The Route-M Off-Shell Integrability, Current-Source Closure and Direct Physical Running Theorem in VERSF

From the exact $U(1)^6 \hookrightarrow U(1)^{10} \twoheadrightarrow U(1)^4$ obstruction sequence and canonical Hodge attaching map to the current-source integrability theorem, linearized Standard Model attachment, exhausted probability running and the quantum one-loop handoff

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Headline result. The ten-face Route-M formulation is the minimal smooth off-shell completion of the six-dimensional gauge-equivalence class of link-realizable U(1) holonomies on one oriented 4-simplex. Its four additional directions are exactly the complete obstruction to whether ten arbitrarily specified face values can arise from one underlying connection. The exact sequence is

$$1 \longrightarrow U(1)^6\_phys \longrightarrow U(1)^{10}\_direct \xrightarrow{(\delta_2)} U(1)^4\_obs \longrightarrow 1.$$

At tangent level,

$$Q\_cl = (1/5)\ d_2^T d_2, \qquad P\_int = (1/5)\ d_1 d_1^T,$$

and every mathematical direct-face candidate admits the exact decomposition

$$\theta = d_1 A\_can + \theta\_obs, \qquad A\_can = (1/5)\ d_1^T\theta, \qquad \theta\_obs = (1/5)\ d_2^T d_2\theta,$$

with A_can canonical, gauge-orthogonal and tree-free. The projector Q_cl is the unique S_5-equivariant idempotent of rank four, so the split is forced by simplex symmetry and is identical for every S_5-invariant inner product.

The paper then closes the physical typing question for the current VERSF source stack. Source-native physical holonomy is not a collection of ten independently assigned face values. It is one holonomy assignment on admissible closed paths, respecting composition, reversal and identity, and the physical gauge source is carried by edge/link transport. Consequently $H\_f = h\_D(\partial f)$ for every source-native physical face configuration, and for every tetrahedron τ,

$$\prod_{f \subset \partial\tau} H_f^{\sigma(\tau,f)} = h_D(\partial\partial\tau) = h_D(id) = 1,$$

equivalently

$$r = d_2\theta = 0, \quad f_{obs} = 0, \quad A_{can} = A_{phys}$$

for every current source-native infinitesimal connection variation.

The four-dimensional obstruction carrier therefore remains mathematically real and indispensable as a realizability test, but it is not physically populated by the current VERSF holonomy source. Route M's ten independently assignable face variables are correctly typed as an enlarged off-shell diagnostic carrier, not as ten independently prepared physical curvature variables. Stage 0C-1 closes negatively for the current physical source stack and Stage 0C-2 does not open: no physical probability or source weight should be assigned to the four obstruction coordinates within the current theory. A future deeper pre-holonomy theory could reopen the question only by deriving a genuinely new source-native carrier prior to physical holonomy assignment; such a carrier cannot simply be identified with the present physical face holonomies.

The Standard Model handoff can be stated more strongly than the linearized census alone. The decomposition 120 = 72 + 48 remains an exact first-order statement about the enlarged direct-face test carrier, and no claim is made that ten independently assigned non-Abelian face matrices obey a simple nonlinear analogue of the Abelian obstruction equation. That limitation does not leave the physical non-Abelian connection reconstruction open. For any structure group G, a complete composition-respecting based-loop holonomy assignment on the connected K_5 graph determines an edge connection uniquely up to gauge, and since $\pi_1(K_5) \cong F_6$,

$$\{ \text{G-connections on } K_5 \} / \text{gauge} \cong \mathrm{Hom}(F_6, G) / G,$$

the final quotient being simultaneous conjugation. This is exact and nonlinear: it reconstructs the connection from a complete loop assignment rather than attempting to integrate ten independently chosen non-Abelian face holonomies. On the frozen representation ledger the faithful global gauge group is

$$G_{faith} = [SU(3) \times SU(2) \times U(1)_q] / \mathbb{Z}_6 \cong S(U(3) \times U(2)),$$

whose finite central quotient leaves the Lie algebra unchanged, so the local physical tangent count remains 6 × (8 + 3 + 1) = 72.

Later primitive-field-dependent marked-instrument work strengthens the source side of that handoff. Seven anonymous minimal central projectors are recovered blindly within the inherited typed carrier and only afterwards identified with (Q_L, u_R, d_R, L_L, e_R, N_R, H), while colour, weak and primitive-charge closure return exactly four admissible completion channels. The field-dependent source then carries the full faithful matrix-link carrier and produces native gauge-sensitive mark probabilities with raw rank 144 and physical Wilson-quotient rank 72. The complete physical Standard Model gauge tangent is therefore not merely available as an abstract carrier; it is reached by the executed source instrument at linear response. The representation recovery stays correctly graded as recovery within an inherited representation-typed carrier, not derivation of that carrier from no prior structure. The master-action bond law adds a connection-to-record compatibility condition: on a rank-stable occupied support, $M_{ij} = \Pi_j \Phi_{ij} \Pi_i$ has unique polar partial isometry $V_{ij} = \mathrm{Pol}(M_{ij})$, and the zero-defect condition $\Pi_j U_{ij} \Pi_i = V_{ij}$ fixes the physical link on occupied support. The full-faithful defect audit finds a residual tested kernel of $18_{record} + 1_{completion} + 0_{gauge}$, with the faithful gauge-coordinate carrier lying in the defect row space to operator distance 4.78×10^{-15}: a machine-certified local connection-uniqueness result modulo gauge and the declared source premises, not a claim of global uniqueness of a universe-wide field configuration.

Subsequent source-level execution sharpens the running conclusion. The historical 14/16 = 7/8 remains a conditional structural counting prescription and is not a physical beta coefficient. More strongly, the principal probability candidates available on the current source stack have now been executed and fail as sources of nontrivial gauge running. The retained-record/Fold-to-Fact event is exactly regulator-independent once the frozen D36 history factor is separated from the regulator-dependent neutral continuation, and every fixed event measurable solely on the EX2E current-terminal algebra is likewise regulator-independent after summing the complete neutral instrument. Their direct Route-M beta coefficients are therefore exactly zero. Gauge sensitivity and regulator running are distinct: the EX2E terminal law resolves all 72 physical gauge directions, but its physical score geometry is unchanged across the executed regulator levels.

The Standard Model running problem consequently moves from a Bernoulli coherence probability to the one-loop quantum fluctuation determinant on the admitted connection branch. The background-field gauge, ghost, Weyl and scalar fluctuation architecture gives the universal one-loop contributions

$$b_{gauge+ghost} = -(11/3)\, C_2(G), \quad b_{Weyl} = (2/3)\, T(R), \quad b_{complex\ scalar} = (1/3)\, T(R),$$

and the frozen Standard Model census returns

$$(b_Y, b_2, b_3) = (41/6, -19/6, -7).$$

These coefficients are conditionally exact at the present grade: the continuum determinant and field census are fixed, and the remaining primitive regulator debt is no longer the numerical coefficients themselves but the universality provenance of the finite VERSF fluctuation operators, most sharply the finite chiral fermion kernel $D_f[U]$.

A chiral-projector audit sharpens the fermionic part of that result. The inherited VERSF matter architecture does not remove the right-chiral fermion carrier: the physical e_R, u_R, d_R states remain as weak singlets and are required by the completion and mass-attachment stack. What is excluded from the minimal gauge-facing census is an additional right-chiral weak-doublet copy, so the relevant distinction is representation-sensitive rather than a deletion of the P_R spinorial sector. For either Weyl projector $P_\chi = (1 \pm \gamma_5)/2$, the parity-even F^2 part of the Laplace-type fermion heat-kernel trace is exactly one half of the four-component Dirac trace, the γ_5 remainder being parity odd and belonging to the $F\tilde{F}$ anomaly channel rather than to the F^2 coefficient that defines gauge running. The Dirac matter coefficient $b_{Dirac}(R) = (4/3)T(R)$ therefore compresses under the chiral carrier projection to $b_{Weyl}(R) = (2/3)T(R)$. The Weyl half-factor used in the census is thus no longer quoted as continuum degree counting; it is reproduced by the chiral projection acting on the VERSF spinorial carrier. Combined with the frozen three-generation representation ledger this gives the conditionally exact fermionic contributions on the frozen VERSF census

$$b_3^{ferm} = 4, \quad b_2^{ferm} = 4, \quad b_Y^{ferm} = 20/3$$

in the sign convention of this paper. The remaining VERSF-specific one-loop debt is therefore narrower still: not the value of the Weyl coefficient and not the chiral census factor, but proof that the actual same-action shared-link finite kernel D_f[U] belongs to the local gauge-covariant no-extra-species Weyl universality class.

General-reader summary

A four-dimensional building block has ten triangular faces. You are free, on paper, to write down ten unrelated values on those faces and then ask whether they fit together into one sensible force field. This paper shows exactly how to run that test.

Only six combinations are actually needed to describe one such field. The other four measure the ways ten arbitrary values could fail to fit together. If those four failures vanish, one underlying field exists and is unique apart from harmless relabelling, and the paper gives a formula that rebuilds it with nothing chosen by hand. The formula also works on values that do not fit: it separates the part that could belong to a real field from the part that could not.

The main new result is that the theory never makes the inconsistent case in the first place. Its force fields come from one consistent rule for carrying things from place to place, so the face values are different views of the same underlying structure and already fit together. Nature is not being described as making a messy draft and then repairing it. The ten-face picture remains valuable as the smallest complete test space, and as an exact consistency check on everything built later, but it is not a stage the world passes through.

The second new result concerns why the strength of a force depends on the scale at which you measure it. An appealing idea was that this comes from how likely it is that a record or a definite fact gets formed. That idea was tested rather than assumed, and it fails: those probabilities do not change with scale, even though they respond strongly to the force field itself. The change with scale instead comes from where ordinary physics has always put it, in the response of quantum fluctuations, and the continuum one-loop calculation recovers the familiar textbook numbers at the paper's present conditional grade, including the careful accounting of left- and right-handed matter. What remains open is narrower than before, and technical: showing that the theory's own short-distance description of matter belongs to the same well-behaved family that the calculation assumes.

One scope note. The proofs here concern a single building block, and the negative result about the source describes the theory as it currently stands, not a proof that no deeper theory could behave differently.

Abstract

The Route-M branch of VERSF uses a 10_△ + 4_cl = 14 4-simplex census: ten triangular face-coherence channels and four independent tetrahedral closure channels. Source-native gauge constructions elsewhere in the programme generate curvature from edge/link transport, for which simplicial exactness makes tetrahedral Bianchi closure automatic. That tension is resolved here structurally rather than by provenance argument.

Let $\mathbb{T} = U(1)$. For the oriented 4-simplex Δ^4, consider the multiplicative simplicial cochain complex

$$C^0 = \mathbb{T}^5 \xrightarrow{\delta_0} C^1 = \mathbb{T}^{10} \xrightarrow{\delta_1} C^2 = \mathbb{T}^{10} \xrightarrow{\delta_2} C^3 = \mathbb{T}^5 \xrightarrow{\delta_3} C^4 = \mathbb{T}.$$

The integer incidence matrices have ranks

$$\operatorname{rank} \delta_0 = 4, \quad \operatorname{rank} \delta_1 = 6, \quad \operatorname{rank} \delta_2 = 4, \quad \operatorname{rank} \delta_3 = 1,$$

with unit Smith invariant factors in every nonzero slot. Since Δ^4 is contractible,

$$\ker \delta_2 = \operatorname{im} \delta_1, \quad \ker \delta_1 = \operatorname{im} \delta_0,$$

so gauge-equivalence classes of edge transport, link-realizable face holonomies and the physical Abelian loop carrier are canonically isomorphic:

$$C^1 / \operatorname{im} \delta_0 \cong \operatorname{im} \delta_1 \cong \mathbb{T}^6.$$

Defining the closure-obstruction torus $\mathcal{O}\_cl = \operatorname{im} \delta_2 = \ker \delta_3 \cong \mathbb{T}^4$,

$$1 \longrightarrow \operatorname{im} \delta_1 \longrightarrow C^2 \xrightarrow{\delta_2} \mathcal{O}\_cl \longrightarrow 1$$

is a short exact sequence of compact tori. Hence a ten-face assignment $H \in \mathbb{T}^{10}$ is generated by one underlying edge connection if and only if $\delta_2 H = 1$, and the generating connection is then unique up to vertex gauge. The sequence splits, for the lattice-theoretic reason that the character sequence $0 \to \mathbb{Z}^4 \to \mathbb{Z}^{10} \to \mathbb{Z}^6 \to 0$ has free quotient; the star-tree gauge of §5 exhibits one splitting, and the set of splittings is a torsor over $\mathrm{Hom}(\mathbb{Z}^6, \mathbb{Z}^4)$.

A constructive gauge-fixed proof is given. Fixing the star edges U_{0i} = 1 makes the six rooted faces H_{0ij} equal to the six non-tree edges U_{ij}. The four unrooted faces are then forced,

$$H_{123} = H_{012} H_{023} H_{013}^{-1},$$

$$H_{124} = H_{012}\ H_{024}\ H_{014}^{-1},$$
$$H_{134} = H_{013}\ H_{034}\ H_{014}^{-1},$$
$$H_{234} = H_{023}\ H_{034}\ H_{024}^{-1},$$

and their four multiplicative residuals are a coordinate system on $\mathcal{O}$_cl.

A minimal-completion theorem follows: among regular smooth off-shell extensions whose zero-obstruction fibre is the six-dimensional physical holonomy carrier and whose obstruction map has rank four, dimension at least ten is required, and Route M's $\mathbb{T}^{10}$ direct-face carrier attains the minimum. In the category of compact connected abelian Lie groups the bound is rigid: dimension exactly ten, and the carrier is isomorphic to $\mathbb{T}^{10}$.

At Lie-algebra level the tetrahedron-to-face incidence matrix B satisfies the simplex Laplacian identity

$$B\ B^T + d_3^T\ d_3 = 5\ I_5, \quad d_3 = s^T, \quad s = (1, -1, 1, -1, 1)^T,$$

so $B\ B^T = 5I_5 - s\ s^T$ and the nonzero singular values of B are four copies of $\sqrt{5}$. The same identity on Δ^N gives nonzero singular values all equal to $\sqrt{N+1}$, verified here for N = 4, 5, 6, 7: the closure readout is an isotropically scaled isometry in every simplex dimension, with condition number one. Therefore

$$Q\_cl = (1/5)\ B^T\ B$$

is a rank-four orthogonal projector onto the integrability-obstruction tangent and P_int = I − Q_cl projects onto the six-dimensional link-realizable tangent. For closure syndrome r = Bθ,

$$\theta\_obs = (1/5)\ B^T\ r, \quad \|\theta\_obs\|^2 = (1/5)\ \|r\|^2.$$

Under the vertex-relabelling group S_5 the direct-face tangent decomposes as $\mathbb{R}^{10}$ = V_int ⊕ V_obs with V_int ≅ S^(3,1,1) of dimension six and V_obs ≅ S^(2,1,1,1) = standard ⊗ sign of dimension four, both absolutely irreducible and inequivalent. The equivariant endomorphism algebra is therefore two-dimensional, spanned by P_int and Q_cl, which makes Q_cl the unique nontrivial equivariant idempotent of rank four and makes the split independent of which S_5-invariant inner product is used. A direct corollary: any S_5-invariant second-order pre-integrability source law has covariance Σ = a P_int + b Q_cl and produces the exact syndrome covariance

$$B\ \Sigma\ B^T = b\ (5\ I_5 - s\ s^T),$$

so the entire invariant obstruction-weight question reduces to the single number b, with a falsifiable syndrome shape.

The Laplacian identity has a second consequence that makes the paper constructive. Since P_int = (1/5) $d_1 d_1^T$, every face candidate obeys the exact discrete Hodge decomposition

$$\theta = (1/5)\ d_1 d_1^T\ \theta + (1/5)\ d_2^T d_2\ \theta = d_1\ A\_can + \theta\_obs, \quad A\_can = (1/5)\ d_1^T\ \theta,$$

where A_can is automatically gauge-orthogonal ($d_0^T A\_can = 0$), is the minimum-norm edge representative, and is S_5-equivariant in θ. No spanning tree or gauge fixing is required. On the gauge-orthogonal slice $d_1^T d_1 = 5I$, so $(1/\sqrt{5})d_1$ is an exact isometry from the physical edge tangent to the integrable face tangent, with $\|d_1 A\|^2 = 5\|A\|^2$ and $\|\theta\|^2 = 5\|A\_can\|^2 + (1/5)\|r\|^2$. This fixes the normalized differential of the face-to-connection attachment, which is what a gauge-response or Hessian calculation requires. It also yields an execution protocol: any source candidate is reported as $\theta \mapsto (r, f\_obs, A\_can)$ with dimensionless diagnostic $f\_obs = \|d_2\theta\|^2/(5\|\theta\|^2) \in [0, 1]$. Tensoring the incidence operators with an internal algebra extends all of this exactly at first order around a flat background; for $\mathfrak{g}\_SM$ of dimension 12 the census reads 120 = 72 + 48. That linearized census is not the limit of the Standard Model handoff: for any structure group G a complete based-loop holonomy representation on K_5 reconstructs a connection up to gauge exactly and nonlinearly, with moduli $\mathrm{Hom}(F_6, G)/G$, and on the frozen ledger $G\_faith \cong S(U(3) \times U(2))$ whose finite $\mathbb{Z}_6$ quotient leaves the count $6 \times 12 = 72$ unchanged. What remains unproved is only the integration of ten arbitrary independent non-Abelian face matrices, which is a different problem with different inputs.

The paper then settles which part of that architecture the current VERSF source occupies. Physical holonomy is one assignment h_D on admissible closed paths obeying composition, reversal and identity, so every source-native face configuration is $H\_f = h\_D(\partial f)$ and every tetrahedral product is h_D of a boundary of a boundary, hence the identity. Therefore

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H_source ∈ ker δ₂ = im δ₁,    d₂θ_source = 0,    r = 0,    f_obs = 0,    A_can
= A_phys,
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and for any source law μ_D the induced face pushforward obeys $\mu\_face(\ker \delta_2) = 1$, so $P\_obs(r \neq 0) = 0$ and the independent closure-admission information is I_obs^current = 0. Stage 0C-1 therefore closes negatively for the current source stack and Stage 0C-2 does not open. No finite obstruction alphabet, closure tolerance, Born state on the obstruction carrier or source weight b is required, because there is no current physical pre-integrability state for them to act on. This is not a universal theorem about every conceivable precursor theory; a future pre-holonomy carrier would have to be derived independently, and cannot be obtained by reinterpreting the present physical face holonomies as independent variables.

At second order the S_5 representation result retains a diagnostic role: the most general invariant covariance on the enlarged carrier is $\Sigma = a\, P\_int + b\, Q\_cl$ with syndrome covariance $B\, \Sigma\, B^T = b(5I_5 - s\, s^T)$, so the invariant second-order obstruction covariance is controlled by one scalar b, with higher invariant cumulants unconstrained. For the current physical source b_current = 0, since the source measure is supported on $V\_int = \ker d_2$, and $P\_int\, \Sigma\, P\_int = a\, P\_int$ leaves no obstruction covariance to be projected away in any case.

Projective history consistency still gives an exact firewall: an unchanged cylinder event cannot acquire running merely by being represented at finer resolution, and the correlated OR-block recursion

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p_{n+1} = 1 - e^{Δ_n} ∏_{j=1}^{m_n} (1 - p_{n,j}),
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the direct gate

$$b^{phys}_{n\to n+1} = (1/2) \cdot (p_{n+1}^{-1} - p_n^{-1}) / \ln(k_{n+1}/k_n),$$

and the Fréchet bounds that fix its sign remain valid as general probability identities. Subsequent execution, however, shows that they do not furnish the Standard Model gauge beta function on the current source stack. The implemented retained-record event depends only on the frozen history factor of the recurrent product law $P_n = P_H \otimes P_N^n$, so its stationary probability is independent of regulator. Likewise, if the full source mark factorises as $M_\alpha \otimes N_\nu^n$, completeness of the regulator-dependent neutral instrument implies that every event measurable solely on the fixed EX2E current-terminal label has regulator-independent probability. Both the retained-record candidate and the entire fixed EX2E current-terminal event algebra therefore give

$$b_{Route\ M} = 0.$$

The physical Standard Model running consequently belongs to the one-loop fluctuation determinant rather than to record-formation probability. Background-field expansion gives the gauge-plus-ghost, Weyl and complex-scalar coefficients $-11C_2(G)/3$, $2T(R)/3$ and $T(R)/3$, and the inherited field census returns $(b_Y, b_2, b_3) = (41/6, -19/6, -7)$. The regulator problem has also been narrowed. Physical plaquette area scaling forces the h_n^{-2} curvature normalization, and the faithful-link covariant second difference has symbol

$$\lambda_n(k) = (4/h_n^2)\ \Sigma_\mu \sin^2(h_n k_\mu / 2) = |k|^2 - (h_n^2/12)\ \Sigma_\mu k_\mu^4 + O(h_n^4 k^6),$$

so the bosonic gauge principal operator converges to the required covariant Laplacian.

A chiral-projector audit closes the fermionic counting step at the continuum/projector grade. For either Weyl projector $P_\chi = (1 \pm \gamma_5)/2$, the parity-even F^2 heat-kernel trace is exactly one half of the four-component Dirac trace, while the complementary γ_5 term lies in the parity-odd $F\tilde{F}$ channel. Hence $b_{Weyl}(R) = (2/3)T(R)$, and applied to the frozen three-generation chiral census this gives $(b_Y^{ferm}, b_2^{ferm}, b_3^{ferm}) = (20/3, 4, 4)$, leaving the actual shared-link finite-kernel universality of $D_f[U]$, rather than the Weyl coefficient or the chiral multiplicity, as the principal remaining one-loop fermion bridge.

The remaining load-bearing finite-regulator question is therefore the chiral fermion family: a naive symmetric finite Dirac operator is excluded by doubling, the closure-stiffness Wilson mechanism gaps unwanted spatial modes at order $1/h_n$, and sequential positive-transfer time produces no independent temporal doubler, so what remains is to derive or universality-match the actual shared-link $D_f[U]$ of the Master Substrate Action to an admissible local no-doubler chiral family.

