mathrm{Tr}(D_M R_C) = C.$

Proof. Suppose an optimum satisfies $\mathrm{Tr}(D_M R_C) < C$. Since $C < C_{full}$, $R_C \neq I$, so $\mathrm{Tr}(I - R_C) > 0$. For sufficiently small $\varepsilon > 0$,

$R_\varepsilon = R_C + \varepsilon(I - R_C)$

remains a positive contraction; by continuity of the linear cost, R_ε satisfies the capacity constraint for ε small; and

$\mathrm{Tr}\, R_\varepsilon = \mathrm{Tr}\, R_C + \varepsilon\, \mathrm{Tr}(I - R_C) > \mathrm{Tr}\, R_C.$

This contradicts optimality. ■

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7. Strong Duality and the Variational Threshold Certificate

7.1 Lagrangian

For a multiplier $\nu \geq 0$, define

$\mathscr{L}(R, \nu) = \mathrm{Tr}\, R + \nu[\, C - \mathrm{Tr}(D\_M R)\,] = \nu C + \mathrm{Tr}[\, (I - \nu D\_M) R\,]$.

7.2 Positive part

For a Hermitian operator A, define

$A_+ = (|A| + A)/2$.

Its trace is the sum of the positive eigenvalues of A.

7.3 Lemma 3 — Operator-Interval Maximisation

For Hermitian A,

$\max\_\{0 \leq R \leq I\}\ \mathrm{Tr}(AR) = \mathrm{Tr}\, A_+$.

Every maximiser has the form

$R = P\_\{A>0\} + X,\ 0 \leq X \leq P\_\{A=0\}$.

Proof. Let $A = \Sigma_i\, a_i\, |i\rangle\langle i|$ be a spectral decomposition. For any $0 \leq R \leq I$,

$\mathrm{Tr}(AR) = \Sigma_i\, a_i\, \langle i|R|i\rangle,\ 0 \leq \langle i|R|i\rangle \leq 1$,

so

$\mathrm{Tr}(AR) \leq \Sigma\_\{a_i>0\}\ a_i = \mathrm{Tr}\, A_+$.

Equality forces $\langle i|R|i\rangle = 1$ whenever $a_i > 0$ and $\langle i|R|i\rangle = 0$ whenever $a_i < 0$. These diagonal values then propagate to the full operator: if $\langle i|R|i\rangle = 1$, positivity of $I - R$ gives $\langle i|(I-R)|i\rangle = 0$, and since $I - R \geq 0$ a vanishing diagonal entry forces the whole row and column to vanish, so $R|i\rangle = |i\rangle$; if $\langle i|R|i\rangle = 0$, positivity of R likewise forces $R|i\rangle = 0$. Hence R acts as the identity on the positive spectral subspace, as zero on the negative one, and all cross-terms between these and the kernel of A vanish. Only a positive contraction X on the zero eigenspace remains free, and any such X attains the maximum. ■

7.4 Dual problem

Applying Lemma 3 to $A = I - \nu D_M$ gives the dual objective

$$G_C(\nu) = \nu C + \mathrm{Tr}(I - \nu D_M)_+.$$

The dual problem is

$$\inf_{\nu \geq 0} G_C(\nu).$$

7.5 Theorem 3 — Strong Primal–Dual Equality

For $0 < C < C_{full}$,

$$\mathbf{V(C) = \min_{\nu \geq 0} [\nu C + Tr(I - \nu D_M)_+].}$$

The minimum is attained.

Proof. The primal is a finite-dimensional convex programme: the objective is linear and the feasible set is the intersection of the operator interval $0 \leq R \leq I$ with a linear half-space. For $0 < C < C_{full}$, choose $R = \varepsilon I$ with $0 < \varepsilon < 1$ small enough that $\varepsilon \, \mathrm{Tr}\, D_M < C$. This point lies in the interior of the operator interval and satisfies the scalar capacity inequality strictly, so Slater's condition holds. Strong duality and dual attainment follow; dual attainment is also verified directly in Appendix B.2. ■

7.6 Optimal multiplier is positive

For $0 < C < C_{full}$, every optimal multiplier satisfies $\nu_C > 0$. If $\nu_C = 0$, the Lagrangian maximisation would select $R = I$, which costs $C_{full} > C$ and is infeasible; complementary slackness would then fail against Lemma 2.

7.7 Spectral threshold

Define

$$\lambda_C = 1/\nu_C.$$

Then

$$I - \nu_C D_M > 0 \Leftrightarrow D_M < \lambda_C.$$

7.8 Theorem 4 — Complete Variational Threshold Certificate

For every primal optimum R_C and every optimal multiplier ν_C,

$$\mathbf{R_C = P_{<\lambda_C} + X_C,}$$

where

$P_{<\lambda_C} = \mathbb{1}_{[0, \lambda_C)}(D_M), 0 \le X_C \le P_{=\lambda_C}, P_{=\lambda_C} = \mathbb{1}_{\{\lambda_C\}}(D_M).$

The capacity constraint fixes

$$\mathbf{\operatorname{Tr} X_C = [\, C - \operatorname{Tr}(D_M P_{<\lambda_C})\,] / \lambda_C.}$$

All components connecting the low-demand, threshold, and high-demand blocks vanish.

Proof. Strong duality and Lemma 2 give

$\operatorname{Tr} R_C = \mathscr{L}(R_C, \nu_C) = G_C(\nu_C),$

so R_C maximises the Lagrangian over $0 \le R \le I$. Lemma 3, applied to $A = I - \nu_C D_M$, first fixes the optimal diagonal values — one on the subspace $D_M < \lambda_C$, zero on $D_M > \lambda_C$ — and only then invokes positivity of R_C and $I - R_C$ to annihilate every row, column, and cross-block component attached to those extremal diagonal entries. The remaining freedom is a positive contraction X_C on the threshold eigenspace $D_M = \lambda_C$. Because $D_M = \lambda_C I$ on that block, capacity saturation (Lemma 2) gives

$C = \operatorname{Tr}(D_M P_{<\lambda_C}) + \lambda_C \operatorname{Tr} X_C,$

which is the stated trace formula. ■

Remark (proof order). The block-vanishing conclusion is available only after the extremal diagonal values are established; asserting it from positivity alone, before optimality fixes those values, is the proof-order error catalogued as F19.

7.9 Uniqueness

The optimum is unique if: (i) the threshold subspace is one-dimensional; or (ii) the required threshold occupancy is zero; or (iii) the threshold subspace is fully occupied. Otherwise, the residual freedom is exactly the choice of a positive contraction X_C on the threshold eigenspace with fixed trace. The operator need not be unique, but the spectral form is complete.

7.10 Value-function geometry

Since the dual formula expresses V as a pointwise minimum of affine functions of C, the function V is concave. It is also nondecreasing, continuous, and piecewise linear. Its kink points occur exactly at the cumulative costs of completed *distinct* demand eigenspaces (Appendix B.3–

B.4); a degenerate eigenspace contributes a single extended linear segment, not a breakpoint at every ordered-eigenvalue partial sum. For $D_M = I_2$, for instance, $V(C) = C$ on [0, 2] with no interior kink, even though the ordered-eigenvalue partial sum $C = 1$ lies inside the interval. Where differentiable,

$$V'(C) = \nu_C = 1/\lambda_C.$$

Thus the dual multiplier is the marginal maintained support obtained from one additional unit of capacity. At the finitely many breakpoints, the set of optimal multipliers is the superdifferential interval of V.

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8. The Complementary Closure-Release Problem

8.1 Capacity deficit

Define the full-capacity shortfall

$$\kappa_C = C_{full} - C.$$

8.2 Exact primal transformation

Set $\Delta = I - R$. Then $\mathrm{Tr}\, R = n - \mathrm{Tr}\, \Delta$, and

$$\mathrm{Tr}(D_M R) \le C \Leftrightarrow \mathrm{Tr}(D_M \Delta) \ge \kappa_C.$$

Therefore the maintenance problem is equivalent to

$$\boldsymbol{\Delta_C \in \arg\min \{ \mathrm{Tr}\, \Delta : 0 \le \Delta \le I, \mathrm{Tr}(D_M \Delta) \ge \kappa_C \}.}$$

8.3 Theorem 5 — Closure-Release Equivalence

For $0 < C < C_{full}$, the maps

$$R_C \leftrightarrow \Delta_C = I - R_C$$

give a bijection between primal maintenance optima and minimum-trace closure-release optima. Every optimum satisfies

$$\mathrm{Tr}(D_M \Delta_C) = \kappa_C.$$

Proof. The affine involution $R \mapsto I - R$ maps the operator interval bijectively onto itself, changes the objective by the constant n, and converts the capacity inequality exactly as shown. Equality follows from Lemma 2. ■

8.4 Closure-deficit spectral form

From Theorem 4,

$$\Delta_C = P_{>\lambda_C} + Y_C,$$

where

$$Y_C = P_{=\lambda_C} - X_C, \quad 0 \le Y_C \le P_{=\lambda_C}.$$

Thus unresolved closure fully occupies every mode with demand above the threshold and may partially occupy the threshold eigenspace.

8.5 Highest-demand-first release

As capacity is reduced from C_{full}, unresolved closure enters the highest-demand modes first. This is the exact complement of the lowest-demand-first maintenance theorem.

8.6 Closure dual

The closure-release Lagrangian is

$$\mathscr{L}_\Delta(\Delta, \eta) = \mathrm{Tr}\,\Delta + \eta[\, \kappa_C - \mathrm{Tr}(D_M \Delta)\,], \quad \eta \ge 0.$$

Minimising over $0 \le \Delta \le I$ gives

$$\inf \mathrm{Tr}[\, (I - \eta D_M)\, \Delta\,] = -\mathrm{Tr}(\eta D_M - I)_+.$$

Hence

$$n - V(C) = \max_{\eta \ge 0} [\, \eta\kappa_C - \mathrm{Tr}(\eta D_M - I)_+\,].$$

The optimal closure multiplier equals the optimal maintenance multiplier.

---

9. Strict Tau-Demand Separation and Deficit Localisation

9.1 Branch covariance

Assume

$[D\_M, \Pi\_\tau] = 0$.

Then $D\_M = D_0 \oplus D\_\tau$.

9.2 Premise P6 — Branch covariance

The demand operator must preserve the inherited tau and complement sectors, or a separate theorem must derive a stable physical spectral projector replacing them.

