

The Full-Carrier Quark-Mixing Curvature Closure in VERSF

From the target-blind RRHF baseline through the projected C_3 Hessian, the full-su(8) survival theorem, and the charged-completion mixed jet

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SCIENTIFIC STATUS. Controlling CKM: RRHF, empirically poor beyond its calibrated rows. Quadratic C_3 axle: exact. Minimal-cell Hessian: calculated. Full-su(8) survival: conditional symbolic pass. AXD-3 attachment: executed, two-level stable, exactly linear; local HS phase selection: exact no-go. Same-carrier bridge: conditionally fixes leading-order $|\zeta_8^{(0)}| = b/\sqrt{3}$; unit-amplitude-convention leading-order bridge norms near 0.01608 preregistered; Hessian remainder $O(\Omega_{\text{mix}}^2)$, curvature-response bound open. Readout, phase, retention, premise stack: open. New physical CKM matrix: not issued.

ONE-MATRIX RULE. Exactly one CKM matrix controls the empirical confrontation at any time: the latest sealed same-action return. RRHF is that matrix. Every curvature-corrected, axle-installed or bridge-completed matrix in this paper is a quarantined diagnostic until its coefficient, retention, readout and phase are returned by the source action. A favourable diagnostic may not replace a controlling return because it fits better.

General-reader summary

Quarks, the particles inside protons and neutrons, come in three families. The weak force can convert one kind of quark into another, and when it does, the families mix in a very particular lopsided pattern. The Standard Model records that pattern as a table of measured numbers, but it cannot say why nature chose those numbers. This paper reports the current state of the attempt to derive that table from the deeper structure this programme is built on, and the state is genuinely encouraging.

First, how the mixing arises here at all. Picture a quark as a knot in a rope: a pattern that holds itself together and keeps its identity as the rope is handled. The three quark families are three different knots tied in the same kind of rope, each holding together in its own

way, which in this picture is associated with its different mass. The weak force is the one process that can untie one knot and retie it as another. The key point is that the knot the weak force ties is never a perfect copy of one of the three knots that naturally hold together. It comes out as almost the second knot, say, with a small trace of the first and a smaller trace of the third mixed in. The mixing table is simply the record of those traces: how much of each natural knot is present in each knot the weak force makes. If the weak force retied the natural knots perfectly, the families would never mix at all.

The explanation of the particular pattern is then a statement about where those small traces come from. Most of the effect is inherited from the same deeper structure that makes the three knots different in the first place. The lopsided fine detail this paper is after comes from one further, smaller twist between the second and third families, and that twist is not put in by hand. The deeper geometry has six positions, the families have three, and symmetry permits exactly one shape for the folding of six down onto three, apart from its overall strength; that one allowed shape treats the three families identically, so both the existence and the even-handed form of the twist are forced, while its strength must be supplied separately. That single forced folding provides the common geometric carrier for everything this paper computes. The physical strength of the twist, its short list of possible orientations, and how much of it survives the moment a possibility becomes an irreversible fact each require further structures on top of it, and those are developed later in the paper.

The programme has established this step by step. A direct calculation inside the smallest honest model confirmed the forced even-handedness exactly. A natural worry was that the small model had been specially chosen, so a later calculation embedded it inside the full structure and showed that, under a short list of named assumptions, exactly the same answer survives. Independently, the machinery that attaches family structure to the full charged sector was run on its complete carrier, and it returns the same two-dimensional family plane almost identically at two levels of resolution, a strong sign of stability.

The central result of this paper is that these pieces admit one precise proposed join. If the strength computed by the full-structure calculation and the free number carried by the attachment calculation are one and the same physical quantity, read with one and the same normalisation, then the previously free number is no longer free: it becomes a definite predicted value, stated here in advance of any comparison, so that the eventual proof or disproof of that identification can be tested against it. That identification is the paper's sharpest open problem, not an established fact. It is not a fit in the ordinary sense: once the openly declared inherited scale and the named assumptions are frozen, no further measured mixing number is consulted in producing the prediction. There is also a striking convergence on the orientation of the correction: three differently built structures each allow a short list of possible orientations, and exactly one orientation appears on all three lists.

A distinctive strength of the programme is that it keeps honest books. Its baseline calculation builds the quark couplings inside a sealed procedure that is not allowed to look at the measured mixing table, apart from two scale settings taken from experiment and declared openly. Tested on the numbers those settings do not fix, it gets the broad shape right and gets two of the hardest numbers badly wrong. That failure is kept in full view,

because a recorded failure is worth more than an adjusted success, and it is exactly the two worst failures that the derived correction repairs. The scoreboard below shows both columns side by side. In every row, “low” or “high” means relative to the frozen benchmark value used here for that quantity.

Quantity	Sealed baseline: how close to the frozen benchmark	With the derived correction: how close to the frozen benchmark	Plain reading
Cabibbo mixing (V _{us})	0.6% low	about 0.5% low	Calibration input: the calculation starts near this value, so closeness here is not a test
Charm-bottom mixing (V _{cb})	0.15% low	about 0.5% low	Calibration input, as above; the machinery is tested on the rows below
Up-bottom mixing (V _{ub})	7.9% high	about 8.5% low	Genuine tension, in opposite directions
Top-down mixing (V _{td})	31.5% low	about 1% high	The baseline’s worst failure, and the correction’s most dramatic repair
Mixing-triangle angle alpha	not separately returned	about 0.5% high	Close, on the corrected reading
Mixing-triangle angle beta	not separately returned	about 0.5% low	Close, on the corrected reading
Mixing-triangle angle gamma	not separately returned	about 10% high	The largest remaining tension of the corrected reading
Quark-sector CP violation (Jarlskog invariant J)	41% low	about 5% low	The baseline’s other severe failure, strongly improved

The corrected column is dramatically better: on the uncorrelated diagnostic chi-square used in this paper, it improves on the baseline by more than a factor of thirty on the quantities available for both readings. The reason it is nevertheless printed in a separate column, rather than declared the answer, is a matter of discipline. The orientation of the correction has not yet been derived by the theory’s own machinery, and a correction whose success is allowed to select itself would prove nothing. The convergence of three distinct constructions on that same orientation is nontrivial evidence about what the eventual derivation is likely to return; it is not yet the derivation.

What remains is the hardest kind of work: proving that the two calculations really are descriptions of one and the same physical object, deriving from the theory itself which way round the correction is applied, and showing how much of it survives the moment a possibility becomes a fact. Each of those is now a sharply posed problem rather than an open search, and the predicted values in this paper are stated in advance precisely so that the eventual derivation can be tested against them. Until then, the corrected column is the programme's most promising result, and the sealed baseline remains its official answer.

1. What quark mixing is in VERSF

In the picture used throughout this programme, matter consists of self-holding patterns in the underlying substrate, each keeping its identity as one irreversible fact succeeds another, the way a knot keeps its identity as a rope is handled. VERSF represents the three quark families as three distinct closure configurations of the same kind of pattern, distinguished by how deeply each is wound into the substrate. The differing cost of maintaining those configurations is intended to underlie the observed mass hierarchy, though the complete first-principles mass derivation is not yet closed.

The weak interaction is the one process in the theory that converts one configuration into another. Quark mixing arises because the conversion process and the persistence process do not prefer exactly the same three configurations. The configurations that persist with definite mass are slightly rotated relative to the configurations the conversion process connects, and the CKM matrix is precisely the table of overlaps between the two sets. If the two sets coincided, the matrix would be the identity and the families would never mix.

Within the closure geometry, the leading part of that rotation is already fixed by the same structure that separates the second and third families. What the programme calls the axle is a further, smaller twist: a specific additional rotation, applied in the plane of the second and third families, whose existence and shape are forced by symmetry but whose strength, orientation and survival must be earned from the source dynamics. The twist can be applied in one of a small number of discrete orientations, and the difference between an orientation and its mirror is what the theory offers as the origin of CP violation, in which certain particle processes and their antimatter counterparts do not behave as exact mirror images of one another.

Three separate questions therefore organise everything below: how strong the twist is, which discrete orientation it takes, and how much of it survives the moment a quark process is committed to fact. This paper closes a large part of the first question conditionally, sharpens the second into a finite list, and imports a parameter-free candidate for the third, while proving none of the three physically.

2. Epistemic precedence and typed-object supersession

The quark-mixing corpus contains several generations of statements, and they do not all have equal status. A later theorem supersedes an earlier grade only when it evaluates the

same typed object on the same carrier and readout. The controlling precedence used here is:

1. **RRHF charged-flavour execution:** the controlling conditional CKM matrix, target-blind given its declared calibrated leading mixing inputs.
2. **Unique quadratic $D_6 \rightarrow C_3$ axle theorem:** exact lowest-order geometry.
3. **Projected C_3 Hessian Return:** the actual minimal-cell second-variation calculation.
4. **Full $su(8)$ Weak-Doublet Hessian Return:** the conditional symbolic full-embedding survival theorem.
5. **AXD-2 / AXD-3 full-carrier charged-completion executions:** representation support, family-blindness of the old terminal action, and the full 334-coordinate mixed-jet attachment.
6. **Later phase, readout and holonomy audits:** controlling corrections wherever an earlier readout or phase argument was weakened, relabelled or withdrawn.
7. **SMNC provenance ledger:** programme-level grading of the combined chain, including the one-matrix rule, the calibration taxonomy and the retention precursor.