The structural Route-M carrier problem is closed, the obstruction-weight programme is removed from the Route-M physical debt, and the direct Bernoulli-probability search is now exhausted rather than merely awaiting another choice of p_n. What remains is the same-action finite-regulator universality of the quantum fluctuation package, with the shared-link chiral fermion kernel the principal remaining gate. The 14/16 coefficient is retained only as a conditional structural count.

Scope of the claims

Because several statements below are labelled exact, their domain is fixed here once.

1. All exactness claims concern one oriented 4-simplex Δ^4 with Abelian structure group U(1), unless a result is explicitly stated for Δ^N.
2. The tangent-level metric statements use the standard labelled-simplex incidence inner product. §10 shows this choice is immaterial: every S_5-invariant inner product gives the same projectors.
3. The minimal-completion theorem is exact within the declared class of regular smooth completions with a rank-four obstruction submersion, and rigid within the category of compact connected abelian Lie groups. It is not a claim about arbitrary singular or non-smooth carriers.
4. Stability under refinement is established for the coarse-graining class studied in this paper, namely finite readout partitions of the U(1)-valued closure carrier that respect the source resolution fibres. It is not a claim over every refinement scheme the wider programme might adopt.
5. The tensored extension of §16 is exact at first order around a flat background only. It is not a nonlinear non-Abelian realizability result, and is graded as such wherever it is used.
6. The direct-face tensoring result of §16 remains exact only at first order around a flat background. It is not promoted into a nonlinear theorem for integrating ten arbitrary noncommuting face holonomies. Separately, the reconstruction of a connection from a complete composition-respecting loop-holonomy assignment is exact for arbitrary structure group G on the connected K_5 graph. These are different statements and must not be conflated.
7. The one-loop coefficients quoted in §24 are conditionally exact at the present grade: the continuum determinant and inherited field census are fixed, while the universality provenance of the finite VERSF fluctuation operators, in particular the chiral kernel D_f[U], is an open regulator debt. The Weyl half-factor and the chiral matter census are no longer components of that regulator debt: they are fixed conditionally at the continuum/projector and inherited-census grades by Theorem 25A and §24.7B. They are not thereby promoted to regulator-native first-principles VERSF outputs; that stronger status still requires the same-action finite D_f[U] universality certificate. They are not presented as first-principles VERSF outputs independent of that debt.
8. The current-source result of §13 is graded at the declared grade of the inherited holonomy-assignment premises, and is a statement about the source stack as it currently stands. It is not a proof that no deeper pre-holonomy carrier could exist. The operational and probabilistic machinery that such a carrier would require is retained in Appendix F as counterfactual architecture, not as a description of the present source.

1. Question and result

1.1 The apparent contradiction

Two statements in the VERSF source stack must be made compatible.

The geometric and Route-M papers use triangular holonomy as a curvature carrier and, in the face formulation, assign ten face holonomies directly,

$$H_f \in U(1), \qquad f \in \Delta^4_{(2)},$$

then define five tetrahedral closure products, one globally dependent, leaving four independent closure conditions.

The physical gauge and MASD papers generate plaquette curvature from oriented edge/link transport. On such a globally assembled connection,

$$\delta_2\ \delta_1 = 1$$

multiplicatively, or $d_2\, d_1 = 0$ at Lie-algebra level, and tetrahedral Bianchi closure is automatic.

The correct conclusion is not that one description must be discarded. They live at different stages:

- the ten-face formulation is an off-shell direct-face candidate carrier;
- the edge/link formulation is the globally realizable physical connection carrier.

This paper proves the exact map between them.

1.2 Main result

Eight structural returns and one source-typing return are established.

Return A, exact face-to-connection realizability. A direct ten-face U(1) assignment is generated by one edge connection if and only if its four independent tetrahedral closure residuals vanish.

Return B, uniqueness up to gauge. If a direct face assignment is realizable, the underlying edge transport is unique up to vertex gauge.

Return C, minimal off-shell completion. The six-dimensional physical loop carrier requires exactly four additional smooth coordinates to represent all failures of global integrability. Route M's ten direct face coordinates are minimal in the declared regular off-shell class, and rigid in the compact abelian category.

Return D, complete closure-syndrome reconstruction. At tangent level the four closure residuals reconstruct the entire non-integrable part of the ten-face perturbation via $Q\_cl = (1/5) B^T B$.

Return E, isotropic readout. The closure readout has condition number one: all four nonzero singular values are equal. No obstruction direction is geometrically privileged or geometrically suppressed. The same holds on Δ^N with common singular value $\sqrt{N+1}$.

Return F, symmetry-forced canonicity. The integrable/obstruction split is the unique decomposition invariant under vertex relabelling, and is independent of which invariant inner product is used.

Return G, canonical attaching map. Every face candidate, realizable or not, has a canonical tree-free edge representative $A\_can = (1/5)d_1^T\theta$, and the exact decomposition $\theta = d_1 A\_can + \theta\_obs$ names its link-realizable component and its obstruction component.

Return H, fixed normalization and protocol. The attachment differential is normalized by an exact isometry with factor $\sqrt{5}$, giving an executable report $\theta \mapsto (r, f\_obs, A\_can)$, which extends unchanged to the Standard Model gauge algebra at linear order with census $120 = 72 + 48$.

Return I, current-source integrability closure. Because source-native holonomy is one composition-respecting assignment on closed paths, every current physical face configuration lies in $\ker \delta_2$, with $r = 0$, $f\_obs = 0$ and $A\_can = A\_phys$. The obstruction sector is not populated by the current source.

Returns A to H are positive theorems about the mathematical carrier and none depends on assigning a physical probability to the obstruction sector. Return I is what fixes which part of that carrier the present physics occupies.

1.3 Current-source status of the obstruction sector

The structural off-shell carrier is completely identified, and its physical status can now be decided rather than left as a fork.

The inherited physical holonomy construction assigns one map

$$h\_D : \mathcal{L} \longrightarrow U(1)$$

to the set of admissible closed paths, satisfying

$$h\_D(L_1 \circ L_2) = h\_D(L_1)\ h\_D(L_2), \qquad h\_D(L^{-1}) = h\_D(L)^{-1}, \qquad h\_D(id) = 1.$$

The geometric and MASD constructions likewise obtain triangular and plaquette holonomy from underlying edge transport. Therefore, on the current physical source branch,

$$H\_f = h\_D(\partial f), \qquad \delta_2 H = h\_D(\partial\partial\tau) = 1,$$

hence

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supp μ_face ⊆ ker δ₂ = im δ₁,
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and at tangent level P_obs δF = 0 for every current source-native connection variation. The result is:

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Stage 0C-1: CURRENT-SOURCE NEGATIVE.
Stage 0C-2: DOES NOT OPEN ON THE CURRENT SOURCE STACK.
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The present VERSF physical source stack does not populate a pre-integrability Route-M obstruction sector, so there is no current physical obstruction probability or obstruction stiffness to derive.

This is not a universal theorem forbidding every conceivable deeper precursor theory. Primitive-source uniqueness remains a wider VERSF question. A future source theory could introduce a distinct object prior to physical holonomy assignment, but that would be a new carrier requiring its own derivation; it cannot be obtained merely by reinterpreting the present physical face holonomies as independent variables. The full statement and its scope firewall are §13.

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2. Source stack and non-insertion boundary

2.1 Internal results consumed

The following prior VERSF results are consumed at their existing grades.

1. **Geometric Closure.** Triangle holonomy is generated locally from oriented edge transport; local co-face compatibility and the four global 4-simplex Bianchi-type relations are distinct requirements.
2. **Microphysical Foundations of Route M.** Route M uses a face-based formulation in which ten face holonomies are direct variables and five tetrahedral closure functionals obey one global relation, yielding a 10 + 4 structural census.
3. **Holonomy Assignment from Distinguishability.** Physical U(1) closed-loop holonomy obeys composition, reversal and identity, with gauge-invariant content exhausted by closed-loop holonomy on the connected graph branch, conditionally at that paper's declared grade.
4. **Operationally Invisible Sector Boundaries, Operational Geometry.** Finite distinguishability supplies a finite operational quotient/readout and an operational Hilbert geometry; operational structure must descend through resolution fibres.
5. **Universal Measure Principle, ODG probability programme.** The Born rule fixes the operational probability functional on the admitted amplitude carrier but does not select the prepared state.

6. **Primitive Matter Selection.** No-hidden-distinction admission states that an admissible positive distinguishability direction has capacity to form a physical fact unless a source theorem makes it redundant or forbidden. The same programme distinguishes support from probability/stiffness.
7. **Primitive Selection History-to-Curvature and Standard Model Attaching Map Closure.** The local MASD closure differential is an oriented circulation, and the canonical history-to-closure normalization is fixed with unit injection on the declared local carrier.
8. **Projective history-measure and global-measure papers.** Once finite operational marginals are fixed and projectively consistent, no independent post-hoc history weight may be appended.
9. **MASD and full-faithful source papers.** Physical gauge curvature is carried by edge/link transport on the admitted connection branch.
10. **Distinguishability Conservation and Gauge Structure.** Conservation of distinguishability/Fisher geometry requires smooth, equivariant, metric-compatible parallel transport, that is an Ehresmann connection structure. It forces the gauge-connection framework while correctly leaving the particular connection configuration as a physical dynamical variable.
11. **PFDMI source-selection sequence.** The support-changing marked-instrument calculation blindly recovers seven anonymous representation atoms within the inherited typed carrier, returns exactly four completion channels, and subsequent executions supply the full faithful matrix-link source and a noncentral gauge-sensitive marked law with physical Wilson-quotient rank 72.
12. **Faithful Gauge Quotient and Bundle Census.** On the frozen representation ledger the faithful global Standard Model gauge group is uniquely $G_{faith} \cong S(U(3) \times U(2)) = [SU(3) \times SU(2) \times U(1)_q]/\mathbb{Z}_6$, with the global bundle census and lifting obstruction correspondingly typed.
13. **Full-faithful kernel and MASD polar-bond closure.** The physical link is tied on occupied support to the unique polar transport of the record map, and the executed full-faithful residual kernel contains zero gauge directions.
14. **Born/commitment/record closure papers.** The distinguishable pre-commitment alternatives, quadratic Born weighting, stochastic first-entry realisation and persistent record formation are supplied elsewhere in the VERSF programme. They are not outstanding Route-M source debts.

2.2 Non-insertion boundary

The following may not be used to select a result:

- the desired value 7/8;
- the numerator fourteen as an equal-information assumption;
- the denominator sixteen as an already physical UV block;
- the experimental fine-structure constant;
- an assumed finite phase alphabet of size 128;
- a chosen phase tolerance;
- a uniform operational face or obstruction measure;

- independence of blocked coherence cells;
- an unproved identification between the four-simplex obstruction carrier and a later six-mode bath/history carrier;
- an added Gibbs/Boltzmann factor chosen after the finite operational law is fixed.

---

3. The oriented 4-simplex cochain complex

Let $\mathbb{T} = U(1)$ and label the vertices 0, 1, 2, 3, 4. The oriented simplex has

$$N_0 = 5, \quad N_1 = 10, \quad N_2 = 10, \quad N_3 = 5, \quad N_4 = 1,$$

so the multiplicative cochain groups are

$$C^0 = \mathbb{T}^5, \quad C^1 = \mathbb{T}^{10}, \quad C^2 = \mathbb{T}^{10}, \quad C^3 = \mathbb{T}^5, \quad C^4 = \mathbb{T},$$

with coboundaries

$$C^0 \rightarrow(\delta_0)\ C^1 \rightarrow(\delta_1)\ C^2 \rightarrow(\delta_2)\ C^3 \rightarrow(\delta_3)\ C^4.$$

For edge transport $U \in C^1$ the face holonomy is $H = \delta_1 U$. For face holonomy $H \in C^2$ the tetrahedral closure syndrome is $C = \delta_2 H$. Boundary-of-boundary gives

$$\delta_2\ \delta_1 = 1, \quad \delta_3\ \delta_2 = 1,$$

and at Lie-algebra level $d_2\ d_1 = 0$, $d_3\ d_2 = 0$.

Because Δ^4 is contractible, $H^k(\Delta^4; \mathbb{T}) = 0$ for $k > 0$, hence

$$\ker \delta_1 = \operatorname{im} \delta_0, \quad \ker \delta_2 = \operatorname{im} \delta_1, \quad \ker \delta_3 = \operatorname{im} \delta_2.$$

The integer incidence matrices give

$$\operatorname{rank} d_0 = 4, \quad \operatorname{rank} d_1 = 6, \quad \operatorname{rank} d_2 = 4, \quad \operatorname{rank} d_3 = 1,$$

and their nonzero Smith invariant factors are all equal to one (Appendix A). There is therefore no hidden finite torsion factor in the passage from edges to faces or from faces to tetrahedral obstruction.

---

4. Physical loop rank

4.1 Graph-theoretic count

The 1-skeleton of Δ^4 is the complete graph K_5 with E = 10, V = 5. For a connected graph the cycle-space rank is E − V + 1, so

$$\beta_1(K_5) = 10 - 5 + 1 = 6.$$

This is the content expected from the Holonomy Assignment paper: link phases are defined modulo vertex gauge, and physical U(1) information resides in closed-loop holonomy.

4.2 Gauge-quotient count

A vertex gauge transformation is a 0-cochain $g \in C^0 = \mathbb{T}^5$. The common global phase acts trivially, so the effective vertex-gauge group has dimension four and

$$\dim (C^1 / \operatorname{im} \delta_0) = 10 - 4 = 6.$$

Since $\ker \delta_1 = \operatorname{im} \delta_0$, the first isomorphism theorem gives

$$C^1 / \operatorname{im} \delta_0 \cong \operatorname{im} \delta_1,$$

so the link-realizable face-holonomy carrier is

$$\mathcal{H}\_\text{phys} := \operatorname{im} \delta_1 \cong \mathbb{T}^6.$$

Theorem 1: Physical Abelian loop-rank theorem

[EXACT]

On one 4-simplex, the physical U(1) face-holonomy information generated by link transport has six independent gauge-invariant degrees of freedom:

$$\mathcal{H}\_\text{phys} \cong U(1)^6.$$

This follows independently from the K_5 cycle rank 10 − 5 + 1 = 6 and from the exact cochain quotient $C^1 / \operatorname{im} \delta_0 \cong \operatorname{im} \delta_1$.

Consequence. The ten triangular loops of the face formulation cannot represent ten independent physical link-holonomy degrees of freedom. This does not invalidate the ten-face formulation; it fixes its type as strictly larger than the physical connection image.

---

5. Constructive face-to-connection reconstruction

5.1 Star-tree gauge

Choose vertex 0 as a temporary base and use vertex gauge freedom to set

$$U_{01} = U_{02} = U_{03} = U_{04} = 1.$$

This is a spanning-tree gauge choice and fixes no physical holonomy. The six remaining edge transports are U_{12}, U_{13}, U_{14}, U_{23}, U_{24}, U_{34}. For each triangle containing vertex 0,

```
H_{0ij} = U_{0i} U_{ij} U_{j0} = U_{ij},
```

so

$$U_{12} = H_{012}, \quad U_{13} = H_{013}, \quad U_{14} = H_{014},$$
$$U_{23} = H_{023}, \quad U_{24} = H_{024}, \quad U_{34} = H_{034}.$$

The six rooted face holonomies reconstruct the six physical non-tree links directly.

5.2 The four remaining faces are forced

The face 123 must satisfy $H_{123} = U_{12} U_{23} U_{31}$, so

$$H_{123} = H_{012} H_{023} H_{013}^{-1},$$
$$H_{124} = H_{012} H_{024} H_{014}^{-1},$$
$$H_{134} = H_{013} H_{034} H_{014}^{-1},$$
$$H_{234} = H_{023} H_{034} H_{024}^{-1}.$$

These are four independent global realizability conditions.

5.3 The four obstruction coordinates

Define

$$R_{0123} = H_{013} H_{123} H_{012}^{-1} H_{023}^{-1},$$
$$R_{0124} = H_{014} H_{124} H_{012}^{-1} H_{024}^{-1},$$
$$R_{0134} = H_{014} H_{134} H_{013}^{-1} H_{034}^{-1},$$
$$R_{0234} = H_{024} H_{234} H_{023}^{-1} H_{034}^{-1}.$$

Then R_{0ijk} = 1 is exactly the corresponding tetrahedral closure condition. The fifth tetrahedral residual is fixed by the global boundary relation $s = (1, -1, 1, -1, 1)^T$ and supplies no fifth independent obstruction.

Theorem 2: Constructive face-to-connection reconstruction theorem

[EXACT]

A direct face assignment $H \in U(1)^{10}$ arises from one globally defined edge transport if and only if

$$R_{0123} = R_{0124} = R_{0134} = R_{0234} = 1.$$

When these hold, the underlying edge transport is reconstructed by the star-tree formulas of §5.1 and is unique up to vertex gauge.

Proof of uniqueness. Suppose $\delta_1 U = H = \delta_1 U'$. Then $\delta_1(U'U^{-1}) = 1$, so $U'U^{-1} \in \ker \delta_1 = \operatorname{im} \delta_0$ and $U' = U \cdot \delta_0 g$ for some vertex gauge g. The face data therefore determine one and only one physical gauge-equivalence class. ■

Significance. The four Route-M tetrahedral conditions are exactly the gate that turns a ten-face local assignment into one globally coherent connection class. They are neither arbitrary extras nor additional post-connection field modes.

---

6. The global obstruction sequence

Define the direct-face carrier, the physical subtorus and the closure-obstruction torus

$$\mathcal{F}\_\mathrm{dir} := C^2 = \mathbb{T}^{10}, \quad \mathcal{H}\_\mathrm{phys} := \operatorname{im} \delta_1, \quad \mathcal{O}\_\mathrm{cl} := \operatorname{im} \delta_2 = \ker \delta_3.$$

Since rank $d_2 = 4$ with unit nonzero Smith invariant factors, $\mathcal{O}\_\mathrm{cl} \cong \mathbb{T}^4$, and exactness at C^2 gives $\ker \delta_2 = \mathcal{H}\_\mathrm{phys}$.

Theorem 3: Route-M global obstruction exact sequence

[EXACT]

There is a short exact sequence of compact Abelian Lie groups

$$1 \longrightarrow \mathcal{H}\_\mathrm{phys} \longrightarrow \mathcal{F}\_\mathrm{dir} \xrightarrow{(\delta_2)} \mathcal{O}\_\mathrm{cl} \longrightarrow 1,$$

with

$$\mathcal{H}\_\mathrm{phys} \cong U(1)^6, \quad \mathcal{F}\_\mathrm{dir} = U(1)^{10}, \quad \mathcal{O}\_\mathrm{cl} \cong U(1)^4,$$

equivalently

$$\mathcal{F}\_\mathrm{dir} / \mathcal{H}\_\mathrm{phys} \cong \mathcal{O}\_\mathrm{cl}.$$

Interpretation. The quotient of the direct-face carrier by physically link-realizable face holonomy is not an arbitrary residual space. It is precisely the four-dimensional tetrahedral closure-obstruction torus.

No torsion. Because the Smith invariant factors are units, the quotient contains no hidden finite torsion sector. In particular the residual is not naturally a 128-state object, a $\mathbb{Z}_{128}$ object, or any other finite group merely because K7 carries 2^7 Boolean configurations.

---

7. Splitting of the sequence

The tree gauge of §5 exhibits a product decomposition. It is worth separating what that construction proves from what the choice of tree contributes, since the two are often conflated.

Let the six rooted-face coordinates and four residuals be

$$A = (H_{012}, H_{013}, H_{014}, H_{023}, H_{024}, H_{034}) \in \mathbb{T}^6,$$
$$R = (R_{0123}, R_{0124}, R_{0134}, R_{0234}) \in \mathbb{T}^4.$$

The remaining faces are

$$H_{123} = R_{0123} \cdot H_{012} H_{023} H_{013}^{-1},$$
$$H_{124} = R_{0124} \cdot H_{012} H_{024} H_{014}^{-1},$$
$$H_{134} = R_{0134} \cdot H_{013} H_{034} H_{014}^{-1},$$
$$H_{234} = R_{0234} \cdot H_{023} H_{034} H_{024}^{-1},$$

an invertible monomial change of torus coordinates whose integer Jacobian is block triangular with unit diagonal blocks and determinant 1 (Appendix B).

Theorem 4: Splitting theorem and its non-canonicity

[EXACT]

(i) The sequence of Theorem 3 splits, and the splitting requires no geometric input. Applying the character functor gives the exact sequence of free abelian groups

$$0 \to \mathbb{Z}^4 \to \mathbb{Z}^{10} \to \mathbb{Z}^6 \to 0,$$

which splits because $\mathbb{Z}^6$ is free. Dualising back,

$$U(1)^{10} \cong U(1)^6\_\mathrm{rooted} \times U(1)^4\_\mathrm{obstruction},$$

with the physical subtorus given by R = (1, 1, 1, 1).

(ii) The splitting is not unique. The set of splittings is a torsor over $\mathrm{Hom}(\mathbb{Z}^6, \mathbb{Z}^4) \cong M_{4\times6}(\mathbb{Z})$. Choosing a base vertex and spanning tree selects one element of that set and nothing more.

(iii) No splitting is simultaneously integral and invariant under vertex relabelling. Writing L_int = im d_1 and L_obs = im d_2^T for the two saturated integral lattices, both have unit Smith invariant factors, but

$$[\ \mathbb{Z}^{10} : \mathrm{L\_int} \oplus \mathrm{L\_obs}\ ] = 125 = 5^3.$$

The equivariant projector Q_cl of §10 therefore has denominators 5, and is integral only after inverting 5.

Reading. The decomposition into physical and obstruction sectors is canonical over $\mathbb{R}$ and over $\mathbb{Z}[1/5]$, and is forced there by symmetry (§10). At tangent level the tree choice can be dispensed with altogether: §12 gives a canonical equivariant formula for the physical edge representative. The clean group-level product decomposition is available but costs a non-equivariant choice. The two facts are consistent and should not be conflated: an argument that uses the product decomposition inherits a tree choice, whereas an argument that uses Q_cl does not.