9.3 Strict tau-demand gap

Define

$d_0^+ = \sup \sigma(D_0)$, $d\_\tau^- = \inf \sigma(D\_\tau)$, $\Delta\_\tau\text{^}gap = d\_\tau^- - d_0^+$.

Assume

$\Delta\_\tau\text{^}gap > 0$.

Every tau demand then exceeds every complement demand.

9.4 Premise P7 — Independent tau-demand separation

The gap must be derived independently of the intended tau suppression and without using the observed tau mass or lifetime as retrospective labels.

9.5 Complement capacity

Define

$C_0 = \mathrm{Tr}(D\_M\ \Pi_0) = \mathrm{Tr}\ D_0$.

This is the cost of fully maintaining the complement. Also,

$C\_full = C_0 + \mathrm{Tr}\ D\_\tau$.

9.6 Theorem 6 — Variational Deficit-Localisation Theorem

Under P1–P7:

Low-capacity regime. For $0 < C \leq C_0$, every optimum has

$\Pi\_\tau R\_C = 0$, equivalently $\Pi\_\tau \Delta\_C = \Pi\_\tau$.

Partial-tau regime. For $C_0 < C < C\_full$, every optimum satisfies

$\Pi_0 R\_C = \Pi_0$,

and

$\Delta\_C = \Pi\_\tau \Delta\_C \Pi\_\tau$.

Moreover, $0 < \mathrm{Tr}\, \Delta\_C < g$.

Full-capacity regime. For $C \geq C\_full$, $R\_C = I$ is optimal and $\Delta\_C = 0$.

Proof. The threshold form fills demand eigenmodes in ascending order, and strict spectral separation places the whole complement spectrum below the entire tau spectrum. The boundary capacity $C = C_0$ requires separate treatment, because the optimal multiplier is nonunique there: the threshold $1/\nu\_C$ may lie anywhere in the closed gap $[d_0^+, d\_\tau^-]$, including at d_τ^- itself.

Interior low-capacity regime. For $0 < C < C_0$, every optimal threshold satisfies $\lambda\_C \leq d_0^+$. Suppose not. If λ_C lies in the open gap (d_0^+, d_τ^-), the threshold eigenspace is empty and Theorem 4 forces $R\_C = P\_\{<\lambda\_C\} = \Pi_0$ exactly, with cost $C_0 > C$ — contradicting capacity saturation (Lemma 2). If $\lambda\_C \geq d\_\tau^-$, then R_C contains Π_0 fully, so its cost is at least $C_0 > C$ — the same contradiction. Hence $\lambda\_C \leq d_0^+ < d\_\tau^-$, every tau mode lies strictly above the threshold, those modes are empty in R_C by Theorem 4, and their rows and columns vanish, giving $\Pi\_\tau R\_C = 0$.

Boundary $C = C_0$. Choose any λ strictly inside the nonempty gap, $d_0^+ < \lambda < d\_\tau^-$, and set $\nu = 1/\lambda$. Then $I - \nu D\_M$ is strictly positive on the complement and strictly negative on the tau sector, so by Lemma 3 the Lagrangian maximiser at this multiplier is *unique*: $R = \Pi_0$, with no threshold freedom. Its dual value is

$$G\_\{C_0\}(\nu) = \nu C_0 + \mathrm{Tr}(I - \nu D_0)_+ = \nu C_0 + (h - \nu C_0) = h,$$

which equals the objective $\mathrm{Tr}\, \Pi_0 = h$ of the feasible point Π_0 (cost exactly C_0). Hence Π_0 is optimal, and since every primal optimum must maximise the Lagrangian at this ν, it is the unique optimum. Therefore $\Pi\_\tau R\_\{C_0\} = 0$ also holds at the boundary, even though other optimal multipliers place the threshold at the edge d_τ^-.

Partial-tau regime. For $C_0 < C < C\_full$, every optimal threshold satisfies $\lambda\_C \geq d\_\tau^-$. Suppose instead $\lambda\_C \leq d_0^+$ or λ_C lies in the gap: then R_C is supported within the complement (Theorem 4), so its cost is at most $C_0 < C$ — contradicting capacity saturation (Lemma 2). Hence $\lambda\_C \geq d\_\tau^- > d_0^+$, every complement mode lies strictly below the threshold and is fully maintained: Π_0

$R_C = \Pi_0$, with vanishing cross-blocks. Therefore $\Delta_C = \Pi_\tau \Delta_C \Pi_\tau$. The strict inequalities on $\mathrm{Tr}\, \Delta_C$ follow because some but not all tau support is maintained.

Full capacity. For $C \geq C_{full}$, I is admissible and has maximal trace n. ■

9.7 Last-maintained, first-released identity

The tau branch is: last to receive maintenance as C rises; and first to carry unresolved closure as C falls. These are the same variational statement viewed from opposite sides of

$R_C + \Delta_C = I$.

---

10. Exact Tau-Only Suppression Identity

10.1 Tau-only closure deficit

In the partial-tau regime, $\delta_0 = 0$. Let

$\delta_C = \mathrm{Tr}\, \Delta_C = \delta_\tau$.

Then $0 < \delta_C < g$.

10.2 Theorem 7 — Exact Tau-Only Closure-Suppression Identity

For $C_0 < C < C_{full}$,

$$\mathbf{g/n - f_\tau(C) = h\, \delta_C / [\, n(n - \delta_C)\,],}$$

equivalently

$$\mathbf{f_\tau(C) = (g - \delta_C) / (n - \delta_C).}$$

Proof. Set $\delta_0 = 0$ in Theorem 1. ■

10.3 Exact inversion

Let

$S_\tau(C) = g/n - f_\tau(C)$.

Then $S_\tau = h\, \delta_C / [\, n(n - \delta_C)\,]$. Solving for δ_C,

$$\boldsymbol{\delta_C = n^2 S_\tau / (h + n S_\tau).}$$

Thus a claimed suppression magnitude uniquely specifies the unresolved closure trace required in the tau-only regime. This is an inverse audit, not a prediction.

10.4 Monotonicity and convexity in closure

Define

$$S(\delta) = h\delta / [\, n(n - \delta)\,].$$

Then

$$dS/d\delta = h / (n - \delta)^2 > 0, \quad d^2S/d\delta^2 = 2h / (n - \delta)^3 > 0.$$

Suppression increases monotonically and convexly with unresolved tau closure.

10.5 Proposition — Continuity and strict recovery in capacity

Across the partial-tau regime, $\delta_C = n - V(C)$ is continuous, piecewise linear, convex, and strictly decreasing in C (its slope is $-\nu_C < 0$ between breakpoints). Composing with the strictly increasing map $\delta \mapsto S(\delta)$ shows that $f_\tau(C)$ is continuous and strictly increasing throughout $C_0 < C < C_{full}$, with

$$f_\tau(C) \downarrow 0 \text{ as } C \downarrow C_0, \quad f_\tau(C) \uparrow g/n \text{ as } C \uparrow C_{full}.$$

The tau fraction therefore recovers monotonically toward census, with no plateaux and no overshoot.

10.6 Physical three-branch identity

For $g = 1$, $h = 2$, $n = 3$,

$$\boldsymbol{1/3 - f_\tau(C) = 2\delta_C / [\, 3(3 - \delta_C)\,].}$$

The inverse is

$$\boldsymbol{\delta_C = 9(1/3 - f_\tau) / [\, 2 + 3(1/3 - f_\tau)\,].}$$

---

11. Exact Capacity-Deficit Conversion

11.1 Capacity deficit carried by unresolved closure

For every saturated optimum,

$\kappa_C = C_{full} - C = \mathrm{Tr}(D_M \Delta_C)$.

11.2 Deficit-weighted mean demand

When $\Delta_C \neq 0$, define

$\bar{d}_\Delta(C) = \mathrm{Tr}(D_M \Delta_C) / \mathrm{Tr}\, \Delta_C$,

so that $\kappa_C = \bar{d}_\Delta \delta_C$. In the tau-only deficit regime,

$d_\tau^- \leq \bar{d}_\Delta \leq d_\tau^+$, where $d_\tau^+ = \sup \sigma(D_\tau)$,

because $\bar{d}_\Delta$ is a Δ_C-weighted average of tau-sector demand eigenvalues.

11.3 Theorem 8 — Exact Capacity-to-Suppression Identity

For $C_0 < C < C_{full}$,

$$\mathbf{g/n - f_\tau(C) = h\, \kappa_C / \{ n[n\, \bar{d}_\Delta(C) - \kappa_C] \}.}$$

Proof. The open interval guarantees $\Delta_C \neq 0$, so $\bar{d}_\Delta$ is defined (at $C = C_{full}$, excluded here, $\Delta_C = 0$ and the ratio degenerates). Since $\delta_C = \kappa_C / \bar{d}_\Delta$, substitute into Theorem 7:

$$h(\kappa_C/\bar{d}_\Delta) / [n(n - \kappa_C/\bar{d}_\Delta)] = h\, \kappa_C / [n(n\, \bar{d}_\Delta - \kappa_C)].$$

The denominator is positive because $\delta_C < n$. ■

11.4 Meaning

The scalar capacity deficit becomes predictive only after the variational solution specifies its mean demand conversion scale. The factor $\bar{d}_\Delta$ converts capacity units into unresolved carrier support.

11.5 Rank-one tau demand

If the tau sector is one-dimensional,

$D_\tau = d_\tau \Pi_\tau$,

then $\bar{d}_\Delta = d_\tau$ throughout the partial-tau regime. Therefore

$$1/3 - f_\tau(C) = 2(C_{full} - C) / \{ 3[3d_\tau - (C_{full} - C)] \}.$$

11.6 Direct fraction formula

Let $C_0 = d_e + d_\mu$ in the one-dimensional three-branch model. Since

$\delta_C = (C_{full} - C)/d_\tau$,

the maintained tau occupancy is

$1 - \delta_C = (C - C_0)/d_\tau$.

Hence

$$f_\tau(C) = (C - C_0) / (2d_\tau + C - C_0)$$

for $C_0 < C < C_{full}$.

11.7 Exact rank-one inversion

Let $S_\tau = 1/3 - f_\tau$. Then

$$C_{full} - C = 9\, d_\tau S_\tau / (2 + 3S_\tau).$$

Again, this reconstructs the required capacity deficit from a supplied suppression. It does not predict the suppression unless C and d_τ are independently derived.

