The World-to-Framework Occupancy Map for Standard Model Matter in VERSF

▲ Programme Milestone — Standard Model Census Series Gate SC-2 / Matter Occupancy and Representation-Map Closure

Assigning quarks, leptons, chiralities, charges, colours, generation layers, and Higgs-closure routes to their terminal-record framework slots

Keith Taylor VERSF Theoretical Physics Programme — Standard Model Census Series Tier A occupancy-stability gate paper, SC-2

General Reader Summary

The previous census paper answered a necessary question:

What Standard Model matter slots must exist inside VERSF?

It derived the minimal matter census as three complete generations, each containing

Q_L, u_R, d_R, L_L, e_R,

with one Higgs doublet acting as the universal closure carrier.

But a census is only a list of rooms. The next question is the seating chart:

Which actual Standard Model matter fields occupy which VERSF framework slots?

This paper supplies that map.

The map is called the **world-to-framework occupancy map**. It takes each world-side Standard Model matter field — electron, muon, tau, quarks, neutrino weak components, chiralities, colours, and generations — and assigns it to a precise framework address.

A framework address records:

- generation layer;
- species role;
- colour status;
- weak representation;
- weak component;
- chirality;

- hypercharge;
- electric charge;
- Higgs closure route;
- anomaly role;
- minimal or extension status.

For example, e_R is not merely "the right-handed electron." In the framework map it is:

first-generation, colourless, weak-singlet, right-terminal charged-lepton closure species, with $Y = -2$, $Q = -1$, closed by H, and required for charged-lepton mass admissibility.

Likewise, u_R is:

first-generation, colour-triplet, weak-singlet, right-terminal upper quark closure species, with $Y = +4/3$, $Q = +2/3$, closed by $\tilde{H}$, and required for up-type quark mass admissibility.

The paper's key claim is simple:

Every minimal Standard Model matter field occupies exactly one VERSF framework slot, and every minimal VERSF matter slot is occupied exactly once, up to colour multiplicity, flavour-basis rotation, and inherited generation-layer labelling. Within a fixed generation layer, the assignment is forced by representation data alone. Across the three representation-identical generations, the assignment is unique relative to the SC-1 generation-layer ledger, and otherwise unique only up to generation relabelling and permitted flavour-basis rotations.

This matters because later VERSF mass and mixing papers must not predict values for movable labels like "electron" or "top quark" without first stating which framework slot is being targeted. SC-2 fixes the address system.

In one sentence:

The Standard Model matter fields now have fixed VERSF framework addresses, so later mass, hierarchy, CKM, PMNS, Yukawa, and coupling derivations must target specific occupied slots rather than movable particle labels.

Scope Box — What SC-2 Closes and What It Does Not

Claim	Status
World-side Standard Model matter fields identified	Closed
Framework-side matter slots defined	Closed
Occupancy tuple Ω defined	Closed

Claim	Status
Species-slot uniqueness within each generation	Closed
Generation-layer keying (I4″, OM-11)	Closed
Uniqueness up to generation relabelling	Closed
Gauge representation alone distinguishing generations	Not claimed
Mass ordering as generation definition	Not claimed
Quark doublet occupancy	Closed
Up-type singlet occupancy	Closed
Down-type singlet occupancy	Closed
Lepton doublet occupancy	Closed
Charged-lepton singlet occupancy	Closed
Left-neutrino weak-component occupancy	Closed
Sterile ν_R status	Closed as absent from minimal map; extension allowed
Colour multiplicity handling	Closed
Weak-basis versus mass-basis distinction	Closed
Higgs closure routes (H, H̃) assigned	Closed
Anomaly-ledger conjugation map	Closed
Three-generation occupancy table	Closed
No empty minimal slot	Closed
No duplicate minimal occupancy	Closed
Fermion mass values	Not closed
Yukawa eigenvalues	Not closed
CKM numerical entries	Not closed
PMNS numerical entries	Not closed
Neutrino-mass mechanism	Not closed
Gauge-coupling values	Not closed

SC-2 is an **occupancy and representation-map theorem**. It does not predict masses, mixings, or coupling strengths.

Abstract

The Generation and Species Census Theorem fixed the minimal Standard Model matter census inside VERSF: three complete anomaly-stable generations, each containing the species package

Q_L, u_R, d_R, L_L, e_R,

with one Higgs doublet as universal closure carrier and no sterile right-handed neutrino in the minimal census. A census, however, is not yet an operational map. Later quantitative gates require a precise assignment of world-side matter fields to framework-side terminal-record slots.

This paper defines the **world-to-framework occupancy map**

$\Omega : \mathcal{W}\_SM \rightarrow \mathcal{F}\_VERSF$,

where $\mathcal{W}$_SM is the world-side minimal Standard Model matter inventory and $\mathcal{F}$_VERSF is the VERSF framework slot ledger. Each occupied framework address is a tuple

$\Omega(f) = (i, \sigma, \mathcal{C}, \mathcal{W}, T_3, \chi, Y, Q, \mathcal{H}, \mathcal{A}, \mathcal{E})$,

where i is the generation layer, σ the species role, $\mathcal{C}$ the colour status, $\mathcal{W}$ the weak representation, T_3 the weak component where applicable, χ the chirality or attachment status, Y the hypercharge, Q the electric charge, $\mathcal{H}$ the Higgs closure route, $\mathcal{A}$ the anomaly-ledger role, and $\mathcal{E}$ the extension status.

The theorem proves that, in the minimal ledger, Ω is:

1. **total:** every minimal world-side matter field has a framework address;
2. **injective at component-address level:** no two inequivalent matter components occupy the same full address;
3. **surjective onto the minimal matter-slot ledger:** every required VERSF species slot is occupied;
4. **representation-preserving:** colour, weak representation, hypercharge, electric charge, chirality, Higgs closure route, and anomaly role are preserved;
5. **unique on the keyed ledger:** the assignment is forced within each keyed generation layer by representation data; across the three representation-identical layers it is unique on the keyed generation ledger inherited from SC-1 and restated in I4″ (OM-11), and otherwise unique only up to generation relabelling and permitted flavour-basis rotations, where permitted rotations are the unitary generation-space redefinitions commuting with the gauge action (Lemma 3);
6. **basis-aware:** weak-basis and mass-basis rotations change flavour coordinates but not occupied representation slots.

The paper supplies the full three-generation occupancy table. It distinguishes multiplet occupancy from component occupancy, physical right-handed notation from left-handed anomaly-conjugate notation, and minimal matter occupancy from sterile or neutrino-mass extensions.

SC-2 therefore closes the matter occupancy gate. Later VERSF papers may predict masses, Yukawas, hierarchy exponents, CKM angles, PMNS angles, or CP phases only after specifying which Ω-address is being targeted.

Contents

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0. Inheritance, Firewalls, and Aim

0.1 Inherited results

SC-2 inherits the following.

I1 — QN-1 κ-ledger discipline. Localization stiffness, probability exponents, reciprocal compliance, channel-log weights, gap scales, and response couplings are typed separately. Bare κ is inadmissible.

I2 — QN-2 Standard Model convention ledger. SC-2 uses the canonical convention

$Q = T_3 + Y/2$.

The Higgs doublet is

$H = (H^+, H^0)^T$, $Y\_H = +1$.

The canonical charged-fermion closure relations use

$\bar{Q}\_L\ H\ d\_R$, $\bar{Q}\_L\ \tilde{H}\ u\_R$, $\bar{L}\_L\ H\ e\_R$.

I3 — QN-3 anomaly-descent result. Terminal physical support is rank-three, and the VERSF localization stiffness κ⋆ = 8/3 descends from four substrate record obligations into the rank-three physical support quotient. SC-2 does not invoke I3 directly; it is carried because inheritance blocks in this series are cumulative and downstream hierarchy and Yukawa gates require it.

I4 — SC-1 species and generation census. The minimal Standard Model matter census inside VERSF is three complete generations, each containing

Q_L, u_R, d_R, L_L, e_R.

The sterile right-handed neutrino is not part of the minimal census but may appear in an extension ledger.

I4′ — SC-1 slot representation content. SC-1 fixes not only the species census but the framework-side representation content of each slot — the triple ($\mathcal{C}$, $\mathcal{W}$, Y), the chirality class, and the closure role — through its anomaly-closure derivation, prior to and independently of any world-side occupancy. The slot ledger of §3.2 therefore carries its representation data on the framework side; SC-2 does not stipulate that data by copying the world-side field table. (Provenance: SC-1, anomaly-closure representation ledger. The keyed restatement I4″ below is the formal SC-2 input, so SC-2's internal closure does not wait on this cross-reference; insert it at assembly for provenance completeness.)

I4″ — Keyed framework slot ledger restatement. For purposes of SC-2, the framework-side matter ledger is restated explicitly rather than left implicit. For each generation-layer key i = 1, 2, 3, the VERSF matter ledger contains five keyed framework slots:

Q_i, U_i, D_i, L_i, E_i.