Haar diagnostic. Because the coordinate transform is unimodular, normalized Haar measure on the raw direct-face torus factorizes as product Haar in (A, R) coordinates. This is a mathematical diagnostic only. It does not imply that the physical VERSF pre-integrability state is Haar on the direct-face carrier. If raw continuous Haar were nevertheless imposed, exact closure R = 1 would have Haar measure zero. That is a property of the mathematical carrier and not of the physical source: by Theorem 12 the current source measure is supported on ker δ_2 exactly, so the closure event has source probability one rather than Haar probability zero.

---

8. Minimal off-shell completion

The exact sequence lets the phrase "pre-integrability carrier" be made precise. Consider a smooth regular off-shell completion $\tilde{\mathscr{F}}$ with:

1. a physical zero-obstruction fibre identified with $\mathscr{H}$_phys $\cong U(1)^6$;
2. a smooth obstruction map $\Omega : \tilde{\mathscr{F}} \to \mathcal{O}$_cl $\cong U(1)^4$;
3. full rank, rank $D\Omega = 4$, in a neighbourhood of the physical fibre;
4. zero fibre $\Omega^{-1}(1) = \mathscr{H}$_phys.

The regular level-set theorem gives

$$\dim \tilde{\mathcal{F}} = \dim \Omega^{-1}(1) + \mathrm{rank}\ D\Omega \geq 6 + 4 = 10,$$

while Route M has dim $\mathscr{F}$_dir = 10.

Theorem 5: Minimal off-shell completion theorem

[EXACT IN THE DECLARED REGULAR SMOOTH CLASS; RIGID IN THE COMPACT ABELIAN CATEGORY]

(i) Any regular smooth carrier that retains all six physical Abelian loop degrees of freedom while allowing all four independent global integrability failures has dimension at least ten, and Route M's direct-face carrier $\mathscr{F}$_dir = $U(1)^{10}$ attains the bound.

(ii) If the completion is additionally required to be a compact connected abelian Lie group with a surjective homomorphism onto $\mathcal{O}$_cl whose kernel is $\mathscr{H}$_phys, the bound is rigid: dim $\tilde{\mathscr{F}} = 10$ exactly, and $\tilde{\mathscr{F}} \cong U(1)^{10}$. This follows because an extension of a torus by a torus is a compact connected abelian Lie group of dimension dim $\mathscr{H}$_phys + dim $\mathcal{O}$_cl, hence a torus of dimension ten.

Consequence. The question "why should a ten-direct-face pre-integrability carrier exist?" is answered structurally: if VERSF admits a regular off-shell layer that can represent all four global closure failures while retaining the full physical loop carrier, ten is the minimum possible dimension, the Route-M face carrier realizes it, and in the group category it is the only possibility up to isomorphism. What is left is source occupation and weight, not carrier size or geometry.

---

9. The simplex Laplacian and isotropy of the closure readout

The spectral facts used below are not accidents of the 4-simplex. They follow from one identity.

9.1 Oriented tetrahedron-to-face matrix

Order the triangular faces as

```
012, 013, 014, 023, 024, 034, 123, 124, 134, 234
```

and the tetrahedra as

```
0123, 0124, 0134, 0234, 1234.
```

The closure differential is

```
B = d₂ =
  ( −1   1   0  −1   0   0   1   0   0   0 )
```

$$
\begin{pmatrix}
-1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1
\end{pmatrix}
$$

with $s = (1, -1, 1, -1, 1)^T$, $d_3 = s^T$ and $s^T B = 0$.

Lemma 9.1: Simplex Laplacian identity

[EXACT]

On the full simplex Δ^N with $n = N + 1$ vertices, the simplicial coboundary operators satisfy, on each cochain group C^k with the standard incidence inner product and $1 \leq k \leq N$,

$$d_k^T d_k + d_{k-1} d_{k-1}^T = n \cdot I$$

(with d_N taken to be zero). The same identity holds at $k = 0$ once the augmentation $d_{-1} : \mathbb{R} \to C^0$ is adjoined, that is on the reduced complex; without it the $k = 0$ case fails by exactly the rank-one augmentation term, which is why the statement is used here only for $k \geq 1$.

Proof. For an oriented k-face σ, the up-Laplacian term $d_k^T d_k \sigma$ sums over the $n - k - 1$ vertices not in σ, contributing $(n - k - 1)\sigma$ diagonally plus off-diagonal terms; the down-Laplacian term $d_{k-1} d_{k-1}^T \sigma$ sums over the $k + 1$ vertices of σ, contributing $(k + 1)\sigma$ diagonally plus off-diagonal terms with the opposite sign. On the full simplex, in which every subset of vertices is a face, the off-diagonal contributions cancel identically and the diagonal contributions sum to $(n - k - 1) + (k + 1) = n$. ■

Certificate. Verified by explicit construction for $N = 4, 5, 6, 7$; see Appendix C.

Corollary 9.2: Exact Gram identity at the closure level

Taking $k = 3$ and transposing,

$$B B^T + d_3^T d_3 = 5 I_5,$$

that is

$$B B^T = 5 I_5 - s s^T.$$

Since $\|s\|^2 = 5$, the spectrum of $B B^T$ is $\{5, 5, 5, 5, 0\}$ and the four nonzero singular values of B are

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sqrt{5}.$$

The identity $B B^T = 5I_5 - s s^T$ is therefore a specialisation of a general structural theorem rather than a coincidence of the chosen face and tetrahedron ordering, and it does not need to be established by direct matrix multiplication.

Theorem 6: Isotropic closure-readout theorem

[EXACT, ALL SIMPLEX DIMENSIONS]

On Δ^N with $n = N + 1$ vertices, the closure differential d_2 has all nonzero singular values equal to $\sqrt{n}$. Consequently

$$d_2 \, d_2^T = n \cdot \Pi,$$

with Π the orthogonal projector onto the syndrome hyperplane, and the restriction of d_2 to the obstruction tangent is a scaled isometry with condition number one.

For $N = 4$ this gives rank four and common singular value $\sqrt{5}$. Verified numerically for $N = 4, 5, 6, 7$ in Appendix C, with ranks $C(N, 3)$ matching the closed form of Proposition 9.3.

Physical reading. The four closure readouts sample the four obstruction directions with exactly equal sensitivity. There is no incidence-geometric direction that is loud and none that is faint. Any asymmetry among the four obstruction directions in a physical result must therefore be imposed by the source law, and cannot be inherited from the simplex. This converts a possible objection ("perhaps some obstruction directions are nearly invisible to the tetrahedral test") into a proved negative.

Proposition 9.3: General-simplex census

[EXACT]

On Δ^N the direct-face carrier, physical carrier and obstruction torus have dimensions

$$\dim \mathcal{F}\_\mathrm{dir} = C(N+1, 3), \quad \dim \mathcal{H}\_\mathrm{phys} = C(N, 2), \quad \dim \mathcal{O}\_\mathrm{cl} = C(N, 3),$$

with $C(N+1, 3) = C(N, 2) + C(N, 3)$. For $N = 4$ this is $10 = 6 + 4$. For $N = 5$ it is $20 = 10 + 10$, and for $N = 6$ it is $35 = 15 + 20$. The Route-M 4-simplex is therefore the unique simplex dimension in which the obstruction sector is strictly smaller than the physical sector by exactly the count that produces the 10 + 4 census.

---

10. Canonical tangent obstruction geometry

10.1 The projector

Define

$$Q\_cl = (1/5)\ B^T\ B.$$

Because $s^T B = 0$, Corollary 9.2 gives $B\ B^T\ B = 5B$, hence

$$Q\_cl^2 = (1/25)\ B^T\ (5I - s\ s^T)\ B = (1/5)\ B^T\ B = Q\_cl,$$

and $Q\_cl^T = Q\_cl$. So Q_cl is an orthogonal projector of rank four. Define

$$P\_int = I_{10} - Q\_cl, \quad \text{rank } P\_int = 6.$$

Moreover ker B = im P_int and im B^T = im Q_cl, and by simplicial exactness ker B = im d_1.

Theorem 7: Canonical integrable/obstruction tangent decomposition

[EXACT]

The ten-dimensional direct-face tangent decomposes orthogonally as

$$\mathbb{R}^{10} = V\_int \oplus V\_obs,$$

where

$$\begin{aligned} V\_int &= \ker B = \text{im } d_1, & \dim V\_int &= 6, \\ V\_obs &= \text{im } B^T, & \dim V\_obs &= 4, \end{aligned}$$

with canonical projectors

$$P\_int = I - (1/5)\ B^T\ B, \quad Q\_cl = (1/5)\ B^T\ B.$$

This is an incidence theorem, not a fitted Hessian decomposition.

10.2 Symmetry forces the same split

The vertex-relabelling group S_5 acts on oriented faces and tetrahedra by permutation with the induced orientation sign. Write ρ_2 and ρ_3 for those representations. Direct computation gives exact equivariance,

$$B\ \rho_2(g) = \rho_3(g)\ B \quad \text{for all } g \in S_5,$$

so Q_cl commutes with ρ_2.

Theorem 8: Symmetry-canonicity theorem

[EXACT]

Under the S_5 action on the direct-face tangent:

(i) V_int and V_obs are absolutely irreducible and mutually inequivalent, with

```
V_int ≅ S^(3,1,1)   (dimension 6),
V_obs ≅ S^(2,1,1,1) = standard ⊗ sign   (dimension 4).
```

Their characters on the classes (1^5, 2, 3, 2^2, 4, 3·2, 5) are

```
χ_int = (6, 0, 0, −2, 0, 0, 1),
χ_obs = (4, −2, 1, 0, 0, 1, −1).
```

(ii) The algebra of S_5-equivariant endomorphisms of $\mathbb{R}^{10}$ is two-dimensional, spanned by P_int and Q_cl. Hence Q_cl is the unique S_5-equivariant idempotent of rank four, and the only equivariant idempotents at all are 0, P_int, Q_cl and I.

(iii) The space of S_5-invariant bilinear forms on $\mathbb{R}^{10}$ is two-dimensional. Every S_5-invariant inner product is of the form $a\langle\cdot,\cdot\rangle|V\_int \oplus b\langle\cdot,\cdot\rangle|V\_obs$ with $a, b > 0$, and every one of them yields the same orthogonal projectors P_int and Q_cl.

Certificate. Equivariance verified to machine zero for all 120 group elements; equivariant endomorphism dimension 2; invariant bilinear form dimension 2. See Appendix C.

Reading. The split of §10.1 is not an artefact of choosing the standard incidence inner product. It is the only decomposition compatible with the symmetry of the labelled simplex, and it is metric-independent within the invariant class. An objection of the form "a different natural metric would mix the sectors" is therefore closed: no invariant metric does.

---

11. Complete closure-syndrome reconstruction

Let $\theta \in \mathbb{R}^{10}$ be a small direct-face phase perturbation with five-component oriented tetrahedral syndrome

```
r = B θ.
```

Automatically $s^T r = 0$, so r lies in the four-dimensional obstruction hyperplane s⊥. Define θ_obs = Q_cl θ and θ_int = P_int θ, so that B θ_int = 0. On s⊥, Corollary 9.2 gives $B B^T r = 5r$, hence the Moore-Penrose inverse on the physical codomain is $B^+ = (1/5)\, B^T$ and

```
θ_obs = B⁺ r = (1/5) Bᵀ r,
```

with

$$\|\theta\_obs\|^2 = (1/25)\ r^T B B^T r = (1/5)\ \|r\|^2,$$

that is

$$\|\theta\_obs\|^2 = (1/5)\ \|B\ \theta\|^2.$$

Theorem 9: Closure-syndrome completeness theorem

[EXACT]

At the linearised 4-simplex grain:

1. the four independent tetrahedral closure residuals contain exactly the information needed to reconstruct the four-dimensional non-integrable face perturbation;
2. no obstruction direction is invisible to the closure syndrome;
3. no closure-syndrome direction represents an additional independent face mode;
4. the map from V_obs to the obstruction hyperplane is a scaled isometry with singular value √5, so the reconstruction is perfectly conditioned and error in the syndrome propagates to θ_obs with a single uniform factor 1/√5.

Consequence for the 6 + 4 + 4 architecture. The labelled decomposition

```
14 = 6 + 4 + 4
```

has a precise interpretation:

- 6: link-realizable physical face-holonomy tangent;
- first 4: off-shell integrability-obstruction tangent;
- second 4: closure-syndrome readout of that same obstruction tangent.

The two fours are canonically paired by $B^+ = (1/5)B^T$. Therefore fourteen is not fourteen independent physical modes. Neither is it mere bookkeeping. It is a state-plus-complete-obstruction-readout architecture, and the pairing is now explicit rather than asserted.

---

12. The canonical face-to-edge attaching map

The reconstruction of §5 used a spanning tree, and §7 showed that the resulting product decomposition costs a non-equivariant choice. The Laplacian identity removes that cost entirely at tangent level. There is a canonical, tree-free, gauge-fixing-free formula that takes any ten-face candidate and returns its physical edge-connection representative.

12.1 The complementary projector

Lemma 9.1 at k = 2 reads

$$d_1 d_1^T + d_2^T d_2 = 5 I_{10}$$

on the direct-face tangent. Since Q_cl = (1/5) $d_2^T d_2$, the complementary projector is not merely I − Q_cl in the abstract; it is itself an incidence object,

$$P\_int = (1/5)\ d_1 d_1^T.$$

This is verified as an exact identity (Appendix C.7), and it makes both halves of the decomposition explicitly constructed from incidence data rather than one being defined as the complement of the other.

Theorem 10: Exact discrete Hodge decomposition and canonical attaching map

[EXACT]

Every ten-face perturbation $\theta \in \mathbb{R}^{10}$ decomposes exactly as

$$\theta = (1/5)\ d_1 d_1^T \theta + (1/5)\ d_2^T d_2 \theta,$$

the first term being the link-realizable physical curvature and the second the integrability failure. Defining the canonical edge candidate

$$A\_can = (1/5)\ d_1^T \theta,$$

the decomposition takes the closed form

$$\theta = d_1 A\_can + (1/5)\ d_2^T (d_2 \theta),$$

with

$$d_1 A\_can = P\_int\ \theta.$$

Moreover:

(i) **Gauge-orthogonality.** A_can satisfies d_0^T A_can = 0 automatically, since $d_0^T d_1^T = (d_1 d_0)^T = 0$. It therefore lands in the gauge-orthogonal edge slice V_edge^phys = ker d_0^T of dimension six, with no tree, no base vertex and no gauge condition imposed by hand.

(ii) **Minimum norm.** A_can is the Moore-Penrose solution, A_can = d_1^+(P_int θ), hence the unique edge representative of minimal norm among all A with d_1A = P_int θ. The remaining representatives are A_can + d_0g.

(iii) **Equivariance.** The map θ ↦ A_can intertwines the S_5 actions on faces and edges exactly, so the assignment of a physical edge connection to a face candidate is covariant under vertex relabelling.

(iv) **Completeness.** The pair

$$\theta \mapsto (A\_can,\ r), \qquad r = d_2\theta,$$

is a linear isomorphism from $\mathbb{R}^{10}$ onto V_edge^phys ⊕ s⊥, so it is a complete equivariant coordinate encoding of θ and nothing about the candidate is lost by reporting it.

Certificate. All four items verified to machine zero, including the equivariance check over all 120 elements of S_5 (Appendix C.7).

Reading. The existence-and-uniqueness statement of Theorem 2 says that an edge connection exists precisely when the four obstructions vanish. Theorem 10 supplies the map itself, and supplies it for candidates that do not satisfy the closure conditions: the link-realizable component is extracted regardless, and the obstruction component is named. The star-tree construction of §5 remains the constructive proof of realizability; it is not needed to write down the representative. For a current source-native state the obstruction component vanishes identically (Theorem 12), so the map returns the physical edge tangent itself.

Theorem 11: Face/edge normalization isometry

[EXACT]

Lemma 9.1 at k = 1 reads $d_1^T d_1 + d_0 d_0^T = 5I_{10}$ on the edge tangent. On the gauge-orthogonal slice ker d_0^T the second term vanishes, so

$$d_1^T\ d_1 = 5\ I \qquad \text{on V\_edge\^{}phys},$$

and therefore

$$(1/\sqrt{5})\ d_1 : \text{V\_edge\^{}phys} \longrightarrow \text{V\_face\^{}int}$$

is an exact isometry of six-dimensional spaces, with

$$\|d_1\ A\|^2 = 5\ \|A\|^2 \qquad \text{for all A} \in \text{V\_edge\^{}phys}.$$

The corresponding norm identity for a general face candidate is

$$\|\theta\|^2 = 5\ \|A\_can\|^2 + (1/5)\ \|r\|^2.$$

On Δ^N the same statements hold with 5 replaced by $N + 1$.

Certificate. Ratio $\|d_1A\|^2/\|A\|^2$ returned as 5.000000 on random elements of the gauge-orthogonal slice; the restricted Gram $d_1^T d_1$ equals $5I_6$ to 2.7×10^{-15}; the norm identity holds to machine precision (Appendix C.7).

Why this matters downstream. Existence and uniqueness of the face-to-connection attachment is a statement about which objects correspond. Theorem 11 supplies the normalized differential of that correspondence, which is the object a gauge-response or Hessian calculation actually needs: a quadratic form written on face curvature converts to one written on the edge gauge tangent with the single fixed factor 5, and back with 1/5. No fitted constant enters, and the factor is fixed by the simplex, not chosen.

---

13. Current-source integrability closure

The previous sections are statements about an arbitrary mathematical face candidate. This section asks a different question: what does the VERSF source stack, as it currently stands, actually produce on the direct-face carrier? The answer is sharper than a conditional fork, and it is forced by premises already in the corpus.

13.1 The inherited holonomy typing

The Holonomy Assignment construction supplies one map on the admissible closed paths of a connected branch,

$$h\_D : \mathcal{L} \longrightarrow U(1),$$

satisfying composition, reversal and identity,

$$h\_D(L_1 \circ L_2) = h\_D(L_1)\ h\_D(L_2), \qquad h\_D(L^{-1}) = h\_D(L)^{-1}, \qquad h\_D(id) = 1.$$

The geometric and MASD constructions likewise obtain triangular and plaquette holonomy from underlying edge transport. Source-native physical holonomy is therefore not a collection of ten independently assigned face values. It is one assignment, restricted to ten different boundary loops.

Theorem 12: Current-source integrability closure theorem

[EXACT CONSEQUENCE OF THE CURRENT PHYSICAL HOLONOMY TYPING, AT THE DECLARED GRADE OF THE INHERITED HOLONOMY-ASSIGNMENT PREMISES]

Let h_D be the source-determined physical U(1) holonomy assignment on the admissible closed paths of one connected 4-simplex branch, satisfying composition, reversal and identity, and define the face holonomy on each oriented triangle f by

$$H_f = h_D(\partial f).$$

Then for every oriented tetrahedron τ,

$$\prod_{f \subset \partial\tau} H_f^{\sigma(\tau,f)} = h_D(\partial\partial\tau) = h_D(id) = 1,$$

so

$$H_{source} \in \ker \delta_2 = \mathrm{im}\ \delta_1,$$

and at tangent level, for every physical source variation,

$$d_2\ \theta_{source} = 0.$$

The execution protocol of §16 consequently returns

$$r = 0, \quad f_{obs} = 0, \quad A_{can} = A_{phys}.$$

Proof. The oriented product of the face-boundary loops of one tetrahedron is the boundary of its boundary. Every oriented edge of that combined loop occurs twice with opposite orientation, so composition and reversal reduce it to the identity path, whose holonomy is 1. The tangent statement is $d_2 d_1 = 0$ applied to the variation of the underlying transport. ■

Corollary 12.1: No current physical obstruction measure

[EXACT AT THE DECLARED SOURCE GRADE]

Let μ_D be any source probability law on the current physical holonomy branch and μ_{face} its induced face-holonomy pushforward. Then

$$\mathrm{supp}\ \mu_{face} \subseteq \ker \delta_2, \quad \mu_{face}(\ker \delta_2) = 1,$$

hence

$$P_{obs}(r \neq 0) = 0$$

at this physical grain, and there is no independent current-source Route-M closure-admission information,

$$I_{obs}^{current} = 0.$$

13.2 Scope firewall

Three things this does not say.

It does not say that curvature vanishes. Nontrivial individual triangle holonomies remain allowed, and the six-dimensional physical carrier is fully occupied.

It does not say that every connection on an arbitrary larger complex is globally gauge-trivial. Nontrivial holonomy around non-bounding global cycles remains possible; the statement is local to each 4-simplex.

It does not exclude every conceivable deeper theory. It closes the question for the source stack as it currently stands. A future theory could introduce a genuinely pre-holonomy carrier, but that carrier would require its own derivation: it cannot be obtained by declaring the present physical face holonomies independent, because their composition law already forces tetrahedral consistency. Primitive-source uniqueness remains a wider VERSF question and is not settled here.

13.3 Consequence for the two stages

```
Stage 0C-1: CURRENT-SOURCE NEGATIVE.
Stage 0C-2: DOES NOT OPEN ON THE CURRENT SOURCE STACK.
```

The present VERSF physical source stack does not populate a pre-integrability Route-M obstruction sector, so there is no current physical obstruction probability, Born state, finite closure alphabet, tolerance or stiffness to derive. The obstruction sector remains mathematically real and indispensable as a realizability test. It is not a physically populated sector of the current source.

---

14. From local curvature to the full physical Standard Model connection

Prior PSAC work closes the local face-curvature differential. For a face p, perturbing the oriented boundary links gives the MASD closure differential

```
Ḋ_C(p) = (1/|p|) Σ_{e ∈ ∂p} σ(p, e) Ad_{Q_{e→p}}(A_e),
```

and on the scalar oriented closure slice the integrated curvature coordinate has differential equal to the canonical oriented closure score, with exact right-inverse normalization λ_C = 1 on the declared local carrier. The present geometry closes the Abelian local-to-global face consistency problem exactly. Later VERSF work then permits the physical Standard Model handoff to be stated more strongly than the linearized census of §16.

14.1 Abelian direct-face theorem

Local face curvature values cannot be assembled arbitrarily across the 4-simplex; their global compatibility is exactly $\delta_2 H = 1$.

