The VERSF Chiral Regulator Universality-Matching Theorem

From the Substrate-Derived Wilson Stiffness and Shared Faithful Link through the One-Species Overlap Window, Exact Finite Modified Chirality, Index Stability and Standard Model One-Loop Gauge Running

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Central executed result

source-explicit CAR/Fock edge transport ⇒ source-matched Wilson seed ⇒ $\rho\bigstar = r$ ⇒ $\delta\bigstar = \min(r, 1)$ ⇒ nonzero faithful-gauge overlap neighbourhood ⇒ normalised Hamiltonian-transfer Weyl pole ⇒ UV-local regulator remainder ⇒ conditional local one-loop fermion universality

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General Reader Summary

When physicists write the equation for matter particles on a fine grid instead of smooth space, the mathematics produces unwanted ghost copies of every particle. There is a classic cure: add a term that makes the ghost copies enormously heavy, so they fall out of view. This programme had already found that its own rules generate exactly such a term on their own, because rapidly varying configurations carry a natural cost. Nothing had to be borrowed to remove the ghosts.

But real matter has a handedness: left-handed and right-handed particles feel the weak force differently, and the classic cure seems at first to spoil handedness. There is a known mathematical resolution in which handedness is not destroyed but gently bent at finite resolution, straightening out perfectly as the grid becomes fine. The open question was whether the term this programme derived is of the right kind to fit into that resolution, or whether an unrelated construction would have to be bolted on.

This paper shows that it fits, and more. The derived term automatically separates the one real particle from all of its ghost copies, and the one remaining dial in the construction has a naturally best setting, chosen by the mathematics itself rather than by consulting experiment. Around that best setting the construction is robust: modest changes in the background force fields cannot break it, the size of change that is guaranteed safe is given by a single explicit formula that uses nothing measured in a laboratory, the bookkeeping that tracks handedness stays exact, and the theory remains local, meaning distant points do not directly influence one another. What remains open is extending this guarantee from a neighbourhood of simple backgrounds to every allowed background in every topological sector.

The paper can now go one step further. VERSF does not treat time as a fourth microscopic grid direction; it treats temporal development as sequential transfer. The remaining question was therefore whether the genuine three-space-plus-sequential-transfer description still produces the same low-energy chiral particle as the Euclidean overlap comparison regulator, and whether its finite-resolution corrections could alter the familiar logarithmic running of the gauge forces. The answer to the first is yes, and to the second the evidence so far says no. After separating the positive particle/antiparticle excitation spectrum from the signed local fermion operator, the sequential kernel returns the correctly normalised Weyl equation with coefficient exactly one. Its remaining finite-resolution terms carry additional powers of the regulator scale and therefore contribute only finite matching shifts and higher-derivative local corrections, which cannot generate a second infrared logarithm. One step of that bridge remains to be written out in full: the version of the sequential operator that carries the force fields explicitly, rather than only the free particle. Until that is done the running-of-forces conclusion is provisional, though everything checked so far points the same way. The stronger task of certifying every complete topological background component also remains open.

Headline Result

The VERSF Standard Model programme already contains four ingredients that were previously developed in separate parts of the corpus:

- a first-order closure-graph Dirac structure;
- an entropy-derived Wilson stiffness that removes lattice doublers;
- a shared faithful gauge link U in the gauge and fermion sectors of the Master Substrate Action;
- an admissible finite chiral-measure architecture whose reference representative is of Ginsparg-Wilson/overlap type.

The outstanding D-SG6 question is therefore not whether a mathematically consistent chiral regulator exists. It is whether the VERSF-derived fermionic regulator data can be derived or universality-matched to that admissible index-correct chiral family without using Standard Model target values.

This paper executes the first nontrivial form of that match.

The VERSF closure-Wilson branch supplies a positive dimensionless stiffness coefficient $r > 0$. The corresponding dimensionless Wilson seed has the universal momentum-space mass structure

$$\mathbf{X_\rho(q) = i \Sigma_\mu \gamma_\mu \sin q_\mu + r W(q) - \rho}$$

where

$W(q) = \Sigma_\mu (1 - \cos q_\mu)$

and ρ is the negative kernel-mass parameter of the overlap construction. The Hermitian kernel $H_\rho(q) = \gamma_5 X_\rho(q)$ satisfies

$$\mathbf{H_\rho(q)^2 = \Sigma_\mu \sin^2 q_\mu + [r\, W(q) - \rho]^2}$$

At a Brillouin-zone corner containing n_π components equal to π, $W = 2n_\pi$ and the Wilson mass is $M(n_\pi) = 2r\, n_\pi - \rho$. Therefore the interval

$$\mathbf{0 < \rho < 2r}$$

has a distinguished property: $M(0) = -\rho < 0$, while every doubler corner $n_\pi \geq 1$ has $M(n_\pi) \geq 2r - \rho > 0$. The Hermitian Wilson seed has opposite spectral sign on exactly one branch: the physical branch.

This produces the overlap/Ginsparg-Wilson operator

$$\mathbf{D_{ov} = (\rho/a)\,[1 + \gamma_5\, \mathrm{sgn}(H_\rho)] = (\rho/a)\,[1 + X_\rho\, (X_\rho^\dagger X_\rho)^{(-1/2)}]}$$

Writing $\bar{a} = a/\rho$, the operator satisfies the exact finite-regulator Ginsparg-Wilson relation

$$\mathbf{\gamma_5 D_{ov} + D_{ov} \gamma_5 = \bar{a}\, D_{ov} \gamma_5 D_{ov}}$$

The associated modified finite chirality operator is

$$\mathbf{\hat{\gamma}_5 = \gamma_5 (1 - \bar{a}\, D_{ov}) = -\mathrm{sgn}(H_\rho)}$$

with $\hat{\gamma}_5^\dagger = \hat{\gamma}_5$ and $\hat{\gamma}_5^2 = 1$, so $\hat{P}_\pm = (1 \pm \hat{\gamma}_5)/2$ are exact finite-regulator chiral projectors. The overlap operator intertwines the finite and ordinary chiralities exactly:

$$\mathbf{D_{ov}\, \hat{\gamma}_5 = -\gamma_5\, D_{ov}}$$

The operator is normal and γ_5-Hermitian, and its entire spectrum lies exactly on the circle $|\lambda - \rho/a| = \rho/a$. At the physical corner $D_{ov}(0) = 0$; at every former doubler corner in the interval $0 < \rho < 2r$, $D_{ov} = 2\rho/a$, the antipode of the physical zero on that circle. There is exactly one light species.

For small physical momentum $q_\mu = a\, p_\mu$,

$$\mathbf{D_{ov}(p) = i\, \gamma_\mu p_\mu + O(a\, p^2)}$$

while $\hat{\gamma}_5 = \gamma_5 + O(ap)$, so $\hat{P}_- \to P_L$ in the continuum orientation convention used by the VERSF matter ledger. The index is

$$\mathbf{index}\ D\_ov = \tfrac{1}{2}\,\mathrm{Tr}\,\hat{\gamma}_5 = -\tfrac{1}{2}\,\mathrm{Tr}\,\mathrm{sgn}(H\rho)$$

up to the declared orientation convention, and is stable under every continuous gauge-background deformation that does not close the Hermitian-kernel gap. On the flat background the kernel is pointwise traceless, so the flat-sector index is exactly zero; the corpus index-one witness lives in a topologically nontrivial background of the same architecture.

The parameter ρ is not a new physical Standard Model parameter. Every value inside $0 < \rho < 2r$ lies in the same one-species Wilson/overlap phase. Within that phase the regulator freedom is reduced structurally: the midpoint $\rho\bigstar = r$ is a max-gap representative, with exact flat Hermitian-kernel gap

$$\delta\bigstar = \mathrm{gap}\ Hr[I] = \min(r, 1)$$

The primitive Master-Action fermion transport is source-explicit. Its oriented edge difference B_U supplies a positive gauge-covariant roughness $L\_U = B\_U^\dagger B\_U$ whose flat symbol reproduces the Wilson mass function exactly after multiplication by r/2, so the source-matched seed is $X\rho[U] = K\_U + (r/2)L\_U - \rho I$. Weyl spectral perturbation then yields an explicit sufficient physical-background gap condition with a strictly positive critical link radius ε_crit, establishing a nonzero full-dimensional faithful-gauge neighbourhood in which the overlap index is stable and the overlap kernel is exponentially local, with an explicit computable decay rate.

On the four-dimensional Euclidean reference regulator the constants entering that bound are now evaluated: $\lambda\_max = 16$ and $d_+ = 4$ exactly, and $C\_kin \leq 4$ at canonical-reference grade. This gives the parameter-only safe radius

$$\varepsilon\_safe(r) = 2 \min(r, 1) / [4 + 8r + \sqrt{(4 + 8r)^2 + 8r \min(r, 1)}]$$

containing no Standard Model observable and no fitted quantity.

The Euclidean overlap calculation is only the chiral universality representative. The microscopic VERSF architecture instead retains three spatial closure directions together with sequential positive transfer. A further calculation now connects these two descriptions without identifying their ontologies.

For a massless physical fermion branch define the signed spatial Wilson operator, in frequency units,

$$K\_n(q) = (c/h\_n)\,[\Sigma i\ \alpha i \sin qi + r\,\beta\,W(q)]$$

with

$$W(q) = \Sigma i\,(1 - \cos qi)$$

Its square is scalar in spinor space,

$$K_n(q)^2 = (c^2/h_n^2)\,[\Sigma_i \sin^2 q_i + r^2\, W(q)^2]\, I$$

so $|K_n|$ reproduces the positive excitation spectrum but not the Clifford orientation. The positive particle/antiparticle Fock Hamiltonian may therefore be constructed from $|K_n|$, while the signed operator K_n must be retained in the local Grassmann kernel. The two roles must not be conflated.

Using the independently completed particle/antiparticle CAR sectors, the sequential local quadratic kernel is

$$D_n^{HT}(\omega, q) = [I - \exp(-a_t K_n(q)) \exp(i\omega a_t)] / a_t$$

at the standard fermionic coherent-state reconstruction grade. Expanding about the unique physical pole gives

$$D_n^{HT} = -i\omega I + c\, \alpha\cdot p + O(c\, h_n p^2) + O[a_t(\omega^2 + c^2p^2)]$$

Multiplication by γ^0/c therefore gives, up to the declared Fourier/Wick convention,

$$(\gamma^0/c)\, D_n^{HT} = i\gamma^\mu p_\mu + O[(h_n + c\, a_t)\, p^2]$$

Consequently the physical chiral blocks satisfy

$$D_{L,n}(p) = i\, \bar{\sigma}^\mu p_\mu + O[(h_n + c\, a_t)\, p^2]$$

$$D_{R,n}(p) = i\, \sigma^\mu p_\mu + O[(h_n + c\, a_t)\, p^2]$$

The coefficient of the continuum Weyl pole is exactly unity. The free sequential kernel also has an exact zero census: $D_{HT}(\omega, q) = 0$ if and only if $(\omega, q) = (0, 0)$ within the fundamental frequency/momentum zones, so sequential transfer introduces no independent temporal zero. If the refinement aspect ratio $c\, a_t/h_n$ remains bounded, this reduces to the single-cutoff form $O(h_n p^2)$ used in the earlier one-loop universality criterion. More generally the two-cutoff result is sufficient under simultaneous spatial and sequential refinement.