Their representation data are:

Framework slot	$\mathcal{C}$	$\mathcal{W}$	Y	Chirality class	Closure role
Q_i	3	2	+1/3	left-attached	coloured weak doublet bundle
U_i	3	1	+4/3	right-terminal	upper coloured closure singlet

Framework slot	$\mathcal{C}$	$\mathcal{W}$	Y	Chirality class	Closure role
D_i	3	1	−2/3	right-terminal	lower coloured closure singlet
L_i	1	2	−1	left-attached	colourless weak doublet bundle
E_i	1	1	−2	right-terminal	charged-lepton closure singlet

This table is the formal framework-side input to SC-2. SC-2 does not rederive the existence of this ledger; that is inherited from SC-1. But SC-2 does restate the ledger explicitly, so its occupancy theorem is internally closed once this table is admitted as the inherited framework slot ledger.

A challenge to the existence of these slots is a challenge to SC-1. A challenge to the assignment of world-side fields to these slots is the subject of SC-2 and is closed here.

I5 — Physical-sector positivity. Unphysical adjoint residue, negative-norm leakage, ghosts, and gauge artifacts do not occupy matter slots.

I6 — Bridge discipline. A VERSF-to-Standard-Model prediction must state its target occupancy slot. Numerical similarity, naming convention, or mass ordering does not define a bridge.

I7 — Generation-layer keying discipline. SC-2 inherits the existence of three generation layers from SC-1. The index i = 1, 2, 3 is a framework-layer coordinate, not a mass prediction and not a claim that gauge representation data alone distinguishes empirical generations. The Standard Model generations carry identical gauge representation content: representation data separate species roles within a generation, but not the three generation copies from one another. SC-2 therefore proves uniqueness relative to the inherited SC-1 generation-layer ledger; without that ledger, the occupancy map is unique only up to generation relabelling and permitted flavour-basis rotations — the unitary generation-space redefinitions commuting with the gauge action (Lemma 3).

0.2 Firewalls

This paper does **not** claim:

- that the electron mass is derived;
- that the top mass is derived;
- that Yukawa eigenvalues are derived;
- that CKM entries are derived;
- that PMNS entries are derived;
- that neutrino masses are derived;
- that gauge couplings are derived;
- that mass ordering defines generation structure;
- that weak-basis labels are identical to mass-basis labels;
- that colour triplicity is generation triplicity;
- that a sterile neutrino is forbidden in all extensions;
- that a field label is physical without a framework address;

- that gauge representation data alone distinguishes the first, second, and third generations;
- that generation labels are derived from mass ordering in SC-2;
- that the weak-basis labels u, c, t or e, μ, τ are already mass-eigenvalue predictions;
- that the occupancy map fixes CKM, PMNS, Yukawa eigenvalues, or hierarchy ordering;
- that uniqueness across generations is stronger than uniqueness relative to the inherited SC-1 layer ledger.

It proves a narrower map theorem:

Given the SC-1 census, the inherited generation-layer ledger, and the QN-2 Standard Model convention ledger, every minimal chiral Standard Model matter field occupies one and only one VERSF framework address, every minimal VERSF matter address is occupied, and no rival representation-preserving species assignment exists inside a fixed generation layer. Across the three representation-identical layers, the map is unique on the keyed generation ledger inherited from SC-1 and restated in I4″, and otherwise unique only up to generation relabelling and permitted flavour-basis rotations.

0.3 Aim

SC-1 gave the list of required rooms.

SC-2 gives the seating chart — and proves it is the only admissible seating chart within each keyed generation layer, keyed across layers by the SC-1 ledger restated in I4″, with the residual relabelling freedom typed and declared rather than hidden.

The question is:

Where does each Standard Model matter field sit inside the VERSF framework?

0′. Predictive-Content Ledger

Object	Prior status	Status here
World-side matter inventory	Empirical Standard Model list	Defined
Framework-side species slots	Derived in SC-1	Addressed and occupied
Occupancy map Ω	Absent	Defined as (Ω_mult, Ω_comp); proved unique on the keyed ledger, quotient-unique when keys are suppressed
Generation labels	Implicitly empirical	Typed as ledger keys inherited from SC-1 and restated in I4″ (I7, OM-11)

Object	Prior status	Status here
Multiplet occupancy	Implicit	Defined
Component occupancy	Implicit	Defined
Colour multiplicity	Potential source of overcounting	Ledgered
Q_L	Species slot	Mapped to coloured weak-attachment bundle
u_R	Species slot	Mapped to upper coloured closure singlet
d_R	Species slot	Mapped to lower coloured closure singlet
L_L	Species slot	Mapped to colourless weak-completion bundle
e_R	Species slot	Mapped to charged-lepton closure singlet
ν_L	Component of L_L	Mapped to neutral upper lepton weak component
ν_R	Extension candidate	Excluded from minimal occupancy
Higgs H	Closure carrier	Mapped as universal non-matter closure carrier
Weak basis	Often conflated with mass basis	Separated
Mass basis	Later diagonalisation basis	Typed as rotation of occupancy coordinates
CKM	Later mixing data	Occupancy-compatible but not predicted
PMNS	Extension-dependent	Not active in minimal map
Anomaly conjugates	Often confusing	Related by conjugation ledger
Full three-generation table	Absent	Supplied
Later mass predictions	Vulnerable to slot drift	Must target Ω-addresses

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1. The Occupancy Problem

1.1 Why an occupancy map is needed

A later VERSF mass paper cannot merely say:

"We predict the electron mass."

It must say which terminal-record object is being predicted:

- e_R — the closure slot?
- e_L — the component inside L_L?

- m_e — a singular value of the charged-lepton mass matrix?
- y_e(μ) — a running Yukawa eigenvalue?
- m_e^pole — a pole observable?

These are related but not identical. SC-2 does not solve the mass problem. It fixes the address problem, so that the mass problem is well-posed.

1.2 The difference between species and field components

A species multiplet may contain several field components.

For example,

Q_L = (u_L, d_L)^T

is one species multiplet role, but it contains two weak components and three colour branches:

u_L^r, u_L^g, u_L^b, d_L^r, d_L^g, d_L^b.

Thus Q_L is one multiplet slot but six left-handed Weyl components.

Similarly,

L_L = (ν_L, e_L)^T

is one multiplet slot but two left-handed Weyl components.

SC-2 distinguishes:

- **multiplet occupancy:** where Q_L, u_R, d_R, L_L, e_R sit;
- **component occupancy:** where each colour and weak component sits.

Conflating the two levels is the single most common source of overcounting in census arguments, and it is blocked here by construction.

1.3 The result in one line

For each generation i = 1, 2, 3, the world-side matter fields occupy

Ω_i : Q_L$_i$ ↦ Q_i, u_R$_i$ ↦ U_i, d_R$_i$ ↦ D_i, L_L$_i$ ↦ L_i, e_R$_i$ ↦ E_i.

The component expansion gives fifteen chiral Weyl matter components per generation, stated in physical-chirality notation, or forty-five in the minimal three-generation matter census. If right-handed physical fields are converted into left-handed anomaly-conjugate notation, the same physical occupancy content is rewritten rather than enlarged.

2. Named Premises of the Occupancy Theorem

OM-1 — Census inheritance

The minimal matter species census is inherited from SC-1:

$3 \times \{Q_L, u_R, d_R, L_L, e_R\}$.

Falsifier: the SC-1 census is invalid or incomplete.

OM-2 — Convention inheritance

The active Standard Model convention is

$Q = T_3 + Y/2$.

All hypercharges and electric charges in SC-2 use this convention.

Falsifier: an occupancy assignment mixes hypercharge conventions without conversion.

OM-3 — Framework address completeness

A matter occupancy address must specify enough data to determine representation, electric charge, chirality, closure route, and anomaly role.

Falsifier: two inequivalent fields share the same complete address.

OM-4 — Representation preservation

The occupancy map must preserve colour representation, weak representation, hypercharge, electric charge, and chirality.

Falsifier: a world field is assigned to a framework slot with different representation data.

OM-5 — Closure-route preservation

A charged terminal matter field must occupy the closure route compatible with its Higgs Yukawa map:

$u_R : \tilde{H}$, $d_R : H$, $e_R : H$.

Falsifier: a mass-closure route is assigned inconsistently with gauge-invariant Higgs closure.

OM-6 — Multiplet/component distinction

A weak or colour multiplet is not to be confused with an individual field component. Occupancy may be stated at either level, but the level must be named.

Falsifier: a colour multiplicity or weak component is double-counted as a new species.

OM-7 — Basis awareness

Weak-basis and mass-basis fields are related by unitary rotations. Occupancy of representation slots is invariant under these rotations, but flavour coordinates and mixing matrices are not.

Falsifier: a weak-basis pairing is treated as a physical mass-basis prediction without diagonalisation.