---

12. Shadow-Price and Integrated-Closure Identity

12.1 Optimal capacity shadow price

Where V is differentiable,

$V'(C) = \nu_C$.

At the finitely many nondifferentiable capacities, the set of optimal dual multipliers is exactly the superdifferential of the concave value function; any selection from it may be used in the integral below without changing its value.

12.2 Total unresolved closure

Since $V(C) = \mathrm{Tr}\, R_C$ and $\mathrm{Tr}\, \Delta_C = n - \mathrm{Tr}\, R_C$,

$\delta_C = n - V(C)$, $V(C_{full}) = n$.

12.3 Theorem 9 — Integrated Shadow-Price Closure Identity

For every $C \in [0, C_{full}]$,

$$\mathbf{Tr}\, \Delta_C = n - V(C) = \int_C^{C_{full}} \nu_u \, du,$$

where $\nu_u = V'(u)$ at differentiability points. The equality is independent of the values assigned to ν_u on the finite set of spectral kink points.

Proof. The finite-dimensional value function is continuous and piecewise linear (Section 7.10), hence absolutely continuous. Therefore

$$V(C_{full}) - V(C) = \int_C^{C_{full}} V'(u)\, du,$$

with the derivative existing off a finite set. Using $V(C_{full}) = n$ and $V'(u) = \nu_u$ almost everywhere gives the result; altering the integrand on a null set does not change the Lebesgue integral. ■

12.4 Theorem 10 — Variational–Closure Suppression Identity

In the tau-only deficit regime,

$$g/n - f_\tau(C) = h \int_C^{C_{full}} \nu_u \, du \Big/ \left\{ n\left[n - \int_C^{C_{full}} \nu_u \, du \right] \right\}.$$

This is the named gate identity.

Proof. Substitute Theorem 9 into Theorem 7. ■

12.5 Threshold form

Where the threshold is well defined, $\nu_C = 1/\lambda_C$. Thus

$\delta_C = \int_C^{C_{full}} du / \lambda_u$,

and the suppression may be written

$$g/n - f_\tau(C) = h \int_C^{C_{full}} \lambda_u^{-1}\, du \,/\, \{ n[n - \int_C^{C_{full}} \lambda_u^{-1}\, du] \}.$$

12.6 Rank-one tau interval

If the entire partial-tau interval has one tau demand d_τ, then $\lambda_u = d_\tau$ and $\nu_u = 1/d_\tau$. Hence

$\delta_C = (C_{full} - C)/d_\tau$,

recovering the exact rank-one formula.

12.7 Interpretation

The multiplier ν_C is not automatically a physical energy, coupling, temperature, or decay rate. It is the variational marginal value of maintenance capacity:

ν_C = d(maximum maintained support) / d(capacity).

Its integral measures the support lost between C and full maintenance.

---

13. Complete Capacity Regimes

13.1 Theorem 11 — Three-Regime Tau Law

Under strict tau-demand separation:

Regime I — Complement filling. For $0 < C \le C_0$,

$f_\tau(C) = 0$.

The closure data satisfy $\delta_\tau = g$ and $0 \le \delta_0 < h$. The universal identity gives

$g/n - f_\tau(C) = g/n$.

Regime II — Tau-only closure deficit. For $C_0 < C < C_{full}$,

$0 < f_\tau(C) < g/n$,

and

$g/n - f_\tau(C) = h\, \delta_C / [\, n(n - \delta_C)\,].$

Regime III — Full maintenance. For $C \geq C_{full}$,

$R_C = I$, $\Delta_C = 0$, $f_\tau(C) = g/n$.

13.2 Boundary at C = 0

If $D_M > 0$, then the only feasible allocation at $C = 0$ is $R = 0$. The normalised fraction $f_\tau = \mathrm{Tr}(\Pi_\tau R)/\mathrm{Tr}\, R$ is then undefined. The low-capacity result is therefore stated for $C > 0$, with

$\lim_{C\downarrow 0} f_\tau(C) = 0$

under strict tau-last ordering.

13.3 Continuity

The function $f_\tau(C)$ is continuous on $(0, C_{full}]$: it vanishes identically on Regime I, approaches zero as $C \downarrow C_0$ from above, is strictly increasing across Regime II (Proposition 10.5), and approaches g/n as $C \uparrow C_{full}$.

13.4 No overshoot

Under the neutral objective and strict tau-last ordering,

$0 \leq f_\tau(C) \leq g/n$

for every capacity where the fraction is defined. An overshoot would require a different demand ordering, benefit structure, carrier, or participation definition.

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14. Uniqueness, Degeneracy, and Threshold-Sector Purity

14.1 Operator nonuniqueness

If the threshold eigenspace has dimension greater than one and is only partially occupied, the detailed operator X_C is not unique.

14.2 Why branch covariance alone is insufficient

The condition $[D\_M, \Pi\_\tau] = 0$ does not by itself ensure that $Tr(\Pi\_\tau X\_C)$ is fixed across all optimal X_C. A degenerate demand eigenspace may contain both tau and complement directions. Different threshold allocations can then have the same trace and cost but different tau occupancy.

14.3 Counterexample

Let $D\_M = I_2$ and $C = 1$, with one tau and one complement direction. Every positive contraction R satisfying $Tr R = 1$ is optimal, yet $Tr(\Pi\_\tau R)$ can range from zero to one. Therefore branch covariance without cross-sector spectral separation does not guarantee a unique sector fraction.

14.4 Theorem 12 — Aggregate Uniqueness Under Strict Separation

Under strict tau-demand separation, the tau fraction — and with it each sector deficit trace δ_τ, δ_0 — is identical across the entire primal optimum set at every capacity $C > 0$. At $C = 0$ the unique feasible allocation is $R = 0$, so f_τ is undefined (Section 13.2), although the sector deficit traces $\delta\_\tau = g$ and $\delta_0 = h$ remain unique.

Proof. For $0 < C < C_0$, every optimal threshold lies in the complement spectrum by the saturation argument of Theorem 6, and every optimum has zero tau support. At $C = C_0$, the optimum is uniquely $R = \Pi_0$ by the gap-multiplier argument of Theorem 6. In either case $f\_\tau = 0$, $\delta\_\tau = g$, and $\delta_0 = h - V(C)$ are common to all optima.

For $C_0 < C < C\_full$, every optimal threshold eigenspace lies wholly in the tau sector. Since $D\_M = \lambda\_C I$ there, the capacity fixes $Tr X\_C$ (Theorem 4). Because Π_τ acts as the identity on that threshold space,

$Tr(\Pi\_\tau X\_C) = Tr X\_C$

is fixed, hence so are $\delta\_\tau = \delta\_C = n - V(C)$ and $\delta_0 = 0$.

For $C \geq C\_full$, the optimum is unique outright: $Tr R = n$ with $R \leq I$ forces $R = I$, giving $f\_\tau = g/n$ and $\delta\_\tau = \delta_0 = 0$.

Thus the detailed threshold operator may vary, but the aggregate sector data do not. ■

14.5 Correct uniqueness statement

The appropriate claim is:

Under strict cross-sector spectral separation, threshold degeneracy may leave the detailed operator nonunique, but it does not make the aggregate tau-maintenance fraction — or the sector deficit traces entering the suppression identity — nonunique.

Without strict separation, this conclusion fails.

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15. Sharp Bounds and Recovery Laws

15.1 Demand bounds

In the tau-only regime,

$d_\tau^- \leq \bar{d}_\Delta \leq d_\tau^+$.

Since $\delta_C = \kappa_C / \bar{d}_\Delta$,

$\kappa_C/d_\tau^+ \leq \delta_C \leq \kappa_C/d_\tau^-$.

15.2 Theorem 13 — Sharpened Lower Bound

For $C_0 < C < C_{full}$,

$$\mathbf{g/n - f_\tau(C) \geq h\, \kappa_C / [\, n(n\, d_\tau^+ - \kappa_C)\,].}$$

Proof. The function $S(\delta) = h\delta/[n(n - \delta)]$ is increasing. Use $\delta_C \geq \kappa_C/d_\tau^+$. ■

15.3 Earlier linear floor as a corollary

Since $n - \delta_C \leq n$,

$$\mathbf{g/n - f_\tau(C) \geq h\, \kappa_C / (n^2\, d_\tau^+).}$$

This is the earlier capacity-controlled floor, now derived from the exact identity, which shows it is never tight except at $\kappa_C = 0$.

15.4 Conditional upper bound

If $\kappa_C < n\, d_\tau^-$, then

$$\mathbf{g/n - f_\tau(C) \leq h\, \kappa_C / [\, n(n\, d_\tau^- - \kappa_C)\,].}$$

In all cases,

$g/n - f_\tau(C) < g/n$

throughout the partial-tau regime.

15.5 Rank-one derivatives

For a single tau demand d_τ, define $\kappa = C_{full} - C$. Then

$S(\kappa) = h\kappa / [\, n(n\, d_\tau - \kappa)\,]$,

with

$dS/d\kappa = h\, d_\tau / (n\, d_\tau - \kappa)^2 > 0$, $d^2S/d\kappa^2 = 2h\, d_\tau / (n\, d_\tau - \kappa)^3 > 0$.

Since $d\kappa/dC = -1$,

$df_\tau/dC = h\, d_\tau / (n\, d_\tau - \kappa)^2 > 0$,

and

$d^2f_\tau/dC^2 = -2h\, d_\tau / (n\, d_\tau - \kappa)^3 < 0$.

The tau fraction therefore recovers monotonically but *concavely* toward census as capacity increases: the marginal recovery per unit of restored capacity diminishes as full maintenance is approached. Equivalently, read in the opposite direction of variation, suppression grows at an accelerating (convex) rate as capacity is withdrawn.

15.6 Full-capacity expansion

For small capacity deficit κ, in the rank-one tau case,

$g/n - f_\tau(C) = [\, h/(n^2\, d_\tau)\,]\, \kappa + O(\kappa^2)$.

Thus the previous linear deficit floor is also the leading-order recovery law near full maintenance.

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16. Soft Saturation and Regularised Closure

16.1 Soft maintenance allocation

Let $s : [0, \infty) \to [0, 1]$ be a non-increasing function and define

$R_s = s(D_M)$, $\Delta_s = I - s(D_M)$.