Finally, writing $\ell_n = \max(h_n, c\, a_t)$, the Hamiltonian-transfer kernel and the overlap representative differ around their common Weyl pole only by

$$D_{HT,\chi} - D_{ov,\chi} = O(\ell_n p^2)$$

At the free-pole/irrelevant-remainder grade, splitting the parity-even determinant into a scaling region around the unique Weyl pole and a complementary gapped regulator region shows why no second infrared logarithm can arise; promotion of this result to the gauged determinant is conditional on the locality, covariance and regularity conditions stated below. In the scaling region every regulator correction carries at least one factor $\ell_n k$, so instead of the logarithmic shell $\int dk/k$ one obtains

$$\int(dk/k)(\ell_n k)^m = [\ell_n^m/m](\Lambda^m - p_{ext}^m),\ m \geq 1$$

which contains no log p_ext. In the complementary region the absence of additional poles together with locality makes the effective action analytic in small external momentum. Its contribution is therefore a local derivative expansion.

Provided the gauged Hamiltonian-transfer extension satisfies the locality, covariance and regularity conditions of the one-loop power counting,

$$\Gamma_+^{HT} - \Gamma_+^{ov} = c_F \int \mathrm{tr}\, F^2 + \Sigma_{\Delta>4}\, c_\Delta\, \ell_n^{(\Delta-4)} \int O_\Delta$$

apart from the separately typed parity-odd/anomaly channel. The first term is a finite regulator matching shift and the remaining terms are higher-dimension local operators. Under the same conditions there is no second $F^2 \log \mu$ contribution.

The constructed Hamiltonian-transfer/shared-link family therefore has the same parity-even one-loop coefficient as the overlap representative:

$$b_{Weyl}^{HT}(R) = (2/3)\, T(R)$$

at conditional grade: the free-pole and irrelevant-remainder structure is established, while the fully gauged Hamiltonian-transfer operator D_HT[U], with its background-gauge transformation law and fermion-gauge vertices, remains to be written out and executed.

This upgrades the local D-SG6 universality bridge from a flat-symbol construction to a source-matched faithful-background theorem, and advances the local one-loop fermion-universality bridge to conditional-pass grade at the constructed Hamiltonian-transfer level. What remains is the component-wise global certificate over every admitted fixed-index background component.

Abstract

The VERSF fermion programme independently derives a first-order closure-graph Dirac structure and an entropy-generated Wilson stiffness $H_W = (r/2)\, \hbar c\, \xi\, (D_G^2 \otimes \beta)$, $r > 0$, whose high-frequency contribution is $O(\hbar c/\xi)$ and removes the ordinary discrete Dirac doublers. The Master Substrate Action separately requires the finite fermion kernel to use the same faithful physical link U as the gauge sector, and specifies the primitive fermion kinetic transport explicitly: matter crosses an oriented edge through the shared-link difference $\Psi t(e) - Ue\, \Psi s(e)$. The global gauge-measure programme further specifies an admissible finite chiral Dirac family and identifies Ginsparg-Wilson/overlap regularisation as an explicit reference realisation. The remaining D-SG6 debt is to derive or universality-match the substrate fermion regulator to such a local index-correct family.

This paper executes that match at the local source-matched grade.

The flat spectral part is exact. With $X\rho(q) = i\gamma\mu \sin q\mu + r\Sigma\mu(1 - \cos q\mu) - \rho$ and $H\rho = \gamma_5 X\rho$, the kernel obeys $H\rho^2 = \Sigma\mu \sin^2 q\mu + [rW(q) - \rho]^2$ and can become singular only at Brillouin corners,

where the scalar Wilson mass is $2r\, n\_\pi - \rho$. Hence for every $r > 0$ the interval $0 < \rho < 2r$ is a nonempty one-species window: the physical $n\_\pi = 0$ branch has negative kernel mass while every doubler branch has positive mass. The overlap operator $D\_ov = (\rho/a)[1 + \gamma_5 \operatorname{sgn} H\rho]$ then has exactly one zero, all doubler corners at eigenvalue $2\rho/a$, satisfies the exact Ginsparg-Wilson relation $\gamma_5 D\_ov + D\_ov\gamma_5 = \bar{a}D\_ov\gamma_5 D\_ov$ with $\bar{a} = a/\rho$, has exact finite chirality $\hat{\gamma}_5 = \gamma_5(1 - \bar{a}D\_ov) = -\operatorname{sgn} H\rho$ with $\hat{\gamma}_5^2 = 1$, and intertwines exactly, $D\_ov\ \hat{\gamma}_5 = -\gamma_5\ D\_ov$. It is moreover normal and γ_5-Hermitian with spectrum exactly on the circle $|\lambda - \rho/a| = \rho/a$. The low-momentum expansion returns $D\_ov = i\not{p} + O(ap^2)$ and $\hat{\gamma}_5 = \gamma_5 + O(ap)$, recovering the inherited VERSF Weyl projector. The finite index is index $D\_ov = -\frac{1}{2} \operatorname{Tr} \operatorname{sgn} H\rho$, locally constant while the Hermitian-kernel gap stays open; the flat-sector index vanishes exactly because $H\rho(q)$ is pointwise traceless with scalar square.

The source-matched part consumes the primitive fermion kinetic term of the Master Substrate Action and the executed full-faithful CAR/Fock gauge jet. Defining $B\_U\Psi$ on each oriented edge by the exact transported difference gives the positive gauge-covariant roughness $L\_U = B\_U^\dagger B\_U$, whose flat symbol reproduces the Wilson mass function exactly after multiplication by $r/2$. The source-matched seed is therefore $X\rho[U] = K\_U + (r/2)L\_U - \rho I$, with no second matter link.

Two structural calculations then fix the remaining regulator data without empirical input. First, the midpoint $\rho\bigstar = r$ maximises the flat Hermitian-kernel gap throughout the one-species phase, with the exact return

gap $Hr[I] = \delta\bigstar = \min(r, 1)$

Second, Weyl spectral perturbation together with the source edge operator yields an explicit sufficient physical-background gap condition: if

$[C\_kin + r\sqrt{(\lambda\_max\ d_+)}]\ \varepsilon\_U + (r\ d_+/2)\ \varepsilon\_U^2 < \min(r, 1)$

then $Hr[U]$ remains gapped. The corresponding critical radius has the closed form $\varepsilon\_crit = 2\delta\bigstar / (A + \sqrt{(A^2 + 2r\ d_+\ \delta\bigstar)})$ with $A = C\_kin + r\sqrt{(\lambda\_max\ d_+)}$, and is strictly positive, establishing a nonzero full-dimensional faithful-gauge neighbourhood in which the overlap index is stable and the overlap kernel is exponentially local with computable rate. The neighbourhood is a fixed-gauge link-chart statement; because the gap and index are gauge invariant, it extends verbatim to whole gauge orbits through the orbit radius $\varepsilon\_orb(U) = \inf$ over G of $\max\_e \|R\_f(U\_e^{\wedge}G) - I\|$, and a whole-ledger radius $\varepsilon\_orb^{\wedge}SM$, in which one gauge transformation serves every fermion representation simultaneously, certifies the entire frozen ledger at once.

On the four-dimensional Euclidean reference regulator the constants are evaluated. The flat roughness symbol $L\_I(q) = 4\Sigma\mu \sin^2(q\mu/2)$ attains its maximum 16 at $q = (\pi, \pi, \pi, \pi)$, so $\lambda\_max = 16$ exactly; one oriented forward edge per regulated direction gives $d_+ = 4$; and the canonical nearest-neighbour covariant representative of the frozen flat kinetic symbol gives $C\_kin \le 4$. The sufficient condition therefore sharpens to $(4 + 8r)\varepsilon\_U + 2r\varepsilon\_U^2 < \min(r, 1)$, with the explicit parameter-only radius

$\varepsilon\_safe(r) = 2 \min(r, 1) / [4 + 8r + \sqrt{(4 + 8r)^2 + 8r \min(r, 1)}]$

A carrier-mixing firewall forbids importing the source-native diamond value $\lambda\_max = 4$ into this bound: constants and gaps must be evaluated on the same declared carrier.

A further layer then connects the Euclidean representative to the genuine VERSF three-space-plus-sequential-transfer architecture without identifying their ontologies. The independently completed particle/antiparticle CAR structure separates the positive Fock excitation spectrum from the signed local Clifford operator, so the signed spatial Wilson operator K_n may be retained in the local Grassmann kernel. The sequential coherent-state kernel D_n^HT = $[I - \exp(-a\_t K\_n) \exp(i\omega a\_t)]/a\_t$ then returns the correctly normalised Weyl pole, $D\_L,n = i\bar{\sigma}\mu p\mu + O[(h\_n + ca\_t)p^2]$ and $D\_R,n = i\sigma\mu p\mu + O[(h\_n + ca\_t)p^2]$, with unit coefficient, and has an exact free zero census: $D\_HT(\omega, q) = 0$ if and only if $(\omega, q) = (0, 0)$ in the fundamental zones, so sequential transfer introduces no independent temporal zero. Around the common pole the difference from the overlap representative is $O(\ell\_n p^2)$ with $\ell\_n = \max(h\_n, ca\_t)$, and a scaling/gapped-region split shows the regulator remainder produces only finite gauge-coupling matching and higher-derivative local operators, never a second parity-even F^2 infrared logarithm. The constructed Hamiltonian-transfer/shared-link family therefore shares the local one-loop coefficient $b\_Weyl(R) = (2/3)T(R)$ at conditional-pass grade: the free pole and irrelevant-remainder structure are established, while the fully gauged Hamiltonian-transfer operator D_HT[U], with its background-gauge transformation law and fermion-gauge vertices, remains to be constructed and executed.

The paper therefore supplies an explicit VERSF-seeded Ginsparg-Wilson universality representative and closes the local source-matched part of the D-SG6 bridge, together with a conditional local one-loop fermion-universality result for the constructed Hamiltonian-transfer family. The remaining strict returns are the gauged Hamiltonian-transfer execution, source identity, freezing the complete physical Master-Action K_U/D_f[U] family, and the global certificate: a positive uniform gap on each complete admitted fixed-index component of the physical faithful-group background space, with spectral crossings confined to component boundaries.

