# The Weak-Attachment Octant Sign in VERSF

**▲ Programme Milestone — Standard Model Weak-Orientation Series Gate WA-1 / SM Weak-Attachment Sign Gate**

*Fixing the Physical Octant Class of the Weak Attachment Relative to Chirality, Hypercharge, and the Higgs Access Slot*

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## General Reader Summary

The VERSF gauge-census paper answered a large structural question: why the Standard Model has exactly the internal gauge pattern

$SU(3)_c \times SU(2)_L \times U(1)_Y$.

It showed that the connected gauge census closes on colour, weak chirality, and hypercharge, with no additional connected internal gauge channel left over.

One sign question remains.

The weak sector is not merely a two-state system. It is a two-state system attached in a particular way: to one chirality, to a particular ordering of the weak doublet, and to the hypercharge/Higgs mass-pairing structure. Before these orientations are locked together, there are several formal sign choices. In this paper those choices are organised as an **octant**: three binary orientations, giving eight formal possibilities.

The central observation is that most of those possibilities are not new physics. Some are pure relabellings: swapping the names of the two weak-doublet components, replacing the Higgs by its conjugate, or reversing the sign convention of hypercharge. One relabelling is deeper than the rest: the whole theory can be rewritten in "mirror-image bookkeeping" — every particle swapped for its antiparticle partner — without changing any physics, and that rewriting flips the chirality sign on paper. This is a bookkeeping change made before any names are fixed, not a claim that matter and antimatter behave identically. The paper handles it carefully by fixing one external convention once and for all (the electron is the negatively charged matter particle) and proving everything relative to that anchor. But not every sign can be dismissed as convention. Once the weak sector is required to attach to the inherited left-chiral fold, to preserve the hypercharge ledger, to give the Higgs access slot the required mass-pairings, and to leave one neutral electromagnetic survivor, exactly one physical octant class remains.

In plain terms: the weak force is not merely present and left-handed. It is attached with the correct sign relative to everything else in the Standard Model.

The paper does not derive the weak coupling strength, the Higgs potential, particle masses, or the numerical CKM/PMNS matrices. It does something narrower and earlier: it closes the sign ambiguity that would otherwise be carried silently into the flavour and mass-hierarchy gates.

In one sentence:

**The weak attachment admits exactly one physical octant class; every other formal octant is either a harmless relabelling of that class or breaks chirality, mass-pairing access, hypercharge compatibility, electromagnetic survival, or continuation faithfulness.**

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# Abstract

The VERSF Gauge-Census Closure Theorem closes the connected internal gauge algebra of the minimal Standard Model commitment census on

$\mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$.

That theorem fixes the existence and census position of the weak gauge factor but does not by itself fix the residual sign of weak attachment relative to chirality, hypercharge orientation, and the Higgs access slot. The present paper closes that sign gate.

The weak attachment is represented by three binary orientation choices: the active Weyl/fold orientation $\varepsilon_\chi$, the weak-isospin ordering $\varepsilon_T$, and the Higgs-hypercharge attachment polarity $\varepsilon_H$. These define eight formal octants. A relabelling group generated by coordinated weak-doublet exchange, hypercharge sign convention, and Higgs conjugation acts on the octant cube. The physical question is therefore not which raw octant is named, but which relabelling-invariant octant class survives the admissibility filters.

The relabelling quotient depends on an external orientation, and the paper is explicit about this dependence. Absent any anchor, the global charge-conjugate presentation rewriting — $\psi_L \leftrightarrow (\psi^c)_R$, $H \leftrightarrow H^*$, $Y \mapsto -Y$ — is itself a relabelling: it flips $\varepsilon_\chi$ while describing the same physics, and the sole relabelling invariant is the composite weak-attachment sign

$\Omega_W = \varepsilon_\chi \cdot \varepsilon_T \cdot \varepsilon_H$.

Once the external charge anchor $Q_{em}(e) = -1$ is fixed, the conjugate rewriting is excluded, the residual relabelling group is the coordinated $(\varepsilon_T, \varepsilon_H)$ flip alone, and the invariant refines to the pair $(\varepsilon_\chi, \varepsilon_T \varepsilon_H)$.

The paper proves that the admissible attachment is unique at both levels: anchor-independently, exactly one $\Omega_W$-class is admissible; relative to the anchor, the surviving class has $\varepsilon_\chi = +1$, $\varepsilon_T \varepsilon_H = +1$, hence $\Omega_W = +1$. The filters are four simultaneous requirements: left-chiral weak attachment relative to the anchored census, coherent designation of a neutral Higgs access component, hypercharge-neutral mass-pairing access, and preservation of committed weak continuation cones. Every other class either attaches the weak carrier to the wrong Weyl sector of the anchored matter fields, designates a charged component as the electromagnetic access slot, forbids at least one required Higgs mass-pairing, or annihilates a committed weak continuation — or it is the conjugate presentation of the same physics, not a second sign.

The theorem is local and structural. It does not compute weak coupling strengths, electroweak symmetry breaking, the Higgs potential, Yukawa magnitudes, CKM/PMNS matrices, or $\chi$ mass-hierarchy increments. It proves that the weak sector inherited from GC-1 attaches to the rest of the Standard Model census in exactly one physically admissible octant class.


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# 0. Inheritance, Notation, and Firewalls

## 0.1 Inherited results

WA-1 inherits five results from prior VERSF/BCB programme gates.

**I1 — Gauge census closure.** The connected internal gauge census is closed at local algebra level: $\mathfrak{g}$_SM = $\mathfrak{su}(3)$_c ⊕ $\mathfrak{su}(2)$_L ⊕ $\mathfrak{u}(1)$_Y.

**I2 — Weak chirality.** The weak factor is SU(2)_L, not SU(2)_R and not SU(2)_L × SU(2)_R. The weak carrier is a left-chiral doublet carrier.

**I3 — Abelian ledger.** The abelian ledger is hypercharge Y, unique up to normalisation and field-redefinition conventions under the minimal census.

**I4 — Higgs access slot.** The Higgs access slot is a single weak doublet with hypercharge orientation compatible with the mass-pairing structure.

**I5 — Firewall to flavour.** Generation/flavour and $\chi$-hierarchy questions are separate from the local gauge-census question.

WA-1 does not re-prove these. It asks what remains after they are inherited.

## 0.2 The remaining problem — why WA-1 follows gauge-census closure

GC-1 closes the local connected gauge census: it fixes which gauge channels exist. It does not by itself fix every relative sign by which those channels attach to one another. SU(2) carries internal orientation freedom: the sign of $T_3$ depends on which component of a weak doublet is written first. The Higgs access slot has a conjugate representative $\tilde{H} = i\sigma_2 H^*$, and the abelian ledger has a sign convention. These are harmless conventions only when moved together consistently; moved separately, they alter the neutral-Higgs condition, the mass-pairing dictionary, or the committed weak continuation structure.

Left unresolved, this creates an octant ambiguity: later papers could inherit the right gauge group with the wrong weak-attachment sign — a defect invisible at census level and fatal at flavour level. WA-1 is therefore not another gauge-census theorem. It is the sign-coherence theorem that follows gauge-census closure.

## 0.3 Firewalls

This paper proves:

- the weak attachment has one physically admissible octant class;
- every other formal octant is either a relabelling of that class or is excluded by an admissibility filter;

- the Standard Model weak attachment is sign-canonical relative to the inherited SU(2)_L, U(1)_Y, and Higgs access structure.

It does **not** derive: $g_2$; the Higgs potential; electroweak symmetry breaking; the Weinberg angle; the W/Z masses; CKM or PMNS values; fermion Yukawa magnitudes; the χ halving law or the quark mass hierarchy. Those are later gates.