Theorem 13: Local-to-global Abelian gauge attaching theorem

[STRUCTURAL EXACT, CONSUMING THE PRIOR LOCAL NORMALISATION AT ITS DECLARED GRADE]

Once local Route-M/MASD-compatible U(1) face holonomies are supplied on the ten faces of one 4-simplex, the passage to a globally realizable physical connection is completely determined:

```
local face holonomies →(δ₂) four global obstructions →(= 1) one edge-connection gauge class,
```

that is

$$H \in \ker \delta_2 \iff H = \delta_1 U,$$

with U unique modulo $\delta_0 g$. Existence is equivalent to zero obstruction; uniqueness is up to vertex gauge.

14.2 Non-Abelian loop-to-connection reconstruction

Let G be any group. Choose one base vertex and one spanning tree of the connected graph K_5. Its fundamental group is free on six generators,

$$\pi_1(K_5) \cong F_6,$$

the rank agreeing with the cycle count $E - V + 1 = 6$ of §4. Let

$$\rho : F_6 \longrightarrow G$$

be a composition-respecting based-loop holonomy assignment. Gauge-fix the four tree links to the identity and assign the six remaining links to the images under ρ of the six fundamental loops. Every closed loop is a word in those six generators, so its link holonomy reproduces ρ exactly. Hence $\rho \longmapsto [U_\rho]$ is an exact reconstruction of one connection gauge class. Changing the base frame conjugates all six fundamental holonomies simultaneously, so the gauge-equivalence space is $\mathrm{Hom}(F_6, G)/G$.

Theorem 13A: Non-Abelian holonomy-to-connection reconstruction

[EXACT, NONLINEAR, FOR A COMPLETE BASED-LOOP ASSIGNMENT ON THE CONNECTED K_5 GRAPH]

A complete G-valued loop-holonomy representation on K_5 determines one edge connection up to vertex gauge, and

$$\{ \text{G-connections on } K_5 \} / \text{gauge} \cong \mathrm{Hom}(F_6, G) / G.$$

What this is not. This is not a nonlinear extension of the direct-face equation $d_2\theta = 0$. It bypasses arbitrary non-Abelian face integrability by reconstructing the connection from the complete loop assignment itself. The two statements have different inputs: a complete loop representation in one case, ten independently chosen face matrices in the other. Only the second remains unproved.

14.3 Faithful Standard Model group

For the frozen VERSF representation ledger the physical global group is

$$G_{faith} = [SU(3) \times SU(2) \times U(1)_q] / \mathbb{Z}_6 \cong S(U(3) \times U(2)).$$

The finite quotient changes the global topology but not the Lie algebra,

$$\mathfrak{g}_{faith} = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1),$$

so the six physical graph-loop directions give

$$6 \times 12 = 72$$

local physical Standard Model gauge tangents. The 120 = 72 + 48 calculation of §16 remains the exact first-order decomposition of the enlarged direct-face test carrier and should not be confused with the nonlinear connection-moduli statement above.

14.4 Source-side completion

Later PFDMI executions substantially close the connection between the primitive marked source and this faithful gauge carrier.

Seven anonymous minimal central projectors are recovered within the inherited graded representation carrier before particle labels are opened, and subsequently identify exactly with

$$(Q_L,\ u_R,\ d_R,\ L_L,\ e_R,\ N_R,\ H),$$

with three copies on the fermionic blocks. Exactly four admissible completion channels are recovered from the representation constraints, and the typed marked instrument has the corresponding stable terminal species orbits. This is blind recovery inside an inherited typed carrier, not a derivation of that carrier from no prior representation structure.

The field-dependent instrument then contains the full faithful matrix links and returns a native field-sensitive marked law. Its probability Jacobian has raw gauge rank 144 and Wilson-quotient rank 72, and the seven terminal-type populations alone locally identify all 72 physical gauge directions at linear order.

14.5 Conditional local connection uniqueness

The MASD bond defect is

```
D_B(i, j) = Π_j U_ij Π_i − Pol(Π_j Φ_ij Π_i),
```

so on a zero-defect stationary solution

```
Π_j U_ij Π_i = Pol(Π_j Φ_ij Π_i),
```

and the realised record transport determines the physical link uniquely on occupied support. The complete full-faithful defect audit then gives

```
19 = 18_record + 1_completion + 0_gauge,
```

with the full gauge-coordinate carrier lying in the defect row space to numerical precision 4.78×10^{-15}. Therefore, in the executed local chart,

```
full realised state fixed ⟹ no surviving infinitesimal null freedom changes
U_phys.
```

14.6 Grades

Statement	Grade
Connection structure required by distinguishability conservation	STRUCTURAL THEOREM
Complete loop holonomy → connection up to gauge	EXACT NONLINEAR
Full faithful matrix-link carrier	EXECUTED PASS
Physical gauge rank 72 in the native marked law	EXECUTED PASS
Record transport → occupied-support link	CONDITIONAL EXACT
Zero residual gauge content of the full tested kernel	MACHINE-CERTIFIED LOCAL PASS
Global uniqueness of a universe-wide gauge configuration independent of realised state and history	NOT CLAIMED AND NOT REQUIRED

Standard Model significance. The geometric attachment between local closure/curvature structure and the edge/link connection carrier is closed in the Abelian direct-face problem by Theorem 13, and the physical non-Abelian connection is reached exactly by Theorem 13A together with the executed source results above. What the paper does not derive is how the primitive history source would prepare an arbitrary ten-face off-shell candidate, which by §13 it does not do in any case.

15. Execution protocol

Theorems 9, 10 and 11 make the paper executable rather than only structural. Any calculation that outputs a direct-face candidate, physical or hypothetical, can be required to report a fixed triple, computed by formula with no fitting and no gauge choice.

Given a candidate $\theta \in \mathbb{R}^{10}$ at the linearised grain:

closure syndrome	$r = d_2\,\theta$	$\in s^\perp \subset \mathbb{R}^5$
non-integrable component	$\theta\_obs = (1/5)\,d_2^T\,r$	$\in V\_obs$
physical edge connection	$A\_can = (1/5)\,d_1^T\,\theta$	$\in V\_edge\text{^}phys$

with the exact reconstruction $\theta = d_1 A\_can + \theta\_obs$.

Define the dimensionless integrability diagnostic

$$f\_obs = \|\theta\_obs\|^2 / \|\theta\|^2 = \|d_2\,\theta\|^2 / (5\,\|\theta\|^2).$$

Theorem 14: Integrability diagnostic

[EXACT]

For every nonzero $\theta \in \mathbb{R}^{10}$,

$$0 \le f\_obs \le 1,$$

with $f\_obs = 0$ exactly on the link-realizable tangent V_int and $f\_obs = 1$ exactly on the obstruction tangent V_obs. The complementary fraction $1 - f\_obs = 5\|A\_can\|^2/\|\theta\|^2$ is the share of the candidate that survives as physical gauge field. Because both projectors are S_5-equivariant, f_obs is invariant under vertex relabelling, so it is a property of the candidate and not of the labelling.

Certificate. f_obs evaluated as 5.7×10^{-33} on random elements of V_int and 1.0000000 on random elements of V_obs (Appendix C.7).

Reporting requirement

Any source calculation claiming to produce a Route-M face candidate should report

$$\theta \longrightarrow (\,r,\ f\_obs,\ A\_can\,),$$

together with the multiplicity and regulator data required by §21 if it also claims a running consequence. This is a complete report: by Theorem 10(iv) the pair (A_can, r) determines θ, and f_obs is the single scalar that says how much of the candidate is rejected by integrability.

For a current source-native physical state the report is fixed in advance by Theorem 12:

$$\theta_{\text{source}} \mapsto (A_{\text{phys}}, 0), \qquad f_{\text{obs}} = 0.$$

The protocol therefore functions as a consistency certificate on the physical branch rather than as a description of a filtering process. A returned $f_{\text{obs}} \neq 0$ on a configuration claimed to be source-native indicates an error in the construction, not a physical admission cost.

---

16. Linearized non-Abelian extension and the Standard Model carrier

The whole tangent-level construction is a statement about incidence operators, so at first order around a flat connection it tensors with any internal Lie algebra. The grading of this section is deliberately narrow: it is an exact linearized flat-background extension, not a proof of the full nonlinear non-Abelian result.

Let $\mathfrak{g}$ be a compact Lie algebra of dimension D and replace each incidence operator by

$$d_k \to d_k \otimes I_{\mathfrak{g}}.$$

Because the Laplacian identity of Lemma 9.1 is an identity of incidence operators, it survives the tensor product unchanged:

$$(d_1 d_1^T + d_2^T d_2) \otimes I_{\mathfrak{g}} = 5\, I_{10} \otimes I_{\mathfrak{g}}.$$

Theorem 15: Linearized flat-background gauge extension

[EXACT AT FIRST ORDER AROUND A FLAT CONNECTION; NOT A NONLINEAR NON-ABELIAN RESULT]

For internal algebra $\mathfrak{g}$ of dimension D, the linearised direct-face tangent on one 4-simplex is $\mathbb{R}^{10} \otimes \mathfrak{g}$ of dimension 10D, and it decomposes orthogonally as

$$10D_{\text{face}} = 6D_{\text{physical}} \oplus 4D_{\text{obstruction}},$$

with projectors

$$P_{\mathfrak{g}} = P_{\text{int}} \otimes I_D, \qquad Q_{\mathfrak{g}} = Q_{\text{cl}} \otimes I_D,$$

canonical attaching map

$$A\_can^{\mathfrak{g}} = (1/5)\ (d_1^T \otimes I\_D)\ \Theta,$$

and unchanged normalization $\|(d_1 \otimes I\_D) A\|^2 = 5\|A\|^2$ on the gauge-orthogonal slice. All singular values remain √5, so isotropy is preserved and no internal direction is privileged.

For the Standard Model algebra

$$\mathfrak{g}\_SM = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1), \quad D = 8 + 3 + 1 = 12,$$

this gives

$$120\_face = 72\_physical + 48\_obstruction,$$

since 10 × 12 = 120, 6 × 12 = 72 and 4 × 12 = 48, and

$$A\_SM^{can} = (1/5)\ (d_1^T \otimes I_{12})\ \Theta.$$

Grade and limits. The extension is exact for the linearised theory around a flat background, where the non-Abelian coboundary reduces to the Abelian one tensored with the adjoint index. It does not establish the nonlinear statement: at finite field strength the non-Abelian face holonomies do not commute, path-ordering matters, and the realizability condition is not simply the vanishing of a linear syndrome. Nothing in this section should be read as a non-Abelian realizability theorem. What it does establish is that the carrier census, the projectors, the normalization and the execution protocol carry over to the Standard Model gauge algebra without modification at the grain where they are usually applied.

Sector split. Because $P\_\mathfrak{g}$ and $Q\_\mathfrak{g}$ act as the identity on the internal index, the split never mixes colour, weak and hypercharge directions. The obstruction sector of the Standard Model carrier is 48-dimensional and decomposes as 4 × (8 + 3 + 1), so each internal generator carries its own copy of the same four-dimensional obstruction geometry, with identical isotropy and identical readout normalization.

Current-source occupation and later nonlinear completion. The 120 = 72 + 48 result is an exact first-order statement about the mathematical direct-face carrier, and by Theorem 12 the source-native physical tangent has zero rank into its 48-dimensional obstruction sector,

$$\mathrm{rank}\ Q\_\mathfrak{g}\ D\Theta\_source = 0.$$

Separately, the physical non-Abelian connection is not limited to this linearized statement. A complete G_faith-valued loop assignment on K_5 reconstructs a connection up to gauge nonlinearly (Theorem 13A), and later PFDMI work supplies the full faithful matrix-link source and returns all 72 Wilson-quotiented physical gauge directions in native marked probabilities. The limitation of this section is therefore specifically the nonlinear independent-face integrability problem, not the existence or reconstruction of the physical non-Abelian connection itself. The two must not be conflated: "not a nonlinear non-Abelian face theorem" does not mean "the non-Abelian physical connection is still missing."

---

17. Off-shell invariant covariance as a diagnostic

The S_5 representation result of §10 remains useful, but its role is diagnostic rather than a description of a physical source law.

Theorem 16: Invariant second-order covariance

[EXACT, CONDITIONAL ONLY ON S_5 INVARIANCE OF THE ENSEMBLE]

For an arbitrary mathematical ensemble on the enlarged direct-face carrier with finite second moments and invariance under vertex relabelling, the most general covariance is

$$\Sigma = a\ P\_int + b\ Q\_cl, \qquad a,\ b \geq 0,$$

and the induced tetrahedral syndrome covariance is exactly

$$B\ \Sigma\ B^T = b\ (5\ I_5 - s\ s^T).$$

Certificate. Verified to machine zero for arbitrary a, b (Appendix C.4).

What this does and does not say. It is an exact statement about second moments on the off-shell carrier. It does not imply that an arbitrary S_5-invariant probability law is determined by one number, because higher-order invariant cumulants may remain unconstrained. The correct statement is narrower: the S_5-invariant second-order obstruction covariance is controlled by one scalar b.

Consequences on the mathematical carrier.

1. Under simplex symmetry the four obstruction directions cannot carry unequal second-order weight. Any claim of differential occupation among them is a claim that the ensemble breaks S_5.
2. The syndrome covariance has a fixed shape, proportional to $5I_5 - s\ s^T$ with no free direction. Any returned syndrome covariance of another shape falsifies either S_5 invariance or the identification of the syndrome with B applied to a direct-face perturbation.
3. The parameter a is invisible to the tetrahedral readout, since B P_int = 0. Closure statistics constrain b only.

17.1 The current physical value

For the current physical VERSF source,

```
b_current = 0,
```

because by Theorem 12 the source measure is supported on $V_{int} = \ker d_2$. Hence

```
Σ_current = P_int Σ_current P_int,
```

and there is no obstruction covariance in the first place.

Corollary 17.1: The obstruction weight decouples in any case

[EXACT]

Even for a hypothetical ensemble with $b \neq 0$, projection onto the physical gauge carrier gives

```
P_int Σ P_int = a P_int,
```

since $P_{int} Q_{cl} = 0$. The obstruction weight does not appear. In the tensored Standard Model form of §16 the same holds with $P_\mathfrak{g}$ in place of P_{int}, so b is absent from the 72-dimensional gauge covariance as well.

Reading. Two separate facts point the same way. On the current source stack there is no obstruction weight at all. And even if a future carrier supplied one, it could not reach the physical gauge covariance by projection at second order. An effect on a Standard Model coupling would require a genuinely dynamical route: reweighting by an admission event on an independently derived pre-holonomy carrier, non-Gaussian structure about which the second-moment argument says nothing, scale-dependent blocking of the kind isolated in §19 and quantified in §22, or explicit S_5 breaking, which Theorem 16 shows is required for any differential weighting among the four directions.

The $b \neq 0$ family is therefore retained as an off-shell diagnostic and as the covariance structure that a future independently derived pre-holonomy source would have to satisfy if it were S_5-invariant.

18. Physical status of the Route-M obstruction sector

This section states the settled position on physical occupation, and records what happens to the machinery that was previously needed to keep the question open.

18.1 Status of the four-dimensional carrier

The mathematical carrier $U(1)^{10}$_direct admits configurations away from ker δ_2. The current physical source does not produce them: every source-native holonomy configuration comes from one composition-respecting transport assignment and therefore satisfies $\delta_2 H = 1$ (Theorem 12).

Property of the four-dimensional obstruction torus	Status
Mathematical off-shell carrier	YES
Complete realizability diagnostic	YES
Minimal smooth completion of the physical carrier	YES
Current source-native physical tangent sector	NO
Current physical probability distribution	NO
Current independent admission information	NO

18.2 What this retires from the physical chain

The finite-resolution operational construction is not required for the present physical source. There is no need to choose a finite obstruction alphabet, a closure tolerance, a Born state on the obstruction carrier, or a source weight b, because there is no current physical pre-integrability state on which those objects would act.

Three previously load-bearing results therefore change role rather than being discarded, and are collected in Appendix F:

- the no-finite-quotient theorem remains mathematically correct and is retained as a firewall against future misuse of a finite obstruction alphabet, but it is no longer a step in the current physical Route-M chain;
- the Born non-identifiability result remains a valid counterfactual: if a future independently derived pre-holonomy operational carrier contained both integrable and non-integrable states, the Born functional alone would not select their relative occupation. That counterfactual does not describe the presently sourced gauge carrier;
- the conditional operational-quotient construction and the obstruction-capacity argument survive as the architecture such a future carrier would have to realize, not as descriptions of the current source.

18.3 Information

With I_obs^current = 0 by Corollary 12.1, the inherited information identity

```
I_RM = 70 ln 2 + Δ_T + I_int^op
```

carries no independent third term on the current branch. Whatever departure from naive seventy-bit factorisation exists is carried by Δ_T, on the admitted connection carrier, and not by a separate four-direction closure-admission channel. The typing statement that any such term, if it existed, would live on the obstruction carrier rather than on reused K7 bits is retained in Appendix F.

18.4 Post-source closure probability

On the source-native branch the tetrahedral closure test is satisfied identically, so its probability is 1 and its information content is 0. This is now a direct consequence of Theorem 12 rather than a statement about what happens after a gate is imposed. There is no pre-gate physical stage from which to fall.

---

19. Projective consistency and the running firewall

Let adjacent operational resolutions be related by $X_{n+1} \xrightarrow{\rho_{n+1,n}} X_n$ with projectively consistent measures

$$(\rho_{n+1,n})_* \mu_{n+1} = \mu_n.$$

For any event $E_n \subseteq X_n$, $\mu_{n+1}(\rho_{n+1,n}^{-1} E_n) = \mu_n(E_n)$.

Theorem 17: Cylinder-event invariance theorem

[EXACT]

The probability of one unchanged operational event cannot run merely because it is represented at a finer regulator by pullback. A nonzero Route-M beta function therefore requires a genuinely scale-dependent blocked coherence observable,

$$C_{n+1} \neq \rho_{n+1,n}^{-1}(C_n)$$

up to null sets, with exact difference

$$p_{n+1} - p_n = \int_{X_{n+1}} [1_{C_{n+1}} - 1_{\rho_{n+1,n}^{-1} C_n}] \, d\mu_{n+1}.$$

This identifies the beta function with observable deformation rather than with a change in the probability of an unchanged event.

---

20. Exact correlated Route-M block recursion

Route M defines a block as coherent if at least one of its fine cells is coherent. Let $C_1, \ldots, C_m \in \{0, 1\}$ be the fine coherence indicators, so that

$$C_B = 1 - \prod_{j=1}^m (1 - C_j),$$

and taking expectations,

$$p_B = 1 - P(C_1 = 0, \ldots, C_m = 0).$$

With $p_j = P(C_j = 1)$, define the exact joint-failure correlation defect

$$\Delta_m = \ln \left[P(C_1 = 0, \ldots, C_m = 0) / \prod_{j=1}^m (1 - p_j) \right].$$

Then identically

$$p_B = 1 - e^{\Delta_m} \prod_{j=1}^m (1 - p_j),$$

and for equal marginals $p_j = p$,

$$p_B = 1 - (1 - p)^m e^{\Delta_m}.$$

Theorem 18: Correlation-complete block theorem

[EXACT]

Route M's mean-field recursion $p_B = 1 - (1 - p)^m$ is exactly the conditional case $\Delta_m = 0$. Exchangeability or cell symmetry does not imply $\Delta_m = 0$; the physical joint law must determine it. The defect is not sign-constrained: positively dependent cells give $\Delta_m > 0$ and negatively dependent cells give $\Delta_m < 0$.

---

21. Model-free bounds on the block probability and the beta gate

The correlation defect is unknown, but it is not unconstrained. Fréchet bounds on the joint failure event give a two-sided interval that requires only the marginals.

Theorem 19: Fréchet bound and sign theorem for the direct gate

[EXACT]

Let $p_1, \ldots, p_m$ be the fine coherence marginals at regulator n and let p_B be the blocked probability at regulator n + 1. Then, with no assumption whatever on the joint law,

$$\max_j p_j \leq p_B \leq \min(1, \Sigma_j p_j),$$

equivalently

$$\ln [\max(0, 1 - \Sigma_j p_j) / \prod_j (1 - p_j)] \leq \Delta_m \leq \ln [\min_j (1 - p_j) / \prod_j (1 - p_j)].$$

Consequently:

(i) **Sign.** The numerator of the direct beta gate obeys

$$p_{n+1}^{-1} - p_n^{-1} \leq 0$$

whenever $p_n \leq \max_j p_{n,j}$, in particular for equal fine marginals $p_{n,j} = p_n$. OR-blocking cannot decrease the coherence probability, so the sign of $b^{phys}_{n\to n+1}$ is fixed entirely by the direction of the regulator step and never by the unknown correlation defect.

(ii) **Bound.** For $L = \ln(k_{n+1}/k_n) > 0$ and equal fine marginals,

$$(1/(2L)) [1/\min(1, m p_n) - 1/p_n] \leq b^{phys}_{n\to n+1} \leq 0,$$

with the lower bound attained in the maximally anti-dependent limit and the upper bound attained in the comonotone limit $p_B = p_n$.

Certificate. Bounds and the exact recursion checked against randomly drawn joint laws on $m \in \{2, \ldots, 5\}$ binary cells; all trials satisfy the interval and reproduce p_B from Δ_m to machine precision.

Reading. R2 of §24 is therefore a bounded return, not an unbounded one. Even before the joint law is known, the direct gate is confined to a computable interval determined by the marginals and the multiplicity, and its sign is not in question. Any proposed Route-M running coefficient falling outside that interval is falsified by the marginals alone.

---

22. Direct physical beta gate

Define the physical coherence coupling at regulator n by $g_n^2 = p_n$. For adjacent physical regulator scales k_n, k_{n+1}, the direct finite-step beta estimator is

$$b^{\text{phys}}_{n\to n+1} = (1/2) \cdot (p_{n+1}^{-1} - p_n^{-1}) / \ln(k_{n+1}/k_n).$$

Using the exact block recursion,

$$b^{\text{phys}}_{n\to n+1} = 1 / (2 \ln(k_{n+1}/k_n)) \cdot [\ 1 / (1 - e^{\Delta_n} \prod_j (1 - p_{n,j})) - 1/p_n\ ],$$

and for equal fine marginals,

$$b^{\text{phys}}_{n\to n+1} = 1 / (2 \ln(k_{n+1}/k_n)) \cdot [\ 1 / (1 - (1 - p_n)^{m_n} e^{\Delta_n}) - 1/p_n\ ].$$

Theorem 20: Direct physical running theorem

[EXACT CONDITIONAL ON THE BLOCK EVENT AND STATE BEING PHYSICALLY TYPED]

The Route-M physical running is completely determined by the source-selected fine coherence marginals, the same-carrier block multiplicity, the joint-failure correlation defect, and the physical regulator ratio. No insertion of 14, 16 or 14/16 is required in the direct estimator, and by Theorem 19 the result is confined to a marginal-determined interval before the defect is known.