1. Problem Statement

The relevant VERSF chain is now

closure dynamics → Dirac carrier → Wilson stiffness → chiral matter ledger → finite chiral regulator → fermion determinant

The first four arrows have substantial prior support. The last two have remained separated.

The Wilson paper answers: *how are spurious discrete Dirac species removed?*

The chiral gauge-measure papers answer: *what properties must the finite Standard Model fermion regulator possess?*

The present paper asks:

Does the VERSF Wilson seed occupy the correct overlap/GW phase, and can the seed itself be written from the source action?

If yes, the two strands can be joined by universality rather than by constructing a completely unrelated fermion regulator.

2. Source-Locked Inputs

2.1 First-order closure Dirac structure

The VERSF closure-graph fermion programme derives the first-order structure whose locally regular continuum form is Dirac. At the free regular-lattice symbol level,

$$D_{\text{naive}}(q) \sim (i/a)\, \Sigma_\mu\, \gamma_\mu \sin q_\mu$$

The sine structure is responsible for discrete doubling.

2.2 Closure entropy gives the Wilson mass structure

$$H_W = (r/2)\, \hbar c\, \xi\, (D_G^2 \otimes \beta),\ r > 0$$

On a regular lattice branch this supplies $r(1 - \cos q)/a$ per regulated direction. Thus the universal dimensionless Wilson mass function is $r\, W(q)$ with $W(q) = \Sigma_\mu(1 - \cos q_\mu)$.

The coefficient r is not fitted to the Standard Model. Its form is derived from closure roughness; its numerical normalisation remains a separate closure-entropy calculation.

For the present theorem only $r > 0$ is required.

2.3 The physical gauge link is shared

$$\Gamma_{\text{CAR}} = \Sigma_f\, \Psi_f\, D_f[U]\, \Psi_f$$

The same physical faithful link U appears in the gauge sector, and the Master Substrate Action specifies the primitive kinetic transport explicitly: on each oriented edge $e: i \to j$ the matter difference is $\Psi_j - U_e \Psi_i$. A valid fermion universality representative must therefore use this same gauge transport, in the representation carried by each fermionic species.

2.4 The chiral regulator architecture already exists

The global quantum programme requires, on every regulated faithful-group bundle, a chiral Dirac family that is:

1. gauge covariant;
2. local;
3. faithful-quotient compatible;
4. continuous on admissible gauge backgrounds;
5. index-correct;
6. CAR/Fock compatible;
7. regulator-removal stable.

The corpus already identifies a Ginsparg-Wilson/overlap family as an admissible reference implementation. Thus overlap mathematics is not introduced here as a new physical postulate. It is the already-declared comparison class.

3. Scope Firewall: Euclidean Regulator Versus Primitive Time

The overlap calculation below uses the standard finite Euclidean Wilson kernel. This must not be misread as a claim that VERSF has four primitive symmetric lattice directions. The strict VERSF microscopic description retains

3 spatial closure directions + sequential positive transfer

The four-dimensional Wilson momentum variable used below belongs to the Euclidean regulator representative. The purpose is universality matching:

VERSF Hamiltonian-transfer regulator ~ Euclidean GW/overlap regulator

at the level of local gauge covariance, low-energy pole content, chiral representation, anomaly/index class, and physical renormalised observables. The ontology of the two regulators need not be identical. That distinction is already permitted by the VERSF regulator-independence theorem.

4. The VERSF-Seeded Wilson Kernel

Take a finite Euclidean reference spacing a and set

$s_\mu(q) = \sin q_\mu$, $W(q) = \Sigma_{\mu=1}^4 (1 - \cos q_\mu)$

$$\mathbf{X_\rho(q) = i\,\gamma_\mu\, s_\mu(q) + r\, W(q) - \rho}$$

The parameter ρ is the standard negative kernel-mass coordinate. It is not a physical fermion mass. The physical massless branch will emerge as the zero of the overlap transform. Writing $M_\rho(q) = rW(q) - \rho$,

$X_\rho = i\gamma\cdot s + M_\rho$, $X_\rho^\dagger = -i\gamma\cdot s + M_\rho$

$$X_\rho^\dagger X_\rho = \Sigma_\mu \sin^2 q_\mu + [rW(q) - \rho]^2$$

where $s^2 = \Sigma_\mu \sin^2 q_\mu$. Since $X_\rho^\dagger X_\rho$ is a positive scalar multiple of the identity at every q, all square roots below are unambiguous scalar functions.

5. Hermitian Wilson Kernel

$$H_\rho = \gamma_5 X_\rho$$

Wilson γ_5-Hermiticity gives $X_\rho^\dagger = \gamma_5 X_\rho \gamma_5$, so $H_\rho^\dagger = H_\rho$ and $H_\rho^2 = X_\rho^\dagger X_\rho$:

$$H_\rho^2(q) = \Sigma_\mu \sin^2 q_\mu + [rW(q) - \rho]^2$$

This equation completely determines where the kernel may change spectral sign. Two further exact pointwise facts will be used later. First, $H_\rho(q)$ is traceless for every q:

$$\mathrm{Tr}\, H_\rho(q) = i\, s_\mu\, \mathrm{Tr}(\gamma_5\gamma_\mu) + M_\rho\, \mathrm{Tr}\, \gamma_5 = 0$$

Second, because $H_\rho(q)^2$ is a positive scalar multiple of the identity, the four spinor eigenvalues of $H_\rho(q)$ are $\pm\sqrt{(H_\rho^2(q))}$, each with multiplicity two.

6. The Exact One-Species Window

This section is an executed calculation.

A zero of H_ρ requires simultaneously $\sin q_\mu = 0$ for every μ and $rW(q) - \rho = 0$. The first condition means every momentum component is either $q_\mu = 0$ or $q_\mu = \pi$. Let n_π be the number of π-components. Since $1 - \cos 0 = 0$ and $1 - \cos \pi = 2$,

$$W = 2\, n_\pi$$

and the Wilson mass at that corner is

$$M(n_\pi) = 2r\, n_\pi - \rho$$

For the physical corner, $M(0) = -\rho$. For the nearest doubler corners, $M(1) = 2r - \rho$. Now impose

$$0 < \rho < 2r$$

Then $M(0) < 0$ but $M(1) > 0$, and since $M(n_\pi) \geq M(1)$ for all $n_\pi \geq 1$, every doubler corner has $M(n_\pi) > 0$.

Theorem 1: VERSF One-Species Wilson-Sign Window

[EXACT FOR THE REFERENCE WILSON SYMBOL; DEPENDS ONLY ON r > 0]

If the VERSF closure stiffness satisfies $r > 0$, then the interval $0 < \rho < 2r$ is nonempty and has exactly one negative Wilson-mass corner: $M(0) = -\rho < 0$. Every corner with at least one π-component has $M(n_\pi) > 0$. Thus exactly one branch has the opposite Hermitian-kernel spectral orientation.

No Standard Model mass, coupling, beta coefficient or experimental chirality datum is needed to select this interval.

7. Phase Boundaries and Regulator Meaning of ρ

The sign of a corner changes whenever $2r\, n_\pi - \rho = 0$. Thus the Wilson phase boundaries occur at

$$\rho = 0, 2r, 4r, 6r, 8r$$

for the four-dimensional reference regulator. The first phase, $0 < \rho < 2r$, is the unique corner-sign phase in which only the physical branch has negative Wilson mass. Variation of ρ inside this interval cannot change the corner species census: it is regulator freedom. Crossing a boundary can change the topological/species sector.

Corollary 1.1: ρ Is Not a New Standard Model Parameter

Within $0 < \rho < 2r$, ρ labels a family of regulator representatives with the same low-energy species census. It is therefore not a new physical parameter to be fitted to observation. This is consistent with the VERSF numerical non-insertion firewall. Section 18.2 reduces the residual freedom further, structurally, by exact gap maximisation.

8. Construction of the Overlap Operator

Because $H\rho$ is Hermitian and gapped away from its allowed topological crossings, define

$$\varepsilon\rho = \mathrm{sgn}(H\rho) = H\rho\, (H\rho^2)^{(-1/2)},\ \varepsilon\rho^\dagger = \varepsilon\rho,\ \varepsilon\rho^2 = 1$$

and the unitary

$$V\rho = \gamma_5\, \varepsilon\rho,\ V\rho^\dagger = \gamma_5\, V\rho\, \gamma_5,\ V\rho^\dagger V\rho = 1$$

Now define

$$\mathbf{D_{ov} = (\rho/a)(1 + V\rho) = (\rho/a)[1 + \gamma_5\, \mathrm{sgn}(H\rho)] = (\rho/a)[1 + X\rho\, (X\rho^\dagger X\rho)^{(-1/2)}]}$$

This is the explicit chiral universality representative.

9. Spectral Circle, Normality and Exact Pole Census

Because $V\rho$ is unitary, three exact global spectral facts follow immediately.

First, D_ov is γ_5-Hermitian: $D_{ov}^\dagger = \gamma_5 D_{ov} \gamma_5$, directly from $V\rho^\dagger = \gamma_5 V\rho\gamma_5$.

Second, D_ov is normal, since it is an affine function of a single unitary.

Third, every eigenvalue has the form $\lambda = (\rho/a)(1 + e^{i\theta})$ with $e^{i\theta} \in \text{spec } V\rho$, so

$$|\lambda - \rho/a| = \rho/a$$

Equivalently, the operator identity

$$D_{ov}^\dagger D_{ov} = (\rho/a)(D_{ov} + D_{ov}^\dagger)$$

holds exactly: expanding, $(\rho/a)^2(1 + V\rho^\dagger)(1 + V\rho) = (\rho/a)^2(2 + V\rho + V\rho^\dagger) = (\rho/a)(D_{ov} + D_{ov}^\dagger)$. The whole spectrum lies on the Ginsparg-Wilson circle of radius ρ/a centred at ρ/a. Doubler modes cannot hide at intermediate eigenvalues: the circle pins the spectrum between the physical zero and its antipode $2\rho/a$.

Theorem 2A: Ginsparg-Wilson Circle Theorem

[EXACT FOR THE OVERLAP REPRESENTATIVE]

The VERSF-seeded overlap representative is normal, γ_5-Hermitian, and satisfies $D_{ov}^\dagger D_{ov} = (\rho/a)(D_{ov} + D_{ov}^\dagger)$. Its spectrum lies exactly on the circle $|\lambda - \rho/a| = \rho/a$. In particular no eigenvalue exceeds $2\rho/a$ in modulus, and $\lambda = 0$ and $\lambda = 2\rho/a$ are the only real spectral points available to Brillouin corners.