OM-8 — Minimal-extension separation

A sterile right-handed neutrino, Majorana mass term, additional Higgs doublet, vector-like fermion, or fourth generation is not part of the minimal occupancy map unless an extension ledger is declared.

Falsifier: an extension field is silently inserted into the minimal map.

OM-9 — No empty slot

Every minimal VERSF matter slot required by the census must be occupied by a world-side Standard Model matter field.

Falsifier: a required framework slot is left empty.

OM-10 — No duplicate occupancy

No two inequivalent minimal world-side matter components may occupy the same full framework address.

Falsifier: two distinct fields share the same generation, species role, colour status, weak slot, chirality, hypercharge, charge, and closure route.

OM-11 — Keyed generation-layer ledger

The coordinate $i = 1, 2, 3$ is a ledger key on both the world-side matter inventory and the framework-side slot ledger (I4″). A keyed occupancy map is required to preserve this coordinate:

$Q\_L_i \mapsto \mathsf{Q}_i$, $u\_R_i \mapsto \mathsf{U}_i$, $d\_R_i \mapsto \mathsf{D}_i$, $L\_L_i \mapsto \mathsf{L}_i$, $e\_R_i \mapsto \mathsf{E}_i$.

SC-2 does not claim that gauge representation data alone distinguish $i = 1$, $i = 2$, and $i = 3$. The three generation layers are representation-identical copies. The key i is inherited as part of the SC-1 generation-layer ledger and restated here as part of the SC-2 framework-side input (I4″). The key is shared across species roles: the inherited ledger defines each generation layer as a complete species package, so the alignment of Q_i, U_i, D_i, L_i, and E_i at a common key i is part of the restated input, not a separate derivation.

If the layer keys are deliberately suppressed, generation permutations are not rival physical occupancy maps. They are relabellings of indistinguishable representation copies. In that unkeyed quotient, uniqueness is understood only up to generation relabelling — independent per species role — and permitted flavour-basis rotations.

Falsifier: the inherited ledger does not in fact supply a generation-layer key shared across species roles; or SC-2 claims that $(\mathcal{C}, \mathcal{W}, Y)$ alone distinguishes the three generations; or an unkeyed generation relabelling is treated as a physically distinct rival occupancy map before hierarchy, mass, or mixing data have been introduced.

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3. World Inventory and Framework Slot Ledger

3.1 World-side minimal matter inventory

The minimal Standard Model matter inventory is

$\mathcal{W}\_SM = \{ Q\_L_i, u\_R_i, d\_R_i, L\_L_i, e\_R_i \}, i = 1, 2, 3.$

At the component level:

$Q\_L_i = (u\_L_i, d\_L_i)^T$

with colour branches

$u\_L_i^a, d\_L_i^a, a \in \{r, g, b\}.$

The fields

$u\_R_i^a, d\_R_i^a$

are colour-triplet weak singlets.

$L\_L_i = (\nu\_L_i, e\_L_i)^T$,

and e_R_i is the charged-lepton weak singlet.

Thus each generation contains

6 + 3 + 3 + 2 + 1 = 15

matter components of physical chirality, counted by representation multiplicity.

3.2 Framework-side slot ledger

For each generation layer i, VERSF supplies five matter multiplet slots:

Q_i, U_i, D_i, L_i, E_i.

Their meanings are:

Framework slot	Role
Q_i	coloured weak-attachment doublet bundle
U_i	upper coloured terminal closure singlet
D_i	lower coloured terminal closure singlet
L_i	colourless weak-completion doublet bundle
E_i	charged-lepton terminal closure singlet

The representation content of each slot — the triple ($\mathcal{C}$, $\mathcal{W}$, Y), the chirality class, and the closure role — is framework-side data inherited from SC-1 (I4′) and restated as the formal SC-2 input in I4″. It exists prior to and independently of the occupancy map constructed here: SC-2 matches world fields to it; it does not copy world data into it.

The component-level slots are obtained by adding colour-branch and weak-component coordinates.

3.3 Generation-layer labels

The three generation layers are i = 1, 2, 3. World labels may be assigned in the familiar order:

i = 1 : (u, d, ν_e, e),

i = 2 : (c, s, ν_μ, μ),

i = 3 : (t, b, ν_τ, τ).

The layer coordinate i is inherited from the SC-1 generation-layer ledger (I7, OM-11). The familiar world-side naming shown above follows empirical convention for readability only: nothing in the representation data distinguishes Q_2 from Q_3, and SC-2 does not derive the empirical mass ordering of generations. The naming does not assert that weak-basis doublets are mass-basis pairs without CKM or PMNS rotations, and any later hierarchy gate that derives an ordering must audit its result against this labelling rather than inherit it.

3.4 Keyed ledger versus unkeyed quotient

SC-2 uses a keyed framework ledger. In the keyed ledger, the generation coordinate i is part of the address. A map that sends Q_L_1 to Q_2 does not preserve the keyed address; it is therefore not a different occupancy map of the same keyed ledger.

If the generation keys are suppressed, the three copies of each species role are representation-identical. In that unkeyed quotient, permutations of generation labels are pure relabellings. They do not produce rival species assignments, because Q_L still occupies Q, u_R still occupies U, d_R still occupies D, L_L still occupies L, and e_R still occupies E.

Thus SC-2 proves two related statements:

- **keyed statement:** with generation keys retained, the occupancy map is unique;
- **unkeyed statement:** with generation keys suppressed, the occupancy structure is unique up to generation relabelling and permitted flavour-basis rotation.

This removes the apparent generation-permutation loophole. The loophole arises only if one asks an unkeyed question while demanding a keyed answer.

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4. The Occupancy Tuple

4.1 Definition — occupancy address

A full occupancy address is

$\Omega(f) = (i, \sigma, \mathcal{C}, \mathcal{W}, T_3, \chi, Y, Q, \mathscr{H}, \mathcal{A}, \mathscr{E})$.

The entries are:

Symbol	Meaning
i	generation layer
σ	species role
$\mathcal{C}$	colour status

Symbol	Meaning
$\mathcal{W}$	weak representation
T_3	weak component
χ	chirality or terminal attachment
Y	hypercharge
Q	electric charge
$\mathscr{H}$	Higgs closure route
$\mathcal{A}$	anomaly-ledger role
$\mathscr{E}$	minimal or extension status

The coordinate $\mathcal{A}$ records the field's contribution pattern to the anomaly sums $[SU(3)]^2U(1)$, $[SU(2)]^2U(1)$, $[U(1)]^3$, and $\text{grav}^2 \cdot U(1)$, inherited from the SC-1 anomaly ledger; the symbols $\mathcal{A}$_Q, $\mathcal{A}$_U, $\mathcal{A}$_D, $\mathcal{A}$_L, $\mathcal{A}$_E name the five species-role patterns.

This address is intentionally redundant. Q is determined by T_3 and Y through OM-2; $\mathscr{H}$ is determined by the representation data through gauge invariance, given the one-doublet closure inventory (I4). The redundancy is audit armour, not logical need: it makes silent field-label drift detectable at every coordinate simultaneously.

4.2 Remark — the separating triple and its limit

Although the full address carries eleven coordinates, the triple

$(\mathcal{C}, \mathcal{W}, Y)$

already separates the five species roles within a generation. The five triples are

(3, 2, +1/3), (3, 1, +4/3), (3, 1, −2/3), (1, 2, −1), (1, 1, −2),

and these are pairwise distinct. This observation carries the uniqueness content of the main theorem: any assignment that preserves $(\mathcal{C}, \mathcal{W}, Y)$ has no freedom left at the level of species roles.

The triple has a limit, and the limit must be stated: it separates species roles within a fixed generation layer, but it does not by itself separate the three generation layers, because each generation carries the same Standard Model gauge representation package. Q_L_1, Q_L_2, and Q_L_3 all carry (3, 2, +1/3), and likewise for each species role. The generation coordinate i is therefore supplied by the SC-1 generation-layer ledger (I7, OM-11), not by representation data. Once i is fixed, the separating triple removes all remaining species-assignment freedom. Uniqueness in this paper is accordingly claimed relative to the inherited layer keys (I4″, OM-11), and otherwise up to generation relabelling and permitted flavour-basis rotations.

4.3 Multiplet address versus component address

For a multiplet such as Q_L, the multiplet address leaves T_3 unresolved:

Ω_mult(Q_L_i) = (i, Q, **3**, **2**_L, ∗, L, Y = +1/3, ∗, part{H, H̃}, $\mathcal{A}$_Q, minimal).

The notation part{·} denotes participatory closure involvement of a doublet in a gauge-invariant closure term; it is not a right-terminal mass-closure route (§6.1).

The component addresses resolve T_3 and Q:

$u\_L_i^a$: $T_3 = +½$, Q = +2/3,

$d\_L_i^a$: $T_3 = −½$, Q = −1/3.

Both levels are valid if named.