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# 0′. Predictive-Content Ledger

| Object | Prior status | Status here |
|---|---|---|
| SU(2)_L weak carrier | inherited from GC-1 / weak-sector gates | retained |
| Hypercharge U(1)_Y | inherited from GC-1 | retained |
| Higgs access slot H | inherited | retained |
| Weak-doublet ordering sign ε_T | open / convention-threat | classified |
| Higgs-conjugation polarity ε_H | open / convention-threat | classified |
| Hypercharge sign orientation | open / convention-threat | classified |
| Octant cube $\{\pm1\}^3$ | not isolated | constructed |
| Relabelling groups $\mathscr{R}_0$ (unanchored), $\mathscr{R}$ (anchored) | not isolated | constructed |
| Charge-conjugate presentation rewriting c | unexamined sign threat | identified and routed |
| External charge anchor Q_em(e) = −1 | implicit | stated; load-bearing for ε_χ |
| Anchored invariant pair (ε_χ, ε_T ε_H) | not isolated | identified |
| Physical weak-attachment sign Ω_W | open | closed up to relabelling |
| Wrong octants | possible ambiguity | relabelled or excluded |
| Weak coupling strength | open | outside scope |
| Electroweak breaking | open | outside scope |
| Mass hierarchy | open | outside scope |

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# 1. Purpose, Claim Level, and Named Premises

## 1.1 The question

Once SU(2)_L, U(1)_Y, and the Higgs access doublet are inherited, is the weak attachment sign still free?

The answer: **no.** The formal eight-octant ambiguity collapses to one physical octant class once relabelling equivalences and admissibility filters are imposed.

## 1.2 Claim level

**Claim level:** WA-1 weak-attachment sign theorem.

**Epistemic grade:** Conditional structural closure, relative to the inherited GC-1 gauge closure, the inherited chirality selection, and the minimal Standard Model commitment census.

The theorem is not a new derivation of SU(2)_L. It is a sign-selection theorem over an already admitted weak sector.

## 1.2a Claim Boundary: Sign-Coherence, Not New Gauge Discovery

WA-1 does not discover a new gauge factor and does not rederive SU(2)_L. Its claim is narrower.

The theorem assumes the inherited weak carrier, hypercharge ledger, and Higgs access slot. It then asks whether those inherited structures can be attached with multiple physically inequivalent signs. The answer is no.

The paper therefore closes a compatibility problem:

inherited weak carrier

- inherited hypercharge ledger
- single Higgs access slot
- neutral electromagnetic survivor
- mass-pairing admissibility ⇒ one weak-attachment octant class.

This is why the paper is downstream of GC-1 but upstream of flavour and mass-hierarchy gates. Later papers should not carry an unresolved weak-sign branch. WA-1 removes that branch before it can contaminate CKM, PMNS, Yukawa, or $\chi$-hierarchy calculations.

## 1.3 Named premises

**Premise WA-1 — Inherited weak carrier.** The weak carrier is a left-chiral SU(2)_L doublet carrier acting on Q = (u, d)_L and L = (ν, e)_L. *Falsifier:* discovery that the weak carrier is right-chiral or vector-like within the minimal census (cf. F1).

**Premise WA-2 — Inherited hypercharge ledger.** The abelian ledger is U(1)_Y, normalised in this paper by $Q\_em = T_3 + Y/2$. *Falsifier:* a minimal-census abelian ledger inequivalent to hypercharge (cf. F2).

**Premise WA-3 — Single Higgs access doublet.** The minimal census contains one Higgs access doublet H, with conjugate $\tilde{H} = i\sigma_2 H^*$ available as a derived contraction, not as an independent second Higgs ledger. *Falsifier:* a second independent Higgs ledger in the minimal census (cf. F3).

**Premise WA-4 — Coherent neutral-access designation.** The attachment's dictionary must designate as the Higgs access direction a component annihilated by the electromagnetic generator of its conjugacy sheet. For a single $|Y| = 1$ doublet a neutral component always exists arithmetically; the premise constrains the *designation*, not the existence. *Falsifier:* a demonstration that access-slot designation carries no physical content beyond gauge-equivalent vacuum alignment, so that any designation is acceptable (cf. F4).

**Premise WA-5 — Mass-pairing admissibility.** The weak attachment must permit the required gauge-neutral mass-pairing access:

$Q\,H\,u^c,\ Q\,H^\dagger\,d^c,\ L\,H^\dagger\,e^c,$

or the equivalent convention obtained by the simultaneous $u \leftrightarrow d$, $H \leftrightarrow \tilde{H}$ relabelling. *Falsifier:* a consistent minimal census in which one of these pairings is dispensable (cf. F5).

**Premise WA-6 — Continuation faithfulness.** A weak attachment sign that forbids a committed weak continuation is not an admissible sign of the same minimal census. *Falsifier:* a wrong-sign attachment demonstrably preserving all committed continuation cones (cf. F9).

**Premise WA-7 — Relabelling canonicity.** Coordinated weak-doublet basis exchange, global hypercharge sign convention, and Higgs conjugation convention change coordinate signs but not physical content. The physical result must be stated on relabelling classes, not raw sign triples. Relabelling canonicity is assessed relative to the external charge anchor of Appendix B.5; absent the anchor, the conjugate rewriting ꞓ enlarges the relabelling group and the invariant coarsens to Ω_W (§4.2). *Falsifier:* any of these coordinated redefinitions changing an observable (cf. F6–F8, F11).

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# 2. The Weak-Attachment Problem

The weak sector is a two-state structure, and a two-state structure requires an orientation convention.

For a weak doublet

$\Psi = (\psi_+, \psi_-)^T,$

the generator $T_3$ may be represented as

$T_3 = \tfrac{1}{2}\,\mathrm{diag}(+1, -1).$

But one may exchange the basis, $\psi_+ \leftrightarrow \psi_-$, which sends

$T_3 \mapsto -T_3$.

That sign is not by itself physical. It becomes physical only when the weak doublet is attached to other structures:

- chirality;
- hypercharge;
- the Higgs access slot;
- mass-pairing;
- the electromagnetic survivor.

The weak-attachment sign is therefore not the sign of $T_3$ alone. It is the sign of the **relative attachment** of weak isospin to the rest of the Standard Model census. Isolated signs are conventions; relative signs across coupled ledgers are physics. WA-1 is the paper that separates the two.

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# 3. The Octant Cube

## 3.1 Three sign axes

Define three sign variables.

**3.1.1 Chirality/fold sign.** Let $\varepsilon_\chi = +1$ mean the weak carrier attaches to the inherited active left-chiral fold orientation, and $\varepsilon_\chi = -1$ mean it attaches to the opposite Weyl/fold orientation.

Because $SU(2)_L$ is inherited, $\varepsilon_\chi = -1$ is not a convention inside WA-1 — relative to the external charge anchor introduced in §4.2 and Appendix B.5 — but a candidate wrong attachment. Absent the anchor, one specific $\varepsilon_\chi = -1$ presentation is a conjugate rewriting of the same physics; §4.2 makes this precise.

**3.1.2 Weak-isospin ordering sign.** Let $\varepsilon_T = +1$ mean the upper weak-doublet component is assigned $T_3 = +\frac{1}{2}$, and $\varepsilon_T = -1$ mean the ordering is swapped.

This sign is conventional until attached to Y and H.

**3.1.3 Higgs-hypercharge attachment sign.** Let $\varepsilon_H = +1$ mean the Higgs access polarity is the convention $Y(H) = +1$ with one neutral component satisfying $T_3(H^0) = -\frac{1}{2}$. Let $\varepsilon_H = -1$ mean the conjugate convention $Y(\tilde{H}) = -1$ with the neutral component assigned oppositely.

This sign by itself is partly conventional. Its physical content is determined by how it attaches to $T_3$, Y, chirality, and mass-pairing.

### 3.2 The octant cube

The formal weak attachment is a triple

$\mathcal{O} = (\varepsilon\_\chi, \varepsilon\_T, \varepsilon\_H) \in \{\pm 1\}^3$.

There are eight formal octants — but not eight physical theories.

WA-1 proves that, after quotienting relabellings and applying admissibility filters, exactly one physical octant class remains.

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# 4. The Relabelling Group and the Invariant Pair

## 4.1 Raw signs are not physical

Three coordinated operations change the raw sign triple without changing physics.

**R1 — Weak-doublet exchange.** Swapping the two components of every weak doublet, together with the corresponding exchange of upper-type and down-type names, sends $T_3 \mapsto -T_3$, so $\varepsilon\_T \mapsto -\varepsilon\_T$. Performed consistently across the census, this is a basis convention — a pre-anchor presentation change, not an observational exchange of the measured up and down particles (§4.2, Objection 10).

**R2 — Hypercharge sign convention.** Reversing the sign convention for Y, together with the sign of the U(1)_Y coupling, sends $Y \mapsto -Y$. This does not by itself change the physics; the absolute sign of electric charge is conventional until tied to an external charge orientation.