The profile $1 - s(d)$ is non-decreasing with demand.

16.2 The universal identity remains exact

No optimisation assumption is needed for Theorem 1. Therefore

$$g/n - f_\tau(R_s) = \mathrm{Tr}(Q_\tau \Delta_s) / (n - \mathrm{Tr}\, \Delta_s).$$

16.3 Theorem 14 — Soft Closure-Suppression Theorem

Assume branch covariance, strict tau-demand separation, and $\mathrm{Tr}\, s(D_M) > 0$, so that the normalised fraction is defined (the trivial profile $s \equiv 0$ is excluded). If s is non-increasing, then

$$f_\tau(R_s) \leq g/n.$$

If in addition

$$s(d_0^+) > s(d_\tau^-),$$

then

$$f_\tau(R_s) < g/n.$$

Proof. Every tau eigenvalue has demand greater than every complement eigenvalue. Since s is non-increasing, every complement occupancy is at least $s(d_0^+)$ and every tau occupancy is at most $s(d_\tau^-)$; therefore $\bar{r}_\tau \leq \bar{r}_0$, with strict inequality under the strict-drop condition $s(d_0^+) > s(d_\tau^-)$. The sector-average form of Theorem 1 fixes the sign. ■

The strict-drop condition is the load-bearing form of "s decreases strictly across the separating spectral interval": any strict decrease of s anywhere on $[d_0^+, d_\tau^-]$ implies it, by monotonicity, regardless of where within the interval the decrease occurs.

16.4 Soft profiles need not produce tau-only closure

Unlike the hard capacity optimum, a smooth profile may leave nonzero deficits in both sectors. Therefore the exact general identity

$$(h\, \delta_\tau - g\, \delta_0) / [\, n(n - \delta)\,]$$

must be used unless a separate localisation theorem proves $\delta_0 = 0$.

16.5 Capacity for a soft profile

If one defines

$C_s = \mathrm{Tr}(D_M R_s)$,

then

$C_{full} - C_s = \mathrm{Tr}(D_M \Delta_s)$.

However, this scalar still does not determine the sector suppression unless the deficit location is known.

---

17. General Benefit-to-Demand Extension

17.1 Positive benefit operator

Let $B_M \geq 0$ represent independently derived maintained benefit. Consider

$V_B(C) = \max \{ \mathrm{Tr}(B_M R) : 0 \leq R \leq I, \mathrm{Tr}(D_M R) \leq C \}$.

A cleaner sufficient hypothesis, adopted wherever indicated below, is strict positivity, $B_M > 0$: then every unfilled direction strictly improves the objective, so for every $0 < C < C_{full}$ the capacity constraint is saturated and the optimal multiplier is strictly positive.

17.2 General dual

The Lagrangian is

$\mathscr{L}_B(R, \nu) = \nu C + \mathrm{Tr}[(B_M - \nu D_M) R]$.

Therefore

$$\mathbf{V_B(C) = \min_{\nu \geq 0} [\nu C + \mathrm{Tr}(B_M - \nu D_M)_+]}$$

under the same strict-feasibility conditions.

17.3 General threshold certificate

Every optimum has the form

$$\mathbf{R_C = P_{\{B_M - \nu_C D_M > 0\}} + X_C,\ 0 \leq X_C \leq P_{\{B_M - \nu_C D_M = 0\}}.}$$

When $\nu_C > 0$, complementary slackness gives capacity saturation, $\mathrm{Tr}(D_M R_C) = C$, and the capacity then fixes $\mathrm{Tr}(D_M X_C)$ — not generally $\mathrm{Tr}\, X_C$. When $\nu_C = 0$ — possible when B_M has a kernel, so the objective saturates before the capacity is exhausted — only the capacity *inequality* remains: the optimum need not exhaust C, and zero-benefit threshold directions may be occupied nonuniquely without capacity fixing anything about them. Under $B_M > 0$ this degeneracy is excluded and $\nu_C > 0$ throughout $0 < C < C_{full}$.

17.4 Commuting benefit and demand

If $[B_M, D_M] = 0$, let

$B_M |i\rangle = b_i |i\rangle$, $D_M |i\rangle = d_i |i\rangle$.

A mode is fully maintained when $b_i - \nu_C d_i > 0$. Define its efficiency

$\eta_i = b_i / d_i$.

Modes are filled in decreasing order of η_i.

17.5 Strict tau-efficiency separation

Assume, in addition to $[B_M, D_M] = 0$, the branch-covariance conditions

$[B_M, \Pi_\tau] = 0$, $[D_M, \Pi_\tau] = 0$,

so that the efficiency operator $B_M D_M^{-1}$ is well defined on the invariant tau and complement sectors; without these, the restrictions below are not defined on physical sectors. Suppose then

$\sup \sigma(B_M D_M^{-1}|_{\mathcal{H}_\tau}) < \inf \sigma(B_M D_M^{-1}|_{\mathcal{H}_0})$.

Then every complement mode is more efficient than every tau mode. The tau branch is admitted after the complement, and the universal closure identity again yields

$f_\tau < g/n$

throughout incomplete tau maintenance.

17.6 Degenerate efficiency caveat

A threshold-efficiency subspace may contain modes with different demands. In that case, capacity fixes $\mathrm{Tr}(D_M X_C)$, but not necessarily $\mathrm{Tr}\, X_C$. Even if the threshold subspace lies wholly in the tau sector, different optimal threshold allocations may then produce different unweighted tau-maintenance fractions.

Aggregate uniqueness requires an additional condition, such as: D_M proportional to the identity on the threshold-efficiency space; a one-dimensional threshold; a secondary variational principle fixing X_C; or a participation observable using the same weighted benefit measure.

17.7 Neutral case

For $B_M = I$, the threshold kernel is an eigenspace of D_M, so D_M is automatically proportional to the identity there. The neutral theorem therefore retains aggregate uniqueness under strict cross-sector separation.

17.8 Benefit-weighted shadow identity

The unweighted identity $\mathrm{Tr}\,\Delta_C = \int_C^{C_{full}} \nu_u\,du$ of Theorem 9 is specific to the neutral objective $B_M = I$. For a general benefit objective the correct analogue weights the closure deficit by the benefit, and its regularity argument differs from the neutral case: when $[B_M, D_M] \neq 0$, the eigenvectors of $B_M - \nu D_M$ can rotate continuously with ν, and the optimal benefit frontier can be curved rather than piecewise linear. Piecewise linearity may therefore not be assumed. Concavity and a Lipschitz bound suffice instead.

Define the maximal benefit-to-demand ratio

$$\eta_{max} = \| D_M^{-1/2} B_M D_M^{-1/2} \|_\infty.$$

Then

$$0 \leq B_M \leq \eta_{max} D_M,$$

so an extra unit of capacity can purchase at most η_{max} units of benefit: V_B is finite, nondecreasing, concave, and Lipschitz with constant η_{max} on $[0, C_{full}]$. It is therefore absolutely continuous and differentiable almost everywhere. Since $V_B(C_{full}) = \mathrm{Tr}\,B_M$ and $V_B'(u) = \nu_u$ wherever differentiable,

$$\mathbf{Tr(B_M\,\Delta_C) = Tr\,B_M - V_B(C) = \int_C^{C_{full}} \nu_u\,du.}$$

If $[B_M, D_M] = 0$, the problem reduces to a finite fractional-knapsack ordering and V_B is additionally piecewise linear, recovering the neutral-case geometry.

The integral therefore measures accumulated *benefit-weighted* unresolved closure, not the unweighted trace $\mathrm{Tr}\,\Delta_C$; and by Section 17.6 the unweighted tau fraction may remain nonunique under threshold-efficiency degeneracy.

The division of labour is therefore:

- **Neutral objective ($B_M = I$):** the full VCS chain holds — exact tau-only identity, unweighted capacity conversion, and unweighted shadow integral (Theorems 7–10), with aggregate uniqueness under strict separation.
- **General strictly positive benefit objective:** strict tau-efficiency separation still yields the suppression sign, and the shadow integral yields the benefit-weighted closure identity

above; the unweighted magnitude is fixed only under the additional threshold conditions of Section 17.6.

---

18. Basis Covariance and Perturbative Stability

18.1 Theorem 15 — Basis Covariance

Under a unitary transformation

$D_M \mapsto U D_M U^\dagger$, $R_C \mapsto U R_C U^\dagger$, $\Delta_C \mapsto U \Delta_C U^\dagger$, $\Pi_\tau \mapsto U \Pi_\tau U^\dagger$,

all of the following are invariant: the capacity; the objective; the primal and dual optimum values; the threshold spectrum; the closure-deficit trace; the tau-maintenance fraction; the universal suppression identity.

Proof. Unitary conjugation preserves positivity, operator order, trace, spectrum, and trace pairings. ■

18.2 Inconsistent transformation

Transforming D_M or R_C while leaving Π_τ fixed changes the physical branch relation. It is not a passive basis transformation.

18.3 Branch-preserving perturbation

Let

$\tilde{D}_M = D_M + E$, $E = E^\dagger$, $[E, \Pi_\tau] = 0$.

18.4 Theorem 16 — Gap and Positivity Stability

If

$\|E\|_\infty < \min\{ \lambda_{min}(D_M), \tfrac{1}{2} \Delta_\tau^{gap} \}$,

then the perturbed demand operator remains strictly positive,

$\tilde{D}_M = D_M + E > 0$,

and the strict cross-sector ordering survives,

$\sup \sigma(\tilde{D}_0) < \inf \sigma(\tilde{D}_\tau)$,

with corrected gap at least

$\Delta_\tau^{gap} - 2\|E\|_\infty$.

Proof. Weyl's inequalities give $\lambda_{min}(\tilde{D}_M) \geq \lambda_{min}(D_M) - \|E\|_\infty > 0$, establishing positivity, and

$\sup \sigma(\tilde{D}_0) \leq d_0^+ + \|E\|_\infty$, $\inf \sigma(\tilde{D}_\tau) \geq d_\tau^- - \|E\|_\infty$.

Subtracting gives the gap bound. ■

Remark. The gap condition alone does not suffice: a large negative branch-preserving perturbation can preserve the cross-sector ordering while driving demand eigenvalues negative, destroying the positivity premise P3 on which the entire finite-capacity interpretation rests. The positivity clause is therefore load-bearing, not decorative.