4.4 Definition — occupied, empty, overoccupied

A framework slot is **occupied** if a world-side matter field maps to it with matching representation, charge, chirality, and closure data.

A slot is **empty** if it is required by the census but no world-side matter field occupies it.

A slot is **overoccupied** if two inequivalent world-side matter fields share the same full component address.

SC-2 proves that the minimal map has no empty and no overoccupied slots.

4.5 Two-level occupancy maps

Strictly, SC-2 uses two related maps.

The multiplet-level map is

Ω_mult : $\mathcal{W}$_SM^mult → $\mathcal{F}$_VERSF^mult,

where

Ω_mult(Q_L_i) = Q_i, Ω_mult(u_R_i) = U_i, Ω_mult(d_R_i) = D_i, Ω_mult(L_L_i) = L_i, Ω_mult(e_R_i) = E_i.

The component-level map is

Ω_comp : $\mathcal{W}$_SM^comp → $\mathcal{F}$_VERSF^comp, where weak components, colour branches, chirality, hypercharge, electric charge, closure route, anomaly role, and extension status are resolved.

The symbol Ω denotes the combined occupancy ledger Ω = (Ω_mult, Ω_comp). When the paper discusses uniqueness at species level, it refers to Ω_mult. When it discusses no duplicate component occupancy, it refers to Ω_comp. This distinction prevents a multiplet-level statement from being mistaken for a component-level count, and prevents a component expansion from being mistaken for new species content. Where the level is unambiguous from the argument — a multiplet on the left and a multiplet slot on the right, or a fully resolved component address — the bare symbol Ω may be used as an abbreviation for the map at that level.

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5. One-Generation Multiplet Occupancy

For each generation i, the multiplet-level occupancy map is:

World field	Framework slot	Representation	Y	Role
Q_L_i	Q_i	(3, 2)	+1/3	coloured weak-attachment doublet
u_R_i	U_i	(3, 1)	+4/3	upper coloured closure singlet
d_R_i	D_i	(3, 1)	−2/3	lower coloured closure singlet
L_L_i	L_i	(1, 2)	−1	colourless weak-completion doublet
e_R_i	E_i	(1, 1)	−2	charged-lepton closure singlet

This table is the core one-generation occupancy map.

5.0 Lemma 0 — Keyed slot-ledger closure

Lemma 0. Given the keyed framework slot ledger of I4″, the SC-2 assignment problem is internally well-posed: each world-side matter multiplet has a target class with matching colour representation, weak representation, hypercharge, chirality class, and closure role.

Proof. The framework-side ledger supplies five keyed slots per generation layer: Q_i, U_i, D_i, L_i, E_i. The world-side minimal chiral inventory supplies five matter multiplets per generation layer: Q_L_i, u_R_i, d_R_i, L_L_i, e_R_i. The representation triples on the world side are

(3, 2, +1/3), (3, 1, +4/3), (3, 1, −2/3), (1, 2, −1), (1, 1, −2),

and the keyed framework ledger of I4″ carries the same five triples, attached respectively to Q_i, U_i, D_i, L_i, E_i. Therefore the assignment problem is not circular: the world-side inventory and the framework-side ledger are separately stated, and the task of SC-2 is to prove that the representation-preserving matching between them is unique. ■

5.1 Lemma 1 — One-generation multiplet occupancy

Lemma 1. For each fixed generation layer i, the five world-side multiplets Q_L_i, u_R_i, d_R_i, L_L_i, e_R_i occupy the five framework slots Q_i, U_i, D_i, L_i, E_i uniquely. No alternative assignment preserves colour representation, weak representation, hypercharge, chirality, and closure route within that fixed layer.

Proof. By the separating-triple remark (§4.2), the invariant data ($\mathcal{C}$, $\mathcal{W}$, Y) take five pairwise distinct values across the five world multiplets and five pairwise distinct values across the five framework slots, and the two value sets coincide:

Q_L ↔ (3, 2, +1/3), u_R ↔ (3, 1, +4/3), d_R ↔ (3, 1, −2/3), L_L ↔ (1, 2, −1), e_R ↔ (1, 1, −2).

A representation-preserving map must match these triples. Since the triples are pairwise distinct on both sides, matching is possible in exactly one way. Each world multiplet therefore occupies exactly one framework slot, each slot is filled exactly once, and no rival assignment is representation-preserving. Chirality and closure route then follow automatically: the doublets are left-attached, the singlets right-terminal, and the closure routes are the unique gauge-invariant ones fixed in §8. ■

The lemma is intentionally a fixed-layer result. It does not claim that gauge representation data alone distinguishes i = 1 from i = 2 from i = 3; that layer coordinate is inherited from SC-1 (I7, OM-11).

---

6. Component-Level Occupancy

6.1 Quark doublet components

For each generation i and colour branch a ∈ {r, g, b}:

$u\_L_i^a \mapsto$ (i, Q, a, **2**_L, +½, L, Y = +1/3, Q = +2/3, part{H, H̃}, $\mathcal{A}$_Q, minimal),

$d\_L_i^a \mapsto$ (i, Q, a, **2**_L, −½, L, Y = +1/3, Q = −1/3, part{H, H̃}, $\mathcal{A}$_Q, minimal).

For doublet components the $\mathscr{H}$ coordinate is typed participatory: part{·} lists the closure carriers appearing in gauge-invariant closure terms that involve the component. A closure route proper is a property of a right-singlet channel (U_i : H̃; D_i : H), not of an individual doublet component — both quark doublet components appear in both closure terms through the SU(2) contraction, so the participatory set is {H, H̃} for each.

6.2 Up-type singlets

$u\_R_i^a \mapsto (i, \mathsf{U}, a, \mathbf{1}, 0, R, Y = +4/3, Q = +2/3, \tilde{H}, \mathcal{A}\_U, \text{minimal})$.

6.3 Down-type singlets

$d\_R_i^a \mapsto (i, \mathsf{D}, a, \mathbf{1}, 0, R, Y = -2/3, Q = -1/3, H, \mathcal{A}\_D, \text{minimal})$.

6.4 Lepton doublet components

$\nu\_L_i \mapsto (i, \mathsf{L}, \emptyset, \mathbf{2}\_L, +½, L, Y = -1, Q = 0, \text{part}\{H\}, \mathcal{A}\_L, \text{minimal})$,

$e\_L_i \mapsto (i, \mathsf{L}, \emptyset, \mathbf{2}\_L, -½, L, Y = -1, Q = -1, \text{part}\{H\}, \mathcal{A}\_L, \text{minimal})$.

Both lepton doublet components participate in the single minimal closure term $\bar{L}\_L\ H\ e\_R$ through the SU(2) contraction; hence part{H} for each. The neutral component nonetheless has no right-singlet closure channel of its own in the minimal massless-neutrino ledger: participation in the E_i channel is not a mass closure for ν_L.

6.5 Charged-lepton singlets

$e\_R_i \mapsto (i, \mathsf{E}, \emptyset, \mathbf{1}, 0, R, Y = -2, Q = -1, H, \mathcal{A}\_E, \text{minimal})$.

6.6 Count check

Per generation:

Q_L : 3 × 2 = 6, u_R : 3, d_R : 3, L_L : 2, e_R : 1.

Total:

6 + 3 + 3 + 2 + 1 = 15.

For three generations:

3 × 15 = 45.

This count excludes sterile right-handed neutrinos.

6.7 Lemma 1′ — Component-address injectivity

Lemma 1′. Within the minimal three-generation ledger, no two inequivalent matter components share a full component address.

Proof. Fix two inequivalent components. If they differ in generation, the coordinate i separates them. If they share a generation but differ in species role, the triple $(\mathcal{C}, \mathcal{W}, Y)$ separates them by

Lemma 1. If they share a species role but differ in colour branch, the coordinate a separates them. If they share a colour branch inside a weak doublet, the coordinate T_3 separates them, and with it Q. No further inequivalent pairs exist in the minimal ledger. Hence every pair of inequivalent components differs in at least one address coordinate, and the component map is injective. ■

---

7. Three-Generation Occupancy Map

7.1 Generation layer 1

World field	Framework slot
$Q\_L_1 = (u\_L, d\_L)^T$	Q_1
u_R_1	U_1
d_R_1	D_1
$L\_L_1 = (\nu\_eL, e\_L)^T$	L_1
e_R_1	E_1

7.2 Generation layer 2

World field	Framework slot
$Q\_L_2 = (c\_L, s\_L)^T$	Q_2
c_R	U_2
s_R	D_2
$L\_L_2 = (\nu\_\mu L, \mu\_L)^T$	L_2
μ_R	E_2

7.3 Generation layer 3

World field	Framework slot
$Q\_L_3 = (t\_L, b\_L)^T$	Q_3
t_R	U_3
b_R	D_3
$L\_L_3 = (\nu\_\tau L, \tau\_L)^T$	L_3
τ_R	E_3

7.4 Basis warning

The tables above are occupancy tables, not mass-diagonal charged-current predictions.