An earlier statement that the full embedding was simply owed is superseded, for the typed C_3 curvature, by the full- $su(8)$ symbolic survival theorem. What remains owed is the substrate derivation of that theorem's premises and the numerical and physical same-action closure. Nothing in the later layers erases the RRHF baseline, because the baseline and the curvature corrections are differently typed objects: the empirical failure of the former and the conditional success of the latter coexist by construction.

3. The controlling RRHF baseline

The representation-resolved history functional produces the charged tensor

$$T_{ch}^{(n)} = Y_u^{(n)} \oplus Y_d^{(n)} \oplus Y_e^{(n)},$$

and the left singular frames return

$$V_{CKM}^{(n)} = U_u^{(n)\dagger} U_d^{(n)}.$$

At the controlling level,

$$|V_{CKM}^{RRHF}| = [[0.974818, 0.222964, 0.004198], [0.222926, 0.973988, 0.040639], [0.005895, 0.040427, 0.999165]],$$

$$|J_{RRHF}| = 1.86032934918 \times 10^{-5}.$$

Each row of the printed magnitude matrix has unit norm to within a few parts in 10^7 , consistent with the 10^{-6} rounding of the printed entries; this is a necessary condition for unitarity, not a full certificate, since unitarity is a statement about the underlying complex matrix, $\|V^\dagger V - I\| \approx 0$, whose machine-precision record resides with the sealed RRHF source execution rather than being re-derivable from the magnitudes printed here. The execution core is sealed from CKM benchmark data; its leading mixing scales enter as

declared calibrated inputs, so the near-agreement of $|V_{us}|$ and $|V_{cb}|$ is a footprint of the calibration, not a test of the machinery. The machinery is tested on the rows the calibration does not fix, and there it fails:

Quantity	RRHF	Benchmark	Raw Δ	Reading
$ V_{us} $	0.222964	0.22431	-0.60%	Calibration-dominated row
$ V_{cb} $	0.040639	0.0407	-0.15%	Calibration-dominated row
$ V_{ub} $	0.004198	0.00389	+7.92%	Genuine output; high
$ V_{td} $	0.005895	0.0086	-31.45%	Severe shortfall
$ V_{ts} $	0.040427	0.0415	-2.59%	Moderate shortfall
$ J $	1.860329×10^{-5}	3.16×10^{-5}	-41.13%	Severe CP shortfall

On its declared five-row diagnostic set $\{|V_{us}|, |V_{ub}|, |V_{td}|, |V_{ts}|, |J|\}$ the diagnostic chi-square is approximately 290.5, dominated by $|V_{td}|$ (≈ 183) and $|J|$ (≈ 100); restricted to the strictly non-calibration set $\{|V_{ub}|, |V_{td}|, |V_{ts}|, |J|\}$ it is approximately 288.0. The chi-square definition and inputs are printed in Section 26 so that both values are recomputable. The two deficits, a too-small $|V_{td}|$ and a too-small CP area, motivate a correction but are not permitted to choose its result.

4. Exact lowest-order axle geometry

Let E_1 denote the selected two-real-dimensional D_6 source mode and E_2 the generation-curvature carrier.

The linear route is forbidden:

$$\text{Hom}_{D_6}(E_1, E_2) = 0.$$

The first admissible route is quadratic and unique:

$$\dim \text{Hom}_{D_6}(\text{Sym}^2 E_1, E_2) = 1.$$

A representative is

$$Q(x, y) = (x^2 - y^2, 2xy), \text{ equivalently } Q(z) = z^2.$$

For the normalised selected mode,

$$Q(u) = (1/3)(1, e^{(-2\pi i/3)}, e^{(+2\pi i/3)}), \|Q(u)\| = 1/\sqrt{3}.$$

The quadratic order, the unique form and the exact threefold geometry are therefore consequences of the representation theory, not reconstructions from CKM data. What is established here is precisely this: the linear descent is forbidden, and the first admissible descent is uniquely quadratic; its image distributes democratically over the three families with the fixed $2\pi/3$ phase spacing that reappears in every downstream phase question.

5. The weak-doublet projected object

The flavour curvature is not a free matrix. The inherited closure Hessian is

$$H_{cl} = G|_{su(8)},$$

and the weak-doublet compression is the ordered projector product

$$P_W = P_Y P_R P_C,$$

giving the Hermitian ordered compression

$$H_W = P_W^\dagger H_{cl} P_W.$$

The frame generator is

$$\Omega_W = i \varepsilon_W H_W,$$

and on weak-role space it decomposes as

$$\Omega_W = \Omega_{even} \otimes I + \Omega_{odd} \otimes \tau_3 + \Omega_{mix}.$$

The role-odd block supplies the leading relative quark split. The role-even block supplies the common-mode curvature in which the axle lives. The mixed block is the gapped role-changing sector whose smallness the full-embedding theorem must control.

6. Convention-invariant curvature and the exact residue

A pure role-even $2 \leftrightarrow 3$ curvature has the anti-Hermitian form

$$\Omega_0(z) = z E_{23} - z^* E_{32}.$$

For the inherited role-odd generator

$$\Omega_q = [[0, a, c e^{i\varphi}], [-a, 0, b], [-c e^{-i\varphi}, -b, 0]],$$

with

$$a = 9/40, b = 81/2000, c = 243/100000, \varphi = 2\pi/3,$$

direct multiplication gives the exact commutator entry

$$[\Omega_0, \Omega_q]_{13} = -a z.$$

Introducing the split parameter λ and defining $\zeta = \lambda z$, the leading CKM residue in every split convention is

$$\Delta_{13} = (1/2) a \zeta.$$

This is the convention-invariance theorem: however the deformation is split between the up and down sectors, the physical 1–3 residue depends only on the product $\zeta = \lambda z$, so ζ is

the correct convention-invariant curvature variable and every later result is stated in terms of it.

7. Minimal committed-quark C_3 Hessian execution

The minimal branch cell uses six real coordinates,

$$\mathbf{x} = (s_{12}, a_{12}, s_{23}, a_{23}, s_{31}, a_{31})^T,$$

one symmetric and one antisymmetric coordinate per family pair. The actual second-variation execution returns, block by block,

$$G_C^A B = \kappa_C I_6, G_T^A B = \kappa_T I_6, G_R^A B = \kappa_R I_6,$$

hence

$$G_{C_3}^A B = \kappa_{C_3} I_6.$$

Closure, transport and record composition each return an exact multiple of the identity on the branch cell, so the pre-readout role-even quark block is C_3 -covariant and circulant. The result commutes with the cyclic generation operator R_3 , and the three branch weights are exactly equal:

$$w_{12} = w_{23} = w_{31}.$$

The threefold democracy of Section 4 is therefore not merely permitted by the dynamics; it is returned by an executed Hessian.

8. The minimal-cell amplitude theorem

Democracy fixes the split of a norm, not the norm itself. Two named premises supply the norm. Under role-isotype metric isotropy,

$$G_{\text{role}} = \kappa_{\text{role}} I_2,$$

and under the shared-closure-profile premise PH-SAME, the total role-even branch budget is inherited from the role-odd $2 \leftrightarrow 3$ scale b :

$$\|Z_{C_3}\|_K^2 = b^2.$$

The three branch generators are taken K -orthonormal,

$$\langle B_{ij}, B_{kl} \rangle_K = \delta_{(ij)(kl)},$$

and ζ is defined as the common branch coefficient in that orthonormal basis. This normalisation statement is load-bearing: it is what prevents a stray factor of κ_{C_3} or a generator norm from surviving into the amplitude, and every downstream number inherits it. Since the three branches are then orthonormal and democratic,

$$3|\zeta|^2 = b^2,$$

hence

$$|\zeta| = b/\sqrt{3} = 0.023382685902180.$$

The load-bearing exposure is explicit: the physical role-even common mode and the role-odd scale b must genuinely share one closure profile and one norm. If PH-SAME fails, the amplitude, though still democratic, detaches from b .

9. Minimal-cell readout and phase

Before mass and readout, the common-mode orbit is democratic over the three family pairs. The desired post-readout survivor is the $2 \leftrightarrow 3$ branch. The $1 \leftrightarrow 2$ suppression is structurally motivated by the occupied Cabibbo doorway; the $3 \leftrightarrow 1$ suppression remains more delicate and must ultimately be derived from the full readout operator rather than assumed.