---

23. The status of 14/16

The historical Route-M coefficient is $b_{\text{count}} = 14/16 = 7/8$. This paper does not call that arithmetic wrong; it fixes its grade.

The numerator fourteen is now understood exactly as

$$14 = 6_{\text{physical face}} + 4_{\text{off-shell obstruction}} + 4_{\text{obstruction readout}},$$

with the two four-dimensional pieces canonically paired by $B^+ = (1/5)B^T$ (Theorem 9) and not independent physical field carriers. The denominator sixteen remains a coarse-block prescription unless the same physical carrier and primitive ordering/refinement law derive it at the UV grain.

Theorem 21: Route-M counting/beta separation theorem

[EXACT TYPING RESULT]

14/16 is a valid structural counting ratio in the declared Route-M blocking prescription. It is not, by that fact alone, the physical VERSF beta coefficient. A physical identification b_RM^phys = 7/8 would have required an independent same-carrier return from the direct probability-running law of §22, inside the Fréchet interval of Theorem 19. §24 shows that the candidate events supplied by the current source stack are regulator invariant, so that route is exhausted and any comparison must now be made against the one-loop determinant coefficients.

---

24. Probability-running closure and the quantum gauge-running handoff

The general probability-running identities of §§19 to 22 are correct. The question this section settles is whether they are the mechanism by which Standard Model couplings run on the current source stack. They are not, and the principal candidates have now been executed rather than assumed.

24.1 The retained-record candidate is regulator invariant

The Fold-to-Fact and Ordered-Record execution returns a persistent ledger alphabet

```
(e, r) ∈ {1, …, 12} × {1, …, 7},
```

giving 84 retained record classes. At the microscopic 319-mark grain the seven self-history marks produce no new ledger entry, while the 24 nonself history marks each carry thirteen current subchannels,

```
7 + 24 × 13 = 319,
```

with 312 record-producing microscopic marks. The no-record map has spectral radius

```
r(N_R) = 0.200305886206958 < 1,
```

so first entry into the ledger occurs almost surely under the declared renewal architecture.

The frozen D36 history kernel has

```
D(α) = 2 + 2 cosh α,
```

with stationary hub probability

```
π_h = 7 / (7 + 6D),
```

and at the archived root π_h = 0.189423730905787. A new ledger entry occurs precisely on a nonself history transition, so the stationary one-opportunity retained-record probability is

$$p\_rec = 1 - \pi\_h = 6D / (7 + 6D) = 0.810576269094213.$$

The full recurrent source factorises as

$$P\_n = P\_H \otimes P\_N\hat{}n,$$

where P_H is the frozen D36 history factor and the regulator dependence lies entirely in the neutral continuation P_N^n. Because the retained-record event depends only on the self versus nonself history transition and sums over the complete neutral outcomes,

$$p\_rec^3 = p\_rec^4 = p\_rec^5 = p\_rec^6,$$

and therefore b_rec^phys = 0.

Theorem 22: Retained-record no-running theorem

[EXACT FOR THE EXECUTED FACTORISED SOURCE ARCHITECTURE]

The implemented retained-record probability is a well-defined physical probability, but it is regulator invariant. It cannot supply nonzero Standard Model gauge running.

The record architecture and its 7/312 split are executed results rather than an assumed partition, and the regulator-dependent neutral factor and the factorisation are independently explicit in the full-carrier provenance theorem.

24.2 Fixed current-terminal events cannot run either

Let M_α denote a mark of the fixed history/current terminal instrument and N_ν^n a regulator-dependent neutral mark, so that on the factorised branch

$$M\_\alpha\nu\hat{}n = M\_\alpha \otimes N\_\nu\hat{}n.$$

Let E be any event defined solely by the fixed current-terminal label α. Then

$$p\_n(E) = \Sigma\_\{\alpha \in E\}\ \Sigma\_\nu\ \mathrm{Tr}[\ \rho\_n\ M\_\alpha\dagger\ M\_\alpha\ N\_\nu\hat{}(n)\dagger\ N\_\nu\hat{}n\ ].$$

Completeness of the neutral instrument,

$$\Sigma\_\nu\ N\_\nu\hat{}(n)\dagger\ N\_\nu\hat{}n = I,$$

removes the entire regulator-dependent factor, giving

$$p\_n(E) = \Sigma\_\{\alpha \in E\}\ \mathrm{Tr}[\ \rho\_current\ M\_\alpha\dagger\ M\_\alpha\ ],$$

hence $p_3(E) = p_4(E) = p_5(E) = p_6(E)$.

Theorem 23: Current-terminal cylinder no-running theorem

[EXACT FOR THE EXECUTED FACTORISED SOURCE ARCHITECTURE]

Every fixed event measurable solely on the EX2E current-terminal algebra has zero direct beta coefficient.

This holds although the EX2E probabilities are genuinely gauge-sensitive. The native accepted-current hazard is nontrivial and the conditional terminal law has physical Jacobian rank 72, yet the same physical ranks are returned at regulator levels 3, 4, 5 and 6 with stored Gram drift exactly zero. Therefore

```
gauge-sensitive ⇏ regulator-running,
```

which is a sharper statement than the cylinder-invariance firewall of §19: here the event genuinely resolves the gauge directions and still does not run.

24.3 Status of the direct Route-M probability beta theorem

The exact identities of §§19 to 22 remain mathematically correct. In particular, for any independently derived blocked event,

```
b^phys_{n→n+1} = (1/2) · (p_{n+1}⁻¹ − p_n⁻¹) / ln(k_{n+1}/k_n)
```

remains an exact finite-step probability-running identity, with the Fréchet bounds of Theorem 19 still constraining it.

What changes is its physical role. The principal candidate event families supplied by the present source stack have been executed and found regulator invariant, so the direct Bernoulli route is no longer the outstanding Standard Model gauge-running programme:

```
CURRENT ROUTE-M BERNOULLI-PROBABILITY SEARCH: EXHAUSTED.
```

A future independently derived source could still provide a genuinely deformed blocked event not equivalent to any event retired here. No such event is currently returned, and none should be invented for the purpose of reproducing a target coefficient.

24.4 One-loop running on the admitted gauge branch

For background fields φ let H_gf = δ²S_gf / δη δη at φ. The one-loop Euclidean effective action is

```
Γ_E^(1) = (1/2) STr log H_gf,
```

and for a Laplace-type operator $\Delta = -(D^2 + E)$ the logarithmic F^2 term gives the universal contributions

$$b_{gauge+ghost} = -(11/3)\, C_2(G),$$
$$b_{Weyl} = (2/3)\, T(R),$$
$$b_{complex\ scalar} = (1/3)\, T(R).$$

Applying the frozen Standard Model census,

$$b_3 = -(11/3)(3) + (2/3)(6) = -7,$$
$$b_2 = -(11/3)(2) + (2/3)(6) + (1/3)(1/2) = -19/6,$$
$$b_Y = (2/3)(10) + (1/3)(1/2) = 41/6,$$

so

$$(b_Y, b_2, b_3) = (41/6, -19/6, -7).$$

Theorem 24: Standard Model one-loop determinant census

[CONDITIONAL EXACT AT THE PRESENT REGULATOR GRADE]

The one-loop coefficients are fixed by the inherited Standard Model field content and the background-field fluctuation architecture. Their remaining VERSF debt is regulator provenance, not numerical freedom.

24.5 Refinement to the Laplace principal symbol

Let h_n be the physical refinement scale and $U_\mu(x)$ the faithful background link. Define the adjoint covariant second difference

$$(\Delta_U^n X)(x) = (1/h_n^2)\, \Sigma_\mu\, [\, 2X(x) - \mathrm{Ad}_{U_\mu(x)}\, X(x + h_n e_\mu) - \mathrm{Ad}_{U_\mu(x - h_n e_\mu)^{-1}}\, X(x - h_n e_\mu)\,].$$

At flat background its symbol is

$$\lambda_n(k) = (4/h_n^2)\, \Sigma_\mu \sin^2(h_n k_\mu / 2) = |k|^2 - (h_n^2/12)\, \Sigma_\mu k_\mu^4 + O(h_n^4 k^6),$$

so $\Delta_U^n \longrightarrow -D^2$. The h_n^{-2} normalization is physically forced by plaquette area scaling,

$$U_{\mu\nu} = \exp[\, i\, h_n^2\, F_{\mu\nu} + O(h_n^3)\,],$$

rather than inserted to manufacture a continuum Laplacian. The unrescaled Stage-IX transport adjacency is not itself the quantum regulator: its vacuum-subtracted quadratic symbol vanishes as $O(h_n^2 k^2)$. The required object is the h_n^{-2}-normalised covariant second-difference/Hodge operator.

24.6 Gauge and ghost Hessian package

The finite plaquette functional reaches the continuum Yang-Mills action. Expanding about A_μ and applying background Feynman gauge gives

$$H^{gauge}_{\mu\nu} = Z_A [-D^2 \delta_{\mu\nu} - 2\, \mathrm{ad}(F_{\mu\nu})], \qquad M_{FP} = -D^2,$$

so the bosonic principal-symbol side required by the one-loop heat-kernel calculation is structurally closed.

24.7 Fermion spectral universality

The Master Substrate Action uses the same physical faithful link in the fermion and gauge sectors,

$$\Gamma_{CAR} = \Sigma_f\, \bar{\Psi}_f\, D_f[U]\, \Psi_f,$$

but the exact finite chiral kernel $D_f[U]$ is not yet explicitly source-returned. That is the remaining D-SG6 regulator debt.

A naive symmetric finite Dirac derivative is inadmissible because it produces additional light zeros. VERSF nevertheless contains a source-motivated doubler-lifting branch. The closure-graph Wilson stiffness gives

$$H_W = (r/2)\, \hbar c\, h_n\, (D_G^{phys})^2 \otimes \beta, \qquad r > 0,$$

and for the massless spatial branch the dispersion can be written

$$E_n^2(q) = (\hbar c/h_n)^2 \left[\Sigma_{i=1}^{3} \sin^2 q_i + r^2 \left(\Sigma_{i=1}^{3} (1 - \cos q_i) \right)^2 \right].$$

Its only zero is $q = 0$, and the lightest unwanted spatial mode has gap

$$E_{gap} = 2r\, \hbar c / h_n.$$

The Wilson-term form is derived from closure roughness in the earlier VERSF paper, although the numerical value of r remains tied to an unexecuted entropy calibration.

Time is not a fourth primitive lattice direction in this architecture. Under positive transfer $T_{a_t} = e^{-a_t H}$, the sequential Euclidean temporal residual for energy E and frequency ω is

$$D_t(\omega, E) = [1 - e^{-a_t E} e^{i \omega a_t}] / a_t,$$

whose modulus satisfies

$$a_t^2 |D_t|^2 = (1 - e^{-a_t E})^2 + 4 e^{-a_t E} \sin^2(\omega a_t / 2),$$

so $D_t(\omega, E) = 0$ if and only if $(E, \omega) = (0, 0)$. There is no temporal doubler, and at the temporal zone edge

$$| D_t(\pi/a_t, E) | = (1 + e^{-a_t E}) / a_t \geq 1/a_t.$$

The combined Hamiltonian-transfer branch therefore has one and only one light (3+1)-dimensional pole per admitted fermionic carrier and representation branch, conditional on $r > 0$. The statement concerns removal of regulator-generated copies, not the number of distinct physical fermion representations in the frozen matter census.

Theorem 25: Hamiltonian-transfer no-extra-pole theorem

[CONDITIONAL EXACT ON THE CLOSURE-WILSON AND POSITIVE-TRANSFER BRANCH]

The spatial Wilson stiffness removes the seven unwanted spatial lattice zeros, and sequential positive-transfer time introduces no additional temporal zero. The candidate branch therefore satisfies the no-extra-light-species condition required for one-loop universality: each admitted physical fermionic carrier retains its intended low-energy pole while no additional lattice-generated pole survives.

The positive-transfer construction is explicitly part of the later global quantum architecture, and time is separately typed in the source programme as sequential commitment rather than primitive geometric adjacency.

24.7A Chiral-projector trace and the Weyl coefficient

The no-extra-pole result of §24.7 settles the species-count side of the candidate Hamiltonian-transfer branch. A logically distinct question remains: given one admitted low-energy spinorial carrier, does the gauge-running trace count a four-component Dirac carrier or the correct chiral Weyl representation attached by the VERSF matter ledger? That must be answered representation by representation.

The inherited matter architecture contains both chiralities. It does not assert $P_R \Psi = 0$. The charged right-chiral carriers e_R, u_R, d_R are physical. What the electroweak representation stack establishes, at its declared grade, is that the nontrivial weak branch is supported only on the left orientation,

$$P_W = P_L,$$

while the corresponding right-chiral charged carriers are weak singlets. A "mirror" relevant to the weak-running problem therefore means an additional right-chiral weak doublet, not the ordinary right-handed singlet carriers required by the Standard Model census.

Parity-even heat-kernel trace. Let the continuum-limit gauge-covariant Dirac operator on representation R be

$$D\!\!\!/ = \gamma^\mu \nabla_\mu, \quad \nabla_\mu = \partial_\mu + A_\mu, \quad \Omega_{\mu\nu} = [\nabla_\mu, \nabla_\nu].$$

Its positive Laplace-type square has the standard form $D\!\!\!/^\dagger D\!\!\!/ = -(\nabla^2 + E)$, with E linear in $\gamma^{\mu\nu}\Omega_{\mu\nu}$. In flat four-dimensional background the gauge-curvature part of the logarithmic heat-kernel coefficient is controlled by spin traces of

$$(1/2)\, E^2 + (1/12)\, \Omega_{\mu\nu} \Omega^{\mu\nu}.$$

Insert one chiral projector $P_\chi = (1 \pm \gamma_5)/2$. For the spin-trivial connection-curvature term $\Omega_{\mu\nu} \Omega^{\mu\nu}$,

$$\mathrm{tr}_{spin}\, P_\chi = 2 = (1/2)\, \mathrm{tr}_{spin}\, I_4,$$

and for the E^2 term,

$$\mathrm{tr}_{spin}(P_\chi E^2) = (1/2)\, \mathrm{tr}_{spin}(E^2) \pm (1/2)\, \mathrm{tr}_{spin}(\gamma_5 E^2).$$

The second term is proportional to the parity-odd contraction $F\tilde{F}$. It therefore belongs to the chiral/anomaly channel and does not contribute to the parity-even $F_{\mu\nu} F^{\mu\nu}$ coefficient defining the gauge beta function, so

$$\mathrm{tr}_{spin}(P_\chi E^2)|_{F^2} = (1/2)\, \mathrm{tr}_{spin}(E^2)|_{F^2}.$$

Both contributions therefore carry the same half-factor, hence

$$a_{4,\chi}|_{F^2} = (1/2)\, a_{4,\mathrm{Dirac}}|_{F^2}.$$

Since the Dirac determinant gives $b_{Dirac}(R) = (4/3)T(R)$, one chiral carrier gives

$$b_{Weyl}(R) = (1/2)\, b_{Dirac}(R) = (2/3)\, T(R).$$

The result is identical for left- and right-chiral Weyl carriers in the parity-even beta coefficient. Their difference lies in their representation assignments and in the parity-odd chiral structure, not in the magnitude of the F^2 screening coefficient.

Theorem 25A: VERSF chiral-projector one-loop coefficient theorem

[CONDITIONAL EXACT ON THE INHERITED CHIRAL-CARRIER PROJECTORS AND THE LAPLACE-TYPE CONTINUUM LIMIT]

For a VERSF fermionic carrier whose continuum pole is projected onto one Weyl orientation P_χ, the parity-even logarithmic gauge-curvature trace is exactly one half of the corresponding Dirac trace, so

$$b_{Weyl}(R) = (2/3)\, T(R).$$

The factor 1/2 is an output of the chiral projector trace and need not be inserted as an independent counting prescription.

This theorem does not claim that the closure-Wilson Dirac operator is already a complete finite chiral gauge regulator. It establishes the continuum and projector coefficient. Exact finite-lattice chirality, a Ginsparg-Wilson relation, determinant-line structure, reflection positivity and the full nonperturbative chiral construction remain outside it and are retained under D-SG6/D-SG8.

24.7B Application to the frozen VERSF matter census

For one generation the inherited chiral gauge ledger is

```
Q_L : (3, 2, 1/6),    u_R : (3, 1, 2/3),    d_R : (3, 1, -1/3),
L_L : (1, 2, -1/2),   e_R : (1, 1, -1),
```

with the neutral extension N_R : (1, 1, 0), when present, contributing zero to all three gauge-running sums.

For SU(3), one generation contains 2T(3) + T(3) + T(3) = 2, so three generations give Σ_Weyl $T_3(R) = 6$ and

```
b₃^ferm = (2/3)(6) = 4.
```

For SU(2), the coloured quark doublet contributes 3T(2) = 3/2 and the lepton doublet contributes T(2) = 1/2, giving two per generation, hence Σ_Weyl $T_2(R) = 6$ and

```
b₂^ferm = (2/3)(6) = 4.
```

For hypercharge, summing Y^2 over the Weyl states of one generation,

```
6(1/6)² + 3(2/3)² + 3(1/3)² + 2(1/2)² + 1(1)² = 1/6 + 4/3 + 1/3 + 1/2 + 1 =
10/3,
```

so Σ_Weyl $Y^2 = 10$ over three generations and

```
b_Y^ferm = (2/3)(10) = 20/3.
```

Adding the inherited gauge-plus-ghost and single-complex-Higgs contributions,

```
b₃ = −11 + 4 = −7,
b₂ = −22/3 + 4 + 1/6 = −19/6,
b_Y = 20/3 + 1/6 = 41/6,
```

hence again $(b_Y, b_2, b_3) = (41/6, -19/6, -7)$.

The significance of this repetition is provenance rather than arithmetic. The fermionic part is now obtained through the chain

$$\text{VERSF chiral carrier} \rightarrow P_\chi \rightarrow (1/2)(\text{Dirac } F^2 \text{ trace}) \rightarrow (2/3)T(R) \rightarrow \text{frozen VERSF census},$$

rather than by importing the Weyl half-factor as an unexplained continuum multiplicity.

Mirror-sector counterfactual. The ordinary P_R singlets are required and are already included above. A complete additional mirror copy coupled to the same gauge fields would add a second full set of Weyl contributions, and holding the single Higgs contribution fixed the coefficients would become

$$(b_Y, b_2, b_3)_{\text{coupled mirror}} = (27/2, 5/6, -3)$$

rather than the frozen-census result. This is a counterfactual running audit, not a proof that no decoupled mirror world can exist: a completely gauge-decoupled sector contributes nothing to the present gauge determinant by definition. The narrower statement relevant here is that no additional coupled mirror weak-doublet family is present in the frozen VERSF gauge-facing census used by the determinant.

24.8 Minimal beta-universality certificate

The chiral-projector calculation above closes the continuum half-factor. The remaining finite-regulator question is whether the actual shared-link kernel appearing in the frozen Master Substrate Action realizes that same chiral universality class. It is sufficient to establish:

1. **Gauge-covariant locality.** $D_f^n[U]$ is local on the finite VERSF carrier and transforms covariantly under the faithful gauge link.
2. **Correct normalized chiral continuum pole.** On each frozen Weyl representation the corresponding two-component block satisfies
3. $D_{\{f,L\}}^n(p) = i \bar{\sigma}^\mu p_\mu + O(h_n p^2)$

   for a left-handed carrier, or

   $$D_{\{f,R\}}^n(p) = i \sigma^\mu p_\mu + O(h_n p^2)$$

   for a right-handed carrier, with its gauge representation and chiral projector agreeing with the independently frozen matter ledger. This condition concerns the physical Weyl pole of each representation and does not identify the left- and right-handed gauge representations with one another.

4. **No additional light species outside the frozen census.** Equivalently, after the intended physical poles are removed from the test region, $\inf_{\{p \notin B_\varepsilon\}} h_n \sigma_{\min}(D_f^n(p)) > 0$. This condition excludes lattice doublers and additional coupled mirror carriers. It does not remove the physical right-handed singlets already present in the frozen census.
5. **Local regulator-stable UV remainder.** High-momentum deviations from the continuum chiral operator produce finite matching terms and higher-derivative corrections rather than an additional infrared logarithm.

Theorem 26: One-loop fermion universality theorem

[CONDITIONAL EXACT ON THE FOUR CONDITIONS ABOVE, WITH THE WEYL HALF-FACTOR FIXED BY THEOREM 25A]

Under these conditions the coefficient of the parity-even fermionic $F^2 \log \mu$ term is regulator universal and equals

```
b_Weyl(R) = (2/3) T(R).
```

Theorem 25A fixes why one Weyl carrier contributes one half of the Dirac coefficient. The present theorem has a different role: it states when the actual finite VERSF kernel is guaranteed to reproduce that already-fixed coefficient. A different admissible finite stencil may change finite matching constants and higher-order cutoff terms, but cannot change the one-loop logarithmic coefficient.

The one-loop fermion problem therefore separates into two questions:

```
coefficient and chiral counting:        CLOSED at the continuum/projector
grade,
actual D_f[U] finite-regulator provenance:  OPEN.
```

24.9 Remaining returns

The earlier Route-M Bernoulli programme is retired on the present source stack. The remaining quantum-running returns are these.

R1Q, shared-link fermion universality. Show that the D_f[U] actually appearing in the Master Substrate Action satisfies the locality, faithful-link gauge covariance, projected Weyl continuum-pole, no-additional-light-species and UV-locality conditions of Theorem 26, or derive an equivalent admissible chiral-regulator universality match. The factor-of-two Weyl coefficient is no longer part of this debt: Theorem 25A has already fixed b_Weyl(R) = (2/3)T(R) from the chiral projector trace, so R1Q asks only whether the actual finite VERSF operator belongs to that universality class. This is the load-bearing bridge.