Now execute the corner census. At the physical corner $q = 0$, $X\rho(0) = -\rho$, so $X\rho/\sqrt{(X\rho^\dagger X\rho)} = -1$ and

$$D_{ov}(0) = 0$$

At a doubler corner in the one-species window, the Wilson mass is positive, $X\rho = 2r\, n_\pi - \rho > 0$, so $X\rho/\sqrt{(X\rho^\dagger X\rho)} = +1$ and

$$D_{ov} = 2\rho/a$$

at every former doubler corner: the unwanted modes sit at the regulator scale, at the antipode of the physical zero on the spectral circle.

Theorem 2: Exact Overlap One-Zero Theorem

[EXACT ON THE FLAT REFERENCE BRANCH FOR $0 < \rho < 2r$]

The overlap operator generated from the VERSF-seeded Wilson mass structure has exactly one zero in the Brillouin zone. The physical branch satisfies D_ov(0) = 0, while every Wilson doubler corner satisfies |D_ov| = 2ρ/a. Thus the overlap representative contains exactly one light fermion species per admitted representation.

10. Exact Ginsparg-Wilson Relation

Set

ā = a/ρ

so D_ov = (1/ā)(1 + V) with V = Vρ. The unitary satisfies $V\gamma_5 V = \gamma_5$, because $V\gamma_5 V = \gamma_5\varepsilon\, \gamma_5\, \gamma_5\varepsilon = \gamma_5\varepsilon^2 = \gamma_5$. Therefore

$\bar{a}\, D\, \gamma_5\, D = (1/\bar{a})(1 + V)\gamma_5(1 + V) = (1/\bar{a})(\gamma_5 + \gamma_5 V + V\gamma_5 + V\gamma_5 V) = (1/\bar{a})(2\gamma_5 + \gamma_5 V + V\gamma_5)$

But $\gamma_5 D + D\gamma_5 = (1/\bar{a})[\gamma_5(1 + V) + (1 + V)\gamma_5]$ is exactly the same expression. Hence

γ_5 D_ov + D_ov γ_5 = ā D_ov γ_5 D_ov

Theorem 3: Exact Finite Ginsparg-Wilson Theorem

[EXACT FOR THE OVERLAP REPRESENTATIVE]

The VERSF-seeded overlap representative satisfies the Ginsparg-Wilson relation exactly at finite regulator. No ordinary finite-cutoff anticommutation relation $\{\gamma_5, D\} = 0$ is required. This is the correct resolution of the naive Wilson/chirality tension.

11. Why the Naive P_L Projection Was the Wrong Test

The Wilson kernel does not preserve ordinary continuum chirality at finite regulator. Thus the statement P_L D_W P_L is not the correct finite chiral regulator. The previous apparent conflict, that Wilson lifting needs a chirality-breaking mass channel while the Standard Model needs chirality, is precisely what the Ginsparg-Wilson relation resolves. Finite chirality is deformed rather than destroyed.

Firewall FW-CWGW1: Naive Projection Firewall

The ordinary continuum projector P_L = $(1 - \gamma_5)/2$ must not be imposed directly on the Wilson seed and then used to claim a complete finite chiral theory. The correct finite projector is derived from the GW operator itself.

12. Exact Modified Chirality

$\hat{\gamma}_5 = \gamma_5$ (1 − ā D_ov)

Since $\bar{a}\, D\_ov = 1 + V$, we have $1 - \bar{a}\, D\_ov = -V$, so $\hat{\gamma}_5 = -\gamma_5 V$. But $V = \gamma_5\, \mathrm{sgn}(H\rho)$, hence

$$\boldsymbol{\hat{\gamma}_5 = -\mathrm{sgn}(H\rho)}$$

This is an unusually transparent result: finite chirality is exactly the spectral orientation of the Hermitian Wilson kernel. Immediately,

$$\boldsymbol{\hat{\gamma}_5^\dagger = \hat{\gamma}_5,\ \hat{\gamma}_5^2 = 1}$$

$$\boldsymbol{\hat{P}_\pm = (1 \pm \hat{\gamma}_5)/2}$$

are exact finite-regulator chiral projectors.

Theorem 4: Hermitian-Sign Chirality Theorem

[EXACT]

For the overlap representative, $\hat{\gamma}_5 = -\mathrm{sgn}(H\rho)$. Thus the same Wilson-kernel spectral sign that distinguishes the physical branch from doublers also defines the exact finite-regulator chirality. The doubler-removal problem and the finite-chirality problem are therefore not separate mechanisms in the overlap representative. They are two consequences of the same Hermitian-kernel sign structure.

13. Exact Chiral Intertwining

Using the GW relation, $D\gamma_5 + \gamma_5 D = \bar{a} D\gamma_5 D$, compute

$$D\, \gamma_5\, (1 - \bar{a}D) = D\gamma_5 - \bar{a}D\gamma_5 D = -\gamma_5 D$$

Hence

$$\boldsymbol{D\_ov\ \hat{\gamma}_5 = -\gamma_5\ D\_ov}$$

Equivalently,

$$D\, \hat{P}_- = P_+\, D,\quad D\, \hat{P}_+ = P_-\, D$$

This is the finite chiral mapping structure required by the regulated Weyl theory.

Theorem 5: Exact Finite Weyl Intertwining

[EXACT]

The overlap operator maps the modified finite chiral subspace into the ordinary opposite-spinor codomain exactly. Thus finite Weyl fermions are well defined without demanding that the Wilson seed itself commute or anticommute with ordinary γ_5.

14. Continuum Limit

With $q_\mu = a p_\mu$,

$\sin q_\mu = a p_\mu + O(a^3p^3)$, $1 - \cos q_\mu = \frac{1}{2} a^2 p_\mu^2 + O(a^4p^4)$

Therefore

$X_\rho = -\rho + i a \gamma_\mu p_\mu + \frac{1}{2} r a^2 p^2 + O(a^3p^3)$

$(X_\rho^\dagger X_\rho)^{(1/2)} = \rho + O(a^2p^2)$

$X_\rho/(X_\rho^\dagger X_\rho)^{(1/2)} = -1 + (ia/\rho) \gamma_\mu p_\mu + O(a^2p^2)$

Substituting into D_{ov},

$\mathbf{D_{ov} = i \gamma_\mu p_\mu + O(a p^2)}$

The continuum coefficient is automatically unity. No kinetic normalisation depending on ρ survives.

15. Continuum Limit of the Modified Projector

Since $\hat{\gamma}_5 = \gamma_5(1 - \bar{a}D)$ and $\bar{a}D = O(ap)$,

$\boldsymbol{\hat{\gamma}_5 = \gamma_5 + O(ap)}$, $\boldsymbol{\hat{P}_\pm = P_\pm + O(ap)}$

Therefore the finite overlap chirality tends directly to the continuum Weyl chirality used by the VERSF matter representation theorem. In the inherited naming convention $P_W = P_L$, so

$\hat{P}_- \to P_L$

for the weak-active matter orientation.

Theorem 6: VERSF Chiral-Ledger Matching Theorem

[CONDITIONAL EXACT ON THE INHERITED P_W = P_L REPRESENTATION RESULT]

The exact finite overlap projector tends to the VERSF weak-active chiral projector: $\hat{P}_- \to P_L = P_W$ on the frozen weak-active matter sector. The physical right-handed singlet carriers remain represented independently by their corresponding right-handed Weyl sectors. No physical u_R, d_R or e_R state is removed by the no-doubler construction.

16. Index Formula and the Flat-Sector Index

For a GW operator, on a finite regulator carrier with the standard balanced spin trace convention,

index D = ½ Tr $\hat{\gamma}_5$

Since $\hat{\gamma}_5 = -\mathrm{sgn}(H\rho)$,

index D_ov = −½ Tr sgn(Hρ)

This is the spectral-flow form of the index. If the gauge background changes continuously while zero remains excluded from the spectrum of Hρ, no eigenvalue of sgn(Hρ) can change from +1 to −1, so the index remains constant.

Theorem 7: Gap-Protected Index Stability

[EXACT]

On any connected set of gauge backgrounds for which $0 \notin$ spec Hρ[U], the overlap index is constant. A topology change requires a Hermitian-kernel spectral crossing. This is precisely the structure required for the regulated topological sectors used by the wider VERSF global gauge-measure programme.

The flat sector can now be evaluated exactly. By the two pointwise facts of Section 5, Hρ(q) is traceless and its eigenvalues are $\pm\sqrt{(H\rho^2(q))}$ with equal multiplicity two. Hence Tr sgn Hρ(q) = 0 at every momentum, and the momentum integral of zero is zero:

index D_ov[I] = 0

Theorem 7A: Flat-Sector Index Zero

[EXACT IN THE DECLARED BALANCED TRACE CONVENTION]

On the flat shared-link background the Hermitian Wilson kernel is pointwise traceless with scalar square, so sgn Hρ(q) is pointwise traceless and the overlap index vanishes identically for every ρ in the one-species window. The flat branch is the trivial topological sector. The corpus index-one witness therefore necessarily lives in a topologically nontrivial background of the same architecture, exactly as required for consistency between the local analysis here and the global gauge-measure construction.

17. Shared-Link Gauge Covariance

Let G_R(g) denote the gauge transformation in fermion representation R. Assume the Wilson seed uses the same faithful physical link U and transforms as

Xρ[U^g] = G_R(g) Xρ[U] G_R(g)^(−1)

Since gauge transformations act on internal representation space and commute with γ_5,

$H\rho[U^g] = G_R(g)\, H\rho[U]\, G_R(g)^{-1}$

Functional calculus respects conjugation, so $\operatorname{sgn} H\rho[U^g] = G_R(g) \operatorname{sgn} H\rho[U]\, G_R(g)^{-1}$. Therefore

$$\mathbf{D_{ov}[U^g] = G_R(g)\, D_{ov}[U]\, G_R(g)^{-1}}$$

Theorem 8: Shared-Link Overlap Covariance

[EXACT GIVEN THE GAUGE-COVARIANT SHARED-LINK WILSON SEED]

The overlap transform preserves the exact gauge covariance of the Wilson seed. Thus no second gauge connection is introduced by the chiral completion. The fermion regulator remains a function of the same physical link U as the gauge sector.

18. Source-Matched Faithful-Background Gap and Locality Theorem

The flat analysis above reaches the overlap construction from the Wilson symbol alone. The Master-Action and full-faithful source papers permit a stronger return, because the primitive finite fermion transport is source-explicit. This section constructs the seed from the source action, fixes the regulator representative by exact gap maximisation, and proves a nonzero faithful-background gap and locality neighbourhood.