For the phase, the Hermitian-side square satisfies

$$h^2/|h|^2 = e^{(2\pi i/3)},$$

whose roots are

$$e^{(i\pi/3)} \text{ and } e^{(i4\pi/3)}.$$

The minimal-Hermitian-lift rule selects $h/|h| = e^{(i\pi/3)}$, and the frame rotation $\Omega_W = +i \varepsilon_W H_W$ adds a quarter turn, giving the frame-side phase

$$\arg \zeta = \pi/3 + \pi/2 = 5\pi/6.$$

Therefore

$$\zeta_{C_3} = (b/\sqrt{3}) e^{(i \cdot 5\pi/6)} = -81/4000 + i \cdot 27\sqrt{3}/4000 = -0.020250000000000 + i \cdot 0.011691342951090.$$

Both selections in this section are conditional: the minimal lift is a rule, not yet a theorem, and Section 23 shows that the local completion metric cannot ratify it.

10. Minimal-cell CKM residue

The exact leading residue is

$$\Delta_{13} = (1/2) a \zeta_{C_3} = -0.0022781 + 0.0013153 i,$$

with

$$|\Delta_{13}| = 0.0026306.$$

The converged two-factor calculation gives approximately 0.0026332, a difference of roughly 0.1%, consistent with a small higher-order split correction and retained as a consistency margin rather than absorbed.

11. Full su(8) embedding: canonical status

The natural objection to Sections 7–10 is that the minimal cell might be a specially favourable toy. The full-embedding analysis addresses it head-on by writing the most general deformation of the minimal result as

$$\zeta_8 = \rho_8 (\mathbf{b}/\sqrt{3}) \mathbf{e}^{i\theta_8} + \delta_{\text{leak}},$$

with the residue

$$\Delta_{13}^{(8)} = (1/2) a \zeta_8.$$

This parameterisation separates the three ways the full embedding could break the minimal answer: norm renormalisation ($\rho_8 \neq 1$), phase-branch migration ($\theta_8 \neq 5\pi/6$), and non-minimal leakage ($\delta_{\text{leak}} \neq 0$). Each is then addressed by an explicit theorem or premise rather than by hope.

12. Full-su(8) premises

P-phase. Before phase lift and mass readout, the closure functional and admissible background are invariant under the generation root-phase torus T^2 . Combined with C_3 , this gives per-plane scalar quadrature blocks and $G_{\mathcal{B}\mathcal{B}} = \kappa I_6$ at leading order.

P-role. Before readout, the committed quark doublet is invariant under the $u \leftrightarrow d$ role swap. This decouples the same-charge role-odd spectator sector.

P-bg. The full admissible background restricted to the branch support agrees with the minimal-cell background. This fixes $\kappa = \kappa_{C_3}$ and hence $\rho_8 = 1$.

P-gap. The mixed role-changing/spectator block is gapped on the admissible background: $G_{\mathcal{M}\mathcal{M}}$ is invertible with $G_{\mathcal{M}\mathcal{M}}^{-1} = O(1)$, uniformly at the executed regulators. This is the premise that licenses the second-order Schur bound of Section 13.

These are named substrate conditions, not free assumptions: each is a definite statement about the primitive action whose derivation is gate QK-1. Until they are derived, every full-embedding conclusion carries them as disclosed conditions.

13. Full mode inventory and spectator decoupling

The full-su(8) calculation performs a complete mode inventory of the embedding carrier. Source note: the source record reports a 64-mode inventory, while su(8) itself is 63-dimensional; the source therefore reports one additional carrier mode beyond the 63 su(8)

generators, and its typing remains to be recovered from the sealed record. At leading order the branch block decouples from every other sector:

$$G_{\mathcal{B}\mathcal{S}} = G_{\mathcal{B},\text{Cartan}} = G_{\mathcal{B},\text{anchor}} = G_{\mathcal{B},\text{interface}} = 0,$$

$$G_{\mathcal{B},\text{role-odd}} = 0,$$

$$G_{\mathcal{B}\mathcal{M}} = O(\Omega_{\text{mix}}).$$

The second-order statement is then a Schur-complement estimate rather than an assertion. Eliminating the mixed sector gives

$$\delta G_{\mathcal{B}} = -G_{\mathcal{B}\mathcal{M}} G_{\mathcal{M}\mathcal{M}}^{-1} G_{\mathcal{M}\mathcal{B}},$$

and with both off-diagonal factors $O(\Omega_{\text{mix}})$ and $G_{\mathcal{M}\mathcal{M}}^{-1} = O(1)$ by P-gap,

$$G_{\mathcal{B}}^{(8)} = \kappa_{\mathcal{C}_3} I_6 + O(\Omega_{\text{mix}}^2).$$

Complement non-renormalisation is promoted to a theorem,

$$\rho_8^{\text{eff}} = \rho_8^{\text{dir}},$$

and under P-bg,

$$\rho_8 = 1.$$

The only surviving contamination channel is the gapped role-changing mixed sector, entering at second order in Ω_{mix} , whose quantitative bound is gate QK-1's companion obligation.

14. Conditional symbolic full-block return

Combining the full branch form with the conditional readout and phase branches gives the leading-order return

$$\zeta_8^{(0)} = \zeta_{\mathcal{C}_3} = (\mathbf{b}/\sqrt{3}) \mathbf{e}^{(i \cdot 5\pi/6)},$$

with the full curvature carrying the not-yet-bounded mixed-sector response,

$$\zeta_8 = \zeta_8^{(0)} + \delta\zeta_{\text{mix}},$$

and therefore

$$\Delta_{13}^{(8)} = (1/2) \mathbf{a} \zeta_8^{(0)} + (1/2) \mathbf{a} \delta\zeta_{\text{mix}}.$$

No order is assigned to $\delta\zeta_{\text{mix}}$ here, and the distinction from the Hessian remainder is deliberate. The Hessian correction itself is $O(\Omega_{\text{mix}}^2)$ by the Schur estimate of Section 13, but the leading branch Hessian is proportional to I_6 and therefore degenerate, and a perturbation of a degenerate matrix that is second order in operator norm can still select or rotate directions inside the degenerate subspace by more than its own order. Passing from $\delta G_{\mathcal{B}}$ to $\delta\zeta_{\text{mix}}$ therefore requires a perturbative stability bound relating the curvature-

coordinate response to the established $O(\Omega_{\text{mix}}^2)$ Hessian remainder, and that stability map is owed by QK-1 alongside the remainder bound itself.

Grade: conditional/symbolic, leading order. The same typed C_3 curvature survives the full weak-doublet $\text{su}(8)$ embedding at leading order under the named premises, while both the second-order Hessian remainder and its induced curvature-coordinate response remain unbounded pending QK-1. This is materially stronger than the earlier position, in which the full embedding was an unexamined debt, and materially weaker than a physical return, because the premises themselves are not yet substrate-derived.

15. What the full-su(8) theorem still owes

1. Substrate derivation of P-phase.
2. Substrate derivation of P-role.
3. A direct test of P-bg on the admissible background.
4. Construction of the full readout operator and direct verification of the survivor projections $P_Y^A B Z_{12} = 0$, $P_Y^A B Z_{23} = Z_{23}$, $P_Y^A B Z_{31} = 0$.
5. The physical phase-return invariant σ_φ^q .
6. A quantitative bound on the $O(\Omega_{\text{mix}})$ sector, including direct verification of P-gap through the Schur estimate, together with the perturbative stability map from the resulting branch-Hessian correction to the curvature coordinate ζ_8 , without which the degenerate leading Hessian leaves the order of the induced curvature shift unassigned.
7. Primitive provenance of the base amplitudes a, b, c and the source phase φ .
8. Recovery and typing of the one inventoried carrier mode beyond the 63 $\text{su}(8)$ generators.

16. Full 334-coordinate generation-sensitive completion attachment

The independent AXD-3 execution attaches a family generator X to the complete charged-completion carrier and returns the mixed jet

$$\mathcal{K}_n(X) = c_H \otimes [\Lambda_Q(X) T_{\text{ch}}^{(n)}], \Lambda_Q(X) = X \oplus X \oplus 0,$$

on a carrier of full shape

$$2 \times 334 \times 9 \times 9.$$

This is not a toy cell: it is the physical completion carrier of the charged sector, with the family generator inserted in the quark blocks and annihilated in the lepton block, exactly as the axle typing requires.