18.5 Stability of the suppression identity

The algebraic identity itself is exact under every perturbation because it depends only on R, Δ, and Π_τ. What perturbations may change is the variational localisation of Δ. Under Theorem 16, the tau-only localisation and suppression sign persist, provided the capacity remains in the corrected partial-tau interval

$\tilde{C}_0 < C < \tilde{C}_{full}$,

whose endpoints shift by at most $h\|E\|_\infty$ and $n\|E\|_\infty$ respectively.

18.6 Branch-mixing corrections

If $[E, \Pi_\tau] \neq 0$, small norm alone does not preserve the original branch projector. A complete analysis then requires: spectral-subspace perturbation theory; a physical rule identifying the corrected tau sector; control of the angle between original and corrected projectors; observable matching in the corrected frame.

VCS-1 does not silently include that bridge.

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19. Non-Insertion and Underdetermination Theorems

19.1 Identity versus prediction

The universal identity determines the suppression once Δ is known. It does not derive Δ. The variational theorem derives Δ only after specifying: the carrier; the demand operator; the capacity; the objective; the sector projectors.

19.2 Theorem 17 — Capacity-Only Sign No-Go

A fixed scalar capacity deficit does not determine whether the tau fraction is suppressed, unchanged, or enhanced.

Proof by construction. Take $D_M = I_n$ and choose $0 < \kappa \leq \min(g, h)$. All three closure operators below satisfy $\mathrm{Tr}(D_M \Delta) = \kappa$.

Tau-localised deficit. $\Delta_\tau = (\kappa/g) \Pi_\tau$. Then $\delta_\tau/g = \kappa/g$ and $\delta_0/h = 0$, so the tau fraction is suppressed.

Isotropic deficit. $\Delta_{iso} = (\kappa/n) I$. Then $\delta_\tau/g = \delta_0/h = \kappa/n$, so the census fraction is unchanged.

Complement-localised deficit. $\Delta_0 = (\kappa/h) \Pi_0$. Then $\delta_\tau/g = 0$ and $\delta_0/h = \kappa/h$, so the tau fraction is enhanced.

All three have the same capacity deficit. ■

19.3 Consequence

The scalar shortfall $C_{full} - C$ cannot determine the sector contrast without a localisation principle. The variational demand ordering supplies that missing information.

19.4 Theorem 18 — Constructive Magnitude Underdetermination

Every target suppression

$S\star \in [0, g/n)$

can be realised by a tau-only closure deficit satisfying $0 \leq \Delta < I$.

Proof. Set

$\delta\bigstar = n^2 S\bigstar / (h + n S\bigstar)$.

For $0 \le S\bigstar < g/n$, the map $S \mapsto n^2S/(h + nS)$ is increasing with value 0 at $S = 0$ and value g at $S = g/n$, so $0 \le \delta\bigstar < g$. Choose

$\Delta\bigstar = (\delta\bigstar/g)\, \Pi_\tau$.

Then Theorem 7 gives exactly $S\bigstar$. ■

19.5 Corollary

The closure-suppression identity fixes the conversion law, not the numerical input. A numerical prediction requires an independent derivation of the closure deficit or of all quantities determining it.

19.6 Theorem 19 — Rank-One Inverse Reconstruction Is Not Prediction

In the one-dimensional tau model, any target $f\bigstar \in [0, 1/3)$ determines a required capacity

$C = C_{full} - d_\tau \cdot 9(1/3 - f\bigstar) / [\, 2 + 3(1/3 - f\bigstar)\,]$.

Selecting C from the desired $f\bigstar$ is inverse reconstruction, not first-principles prediction.

19.7 No-target-fitting requirement

A predictive application requires

$\{ m_\tau, \tau_\tau, \Gamma_\tau, y_\tau, f_\tau^{observed} \} \cap \mathrm{Inputs}(D_M, C, B_M, \Pi_\tau) = \emptyset$,

unless the relevant quantity is explicitly declared as empirical calibration.

19.8 Independence grades

VCS-1 distinguishes:

Grade I — Structural identity. The universal identity is exact once R is given.

Grade II — Sign prediction. The sign is predictive if the tau-last demand or efficiency ordering is independently derived.

Grade III — Magnitude floor. A lower bound becomes predictive if a capacity margin and an upper demand bound are independently established.

Grade IV — Numerical internal prediction. A numerical maintenance fraction requires the demand spectrum, capacity, objective, and threshold conventions.

Grade V — Physical observable prediction. An experimental number additionally requires a gauge-invariant observable bridge, state, scale, and measurement functional.

---

20. Mass, Lifetime, and Observable Firewalls

20.1 Closure suppression is not particle disappearance

Even if $\Pi_\tau R_C = 0$, the tau sector remains in the carrier: $\Pi_\tau \neq 0$. The result means that the declared finite-capacity maintenance allocation assigns no continuing support to it. It does not remove the tau field from the theory.

20.2 Closure deficit is not mass

The operator Δ_C is dimensionless under the present normalisation. A physical mass requires an independently derived dimensionful map, for example

$$M_{phys} = \mathscr{F}(\Delta_C, D_M, \ldots),$$

with field normalisation, electroweak symmetry breaking, and renormalisation specified. No such map is derived here.

20.3 Conditional mass bridge

If a later gate independently proves a monotone relation

$$m_a = \mathscr{M}(\bar{r}_a), \quad d\mathscr{M}/d\bar{r} < 0,$$

then $\bar{r}_\tau < \bar{r}_0$ would imply a larger tau mass scale. This remains conditional.

20.4 Closure deficit is not a lifetime

A lifetime requires: a Hamiltonian or effective action; transition amplitudes; available final states; phase-space integration; coupling normalisation; radiative corrections; an asymptotic-time interpretation. The maintenance deficit supplies none of these by itself.

20.5 Closure deficit is not a Yukawa coupling

The charged-lepton Yukawa matrix acts between left-handed and right-handed electroweak representations and the Higgs sector. A maintenance operator on a finite carrier cannot be identified with a Yukawa eigenvalue without deriving: chiral embedding; Higgs-radial coupling; field normalisation; scale and renormalisation scheme; mass-frame diagonalisation.

20.6 Static optimisation is not dynamics

The variational principle specifies an optimum. It does not yet specify: a relaxation equation for the allocation along the emergent ordering parameter; a mechanism driving the allocation toward the optimum; an attractor proof; convergence rates; fluctuations around the optimum; a conservation law generating C.

The correct present description is finite-capacity maintenance optimisation, not yet a complete maintenance dynamics.

20.7 Observable extraction

A measurable tau suppression requires a gauge-invariant observable $\mathcal{O}$ and a derived decomposition

$$\mathcal{O} = \mathcal{O}_\tau + \mathcal{O}_0$$

such that $\mathcal{O}_\tau/\mathcal{O}$ is proven to equal or depend controllably on $f_\tau(C)$. No such observable is supplied by VCS-1.

20.8 Electroweak representation firewall

The physical charged leptons involve different left- and right-handed gauge representations. A complete application must state: whether the carrier is pre- or post-electroweak breaking; whether chirality is explicit; how Π_τ spans left and right components; how gauge covariance is maintained; how the physical mass frame is selected.

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21. Premise-Dependency Map

21.1 Named premises

Premise	Content
P1	One consistently counted finite maintenance carrier

Premise	Content
P2	Physical tau and complement projectors
P3	Independently constructed positive demand operator
P4	Independently specified finite capacity
P5	Neutral maintained-support objective
P6	Branch covariance of the demand operator
P7	Strict tau-demand separation
P8	Maintenance fraction is the declared internal response quantity

21.2 Result dependencies

Result	P1	P2	P3	P4	P5	P6	P7	P8
Universal closure identity	●	●						●
Exact sign criterion	●	●						●
Primal existence	●		●	●	●			
Capacity saturation	●		●	●	●			
Strong duality	●		●	●	●			
Threshold certificate	●		●	●	●			
Closure-release equivalence	●		●	●	●			
Tau localisation	●	●	●	●	●	●	●	
Exact tau-only identity	●	●	●	●	●	●	●	●
Strict recovery monotonicity	●	●	●	●	●	●	●	●
Capacity conversion	●	●	●	●	●	●	●	●
Shadow-price identity	●		●	●	●			
Named VCS identity	●	●	●	●	●	●	●	●
Aggregate uniqueness	●	●	●	●	●	●	●	●
Soft sign theorem	●	●	●			●	●	●
Perturbative sign stability	●	●	●	●	●	●	●	●
Capacity-only no-go	audit theorem; requires no physical premises							
Magnitude underdetermination	audit theorem; requires no physical premises							

21.3 Load-bearing physical premises

The most important physical dependencies are:

1. **P3:** D_M must have an independent VERSF origin;
2. **P4:** C must have an independent VERSF origin;

3. **P5:** the objective must be physically justified;
4. **P7:** the tau-last demand ordering must be derived rather than inferred from observation.

The universal identity itself is stronger than these premises. It remains true even if the VERSF interpretation fails.