R2Q, same-action finite fluctuation package. Show that the finite gauge, ghost, scalar and fermion Hessians arise from the same frozen VERSF action and regulator family and converge to

```
H_gauge = −D² δ − 2 ad F,    M_FP = −D²,    H_Weyl = −D² + (1/2) σ^{μν} F_μν,
```

with the corresponding scalar operator.

R3Q, full nonperturbative chiral completion. The stronger determinant-line, finite-index, faithful-bundle, global-anomaly and reflection-positivity problem remains D-SG6/D-SG8 territory. It is required for complete nonperturbative Standard Model quantisation but is logically stronger than the one-loop beta-universality certificate.

25. What this closes in the Standard Model programme

It reconciles the face and link descriptions, and then decides between them. The exact sequence

$$1 \to U(1)^{6}_{\mathrm{phys}} \to U(1)^{10}_{\mathrm{direct}} \to U(1)^{4}_{\mathrm{obs}} \to 1$$

describes the relationship between the physical connection image and the smallest mathematical carrier capable of representing every failure of integrability. The current-source result adds

$$\text{current physical VERSF source} \subset U(1)^{6}_{\mathrm{phys}},$$

so the direct-face carrier is not an additional physical stage through which the present source must dynamically travel.

It changes the attaching map from a repair mechanism into a certificate. For an arbitrary mathematical candidate,

$$\theta \longmapsto (A_{\mathrm{can}}, r)$$

is a complete equivariant coordinate encoding. For a current source-native physical state,

$$\theta_{\mathrm{source}} \longmapsto (A_{\mathrm{phys}}, 0).$$

The interface therefore tests source consistency and fixes the face/edge normalization, rather than describing a physical process that discards inconsistent pieces.

It closes a potential source of extra gauge physics. The four obstruction directions are not four extra physical fields, not four hidden physical modes and not four additional statistical channels of the current source. At the linearized Standard Model grade,

$$120 = 72 + 48$$

remains the exact off-shell decomposition of the mathematical test carrier, but the current physical link source has zero tangent rank into the 48-dimensional obstruction sector,

$$\operatorname{rank} Q_{\mathrm{SM}}\, D\Theta_{\mathrm{source}} = 0,$$

so the physical Standard Model gauge tangent is the 72-dimensional integrable part.

It gives the later gauge-response calculation an exact normalized input. The canonical identities

$$A\_can = (1/5)\, d_1^T\, \theta, \qquad \|d_1 A\|^2 = 5\|A\|^2$$

remain directly useful to the MASD, gauge-response and Hessian chain. On a genuine source-native connection they reproduce the physical edge tangent with the correct normalization and no fitted constant.

It fixes the Standard Model handoff. The physical chain is

```
primitive/history source → physical edge/link transport → Bianchi-compatible
face curvature ⟶RMOP (A_phys, 0) → MASD Schur response → (Z₃, Z₂, Z_q).
```

The separate mathematical diagnostic branch is

```
θ_arbitrary ⟼ (A_can, r),
```

which is not asserted to be physically populated.

It distinguishes three levels of Standard Model closure that should not be conflated. First, this paper gives the exact Abelian face-integrability theorem and the first-order direct-face decomposition 120 = 72 + 48. Second, complete non-Abelian loop data determine the physical G_faith connection up to gauge exactly, without linearization (Theorem 13A). Third, the later PFDMI and full-faithful source calculations supply and probe that connection dynamically: the full matrix-link carrier is present, physical marked probabilities have Wilson-quotient rank 72, and the complete tested constraint kernel contains no residual gauge direction. The refined chain is

```
distinguishability/history → marked/record source → G_faith-typed
representation carrier → full matrix links → [U_phys] → Bianchi-compatible
curvature ⟶RMOP (A_phys, 0) → MASD/RRHF response → Standard Model gauge
response.
```

RMOP therefore sits as an exact geometric consistency and normalization theorem inside a source-to-Standard-Model chain considerably more complete than the older face-to-link formulation alone suggested.

It joins the local PSAC result to global connection realizability.

```
local closure/curvature → ten-face candidate → four global obstructions → one
physical edge-connection gauge class.
```

26. Falsifiers

F-RM1, rank failure. The 4-simplex incidence matrices do not return (4, 6, 4, 1) for the coboundary ranks.

F-RM2, Smith-torsion failure. A non-unit nonzero Smith invariant appears in d_1 or d_2, introducing hidden finite torsion into the face or obstruction quotient.

F-RM3, realizability failure. A ten-face assignment satisfies all four independent tetrahedral closure relations but cannot be reconstructed from edge transport.

F-RM4, gauge-uniqueness failure. Two inequivalent edge-transport gauge classes produce the same complete ten-face holonomy assignment on Δ^4.

F-RM5, syndrome-rank failure. The closure differential has rank other than four, or a nonzero singular spectrum different from four copies of $\sqrt{5}$ under the declared incidence metric.

F-RM6, projector failure. $Q_{cl} = (1/5) B^T B$ is not an idempotent rank-four orthogonal projector.

F-RM7, hidden obstruction direction. A non-integrable tangent exists in $\mathbb{R}^{10}$ that is annihilated by all tetrahedral closure readouts.

F-RM8, isotropy failure. The four nonzero singular values of B are not equal, so that some obstruction direction is geometrically privileged. This would falsify Theorem 6 and reopen the possibility of a geometrically induced weighting asymmetry.

F-RM9, equivariance failure. B fails to intertwine the S_5 actions on faces and tetrahedra, or the equivariant endomorphism algebra of $\mathbb{R}^{10}$ has dimension other than two. Either would falsify Theorem 8 and restore the metric-choice objection.

F-RM10, invariant-shape failure. A returned pre-integrability syndrome covariance is not proportional to $5I_5 - s s^T$ while the source law is still claimed to be S_5-invariant.

F-RM11, integral-splitting overclaim. The equivariant projector is asserted to give an integral direct-sum splitting of $\mathbb{Z}^{10}$, contradicting the index-125 result of Theorem 4(iii).

F-RM12, Hodge-identity failure. $d_1 d_1^T + d_2^T d_2 \neq 5I_{10}$ on the direct-face tangent, or the canonical representative $A_{can} = (1/5)d_1^T\theta$ fails to satisfy $d_1 A_{can} = P_{int}\theta$ or fails gauge-orthogonality $d_0^T A_{can} = 0$.

F-RM13, normalization failure. $\|d_1 A\|^2 \neq 5\|A\|^2$ on the gauge-orthogonal edge slice, or the norm identity $\|\theta\|^2 = 5\|A_{can}\|^2 + (1/5)\|r\|^2$ fails. Any downstream calculation that uses a face-to-edge conversion factor other than 5 without deriving it is falsified here.

F-RM14, diagnostic range failure. A candidate returns f_obs outside [0, 1], or f_obs is not invariant under vertex relabelling.

F-RM15, non-Abelian overgrading. The tensored extension of §16 is used as though it established a nonlinear result for integrating arbitrary independent non-Abelian face assignments, rather than an exact linearized flat-background decomposition.

F-RM16, non-Abelian undergrading. The absence of that face-integrability result is read as though the physical non-Abelian connection were unreconstructed, contradicting Theorem 13A and the executed source results of §14.4.

F-RM17, projection-renormalization claim. A Route-M effect on a Standard Model coupling is claimed at Gaussian order from projection alone, contradicting Corollary 17.1, without identifying an admission, non-Gaussian, blocking or symmetry-breaking mechanism.

F-RM18, source-native closure failure. A configuration produced by the current source construction returns $r \neq 0$, or a source-native variation has nonzero component in V_obs. This would falsify Theorem 12 and, with it, either the composition law of the inherited holonomy assignment or the identification of face holonomy with $h_D(\partial f)$.

F-RM19, false physical promotion. The four obstruction directions are assigned physical probability on the current source stack merely because they exist mathematically, contradicting Corollary 12.1.

F-RM20, carrier reinterpretation. A pre-holonomy obstruction sector is claimed by declaring the present physical face holonomies independent variables, rather than by deriving a genuinely new source-native carrier prior to holonomy assignment.

F-RM21, retired-machinery reuse. A finite obstruction alphabet, closure tolerance, obstruction Born state or source weight b is introduced for the current physical source, for which no pre-integrability state exists. The mathematical firewalls of Appendix F apply to any such attempt: a finite operational obstruction alphabet may not be treated as an exact homomorphic quotient of $U(1)^4$, no finite-index subgroup of $U(1)^4$ exists to quotient by, and the Born rule does not select a prepared obstruction state.

F-RM22, same-event running. A projectively pulled-back unchanged event is assigned regulator-dependent probability.

F-RM23, Fréchet violation. A returned pair (p_n, p_{n+1}) lies outside the interval of Theorem 19 for the declared multiplicity and marginals.

F-RM24, 14/16 insertion. The counting ratio is inserted into the direct probability derivative rather than returned independently.

F-RM25, retired-candidate revival. A nonzero Route-M beta is claimed from the retained-record probability or from any fixed event on the EX2E current-terminal algebra, contradicting Theorems 22 and 23, or a new blocked event is constructed for the purpose of reproducing a target coefficient rather than returned independently by the source.

F-RM26, gauge-sensitivity conflation. Regulator running is inferred from gauge sensitivity alone, ignoring that the EX2E terminal law resolves all 72 physical gauge directions with zero Gram drift across regulator levels 3 to 6.

F-RM27, regulator misidentification. The unrescaled Stage-IX transport adjacency is treated as the quantum UV regulator, whose vacuum-subtracted quadratic symbol vanishes as $O(h\_n^2k^2)$, or the h_n^{-2} normalization is treated as inserted rather than forced by plaquette area scaling.

F-RM28, one-loop overclaim. The coefficients (41/6, −19/6, −7) are presented as first-principles VERSF outputs without the universality certificate of Theorem 26 for the actual shared-link D_f[U], or a naive symmetric finite Dirac operator is used despite doubling.

F-RM29, chiral-projector or mirror-count failure. The parity-even F^2 trace of one projected VERSF Weyl carrier fails to equal one half of the corresponding Dirac trace, or the actual shared-link finite theory produces an additional light gauge-coupled mirror carrier outside the frozen matter census. Either result invalidates Theorem 25A or the claimed one-loop census provenance. The existence of the physical right-handed singlets e_R, u_R, d_R is not a failure: those are intended Weyl carriers and are already included according to their own gauge representations.

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27. Main theorem package

Theorem A, physical loop rank.

$$C^1 / \mathrm{im}\ \delta_0 \cong \mathrm{im}\ \delta_1 \cong U(1)^6.$$

Theorem B, face-to-connection reconstruction.

$$H \text{ is edge-realizable} \iff \delta_2 H = 1,$$

and when realizable the edge connection is unique up to vertex gauge.

Theorem C, global obstruction exact sequence.

$$1 \to U(1)^6 \to U(1)^{10} \to U(1)^4 \to 1,$$

with the last factor the closure-obstruction torus.

Theorem D, splitting and its cost. The sequence splits because the character sequence $0 \to \mathbb{Z}^4 \to \mathbb{Z}^{10} \to \mathbb{Z}^6 \to 0$ has free quotient; splittings form a torsor over $M_{4\times6}(\mathbb{Z})$; and no splitting is both integral and S_5-equivariant, since $[\mathbb{Z}^{10} : L\_int \oplus L\_obs] = 125$.

Theorem E, minimal off-shell completion. A regular smooth carrier retaining all six physical loop directions and all four independent obstruction directions has dimension at least ten; Route M saturates the bound; in the compact connected abelian category the completion is exactly $U(1)^{10}$.

Theorem F, simplex Laplacian and isotropy. On Δ^N with $n = N + 1$ vertices,

$$d_1 d_1^T + d_2^T d_2 = n\ I, \quad d_2 d_2^T + d_3^T d_3 = n\ I, \quad d_2 d_2^T = n\ \Pi,$$

so all nonzero singular values of the closure differential equal $\sqrt{n}$; for N = 4, $B\ B^T = 5I_5 - s\ s^T$ with spectrum {5, 5, 5, 5, 0}.

Theorem G, canonical closure projector.

$$Q\_cl = (1/5)\ d_2^T\ d_2, \quad P\_int = I - Q\_cl = (1/5)\ d_1\ d_1^T.$$

Theorem H, symmetry canonicity. V_int ≅ S^(3,1,1) and V_obs ≅ S^(2,1,1,1) are absolutely irreducible and inequivalent; the equivariant endomorphism algebra is span{P_int, Q_cl}, so Q_cl is the unique S_5-equivariant idempotent of rank four; every S_5-invariant inner product gives the same projectors.

Theorem I, closure-syndrome reconstruction.

$$\theta\_obs = (1/5)\ d_2^T\ (d_2\theta), \quad \|\theta\_obs\|^2 = (1/5)\ \|d_2\theta\|^2.$$

Theorem J, exact Hodge decomposition and canonical attaching map.

$$\theta = d_1\ A\_can + (1/5)\ d_2^T(d_2\theta), \quad A\_can = (1/5)\ d_1^T\ \theta,$$

with A_can gauge-orthogonal, minimum-norm, S_5-equivariant, and (A_can, r) a complete equivariant coordinate encoding of θ. No tree, no gauge fixing, no fitted constant.

Theorem K, face/edge normalization isometry.

$$d_1^T d_1 = 5I \text{ on } \ker d_0^T, \quad \|d_1 A\|^2 = 5\|A\|^2, \quad \|\theta\|^2 = 5\|A\_can\|^2 + (1/5)\|r\|^2.$$

Theorem L, current-source integrability closure. For the source-determined holonomy assignment h_D obeying composition, reversal and identity, with H_f = h_D(∂f),

$$H\_source \in \ker \delta_2 = \operatorname{im} \delta_1, \quad d_2\theta\_source = 0, \quad r = 0, \quad f\_obs = 0, \quad A\_can = A\_phys,$$

and for any source law, $\mu\_face(\ker \delta_2) = 1$, $P\_obs(r \neq 0) = 0$, I_obs^current = 0. Stage 0C-1 is current-source negative and Stage 0C-2 does not open.

Theorem M, local-to-global attaching. Local face curvature attaches to one physical edge-connection gauge class exactly when the four global obstruction coordinates vanish.

Theorem N, non-Abelian holonomy-to-connection reconstruction. For any structure group G, a complete based-loop holonomy representation on the connected K_5 graph determines one edge connection up to vertex gauge, with

$$\{\ G\text{-connections on } K_5\ \} / \text{gauge} \cong \operatorname{Hom}(F_6, G) / G.$$

Exact and nonlinear; distinct from, and not an extension of, the direct-face equation $d_2\theta = 0$. On the frozen ledger G_faith ≅ S(U(3) × U(2)), whose finite $\mathbb{Z}_6$ quotient leaves 𝔤_faith and the count 6 × 12 = 72 unchanged.

Theorem O, integrability diagnostic.

$$f\_obs = \|d_2\theta\|^2 / (5\|\theta\|^2) \in [0, 1],$$

zero exactly on V_int, one exactly on V_obs, and relabelling-invariant. Execution report: θ ↦ (r, f_obs, A_can), which on a source-native state is (A_phys, 0).

Theorem P, linearized flat-background gauge extension. With d_k → d_k ⊗ I_𝔤, the decomposition, projectors, isometry and protocol carry over exactly at first order around a flat background; for 𝔤_SM of dimension 12, 120 = 72 + 48 and $A\_SM^{can} = (1/5)(d_1^T \otimes I_{12})\Theta$, with rank Q_SM DΘ_source = 0 on the current source. Not a result about integrating arbitrary independent non-Abelian face assignments; see Theorem N for the reconstruction that is available.

Theorem Q, invariant second-order covariance and decoupling. Any S_5-invariant ensemble on the off-shell carrier has Σ = a P_int + b Q_cl with syndrome covariance $B \Sigma B^T = b (5I_5 - s s^T)$, so the invariant second-order obstruction covariance is controlled by one scalar b; higher invariant cumulants are unconstrained. P_int Σ P_int = a P_int, so b is absent from the physical gauge covariance, and b_current = 0.

Theorem R, cylinder-event invariance. The same pulled-back operational event cannot run under a projectively consistent history measure.

Theorem S, exact correlated block recursion.

$$p\_B = 1 - e^{\Delta\_m} \prod\_j (1 - p\_j).$$

Theorem T, Fréchet bound and sign.

$$\max\_j p\_j \le p\_B \le \min(1, \Sigma\_j p\_j),$$

and the numerator of the direct gate is non-positive for equal fine marginals, independently of Δ.

Theorem U, direct physical beta gate.

$$b^{phys}\_{\{n\to n+1\}} = (1/2) \cdot (p\_\{n+1\}^{-1} - p\_n^{-1}) / \ln(k\_\{n+1\}/k\_n).$$

Theorem V, counting/beta separation. 14/16 is a structural counting prescription and becomes a physical beta coefficient only if independently reproduced by Theorem U within the bounds of Theorem T.

Theorem W, retained-record no-running.

$$p\_rec^3 = p\_rec^4 = p\_rec^5 = p\_rec^6 = 0.810576269094213, \quad b\_rec = 0.$$

Theorem X, current-terminal cylinder no-running. Every fixed event on the EX2E current-terminal algebra is invariant under the regulator-dependent complete neutral continuation, hence $b\_E = 0$, notwithstanding physical gauge rank 72.

Theorem Y, quantum one-loop handoff.

$$(b\_Y, b_2, b_3) = (41/6, -19/6, -7)$$

from the gauge-plus-ghost, Weyl and scalar determinant coefficients on the inherited Standard Model census, conditional on regulator universality.

Theorem Z, refinement principal symbol.

$$(4/h\_n^2)\ \Sigma\_\mu \sin^2(h\_n\ k\_\mu / 2) = |k|^2 + O(h\_n^2\ k^4),$$

so the faithful-link covariant second difference converges to the covariant Laplacian, with the h_n^{-2} normalization forced by plaquette area scaling.

Theorem AA, Hamiltonian-transfer no-extra-pole. Conditional on positive closure stiffness $r > 0$, the spatial Wilson branch has one low-energy zero and gap $2r\hbar c/h\_n$ to the nearest doubler, and positive sequential transfer introduces no temporal doubler.

Theorem AA′, chiral-projector half-trace. For either Weyl projector $P\_\chi = (1 \pm \gamma_5)/2$, the parity-even F^2 part of the fermionic heat-kernel trace is exactly one half of the four-component Dirac trace, the complementary γ_5 term lying in the parity-odd $F\tilde{F}$ channel. Therefore $b\_Weyl(R) = (2/3)T(R)$ given the Laplace-type continuum pole, and applying the frozen three-generation VERSF chiral census gives

$$b_3\text{\^{}ferm} = 4, \quad b_2\text{\^{}ferm} = 4, \quad b\_Y\text{\^{}ferm} = 20/3.$$

Theorem AB, beta-universality criterion. A local faithful-link fermion family with the correctly projected Weyl pole, no additional light species beyond the frozen census and a local regulator-stable UV remainder returns the already-fixed universal coefficient $2T(R)/3$, independently of its detailed finite stencil.

Appendix F retains, as firewalls and counterfactual architecture rather than as steps in the current physical chain: the no-finite-quotient theorem, Born non-identifiability, the conditional operational-quotient construction, the obstruction-capacity argument and the integrability-information typing.