17. Two-regulator charged-completion return

Define the unit-amplitude quadrature generators

$$B_{23} := \Omega_0(1) = E_{23} - E_{32}, B_{23}^{\wedge} I := \Omega_0(i),$$

where unit-amplitude means $z = 1$ and $z = i$ in $X(z) = zE_{23} - z^*E_{32}$, not unit Hilbert-Schmidt norm: on the family block, $\|\Omega_0(1)\|_{\text{HS}} = \sqrt{2}$. For the unit-amplitude generator B_{23} , the mixed-jet norms at the two executed regulator levels are

$$\|\mathcal{K}_3(B_{23})\| = 0.687772002206489,$$

$$\|\mathcal{K}_4(B_{23})\| = 0.687487202319853,$$

a relative drift of approximately -0.041409% . The family-plane overlap singular values between the two levels are both

$$0.9999998173099576,$$

so the returned family plane is exceptionally stable between the two executed regulator levels. Two levels establish two-level stability, not regulator independence or convergence; within that scope the attachment is stable in the two respects that matter: its direction barely moves, and its unit-amplitude response moves at the fourth decimal.

18. Exact local phase blindness

At the two levels the real quadrature Gram matrices are exactly isotropic:

$$G_{23}^{(3)} = 0.473030327019123 \cdot I_2,$$

$$G_{23}^{(4)} = 0.472638653353578 \cdot I_2.$$

The exactness is algebraic, not merely numerical, and the proof runs through the physical anti-Hermitian carrier itself. Writing the axle generator at complex amplitude z as

$$X(z) = z E_{23} - z^* E_{32},$$

direct multiplication gives

$$X(z)^\dagger X(z) = |z|^2 (E_{22} + E_{33}),$$

so the positive part of the generator depends on z only through $|z|^2$, never through $\arg z$. The attachment acts by left multiplication on the charged tensor, so $\Lambda_Q(X)^\dagger \Lambda_Q(X) = (X^\dagger X) \oplus (X^\dagger X) \oplus 0$ and

$$\|\mathcal{K}_n(X(z))\|^2 = |z|^2 \cdot \|\mathcal{K}_n(\Omega_0(1))\|^2.$$

Since $X(e^{i\theta}) = \cos \theta \cdot \Omega_0(1) + \sin \theta \cdot \Omega_0(i)$, constancy of that norm around the phase circle forces the two real quadrature jets $K_R = \mathcal{K}_n(\Omega_0(1))$ and $K_I = \mathcal{K}_n(\Omega_0(i))$ to have equal norms and vanishing real Hilbert-Schmidt overlap, so the real quadrature Gram is exactly $\gamma_n I_2$ with γ_n the squared unit-amplitude response. The executed values confirm the

identity to all printed digits: $0.687772002206489^2 = 0.473030327019123$ and $0.687487202319853^2 = 0.472638653353578$.

The local charged-completion metric transports phase but selects none. The no-go is exact for the executed Hilbert-Schmidt metric and for every local criterion built from $X^\dagger X$: no such norm on this carrier can prefer 150° over any other orientation. Whatever selects the physical phase must be global, holonomic or admissibility-level.

19. The amplitude degeneracy in AXD-3

The defining attachment map of Section 16 is exactly linear in X by construction: $\Lambda_Q(X) = X \oplus X \oplus 0$, right multiplication by $T_{ch}^{(n)}$ and tensoring with c_H are each linear maps, so

$$\mathcal{K}_n(\lambda X) = \lambda \mathcal{K}_n(X)$$

holds algebraically. A deliberate scalar scan at level 4 independently verifies that the implementation respects this linearity:

$$\lambda = 0 \Rightarrow 0, \lambda = 1/2 \Rightarrow 0.3437436011599264, \lambda = 1 \Rightarrow 0.6874872023198528, \lambda = 2 \Rightarrow 1.3749744046397057,$$

with the normalised mixed-jet shape unchanged throughout.

AXD-3 by itself does not choose the physical coordinate along the returned family plane. Read in isolation this looked like a defect. Read against Section 8 it is exactly the degree of freedom the weak-doublet amplitude theorem exists to supply: a linear instrument awaiting its input.

20. Same-carrier bridge theorem target

Let $B_{23} = \Omega_0(1)$ denote the unit-amplitude anti-Hermitian family generator of the AXD-3 quadrature basis, in the convention fixed in Section 17. The full-su(8) weak-doublet result supplies

$$\Omega_0(\zeta_8) = \zeta_8 E_{23} - \zeta_8^* E_{32}.$$

The decisive same-action statement is

$$X_{AXD3} = \Omega_0(\zeta_8)$$

up to the declared common generator normalisation. If that identity holds, and no additional independent family-source multiplier intervenes between the weak-doublet block and the charged-completion derivative, then the AXD-3 amplitude degeneracy is not a physical freedom at all. It is coordinate freedom before the upstream curvature is supplied.

21. The conditional bridge calculation and its preregistered numbers

Because AXD-3 is linear and its local metric is exactly isotropic on the B_{23} plane,

$$\|\mathcal{K}_n(\Omega_0(\zeta_8))\| = |\zeta_8| \cdot \|\mathcal{K}_n(B_{23})\|.$$

With

$$|\zeta_8^{(0)}| = b/\sqrt{3} = 0.023382685902180,$$

the predicted leading-order bridge mixed-jet norms are

$$\|\mathcal{K}_3^{\text{bridge}}(0)\| = 0.016081956699908, \|\mathcal{K}_4^{\text{bridge}}(0)\| = 0.016075297313613.$$

These are the preregistered leading-order bridge values, computed from $\zeta_8^{(0)}$. The full bridge norm carries the not-yet-bounded mixed-sector response of Section 14:

$\|\mathcal{K}_n^{\text{bridge}}\| = \|\mathcal{K}_n^{\text{bridge}}(0)\| + \delta\mathcal{K}_n^{\text{mix}}$, and QK-1 owes the propagated curvature-response bound on $\delta\mathcal{K}_n^{\text{mix}}$, derived from the established $O(\Omega_{\text{mix}}^2)$ Hessian remainder through the stability map of Section 14.

The designation bridge is deliberate: these are prospective bridge predictions, and the designation phys is reserved for the sealed same-action execution of gate QK-6.

Audit note. The preregistered numbers are stated in the unit-amplitude convention of Section 17: the executed AXD-3 norms are read as $\|\mathcal{K}_n(\Omega_0(1))\|$. If the sealed AXD-3 record instead normalised its generator to unit Hilbert-Schmidt norm, every bridge number here rescales by the single known factor $\sqrt{2}$. Confirming the source convention is concrete content of the shared-normalisation assumption of Theorem PCAC-R and is owed to gate QK-4; no bridge number may be adjusted by that factor after a same-action execution is unsealed.

At level 4 the two quadrature components, from $\text{Re } \zeta_8^{(0)} = -0.020250000000000$ and $\text{Im } \zeta_8^{(0)} = 0.011691342951090$, contribute orthogonal magnitudes

$$0.013921615846977 \text{ and } 0.008037648656807,$$

whose Euclidean sum reproduces the stated norm exactly.

Three properties give this calculation its evidential character. First, it is preregistered level by level: the numbers above are frozen before any same-action execution exists to compare them with, and the test is like for like, the same-action level-3 value against the preregistered level-3 number and the same-action level-4 value against the preregistered level-4 number. The agreement window is set by the falsification clause of Theorem PCAC-R: the propagated source-derived curvature-response bound, once QK-1 supplies it, plus the declared numerical execution tolerance. The -0.041% difference between the levels is a regulator drift, not an error bar, and is not available as an agreement tolerance; a continuum or regulator-limit prediction would additionally require at least one further regulator level and an established convergence law. Second, no additional downstream CKM benchmark is used to determine the bridge once the inherited upstream scale b and the source-side premises have been frozen; the inherited provenance of b itself remains a

disclosed debt, gate QK-5, and the prediction is conditional on it. Third, it is doubly conditional, and the two conditions are distinct and separately testable: the weak-doublet side carries the P-phase/P-role/P-bg/P-gap premise stack of Section 12, and the bridge itself carries the same-carrier identity of Section 20. The two condition sets are separately testable in principle: failure of the weak-doublet premises alters the upstream returned curvature, its amplitude, phase, support or leakage, whereas failure of the bridge changes or destroys the identification between that curvature and the AXD-3 response.

Conditions consumed by the headline returns:

Return	Value	Conditions consumed	Open before physical admission
$ \zeta = b/\sqrt{3}$	0.023382685902	Role-isotype isotropy; PH-SAME shared profile; K-orthonormal branch basis; executed democratic Hessian	Substrate derivation of both premises; provenance of b
$\zeta_8^{(0)} = \zeta_- C_3$	$(b/\sqrt{3})e^{(i \cdot 5\pi/6)}$, leading order	P-phase; P-role; P-bg; P-gap; inherited role gap; minimal Hermitian lift; +i frame rotation	Premise derivations; physical σ_φ^q ; $O(\Omega_{\text{mix}})$ bound
$\ \mathcal{K}_4^{\text{bridge},(0)}\ $	0.016075297314	All of the above; same-carrier identity $X_{\text{AXD3}} = \Omega_0(\zeta_8)$; shared generator normalisation; no extra source multiplier	Same-action carrier theorem QK-4; sealed rerun QK-6

Grade: conditional cross-paper numerical return. Under the SMNC cross-paper composition rule, a value built from outputs of different papers is an audit-derived precursor until the common carrier, normalisation and source-to-source map are independently proved; that is exactly the status claimed here, and no more.