In the quark sector, the weak-basis doublets and mass-basis fields are related by unitary rotations. The CKM matrix arises from the mismatch between the up-type and down-type left diagonalisation frames:

$V\_CKM = U\_uL^\dagger U\_dL$.

The occupancy table fixes representation slots, not CKM numerical values.

7.5 Generation relabelling firewall

The three generation layers carry identical gauge representation packages. Representation data alone therefore cannot distinguish the first, second, and third generation copies.

SC-2 treats the layer index i as inherited from the SC-1 generation census. Once that layer ledger is fixed, each layer has a unique species occupancy:

$Q\_L_i \mapsto Q_i$, $u\_R_i \mapsto U_i$, $d\_R_i \mapsto D_i$, $L\_L_i \mapsto L_i$, $e\_R_i \mapsto E_i$.

Before hierarchy, mass, or mixing data are introduced, a permutation of the generation labels is not a new physical occupancy map. It is a relabelling of identical representation layers. Later mass and hierarchy gates may give empirical meaning to the labels "first," "second," and "third," but SC-2 itself does not use mass ordering to define them.

Two refinements sharpen the firewall. First, assignment rivals are discrete: a rival occupancy map permutes multiplets among layers; it does not superpose them. The discrete rival set is generated by independent generation permutations within each species-role stack — one S_3 per role, five roles in all — and this discrete set sits inside the continuous flavour group $U(3)^5$, whose continuous part is field-redefinition gauge handled by Lemma 3, not assignment freedom. Second, the alignment of layers across species roles is itself inherited, not derived: because the permutation freedom is independent per species role, representation data cannot tie U_2 to the same generation layer as Q_2, D_2, L_2, and E_2. The SC-1 ledger, restated in I4″, supplies that alignment by defining each generation layer as a complete species package; anomaly cancellation holds for any alignment of representation-identical layers and therefore does not select one (OM-11). If the layer keys are suppressed, the residual relabelling freedom accordingly enlarges from a single common S_3 to the full independent product S_3^5 — still pure relabelling, still not a rival species occupancy structure.

This firewall blocks two opposite errors: claiming that SC-2 derives generation mass ordering, and treating a mere generation relabelling — common or per-role — as a rival representation-preserving occupancy map.

7.6 Lemma 3′ — Layer key versus relabelling

Lemma 3′. A generation permutation is not a rival keyed occupancy map. It is either a failure to preserve the keyed ledger or, if the keys are suppressed, a relabelling of representation-identical copies.

Proof. The full occupancy address contains the coordinate i, and in the keyed ledger this coordinate is part of the address (OM-11). A map such as $Q\_L_1 \mapsto Q_2$ therefore does not preserve the full keyed address and is not an automorphism of the keyed occupancy map.

If the layer coordinate is suppressed, the three copies of each species role are representation-identical. A permutation among them changes labels but not species role, representation data, chirality class, closure route, anomaly role, or extension status. It therefore does not generate a rival species occupancy structure; it is a relabelling within the unkeyed quotient (§3.4, §7.5).

Thus generation relabelling cannot falsify the keyed SC-2 occupancy theorem. ■

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8. Higgs Closure Routes

8.1 Universal closure carrier

The Higgs doublet is not repeated by generation. It is the universal closure carrier:

$H = (H^+, H^0)^T$, Y_H = +1.

Its conjugate is

$\tilde{H} = i\sigma_2 H*$, $Y\_\tilde{H} = -1$.

8.2 Closure table

Terminal mass channel	Gauge-invariant closure term	Framework closure route
up-type quarks	$\bar{Q}\_L \tilde{H} u\_R$	$\tilde{H}$
down-type quarks	$\bar{Q}\_L H d\_R$	H
charged leptons	$\bar{L}\_L H e\_R$	H
neutrinos, minimal ledger	none	no minimal right closure
neutrinos, extension	$\bar{L}\_L \tilde{H} \nu\_R$ or Weinberg operator	extension

8.3 Lemma 2 — Closure-route preservation

Lemma 2. The SC-2 occupancy map preserves the minimal Higgs closure routes for all charged terminal matter fields, and these routes are forced, not chosen.

Proof. Hypercharge additivity fixes each route. The up-type bilinear $\bar{Q}_L u_R$ carries net hypercharge $-1/3 + 4/3 = +1$, so gauge invariance requires the $Y = -1$ carrier $\tilde{H}$. The down-type bilinear $\bar{Q}_L d_R$ carries $-1/3 - 2/3 = -1$, requiring the $Y = +1$ carrier H. The charged-lepton bilinear $\bar{L}_L e_R$ carries $+1 - 2 = -1$, again requiring H. In each case exactly one of $\{H, \tilde{H}\}$ closes the channel; the other is gauge-forbidden. The occupancy map assigns U_i to $\tilde{H}$ closure, D_i to H closure, and E_i to H closure, matching the forced routes. Therefore closure routes are preserved, and no closure-route freedom exists to exploit. ■

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9. Weak Basis, Mass Basis, and Mixing Firewalls

9.1 The occupancy map is not a mass-basis claim

The occupancy map fixes representation addresses. It does not fix the numerical Yukawa matrices.

Before diagonalisation, the Yukawa matrices are

Y_u, Y_d, Y_e.

Mass matrices are

$M_f = (v/\sqrt{2}) Y_f$.

Diagonalisation uses unitary rotations:

$U_{fL}^\dagger M_f U_{fR} = D_f$.

These rotations change flavour coordinates, not species occupancy.

9.2 Quark mixing

The quark charged-current matrix is

$V_{CKM} = U_{uL}^\dagger U_{dL}$.

SC-2 fixes the fact that up-type and down-type quark fields occupy U_i, D_i, and Q_i slots. It does not fix U_{uL}, U_{dL}, or V_{CKM}.

9.3 Lepton mixing

In the minimal massless-neutrino ledger, PMNS mixing is not active as a physical mass-mixing object.

If a neutrino-mass extension is declared, then

$$U_{PMNS} = U_{eL}^\dagger U_{\nu L}.$$

SC-2 supplies the lepton occupancy slots. It does not supply the neutrino mass mechanism or PMNS numerical entries.

9.4 Lemma 3 — Occupancy invariance under flavour rotations

Lemma 3. Flavour-basis rotations change coordinates inside generation space but do not change the occupied representation slots.

Proof. A flavour rotation such as

$$Q_L \to U_Q Q_L$$

acts on the generation index while commuting with colour representation, weak representation, hypercharge, chirality, and closure class. It therefore permutes and superposes occupants within a fixed slot bundle without altering which slot bundle is occupied. Physical mixing matrices arise from relative rotations between closure sectors — the mismatch $U_{uL}^\dagger U_{dL}$ — not from any change in the occupancy ledger. The ledger is a rotation invariant. ■

---

10. Anomaly-Ledger Conjugation Map

10.1 Physical notation versus anomaly notation

SC-2 uses physical right-handed fields in the occupancy map:

$u_R, d_R, e_R.$

Anomaly calculations are usually performed using left-handed Weyl fields only. The right-handed fields are replaced by left-handed conjugates:

$u_R \mapsto u_L^c,\ d_R \mapsto d_L^c,\ e_R \mapsto e_L^c.$

The conjugation map reverses representation and hypercharge:

$(3, 1)_Y \mapsto (\bar{3}, 1)_{-Y},$

$(1, 1)_Y \mapsto (1, 1)_{-Y}$.

10.2 Conjugate occupancy table

Physical field	Anomaly-ledger field	Physical rep	Anomaly rep
u_R	u_L^c	(3, 1), Y = +4/3	($\bar{3}$, 1), Y = −4/3
d_R	d_L^c	(3, 1), Y = −2/3	($\bar{3}$, 1), Y = +2/3
e_R	e_L^c	(1, 1), Y = −2	(1, 1), Y = +2

10.3 Lemma 4 — Conjugation preserves occupancy content

Lemma 4. The anomaly-ledger conjugation map changes calculation notation but preserves physical occupancy content.

Proof. A right-handed physical field and its left-handed conjugate describe the same physical species role in conjugate representation notation; the map is an involution on descriptions, not an operation on slots. It reverses gauge representation and hypercharge as anomaly bookkeeping requires, but it neither introduces nor removes a species slot, and composing it with itself returns the physical notation. Therefore anomaly notation is a ledger transform, not a change of occupancy, and F13-type errors — mixing the two notations in one calculation — are detectable through the χ coordinate together with the sign reversal of ($\mathcal{C}$, Y): a conjugate-notation field carries χ = L with reversed colour representation and hypercharge, and no physical-notation address matches that combination. ■

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11. Sterile and Neutrino-Extension Slots

11.1 Minimal neutrino occupancy

Each generation contains a neutral lepton component:

$\nu\_L_i \subset L\_L_i$.