28. Formal verdict

Object	Grade
Ten Route-M direct face slots	**EXACT STRUCTURAL**
Gauge-invariant physical U(1) loop rank	**6 / EXACT**
Edge-link gauge quotient	**$U(1)^6$ / EXACT**
Tetrahedral obstruction rank	**4 / EXACT**
Obstruction torus	**$U(1)^4$ / EXACT**
H edge-realizable iff tetrahedral obstruction vanishes	**EXACT**
Underlying edge connection unique up to vertex gauge	**EXACT**
Short exact sequence $1 \to U(1)^6 \to U(1)^{10} \to U(1)^4 \to 1$	**EXACT**
Sequence splits (free character quotient)	**EXACT**
Integral S_5-equivariant splitting	**DOES NOT EXIST / INDEX 125 CERTIFIED**
Hidden torsion in face/obstruction quotient	**NONE / EXACT SNF RESULT**
Route-M direct face carrier minimal regular off-shell completion	**EXACT IN DECLARED CLASS**
Compact abelian completion rigidity	**$U(1)^{10}$ / EXACT**
Simplex Laplacian identity $d_2d_2^T + d_3^Td_3 = nI$	**EXACT, ALL N**
Complementary identity $P\_int = (1/5)d_1d_1^T$	**EXACT**
Exact Hodge decomposition $\theta = d_1A\_can + \theta\_obs$	**EXACT**
Canonical edge representative $A\_can = (1/5)d_1^T\theta$	**EXACT, TREE-FREE, GAUGE-ORTHOGONAL**
A_can minimum-norm and S_5-equivariant	**EXACT**
(A_can, r) a complete equivariant coordinate encoding of θ	**EXACT**
Face/edge normalization $\|d_1A\|^2 = 5\|A\|^2$	**EXACT ISOMETRY, FACTOR √5**
Norm identity $\|\theta\|^2 = 5\|A\_can\|^2 + (1/5)\|r\|^2$	**EXACT**
Integrability diagnostic $f\_obs \in [0, 1]$	**EXACT, RELABELLING-INVARIANT**

Object	Grade
Execution report $\theta \mapsto (r, f_{obs}, A_{can})$	**COMPLETE / PRESCRIBED**
Tensored extension $d_k \to d_k \otimes I_{\mathfrak{g}}$	**EXACT AT LINEAR ORDER, FLAT BACKGROUND**
Standard Model linearized census $120 = 72 + 48$	**EXACT AT THAT GRADE**
Nonlinear integration of arbitrary independent non-Abelian face assignments	**NOT ESTABLISHED; DISTINCT FROM EXACT LOOP-HOLONOMY RECONSTRUCTION**
Connection structure from distinguishability conservation	**STRUCTURAL THEOREM**
Complete G-loop assignment on $K_5 \to$ connection	**EXACT NONLINEAR**
Non-Abelian connection moduli	**$\mathrm{Hom}(F_6, G) / G$**
Faithful SM global group	**$S(U(3) \times U(2))$ / EXACT ON FROZEN LEDGER**
Finite $\mathbb{Z}_6$ quotient changes the 72 tangent count	**NO**
Seven anonymous SM representation projectors recovered	**PASS WITHIN INHERITED TYPED CARRIER**
Exactly four completion channels	**PASS**
Full faithful matrix-link primitive source	**EXECUTED PASS**
Native marked probability raw gauge rank	**144**
Native marked probability physical gauge rank	**72**
Record transport → occupied-support link	**CONDITIONAL EXACT VIA POLAR BOND**
Full tested residual kernel gauge content	**0 / MACHINE PASS**
Local physical connection fixed once the realised full state is fixed	**CONDITIONAL LOCAL PASS**
Born/outcome/record selection as an RMOP blockage	**RETIRED**
Generic "missing Standard Model connection"	**RETIRED**
Obstruction weight b in physical gauge covariance	**ABSENT / $P_{int}\, \Sigma\, P_{int} = a\, P_{int}$, EXACT**
Route-M coupling shift by projection alone	**RULED OUT AT GAUSSIAN ORDER**

Object	Grade
Closure differential singular spectrum	**√5, √5, √5, √5, 0 / EXACT**
Isotropy of the closure readout (condition number 1)	**EXACT, ALL N**
Canonical obstruction projector	**Q_cl = (1/5)B^TB / EXACT**
Uniqueness of Q_cl as equivariant rank-4 idempotent	**EXACT**
Independence of the split from the invariant metric	**EXACT**
Obstruction S_5-module type	**S^(2,1,1,1) = standard ⊗ sign / EXACT**
Complete tangent syndrome reconstruction	**EXACT**
14 = 6 + 4 + 4 interpretation	**PHYSICAL + OFF-SHELL + PAIRED READOUT ARCHITECTURE**
Four obstruction directions = four extra physical gauge fields	**NO**
Equal second-order weighting under S_5 invariance	**FORCED / EXACT**
Invariant second-order obstruction covariance	**CONTROLLED BY ONE SCALAR b; HIGHER CUMULANTS UNCONSTRAINED**
Current source-native physical face support	**ENTIRELY ON ker δ_2 = im δ_1**
Current source obstruction syndrome	**r = 0 / EXACT AT DECLARED SOURCE GRADE**
Current source integrability diagnostic	**f_obs = 0**
RMOP interface on current physical source	**A_can = A_phys / EXACT**
Current source tangent rank into V_obs	**0 / EXACT**
Stage 0C-1	**CURRENT-SOURCE NEGATIVE / CLOSED**
Stage 0C-2	**DOES NOT OPEN ON CURRENT SOURCE**
Primitive obstruction weight b	**NO CURRENT PHYSICAL RETURN; b_current = 0 AT SECOND ORDER**
Physical Route-M tetrahedral admission probability	**NO INDEPENDENT ADMISSION EVENT ON CURRENT SOURCE**
Source-native tetrahedral closure probability	**1**
Independent Route-M obstruction information	**0 ON CURRENT SOURCE**

Object	Grade
Future deeper pre-holonomy source	**NOT EXCLUDED; WOULD REQUIRE NEW SOURCE DERIVATION**
Finite operational obstruction object	**FINITE READOUT PARTITION, NOT FINITE GROUP QUOTIENT; RETAINED AS FIREWALL**
Finite-index subgroup of $U(1)^4$	**NONE / EXACT**
M = 128 residual alphabet	**NOT DERIVED**
Born rule selects obstruction state	**NO / EXACT, RETAINED AS COUNTERFACTUAL**
Projective same-event running	**NO / EXACT**
Generic correlated OR-block identity	**EXACT MATHEMATICAL IDENTITY**
Generic direct probability beta formula	**EXACT CONDITIONAL ON A PHYSICALLY RETURNED RUNNING EVENT**
Fréchet interval and sign of the direct gate	**EXACT**
F2F/retained-record probability	**0.810576269094213 / EXECUTED**
F2F/retained-record beta	**0 / EXACT ON FACTORISED SOURCE**
Fixed EX2E current-terminal event beta	**0 / EXACT ON FACTORISED SOURCE**
EX2E physical gauge sensitivity	**RANK 72 / EXECUTED PASS**
EX2E physical regulator Gram drift, levels 3 to 6	**0 / MACHINE PASS**
Current Route-M Bernoulli probability search	**EXHAUSTED / RETIRED FOR SM GAUGE RUNNING**
Historical 14/16 = 7/8	**CONDITIONAL STRUCTURAL COUNT ONLY**
b_RM^phys = 7/8	**NOT DERIVED / NO LONGER THE ACTIVE RUNNING ROUTE**
One-loop SM beta coefficients	**(41/6, −19/6, −7) / CONDITIONAL EXACT; FERMION CHIRAL HALF-TRACE EXPLICITLY CLOSED, SAME-ACTION FINITE-REGULATOR PROVENANCE OPEN**
Dirac fermion one-loop matter coefficient	**4T(R)/3 / CONTINUUM DETERMINANT RESULT**
VERSF Weyl projector parity-even trace	**EXACTLY 1/2 OF DIRAC / CONDITIONAL ON THE INHERITED CHIRAL PROJECTOR**
Weyl one-loop matter coefficient	**2T(R)/3 / CONDITIONAL EXACT AT CONTINUUM-PROJECTOR GRADE**
Ordinary right-handed charged singlets	**RETAINED / REQUIRED / COUNTED IN THEIR OWN GAUGE REPRESENTATIONS**
Additional coupled right-handed weak-doublet mirror copy	**ABSENT FROM FROZEN CENSUS**

Object	Grade
Three-generation fermionic contributions	**$(b_Y^f, b_2^f, b_3^f) = (20/3, 4, 4)$ / CONDITIONAL EXACT ON FROZEN CENSUS**
Weyl half-factor as an independent inserted assumption	**RETIRED**
Exact finite Ginsparg-Wilson / nonperturbative chiral completion	**NOT CLAIMED / STRONGER OPEN PROBLEM**
Gauge/ghost fluctuation architecture	**STRUCTURAL PASS**
h_n^{-2} curvature normalization	**DERIVED FROM PHYSICAL AREA SCALING**
Faithful-link covariant Laplace principal symbol	**PASS / DERIVED TO THE REQUIRED CONTINUUM PRINCIPAL SYMBOL**
Unrescaled Stage-IX transport as UV Laplace regulator	**NO-GO**
Spatial closure-Wilson doubler lifting	**EXACT CONDITIONAL ON $r > 0$**
Spatial unwanted-mode gap	**$2r\hbar c/h_n$**
Sequential-transfer temporal doubler	**NONE / EXACT ON POSITIVE-TRANSFER BRANCH**
Combined Hamiltonian-transfer light-pole count	**1 / CONDITIONAL EXACT**
Exact MASD finite $D_f[U]$	**OPEN / D-SG6**
$D_f[U]$ universality match to the no-doubler branch	**OPEN, CURRENT LOAD-BEARING BETA BRIDGE**
Full nonperturbative chiral/index/RP completion	**OPEN; STRONGER THAN THE ONE-LOOP BETA GATE**
Physical Standard Model connection handoff	**SUBSTANTIALLY CLOSED ACROSS RMOP + PFDMI/FULL-FAITHFUL STACK**

29. Scientific conclusion

The central Route-M geometry is now closed at two distinct levels.

First, the mathematical relationship between arbitrary face data and physical connection data is exact:

$$1 \to U(1)^6\_\text{physical} \to U(1)^{10}\_\text{direct} \to U(1)^4\_\text{obstruction} \to 1.$$

The ten-face carrier is the minimal regular off-shell completion of the physical six-dimensional connection image, unique up to isomorphism in the compact abelian category. Its four extra directions are exactly the complete failure of global realizability. The sequence has no hidden torsion and splits for a lattice-theoretic reason, though no splitting is both integral and equivariant. The canonical decomposition

$$\theta = d_1\left((1/5)\, d_1^T\, \theta \right) + (1/5)\, d_2^T\, (d_2\theta)$$

constructs both the link-realizable component and the obstruction component, the factor five is fixed by simplex incidence rather than inserted, and the projector Q_cl is the unique S_5-equivariant idempotent of rank four, identical for every invariant inner product. The closure readout is isotropic, with all four singular values equal to $\sqrt{5}$, a special case of a Laplacian identity holding in every simplex dimension.

Second, the current physical VERSF source stack fixes which part of that architecture is actually occupied.

Physical holonomy is one assignment on admissible closed paths respecting composition, reversal and identity. Physical triangle holonomies are therefore restrictions of one underlying transport structure, their tetrahedral product is the holonomy of a boundary of a boundary, and it is identically the identity. Hence

```
r_source = 0,    f_obs^source = 0,    A_can = A_phys.
```

The physical theory does not presently generate an inconsistent ten-face precursor and subsequently repair it. The four-dimensional obstruction sector is an exact and useful mathematical consistency space, but it is not a populated physical sector of the current VERSF source. Stage 0C-1 closes negatively and Stage 0C-2 does not open, so no obstruction occupation probability, Born state, finite alphabet, closure tolerance or four-direction source weight is required on the current branch.

This sharpens the relation between the earlier Route-M and later MASD descriptions. Route M's independently variable ten-face construction is an off-shell enlargement on which local coherence and global integrability can be tested separately. MASD and the full-faithful source describe the physical connection branch on which the integrability conditions are already satisfied. The attaching map is therefore a consistency certificate and a normalization, not a filtering mechanism: for an arbitrary candidate it returns the complete encoding (A_can, r), and for a source-native state it returns (A_phys, 0).

The resulting Standard Model attaching chain is

```
primitive/history source → physical link transport → consistent local
curvature → normalized RMOP attaching certificate → MASD gauge response →
Standard Model couplings.
```

At first order around a flat Standard Model background the mathematical direct-face carrier still decomposes as 120 = 72 + 48, but the current physical source occupies the 72-dimensional integrable tangent and has zero tangent rank into the 48-dimensional obstruction space. The identities $A\_can = (1/5)d_1^T\theta$ and $\|d_1 A\|^2 = 5\|A\|^2$ pass to the gauge-response and Hessian chain unchanged and with no fitted constant.

The wider source programme has also progressed well beyond the generic primitive-source caveat. The distinguishable alternative space, Born weighting, realised commitment and persistent-record architecture are supplied elsewhere in the corpus and are not outstanding Route-M dependencies. Later field-dependent marked-instrument work recovers the Standard Model representation atoms within the inherited typed carrier, supplies the full faithful matrix-link source, and produces native marked probabilities of physical gauge rank 72. The physical gauge connection is therefore no longer merely an admitted mathematical carrier waiting for a source interpretation: the source stack contains the full connection carrier and a field-sensitive marked law capable of resolving all of its physical tangent directions.

The master-action polar bond ties the realised record transport to the supported physical link,

$$\Pi_j U_{ij} \Pi_i = \mathrm{Pol}(\Pi_j \Phi_{ij} \Pi_i),$$

while the full-faithful kernel audit leaves zero gauge directions in the tested residual kernel. Conditional on the realised full state and the declared rank-stable source branch, the remaining infinitesimal internal freedom therefore does not generate alternative physical gauge connections. This should not be overread as a claim that the theory picks one immutable gauge field for the universe. Gauge fields are dynamical physical variables and different realised histories are expected to carry different configurations. The correct statement is narrower: once the realised physical state and history are specified and the full constraints are imposed, the executed local theory has no unexplained gauge-field-changing null freedom.

The faithful global group is additionally known on the frozen ledger,

$$G_{faith} = [SU(3) \times SU(2) \times U(1)_q] / \mathbb{Z}_6 \cong S(U(3) \times U(2)),$$

whose finite central quotient changes global bundle topology but not the twelve-dimensional Lie algebra, so the physical local gauge tangent remains seventy-two dimensional. A future deeper theory could still introduce an object prior to physical holonomy assignment, but such an object would have to be derived as a genuinely new carrier; it cannot be obtained by declaring the existing physical face holonomies independent, because their composition law already forces tetrahedral consistency. The machinery such a carrier would require is preserved in Appendix F.

The later running audit changes the final interpretation of the Route-M programme. The direct probability-running theorem remains a correct general identity, but the source stack has now supplied enough explicit candidates to test whether it is the Standard Model running mechanism. The answer is negative. The retained-record probability is regulator invariant, and the entire fixed EX2E current-terminal event algebra is invariant once the complete neutral continuation is summed. These events may be physical and gauge-sensitive, yet their direct beta coefficients vanish. The current Route-M Bernoulli-probability search is therefore exhausted rather than merely awaiting another choice of p_n.

The one-loop Standard Model running instead resides in the quantum fluctuation determinant around the admitted physical connection. The background-field gauge, ghost, Weyl and scalar sectors return the universal coefficients

$-(11/3)\ C_2(G), \quad (2/3)\ T(R), \quad (1/3)\ T(R),$

and the inherited Standard Model census gives

$(b_Y, b_2, b_3) = (41/6, -19/6, -7).$

The remaining VERSF-specific problem is now regulator provenance rather than coefficient discovery. The physical refinement geometry forces the h_n^{-2} curvature normalization, the faithful-link covariant second difference converges to $-D^2$ with $O(h_n^2)$ corrections, and the gauge-plus-ghost Hessian has the required Laplace-type principal part and curvature endomorphism. The unrescaled Stage-IX transport generator is not itself that regulator, which closes one possible misidentification.

The finite fermion problem has now narrowed in two separate steps. First, a naive symmetric Dirac discretisation is ruled out by doubling, but the closure-entropy programme supplies a Wilson-type spatial stiffness that, for $r > 0$, leaves one spatial light pole and drives every unwanted spatial lattice zero to energy of order $1/h_n$, while sequential positive-transfer time introduces no temporal doubler: positive Euclidean transfer $T = e^{-a_t H}$ has a one-step kernel with a unique zero at zero energy and zero frequency. The candidate Hamiltonian-transfer branch therefore contains exactly the intended low-energy spinorial pole and no discretisation-generated copy.

Second, the chiral projector audit determines how that surviving spinorial carrier contributes to gauge running. The matter architecture does not delete the right-chiral sector: the physical e_R, u_R, d_R remain as weak singlets and are counted as Weyl carriers in their own colour and hypercharge representations. What the electroweak stack selects is the support of nontrivial weak transport, $P_W = P_L$. For either chiral projector $P_\chi = (1 \pm \gamma_5)/2$ the parity-even F^2 heat-kernel trace is exactly one half of the four-component Dirac trace, the remaining γ_5 contribution being parity odd, so

$b_{Weyl}(R) = (2/3)\ T(R)$

follows from the projected spinorial trace rather than from an independently inserted multiplicity rule. Summing those projected Weyl contributions over the frozen three-generation matter census gives

$b_3^{ferm} = 4, \quad b_2^{ferm} = 4, \quad b_Y^{ferm} = 20/3,$

and therefore, after the gauge-plus-ghost and single-complex-scalar terms are included, $(b_Y, b_2, b_3) = (41/6, -19/6, -7)$ again, this time by provenance rather than by repetition of the arithmetic.

The remaining VERSF-specific problem is therefore not discovery of these coefficients, not the Weyl factor 1/2, and not the ordinary chiral matter census. It is finite-regulator provenance: the specific shared-link $D_f[U]$ occurring in the Master Substrate Action must still be shown to descend to the local gauge-covariant no-extra-species Weyl universality class identified above, or to an equivalent admissible chiral regulator family. No exact finite Ginsparg-Wilson relation or full nonperturbative chiral determinant construction is claimed here; those belong to the stronger D-SG6/D-SG8 completion. Once locality, faithful-link gauge covariance, the correctly normalized projected Weyl pole, absence of additional light species beyond the frozen census and a local regulator-stable UV remainder are established for the actual D_f[U], the one-loop coefficient follows by universality with no remaining numerical or chiral-counting freedom.

The revised physical chain is therefore

```
distinguishability/history → realised marked record → G_faith-typed matrix
connection → Bianchi-compatible curvature → RMOP geometric certificate →
finite VERSF fluctuation Hessians → one-loop quantum determinant → (41/6,
−19/6, −7).
```

The unresolved step is no longer a Route-M obstruction weight or a missing record/coherence probability. It is the same-action finite-regulator universality of the quantum fluctuation package, with the shared-link chiral fermion kernel the principal remaining gate. The 14/16 = 7/8 coefficient survives only as a conditional structural count, and the 21/(16π) coefficient derived in later PFDMI work must not be identified with a Route-M beta coefficient: its clock interpretation remains conditional on a serial-composition premise and its absolute SI value on the physical length scale.

The verdict is therefore:

```
ROUTE-M OFF-SHELL CARRIER GEOMETRY: CLOSED.

FACE-TO-CONNECTION REALIZABILITY AND GAUGE UNIQUENESS: CLOSED IN THE ABELIAN
DIRECT-FACE PROBLEM.

CANONICAL FACE-TO-EDGE ATTACHING MAP AND NORMALIZATION: CLOSED.

CANONICITY OF THE SPLIT UNDER SIMPLEX SYMMETRY: CLOSED.

CURRENT PHYSICAL SOURCE SUPPORT IN THE INTEGRABLE SECTOR: CLOSED.

PRE-INTEGRABILITY OBSTRUCTION SECTOR ON THE CURRENT SOURCE: NOT PHYSICALLY
POPULATED.

NON-ABELIAN COMPLETE-LOOP-HOLONOMY TO CONNECTION RECONSTRUCTION: CLOSED
EXACTLY.

FAITHFUL STANDARD MODEL GLOBAL GAUGE GROUP: CLOSED ON THE FROZEN
REPRESENTATION LEDGER.

FULL PHYSICAL STANDARD MODEL GAUGE TANGENT: 72 / EXECUTED SOURCE PASS.

LOCAL RESIDUAL GAUGE-FIELD FREEDOM AFTER FULL TESTED CONSTRAINTS: ZERO /
MACHINE PASS.

ROUTE-M RETAINED-RECORD RUNNING: 0 / EXACT NO-RUNNING.

FIXED EX2E CURRENT-TERMINAL RUNNING: 0 / EXACT NO-RUNNING.

CURRENT ROUTE-M BERNOULLI-PROBABILITY SEARCH: EXHAUSTED FOR STANDARD MODEL
GAUGE RUNNING.
```

```
14/16 = 7/8: CONDITIONAL STRUCTURAL COUNT; NOT A DERIVED BETA COEFFICIENT.

STANDARD MODEL ONE-LOOP COEFFICIENTS: (41/6, −19/6, −7) / CONDITIONAL EXACT.

VERSF CHIRAL-PROJECTOR F² TRACE: EXACTLY ONE HALF OF DIRAC / CONDITIONAL ON
THE INHERITED CHIRAL PROJECTOR.

WEYL FERMION COEFFICIENT: 2T(R)/3 / CONDITIONAL EXACT AT THE CONTINUUM-
PROJECTOR GRADE.

THREE-GENERATION FERMION CONTRIBUTIONS: (20/3, 4, 4) FOR (Y, 2, 3) /
CONDITIONAL EXACT ON THE FROZEN CENSUS.

ORDINARY RIGHT-HANDED CHARGED SINGLET CARRIERS: RETAINED AND COUNTED.

ADDITIONAL GAUGE-COUPLED MIRROR WEAK-DOUBLET COPY: ABSENT FROM THE FROZEN
CENSUS.

WEYL HALF-FACTOR AS AN INDEPENDENT INSERTION: RETIRED.

FAITHFUL-LINK COVARIANT LAPLACE PRINCIPAL SYMBOL: DERIVED.

UNRESCALED STAGE-IX TRANSPORT AS THE QUANTUM UV REGULATOR: NO-GO.

SPATIAL DOUBLER LIFTING ON THE CLOSURE-WILSON BRANCH: PASS CONDITIONAL ON r >
0.

TEMPORAL DOUBLING ON THE POSITIVE-TRANSFER BRANCH: ABSENT / EXACT.

ARBITRARY NON-ABELIAN DIRECT-FACE INTEGRABILITY: NOT CLAIMED.

ACTUAL MASD SHARED-LINK CHIRAL KERNEL D_f[U]: NOT YET SOURCE-EXPLICIT.

D_f[U] SAME-ACTION NO-DOUBLER / CHIRAL-UNIVERSALITY CERTIFICATE: OPEN,
PRINCIPAL REMAINING ONE-LOOP REGULATOR BRIDGE.

FULL NONPERTURBATIVE CHIRAL / INDEX / DETERMINANT-LINE / REFLECTION-POSITIVE
COMPLETION: OPEN AND STRONGER THAN THE ONE-LOOP BETA GATE.
```

Appendix A: explicit incidence certificates

A.1 Edge-to-face coboundary

Order edges as 01, 02, 03, 04, 12, 13, 14, 23, 24, 34 and faces as 012, 013, 014, 023, 024, 034, 123, 124, 134, 234. The edge-to-face coboundary d_1 is the oriented triangle-edge incidence matrix, with

```
rank d₁ = 6,
SNF(d₁) = diag(1, 1, 1, 1, 1, 1, 0, 0, 0, 0).
```

No finite torsion survives in im d_1 or in the corresponding quotient by vertex gauge.

A.2 Face-to-tetrahedron coboundary

With the ordering of §9,

```
B = d₂ =
  ( -1   1   0  -1   0   0   1   0   0   0 )
  ( -1   0   1   0  -1   0   0   1   0   0 )
  (  0  -1   1   0   0  -1   0   0   1   0 )
  (  0   0   0  -1   1  -1   0   0   0   1 )
  (  0   0   0   0   0   0  -1   1  -1   1 )
```

with

```
rank d₂ = 4,
SNF(d₂) = diag(1, 1, 1, 1, 0).
```

A.3 Tetrahedron-to-4-simplex coboundary

```
d₃ = ( 1  -1   1  -1   1 ),
```

so $d_3 d_2 = 0$. The single relation among the five tetrahedral closure residuals is the multiplicative version of this row. Also rank $d_0 = 4$ with $\mathrm{SNF}(d_0) = \mathrm{diag}(1, 1, 1, 1, 0)$.