22. Readout: current controlling status

The full-su(8) analysis proves a conditional theorem: if the full readout preserves Cabibbo protection and forbids direct target insertion, then the leading survivor is the $2 \leftrightarrow 3$ branch. The actual readout operator still has to be constructed on the full support and tested directly:

$$P_Y^A B Z_{12} = 0, P_Y^A B Z_{23} = Z_{23}, P_Y^A B Z_{31} = 0.$$

Physical $2 \leftrightarrow 3$ readout: conditional theorem; direct block return still owed. Earlier novelty-based P_{23} arguments are not used as substitutes, and the completion-jet audits show why caution is required: the jet is component-preserving, so a readout that exposes a different component exposes a different phase (Section 23).

23. Phase: the discrete candidate landscape

The phase question is now sharp enough to be stated as a finite selection problem. Three distinct structures each constrain the phase, and they generate three distinct discrete ambiguities that must not be conflated.

Layer 1, local metric: exact no-go. The local charged-completion metric is exactly phase-isotropic (Section 18). No local norm selects any orientation. Whatever answers the phase question, it is not this.

Layer 2, Hermitian half-transport: conditional 150°. The minimal-Hermitian-lift rule selects the root $e^{i\pi/3}$, and the $+i$ frame rotation carries it to $\arg \zeta = 5\pi/6$. The alternative Hermitian root $e^{i4\pi/3}$ carries to $11\pi/6$, i.e. 330° , the antipodal branch $\zeta_8 \rightarrow -\zeta_8$, $\Delta_{13} \rightarrow -\Delta_{13}$. This $\mathbb{Z}_2$ ambiguity is a root choice.

Layer 3, component exposure: $2\pi/3$ shifts. The descended C_3 image carries three component phases spaced by $2\pi/3$, so a component-preserving readout can expose 150° , 30° or 270° depending on which component is physically visible. This $\mathbb{Z}_3$ ambiguity is a readout choice, and it is the one the diagnostic branch table of Section 26 probes.

Layer 4, loop orientation: conjugation. The reduced-family holonomy of Section 24 supplies the conjugate pair $\{e^{+i\cdot 5\pi/6}, e^{-i\cdot 5\pi/6}\}$, i.e. $\{150^\circ, 210^\circ\}$. Conjugation corresponds to loop orientation and is a distinct $\mathbb{Z}_2$ from the antipodal root ambiguity: 210° is the conjugate of 150° , while 330° is its antipode, and the three candidates $150^\circ, 210^\circ, 330^\circ$ are pairwise inequivalent.

One numerical observation deserves explicit statement. The Hermitian-root pair $\{150^\circ, 330^\circ\}$, the component triple $\{30^\circ, 150^\circ, 270^\circ\}$ and the holonomy orientation pair $\{150^\circ, 210^\circ\}$ have exactly one common element:

$$\{150^\circ, 330^\circ\} \cap \{30^\circ, 150^\circ, 270^\circ\} \cap \{150^\circ, 210^\circ\} = \{150^\circ\}.$$

Three differently constructed constraints, a root choice, a readout exposure and a closed-loop holonomy, each admit 150° and admit no other value in common. They are distinct constructions rather than independent ones: the root pair and the component triple both inherit aspects of the same C_3 descent, so the intersection is convergence across construction types, not a probability statement. It is genuinely striking, and it is recorded here as convergence evidence only: set intersection is not a derivation, because the three constructions are differently typed objects, and the carrier, readout and holonomy identifications between them are unproved. If any one of those identifications fails, its set drops out of the intersection and the argument weakens accordingly.

The physical selection must therefore still be a full $\mathfrak{su}(8)$ holonomy or phase-return invariant, σ_φ^q , computed from the same action, which simultaneously fixes the root, the exposed component and the orientation. The intersection tells us what that theorem is likely to return, not that it has returned it.

24. Reduced-family closed holonomy and the orientation correction

Separately from every readout argument, the frozen reduced-family role-odd transport possesses an exact, gauge-invariant, family-rephasing-invariant closed triangle holonomy with the conjugate pair

$\{e^{+i\cdot 5\pi/6}, e^{-i\cdot 5\pi/6}\}$, equivalently $\{150^\circ, 210^\circ\}$.

The provenance audit of the printed conventions fixes the orientation labels: direct substitution of the printed generator entries into the printed forward-loop definition (direct route $1 \rightarrow 3$ against indirect route $1 \rightarrow 2 \rightarrow 3$) evaluates to $\arg W_{\text{forward}} = 7\pi/6$, that is 210° , with 150° on the reversed loop. The conjugate-pair content is unchanged; the audit corrects only which label attaches to which orientation, and any use of the pair must carry the corrected assignment.

The remaining inheritance statement is

$h(L_{\text{ax}}) \triangleq W_{\Delta}$:

does the physical co-terminal charged-completion comparison loop inherit this family holonomy, and if so with which orientation? No same-action theorem yet answers this, and the holonomy route and the Hermitian minimal-lift route should ultimately agree if they describe one physical phase. Their present convergence on the same conjugate pair, from two differently typed constructions, is the programme's most substantial phase evidence; it remains evidence about candidates, not a selection.

25. Retention and the commitment-survival precursor

The physical installation of the axle contains a third question, independent of strength and orientation: how much of the axle's continuous phase carrier survives commitment, the moment at which a possibility becomes an irreversible fact. Earlier treatments carried this as an unconstrained retention scalar in $[0, 1]$. The commitment-survival programme has since narrowed it to a parameter-free structural candidate, $s_{\text{old}} = \sqrt{5/6} = 0.912870929175$ on the minimal private-rim selector, together with a discrete multi-trap staircase in place of a free continuous parameter; the construction and its arithmetic are set out in Appendix E. The value is exact on its declared selector and is not a CKM input: the common action has not yet generated the selector, and the surviving phase carrier has not been identified with the physical charged-completion axle-retention carrier. Accordingly no jet norm, no diagnostic matrix and no residue in this paper is rescaled by $\sqrt{5/6}$. Its role is that of a preregistered check for gate QK-7, which must reproduce it, supersede it with a different returned value, or reject the carrier identification; any of those three outcomes is informative, and firewall F-QK-10 governs every use of the number.

26. CKM curvature diagnostic (quarantined)

Using the conditional C_3 curvature on its historically favourable 150° branch, the downstream curved CKM diagnostic gives approximately

$$|V_{us}| = 0.2231, |V_{cb}| = 0.0405, |V_{ub}| = 0.00356, |V_{td}| = 0.00869, |J| = 3.00 \times 10^{-5},$$

with triangle angles

$$\alpha = 84.5^\circ, \beta = 22.5^\circ, \gamma = 73.0^\circ.$$

The first two entries are inherited or assembled and are not counted as new predictive evidence. Relative to RRHF the curvature strongly repairs $|V_{td}|$ and $|J|$, lowers $|V_{ub}|$ into tension of the opposite sign, and leaves γ roughly 10% high. Audit note: the triangle-angle comparators behind the general-reader scoreboard rows for α , β and γ are source-reported; the frozen benchmark angles are not printed in the ledger extract available to this paper, so those three rows are not recomputable from the data printed here.

The branch comparison quantifies why phase provenance is load-bearing. On the four quantities printed for both objects, $\{|V_{us}|, |V_{ub}|, |V_{td}|, |J|\}$, the 150° branch values above give

$$\chi^2(150^\circ, \text{four-quantity}) \approx 8.0 \text{ against } \chi^2(\text{RRHF}, \text{four-quantity}) \approx 289.1,$$

an improvement of more than a factor of thirty, recomputable term by term from the inputs below. The source diagnostic record additionally reports five-quantity branch chi-squares of approximately 5.3 (150°), 539 (30°) and 1032 (270°). Audit note: the reported 5.3 is not reproducible from the rounded branch values and benchmark inputs printed in this paper, which give ≈ 8.0 before $|V_{ts}|$ is included; the branch value of $|V_{ts}|$ and the full 30° and 270° observable sets are not printed in any record available here. The five-quantity branch chi-squares are therefore carried as quarantined source-reported values, a re-executed three-branch table printing all five observables per branch is recorded as owed, and the recomputable four-quantity comparison is the controlling statement of this section. The reproducible 150° calculation shows that the axle can improve the confrontation dramatically; the quarantined source record additionally reports severe worsening on the 30° and 270° branches. Which of these obtains depends on a readout phase the physical action has not selected. That is precisely why the 150° matrix is quarantined rather than controlling: its empirical success is real, and it is not permitted to select itself.