Its minimal occupancy address is

(i, L, ∅, $\mathbf{2}$_L, +½, L, Y = −1, Q = 0, part{H}, $\mathcal{A}$_L, minimal).

This is a real minimal matter component. It participates in the charged-lepton closure term but possesses no right-singlet closure channel of its own; absence of an own-channel closure route is not absence of occupancy.

11.2 Sterile right-handed neutrino

A sterile right-handed neutrino would carry

$\nu_R \sim (1, 1), Y = 0$.

Its framework address would be

(i, N, ∅, **1**, 0, R, Y = 0, Q = 0, H̃ or Majorana, $\mathcal{A}$_sterile, extension).

But N_i is not part of the minimal SC-2 ledger.

11.3 Extension rule

A sterile neutrino may be admitted only by declaring an extension ledger, such as:

- Dirac neutrino extension;
- Majorana mass extension;
- seesaw extension;
- sterile completion-sector bridge;
- VERSF neutrino-mass gate.

Without such a declaration, ν_R is not an empty minimal slot. It is an extension slot outside the minimal occupancy map. The distinction matters: an empty minimal slot would falsify SC-2 through F3, whereas an undeclared extension slot is simply out of scope.

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12. The World-to-Framework Occupancy Map Theorem

12.1 Statement

Theorem 1 — World-to-Framework Occupancy Map Theorem. Under premises OM-1 through OM-11, and using the keyed framework slot ledger restated in I4″, there exists a unique keyed representation-preserving occupancy ledger

$\Omega = (\Omega_{mult}, \Omega_{comp})$

assigning the minimal chiral Standard Model matter fields to the VERSF framework matter slots. For each keyed generation layer i = 1, 2, 3,

Ω_mult(Q_L$_i$) = Q_i, Ω_mult(u_R$_i$) = U_i, Ω_mult(d_R$_i$) = D_i, Ω_mult(L_L$_i$) = L_i, Ω_mult(e_R$_i$) = E_i.

At component level, Ω_comp assigns the fifteen minimal chiral matter components per generation to distinct addresses labelled by generation key, species role, colour branch, weak component, chirality, hypercharge, electric charge, Higgs closure route, anomaly role, and extension status.

The map is:

1. total on the minimal chiral world-side matter inventory;
2. injective at component-address level;
3. surjective onto the keyed minimal framework matter-slot ledger;
4. representation-preserving;
5. closure-route-preserving;
6. unique at species level within each keyed generation layer;
7. unique across the full keyed three-generation ledger;
8. unique only up to generation relabelling and permitted flavour-basis rotation if the generation keys are deliberately suppressed;
9. invariant under permitted flavour-basis rotations as an occupancy ledger;
10. minimal-extension aware.

12.2 Proof

Keyed ledger input. By I4″, the framework-side slot ledger is explicitly restated inside SC-2. The proof therefore does not depend on an unstated or deferred representation table. SC-1 supplies the provenance of the ledger; I4″ supplies the formal SC-2 input. The internal SC-2 claim is consequently closed: given the keyed slot ledger, the occupancy map is forced.

By OM-1, the inherited minimal matter census contains, for each generation, exactly

Q_L, u_R, d_R, L_L, e_R.

By OM-2, hypercharge and electric charge are computed using $Q = T_3 + Y/2$.

Existence and uniqueness at multiplet level. By OM-3 and OM-4, an occupancy address must preserve representation, charge, chirality, and species role. Within a fixed generation layer, the separating triple ($\mathcal{C}$, $\mathcal{W}$, Y) takes five pairwise distinct values on the world multiplets and, by the keyed slot ledger (I4″, provenance I4′), the same five values on the framework slots (§4.2, §5, Lemma 0). A representation-preserving map must match these triples. Distinctness on both sides forces exactly one matching inside that layer. Lemma 1 records this: each world multiplet occupies exactly one framework slot, each slot is filled once, and no rival fixed-layer species assignment is representation-preserving.

Layer-key closure. Repeating the fixed-layer assignment over i = 1, 2, 3 gives the full keyed three-generation ledger. By OM-11, the coordinate i is part of the keyed address. A cross-layer permutation does not preserve the keyed address and is therefore not a different keyed occupancy map. If the keys are suppressed, Lemma 3′ shows that generation permutations are relabellings of representation-identical copies — discrete elements sitting inside the flavour group whose continuous part is the field-redefinition gauge recorded in Lemma 3 — rather than rival species occupancy structures (§3.4, §7.5). Thus uniqueness is exact for the keyed ledger and quotient-unique for the unkeyed ledger.

Closure preservation. By OM-5 and Lemma 2, the charged closure routes are forced by hypercharge additivity and preserved:

u_R : H̃, d_R : H, e_R : H.

Component injectivity. By OM-6, component expansion does not create new species; it fills colour and weak-component coordinates inside the same multiplet slots. Lemma 1′ shows that any two inequivalent components differ in at least one of the coordinates i, ($\mathcal{C}$, $\mathcal{W}$, Y), a, T_3. Hence no two inequivalent components share a full address, satisfying OM-10.

Surjectivity. The framework ledger contains, per generation, exactly the five slots of §3.2 and their component expansions. Every one of them receives an occupant in §§5–6, satisfying OM-9. No slot is empty.

Basis awareness. By OM-7 and Lemma 3, flavour-basis rotations change coordinates inside generation space but not representation-slot occupancy. The map is therefore basis-aware and does not overclaim CKM or PMNS values.

Extension separation. By OM-8 and §11, sterile neutrinos and other extension species remain outside the minimal map unless an extension ledger is declared.

Assembling the parts: Ω is total, injective at component-address level, surjective onto the keyed minimal slot ledger, representation- and closure-route-preserving, unique at species level inside each keyed generation layer, unique across the full keyed ledger, quotient-unique up to relabelling when keys are suppressed, invariant as a ledger under flavour rotations, and minimal-extension aware. ■

12.3 Corollary 1 — No empty minimal matter slots

Every minimal VERSF matter slot required by the census is occupied by a Standard Model matter field.

12.4 Corollary 2 — No duplicate minimal occupancy

No two inequivalent minimal matter components occupy the same full framework address.

12.5 Corollary 3 — No rival fixed-layer seating chart

Within each keyed generation layer, any representation-preserving assignment of the five minimal matter species to the five VERSF matter slots coincides with Ω. Across the full keyed three-generation ledger, Ω is unique. If generation keys are deliberately suppressed, the remaining ambiguity is only generation relabelling (Lemma 3′), not a rival species occupancy structure.

12.6 Corollary 4 — Later predictions must target addresses

A later VERSF paper may not merely predict "a quark mass" or "a lepton mass." It must specify the target address, for example

$\Omega(t_R) = U_3$,

or the mass eigenvalue derived from the closure coupling associated with U_3.

---

13. Why This Is Not a Mass Derivation

13.1 Occupancy is not hierarchy

The occupancy map tells us where the top quark sits. It does not tell us why the top quark is heavy.

It tells us where the electron sits. It does not tell us why the electron is light.

It tells us where the three charged-lepton closure slots are. It does not derive the three charged-lepton Yukawa eigenvalues.

13.2 Occupancy is not mixing

The occupancy map tells us that up-type and down-type quarks occupy corresponding framework sectors. It does not give the CKM matrix.

The CKM matrix comes from relative diagonalisation:

$V_{CKM} = U_{uL}^\dagger U_{dL}$.

SC-2 fixes the slot labels that later mixing theorems must use.

13.3 Occupancy is not neutrino mass

The map includes three left-handed neutrino weak components. It does not decide whether neutrino masses are Dirac, Majorana, seesaw-generated, VERSF-completion induced, or absent in the minimal ledger.

13.4 The value of SC-2

SC-2's value is that it prevents later derivations from moving targets.

A later mass or mixing paper cannot silently switch from e_R to e_L, or from weak basis to mass basis, or from a minimal neutrino slot to a sterile extension slot. The address must be stated, and by Corollary 3 there is only one admissible address map to state it against on the keyed ledger.

---

14. Relation to Later Yukawa, CKM, PMNS, and Hierarchy Gates

14.1 Yukawa gates

A future Yukawa gate must define a function from occupancy addresses to Yukawa structures:

$\mathcal{Y} : \Omega(f) \mapsto y\_f$,

or, more generally,

$\mathcal{Y} : S_i \to Y\_f^{ij}$.

It must state whether it predicts a matrix entry, an eigenvalue, a ratio, a pole mass, a running mass, or a hierarchy exponent.

14.2 CKM gates

A CKM gate must operate on the relative orientation of the up-type and down-type left closure frames, U_uL and U_dL. SC-2 supplies the occupied slots. It does not supply the rotations.

14.3 PMNS gates

A PMNS gate must first declare a neutrino-mass extension. The minimal occupancy map contains ν_L components but no minimal ν_R slot.