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22. Falsification Conditions

#	Failure condition	Consequence
F1	The carrier is not explicitly defined	No valid census or denominator
F2	Tau and complement sectors use different hidden multiplicities	Rank comparison invalid
F3	Π_τ is a coordinate label rather than a physical projector	Basis-dependent result
F4	R is not positive	Maintenance interpretation fails
F5	$R \npreceq I$ without a replacement interpretation	$\Delta = I - R$ need not be positive
F6	Tr R = 0 but a normalised fraction is quoted	Division by zero
F7	Δ is called a missing-particle operator	Category error
F8	A finite capacity is assumed to imply tau suppression without localisation	Theorem 2 violated
F9	The universal identity is presented as a derivation of Δ	Identity/prediction confusion
F10	The maintenance-load map is not specified	Demand origin unexplained
F11	The response metric is not positive	Demand positivity fails
F12	D_M is fitted from the desired tau result	Hidden insertion
F13	Observed mass ordering is used to define the tau-demand ordering	Circularity
F14	Capacity is chosen to reproduce a target suppression	Magnitude insertion
F15	Capacity exceeds full-maintenance cost but saturation is still claimed	Wrong regime
F16	An optimum below full capacity does not exhaust capacity	Violates Lemma 2
F17	The dual positive-part term is omitted	Incorrect dual problem
F18	The multiplier threshold is asserted without strong duality or KKT conditions	Incomplete proof
F19	Off-block vanishing is attributed to positivity before optimal diagonal values are established	Proof-order error

#	Failure condition	Consequence
F20	Every optimum is claimed unique despite partial degenerate threshold occupancy	Uniqueness overclaim
F21	Branch covariance alone is claimed to fix the tau fraction across a mixed threshold eigenspace	False
F22	Strict cross-sector separation is absent	Tau-only localisation may fail
F23	Tau and complement demand spectra overlap	Strict tau-last theorem fails
F24	The demand gap is smaller than uncontrolled corrections	Sign instability
F25	A branch-mixing perturbation is treated as covered by the branch-preserving theorem	Projector ambiguity
F26	A soft profile is chosen from the desired result	Profile insertion
F27	A soft profile increasing with demand is called maintenance suppression	Physical inconsistency
F28	The tau-only identity is applied when $\delta_0 \neq 0$	Wrong formula
F29	The capacity identity is applied without defining d̄_Δ	Missing conversion scale
F30	A finite capacity gap is said to guarantee a finite suppression floor while d_τ^+ is uncontrolled	Bound overclaim
F31	A one-sided bound is called two-sided control	Mathematical overstatement
F32	The shadow price is called a physical energy without a dimensional bridge	Type error
F33	Nondifferentiable value-function points are treated as having a unique multiplier	Dual overclaim
F34	The integrated multiplier is evaluated without respecting the almost-everywhere derivative	Calculus error
F35	In the benefit model, capacity is said to fix Tr X when only Tr(D_M X) is fixed	Degeneracy error
F36	Unequal benefit is ignored despite being dynamically present	Objective incompleteness
F37	Tau efficiency is not lower in the benefit model	Suppression need not follow
F38	Exact zero maintenance is interpreted as tau nonexistence	Category error
F39	Maintenance suppression is called a mass theorem	Missing mass bridge
F40	Maintenance suppression is called a lifetime theorem	Missing decay bridge
F41	Maintenance suppression is identified with a Yukawa coupling	Operator-type error
F42	Static optimisation is described as a derived evolution law	Dynamics overclaim

#	Failure condition	Consequence
F43	A physical observable is claimed without a measurement functional	Observable failure
F44	Left- and right-handed sectors are combined inconsistently	Electroweak counting failure
F45	The carrier changes with scale without rematching	Scale mismatch
F46	A fitted R is renamed a variational prediction	Hidden construction
F47	The same capacity deficit is assumed to produce one universal sector contrast	Refuted by Theorem 17
F48	Passing VCS-1 is described as empirical confirmation	Admissibility/truth confusion
F49	A numerical suppression is quoted without independent D_M, C, and objective data	Underdetermination
F50	The inverse reconstruction formula is presented as forward prediction	Direction-of-inference error
F51	Ordered-eigenvalue partial sums inside a degenerate eigenspace are treated as value-function kinks	Degenerate-spectrum error
F52	The unweighted shadow-price identity is applied under a non-neutral benefit objective	Wrong shadow identity; use Tr(B_M Δ_C)
F53	The gap bound is applied to a perturbation violating demand positivity	Premise P3 destroyed
F54	A noncommuting benefit problem is assumed to have a piecewise-linear value function	Incorrect value-function geometry; use concavity and Lipschitz continuity

23. VCS-1 Closure Certificate

VCS-1 requirement	Result
Finite carrier typed	Closed conditionally
Tau and complement projectors defined	Closed conditionally
Raw census identified	Closed
Positive maintenance allocation defined	Closed
Positive closure-deficit operator defined	Closed
Tau and complement closure traces defined	Closed
Universal suppression identity proved	Closed
Centred-projector form proved	Closed

VCS-1 requirement	Result
Mean-deficit anisotropy form proved	Closed
Suppression criterion proved	Closed
Census-equality criterion proved	Closed
Enhancement criterion proved	Closed
Uniform-deficit invariance proved	Closed
Positive demand architecture supplied	Closed structurally
Finite-capacity primal problem defined	Closed
Existence proved	Closed
Capacity saturation proved in saturated regime	Closed
Strong duality proved	Closed
Dual positive-part formula proved	Closed
Threshold multiplier identified	Closed
Complete optimum-set form proved	Closed
Proof-order safeguard against F19 installed	Closed
Closure-release duality proved	Closed
Highest-demand-first deficit localisation proved	Closed
Strict tau-gap premise isolated	Closed
Zero tau support below complement saturation proved	Closed
Tau-only deficit above complement saturation proved	Closed
Exact tau-only suppression identity proved	Closed
Strict recovery monotonicity proved	Closed
Exact inversion proved	Closed
Exact capacity conversion proved	Closed
Rank-one charged-lepton identity proved	Closed
Integrated shadow-price closure identity proved (neutral objective)	Closed
Benefit-weighted shadow identity proved (general objective)	Closed
Noncommuting benefit value-function regularity established via Lipschitz bound	Closed
Kink-point multiplier ambiguity resolved by superdifferential statement	Closed
Degenerate-spectrum kink structure corrected (distinct-eigenvalue form)	Closed
Boundary uniqueness at $C = C_0$ proved via gap multiplier	Closed
Named VCS identity proved	Closed
Capacity regimes completed	Closed
Threshold-degeneracy caveat corrected	Closed
Aggregate uniqueness under strict separation proved	Closed
Sector-deficit uniqueness under strict separation proved	Closed
Sharpened lower bound proved	Closed

VCS-1 requirement	Result
Earlier linear floor recovered	Closed
Soft-profile identity proved	Closed
General benefit dual supplied	Closed
Efficiency-ordering extension supplied	Closed
Benefit-threshold degeneracy caveat supplied	Closed
Benefit slackness caveat ($\nu_C = 0$ regime) supplied	Closed
Basis covariance proved	Closed
Branch-preserving gap and positivity stability proved	Closed
Regime-endpoint perturbation bounds stated	Closed
Capacity-only sign no-go proved	Closed
Numerical magnitude underdetermination proved	Closed
Inverse-reconstruction firewall installed	Closed
Mass firewall installed	Closed
Lifetime firewall installed	Closed
Yukawa firewall installed	Closed
Static-optimisation firewall installed	Closed
Observable firewall installed	Closed
Numerical demand spectrum from VERSF primitives	Open
Numerical capacity from VERSF primitives	Open
Physical objective derivation	Open
Dynamical relaxation law	Open
Charged-lepton mass bridge	Open
Tau lifetime and decay bridge	Open
Electroweak observable extraction	Open
Empirical confirmation	Open

Closure grade

VCS-1 is mathematically closed as an exact operator identity, finite-capacity primal–dual theorem, closure-release localisation theorem, tau-only suppression identity, capacity-conversion identity, shadow-price integral identity, threshold-degeneracy audit, and numerical non-insertion theorem. It is physically predictive at the suppression-sign level only if VERSF independently derives the carrier, branch projector, maintenance objective, finite capacity, and strict tau-last demand or efficiency ordering. It becomes numerically predictive for the internal maintenance fraction only when the demand spectrum and capacity are independently fixed. It is not a tau-mass, lifetime, decay-width, Yukawa, dynamical-evolution, or experimental-observable theorem.

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24. Conclusion

The raw tau census fraction is

$g/(g+h)$.

A nonuniform maintenance allocation changes that fraction. The exact object controlling the change is the positive closure-deficit operator

$\Delta = I - R$.

Resolving the closure deficit into tau and complement components,

$\delta_\tau = \mathrm{Tr}(\Pi_\tau \Delta)$, $\delta_0 = \mathrm{Tr}(\Pi_0 \Delta)$,

gives the universal identity

$$\mathbf{g/(g+h) - f_\tau = (h\,\delta_\tau - g\,\delta_0) / [\,(g+h)(g+h - \delta_\tau - \delta_0)\,].}$$

This identity completely classifies the sign. Tau suppression occurs exactly when

$\delta_\tau/g > \delta_0/h$.

The census is recovered exactly when the unresolved closure is sector-uniform on average. Tau enhancement occurs when unresolved closure is more concentrated in the complement.

Finite capacity alone therefore has no sector sign. The sign comes from localisation.

VCS-1 derives that localisation from the finite-capacity variational problem

$\max \{ \mathrm{Tr}\, R : 0 \le R \le I,\ \mathrm{Tr}(D_M R) \le C \}$.

Strong duality gives

$V(C) = \min_{\nu \ge 0} [\, \nu C + \mathrm{Tr}(I - \nu D_M)_+ \,]$.

The dual multiplier defines a spectral threshold $\lambda_C = 1/\nu_C$. Every optimum has the exact form

$R_C = P_{<\lambda_C} + X_C$,

and every closure-deficit optimum has the complementary form

$\Delta_C = P_{>\lambda_C} + (P_{=\lambda_C} - X_C)$.

Thus maintenance fills the lowest-demand modes first, while unresolved closure occupies the highest-demand modes first.

If the entire tau-demand spectrum lies above the entire electron–muon spectrum, unresolved closure enters the tau sector before the complement loses support. In the partial-tau regime,

$$\Delta_C = \Pi_\tau \Delta_C \Pi_\tau,$$

and the universal identity becomes

$$\mathbf{g/(g+h) - f_\tau(C) = h\, Tr\, \Delta_C / [\, (g+h)(g+h - Tr\, \Delta_C)\,].}$$

The suppression is therefore the exact normalised image of unresolved tau closure. Converting the capacity shortfall through the deficit-weighted mean demand, and expressing the total unresolved closure as the integrated shadow price, then give the equivalent forms of Theorems 8 and 10 — for the neutral objective — of which the strongest is

$$\mathbf{g/(g+h) - f_\tau(C) = h \int_C^{C_{full}} \nu_u\, du \,/\, \{\, (g+h)[\, g+h - \int_C^{C_{full}} \nu_u\, du\,]\, \}.}$$

This is the Variational–Closure Suppression Identity. It establishes that

suppression = normalised unresolved-sector contrast,

while

unresolved closure = integrated marginal loss of maintainable support.

These quantitative identities — capacity conversion, rank-one charged-lepton law, and unweighted shadow integral — belong to the neutral objective. Under a general strictly positive, branch-covariant benefit objective with strictly lower tau efficiency, the suppression *sign* persists and the shadow integral instead measures the benefit-weighted closure $Tr(B_M \Delta_C)$ (Section 17.8); the unweighted magnitude is fixed only under the additional threshold conditions of Section 17.6.