18.1 Source-explicit primitive CAR/Fock difference

$$\Gamma_{kin}^{(0)} = \Sigma_{\{e:\, i\to j\}}\, \bar{\Psi}_j\, \gamma_e\, (\Psi_j - U_e \Psi_i) + \text{h.c.}$$

For fermion representation R_f define the oriented edge operator

$$(B_U^{(f)}\, \Psi)_e = \Psi_{\{t(e)\}} - R_f(U_e)\, \Psi_{\{s(e)\}}$$

Under a faithful local gauge transformation the edge difference transforms covariantly, so the positive operator

$$L_U^{(f)} = B_U^{(f)\dagger} B_U^{(f)} \geq 0$$

obeys $L_U^{(f)}[U^G] = G\, L_U^{(f)}[U]\, G^{-1}$. This is the source-derived gauge-covariant roughness associated with the primitive fermion kinetic transport.

On the regular flat regulator, B_U reduces to the ordinary transported finite difference and

$$L_I(q) = 4\, \Sigma_\mu \sin^2(q_\mu/2) = 2\, \Sigma_\mu\, (1 - \cos q_\mu)$$

Therefore

$(r/2)\, L\_I(q) = r\, \Sigma\mu\, (1 - \cos q\mu)$

exactly reproduces the Wilson mass function used in the one-species calculation. The source-matched shared-link Wilson seed is therefore

$X\rho^{(f)}[U] = K\_U^{(f)} + (r/2)\, L\_U^{(f)} - \rho\, I$

where $K\_U^{(f)}$ is the canonically normalised first-order operator extracted from the same primitive CAR/Fock kinetic term. The full-faithful execution independently confirms that differentiation of this shared-link CAR/Fock transport spans the complete 144-dimensional faithful gauge tangent, rather than a selected Abelian or factorwise subset.

Theorem 9A: Source-Matched Wilson-Seed Theorem

[CONDITIONAL EXACT ON THE INHERITED CLOSURE-WILSON STIFFNESS $r > 0$]

The primitive Master-Action fermion difference canonically supplies a local shared-link gauge-covariant positive roughness $L\_U = B\_U^\dagger B\_U$. Multiplication by $r/2$ reproduces the previously derived Wilson mass function on the flat regular branch. Consequently the overlap seed can be written entirely from source-matched VERSF objects as $X\rho[U] = K\_U + (r/2)L\_U - \rho I$, providing a source-matched representative: matching the flat symbol does not exclude other gauge-covariant irrelevant completions, but no second matter link and no independently normalised Wilson gauge transport is introduced by this one.

18.2 Max-gap representative inside the one-species phase

Theorem 1 fixed the window $0 < \rho < 2r$. The remaining regulator freedom can now be reduced structurally by maximising the flat Hermitian-kernel gap.

Lemma 18.2.1 (trigonometric normal form). Write $x\mu = 1 - \cos q\mu \in [0, 2]$, $S = \Sigma\mu\, x\mu$ and $Q = \Sigma\mu\, x\mu^2$. Then $\sin^2 q\mu = x\mu(2 - x\mu)$, so

$\Sigma\mu \sin^2 q\mu = 2S - Q$

and $S^2 - Q = 2\, \Sigma\_\{\mu<\nu\}\, x\mu\, x\nu \geq 0$, hence $Q \leq S^2$, with equality iff at most one component $x\mu$ is nonzero. Also $Q \leq 2S$, since each $x\mu \leq 2$.

Set the candidate

$\rho\star = r$

At $\rho = r$ the Wilson mass is $M = r(S - 1)$ and

$Hr^2(q) = 2S - Q + r^2(S - 1)^2$

Lower bound, case $r \geq 1$. Then $r^2(S - 1)^2 \geq (S - 1)^2$, so

$$H_r^2 \geq 2S - Q + (S - 1)^2 = 1 + S^2 - Q \geq 1$$

by the Lemma. Equality is attained at momenta with exactly one nonzero component satisfying $x = 1$, for example $q = (\pi/2, 0, 0, 0)$, where $S = 1$, $Q = 1$. Hence gap = 1.

Lower bound, case $0 < r \leq 1$. For $S \geq 2$ the mass term alone gives $H_r^2 \geq r^2(S - 1)^2 \geq r^2$, and $2S - Q \geq 0$ by the Lemma. For $0 \leq S \leq 2$, use $Q \leq S^2$ and set $u = S - 1 \in [-1, 1]$:

$$H_r^2 \geq 2S - S^2 + r^2(S - 1)^2 = 1 - (1 - r^2)\, u^2 \geq r^2$$

with equality at $u = \pm 1$. Both endpoints are attained: at the physical corner ($S = 0$, $H_r^2 = r^2 = \rho^2$) and at the nearest doubler corner ($S = 2$ in one component, $H_r^2 = r^2 = (2r - \rho)^2$). Hence gap = r.

Combining the cases,

$\delta\star := \text{gap } H_r[I] = \min(r, 1)$

Optimality over the window. For arbitrary $0 < \rho < 2r$, three explicit witnesses bound the gap from above:

- the physical corner gives $\text{gap}^2 \leq H_\rho^2(0) = \rho^2$;
- the nearest doubler corner gives $\text{gap}^2 \leq (2r - \rho)^2$;
- the mid-edge momentum $q\star$ with a single component solving $1 - \cos q = \rho/r$ (which exists since $\rho/r < 2$) and all others zero sets the Wilson mass to zero, giving $\text{gap}^2 \leq (\rho/r)(2 - \rho/r) \leq 1$.

Hence for every ρ in the window,

$$\text{gap } H_\rho[I] \leq \min(\rho, 2r - \rho, \sqrt{[(\rho/r)(2 - \rho/r)]}) \leq \min(r, 1)$$

and $\rho = r$ attains the bound.

Theorem 9B: Max-Gap Midpoint Theorem

[EXACT ON THE FLAT OVERLAP REFERENCE BRANCH]

Within the one-species interval $0 < \rho < 2r$, the midpoint $\rho\star = r$ is a max-gap regulator representative. Its exact Hermitian-kernel gap is $\delta\star = \min(r, 1)$, and no other ρ in the window exceeds this value. The choice is structural regulator optimisation and uses no Standard Model observable.

18.3 Faithful-background perturbation bound

Let $H_0 = H_r[I]$. For any Hermitian shared-link background kernel, Weyl spectral perturbation gives

$$\text{gap}\, H_r[U] \geq \text{gap}\, H_0 - \|H_r[U] - H_0\|$$

Since γ_5 is unitary, $\|H_r[U] - H_0\| = \|X_r[U] - X_r[I]\|$. Hence the elementary sufficient condition

$$\mathbf{\|X_r[U] - X_r[I]\| < \min(r, 1)}$$

already guarantees an open overlap gap and fixed index. The source edge difference sharpens this to an explicit link-neighbourhood bound. Define

$$\varepsilon_U = \max_e \|R_f(U_e) - I\|$$

and let d_+ be the maximum number of oriented source edges leaving one regulator vertex. If $C_U = B_U - B_I$, then C_U acts edgewise by $(C_U \Psi)_e = -(R_f(U_e) - I)\, \Psi_{s(e)}$, so

$$\|C_U \Psi\|^2 = \Sigma_e \|(R_f(U_e) - I)\, \Psi_{s(e)}\|^2 \leq \varepsilon_U^2\, d_+ \|\Psi\|^2, \text{ hence } \|C_U\| \leq \sqrt{d_+}\, \varepsilon_U$$

Let $\lambda_{max} = \|B_I^\dagger B_I\|$. Since $L_U - L_I = B_I^\dagger C_U + C_U^\dagger B_I + C_U^\dagger C_U$,

$$\mathbf{\|L_U - L_I\| \leq 2\sqrt{\lambda_{max}\, d_+}\, \varepsilon_U + d_+\, \varepsilon_U^2}$$

Finally define the fixed kinetic link-Lipschitz constant C_{kin} by $\|K_U - K_I\| \leq C_{kin}\, \varepsilon_U$ on the frozen local regulator. This is calculable from the published γe, edge weights and kinetic normalisation and is not a new physical parameter. Therefore

$$\mathbf{\|X_r[U] - X_r[I]\| \leq [C_{kin} + r\sqrt{\lambda_{max}\, d_+}]\, \varepsilon_U + (r\, d_+/2)\, \varepsilon_U^2}$$

A sufficient source-matched faithful-background gap condition is consequently

$$\mathbf{[C_{kin} + r\sqrt{\lambda_{max}\, d_+}]\, \varepsilon_U + (r\, d_+/2)\, \varepsilon_U^2 < \min(r, 1)}$$

Writing $A = C_{kin} + r\sqrt{\lambda_{max}\, d_+}$ and $\delta\bigstar = \min(r, 1)$, the permitted radius is the positive root of the quadratic, with two equivalent closed forms:

$$\mathbf{\varepsilon_{crit} = [-A + \sqrt{A^2 + 2r\, d_+\, \delta\bigstar}] / (r\, d_+) = 2\delta\bigstar / (A + \sqrt{A^2 + 2r\, d_+\, \delta\bigstar})}$$

The second form makes positivity and monotonicity manifest: $\varepsilon_{crit} > 0$ for every $r > 0$, ε_{crit} is strictly increasing in $\delta\bigstar$ and strictly decreasing in A and in d_+, and in the small-quadratic regime $\varepsilon_{crit} \to \delta\bigstar/A$, the pure Lipschitz radius.

Theorem 9C: Local Faithful-Background Gap Theorem

[CONDITIONAL EXACT ON THE SOURCE-MATCHED KINETIC NORMALISATION; FIXED-GAUGE LINK CHART]

At the max-gap representative $\rho = r$, every faithful-link background in the identity link chart satisfying $\varepsilon\_U < \varepsilon\_crit$ retains a nonzero Hermitian-kernel gap, bounded below by $\delta\bigstar - A\,\varepsilon\_U - (r\,d_{+}/2)\,\varepsilon\_U^2 > 0$. The overlap index is therefore constant throughout that connected neighbourhood. Because the executed CAR/Fock gauge jet has full rank on all 144 faithful gauge directions, the result is a full-dimensional local gauge-background theorem rather than a restricted factorwise perturbation.

The chart status matters for interpretation. The radius $\varepsilon\_U = \max\_e \|R\_f(U\_e) - I\|$ is not invariant under local gauge transformations: a gauge transform of the flat connection can carry individual links far from the identity even though the transformed kernel is unitarily equivalent and has exactly the same spectrum. Theorem 9C is therefore a sufficient certificate in a chosen gauge/link coordinate chart around the identity. Its physical content is nevertheless gauge invariant, and can be stated invariantly at once.

Corollary 9C.3: Gauge-Orbit Formulation

[EXACT GIVEN THEOREM 9C]

Since $Hr[U^G] = G\,Hr[U]\,G^{-1}$, the gap and index depend only on the gauge orbit of U. Defining the orbit radius

$\varepsilon\_orb(U)$ = inf over G of $\max\_e \|R\_f(U\_e^G) - I\|$

the sufficient condition $\varepsilon\_orb(U) < \varepsilon\_crit$ delivers the open gap, fixed index and exponential locality for the entire gauge orbit. A stronger eventual formulation would replace the chart radius by a gauge-invariant plaquette admissibility condition, in line with standard overlap-locality constructions on sufficiently smooth gauge fields; that formulation is deferred to the component-wise global programme.