14.4 Hierarchy gates

A hierarchy gate must not predict "generation 1 is light" as a bare statement. It must specify the occupancy layer and closure channel, for example

E_1, E_2, E_3

for charged leptons, or

U_1, U_2, U_3

for up-type quarks.

14.5 Bridge clause

Any VERSF mechanism that maps completion density, localization load, channel census, or anomaly descent into a matter parameter must target an Ω-address. Without an address, it is not a quantitative Standard Model prediction.

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15. Falsification Conditions

#	Failure	Consequence
F1	The SC-1 census fails	SC-2 inheritance fails
F2	A world field has no framework address	Occupancy map incomplete
F3	A required framework slot is empty	Surjectivity fails
F4	Two inequivalent components share the same full address	Injectivity fails
F5	Hypercharge convention is mixed	Charge assignment invalid
F6	Colour multiplicity is counted as separate species	Census overcount
F7	Weak components are treated as separate multiplet species	Species/component confusion
F8	Weak-basis doublets are treated as mass-basis predictions	CKM/PMNS overclaim
F9	e_L and e_R are conflated	Chirality occupancy failure
F10	u_R, d_R, e_R closure routes are assigned wrongly	Higgs closure failure
F11	ν_R is inserted into the minimal map	Extension boundary violation
F12	A sterile extension is forbidden despite a declared bridge	Over-restriction
F13	Right-handed physical fields and left-handed conjugates are mixed	Anomaly ledger error

#	Failure	Consequence
F14	A later mass paper targets a particle label without an Ω-address	Prediction under-specified
F15	A later mixing paper ignores basis rotations	Mixing prediction invalid
F16	A fourth-generation field is inserted without a support-extension ledger	Minimal map violated
F17	A vector-like field is treated as minimal chiral matter	Census boundary violated
F18	The Higgs closure carrier is treated as a matter species generation	Category error
F19	A fixed-layer representation-preserving assignment differing from Ω is exhibited	Fixed-layer uniqueness fails; §4.2 separating triple is wrong
F20	Representation data alone is claimed to distinguish the three generations	Generation-layer overclaim
F21	A generation relabelling is treated as a physically distinct occupancy map before mass or hierarchy data are introduced	False uniqueness challenge
F22	The map is claimed to derive empirical generation ordering	Scope violation
F23	Ω_mult and Ω_comp are conflated	Multiplet/component map ambiguity
F24	Minimal chiral occupancy is treated as a complete neutrino-mass theory	Neutrino-extension overclaim
F25	The I4″ ledger is shown not to match the SC-1-derived slot content	I4″ retyped from restatement to stipulation; derivational status of addresses fails
F26	A cross-layer generation permutation is treated as a rival keyed map despite failing to preserve the i coordinate	Keyed/unkeyed confusion
F27	An unkeyed generation relabelling is treated as a new physical occupancy structure	Relabelling overclaim
F28	SC-2 is asked to rederive the SC-1 census rather than map the inherited slot ledger	Scope confusion

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16. SC-2 Closure Certificate

SC-2 condition	Result
World-side matter inventory defined	Closed
Framework-side slot ledger defined	Closed
Occupancy tuple Ω defined	Closed
Fixed-layer species uniqueness	Closed

SC-2 condition	Result
Keyed slot ledger restated as formal input (I4″)	Closed
Full-ledger uniqueness on the keyed ledger	Closed
Unkeyed quotient uniqueness up to relabelling	Closed
Generation relabelling firewall	Closed
Gauge representation not claimed to distinguish generations	Closed
Ω_mult / Ω_comp distinction	Closed
Physical-chirality versus anomaly-conjugate count	Closed
Multiplet/component distinction stated	Closed
One-generation multiplet map supplied	Closed
Component-level map supplied	Closed
Three-generation map supplied	Closed
Colour multiplicity handled	Closed
Weak component status handled	Closed
Chirality slots separated	Closed
Hypercharge and electric charge preserved	Closed
Higgs closure routes forced and assigned	Closed
Minimal neutrino occupancy stated	Closed
Sterile ν_R typed as extension	Closed
Weak-basis/mass-basis firewall supplied	Closed
CKM not overclaimed	Closed
PMNS typed as extension-dependent	Closed
Anomaly conjugation map supplied	Closed
No empty minimal slot	Closed
No duplicate full address	Closed
Later-gate address requirement stated	Closed
Empirical generation mass ordering	Outside scope
Mass values	Outside scope
Yukawa values	Outside scope
CKM/PMNS numerical values	Outside scope
Neutrino mass mechanism	Extension gate
Gauge-coupling values	Outside scope

SC-2 closure grade: closed as a keyed world-to-framework matter occupancy theorem.

SC-2 does not rederive the SC-1 census or the existence of the framework slot ledger. Instead, it restates the required keyed slot ledger explicitly in I4″ and proves that, once this ledger is admitted, the occupancy assignment is forced. The theorem is therefore internally closed: every minimal chiral Standard Model matter field has a unique keyed VERSF framework address;

every keyed minimal framework matter slot is occupied; no two inequivalent components share a full component address; and any unkeyed generation ambiguity reduces to relabelling rather than rival occupancy.

External provenance note: SC-1 remains the provenance of the slot ledger and generation-layer census. If a later audit revises SC-1, the input ledger I4″ must be updated accordingly. That would revise the inherited premises, not expose an internal gap in the SC-2 occupancy proof.

Master Ledger item (non-blocking for SC-2 internal closure; blocking for derivational-status claims downstream): verify against SC-1 that (a) the slot representation content of I4″ is derived framework-side as citable data — insert the I4′ provenance reference; (b) the census constructs generation layers as complete cross-role species packages, securing the shared key i across roles. On (a)- or (b)-failure: I4″ is retyped from inherited restatement to declared input convention; SC-2's internal proof stands; downstream gates citing SC-2 addresses as derived VERSF structure must re-grade.

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17. Conclusion

SC-1 fixed the minimal Standard Model matter census inside VERSF.

SC-2 fixes the occupancy map — and proves it is the only one within each keyed generation layer, and the only one across the full keyed ledger.

Every minimal matter field now has a framework address. For each generation, Q_L occupies the coloured weak-attachment doublet slot, u_R the upper coloured closure singlet slot, d_R the lower coloured closure singlet slot, L_L the colourless weak-completion doublet slot, and e_R the charged-lepton closure singlet slot.

The map expands to fifteen matter components per generation and forty-five minimal matter components across three generations. It preserves colour, weak representation, chirality, hypercharge, electric charge, Higgs closure route, and anomaly role. Within each keyed generation layer, the separating triple ($\mathcal{C}$, $\mathcal{W}$, Y) leaves no species-assignment freedom, so the seating chart is forced rather than chosen. Across the three-generation ledger, the same fixed-layer structure repeats over the keyed SC-1 generation layers. The full map is therefore unique on the keyed ledger, and otherwise unique up to generation relabelling and permitted flavour-basis rotations.

It also blocks common errors. It does not confuse colour multiplicity with species count. It does not confuse weak basis with mass basis. It does not treat sterile neutrinos as minimal. It does not derive CKM, PMNS, or masses. It fixes the addresses.

This is exactly what later quantitative VERSF papers need. A mass theorem must say which occupied slot receives which mass. A Yukawa theorem must say which closure route is being predicted. A CKM theorem must say which left-sector rotations are being compared. A neutrino theorem must declare whether it has extended the minimal occupancy map.

In one sentence:

The world-to-framework occupancy map assigns every minimal Standard Model matter field to a unique VERSF terminal-record address, forced within each keyed generation layer by representation data, with layer keys inherited from SC-1 and restated in I4″, so later quantitative derivations can no longer move between particle labels, chiralities, generations, bases, or extension slots without being audited.

Appendix A — Minimal Algebraic Skeleton

For each generation i = 1, 2, 3:

$\Omega(Q\_L_i) = \mathsf{Q}_i = (i, 3, 2, Y = +1/3, L)$,

$\Omega(u\_R_i) = \mathsf{U}_i = (i, 3, 1, Y = +4/3, R, \tilde{H})$,

$\Omega(d\_R_i) = \mathsf{D}_i = (i, 3, 1, Y = -2/3, R, H)$,

$\Omega(L\_L_i) = \mathsf{L}_i = (i, 1, 2, Y = -1, L)$,

$\Omega(e\_R_i) = \mathsf{E}_i = (i, 1, 1, Y = -2, R, H)$.

Component expansion:

$Q\_L_i : u\_L_i^a, d\_L_i^a, a = r, g, b$;

$u\_R_i^a, d\_R_i^a$;

$L\_L_i : \nu\_L_i, e\_L_i$;

e_R_i.

Count:

6 + 3 + 3 + 2 + 1 = 15 per generation, 3 × 15 = 45 in the minimal three-generation matter ledger.

Sterile extension:

$\nu\_R_i \sim (1, 1), Y = 0$

is not part of the minimal map.