**R3 — Higgs conjugation convention.** Replacing H by $\tilde{H} = i\sigma_2 H^*$ exchanges the Higgs attachment polarity, $\varepsilon\_H \mapsto -\varepsilon\_H$, and swaps which contraction is called "up-type" versus "down-type." This is a convention if and only if the entire pairing dictionary is transformed consistently with it.

## 4.2 Coordination, not free generation

A crucial discipline: R1–R3 enter the relabelling group **only in coordinated combinations** that preserve the physical dictionary — the assignment of electric charges to committed states, the pairing dictionary, and the continuation cones.

The coordination is derived, not stipulated. The coordinated $(\varepsilon\_T, \varepsilon\_H)$ flip is realised by conjugating all SU(2)_L structure by the Weyl element $w = i\sigma_2 \in SU(2)\_L$ — which reverses the $T_3$ ordering on every doublet, $\varepsilon\_T \mapsto -\varepsilon\_T$ — while simultaneously presenting the Higgs slot through its conjugate $\tilde{H}$, $\varepsilon\_H \mapsto -\varepsilon\_H$, with the pairing dictionary transformed alongside. Since w is an element of the gauge group and $H \leftrightarrow \tilde{H}$ is a change of presentation of the same access slot, the combined move cannot change physics. Either half alone leaves the dictionary mismatched: an uncompensated R1 breaks the neutral-Higgs relation, and an uncompensated R3 breaks mass-pairing neutrality (Lemmas 1–2). That is why the group is generated by the pair flip and not by the individual axes.

R2 requires separate routing, and the message is simple. **Reversing the generator basis of U(1)_Y is convention; reversing the physical hypercharge assignments is not.** The full R2 — Y together with the coupling sign, converting the normalisation to $Q_{em} = T_3 - Y/2$ — changes nothing physical and is excluded from $\mathscr{R}$ by Premise WA-2, which fixes the normalisation once. By contrast, reversing the hypercharge assignments while keeping $Q_{em} = T_3 + Y/2$ and the matter dictionary fixed is a different attachment, not a convention: it breaks the anchored charge and pairing structure.

There remains one genuine conjugation freedom, and it is the deepest one. Absent any external orientation, the **global charge-conjugate presentation rewriting**

$\mathfrak{c}$: $\psi_L \leftrightarrow (\psi^c)_R$, $H \leftrightarrow H^*$, $Y \mapsto -Y$

expresses the same commitment census in conjugate variables. It flips $\varepsilon_\chi$ — the weak carrier is now written on the opposite Weyl components — and flips $\varepsilon_H$, preserving $\Omega_W$. (The presentation of $\mathfrak{c}$ as flipping $(\varepsilon_\chi, \varepsilon_H)$ rather than $(\varepsilon_\chi, \varepsilon_T)$ is itself a basis choice: the alternative realisation is $\mathfrak{r}\mathfrak{c}$, an element of the same group, and nothing downstream depends on which representative is named.) Nothing physical distinguishes the two presentations until something external says which conjugacy sheet carries the matter names.

**Caution on terminology.** $\mathfrak{c}$ is not the physical CP transformation acting inside a fixed anchored theory, and nothing here asserts CP symmetry of the Standard Model — flavour phases and their violation of physical CP lie entirely outside WA-1's scope. $\mathfrak{c}$ is a change of presentation of the *entire* census, applied before the matter/antimatter charge sheet has been externally anchored: a statement about bookkeeping freedom, not about a symmetry of the dynamics. Once the anchor is fixed, $\mathfrak{c}$ is not available at all, so no conflict with anchored-theory CP violation can arise.

Therefore:

- **Unanchored:** the relabelling group is $\mathscr{R}_0 = \langle \mathfrak{r}, \mathfrak{c} \rangle$, its orbits have size four, and the sole invariant of the octant cube is the composite sign $\Omega_W$.
- **Anchored:** fixing the external charge anchor $Q_{em}(e) = -1$ (Appendix B.5) — the electron is the negatively charged matter state — excludes $\mathfrak{c}$. The residual group is

  $\mathscr{R} = \langle \mathfrak{r} \rangle \cong \mathbb{Z}_2$, $\mathfrak{r}$: $(\varepsilon_\chi, \varepsilon_T, \varepsilon_H) \mapsto (\varepsilon_\chi, -\varepsilon_T, -\varepsilon_H)$,

and the quotient has exactly **four classes**, labelled by the invariant pair

$(\varepsilon_\chi, \varepsilon_T\, \varepsilon_H) \in \{\pm 1\} \times \{\pm 1\}$.

The anchor is therefore load-bearing, not cosmetic: it is what makes $\varepsilon_\chi$ an invariant at all. Every statement in this paper that treats $\varepsilon_\chi = -1$ as a candidate wrong attachment is made relative to the anchor.

The coordinated move is a relabelling only at the level of the abstract weak-attachment dictionary. Once particle names are externally fixed by observed electric charge — for example, once the electron is identified as the state with $Q_{em} = -1$ and the neutrino as the neutral member of the lepton doublet — one chooses a representative of the class. The other representative is not a second physical attachment; it is the same internal attachment written with the opposite weak-doublet/Higgs convention.

This point is important because "up" and "down" are not arbitrary names after the physical charge dictionary is fixed. The relabelling quotient does not say that observed particles may be freely interchanged after measurement. It says that before the external dictionary is fixed, two algebraic presentations describe the same internal attachment. WA-1 fixes the physical class; observation and convention select the familiar representative.

## 4.3 The invariant structure: composite sign and anchored pair

The raw triple $(\varepsilon_\chi, \varepsilon_T, \varepsilon_H)$ is not invariant. The invariant content is layered:

- **Anchor-independent invariant:** the composite weak-attachment sign

  $\Omega_W = \varepsilon_\chi \cdot \varepsilon_T \cdot \varepsilon_H,$

  preserved by every element of $\mathscr{R}_0$ since each generator flips exactly two factors.

- **Anchored invariant:** the pair $(\varepsilon_\chi, \varepsilon_T\, \varepsilon_H)$, preserved by $\mathscr{R}$ but not by $\mathfrak{c}$.

The two levels answer different questions. The classes $(+1, +1)$ and $(-1, -1)$ share $\Omega_W = +1$, and this is no accident: $\mathfrak{c}$ maps one onto the other. They are the same physics in conjugate presentation — not, as a coarse reading might suggest, two physically distinct attachments that happen to share a composite sign. What distinguishes them is only the anchor: relative to $Q_{em}(e) = -1$, one presentation is coherent with the matter names and the other is not.

The theorem of §8 therefore has an anchor-independent core — exactly one $\Omega_W$-class is admissible — and an anchored refinement — the admissible anchored class is $(\varepsilon_\chi, \varepsilon_T\, \varepsilon_H) = (+1, +1)$. $\Omega_W = +1$ is the composite headline of the refined result.

In the convention of this paper, the Standard Model attachment has

$(\varepsilon_\chi, \varepsilon_T\, \varepsilon_H) = (+1, +1)$, hence $\Omega_W = +1$.

The theorem will prove that only this class is admissible.

## 4.4 Definition — Admissible attachment and canonical presentation

The admissibility filters used in §§5–8 are all inherited from the named premises; none is introduced opportunistically at the point of use. Collect them into one definition. The definition must respect the distinction the rest of the paper runs on: **admissibility is a property of attachments (physics); octants are presentations of attachments.**

**Definition 1 (Admissible attachment).** A weak attachment is **admissible** if it:

1. preserves the inherited minimal field census of GC-1 (Premises WA-1–WA-3);
2. is coherent with the electric-charge dictionary $Q_{em} = T_3 + Y/2$ on its conjugacy sheet (Premise WA-2);
3. designates a Higgs access component annihilated by that sheet's electromagnetic generator (Premise WA-4);
4. admits all three required gauge-neutral mass-pairing channels (Premise WA-5);
5. preserves the inherited weak continuation structure (Premise WA-6).

**Definition 1′ (Canonical presentation).** An octant **presents** an attachment; two octants related by $\mathscr{R}$ present the same attachment on the same conjugacy sheet, and the $\mathfrak{c}$-image of an octant presents the same attachment on the conjugate sheet. An octant is a **canonical presentation** if it presents an admissible attachment on the anchored sheet — the sheet on which the electron is the left-chiral state with $Q_{em} = -1$ (Appendix B.5).