Theorem 9C and Corollary 9C.3 are species-by-species statements: each fermion representation R_f carries its own chart radius and orbit radius. The frozen matter ledger contains several representations, and a background certified for one species in one gauge slice is not automatically certified for the others in the same slice.

Corollary 9C.4: Whole-Ledger Orbit Radius

[EXACT GIVEN THEOREM 9C, APPLIED SPECIES-BY-SPECIES]

Define the simultaneous orbit radius

$\varepsilon\_orb^{SM}(U)$ = inf over G of max over f, e of $\|R\_f(U\_e^G) - I\|$

where the same gauge transformation G must serve every fermion representation in the frozen ledger at once. If $\varepsilon\_orb^{SM}(U) < \varepsilon\_crit$, then every species in the ledger simultaneously retains its Hermitian-kernel gap, fixed index and exponential locality on a single common gauge slice. On the Euclidean reference carrier the radius ε_crit is representation-uniform at the declared grades, since $d_+ = 4$, $\lambda_{max} = 16$, $\delta^\star = \min(r, 1)$ and the canonical bound $C_{kin} \le 4$ are all representation independent there.

18.4 Explicit reference constants and the closed-form safe radius

The three constants entering Theorem 9C can now be evaluated on the four-dimensional Euclidean overlap reference regulator, the same nearest-neighbour carrier fixed by the prior graph-Dirac construction, with one incidence operator per regulated direction.

Maximal roughness eigenvalue. The flat roughness symbol is $L_I(q) = 4\Sigma_\mu \sin^2(q_\mu/2)$. Each sine-squared term is at most 1, so $L_I(q) \le 16$, with equality at $q = (\pi, \pi, \pi, \pi)$. Hence

$$\lambda_{max}^{ref} = 16$$

exactly, and the first symbolic constant is removed completely.

Reference out-degree. With one oriented forward edge for each of the four Euclidean regulator directions, the same nearest-neighbour directional structure used in the prior graph-Dirac construction,

$$d_+^{ref} = 4$$

The roughness part of the Theorem 9C bound therefore evaluates to $r\sqrt{(\lambda_{max} d_+)} = r\sqrt{64} = 8r$ and $r\, d_+/2 = 2r$, so the source-locked sufficient condition sharpens from its symbolic form to

$$(C_{kin} + 8r)\, \varepsilon_U + 2r\, \varepsilon_U^2 < \min(r, 1)$$

This part is fully source-locked at the Euclidean-reference grade.

Kinetic Lipschitz constant. Here a calculation must be distinguished from a source claim. The prior papers fix the primitive edge action and the flat first-order symbol $K_I(q) = i\, \Sigma_\mu\, \gamma_\mu \sin q_\mu$, but they do not publish a unique explicit interacting four-dimensional matrix K_U with every edge weight written out, so a numerical C_{kin} is not yet a fully native substrate return. For the canonical nearest-neighbour covariant representative of the already-frozen flat symbol,

$$(K_U\, \psi)(x) = \tfrac{1}{2}\, \Sigma_\mu\, \gamma_\mu\, [U_\mu(x)\, \psi(x + \hat{\mu}) - U_\mu^\dagger(x - \hat{\mu})\, \psi(x - \hat{\mu})]$$

the Clifford normalisation gives $\|\gamma_\mu\| = 1$, unitarity of the representation gives $\|R_f(U_e)^\dagger - I\| = \|R_f(U_e) - I\| \le \varepsilon_U$, and each direction supplies two link perturbations of norm at most ε_U, each carrying the factor ½. Hence

$$\|K_U - K_I\| \le \tfrac{1}{2}\, \Sigma_\mu\, (\varepsilon_U + \varepsilon_U) = 4\, \varepsilon_U$$

so

$$C_{kin}^{ref} \le 4$$

This is deliberately a canonical-reference bound, not a source-native equality $C_{kin} = 4$.

Explicit radius. Inserting the conservative value $C_{kin} = 4$ into the sufficient inequality gives

$$(4 + 8r)\, \varepsilon_U + 2r\, \varepsilon_U^2 < \min(r, 1)$$

and the Section 18.3 closed form with $A = 4 + 8r$ yields the explicit guaranteed radius

$$\varepsilon_{safe}(r) = 2 \min(r, 1) / [4 + 8r + \sqrt{(4 + 8r)^2 + 8r \min(r, 1)}]$$

Theorem 9D: Explicit Reference Admissibility Radius

[λ_{max} AND d_+ EXACT AT THE EUCLIDEAN-REFERENCE GRADE; C_{kin} AT CANONICAL-REFERENCE GRADE]

On the four-dimensional Euclidean overlap reference regulator, $\lambda_{max} = 16$ and $d_+ = 4$ exactly, and the canonical nearest-neighbour covariant kinetic representative satisfies $C_{kin} \leq 4$. Consequently every faithful-link background satisfying $\varepsilon_U < \varepsilon_{safe}(r)$ keeps the Hermitian Wilson kernel gapped at the max-gap representative $\rho = r$, so the overlap index cannot change and the overlap operator remains exponentially local on that reference family. The radius is a parameter-only VERSF-seeded formula: it contains no Standard Model observable and no fitted quantity, and converts the abstract existence of a nonzero local overlap neighbourhood into an explicit closed-form reference bound.

As a pure illustration only, at the conventional value $r = 1$,

$$\varepsilon_{safe} = 2 / (12 + \sqrt{152}) \approx 0.0822$$

This value is not inserted into the theorem as a VERSF prediction, because the programme has derived only $r > 0$; the numerical entropy calibration of r remains open.

Firewall FW-CWGW2: Carrier-Mixing Firewall

A source-native diamond calculation elsewhere in the corpus returns $\mathrm{spec}(B^\dagger B) = \{0, 2, 2, 4\}$, so $\lambda_{max} = 4$ on that diamond carrier. That value must not be inserted into the present bound. The diamond regulator and the four-dimensional Euclidean overlap regulator are different carriers; mixing $\lambda_{max} = 4$ from one with $\delta\bigstar = \min(r, 1)$ from the other would artificially improve the radius. Constants and gaps must always be evaluated on the same declared carrier.

18.5 Exponential locality inside the gapped neighbourhood

The source-matched seed is finite range: K_U is built from the local primitive edge transport (graph range 1 in primitive edge units) and $L_U = B_U^\dagger B_U$ has graph range 2, so the seed $Xr[U]$, and with it $Hr[U]$, has range $R_{seed} = 2$. On any domain where gap $Hr[U] \geq \delta > 0$ and $\|Hr[U]\| \leq \Lambda$, the sign function $Hr(Hr^2)^{(-1/2)}$ is uniformly approximable by polynomials on the gapped spectrum. A degree-n polynomial in Hr^2 has range at most $2n\, R_{seed}$, while the best polynomial approximation error of the inverse square root on $[\delta^2, \Lambda^2]$ falls geometrically with ratio $(\Lambda - \delta)/(\Lambda + \delta)$ per degree. Hence the overlap matrix elements decay exponentially with graph distance.

finite range + uniform Hermitian gap ⇒ exponential overlap locality

Corollary 9C.1: Local Overlap Locality Certificate

[CONDITIONAL EXACT ON THE GAP DOMAIN]

Every connected faithful-background domain obeying the Theorem 9C gap bound supports an exponentially local overlap representative with stable index and exact Ginsparg-Wilson chirality.

The rate is explicit and computable from the same data.

Corollary 9C.2: Quantitative Locality Rate

[PASS AT THE STANDARD ADMITTED FUNCTIONAL-CALCULUS GRADE]

On any connected background domain with $\delta \le |\text{spec } Hr[U]|$ and $\|Hr[U]\| \le \Lambda$, the overlap kernel obeys

$$|D_{ov}(x, y)| \le C \exp(-\mu\, d(x, y)),\quad \mu = (1/(2\, R_{seed})) \ln[(\Lambda + \delta)/(\Lambda - \delta)]$$

with $R_{seed} = 2$ in primitive edge units and C a computable constant. On the flat branch a crude admissible bound is $\Lambda_I^2 \le 4 + 49\, r^2$, since $\Sigma\mu \sin^2 q\mu \le 4$ and $(S - 1)^2 \le 49$; within the Theorem 9C neighbourhood $\Lambda_U \le \Lambda_I + \delta\star$. The rate degrades continuously as the gap shrinks: for $\delta \ll \Lambda$ the expansion of the logarithm gives $\mu \approx \delta/(R_{seed}\, \Lambda)$, so the locality exponent vanishes approximately linearly in δ/Λ, and no locality is claimed where the gap certificate fails.

18.6 Hamiltonian-Transfer Weyl-Pole and UV-Locality Closure

The source-matched overlap construction establishes an admissible local chiral representative, but VERSF microscopic time is sequential rather than a fourth primitive symmetric lattice direction. The next question is therefore whether the three-space-plus-sequential-transfer regulator lies in the same local one-loop universality class.

18.6.1 Signed local operator versus positive excitation transfer

For the spatial massless Wilson branch define

$$K_n(q) = (c/h_n)\, [\Sigma i\ \alpha i \sin qi + r\, \beta\, W(q)]$$

The Clifford algebra gives

$$K_n^2 = (c^2/h_n^2)\, [\Sigma i \sin^2 qi + r^2 W^2]\, I$$

Hence $|K_n|$ is proportional to the identity in spinor space. It correctly returns the positive one-particle energies but removes the sign/orientation information carried by $\alpha\cdot p$. Therefore $|K_n|$ may be used in the positive particle/antiparticle excitation Hamiltonian, but it cannot itself be the local Weyl operator.

The completed particle/antiparticle CAR structure resolves the apparent conflict. After the standard particle-hole reinterpretation the normal-ordered Fock Hamiltonian has nonnegative spectrum,

$$\mathbf{H_{Fock} = \Sigma_\lambda \hbar \varepsilon_\lambda (b_\lambda^\dagger b_\lambda + d_\lambda^\dagger d_\lambda) \geq 0}$$

while the signed K_n remains in the local quadratic fermion kernel. Positive physical excitation energy therefore does not require replacing the signed local Dirac/Weyl operator by its absolute value.

Theorem 9E: Signed-Kernel / Positive-Fock Compatibility Theorem

[CONDITIONAL EXACT ON THE INHERITED PARTICLE/ANTIPARTICLE CAR COMPLETION]

The VERSF spatial Wilson operator may retain its signed Clifford structure in the local fermion action while the associated physical Fock excitation Hamiltonian remains nonnegative. Particle/antiparticle completion removes the false requirement that the local Weyl operator itself be positive.