Chi-square definition and inputs. The diagnostic chi-square used throughout is the uncorrelated form

$$\chi^2 = \sum_i ((x_i - x_i^{\wedge} \text{bench})/\sigma_i)^2,$$

with correlations ignored and no theory covariance, so the values are orientation only and are not likelihoods. The RRHF diagnostics and the source-reported branch diagnostics use the five-quantity set $\{|V_{us}|, |V_{ub}|, |V_{td}|, |V_{ts}|, |J|\}$; the recomputable branch comparison above uses, as explicitly identified there, the four quantities printed for both objects. The frozen benchmark inputs are:

Quantity	Benchmark $x_i^{\wedge}\text{bench}$	σ_i used
V_us	0.22431	0.00085
V_ub	0.00389	0.00016
V_td	0.0086	0.0002
V_ts	0.0415	0.0009
J	3.16×10^{-5}	0.13×10^{-5} (upper uncertainty)

With these inputs the RRHF matrix reproduces $\chi^2 \approx 290.5$ (terms: |V_us| 2.51, |V_ub| 3.71, |V_td| 182.93, |V_ts| 1.42, |J| 99.95), the value used throughout this paper. The four-quantity subtotals used in the branch comparison recompute directly from this table: 289.1 for RRHF (terms 2.5, 3.7, 182.9, 99.9) and 8.0 for the printed 150° branch values (terms 2.0, 4.3, 0.2, 1.5). The ledger freeze date is 17 August 2026. Source note: the external dataset and version behind the frozen values (PDG, CKMfitter, UTfit or another global fit) is not identified in the ledger extract used here; that citation is owed at ledger level.

27. Consolidated provenance ledger

Object	Current result	Grade
RRHF charged tensor	returned	Conditional action-level
RRHF CKM matrix	returned; diagnostic $\chi^2 \approx 290.5$	Controlling, Stage 2
Linear $D_6 \rightarrow C_3$ axle	forbidden	Exact
Quadratic axle $Q(z) = z^2$	unique	Exact
Minimal branch Hessian	$\kappa_{C_3} I_6$	Calculated
Pre-readout covariance	circulant, R_3 -commuting	Calculated in minimal cell
Equal branch weights	$w_{12} = w_{23} = w_{31}$	Exact consequence
Minimal amplitude	$b/\sqrt{3}$	Conditional on isotropy + PH-SAME
Minimal $2 \leftrightarrow 3$ survivor	returned in declared readout	Conditional physical status
Minimal 150° branch	returned under lift premises	Conditional
Full-su(8) branch Hessian	$\kappa_{C_3} I_6 + O(\Omega_{\text{mix}}^2)$	Conditional/symbolic under P-gap Schur bound
Complement non-renormalisation	$\rho_8^{\wedge}\text{eff} = \rho_8^{\wedge}\text{dir}$	Symbolic theorem
Full value $\rho_8 = 1$	under P-bg	Conditional
Full $\zeta_8^{(0)} = \zeta_{C_3}$	under named premises, leading order	Conditional/symbolic

Object	Current result	Grade
Full 2↔3 readout	conditional theorem	Direct P_Y construction owed
Full phase sign	discrete candidate set {150°, 210°, 330°}	Physical invariant owed
Reduced-family holonomy	exact pair {150°, 210°}; 210° on printed forward loop	Exact; inheritance open
AXD-3 mixed-jet shape	2×334×9×9	Executed
AXD-3 B ₂₃ plane	two-level stable, overlaps 0.99999982	Executed
AXD-3 local phase preference	none	Exact no-go via $X^\dagger X = z ^2(E_{22}+E_{33})$
AXD-3 amplitude alone	exactly linear, free scalar	Exact property
$\zeta_8 \rightarrow$ AXD-3 same-carrier bridge	leading-order preregistered norms ≈ 0.01608 (unit-amplitude convention)	Conditional cross-paper return
Retention precursor	$s_{\text{old}} = \sqrt{5/6}$; staircase $\sqrt{(1 - r/6)}$	Exact on constructed selector; carrier bridge open
New physical CKM matrix	not issued	Correctly withheld

28. Reconciliation theorem

Theorem PCAC-R (Weak-Doublet / Charged-Completion Reconciliation)

Assume:

1. the full-su(8) weak-doublet symbolic return holds on the physical branch;
2. its role-even 2↔3 generator $\Omega_0(\zeta_8)$ is the same generator variable X differentiated by the AXD-3 charged-completion map;
3. the generator normalisation is shared;
4. no additional independent source multiplier appears between the weak-doublet block and the charged-completion derivative.

Then the AXD-3 scalar degeneracy is not a physical freedom. It is coordinate freedom before the upstream weak-doublet curvature is supplied, and the bridge prediction for the physical mixed jet is

$$\mathcal{K}_n^{\text{bridge}} = \mathcal{K}_n(\Omega_0(\zeta_8)),$$

with predicted leading-order norms

$$\|\mathcal{K}_3^{\text{bridge}}(0)\| = 0.016081956699908, \|\mathcal{K}_4^{\text{bridge}}(0)\| = 0.016075297313613.$$

The designation phys is reserved for the sealed same-action execution of QK-6; until that execution exists, these are conditional bridge values.

Falsification clause. The theorem is conditional because assumptions 2–4 are not yet established by one common primitive source action. The level-specific leading-order bridge values are frozen in advance. Once QK-1 supplies the propagated curvature-response bound, carried from the established $O(\Omega_{\text{mix}}^2)$ Hessian remainder through the stability map of Section 14, a sealed same-action execution must return a mixed-jet norm lying, at the same regulator level, within that bound plus the declared numerical execution tolerance; if QK-1 proves the induced curvature response vanishes or lies below numerical tolerance, the printed values become exact numerical targets. If the sealed result falls outside that window, then at least one of assumptions 1–4 fails, and the bridge is refused rather than renormalised: no rescaling toward the preregistered value is permitted, no remainder bound may be chosen or adjusted after the same-action result is unsealed, and the failing assumption must be identified before any successor bridge is proposed. Conversely, agreement upgrades the bridge from cross-paper precursor to same-action return only when the carrier identity of assumption 2 is proved, not merely matched numerically, since a numerical coincidence at one regulator pair cannot substitute for the typing theorem.

29. What a formal physical quark-mixing closure still requires

QK-1 (Primitive weak-doublet premises). Derive P-phase, P-role, P-bg and P-gap from the primitive substrate and history law, bound the $O(\Omega_{\text{mix}})$ sector quantitatively through the Schur estimate, derive the perturbative stability map from the resulting effective branch-Hessian correction to the physical curvature coordinate ζ_8 , and recover from the sealed record the typing of the one inventoried carrier mode beyond the 63 su(8) generators.

QK-2 (Full readout operator). Construct P_Y on the full support and verify the survivor projections $P_Y^\Lambda B Z_{12} = 0$, $P_Y^\Lambda B Z_{23} = Z_{23}$, $P_Y^\Lambda B Z_{31} = 0$ directly, rather than by protection arguments.

QK-3 (Full phase invariant). Return σ_φ^q , or an equivalent co-terminal holonomy, from the same action, selecting simultaneously among the root pair $\{150^\circ, 330^\circ\}$, the component set $\{30^\circ, 150^\circ, 270^\circ\}$ and the orientation pair $\{150^\circ, 210^\circ\}$, with the corrected loop-label convention of Section 24.

QK-4 (Same-carrier attachment). Prove $X_{\text{AXD3}} = \Omega_0(\zeta_8)$ with a common generator normalisation and no extra source multiplier.

QK-5 (Base-amplitude provenance). Derive a, b, c and φ from the primitive source rather than inherit them.

QK-6 (Sealed common-action rerun). With all upstream quantities frozen, redifferentiate the charged tensor and issue exactly one new CKM matrix before benchmark access, tested level-for-level against the preregistered bridge numbers.

QK-7 (Retention carrier bridge). Determine from the common action whether the physical axle-retention carrier is the six-rim cycle carrier of the commitment-survival construction and whether the action generates the minimal private-rim trap selector. Only if both identifications pass may the retention amplitude be evaluated, with $s = \sqrt{(5/6)}$ held as a preregistered check and never as an input; a different returned value, or a failed identification, must be retained.

The gates are ordered by dependency: QK-1 licenses the full-embedding premises, QK-2 and QK-3 close readout and phase, QK-4 seals the bridge, QK-5 removes the last inherited amplitudes, QK-7 fixes retention, and QK-6 is the terminal act that produces the single successor matrix which alone may challenge RRHF.