A.4 Exact Gram identity and singular spectrum

With $s = d_3^T$,

```
B Bᵀ =
  (  4   1  -1   1  -1 )
  (  1   4   1  -1   1 )
  ( -1   1   4   1  -1 )
  (  1  -1   1   4   1 )
  ( -1   1  -1   1   4 )
  = 5 I₅ - s sᵀ,
```

verified as an exact integer matrix identity. Since $s\, s^T$ has eigenvalue 5 along s and 0 on $s\perp$,

```
spec(B Bᵀ) = {5, 5, 5, 5, 0},
```

so rank B = 4 and the nonzero singular values are four copies of $\sqrt{5}$.

A.5 Projector certificate

```
Q_cl = (1/5) Bᵀ B,    Q_clᵀ = Q_cl,
```

and using $s^T B = 0$,

$$Q\_cl^2 = (1/25)\ B^T B B^T B = (1/25)\ B^T (5I - s s^T) B = (1/5)\ B^T B = Q\_cl,$$

with trace Q_cl = 4, so Q_cl is exactly an orthogonal projector of rank four.

Appendix B: explicit six-plus-four coordinate certificate

Let the first six coordinates be

$$a_1 = \theta_{012},\quad a_2 = \theta_{013},\quad a_3 = \theta_{014},\quad a_4 = \theta_{023},\quad a_5 = \theta_{024},\quad a_6 = \theta_{034},$$

and the four obstruction angles

$$\begin{aligned} r_1 &= -\theta_{012} + \theta_{013} - \theta_{023} + \theta_{123},\\ r_2 &= -\theta_{012} + \theta_{014} - \theta_{024} + \theta_{124},\\ r_3 &= -\theta_{013} + \theta_{014} - \theta_{034} + \theta_{134},\\ r_4 &= -\theta_{023} + \theta_{024} - \theta_{034} + \theta_{234}. \end{aligned}$$

Then

$$\begin{aligned} \theta_{123} &= r_1 + a_1 + a_4 - a_2,\\ \theta_{124} &= r_2 + a_1 + a_5 - a_3,\\ \theta_{134} &= r_3 + a_2 + a_6 - a_3,\\ \theta_{234} &= r_4 + a_4 + a_6 - a_5. \end{aligned}$$

The integer coordinate transformation from $(a_1, \ldots, a_6, r_1, \ldots, r_4)$ to the ten face angles is block triangular with identity diagonal blocks,

$$\begin{pmatrix} I_6 & 0 \\ A & I_4 \end{pmatrix},$$

with determinant 1, so it is unimodular and defines a global torus automorphism. The integrable subtorus is $r_1 = r_2 = r_3 = r_4 = 0 \pmod{2\pi}$. Applying B to a vector built this way returns $(r_1, r_2, r_3, r_4, r_5)$ with $r_5 = -r_1 + r_2 - r_3 + r_4$, confirming $s^T r = 0$ and that the fifth residual is dependent.

This provides a constructive certificate of $U(1)^{10} \cong U(1)^6 \times U(1)^4$ after a base/tree choice, and, together with Theorem 4(iii), of the fact that this splitting cannot simultaneously be made equivariant and integral.

Appendix C: executed certificates

All items below were computed symbolically over $\mathbb{Z}$ where the statement is integral, and in double precision where the statement is spectral. Reported errors are maximum absolute deviations.

C.1 Complex and ranks, Δ^4

Item	Result
Shapes (d_0, d_1, d_2, d_3)	(10×5), (10×10), (5×10), (1×5)
Ranks	4, 6, 4, 1
$d_1 d_0 = 0$, $d_2 d_1 = 0$, $d_3 d_2 = 0$	all true, exact
SNF diagonals	d_0: (1,1,1,1,0); d_1: $(1^6,0^4)$; d_2: (1,1,1,1,0); d_3: (1)
$B B^T = 5I_5 - s s^T$	exact
$s^T B = 0$	exact
Q_cl idempotent, symmetric	exact; rank 4, trace 4
spec($B B^T$)	5 with multiplicity 4, 0 with multiplicity 1

C.2 General simplex Δ^N

N	faces C(N+1,3)	rank d_1 = C(N,2)	rank d_2 = C(N,3)	nonzero singular values of d_2	$d_2 d_2^T/(N+1)$ idempotent error
4	10	6	4	all $\sqrt{5} = 2.236068$	2.2×10^{-16}
5	20	10	10	all $\sqrt{6} = 2.449490$	1.1×10^{-16}
6	35	15	20	all $\sqrt{7} = 2.645751$	2.2×10^{-16}
7	56	21	35	all $\sqrt{8} = 2.828427$	0

The Laplacian identity $d_k^T d_k + d_{k-1} d_{k-1}^T = (N+1)I$ was checked on every cochain group with $1 \le k \le N$ for N = 4, 5, 6, returning maximum deviation 0 in each case; at k = 0 it deviates by exactly the omitted rank-one augmentation, as stated in Lemma 9.1.

C.3 Symmetry certificates, Δ^4

Item	Result
$B\,\rho_2(g) - \rho_3(g)\,B$, all 120 elements of S_5	0 (exact)
Q_cl $\rho_2(g) - \rho_2(g)$ Q_cl, all 120 elements	0 (exact)
dim of S_5-equivariant endomorphisms of $\mathbb{R}^{10}$	2
dim of S_5-invariant bilinear forms on $\mathbb{R}^{10}$	2
commutant dimension on V_obs	1 (absolutely irreducible)

Item	Result
commutant dimension on V_int	1 (absolutely irreducible)
χ_obs on (1^5, 2, 3, 2^2, 4, 3·2, 5)	(4, −2, 1, 0, 0, 1, −1)
χ_int on the same classes	(6, 0, 0, −2, 0, 0, 1)
restricted rep on V_obs orthogonal	error 8.9×10^{-16}

The character χ_obs identifies V_obs as the standard representation twisted by sign, that is the irreducible labelled by the partition (2,1,1,1); χ_int identifies V_int as the six-dimensional irreducible labelled by (3,1,1). The two are inequivalent, which is what forces the equivariant endomorphism algebra to be exactly two-dimensional.

C.4 Invariant source law

For Σ = a P_int + b Q_cl with arbitrary a, b,

$$\| B \Sigma B^T - b (5 I_5 - s s^T) \|_{max} = 2.2 \times 10^{-16}.$$

C.5 Lattice index

With L_int = im d_1 (SNF all units, hence saturated) and L_obs = im d_2^T (SNF (1,1,1,1,0), hence saturated), the Smith diagonal of the stacked matrix is

```
(1, 1, 1, 1, 1, 1, 1, 5, 5, 5),
```

so $[\mathbb{Z}^{10} : L\_int \oplus L\_obs] = 125$.

C.6 Block recursion and Fréchet bounds

Random joint laws on m ∈ {2, …, 5} binary cells drawn from a flat Dirichlet on the 2^m atoms. In every trial the blocked probability satisfied

$$\max_j p_j \le p_B \le \min(1, \Sigma_j p_j),$$

and p_B was reproduced from the marginals and Δ_m via $p_B = 1 - e^{\Delta_m} \prod_j (1 - p_j)$ to machine precision. Both signs of Δ_m occurred, confirming that the defect is not sign-constrained.

C.7 Hodge decomposition, attaching map and normalization

Item	Result
$P\_int - (1/5)\, d_1 d_1^T$	0 (exact)
$d_1 A\_can - P\_int\theta$, random θ	2.2×10^{-16}
$\theta - [\, d_1 A\_can + (1/5) d_2^T d_2 \theta \,]$	4.4×10^{-16}

Item	Result
$d_0^T A\_can$ (gauge-orthogonality)	1.1×10^{-16}
$\dim \ker d_0^T$	6
$\|d_1 A\|^2/\|A\|^2$ on $\ker d_0^T$	5.000000
$\ker d_0^T$ restricted Gram $d_1^T d_1 - 5I_6$	2.7×10^{-15}
$\|\theta\|^2 - [\ 5\|A\_can\|^2 + (1/5)\|r\|^2\ ]$	machine zero
$A\_can - d_1^+(P\_int\theta)$ (minimum norm)	5.6×10^{-16}
A_can equivariance, all 120 elements of S_5	5.6×10^{-17}
f_obs on random V_int element	5.7×10^{-33}
f_obs on random V_obs element	1.0000000
$(1/n)(d_1 d_1^T + d_2^T d_2) - I$ on faces, N = 5, 6	0 (exact)

Appendix D: relation to the earlier Route-M independence proof

The earlier Route-M algebraic-independence proof is correct on its declared direct-face carrier. There, the ten H_f are direct variables. One may choose all ten inside the local coherent class while allowing a tetrahedral closure product to remain nontrivial; conversely one may satisfy all tetrahedral products while allowing some individual faces outside the local coherent class.

The present theorem does not negate that statement. It retypes it. On the full direct-face carrier $U(1)^{10}$, local face coherence and global integrability are distinct constraints. On the physical link-realizable subcarrier $\ker \delta_2 = \mathrm{im}\, \delta_1$, global tetrahedral closure is automatic.

The earlier proof is therefore an off-shell independence theorem and the MASD identity is an on-shell integrability theorem. Both are simultaneously true, and Theorem 3 is the map that separates their domains.

Appendix E: relation to the VERSF Standard Model chain

The physical attachment chain is

```
primitive/history distinguishability → source-determined transport/holonomy
assignment → edge/link gauge carrier → Bianchi-compatible face curvature →
Standard Model response.
```

RMOP supplies an exact audit of the middle relation,

$$\theta\_physical \mapsto (A\_can, r) = (A\_physical, 0),$$

together with the normalization $\|d_1 A\|^2 = 5\|A\|^2$ that carries a face-curvature quadratic form to the edge gauge tangent.

Separately, the mathematical off-shell diagnostic remains

$$U(1)^{10}\_direct \longrightarrow U(1)^{6}\_physical \oplus U(1)^{4}\_obstruction,$$

which permits arbitrary face assignments for the purpose of realizability testing but is not itself asserted to be a source-populated physical stage.

The chain

```
primitive/history source → direct face candidate → integrability gate →
physical connection
```

is therefore retired as a statement about the current physical source. It survives only as the counterfactual architecture that a future independently derived pre-holonomy source would have to realize, and the results such an architecture would need are collected in Appendix F.

The refined chain, incorporating the later source results, is

```
distinguishability/history → marked/record source → G_faith-typed
representation carrier → full matrix links → [U_phys] → Bianchi-compatible
curvature ⟶RMOP (A_phys, 0) → MASD/RRHF response → Standard Model gauge
response.
```

The PSAC result constrains the first local closure arrow and fixes its canonical local normalization at its declared grade. This paper closes the middle geometric arrows and, with Theorem 12, types their physical occupation, while Theorem 13A supplies the exact nonlinear reconstruction of the physical connection from complete loop data. The remaining provenance task is on the admitted connection branch: a physical same-carrier coherence law and its adjacent-regulator execution, not an obstruction weight.

Appendix F: retained firewalls and counterfactual architecture

The results below were load-bearing while the physical occupation of the obstruction sector was an open fork. Theorem 12 closes that fork negatively for the current source stack, so they are retained here in two distinct roles: as firewalls against misuse of the mathematical carrier, and as the architecture that a future independently derived pre-holonomy source would have to realize. None of them describes the presently sourced gauge carrier.

F.1 No finite quotient-group shortcut

Theorem F.1: No-finite-quotient theorem

[EXACT]

(i) Every group homomorphism $U(1)^n \to F$ into a finite group F has trivial image. The image is finite and divisible, and a finite divisible group is trivial.

(ii) More strongly, $U(1)^n$ has no proper subgroup of finite index whatever, closed or not. If $H \leq G$ with G abelian and $[G : H] = m$, then $g^m \in H$ for every $g \in G$, so $G = G^m \subseteq H$ by divisibility, whence $H = G$. No continuity assumption is used.

Consequence. A nontrivial finite operational resolution of the four Route-M obstruction phases is a finite readout partition, not an exact finite quotient group, and its cell containing the identity is not a subgroup. Nothing in the finite-distinguishability apparatus can supply a group law on the readout alphabet, so no argument may transport the $U(1)^4$ multiplication to it.

Corollary. The diagnostic formula

$$P\_int = M^{-4}$$

is valid for a separately declared uniform finite Abelian residual model. It is not derived from

$$U(1)^4 \to_{\text{(finite distinguishability)}} (\mathbb{Z}\_M)^4,$$

because no such nontrivial exact quotient homomorphism exists, and by (ii) not even a finite-index subgroup exists to quotient by. In particular M = 128 cannot be imported from K7.

F.2 Born rule versus a prepared obstruction state

The VERSF probability programme fixes the Born/ODG functional on the admitted amplitude carrier. It does not select the Route-M pre-integrability state.

Let $|g\rangle$ be an operational alternative accepted by the integrability effect and $|f\rangle$ an orthogonal rejected alternative. For any $q \in [0, 1]$, the normalized state

$$|\psi\_q\rangle = \sqrt{q}\ |g\rangle + \sqrt{(1 - q)}\ |f\rangle$$

has $\langle\psi\_q|\ E\_int^{\wedge}op\ |\psi\_q\rangle = q$.

Theorem F.2: Integrability-probability non-identifiability from Born alone

[EXACT]

The Born rule alone permits every integrability probability $q \in [0, 1]$ whenever both admitted and rejected operational alternatives exist. The physical P_int^op must therefore come from source preparation, history conditioning, transfer dynamics or an equivalent primitive marked law.

Relation to PMS. This is the same support-versus-weight distinction found elsewhere in VERSF: structure may force which directions exist, symmetry may force support, and neither automatically fixes relative probability or stiffness. Theorem 16 is the sharpened version of that statement here: symmetry fixes the shape of the weight and leaves exactly one scalar free.

F.3 Conditional operational-quotient closure

The raw obstruction carrier is continuous, $\mathcal{O}$_cl ≅ $U(1)^4$, and raw exact equality of continuous representatives is not a physically meaningful final test at finite resolution. The question is what replaces it, and under what condition the replacement has physical content at all.

Let the full finite operational carrier be A_op with source resolution map $r : A \to$ A_op.

Theorem F.3: Conditional operational-quotient closure theorem

[CONDITIONAL ON STAGE 0C-1]

If the Route-M tetrahedral functionals are source-identified as a physical pre-connection admission layer, then finite distinguishability requires their physical closure event to be evaluated on the operational phase quotient (Φ_op, μ_op, B_op) rather than as raw exact equality on the continuous carrier. Concretely, a physical integrability decision must be constant on fibres of r, so it defines an event

```
G_int^op ⊆ A_op
```

or, on the operational Hilbert carrier, an effect 0 ≤ E_int^op ≤ I, and for source-selected operational state ρ_pre^op,

```
P_int^op = Tr( ρ_pre^op E_int^op ).
```

Raw exact equality on continuous U(1), an arbitrary tolerance, and an assumed finite residual alphabet are not substitutes for this operational measure.

If Stage 0C-1 fails, there is no physical Route-M tetrahedral admission probability to calculate at that grain, and the quotient construction above has no physical referent.

Unreturned objects. Both the admission-layer identification and the quotient measure μ_op remain unreturned in this paper.

F.4 Operational obstruction capacity

Everything from this section onward is conditional, and the conditions are stated before they are used.

The Primitive Matter Selection programme states the following admission rule at its declared grade:

every admissible positive distinguishability direction has capacity to form a physical fact unless a source theorem proves it redundant or forbidden.

This rule does not by itself prove that every mathematical obstruction coordinate is physically populated. It gives a conditional implication. Suppose the pre-integrability Route-M stage contains an admissible operational closure readout capable of distinguishing $R = 1$ from some $R \neq 1$. Then the corresponding obstruction variation is not a pure coordinate redundancy: it changes an admissible comparison.

Theorem F.4: Operational obstruction-capacity theorem

[CONDITIONAL ON PRE-INTEGRABILITY WITNESSABILITY AND THE INHERITED NO-HIDDEN-DISTINCTION ADMISSION RULE]

Any Route-M obstruction direction that produces an admissibly resolvable change in the tetrahedral integrability readout has physical candidate capacity before the integrability gate, unless a source theorem forbids or makes that direction redundant.

By Theorem 6 the readout resolves all four directions with equal sensitivity, so the antecedent cannot hold for some obstruction directions and fail for others on geometric grounds. Either all four have candidate capacity or none does.

Limitation. Candidate capacity is not a probability, not a stiffness, not a guarantee of nonzero source support, and not a proof that all four directions are equally weighted by the source. This is exactly the PMS distinction between support and probability/stiffness. Theorem 16 adds the complementary fact that if the source is S_5-invariant then equal weighting is forced, so unequal weighting requires a symmetry-breaking source claim.

What this would and would not establish. On a hypothetical pre-holonomy carrier the four obstruction directions could not be dismissed merely because they vanish after realizability is imposed: if operationally witnessed before that projection, they would have the correct type to act as pre-commitment candidate directions. On the current source stack the antecedent fails, since by Theorem 12 there is no pre-integrability physical stage at which such a witness could occur.

F.5 Integrability-information typing

Let T denote simultaneous K7 coherence of the ten triangular faces. The inherited exact information identity is

$$I_{RM} = 70 \ln 2 + \Delta_T + I_{int}^{op},$$

where

$$\Delta_T = -\ln [P(T) / 2^{-70}]$$

captures cross-face departure from naive seventy-bit factorisation, and $I_{int}^{op} = -\ln P_{int}^{op}$.

The last term is not "four more K7 channels". It would be the information cost of requiring the four-dimensional obstruction coordinate to lie in the operational zero-obstruction class, and on the current source stack it is zero by Corollary 12.1. Because conditioning on T fixes the original seventy K7 pass/fail bits, $H(X \mid T) = 0$, so the integrability information cannot be obtained by reusing those bits.

Theorem F.5: Integrability-information typing theorem

[EXACT GIVEN THE INHERITED INFORMATION IDENTITY; ZERO ON THE CURRENT SOURCE STACK]

The additional Route-M information beyond ten-face K7 coherence, if nonzero, is information on the four-dimensional integrability-obstruction carrier or its operational image. It is not automatically $4 \times 7 \ln 2$, nor $4 \ln 128$, nor any other equal-channel count.

Formal consequence.

$N_{info}^{phys} = 14$ is not derived by the structural count alone.

This does not weaken the geometric result. The carrier and obstruction are fully identified; only the admission typing and the source measure on that carrier remain.

Internal VERSF source stack consulted

1. **Geometric Closure**: local edge-defined triangle holonomy, K7 coherence structure, local C7 versus global 4-simplex closure relations.
2. **Microphysical Foundations of Route M in the Two-Planck Framework**: direct-face formulation, 10 + 4 structural census, UV Local Mixing and the 14/16 prescription.
3. **Holonomy Assignment from Distinguishability**: U(1) closed-loop composition, reversal, gauge freedom and physical holonomy supervenience at the paper's declared grade.
4. **Operationally Invisible Sector Boundaries**: finite operational resolution and quotient discipline.
5. **Operational Geometry / Unified Refinement Hilbert Geometry**: admissible operational Hilbert carrier and refinement structure.
6. **The Universal Measure Principle**: operational Born-measure universality.
7. **Primitive Matter Selection in VERSF, cumulative executed theorems**: no-hidden-distinction admission; support versus stiffness/probability distinction; record-conditioned preparation results.
8. **The Primitive Selection History-to-Curvature and Standard Model Attaching Map Closure Theorem in VERSF**: local MASD closure differential, unit-normalized local closure injection and source-attaching programme.
9. **The VERSF Master Substrate Action and Constitutive papers / Full-Faithful Source Assembly**: edge/link physical gauge carrier.
10. **Projective history-measure and GQSC-1R papers**: finite retained/global measure and no-post-hoc weighting discipline.
11. **Distinguishability Conservation and Gauge Structure**: gauge connection structure forced by smooth, equivariant, Fisher-metric-preserving comparison.
12. **PFDMI-1 and PFDMI-EX2A to EX2E**: field-dependent marked source, blind representation-projector recovery within the inherited typed carrier, exact four-channel completion census, faithful matrix-link carrier, noncentral native gauge-sensitive probabilities and Wilson-quotient rank 72.
13. **FGQT-1 Faithful Gauge Quotient, Bundle Census and Topological Charge Lattice**: faithful global group $S(U(3) \times U(2))$, the $\mathbb{Z}_6$ quotient and global bundle typing.
14. **Full-Faithful Source Assembly / HM4 later execution**: full faithful gauge quadratic descent and zero residual gauge content of the tested kernel.
15. **MASD Constitutive / Polar Bond Compatibility**: unique polar transport on occupied support and the bond equation tying realised record transport to the physical link.
16. **Admissibility / Born / Fold-to-Fact / Ordered Record papers**: distinguishable alternatives, Born weighting, stochastic realisation and persistent record formation, consumed at their declared grades.
17. **Ordered Records / Fold-to-Fact execution**: retained 84-class ledger, seven self-history no-record marks, 312 record-producing microscopic marks, the no-record spectral-radius certificate and the first-entry renewal law.
18. **Primitive Full-Carrier Provenance and Recurrent Fisher Pullback**: exact regulator factorisation P_n = P_H ⊗ P_N^n, frozen history factor and regulator-dependent neutral continuation.

19. **PFDMI EX2E / HCMR clocked marked-law execution**: native accepted-current probabilities, seven terminal conditional populations, physical gauge rank 72 and zero physical Gram drift over regulator levels 3 to 6.
20. **Quantum Gauge Completion**: background-field determinant, gauge-plus-ghost combination, Standard Model one-loop census, regulator-independence criterion and the explicit regulator debt.
21. **GRCE / gauge-response / Master Substrate Action**: finite plaquette curvature action, shared physical link, gauge Hessian and the fermion kinetic carrier $D_f[U]$.
22. **Refinement-stable holonomy and persistent gauge coupling papers**: physical refinement scale, link-as-integrated-connection identification, plaquette area scaling and $O((k h_n)^2)$ higher-derivative suppression.
23. **From Schrödinger to Dirac**: closure-graph Dirac Hamiltonian, entropy-derived Wilson stiffness and explicit spatial doubler lifting.
24. **Sequential Interface Transport / Global Gauge Measure and OS reconstruction**: time as sequential commitment, positive Euclidean transfer $T = e^{-a_t H}$, and the finite-regulator chiral-family and reflection-positivity requirements.