18.6.2 Sequential local kernel

At the standard coherent-state reconstruction grade define

$$\mathbf{D_n^{HT}(\omega, q) = [I - e^{(-a_t K_n(q))} e^{(i\omega a_t)}] / a_t}$$

Because K_n commutes with the scalar frequency factor,

$$\mathbf{D_n^{HT} = K_n - i\omega I - (a_t/2)(K_n - i\omega I)^2 + O(a_t^2 p^3)}$$

For $q_i = h_n p_i$,

$$\sin q_i = h_n p_i + O(h_n^3 p_i^3)$$

$$W(q) = \tfrac{1}{2} h_n^2 p^2 + O(h_n^4 p^4)$$

so

$$\mathbf{K_n = c\, \alpha\cdot p + (c\, r\, h_n/2)\, \beta\, p^2 + O(c\, h_n^2 p^3)}$$

Therefore

$$\mathbf{D\_n^{HT} = -i\omega I + c\,\alpha\cdot p + O(c\,h\_n\,p^2) + O[a\_t(\omega^2 + c^2p^2)]}$$

With $p_0 = \omega/c$ and $\alpha_i = \gamma^0\gamma_i$,

$$\mathbf{(\gamma^0/c)\,D\_n^{HT} = i\gamma_\mu p_\mu + O[(h\_n + c\,a\_t)\,p^2]}$$

up to the declared Fourier/Wick orientation convention.

For the two chiral sectors,

$$\mathbf{D\_{L,n} = i\,\bar{\sigma}_\mu p_\mu + O[(h\_n + c\,a\_t)\,p^2]}$$

$$\mathbf{D\_{R,n} = i\,\sigma_\mu p_\mu + O[(h\_n + c\,a\_t)\,p^2]}$$

Theorem 9F: Normalised Hamiltonian-Transfer Weyl-Pole Theorem

[CONDITIONAL EXACT ON THE CONSTRUCTED HAMILTONIAN-TRANSFER KERNEL AND SIMULTANEOUS SPATIAL/SEQUENTIAL REFINEMENT]

The unique physical fermion branch of the constructed VERSF Hamiltonian-transfer regulator returns the correctly normalised continuum Weyl operator with coefficient exactly one. No regulator-dependent kinetic prefactor survives. The rigorous finite-resolution remainder is $O[(h\_n + c\,a\_t)\,p^2]$; the earlier single-cutoff form $O(h\_n\,p^2)$ follows whenever $c\,a\_t/h\_n$ remains bounded.

The zero census of the free sequential kernel can also be executed exactly, before any gauged calculation. Since $K\_n(q)$ is Hermitian, let $\kappa(q)$ be any of its real eigenvalues. The corresponding eigenvalue of the transfer kernel is

$$\mathbf{d(\omega, q) = [1 - e^{(-a\_t\,\kappa(q))}\,e^{(i\omega\,a\_t)}]\,/\,a\_t}$$

A zero $d = 0$ requires $e^{(-a\_t\,\kappa)}\,e^{(i\omega\,a\_t)} = 1$. Taking absolute values gives $e^{(-a\_t\,\kappa)} = 1$, hence $\kappa = 0$. The phase condition then gives $e^{(i\omega\,a\_t)} = 1$, hence $\omega = 2\pi m/a\_t$, so $\omega = 0$ in the principal temporal frequency zone. By the exact scalar square

$$K\_n^2 = (c^2/h\_n^2)\,[\Sigma_i \sin^2 q_i + r^2\,W(q)^2]\,I$$

$\kappa = 0$ requires simultaneously $\sin q_i = 0$ for every i and $W(q) = 0$. Since $r > 0$ and $W = \Sigma_i(1 - \cos q_i) \geq 0$, $W = 0$ forces every $q_i = 0$. Hence the exact free-transfer zero census is

$$\mathbf{D\_HT(\omega, q) = 0 \Leftrightarrow (\omega, q) = (0, 0)}$$

within the fundamental frequency/momentum zones.

Theorem 9H: Exact Free-Transfer Zero Census

[EXACT FOR THE CONSTRUCTED FREE SEQUENTIAL KERNEL]

The free Hamiltonian-transfer kernel has exactly one zero in the fundamental frequency/momentum zones, at $(\omega, q) = (0, 0)$. Sequential transfer introduces no independent temporal zero, and the spatial Wilson stiffness leaves no doubler zero on the free sequential branch. The no-additional-light-species assertion used by Theorem 10 is thereby an executed exact theorem on the free branch rather than an assumption.

18.6.3 UV-local remainder

Let

$$\boldsymbol{\ell_n = \max(h_n, c\, a_t)}$$

Around their common Weyl pole,

$$\boldsymbol{D_{HT,\chi} = D_\chi + R_{HT},\quad D_{ov,\chi} = D_\chi + R_{ov}}$$

with

$$\boldsymbol{R_{HT}, R_{ov} = O(\ell_n p^2)}$$

Hence

$$\boldsymbol{D_{HT,\chi} - D_{ov,\chi} = O(\ell_n p^2)}$$

For the parity-even determinant use

$$\boldsymbol{\Gamma_+[D] = -\tfrac{1}{2} \operatorname{Tr} \log(D^\dagger D)}$$

Near the pole,

$$P_{HT} = D_{HT}^\dagger D_{HT} = p^2 + O(\ell_n p^3)$$

$$P_{ov} = D_{ov}^\dagger D_{ov} = p^2 + O(\ell_n p^3)$$

so

$$\boldsymbol{P_{ov}^{-1} (P_{HT} - P_{ov}) = O(\ell_n p)}$$

Choose an intermediate momentum Λ such that

$$p_{ext} \ll \Lambda \ll 1/\ell_n$$

In the scaling region, the ordinary Weyl logarithm is generated by the shell $\int dk/k$. An m-th order regulator correction instead gives

$$\int_{p_{ext}}^{\Lambda} (dk/k)(\ell_n k)^m = (\ell_n^m/m)(\Lambda^m - p_{ext}^m),\ m \geq 1$$

No regulator correction therefore produces log p_{ext}.

In the complementary regulator region $k \geq \Lambda$, the one-species gap excludes additional singularities and locality makes the determinant analytic in sufficiently small external momenta. The contribution consequently has a local derivative expansion.

Provided the gauged Hamiltonian-transfer extension satisfies the locality, covariance and regularity conditions stated below, the determinant difference therefore has the form

$$\Gamma_+^{HT} - \Gamma_+^{ov} = c_F \int d^4x\, \mathrm{tr}(F_{\mu\nu} F^{\mu\nu}) + \Sigma_{(\Delta>4)}\, c_\Delta\, \ell_n^{(\Delta-4)} \int d^4x\, O_\Delta$$

plus the separately classified parity-odd/anomaly contribution.

The coefficient c_F is a finite matching constant. The terms O_Δ are local higher-dimension operators. Under the same conditions no second nonlocal $F^2 \log(-D^2)F$ or $F^2 \log \mu$ term is generated.

One scope statement must be made explicit before the theorem. The comparison above is executed at the level of the chiral blocks around the common free Weyl pole. The fully gauged sequential operator D_HT[U], with its complete background-gauge transformation law and fermion-gauge vertices, has not yet been written out in this paper. An $O(\ell_n p^2)$ difference between free inverse propagators is not, by itself, a proof that every one-loop gauge vertex differs only by irrelevant local terms. Lattice power counting does establish universality of the logarithmically divergent one-loop terms under locality, continuum-limit and regularity assumptions, but those assumptions must be demonstrated for the actual gauged Hamiltonian-transfer operator rather than inferred from the free pole. The theorem below is therefore stated, and graded, conditionally on that gauged execution.

Theorem 9G: UV-Remainder No-Second-Log Theorem

[CONDITIONAL EXACT AT THE LOCAL/PERTURBATIVE REGULATOR GRADE, PENDING THE FULLY GAUGED HAMILTONIAN-TRANSFER EXECUTION]

For the constructed local Hamiltonian-transfer/shared-link fermion family, once the unique normalised Weyl pole and no-extra-light-species gap are imposed, and provided the fully gauged operator D_HT[U] satisfies the locality, continuum-limit and regularity assumptions of one-loop power counting, all regulator-dependent departures from the continuum Weyl operator contribute only finite local matching and higher-derivative local terms. They cannot alter the coefficient of the parity-even infrared F^2 logarithm. The free-pole and irrelevant-remainder structure is established here; the gauged-vertex demonstration is the open step.

Corollary 9G.1: Local One-Loop Fermion Universality

The constructed Hamiltonian-transfer/shared-link family therefore belongs, at conditional-pass grade, to the same local parity-even one-loop universality class as the source-matched overlap representative:

$b_{Weyl}(R) = (2/3)\, T(R)$

This is a local perturbative universality statement, conditional on the gauged Hamiltonian-transfer execution. It does not assert that the complete unpublished Master-Action $D_f[U]$ has been source-returned, nor does it close the component-wise global determinant-line or reflection-positivity programme.

19. What the Calculation Has Now Closed

The calculation closes a longer source-provenance chain than the flat reference analysis alone:

Master CAR/Fock edge difference → B_U → $L_U = B_U^\dagger B_U$ → source-matched Wilson seed → $\rho^\bigstar = r$ → $\delta^\bigstar = \min(r, 1)$ → nonzero faithful-gauge gap neighbourhood → local GW/overlap Weyl carrier → signed Hamiltonian-transfer kernel → correctly normalised Weyl pole → UV-local regulator remainder → local one-loop fermion universality

Specifically:

- the primitive finite fermion edge transport is source-explicit;
- the fermion and gauge sectors use exactly the same faithful link;
- the full-faithful CAR/Fock gauge derivative spans all 144 local gauge directions;
- the positive source roughness L_U reproduces the closure-Wilson symbol on the flat branch;
- $\rho = r$ is a structurally distinguished max-gap overlap representative;
- the exact flat gap is $\min(r, 1)$;
- an explicit nonzero link-neighbourhood, with closed-form radius ε_{crit}, preserves the Hermitian gap and index;
- on the Euclidean reference regulator the constants are evaluated, $\lambda_{max} = 16$ and $d_+ = 4$ exactly and $C_{kin} \le 4$ at canonical-reference grade, giving the explicit parameter-only radius $\varepsilon_{safe}(r)$;
- exponential overlap locality follows on every uniformly gapped neighbourhood, with an explicit computable rate;
- the independently completed particle/antiparticle CAR structure permits positive physical Fock excitation energies while retaining the signed local Clifford operator;
- the three-space-plus-sequential-transfer kernel has the correctly normalised Weyl pole with unit coefficient;
- the exact free-transfer zero census is executed: $D_{HT}(\omega, q) = 0$ if and only if $(\omega, q) = (0, 0)$ in the fundamental zones;
- the regulator remainder is $O[(h_n + c\, a_t)\, p^2]$ under simultaneous refinement;
- after subtraction against the overlap representative, the scaling-region remainder is infrared-integrable and the gapped regulator region is analytic in external momentum;

- the regulator therefore generates only finite gauge-coupling matching and higher-derivative local operators;
- no second parity-even F^2 logarithm is generated at the free-pole/irrelevant-remainder grade;
- R1Q local one-loop fermion universality is placed at conditional-pass grade: free pole and irrelevant-remainder structure established, full gauged Hamiltonian-transfer execution open.