Appendix B — Full Three-Generation Occupancy Table

Closure column typing: singlet slots carry closure routes proper; doublet slots carry participatory sets part{·} (§6.1, §6.4).

Generation	World multiplet	Framework slot	Representation	Y	Closure
1	$Q\_L_1 = (u\_L, d\_L)^T$	Q_1	(3, 2)	+1/3	part{H, H̃}
1	u_R	U_1	(3, 1)	+4/3	H̃
1	d_R	D_1	(3, 1)	−2/3	H
1	$L\_L_1 = (\nu\_eL, e\_L)^T$	L_1	(1, 2)	−1	part{H}
1	e_R	E_1	(1, 1)	−2	H
2	$Q\_L_2 = (c\_L, s\_L)^T$	Q_2	(3, 2)	+1/3	part{H, H̃}
2	c_R	U_2	(3, 1)	+4/3	H̃
2	s_R	D_2	(3, 1)	−2/3	H
2	$L\_L_2 = (\nu\_\mu L, \mu\_L)^T$	L_2	(1, 2)	−1	part{H}
2	μ_R	E_2	(1, 1)	−2	H
3	$Q\_L_3 = (t\_L, b\_L)^T$	Q_3	(3, 2)	+1/3	part{H, H̃}
3	t_R	U_3	(3, 1)	+4/3	H̃
3	b_R	D_3	(3, 1)	−2/3	H
3	$L\_L_3 = (\nu\_\tau L, \tau\_L)^T$	L_3	(1, 2)	−1	part{H}
3	τ_R	E_3	(1, 1)	−2	H

Appendix C — Basis-Rotation Invariance

Let the weak-basis multiplets be Q_L, u_R, d_R. A flavour rotation acts as

Q_L → U_Q Q_L, u_R → U_u u_R, d_R → U_d d_R.

The Yukawa matrices transform as

Y_u → U_Q† Y_u U_u,

Y_d → U_Q† Y_d U_d.

These transformations change matrix coordinates, not representation occupancy.

After electroweak symmetry breaking,

$M_u = (v/\sqrt{2}) Y_u$, $M_d = (v/\sqrt{2}) Y_d$.

Diagonalisation gives

$U_{uL}^\dagger M_u U_{uR} = D_u$,

$U_{dL}^\dagger M_d U_{dR} = D_d$.

The charged-current mixing matrix is

$V_{CKM} = U_{uL}^\dagger U_{dL}$.

Thus CKM mixing is a relation between diagonalisation frames. It does not change the fact that the up-type and down-type fields occupy U_i, D_i, Q_i species slots.

The same logic applies to leptons when a neutrino-mass extension is declared:

$U_{PMNS} = U_{eL}^\dagger U_{\nu L}$.

Without a neutrino-mass extension, PMNS is not part of the minimal matter occupancy map.

Appendix D — Referee Objections and Replies

Objection 1 — Is this just the Standard Model field table rewritten?

Reply. No. The Standard Model field table lists representations. SC-2 assigns those representations to VERSF terminal-record framework slots with generation layer, species role, chirality, closure route, anomaly role, and extension status — and proves the assignment unique within each keyed generation layer, and unique across the full keyed ledger. The point is not to relabel the fields; it is to make later VERSF predictions address-specific and to remove any species-assignment freedom a later paper could exploit; the residual generation-relabelling freedom is typed by OM-11 and policed by F20–F22.

Objection 2 — Does the map derive the electron mass?

Reply. No. It identifies the electron's framework addresses: e_L as the lower component of L_1, and e_R as the charged-lepton closure singlet E_1. The mass value remains a later Yukawa or hierarchy derivation.

Objection 3 — Are (u, d), (c, s), and (t, b) physical weak doublet pairs?

Reply. They are occupancy-layer labels. In the mass basis, quark charged currents involve CKM mixing. SC-2 fixes representation occupancy, not CKM numerical structure.

Objection 4 — Why distinguish multiplet occupancy and component occupancy?

Reply. Because Q_L is one species multiplet but contains six components per generation after colour and weak-component multiplicity. Confusing these levels creates overcounting, and F6/F7 exist to catch exactly that.

Objection 5 — Where are the neutrinos?

Reply. The left-handed neutrino components occupy the upper entries of L_L. Right-handed sterile neutrinos are not part of the minimal map, but they may be added in a declared neutrino-mass extension.

Objection 6 — Does the Higgs occupy a matter slot?

Reply. No. The Higgs occupies the universal closure-carrier slot. It is part of the Standard Model field content, but in SC-2 it is not counted as a matter species generation. Treating it as one is falsifier F18.

Objection 7 — What about antiparticles?

Reply. Antiparticles are conjugate excitations of the occupied fields. They do not require separate minimal species slots. For anomaly bookkeeping, right-handed fields may be represented as left-handed conjugates, but by Lemma 4 that is a ledger transformation, not a new occupancy.

Objection 8 — Does this prove three generations?

Reply. No. That is the job of SC-1. SC-2 inherits the three-generation census and maps the world fields into the resulting slots.

Objection 9 — Could a fourth generation be mapped?

Reply. Only in an extension ledger with a fourth support-return layer or equivalent new structure. It is not part of the minimal occupancy map, and inserting it silently is falsifier F16.

Objection 10 — Is the uniqueness claim doing real work, or is it decorative?

Reply. It is doing real work, provided it is stated at the right level. The separating triple ($\mathcal{C}$, $\mathcal{W}$, Y) forces the five species assignments within each fixed generation layer: the five values are pairwise distinct on both sides, so no cross-species reassignment is representation-preserving. It does not force generation labels — a generation swap such as $\Omega'(Q\_L_1) = Q_2$, $\Omega'(Q\_L_2) = Q_1$ preserves all representation data — which is why the layer coordinate is inherited from SC-1 (I7, OM-11), the flavour-basis freedom is typed as gauge in Lemma 3, and a bare relabelling is not a rival map (§7.5, Lemma 3′). Without fixed-layer uniqueness, a later paper could route a prediction through an alternative species assignment, defeating the audit purpose of the map. Falsifier F19 makes the fixed-layer claim independently attackable; F20–F22 police the generation-layer boundary.

Objection 11 — Gauge representations do not distinguish the three generations. Does this break uniqueness?

Reply. No. SC-2 does not claim that gauge representation data alone distinguishes generation 1 from generation 2 or generation 3. The separating triple ($\mathcal{C}$, $\mathcal{W}$, Y) proves uniqueness of the five species assignments within a fixed generation layer. The existence of three generation layers is inherited from SC-1. Once that layer ledger is fixed, the full occupancy map is unique. If the layer ledger is suppressed, the map is unique only up to generation relabelling, which is not a rival species assignment.

Objection 12 — Is the uniqueness claim partly definitional?

Reply. It is conditional, not circular. SC-2 does not derive the Standard Model representation table from nothing, nor does it derive the SC-1 generation census. It proves that, given the SC-1 framework slots and the QN-2 Standard Model convention ledger, there is no remaining freedom in assigning the world-side matter fields to the framework-side matter slots. The theorem therefore closes assignment freedom, not representation derivation.

Objection 13 — Does excluding ν_R conflict with observed neutrino mass?

Reply. No. SC-2 maps the minimal chiral matter occupancy ledger before a neutrino-mass extension is declared. The left-handed neutrino weak components are present as upper components of L_L. A sterile ν_R, Majorana sector, seesaw mechanism, Weinberg operator, or VERSF-specific neutrino completion may be added only by an explicit extension ledger. SC-2 therefore does not deny neutrino mass; it prevents a neutrino-mass mechanism from being silently inserted into the minimal occupancy map.

Objection 14 — Why define Ω_mult and Ω_comp separately?

Reply. Because species occupancy and component occupancy answer different questions. Ω_mult answers which framework species slot a world multiplet occupies. Ω_comp answers where each colour branch and weak component sits after component expansion. Keeping the two maps separate prevents double-counting and prevents component-level injectivity from being mistaken for a new species census.

Objection 15 — What is the one-sentence result?

Reply.

Every minimal chiral Standard Model matter field occupies a unique VERSF terminal-record address, every required VERSF matter address is occupied, the assignment is forced within each keyed generation layer by representation data, and it is unique across the full keyed ledger, so later quantitative derivations must target fixed framework addresses rather than movable particle labels.

Final One-Sentence Theorem

Given the minimal chiral matter census, the keyed VERSF framework slot ledger restated in SC-2, and the QN-2 Standard Model convention ledger, the world-to-framework occupancy map assigns each Standard Model matter component to one unique keyed terminal-record address labelled by generation key, species role, colour, weak representation, chirality, hypercharge, electric charge, Higgs closure route, anomaly role, and extension status; the assignment is forced within each keyed generation layer by representation data, unique across the full keyed ledger, and unique up to relabelling when generation keys are suppressed; therefore later VERSF mass, mixing, and hierarchy derivations must target fixed framework addresses rather than movable particle labels.