30. Stage grading

Object	Stage	Reason
RRHF CKM baseline	2	Definite sealed numerical action-level return under frozen premises and declared calibrated inputs
Quadratic axle geometry	2 structural	Exact form; no full physical installation by itself
Minimal C_3 Hessian	3S	Microscopic mechanism and finite carrier actually calculated; norm, readout and phase premises remain conditional
Full-su(8) symbolic survival	3S	Same curvature survives the full weak-doublet embedding at leading order under named substrate premises
Full charged-completion attachment	3S	Physical full carrier executed; its coordinate is not internally source-selected
Same-carrier bridge	3S target	Numerically executable and preregistered; carrier identity unproved
Retention precursor	1	Exact on constructed selector; selector provenance and carrier identification open
Physical CKM curvature ζ_{phys}	below formal 3	Premises, readout, phase and same-action normalisation not all source-returned
Full physical CKM matrix	2 until QK-6	RRHF remains controlling
Quark-mixing sector overall	strong 3S, not formal 3	One common primitive action has not yet returned every physical input and the final sealed matrix

31. Scientific verdict

The VERSF quark-mixing programme is no longer an open search for a correction. It has a definite hierarchy: a target-blind action-level baseline that fails informatively; a symmetry theorem that forbids the linear route and forces one unique quadratic axle; an executed minimal-cell Hessian that returns exact threefold democracy; a conditional symbolic theorem that carries the identical curvature through the full $\text{su}(8)$ weak-doublet embedding; and an executed attachment of family structure to the complete 334-coordinate charged-completion carrier.

The apparent conflict between the layers, one calculation returning $b/\sqrt{3}$ while another left an arbitrary scalar, is reframed as a missing same-action identification rather than an algebraic contradiction. The weak-doublet Hessian conditionally fixes the coordinate along the family plane; the AXD-3 mixed jet propagates that coordinate into the charged-completion tensor; and their conjunction preregisters definite leading-order bridge jet norms, with no downstream CKM benchmark consulted beyond the declared inherited scale. What is missing is the proof that the two calculations read one physical source variable with one normalisation. That same-carrier theorem, QK-4, is now the most important single quark-mixing target, because it converts an entire layer of conditional numerics into same-action returns at a stroke.

The phase problem has meanwhile been reduced from a continuous unknown to a finite discrete landscape: a root pair, a component triple and an orientation pair whose unique common element is 150° , an intersection recorded as convergence evidence rather than selection, with the exact closed holonomy supplying the conjugate pair under corrected orientation labels. The retention problem has been reduced from a free scalar to a parameter-free staircase candidate. Neither reduction is a selection, and the paper treats both accordingly.

The discipline holds throughout: the favourable diagnostic matrix, more than a factor of thirty better than the baseline on the recomputable four-quantity comparison, is quarantined precisely because its success cannot select it, and the empirically poor RRHF matrix remains controlling precisely because it was earned. When the gates close, exactly one successor matrix will be issued, sealed before benchmark access, and only then will the confrontation of Section 26 become evidence.

FINAL STATUS. MINIMAL C_3 HESSIAN: CALCULATED. FULL- $su(8)$ SURVIVAL: CONDITIONAL/SYMBOLIC PASS UNDER NAMED PREMISES INCLUDING P-GAP. FULL CHARGED-COMPLETION ATTACHMENT: EXECUTED, TWO-LEVEL STABLE. LOCAL PHASE SELECTION: EXACT NO-GO VIA THE $X^\dagger X$ IDENTITY. SAME-CARRIER BRIDGE: NUMERICALLY PREDICTIVE, PREREGISTERED LEVEL BY LEVEL IN THE UNIT-AMPLITUDE CONVENTION AT LEADING ORDER, CONDITIONALLY FIXING THE AMPLITUDE, MIXED-SECTOR CURVATURE-RESPONSE BOUND OPEN, PHYSICALLY UNPROVED. PHASE CANDIDATE LANDSCAPE: DISCRETE $\{150^\circ, 210^\circ, 330^\circ\}$ PLUS COMPONENT TRIPLE; UNIQUE COMMON CANDIDATE 150° , CONVERGENCE EVIDENCE ONLY; PHYSICAL INVARIANT OPEN. RETENTION: PARAMETER-FREE $\sqrt{(5/6)}$ PRECURSOR; CARRIER BRIDGE OPEN. READOUT: CONDITIONAL THEOREM; DIRECT CONSTRUCTION OWED. FORMAL QUARK-MIXING STAGE 3: NOT CLAIMED. CONTROLLING CKM MATRIX: RRHF. NEW PHYSICAL CKM MATRIX: NOT ISSUED.

Appendix A: numerical identities

Object	Expression	Value
a	$9/40$	0.225
b	$81/2000$	0.0405
c	$243/100000$	0.00243
φ	$2\pi/3$	120°
$ \zeta_{C_3} $	$b/\sqrt{3} = 0.0405/\sqrt{3}$	0.023382685902180
$\text{Re } \zeta_{C_3}$	$-b/2 = -81/4000$	-0.0202500000000000
$\text{Im } \zeta_{C_3}$	$b/(2\sqrt{3}) = 27\sqrt{3}/4000$	0.011691342951090
Δ_{13}	$(1/2) a \zeta_{C_3}$	-0.0022781 + 0.0013153 i
$ \Delta_{13} $	$(1/2)(9/40) \zeta_{C_3} $	0.0026306
$\ \mathcal{K}_3(B_{23})\ $	AXD-3 level 3	0.687772002206489
$\ \mathcal{K}_4(B_{23})\ $	AXD-3 level 4	0.687487202319853
Regulator drift	relative change level 3 $\rightarrow$ 4	-0.041409%
$\ \mathcal{K}_3^{\text{bridge},(0)}\ $	$0.687772002206489 \times$ 0.023382685902180	0.016081956699908
$\ \mathcal{K}_4^{\text{bridge},(0)}\ $	$0.687487202319853 \times$ 0.023382685902180	0.016075297313613
Real quadrature at level 4	$0.687487202319853 \times$ 0.0202500000000000	0.013921615846977
Imaginary quadrature at level 4	$0.687487202319853 \times$ 0.011691342951090	0.008037648656807
Euclidean recombination	$\sqrt{(0.013921615846977^2 + 0.008037648656807^2)}$	0.016075297313613

Object	Expression	Value
Gram scalar identity, level 3	0.687772002206489^2	0.473030327019123
Gram scalar identity, level 4	0.687487202319853^2	0.472638653353578
Retention precursor	$\sqrt{(5/6)}$	0.912870929175
Commitment principal angle	$\arccos \sqrt{(5/6)}$	24.0948425521°
Retention staircase	$\sqrt{(1 - r/6)}, r/12$	$r = 0 \dots 6$

Appendix B: firewalls

F-QK-1. The RRHF matrix is not replaced because a later branch fits better; only the sealed QK-6 rerun may issue a successor.

F-QK-2. $b/\sqrt{3}$ is not accepted from democracy alone; it requires the role-isotropy and shared-profile norm bridge with the K-orthonormal branch basis, and all of those premises must eventually be substrate-derived.

F-QK-3. The full-su(8) theorem is conditional on P-phase, P-role, P-bg, P-gap and the inherited support, readout and phase premises; none may be silently promoted.

F-QK-4. The AXD-3 Hilbert-Schmidt Gram is not the physical weak-doublet Hessian; equality of their preferred directions is a target, not an identity.

F-QK-5. The local AXD-3 metric cannot select 150°; any phase claim resting on a phase-invariant local norm is refused.

F-QK-6. The reduced-family holonomy pair is not automatically the physical weak-doublet phase; the inheritance statement $h(L_{ax}) = W_{\Delta}$ must be proved, with the corrected orientation labels (210° forward on the printed conventions).

F-QK-7. The $2 \leftrightarrow 3$ readout must be verified from the actual full P_Y ; protection arguments and novelty arguments are motivation only.

F-QK-8. The same-carrier bridge assumes an identical generator normalisation; it does not prove it, and a numerical match cannot substitute for the typing theorem. The unit-amplitude convention of Section 17 is the declared reading, and any $\sqrt{2}$ convention mismatch must be resolved at QK-4, never absorbed post hoc.

F-QK-9. The inherited amplitudes a, b, c, ϕ remain provenance-sensitive until substrate-derived; every downstream number inherits their conditionality.

F-QK-10. The retention value $\sqrt{(5/6)}$ is a preregistered check, never an input: no matrix, norm or residue is rescaled by it before QK-7 closes, and a different returned value must be retained.

F-QK-11. The preregistered leading-order bridge norms may not be rescued: level-matched disagreement beyond the propagated curvature-response bound plus the declared numerical execution tolerance refuses the bridge and forces identification of the failing assumption; the inter-level regulator drift is not an agreement tolerance, and no remainder bound may be chosen or adjusted after the same-action result is unsealed.

F-QK-12. No final physical CKM matrix is issued until every upstream physical quantity is frozen and the QK-6 rerun is sealed before benchmark access.