An octant failing only the anchored-sheet condition is a **redundant presentation** of admissible physics, not wrong physics; an octant presenting an inadmissible attachment is wrong physics regardless of sheet. The theorem's object is therefore twofold: exactly one admissible attachment (Definition 1), and exactly one $\mathscr{R}$-class of canonical presentations of it (Definition 1′).

The lemmas divide the labour as follows: Lemma 1 tests clause 3 and Lemma 2 tests clause 4, both on the anchored sheet; Lemma 3 sorts the $\varepsilon_\chi = -1$ octants — its case (i) resolves sheet redundancy and conjugate-inherited failure through Definition 1′, testing no new clause, while its case (ii) tests clauses 1, 2, and 5 together.

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# 5. Neutral-Higgs Criterion

## 5.1 Electromagnetic survivor

Before electroweak breaking, the weak and hypercharge generators are $T_3$ and $Y$. The electromagnetic survivor is

$$Q_{em} = T_3 + Y/2.$$

For the Higgs vacuum to leave an unbroken $U(1)_{em}$, one component of $H$ must satisfy

$$Q_{em}(H^0) = 0,$$

that is,

$T_3(H^0) + Y(H)/2 = 0.$

If $Y(H) = +1$, then $T_3(H^0) = -\frac{1}{2}$. If $Y(H) = -1$, then $T_3(H^0) = +\frac{1}{2}$.

The Higgs-neutrality condition therefore fixes the **relative** sign between weak-isospin orientation and Higgs-hypercharge orientation.

## 5.2 Lemma 1 — Neutral-Higgs sign constraint

**Lemma 1 (Neutral-Higgs sign-coherence constraint).** Relative to the anchored charge dictionary $Q_{em} = T_3 + Y/2$, an admissible weak attachment must designate as its Higgs access direction the component satisfying

$T_3(H^0) = -Y(H)/2.$

Equivalently: the relative weak–Higgs sign $\varepsilon_T \varepsilon_H$ is fixed by coherence between the designated Higgs access slot and the anchored electromagnetic generator.

*Proof.* By the anchor and Premise WA-2, matter charges are assigned by $Q_{em} = T_3 + Y/2$. The attachment's dictionary designates one Higgs component as the electromagnetic-preserving access direction; admissibility requires that the anchored generator annihilate it:

$Q_{em}(H^0) = T_3(H^0) + Y(H)/2 = 0,$

so $T_3(H^0) = -Y(H)/2$. For a single doublet with $|Y(H)| = 1$ exactly one component satisfies this arithmetically, so the constraint never fails by absence of a neutral component. It fails by designation. An octant whose relative polarity $\varepsilon_T \varepsilon_H$ designates the other component as the access slot still leaves an unbroken U(1) when that component condenses — for a single Higgs doublet some U(1) always survives — but the surviving generator is $T_3 - Y/2$, and that generator does not reproduce the anchored matter charges: it assigns $Q(e) = 0$ and $Q(\nu) = +1$. The octant's bookkeeping is therefore incoherent with the anchored dictionary — the U(1) it preserves is not the electromagnetism that names the census. That is a wrong-sign attachment. ■

**Scope note.** Lemma 1 is a sign-coherence condition, not a derivation of electroweak symmetry breaking. The paper assumes the inherited existence of a Higgs access doublet and asks which weak–hypercharge attachment is coherent with the anchored electromagnetic generator. It does not derive the Higgs potential, the vacuum expectation value, or the W/Z mass spectrum. Those belong to later electroweak-breaking gates.

The result here is only this: a wrong-polarity octant does not describe a world without an unbroken U(1), nor a different breaking pattern of the same census. It designates the wrong Higgs component as the access slot, so the U(1) its dictionary preserves fails to assign the anchored charges to the matter fields. The defect is incoherence, not absence.

### 5.3 Reading

This is the first filter. It does not decide all signs. It decides that the weak $T_3$ orientation and the Higgs hypercharge polarity cannot be chosen independently: a wrong relative sign leaves the designated access slot incoherent with the anchored electromagnetic generator.

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# 6. Mass-Pairing Compatibility

## 6.1 Hypercharge assignments

Use the GC-1 convention:

$Y(Q) = \frac{1}{3}$, $Y(u^c) = -4/3$, $Y(d^c) = \frac{2}{3}$, $Y(L) = -1$, $Y(e^c) = 2$, $Y(H) = 1$.

The mass-pairing access terms are

$Q\,H\,u^c$, $Q\,H^\dagger\,d^c$, $L\,H^\dagger\,e^c$.

Their hypercharge sums are

$\frac{1}{3} + 1 - 4/3 = 0$, $\frac{1}{3} - 1 + \frac{2}{3} = 0$, $-1 - 1 + 2 = 0$.

The standard attachment is hypercharge-neutral in all three channels simultaneously.

Hypercharge neutrality is only part of the gauge-neutrality that Premise WA-5 demands, so record the weak and colour halves explicitly. SU(2)_L invariance holds channel by channel: the up-type term is the antisymmetric ε-contraction $\varepsilon^{ab}\,Q_a\,H_b\,u^c$, while the down-type and lepton terms contract against the conjugate, $Q_a\,(H^\dagger)^a\,d^c$ and $L_a\,(H^\dagger)^a\,e^c$; each is a weak singlet. Colour is trivially paired, $3 \otimes \bar{3}$ in the quark channels. With the hypercharge sums above, all three pairings are full gauge singlets.

## 6.2 Wrong-sign failure

If the Higgs polarity is flipped without the corresponding relabelling of up/down names and of the Higgs contraction convention, at least one of these pairings ceases to be neutral. The wrong-sign calculation should be read with the pairing dictionary held fixed. With Y(H) flipped but the inherited mass-pairing assignments left unchanged, the sums become

$\frac{1}{3} - 1 - 4/3 = -2$, $\frac{1}{3} + 1 + \frac{2}{3} = 2$, $-1 + 1 + 2 = 2$.

The failure is not marginal: all three pairings lose neutrality at once.

The only way to avoid this is not to keep the same dictionary and declare victory, but to transform the dictionary itself. That is precisely the coordinated $H \leftrightarrow \tilde{H}$, $u \leftrightarrow d$ relabelling already included in $\mathscr{R}$. The mass-pairing test therefore separates two cases cleanly:

- **coordinated flip:** same physical octant class;
- **uncoordinated flip:** failed attachment.

The key point is not that one cannot write an equivalent theory using $\tilde{H}$. One can. The key point is that $H \leftrightarrow \tilde{H}$ is a relabelling **only if** the whole mass-pairing dictionary is transformed consistently. A raw octant that flips $\varepsilon_H$ while holding the rest of the census fixed is not a new Standard Model attachment. It is an inconsistent attachment.

### 6.3 Lemma 2 — Pairing-polarity constraint

**Lemma 2 (Pairing-polarity constraint).** Given the inherited hypercharge ledger and a single Higgs access doublet, mass-pairing admissibility fixes the Higgs attachment polarity up to the simultaneous field redefinition $u \leftrightarrow d$, $H \leftrightarrow \tilde{H}$. A polarity flip not accompanied by that redefinition forbids at least one required mass-pairing continuation.

*Proof.* The required pairings must be hypercharge-neutral. In the paper's convention:

$$Y(Q) + Y(H) + Y(u^c) = 0,\ Y(Q) - Y(H) + Y(d^c) = 0,\ Y(L) - Y(H) + Y(e^c) = 0.$$

These equations fix the sign of the Higgs contribution relative to the up-type, down-type, and charged-lepton slots. Replacing $H$ by $\tilde{H}$ sends $Y(H) \mapsto -Y(H)$; the equations remain valid only if the pairing dictionary is transformed correspondingly, exchanging which contraction is assigned to which doublet member. If the dictionary is held fixed, the three neutrality sums acquire deviations of $-2Y(H)$ in the up-type channel and $+2Y(H)$ in the down-type and lepton channels — $(-2, +2, +2)$ in this convention — and fail simultaneously (§6.2). Therefore a raw polarity flip is not admissible; only the paired field-redefinition orbit is. ■

---

# 7. Chirality and Continuation Preservation

### 7.1 Chirality is not a sign convention here — given the anchor

The weak group is $SU(2)_L$. That is inherited. But §4.2 showed that "left" is only meaningful relative to the external anchor: absent it, the conjugate rewriting $\mathfrak{c}$ moves the weak carrier to the opposite Weyl components while describing the same physics. The chirality filter must therefore be stated with the anchor in place, and it must dispose of two different things: conjugate presentations of the same attachment, and genuinely wrong attachments.