The theorem is exact, but it does not select its own numerical inputs. The same capacity shortfall can produce suppression, equality, or enhancement if placed in different sectors. The demand ordering is therefore not decorative. It is the essential localisation principle.

Nor does the theorem claim particle disappearance, mass generation, Yukawa formation, or decay.

Its genuine programme advance is:

sector census → positive maintenance allocation → closure-deficit operator → primal–dual threshold → sector localisation → exact suppression identity.

In one sentence:

Within a consistently typed finite VERSF maintenance carrier, the departure of a rank-g tau sector from its census share $g/(g+h)$ is exactly the unresolved tau closure minus its census-weighted share of total unresolved closure, divided by the maintained support; when an independently derived positive demand operator, finite capacity, and neutral maintained-support objective place all unresolved closure first into the strictly higher-demand tau sector, every optimum yields the exact suppression law $g/(g+h) - f_\tau = h \operatorname{Tr} \Delta_C / [(g+h)(g+h - \operatorname{Tr} \Delta_C)]$, equivalently $h(C_{full} - C)/\{(g+h)[(g+h) \bar{d}_\Delta - (C_{full} - C)]\}$, with $\operatorname{Tr} \Delta_C$ equal to the integral of the optimal capacity shadow price; when the objective is instead a strictly positive, branch-covariant benefit operator with strictly lower tau efficiency, the suppression sign persists and the shadow integral measures the benefit-weighted closure $\operatorname{Tr}(B_M \Delta_C)$; and no numerical mass, lifetime, Yukawa, decay, dynamical, or observable claim is licensed until the demand spectrum, capacity, physical objective, electroweak embedding, and measurement bridge are independently derived.

Appendix A — Minimal Algebraic Skeleton

Carrier:

$\mathscr{H}_\ell = \mathscr{H}_0 \oplus \mathscr{H}_\tau$, $\dim \mathscr{H}_0 = h$, $\dim \mathscr{H}_\tau = g$, $n = g + h$.

Projectors:

$\Pi_0 + \Pi_\tau = I$.

Census:

$c_\tau = g/n$.

Maintenance allocation:

$0 \le R \le I$.

Closure deficit:

$\Delta = I - R$.

Sector deficits:

$\delta_\tau = \operatorname{Tr}(\Pi_\tau \Delta)$, $\delta_0 = \operatorname{Tr}(\Pi_0 \Delta)$, $\delta = \delta_\tau + \delta_0$.

Tau fraction:

$f_\tau = (g - \delta_\tau) / (n - \delta)$.

Centred projector:

$Q_\tau = \Pi_\tau - (g/n) I$.

Universal identity:

$g/n - f_\tau = \mathrm{Tr}(Q_\tau \Delta) / (n - \mathrm{Tr}\,\Delta)$.

Expanded identity:

$g/n - f_\tau = (h\,\delta_\tau - g\,\delta_0) / [\, n(n - \delta)\,]$.

Sign criterion:

$f_\tau < g/n \iff \delta_\tau/g > \delta_0/h$.

Positive demand:

$D_M > 0$.

Primal:

$V(C) = \max \{ \mathrm{Tr}\,R : 0 \le R \le I, \mathrm{Tr}(D_M R) \le C \}$.

Dual:

$V(C) = \min_{\nu \ge 0} [\, \nu C + \mathrm{Tr}(I - \nu D_M)_+ \,]$.

Threshold:

$\lambda_C = 1/\nu_C$.

Complete optimum:

$R_C = P_{<\lambda_C} + X_C, 0 \le X_C \le P_{=\lambda_C}$.

Closure optimum:

$\Delta_C = P_{>\lambda_C} + P_{=\lambda_C} - X_C$.

Capacity deficit:

$\kappa_C = C_{full} - C = \mathrm{Tr}(D_M \Delta_C)$.

Strict tau gap:

$\sup \sigma(D_0) < \inf \sigma(D_\tau)$.

Tau-only deficit:

$C_0 < C < C_{full} \Longrightarrow \Delta_C = \Pi_\tau \Delta_C \Pi_\tau$.

Exact suppression:

$g/n - f_\tau(C) = h \delta_C / [n(n - \delta_C)]$.

Mean deficit demand:

$\bar{d}_\Delta = \kappa_C / \delta_C$.

Capacity identity:

$g/n - f_\tau(C) = h \kappa_C / [n(n \bar{d}_\Delta - \kappa_C)]$.

Shadow-price closure:

$\delta_C = \int_C^{C_{full}} \nu_u \, du$.

Named identity:

$g/n - f_\tau(C) = h \int_C^{C_{full}} \nu_u \, du / \{ n[n - \int_C^{C_{full}} \nu_u \, du] \}$.

Appendix B — Primal–Dual Proof Details

B.1 Spectral maximisation over a positive contraction

Let $A = \Sigma_i a_i |i\rangle\langle i|$. For any $0 \le R \le I$,

$\mathrm{Tr}(AR) = \Sigma_i a_i \langle i|R|i\rangle$, $0 \le \langle i|R|i\rangle \le 1$.

Thus

$\mathrm{Tr}(AR) \le \Sigma_{\{a_i>0\}} a_i = \mathrm{Tr}\, A_+$.

Equality requires

$\langle i|R|i\rangle = 1$ if $a_i > 0$, $\langle i|R|i\rangle = 0$ if $a_i < 0$.

For $a_i > 0$, the operator $I - R$ is positive with vanishing diagonal entry at $|i\rangle$; a positive operator with vanishing diagonal entry has vanishing row and column there, so $R|i\rangle = |i\rangle$. For $a_i < 0$, positivity of R with vanishing diagonal entry forces $R|i\rangle = 0$. Therefore every cross-term involving a positive or negative spectral direction vanishes. Only the zero eigenspace remains free.

B.2 Dual attainment

The function

$$G_C(\nu) = \nu C + \mathrm{Tr}(I - \nu D_M)_+$$

is convex and continuous in ν. At $\nu = 0$, $G_C(0) = n$. As $\nu \to \infty$, because $C > 0$,

$$G_C(\nu) \sim \nu C \to \infty.$$

Therefore the minimum is attained on a compact interval.

B.3 Threshold occupancy (degenerate-safe form)

Ordering individual eigenvalues $d_1 \leq \cdots \leq d_n$ misdescribes the kink structure when eigenvalues repeat, and the sum $\Sigma_{i<k} |i\rangle\langle i|$ is not basis-independent when some of the $i < k$ states share an eigenspace with d_k. The correct grouping is by *distinct* eigenvalues.

Let the distinct demand eigenvalues be

$$0 < \mu_1 < \mu_2 < \cdots < \mu_m,$$

with spectral projectors P_j and multiplicities $r_j = \mathrm{Tr}\, P_j$. Define the cumulative costs and counts

$$K_j = \Sigma_{q\leq j}\, \mu_q r_q,\quad N_j = \Sigma_{q\leq j}\, r_q,\quad K_0 = N_0 = 0,$$

so that $K_m = C_{full}$ and $N_m = n$.

For $K_{j-1} < C < K_j$,

$$\lambda_C = \mu_j,\quad R_C = P_{<\mu_j} + X_j,\quad 0 \leq X_j \leq P_j,\quad \mathrm{Tr}\, X_j = (C - K_{j-1}) / \mu_j,$$

where $P_{<\mu_j} = \Sigma_{q<j} P_q$. This statement is basis-independent and remains complete under arbitrary degeneracy.

B.4 Value-function slopes

Within the same interval $K_{j-1} < C < K_j$,

$V(C) = N_{j-1} + (C - K_{j-1}) / \mu_j$.

Hence

$V'(C) = 1/\mu_j$.

The slopes $1/\mu_j$ strictly decrease in j, proving concavity directly. The kink points of V are therefore exactly the cumulative costs K_j of *completed distinct eigenspaces*, and only these: within a degenerate eigenspace the slope is constant, so ordered-eigenvalue partial sums interior to an eigenspace are not breakpoints (for $D_M = I_2$, $V(C) = C$ on [0, 2] and $C = 1$ is not a kink). At a kink K_j with $0 < j < m$, the superdifferential of V is the interval $[1/\mu_{j+1}, 1/\mu_j]$, which is precisely the optimal-multiplier set at that capacity.

Appendix C — Worked Finite-Dimensional Models

These examples illustrate the theorem. They are not numerical VERSF predictions.

C.1 Uniform closure deficit

Let

$R = (1 - \alpha) I_3, 0 < \alpha < 1.$

Then $\Delta = \alpha I_3$. For one tau direction,

$\delta_\tau = \alpha, \delta_0 = 2\alpha.$

Thus $2\delta_\tau - \delta_0 = 0$, and $f_\tau = 1/3$. The system is less maintained overall but not tau-suppressed.

C.2 Tau-only deficit

Let

$R = \Pi_e + \Pi_\mu + \alpha \Pi_\tau, 0 < \alpha < 1.$

Then $\Delta = (1 - \alpha) \Pi_\tau$, so

$\delta_\tau = 1 - \alpha, \delta_0 = 0.$

The identity gives

$1/3 - f_\tau = 2(1 - \alpha) / [\, 3(2 + \alpha) \,]$.

Since $f_\tau = \alpha/(2 + \alpha)$, the identity is exact.

C.3 Complement-only deficit

Let

$R = \alpha\, \Pi_e + \alpha\, \Pi_\mu + \Pi_\tau$.

Then $\delta_\tau = 0$ and $\delta_0 = 2(1 - \alpha)$. Therefore

$1/3 - f_\tau < 0$,

and the tau fraction is enhanced.

C.4 Ordered maintenance demand

Let

$D_M = 1\cdot\Pi_e + 2\cdot\Pi_\mu + 5\cdot\Pi_\tau$.

Then $C_0 = 3$ and $C_{full} = 8$.

C = 2. The optimum is $R_C = \Pi_e + \frac{1}{2}\Pi_\mu$, giving $f_\tau = 0$.

C = 3. $R_C = \Pi_e + \Pi_\mu$, giving $f_\tau = 0$.