Appendix C: source hierarchy used in this paper

1. Deriving Flavour Mixing from Closure Geometry
2. The Yukawa Operator from Completion-Channel Misalignment in VERSF
3. The Electroweak Flavour-Frame Operator in VERSF
4. The CKM Curvature Residue in VERSF
5. The Projected Weak-Doublet Closure Hamiltonian in VERSF
6. The Weak-Doublet Projection Theorem in VERSF
7. The First Weak-Doublet Projection Audit in VERSF
8. The C_3 Substrate Reduction and Minimal-Hermitian-Lift CKM Branch
9. The Weak-Doublet Hessian Audit Protocol in VERSF
10. The C_3 Common-Curvature Extraction in VERSF
11. The Projected C_3 Hessian Return in VERSF
12. The Quark-Sector Closure Ledger in VERSF
13. The Full $su(8)$ Weak-Doublet Hessian Return in VERSF
14. AXD-2 / AXD-3 generation-sensitive completion executions
15. HCFC and reduced-family holonomy audits, including the orientation-label provenance audit
16. The commitment-survival and refined line-to-loop theorems (retention precursor)
17. SMNC provenance theorem (one-matrix rule, calibration taxonomy, cross-paper composition rule, branch diagnostics)

Appendix D: internal secondary re-execution and verification notes

[This appendix independently re-derives the numerical and exact statements used in the paper, in exact arithmetic where fractions are shown and in double precision otherwise. It records internal secondary re-execution, not external replication. All checks pass.]

1. Curvature components. $\zeta_{C_3} = (b/\sqrt{3})e^{(i \cdot 5\pi/6)}$ gives $\text{Re } \zeta = (b/\sqrt{3})(-\sqrt{3}/2) = -b/2 = -81/4000 = -0.020250000000000$ and $\text{Im } \zeta = (b/\sqrt{3})(1/2) = b/(2\sqrt{3}) = 81\sqrt{3}/12000 = 27\sqrt{3}/4000 = 0.011691342951090$, both exact and matching all printed digits; $|\zeta| = 0.023382685902180$.
2. Residue. $\Delta_{13} = (1/2)(9/40)\zeta_{C_3} = (9/80)\zeta_{C_3}$ has components $-729/320000 = -0.0022781$ and $243\sqrt{3}/320000 = 0.0013153$, magnitude 0.0026306 ; the converged

two-factor comparison value 0.0026332 differs by +0.099%, matching the quoted 0.1% margin.

3. Commutator. With $\Omega_0 = zE_{23} - z^*E_{32}$ and the printed Ω_q , the (1,3) entry of $\Omega_0\Omega_q$ vanishes (row 1 of Ω_0 is zero) while $(\Omega_q\Omega_0)_{13} = (\Omega_q)_{12}(\Omega_0)_{23} = az$; hence $[\Omega_0, \Omega_q]_{13} = -az$ exactly, verified symbolically.
4. Axle image. For the normalised source mode, $Q(u) = (1/3)(1, e^{(-2\pi i/3)}, e^{(+2\pi i/3)})$ has $\|Q(u)\| = \sqrt{(3 \cdot 1/9)} = 1/\sqrt{3}$, and the component phases are spaced by exactly $2\pi/3$, the spacing used in the Section 23 component triple.
5. Phase arithmetic. The Hermitian roots of $e^{(2\pi i/3)}$ are $e^{(i\pi/3)}$ and $e^{(i4\pi/3)}$; the $+i$ frame rotation adds $\pi/2$, carrying them to $5\pi/6 = 150^\circ$ and $11\pi/6 = 330^\circ$ respectively. The conjugate of 150° is $-5\pi/6 \equiv 210^\circ$. The three candidates 150° , 210° , 330° are pairwise distinct, confirming that the orientation $\mathbb{Z}_2$ and the root $\mathbb{Z}_2$ are different ambiguities.
6. Bridge products. $0.687772002206489 \times 0.023382685902180 = 0.016081956699908$ and $0.687487202319853 \times 0.023382685902180 = 0.016075297313613$, reproduced in double precision. The level-4 quadrature components $0.687487202319853 \times 0.020250000000000 = 0.013921615846977$ and $0.687487202319853 \times 0.011691342951090 = 0.008037648656807$ recombine as $\sqrt{(0.013921615846977^2 + 0.008037648656807^2)} = 0.016075297313613$, exactly the direct product, confirming the isotropy used in Section 21.
7. Regulator drift and linearity. $(0.687487202319853 - 0.687772002206489)/0.687772002206489 = -0.041409\%$; the scalar scan values 0.3437436011599264 , 0.6874872023198528 and 1.3749744046397057 are exactly $(1/2)\times$, $1\times$ and $2\times$ the unit value, confirming exact linearity of the attachment.
8. RRHF checks. Each row of the printed $|V_CKM^{RRHF}|$ has unit norm to within a few parts in 10^7 (row deviations 3.5×10^{-7} , 7.7×10^{-8} and 1.1×10^{-7} , consistent with the 10^{-6} rounding of the printed entries); this is a necessary condition for unitarity, not a full certificate, since the full statement $\|V^\dagger V - I\| \approx 0$ concerns the underlying complex matrix and its record resides with the sealed RRHF source execution. The five-row diagnostic chi-square recomposes, from the inputs printed in Section 26, to approximately 290.5 (terms: $|V_{us}|$ 2.51, $|V_{ub}|$ 3.71, $|V_{td}|$ 182.93, $|V_{ts}|$ 1.42, $|J|$ 99.95), the value quoted throughout, dominated by $|V_{td}|$ (≈ 183) and $|J|$ (≈ 100); the non-calibration subset gives approximately 288.0.
9. Retention arithmetic. For the normalised uniform six-rim mode v and one private-rim projector Q , $v^\dagger Qv = 1/6$, so $s_{old} = \sqrt{(5/6)} = 0.912870929175$, $\arccos s_{old} = 24.0948425521^\circ$, $\Delta k = (1 - 5/6)/2 = 1/12$, and the staircase $\sqrt{(1 - r/6)}$, $r/12$ reproduces for $r = 0 \dots 6$. All exact before the displayed decimals.
10. Phase-isotropy identity. Direct multiplication gives $X(z)^\dagger X(z) = |z|^2(E_{22} + E_{33})$, so $\|\mathcal{K}_n(X(z))\|^2 = |z|^2\|\mathcal{K}_n(\Omega_0(1))\|^2$ and the quadrature Gram is exactly the squared

unit-amplitude response times I_2 . Numerically, $0.687772002206489^2 = 0.473030327019123$ and $0.687487202319853^2 = 0.472638653353578$, agreeing with the printed Gram scalars to all digits and confirming the algebraic proof of Section 18.

11. Diagnostic branch table. From the printed 150° branch values and the Section 26 inputs, the four-quantity chi-square over $\{|V_{us}|, |V_{ub}|, |V_{td}|, |J|\}$ recomputes to approximately 8.0 (terms 2.03, 4.25, 0.20, 1.51), against approximately 289.1 for RRHF on the same four quantities (terms 2.51, 3.71, 182.9, 99.9). The source-reported five-quantity branch chi-squares ($\approx 5.3, \approx 539, \approx 1032$) are not reproducible from the data printed in this paper, because the branch $|V_{ts}|$ and the 30° and 270° observable sets are not printed; they are carried as quarantined source-reported values pending the owed three-branch re-execution. The quarantine grading is unchanged by any check in this appendix.

Appendix E: retention precursor detail

The primitive phase-cut census is strikingly negative: every explicitly returned support and completion mechanism carries the selected continuous phase direction through a hard cut of exactly zero, so if those were the complete commitment constraints, retention would be total. The missing object is the primitive full-carrier commitment selector.

The topological commitment line supplies a minimal covariant candidate. One newly trapped triangular face exports exactly one private rim direction, and acting on the normalised uniform six-rim cycle mode this gives the survival amplitude

$$s_{\text{old}} = \sqrt{5/6} = 0.912870929175,$$

with principal commitment angle $\arccos \sqrt{5/6} = 24.0948425521^\circ$ and unit-branch stiffness excess $\Delta k = 1/12$. For r mutually orthogonal private-rim cuts, $0 \leq r \leq 6$, the survival amplitude and excess follow the discrete staircase

$$s_{\text{old}}(r) = \sqrt{(1 - r/6)}, \Delta k(r) = r/12,$$

so the six-rim carrier admits seven discrete retention values rather than a free continuous attenuation parameter.

Two identifications remain unproved: the common action must generate the trap-controlled projector as its physical commitment control, and the surviving phase carrier must be identified with the physical charged-completion axle-retention carrier. Until gate QK-7 closes, firewall F-QK-10 governs every use of these numbers, and no quantity in this paper is rescaled by any entry of the staircase.

End of PCAC-1, full-carrier quark-mixing curvature closure, 20 August 2026

Keith Taylor | 20 August 2026