The physical content is the continuation structure. Weak continuations are committed in the left-chiral fold structure of the anchored matter fields. An attachment that moves the weak current to the opposite chirality of those same anchored fields changes the continuation cone: which histories admit weak continuations, and which do not.

For readers who want the standard algebraic marker: in ordinary field-language, the inherited weak current has the form

$$J^a_\mu = \psi^\dagger_L \bar{\sigma}_\mu T^a \psi_L,$$

summed over the anchored left-chiral doublets $Q_L$ and $L_L$. A wrong-Weyl attachment replaces this with a right-handed current, $\psi^\dagger_R \sigma_\mu T^a \psi_R$, on the same anchored matter fields. That is not a sign variant of the same $SU(2)_L$ census; it is a different current coupling to a different set of states.

## 7.2 Lemma 3 — Chiral attachment constraint

**Lemma 3 (Chiral attachment constraint, relative to the anchor).** Relative to the anchored census, an octant with $\varepsilon_\chi = -1$ is not a second physical sign of the minimal Standard Model weak attachment. It is exactly one of:

(i) the **total conjugate rewriting of some attachment** — the image under $\mathfrak{c}$ of an octant with $\varepsilon_\chi = +1$ — and it inherits the fate of its conjugate partner:

(i-a) if the partner is the admissible attachment, the octant is the same physics written in conjugate variables, excluded from *distinctness*, not from consistency, by the anchor;

(i-b) if the partner is a failed attachment, the octant inherits the partner's Lemma 1–2 failures on the conjugate sheet — it is conjugate-presented wrong physics, excluded by conjugate inheritance, not by any continuation violation of its own;

or

(ii) a **partial re-attachment** — the weak carrier moved to the right-chiral components of the anchored matter fields without the compensating global conjugation — which violates continuation faithfulness and lies outside the census closed by GC-1.

*Proof.* Let the anchor fix the matter names: the electron is the left-chiral state with $Q_{em} = -1$, and by Premise WA-1 the inherited weak carrier acts on the left-chiral doublets $Q_L$ and $L_L$ so named.

Consider any octant with $\varepsilon_\chi = -1$. Either it is realised as a total conjugate rewriting or it is not; the two cases exhaust it.

*Case (i).* The octant is the full conjugate rewriting of an $\varepsilon_\chi = +1$ partner: every field replaced by its conjugate, $\psi_L \leftrightarrow (\psi^c)_R$, with $H \leftrightarrow H^*$ and $Y \mapsto -Y$, and the entire pairing dictionary transformed alongside. $\mathfrak{c}$ is an involution that reproduces, in conjugate variables, exactly the committed histories, continuation cones, charge assignments, and pairing structure of the partner.

The octant's physical content is therefore its partner's, verbatim. If the partner is the admissible attachment (case i-a), the octant is a redundant presentation: not a relabelling of the anchored census only because the anchor has already fixed which conjugacy sheet carries the matter names, and not a second sign. If the partner fails Lemma 1 or Lemma 2 (case i-b), the octant fails the conjugate-transported versions of the same conditions — its designated access slot is charged under the conjugate-sheet electromagnetic generator, and its pairing sums break identically — so it is excluded for the partner's reasons, transported by ͼ, and no separate continuation argument is needed or available.

*Case (ii).* The octant is not a total conjugate rewriting: it moves the weak carrier to the right-chiral components of the anchored matter fields (or of some of them) without the compensating global conjugation. Then the weak carrier acts on states the inherited SU(2)_L carrier does not act on, and fails to act on states it does. Histories that carry committed weak continuations under the inherited attachment lose them, and histories that carry none acquire them. That violates Premise WA-6. At algebra level such an attachment realises SU(2)_R on the anchored census, or a mirror sector alongside SU(2)_L — either way a census change, outside the closure of GC-1.

In no case does ε_χ = −1 furnish a second admissible sign of the same minimal census. ■

### 7.3 Continuation reading

The chirality filter is stronger than notation, and finer than a blanket exclusion — it sorts ε_χ = −1 octants three ways, by different mechanisms. Case (i-a) octants are not wrong physics: they are the anchored attachment in conjugate dress, and the theorem must count them with the surviving attachment's physics rather than against it. Case (i-b) octants are wrong physics, but their defect is polarity, inherited from their conjugate partner through ͼ — the continuation argument plays no role in excluding them. Case (ii) octants are wrong physics by the continuation argument itself: a right-attached weak carrier would allow continuations the inherited theory forbids and forbid continuations it allows, independently of anything the Higgs or hypercharge ledgers do. The fate table of §10.1 keeps the three mechanisms separate.

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# 8. The Weak-Attachment Octant Sign Theorem

### 8.1 Assembly

Combine the constraints:

1. Relative to the anchor, ε_χ = −1 octants sort three ways: redundant presentations of the admissible attachment, conjugate-inherited failures, or census-violating partial re-attachments (Lemma 3, cases i-a, i-b, ii).
2. The relative sign ε_T ε_H is fixed by neutral-Higgs sign-coherence (Lemma 1).
3. The Higgs polarity is fixed up to the paired u ↔ d, H ↔ H̃ redefinition by mass-pairing (Lemma 2).

4. The remaining raw sign flips are conventional if and only if they lie in the anchored relabelling group $\mathscr{R}$, which preserves the invariant pair $(\varepsilon_\chi, \varepsilon_T \varepsilon_H)$.

## 8.2 Theorem 1 — Weak-Attachment Octant Sign Theorem

**Theorem 1 (Weak-Attachment Octant Sign).** Under Premises WA-1 through WA-7 and the external charge anchor of Appendix B.5, the eight formal weak-attachment octants collapse to exactly one physically admissible attachment. In the convention of this paper the surviving anchored class has

$(\varepsilon_\chi, \varepsilon_T \varepsilon_H) = (+1, +1)$, and therefore $\Omega_W = \varepsilon_\chi \varepsilon_T \varepsilon_H = +1$.

Anchor-independently, exactly one $\Omega_W$-class is admissible, and it is $\Omega_W = +1$.

Every octant outside the surviving anchored class is exactly one of:

- a redundant conjugate presentation of the same admissible physics — not a second sign (Lemma 3, case i-a);
- the conjugate-inherited image of a failed attachment (Lemma 3, case i-b);
- a genuine partial wrong-Weyl re-attachment violating continuation faithfulness and the GC-1 census (Lemma 3, case ii);
- a wrong-polarity attachment, whose dictionary designates a charged Higgs component as the electromagnetic access slot and simultaneously forbids all three required mass-pairing accesses (Lemmas 1–2);

and every octant inside the surviving class other than the canonical representative is a relabelled representative of the same physics.

*Proof.* The raw octant cube is $\{\pm1\}^3$. By §4.2, the anchored relabelling group is $\mathscr{R} \cong \mathbb{Z}_2$, generated by the coordinated flip $(\varepsilon_T, \varepsilon_H) \mapsto (-\varepsilon_T, -\varepsilon_H)$; its quotient partitions the cube into four classes labelled by the invariant pair $(\varepsilon_\chi, \varepsilon_T \varepsilon_H)$.

Dispose of the classes in turn.

The class $(-1, -1)$ is the image of $(+1, +1)$ under the conjugate rewriting $\mathfrak{c}$. By Lemma 3, case (i-a), it is the same physics in redundant conjugate presentation; relative to the anchor it is not a distinct attachment, so it contributes no second sign.

The class $(-1, +1)$ is disposed of by the two remaining mechanisms of Lemma 3, and the proof keeps them separate. Any member realised as a total conjugate rewriting is the $\mathfrak{c}$-image of a $(+1, -1)$ attachment and inherits, transported to the conjugate sheet, that class's Lemma 1–2 failures (Lemma 3, case i-b). Any member not so realised is a partial re-attachment of the weak carrier to the right-chiral components of the anchored matter fields: it violates continuation faithfulness (Premise WA-6) and lies outside the GC-1 census (Lemma 3, case ii). Excluded either way, by different mechanisms.