C = 5. The capacity deficit is $\kappa = 8 - 5 = 3$, so the tau closure deficit is $\delta_C = 3/5$. Thus

$f_\tau = (1 - 3/5)/(3 - 3/5) = (2/5)/(12/5) = 1/6$.

The exact capacity identity gives

$1/3 - f_\tau = 2\cdot 3 / [\, 3(15 - 3) \,] = 6/36 = 1/6$. ✓

C = 7.5. $\kappa = 0.5$, $\delta_C = 0.1$. Hence

$f_\tau = 0.9/2.9 \approx 0.310345$,

with suppression

$1/3 - f_\tau = 0.2/8.7 \approx 0.022989$.

C = 8. $R_C = I$, $\Delta_C = 0$, $f_\tau = 1/3$.

C.5 Same capacity deficit, different signs

Let $D\_M = I_3$ and $\kappa = ½$.

Tau-localised. $\Delta\_\tau = ½\ \Pi\_\tau$. This suppresses tau.

Uniform. $\Delta\_iso = (1/6)\ I_3$. This leaves the fraction at one third.

Complement-localised. $\Delta_0 = ¼(\Pi\_e + \Pi\_\mu)$. This enhances tau.

The total capacity deficit is one half in all three cases.

C.6 Mixed threshold degeneracy

Let $D\_M = I_2$ with one tau and one complement direction, and let $C = 1$. Both $R_1 = \Pi\_\tau$ and $R_2 = \Pi_0$ are optimal, but

$f\_\tau(R_1) = 1$, $f\_\tau(R_2) = 0$.

This demonstrates why strict cross-sector separation is required for aggregate uniqueness.

Appendix D — Rank-One Charged-Lepton Formulae

Let

$D\_M = d\_e\ \Pi\_e + d\_\mu\ \Pi\_\mu + d\_\tau\ \Pi\_\tau$, $0 < d\_e \leq d\_\mu < d\_\tau$.

Define

$C_0 = d\_e + d\_\mu$, $C\_full = d\_e + d\_\mu + d\_\tau$.

D.1 Tau occupancy

For $C_0 < C < C\_full$,

$r\_\tau(C) = (C - C_0)/d\_\tau$.

D.2 Tau fraction

$f\_\tau(C) = r\_\tau(C) / [2 + r\_\tau(C)] = (C - C_0) / (2d\_\tau + C - C_0)$.

D.3 Closure deficit

$\delta\_C = 1 - r\_\tau(C) = (C\_full - C)/d\_\tau$.

D.4 Suppression

$1/3 - f\_\tau(C) = 2\delta\_C / [3(3 - \delta\_C)]$,

equivalently

$1/3 - f\_\tau(C) = 2(C\_full - C) / \{ 3[3d\_\tau - (C\_full - C)] \}$.

D.5 Suppression factor

Define

$\mathcal{S}\_\tau = f\_\tau / (1/3) = 3f\_\tau$.

Then

$\mathcal{S}\_\tau(C) = 3(C - C_0) / (2d\_\tau + C - C_0)$.

D.6 Inverse occupancy

Given f_τ,

$r\_\tau = 2f\_\tau / (1 - f\_\tau)$.

D.7 Inverse closure

$\delta\_C = 1 - 2f\_\tau/(1 - f\_\tau) = (1 - 3f\_\tau)/(1 - f\_\tau)$.

D.8 Inverse capacity

$C = C\_full - d\_\tau (1 - 3f\_\tau)/(1 - f\_\tau)$.

These inverse formulae are diagnostic only unless f_τ is derived rather than supplied.

Appendix E — Entropy-Regularised Variational Completion

E.1 Regularised objective

Define the binary operator entropy

$S_{bin}(R) = -\mathrm{Tr}[\, R \ln R + (I - R) \ln(I - R) \,]$.

Consider

$\mathcal{J}_{\{\varepsilon,\nu\}}(R) = \mathrm{Tr}\, R - \nu\, \mathrm{Tr}(D_M R) + \varepsilon\, S_{bin}(R)$,

with $\varepsilon > 0$ and $\nu \geq 0$.

E.2 Stationary condition

In the demand eigenbasis, the problem separates into occupancies r_i. The stationarity equation is

$1 - \nu\, d_i + \varepsilon \ln[\, (1 - r_i)/r_i \,] = 0$,

whence

$r_i = [\, 1 + \exp(\, (\nu\, d_i - 1)/\varepsilon \,)\,]^{-1}$.

Thus

$\mathbf{R_{\{\varepsilon,\nu\}} = [\, I + \exp(\, (\nu D_M - I)/\varepsilon \,)\,]^{-1}}$**.**

E.3 Uniqueness

The entropy term is strictly concave on $0 < R < I$. Therefore $\mathcal{J}_{\{\varepsilon,\nu\}}$ is strictly concave and has a unique maximiser.

E.4 Soft suppression

The profile is strictly decreasing in demand. Under strict tau-demand separation,

$\bar{r}_\tau < \bar{r}_0$,

so the universal identity gives $f_\tau < g/n$.

E.5 Hard-threshold limit

For fixed ν,

$\lim_{\varepsilon\downarrow 0} R_{\varepsilon,\nu}$

is the sharp projector onto $D_M < 1/\nu$, with occupancy ½ exactly on the threshold eigenspace.

One caveat governs capacity matching. For fixed $\varepsilon > 0$, every regularised occupancy satisfies $r_i < 1$, so the achievable regularised cost is bounded above by its $\nu = 0$ value, $Tr(D_M R_{\varepsilon,0}) < C_{full}$. Only capacities below that ceiling can be matched at the given ε. Since $Tr(D_M R_{\varepsilon,0}) \to C_{full}$ as $\varepsilon \downarrow 0$, every fixed $0 < C < C_{full}$ is matchable for all sufficiently small ε, so the hard-threshold limit is unaffected. The zero-capacity endpoint $C = 0$ is approached only in the limit $\nu \to \infty$, not at any finite multiplier.

To recover a prescribed hard-capacity optimum with a different partial threshold occupancy, one must select $\nu = \nu(\varepsilon)$ so that

$Tr(D_M R_{\varepsilon,\nu(\varepsilon)}) = C$

before taking the limit. The regularisation therefore supplies a smooth variational completion, but ε and the multiplier rule must be independently fixed before numerical use.

Appendix F — Open Bridge-Debt Ledger

Debt	Required content	Status
D1 — Carrier-origin debt	Derive the charged-lepton maintenance carrier from VERSF primitives	Open/prior dependency
D2 — Branch-projector debt	Derive Π_e, Π_μ, Π_τ without retrospective observational labelling	Open/prior
D3 — Load-map debt	Derive L_M	Open unless inherited
D4 — Response-metric debt	Derive G_M	Open unless inherited
D5 — Demand-spectrum debt	Derive the eigenvalues of D_M	Open
D6 — Tau-gap debt	Prove strict tau-last demand or efficiency ordering	Central open physical dependency
D7 — Capacity-origin debt	Derive C from VERSF closure structure	Open
D8 — Objective debt	Derive the neutral objective or physical benefit operator	Open

Debt	Required content	Status
D9 — Relaxation debt	Derive dynamics converging to the variational optimum	Open
D10 — Scale debt	Fix the scale at which D_M, C, and R_C are defined	Open
D11 — State debt	Identify the state or ensemble to which maintenance applies	Open
D12 — Chiral debt	Embed left- and right-handed charged leptons consistently	Open
D13 — Higgs-radial debt	Connect maintenance closure to electroweak symmetry breaking	Open
D14 — Mass debt	Derive the charged-lepton mass operator	Open
D15 — Yukawa debt	Derive the charged-lepton Yukawa matrix	Open
D16 — Lifetime debt	Derive tau decay amplitudes and phase space	Open
D17 — Observable debt	Construct a gauge-invariant maintenance-sensitive observable	Open
D18 — Empirical debt	Compare only after all conventions and scales are fixed	Open
D19 — Perturbation debt	Control branch-mixing corrections	Open
D20 — Numerical floor debt	Bound C_full − C and d_τ^+ independently	Open

VCS-1 does not discharge these debts by listing them. It isolates exactly what the next physical derivation must supply.

Appendix G — References

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Final One-Sentence Theorem

Within the VCS-1 gate, let a consistently typed finite VERSF maintenance carrier decompose into a rank-g tau sector and rank-h complement, let R be any nonzero positive contraction and $\Delta = I - R$ its unresolved closure, and define $\delta_\tau = \mathrm{Tr}(\Pi_\tau \Delta)$ and $\delta_0 = \mathrm{Tr}(\Pi_0 \Delta)$; then the complete departure of the normalised tau-maintenance fraction from its census value is the exact identity $g/(g+h) - f_\tau = (h\,\delta_\tau - g\,\delta_0)/[(g+h)(g+h - \delta_\tau - \delta_0)] = \mathrm{Tr}(Q_\tau \Delta)/(g+h - \mathrm{Tr}\,\Delta)$, so suppression, equality, and enhancement occur respectively according as the mean unresolved closure per tau direction is greater than, equal to, or less than that of the complement; and if an independently derived positive maintenance-demand operator, independently fixed finite capacity, branch-covariant carrier structure, and neutral maintained-support objective place every tau demand strictly above every complement demand, then strong primal–dual equality forces every optimum into a lowest-demand threshold form and every complementary closure deficit into a highest-demand threshold form, confines the unresolved closure entirely to the tau sector throughout partial tau maintenance, and yields the exact suppression laws $g/(g+h) - f_\tau = h\,\mathrm{Tr}\,\Delta_C/[(g+h)(g+h - \mathrm{Tr}\,\Delta_C)] = h(C_{full} - C)/\{(g+h)[(g+h)\,\bar{d}_\Delta - (C_{full} - C)]\}$, with $\mathrm{Tr}\,\Delta_C = \int_C^{C_{full}} \nu_u\,du$; if the objective is instead a strictly positive, branch-covariant benefit operator with every tau mode strictly lower in benefit-to-demand efficiency than every complement mode, the suppression sign persists and the shadow integral measures the benefit-weighted closure, $\mathrm{Tr}(B_M \Delta_C) = \int_C^{C_{full}} \nu_u\,du$; while no numerical mass, lifetime, Yukawa, decay, dynamical, or observable result is VERSF-derived until the demand spectrum, capacity, physical objective, electroweak embedding, relaxation law, and measurement bridge are independently fixed without using the intended tau result as input.