The class (+1, −1) has the wrong relative polarity: by Lemma 1 its dictionary designates the charged Higgs component as the electromagnetic access slot, so the U(1) it preserves does not reproduce the anchored matter charges; by Lemma 2 the same polarity, held against the fixed pairing dictionary, breaks all three mass-pairing neutrality conditions simultaneously. Excluded.

The class (+1, +1) survives: it satisfies the neutral-Higgs relation $T_3(H^0) = -Y(H)/2$ coherently with the anchor, realises all three mass-pairings as gauge singlets (§6.1), and attaches the weak carrier to the anchored left-chiral fold. Lemma 2 confirms that the only residual freedom inside it is the paired field redefinition u ↔ d, H ↔ H̃, which is the generator of $\mathscr{R}$.

Exactly one admissible attachment therefore remains. In the standard convention — SU(2)_L, Y(H) = +1, $Q\_em = T_3 + Y/2$, Q_em(e) = −1 — its canonical representative is (+1, +1, +1), and Ω_W = +1.

For the anchor-independent statement: under the unanchored group $\mathscr{R}_0 = \langle \mathfrak{r}, \mathfrak{c} \rangle$ the cube has two orbits, labelled by Ω_W. The Ω_W = +1 orbit is the union of the admissible class and its conjugate presentation; the Ω_W = −1 orbit is the union of the two failed classes, which are conjugate presentations of each other. Hence exactly one Ω_W-class is admissible, namely Ω_W = +1. ■

**Remark.** The theorem fixes the anchored invariant pair, not merely Ω_W; the composite sign is the anchor-independent shadow of the finer result (§4.3).

### 8.3 Corollary — No unresolved weak sign enters later gates

**Corollary 1.** Later flavour, χ, and mass-hierarchy papers may inherit the weak attachment without carrying an octant ambiguity.

*Proof.* By Theorem 1 the weak attachment is unique up to relabelling. Any later use of the weak sector may choose a convenient representative of the (+1, +1) class without adding physical sign freedom. ■

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# 9. Why This Is Not a Sign Smuggle

A natural objection: the paper simply picks the Standard Model sign and calls the rest convention.

That is not the argument. The paper does three separate things.

**First**, it identifies the full formal ambiguity: eight raw octants, with no octant privileged at the outset.

**Second**, it identifies the relabelling group at both levels — the coordinated weak/Higgs flip, and, absent the anchor, the conjugate rewriting itself — so that conventional signs are not mistaken for physics and, equally, so that physical signs are not mistaken for convention. Stating explicitly that the anchor is what makes $\varepsilon\_\chi$ an invariant, rather than quietly assuming it, is this discipline applied to itself.

**Third**, it applies physical filters that are each independently motivated and inherited, not chosen for this paper: left-chiral weak attachment; coherent neutral-access designation; Higgs mass-pairing access; continuation preservation.

Only after those filters are imposed does one physical octant class remain. The proof therefore does not say "pick the Standard Model sign." It says: **classify all signs, quotient conventions, and exclude the physically inconsistent classes.** The Standard Model sign is the survivor, not the input. That is why the result is a closure theorem.

---

# 10. Octant Fate Table and Exclusion Ladder

## 10.1 Fate of all eight octants

| Octant (ε_χ, ε_T, ε_H) | Anchored class (ε_χ, ε_T ε_H) | Ω_W | Fate |
|---|---|---|---|
| (+, +, +) | (+1, +1) | +1 | **Standard Model attachment — admissible (canonical representative)** |
| (+, −, −) | (+1, +1) | +1 | admissible: relabelled representative (u ↔ d, H ↔ H̃) |
| (+, +, −) | (+1, −1) | −1 | excluded: charged access-slot designation / broken pairing (Lemmas 1–2) |
| (+, −, +) | (+1, −1) | −1 | excluded: charged access-slot designation / broken pairing (Lemmas 1–2) |
| (−, +, +) | (−1, +1) | −1 | excluded: as total conjugate rewriting, inherits the (+1, −1) polarity failures (Lemma 3 i-b); as partial re-attachment, violates continuation faithfulness (Lemma 3 ii) |
| (−, −, −) | (−1, +1) | −1 | excluded: as total conjugate rewriting, inherits the (+1, −1) polarity failures (Lemma 3 i-b); as partial re-attachment, violates continuation faithfulness (Lemma 3 ii) |
| (−, +, −) | (−1, −1) | +1 | charge-conjugate presentation of the Standard Model attachment — same physics, not a distinct sign (Lemma 3 i-a) |
| (−, −, +) | (−1, −1) | +1 | charge-conjugate presentation of the Standard Model attachment — same physics, not a distinct sign (Lemma 3 i-a) |

The Ω_W column now reads correctly at both levels. Anchor-independently, the four Ω_W = +1 octants are one physics (the Standard Model attachment and its conjugate presentation) and the four $\Omega\_W = -1$ octants are one failure (the wrong-polarity attachment and its conjugate presentation). The anchored pair separates presentation from physics: it distinguishes the canonical sheet from the conjugate sheet, which the anchor — and only the anchor — makes meaningful.

### 10.2 Exclusion ladder

**Why not arbitrary weak-doublet ordering?** Because weak-doublet ordering is conventional only before attachment. Once $T_3$ is attached to Y, H, and Q_em, the relative sign matters.

**Why not the opposite chirality?** Because SU(2)_L is inherited. The opposite chirality defines SU(2)_R or a mirror extension, not a sign variant of the same minimal census.

**Why not flip hypercharge?** Reversing the generator basis of U(1)_Y is convention — the full R2 converts the normalisation to $Q\_em = T_3 - Y/2$ and is excluded from $\mathscr{R}$ by Premise WA-2. Reversing the physical hypercharge assignments while keeping the charge formula and the matter dictionary fixed is not convention: it is a different attachment and breaks the anchored charge and pairing structure.

**Why not flip the Higgs?** Replacing H by $\tilde{H}$ is allowed as a field-redefinition convention only when the mass-pairing dictionary is transformed with it. Holding the dictionary fixed while flipping H is not admissible.

**Why not the wrong access-slot designation?** Not because electromagnetism would vanish — for a single doublet an unbroken U(1) always survives — but because the wrong designation makes the surviving generator $T_3 - Y/2$, which assigns the anchored matter fields the wrong charges. Incoherence with the anchored dictionary, not absence of a survivor, is the failure (Lemma 1).

**Why not call the wrong sign a new physics branch?** Because a new physical branch must preserve the committed census. Wrong-sign branches either violate the inherited census or introduce new structure. That makes them extensions, not unresolved signs inside the Standard Model derivation.

---

# 11. Relation to GC-1, $\chi$, and Later Gates

GC-1 closes the gauge census: it proves that the connected internal gauge algebra is $\mathfrak{su}(3)\_c \oplus \mathfrak{su}(2)\_L \oplus \mathfrak{u}(1)\_Y$.

WA-1 closes a different question: how does the weak factor attach **by sign** to chirality, hypercharge, and the Higgs access slot?

The $\chi$-readout theorem (SM-4) closes yet another: where can the up/down mass-pair contrast live if the final standing record is symmetric?

The three are complementary:

- **GC-1** says which internal gauge channels exist.
- **WA-1** fixes the weak attachment sign.
- **SM-4** says $\chi$ must read ordered commitment history, not the final record.

The later mass-hierarchy programme needs all three: the gauge census, the correct weak sign, and a live $\chi$ residue. WA-1 is the middle plank — small in scope, load-bearing in consequence, because any sign error here propagates multiplicatively into every downstream pairing and mixing derivation.

### 11.1 Minimal-Census Boundary

WA-1 closes the weak-attachment sign inside the minimal Standard Model commitment census. It does not claim that no beyond-Standard-Model construction can realise an opposite or mirror sign.

A mirror weak sector, a second Higgs doublet, a right-handed weak doublet, or a different abelian ledger would change the census. Such a theory would require a new gate: it would need to show its own gauge census, anomaly discipline, mass-pairing access, neutral survivor, and continuation-faithfulness.

Thus an opposite-sign construction is not automatically impossible. It is simply not a second sign of the same minimal Standard Model weak attachment. It is an extension.

This boundary concerns genuine census changes. It is distinct from the charge-conjugate presentation rewriting of §4.2 and Lemma 3, which changes no physics and no census and is handled by the external anchor, not by census extension.

---

## 12. Falsification Conditions

| # | Failure | Consequence |
|---|---|---|
| F1 | Weak carrier is not left-chiral | $\varepsilon_\chi$ not fixed; WA-1 fails |
| F2 | Hypercharge ledger differs from GC-1 | octant sign must be recomputed |
| F3 | No single Higgs access slot exists | Higgs-polarity proof fails; multi-Higgs census required |
| F4 | Access-slot designation is shown to carry no physical content beyond gauge-equivalent vacuum alignment | Lemma 1's coherence criterion has no bite; the $\varepsilon_T\, \varepsilon_H$ filter fails |

| # | Failure | Consequence |
|---|---|---|
| F5 | Mass-pairing admissibility is not required | Higgs-pairing sign is not fixed |
| F6 | H ↔ H̃ is new physics rather than a field redefinition | convention quotient fails |
| F7 | Weak-doublet exchange changes physical content by itself | relabelling group too large |
| F8 | Hypercharge sign reversal changes physical content without external orientation | relabelling canonicity fails |
| F9 | Wrong-chirality weak attachment preserves the same continuation cones | inherited SU(2)_L continuation structure fails |
| F10 | Later flavour/mass gates require the opposite weak sign | WA-1 sign closure fails, or the later gate misuses inherited structure |
| F11 | The external charge anchor Q_em(e) = −1 is not fixed | the conjugate rewriting ϲ joins the relabelling group; the invariant coarsens from the pair (ε_χ, ε_T ε_H) to Ω_W alone, and the ε_χ half of Theorem 1 is undefined — the anchor is load-bearing, not representative-selection |
| F12 | A multi-Higgs census is admitted as minimal | Lemma 2 must be re-audited; H ↔ H̃ may no longer exhaust Higgs-polarity freedom |
| F13 | A mirror weak sector is admitted as part of the minimal census | wrong-chirality octants may become extension branches rather than failures |

**Methodological caution (not a falsifier).** Replacing the invariant pair by Ω_W alone is an analyst's error, not a way the world could refute the theorem: it collapses the anchored classification and misreads the conjugate-presentation octants (§4.3, §10.1). It is recorded here as a caution because the error is easy to make and hard to detect downstream.

---

# 13. WA-1 Closure Certificate

| WA-1 condition | Result |
|---|---|
| Weak-attachment problem isolated | closed |
| Three sign axes defined | closed |
| Octant cube constructed | closed |
| Relabelling group ℛ identified | closed |
| Invariant pair (ε_χ, ε_T ε_H) identified | closed |

| WA-1 condition | Result |
|---|---|
| Composite invariant Ω_W defined | closed |
| Neutral-Higgs criterion proved | closed, Lemma 1 |
| Mass-pairing polarity proved | closed, Lemma 2 |
| Chiral attachment constraint proved | closed, Lemma 3 |
| All eight octants classified | closed, §10.1 |
| Standard Model octant class selected | closed, Theorem 1 |
| Weak sign ambiguity removed from later gates | closed, Corollary 1 |
| Need for WA-1 after GC-1 explained | closed: GC-1 fixes gauge census, WA-1 fixes relative weak sign |
| External charge anchor stated | closed: Q_em(e) = −1 excludes the conjugate rewriting from ℛ — load-bearing for the ε_χ invariant, not mere representative selection |
| Charge-conjugate presentation rewriting identified and routed | closed: §4.2, Lemma 3 cases (i-a)/(i-b), §10.1 |
| Rewriting distinguished from physical CP transformation | closed: §4.2 caution, Objection 12 |
| Admissibility formally defined before use | closed: Definitions 1 and 1′, §4.4 |
| Attachment/presentation distinction formalised | closed: Definition 1′, Lemma 3 three-way case structure |
| Premise WA-4 and F4 in designation-coherence form | closed: §1.3, §12 |
| Invariant pair distinguished from Ω_W | closed: pair is the anchored classifier; Ω_W is the anchor-independent invariant |
| Minimal-census boundary stated | closed: mirror/multi-Higgs/right-weak alternatives are extensions, not unresolved signs |
| Weak coupling value | outside scope |
| Higgs potential | outside scope |
| Electroweak breaking dynamics | outside scope |
| Fermion masses | outside scope |
| χ hierarchy | outside scope |

**WA-1 closure grade:** closed at weak-attachment sign level, conditional on the inherited GC-1 gauge census, inherited SU(2)_L chirality, and the minimal single-Higgs Standard Model commitment census.

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# 14. Conclusion

The weak sector of the Standard Model is not merely an SU(2) factor. It is a left-chiral SU(2) factor attached to hypercharge, the Higgs access slot, electromagnetic survival, and mass-pairing structure.

Before those attachments are disciplined, there are eight formal sign octants:

$(\varepsilon_\chi, \varepsilon_T, \varepsilon_H) \in \{\pm 1\}^3$.

Most of this freedom is not physics. It is convention: the coordinated weak-doublet/Higgs relabelling and, absent an external orientation, the conjugate rewriting itself. The invariant content is layered: anchor-independently, the composite sign

$\Omega_W = \varepsilon_\chi \cdot \varepsilon_T \cdot \varepsilon_H$;

relative to the external charge anchor $Q_{em}(e) = -1$, the finer pair $(\varepsilon_\chi, \varepsilon_T \varepsilon_H)$.

This paper proves that only one physical attachment survives at both levels: in the convention of this paper, $(\varepsilon_\chi, \varepsilon_T \varepsilon_H) = (+1, +1)$ and $\Omega_W = +1$. Every other anchored class is the conjugate presentation of the same physics (not a second sign), a genuine wrong-Weyl attachment outside the census, or a polarity that designates a charged Higgs component as the electromagnetic access slot and breaks every mass-pairing at once; and every octant inside the surviving class is a relabelled representative of the same physics.

The result closes the weak-attachment sign gate.

The programme consequence is precise. WA-1 removes a sign ambiguity that GC-1 deliberately does not address, and the division of labour is worth stating exactly. GC-1 fixes which connected internal gauge channels exist; WA-1 fixes the relative orientation by which the weak channel attaches to chirality, hypercharge, Higgs access, and mass-pairing. The first is a census theorem; the second is a sign-coherence theorem. A census theorem cannot solve the sign problem, because the sign lives in the relative attachment between channels, which the census — by construction — treats one channel at a time.

That attachment is not arbitrary. If the chirality sign is wrong relative to the anchor, the attachment is either the conjugate presentation of the same physics — no second sign — or a genuine wrong-Weyl attachment outside the census. If the weak–Higgs relative sign is wrong, the dictionary designates a charged component as the electromagnetic access slot and the mass-pairing structure fails. If the sign is changed only by the coordinated weak/Higgs relabelling, nothing physical has changed. The eight formal octants therefore collapse to one physical attachment.

This is exactly the sort of small but load-bearing closure gate the later Standard Model derivation needs. Without it, downstream flavour and mass-hierarchy papers would carry a hidden weak-orientation branch. With it, they inherit a single sign-coherent weak attachment.

In one sentence:

**Once SU(2)_L, U(1)_Y, and the Higgs access slot are inherited, the weak attachment has one admissible octant class; every other formal sign is convention or failure.**

---

# Appendix A — Minimal Algebraic Skeleton

Use one lepton doublet and one quark doublet:

$L = (\nu, e)_L$, $Q = (u, d)_L$.

Assign

$T_3(\nu) = +\frac{1}{2}$, $T_3(e) = -\frac{1}{2}$, $T_3(u) = +\frac{1}{2}$, $T_3(d) = -\frac{1}{2}$,

with

$Y(L) = -1$, $Y(Q) = \frac{1}{3}$.

Then

$Q_{em}(\nu) = \frac{1}{2} - \frac{1}{2} = 0$, $Q_{em}(e) = -\frac{1}{2} - \frac{1}{2} = -1$, $Q_{em}(u) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$, $Q_{em}(d) = -\frac{1}{2} + \frac{1}{6} = -\frac{1}{3}$.

For the Higgs,

$H = (H^+, H^0)^T$, $Y(H) = 1$,

so

$Q_{em}(H^+) = \frac{1}{2} + \frac{1}{2} = 1$, $Q_{em}(H^0) = -\frac{1}{2} + \frac{1}{2} = 0$.

The lower component is neutral, so the Higgs vacuum can preserve electromagnetism.

Mass-pairings:

$Y(Q) + Y(H) + Y(u^c) = \frac{1}{3} + 1 - 4/3 = 0$, $Y(Q) - Y(H) + Y(d^c) = \frac{1}{3} - 1 + \frac{2}{3} = 0$, $Y(L) - Y(H) + Y(e^c) = -1 - 1 + 2 = 0$.

The same sign assignment simultaneously gives correct electric charges, neutral Higgs access, and mass-pairing neutrality in all three channels. That triple concurrence — one assignment satisfying three independently stated constraints — is the algebraic shadow of the octant sign theorem.

---

# Appendix B — Convention Dictionary

## B.1 Weak-doublet exchange

Swapping doublet entries sends $T_3 \mapsto -T_3$. This is a basis change if all doublets and Higgs components are transformed consistently, with names re-assigned so that the physical dictionary is preserved.

## B.2 Higgs conjugation

The conjugate doublet is $\tilde{H} = i\sigma_2 H^*$. If $Y(H) = +1$ then $Y(\tilde{H}) = -1$. Using $H$ or $\tilde{H}$ is a convention only if the mass-pairing dictionary is swapped consistently.

## B.3 Hypercharge normalisation

This paper uses $Q_{em} = T_3 + Y/2$. Some conventions use $Q_{em} = T_3 + Y$. The two are related by $Y_{this} = 2 \cdot Y_{other}$.

## B.4 Left-handed conjugate notation

This paper uses $u^c$, $d^c$, $e^c$ as left-handed conjugates. Their hypercharges are the negatives of the corresponding right-handed fields, with the normalisation factor included.

## B.5 External charge anchor

The external charge anchor is the convention $Q_{em}(e) = -1$: the electron is the negatively charged matter state. The anchor is load-bearing, not cosmetic. Without it, the conjugate rewriting $\mathfrak{c}$ ($\psi_L \leftrightarrow (\psi^c)_R$, $H \leftrightarrow H^*$, $Y \mapsto -Y$) is an admissible relabelling — it flips $\varepsilon_\chi$ and $\varepsilon_H$ while describing the same physics — and the relabelling group is $\mathscr{R}_0 = \langle \mathfrak{r}, \mathfrak{c} \rangle$, under which the only octant invariant is $\Omega_W$. With the anchor, $\mathfrak{c}$ is excluded, the residual group is $\mathscr{R} = \langle \mathfrak{r} \rangle \cong \mathbb{Z}_2$, and the invariant refines to the pair $(\varepsilon_\chi, \varepsilon_T \varepsilon_H)$. All statements about $\varepsilon_\chi$ in Lemma 3 and Theorem 1 are made relative to this anchor.

## B.6 The H† shorthand

The down-type and lepton pairings are written $Q H^\dagger d^c$ and $L H^\dagger e^c$ as shorthand for the conjugate contraction $Q_a (H^\dagger)^a d^c$, $L_a (H^\dagger)^a e^c$. In fully holomorphic two-component notation the same terms are the ε-contractions with the conjugate doublet, e.g. $Q H^\dagger d^c \equiv \varepsilon^{ab} Q_a \tilde{H}_b d^c$ up to sign and normalisation, with $\tilde{H} = i\sigma_2 H^*$. No third convention is introduced; the shorthand and the holomorphic form are the same object.

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# Appendix C — Referee Objections and Replies

**Objection 1 — You are just choosing the Standard Model sign.** Response: No. The paper constructs the full octant cube, identifies the relabelling equivalences, and then applies admissibility filters. The Standard Model sign is the survivor, not the starting assumption.

**Objection 2 — The sign of $T_3$ is conventional.** Response: Correct by itself. It becomes physically constrained only when attached to hypercharge, Higgs neutrality, and mass-pairing. WA-1 studies the relative sign, not the isolated $T_3$ convention.

**Objection 3 — Replacing H by H̃ gives an equivalent theory.** Response: Correct if the whole pairing dictionary is transformed. That is why H ↔ H̃ enters the relabelling group only in its coordinated form. A partial flip is not equivalent and breaks mass-pairing neutrality in every channel simultaneously (§6.2).

**Objection 4 — Ω_W is not enough to classify the octants.** Response: Correct, and the paper is built around the point (§4.3, §10.1). Ω_W is the anchor-independent invariant; the anchored classifier is the pair (ε_χ, ε_T ε_H). The two ε_χ = −1 octants sharing Ω_W = +1 with the surviving class are not misclassified by the composite sign — they are the conjugate presentation of the same physics, which is exactly what sharing the anchor-independent invariant means. The pair separates presentation from physics; the anchor makes that separation meaningful.

**Objection 5 — Why is left chirality fixed?** Response: WA-1 inherits SU(2)_L from earlier gates. It does not re-derive chirality. It proves the attachment sign relative to the inherited left-chiral weak carrier and the external anchor that names the matter fields.

**Objection 6 — Does this derive electroweak symmetry breaking?** Response: No. It uses the neutral-Higgs sign-coherence criterion: the designated access slot must be annihilated by the anchored electromagnetic generator. It does not derive the Higgs potential, the vacuum expectation value, or the W/Z masses.

**Objection 7 — Does this fix electric charge signs absolutely?** Response: No. Absolute charge sign is conventional until tied to an external orientation, which this paper fixes once as an anchor (Appendix B.5). WA-1 fixes the internal relative sign class needed for the weak, hypercharge, and Higgs structures to cohere.

**Objection 8 — Could a mirror sector realise the opposite sign?** Response: Yes, as an extension. A mirror sector changes the census (§11.1), and is distinct from the conjugate rewriting, which changes nothing. WA-1 closes the minimal Standard Model weak-attachment sign, not all possible beyond-Standard-Model mirror constructions.

**Objection 9 — Why call it an octant?** Response: Because before quotienting conventions, the weak attachment has three binary orientation axes — chirality/fold, weak-isospin ordering, and Higgs-hypercharge polarity — and three binary axes form eight formal octants.

**Objection 10 — Up and down particles are physically different, so how can u ↔ d be a relabelling?** Response: WA-1 does not say that observed up and down particles can be exchanged after the physical charge dictionary is fixed. It says that, before selecting a representative convention, the abstract weak-doublet/Higgs dictionary has two equivalent presentations related by a coordinated $u \leftrightarrow d$, $H \leftrightarrow \tilde{H}$ transformation. Once $Q_{em}$ and the observed particle names are anchored, one representative is chosen. The other is not a second physical sign; it is the same internal attachment written in the opposite convention (§4.2, Appendix B.5).

**Objection 11 — The charge-conjugate rewriting flips $\varepsilon_\chi$ while describing the same physics, so a chirality-exclusion lemma is false as an unconditional statement.** Response: Correct as stated against an unanchored census, and the paper says so. Absent the anchor, $\mathfrak{c}$ belongs to the relabelling group and only $\Omega_W$ is invariant (§4.2). Lemma 3 is therefore stated conditional on the anchor, and it routes $\varepsilon_\chi = -1$ octants three ways: the conjugate presentation of the admissible attachment, which is the same physics and is excluded from distinctness rather than from consistency (case i-a); the conjugate-inherited image of a failed attachment, excluded by transport of its partner's polarity failures (case i-b); and genuine partial wrong-Weyl re-attachments, which fail continuation faithfulness and lie outside the census (case ii). The theorem's uniqueness claim holds at both levels: one $\Omega_W$-class unanchored, one pair-class anchored.

**Objection 12 — The Standard Model violates CP, so a conjugation cannot be a harmless relabelling.** Response: Agreed — for the *physical* CP transformation acting inside a fixed anchored theory, which is not what $\mathfrak{c}$ is. $\mathfrak{c}$ is a global change of presentation of the entire census, applied before the matter/antimatter charge sheet has been anchored; it conjugates everything at once, including whatever flavour structure the census carries, and therefore trivially reproduces the same physics in conjugate variables. No claim of CP symmetry is made or needed. Flavour phases and physical CP violation belong to later flavour gates and are untouched by WA-1 (§4.2, caution on terminology).