The remaining debt must now be separated into three questions of increasing strength. First, the narrowest: the fully gauged sequential operator D_HT[U] must be written out with its background-gauge transformation law and fermion-gauge vertices, and the power-counting assumptions demonstrated for it directly. Second, strict source identity requires the complete finite physical Master-Action K_U/D_f[U] family to be frozen and shown to coincide with, or lie in an admissible deformation class of, the constructed source-matched family. Third, global nonperturbative completion requires the gap/admissibility certificate to be extended through every complete admitted fixed-index component of the physical faithful-group background space. None of the stronger requirements is needed to alter the local one-loop coefficient once the local universality conditions above are demonstrated for the gauged operator.

20. Relation to the Existing VERSF Index-One Witness

The global gauge-measure programme already employs an admissible faithful-background configuration $U^{(1)}$ with

$$\text{index } D\_ov[U^{(1)}] = 1$$

It also uses the fact that the overlap index is locally constant inside a sufficiently admissible gauge-background neighbourhood. The present theorem supplies the missing local spectral interpretation of that construction relative to the VERSF Wilson stiffness:

$$\text{index } D\_ov = -\tfrac{1}{2} \text{ Tr sgn } H\rho$$

Theorem 7A sharpens the consistency statement: the flat sector carries index zero exactly, so the index-one witness is necessarily a topologically nontrivial spectral-flow sector of the same Hermitian Wilson kernel whose trivial flat sector has been explicitly analysed above. The local zero-doubler calculation and the global index witness are therefore parts of one regulator architecture, separated by a mandatory Hermitian-kernel spectral crossing.

21. One-Loop Universality

The VERSF Quantum Gauge Completion programme already establishes the regulator criterion: two admissible regulators represent the same renormalised physical theory if their effective actions differ only by finite parameter redefinitions, BRST-exact terms, renormalisation-scheme changes, and explicitly matched EFT coefficients, and they agree on physical anomaly classes.

The overlap representative now has:

- the correct light pole;
- no additional light species;
- exact finite chirality;
- gauge covariance;
- an index;
- the inherited Standard Model representation ledger.

Therefore its parity-even one-loop fermion contribution is the Weyl result

$$\mathbf{b_{Weyl}(R) = (2/3)\ T(R)}$$

This agrees with the independently executed VERSF chiral-projector heat-kernel calculation. The agreement is not used to select ρ. It is an output after the one-species window has already been fixed structurally.

22. Standard Model Census

For three generations, the VERSF frozen matter ledger gives

$$\Sigma_{Weyl}\ T_3(R) = 6,\ \Sigma_{Weyl}\ T_2(R) = 6,\ \Sigma_{Weyl}\ Y^2 = 10$$

Hence

$$\mathbf{b_3^{ferm} = 4,\ b_2^{ferm} = 4,\ b_Y^{ferm} = 20/3}$$

The physical right-handed singlets are included. No mirror-doublet factor is included.

23. Full One-Loop Running

Adding the gauge-plus-ghost terms $-(11/3)\ C_2(G)$ and one complex scalar doublet gives

$$b_3 = -11 + 4 = -7$$

$$b_2 = -22/3 + 4 + 1/6 = -19/6$$

$$b_Y = 20/3 + 1/6 = 41/6$$

Therefore

$$\mathbf{(b_Y,\ b_2,\ b_3) = (41/6,\ -19/6,\ -7)}$$

24. Theorem 10: Local One-Loop Chiral-Regulator Universality Match

[CONDITIONAL EXACT AT THE CONSTRUCTED SOURCE-MATCHED LOCAL/PERTURBATIVE GRADE, PENDING THE FULLY GAUGED HAMILTONIAN-TRANSFER EXECUTION]

The local fermionic regulator problem can now be separated from the stronger global quantisation problem.

For the constructed source-matched Hamiltonian-transfer/shared-link family the four local one-loop conditions are:

1. **Gauge-covariant locality:** inherited from the primitive finite-range shared-link transport and, for the overlap comparison representative, strengthened to exponential locality on the certified gapped faithful-background neighbourhood.
2. **Correct normalised Weyl pole:** the sequential Hamiltonian-transfer kernel returns

   $$D_{L,n} = i \bar{\sigma}^\mu p_\mu + O[(h_n + c\, a_t)\, p^2]$$

   or

   $$D_{R,n} = i \sigma^\mu p_\mu + O[(h_n + c\, a_t)\, p^2]$$

   with coefficient exactly one.

3. **No additional light species:** by Theorem 9H the free sequential kernel vanishes only at $(\omega, q) = (0, 0)$ in the fundamental zones: the positive closure-Wilson stiffness lifts all unwanted spatial branches and sequential transfer introduces no independent temporal zero. This condition is an executed exact theorem on the free branch.
4. **Local regulator-stable UV remainder:** comparison with the overlap representative shows that every regulator-dependent correction carries a positive power of $\ell_n = \max(h_n, c\, a_t)$ in the scaling region, while the complementary high-momentum region is gapped and analytic in external momentum. The difference therefore contains finite local matching and higher-derivative operators but no additional infrared F^2 logarithm.

It follows that the constructed Hamiltonian-transfer regulator and the overlap representative belong to the same local parity-even one-loop universality class and give

$$b_{Weyl}(R) = (2/3)\, T(R)$$

Consequently the finite-regulator ambiguity of this constructed family cannot alter

$$(b_Y, b_2, b_3) = (41/6, -19/6, -7)$$

without creating another light species, destroying locality or the normalised Weyl pole, changing the faithful representation/anomaly class, or otherwise leaving the admitted regulator family.

The present execution establishes conditions 2 through 4 at the free-pole and irrelevant-remainder level. Their demonstration for the fully gauged operator D_HT[U], with its background-gauge transformation law and fermion-gauge vertices written out explicitly, is the remaining step, and the R1Q verdict is graded conditional until it is executed.

Firewall FW-CWGW3: Source-Identity Firewall

Closing local one-loop universality for the constructed source-matched Hamiltonian-transfer family does not by itself prove that the complete unpublished Master-Action D_f[U] is identical to that family. Strict source-level identity remains a separate provenance return until the frozen finite K_U/D_f[U] matrices are published or reconstructed.

Firewall FW-CWGW4: Free-Pole Inference Firewall

An $O(\ell_n p^2)$ difference between free inverse propagators must not be presented as a proof that every one-loop gauge vertex differs only by irrelevant local terms. The gauged Hamiltonian-transfer operator D_HT[U] must be constructed, its transformation law and vertices exhibited, and the locality, continuum-limit and regularity assumptions of the one-loop power counting demonstrated for it directly, before the R1Q gate can be graded a full pass.

25. Status of D-SG6 After This Calculation

The original D-SG6 debt was: derive or universality-match the local index-correct Dirac family. The combined corpus plus the present calculations split that debt into a locally passed part and a genuinely global remainder.

25.1 Closed or substantially closed

- Primitive shared-link CAR/Fock kinetic operator: source-explicit in the Master Substrate Action.
- Same physical link U in gauge, closure and fermion sectors: source-locked.
- Full-faithful CAR/Fock gauge jet: executed rank-144 pass with residual 1.794×10^{-15} and zero branch difference.
- Gauge-covariant source roughness $L_U = B_U^\dagger B_U$: derived from the primitive edge difference.
- Source-matched Wilson seed $X\rho[U] = K_U + (r/2)L_U - \rho I$: constructed at the declared conditional grade.
- One-species overlap interval $0 < \rho < 2r$: exact.
- Spectral circle, normality and γ_5-Hermiticity of D_ov: exact.
- Max-gap representative $\rho\star = r$: exact.
- Exact flat Hermitian gap $\delta\star = \min(r, 1)$: exact.
- Flat-sector index zero: exact.

- Explicit nonzero full-dimensional faithful-gauge neighbourhood, closed-form ε_{crit}, with preserved gap: conditional exact.
- Reference constants $\lambda_{max} = 16$ (exact) and $d_+ = 4$ (source-locked reference structure); kinetic bound $C_{kin} \leq 4$ (canonical-reference grade); explicit parameter-only radius $\varepsilon_{safe}(r)$: pass at declared grades.
- Index stability and exact finite GW chirality inside that neighbourhood: pass.
- Exponential overlap locality on uniformly gapped domains, with explicit rate μ: pass by standard admitted functional calculus.
- Continuum Weyl limit, chiral-ledger match and one-loop coefficient: retained.
- Particle/antiparticle CAR sectors and unitary Fock-space charge conjugation: inherited completion; physical antiparticle structure is not regulator doubling.
- Positive excitation Fock Hamiltonian versus signed local fermion operator: structurally separated.
- Hamiltonian-transfer local quadratic kernel: constructed at standard coherent-state grade.
- Correctly normalised left- and right-Weyl continuum poles: pass.
- Exact free-transfer zero census, single zero at $(\omega, q) = (0, 0)$: exact.
- Two-cutoff remainder $O[(h_n + c\, a_t)\, p^2]$: derived.
- Single-cutoff $O(h_n\, p^2)$ form: conditional on bounded $c\, a_t/h_n$.
- UV-local regulator remainder: pass at local perturbative grade.
- No-second-infrared-log certificate: pass at the free-pole/irrelevant-remainder grade, conditional on the gauged Hamiltonian-transfer execution.
- Fixed-gauge chart status of the ε_U neighbourhood, with its gauge-orbit reformulation $\varepsilon_{orb}(U) < \varepsilon_{crit}$: declared.
- Whole-ledger simultaneous orbit radius ε_{orb}^{SM}, one gauge transformation serving all fermion representations, with representation-uniform ε_{crit} on the reference carrier: defined.
- R1Q local one-loop fermion universality: conditional pass; free pole and irrelevant-remainder structure established, full gauged Hamiltonian-transfer execution open.

25.2 Remaining strict source/global returns

The local one-loop universality calculation for the constructed source-matched Hamiltonian-transfer family is closed at the free-pole and irrelevant-remainder level, but three stronger returns remain.

Gauged Hamiltonian-transfer return. The fully gauged sequential operator D_HT[U] must be written out, with its complete background-gauge transformation law and fermion-gauge vertices, and the locality, continuum-limit and regularity assumptions of the one-loop power counting demonstrated for it directly. This is the narrowest open item and the one that converts the R1Q verdict from conditional to full.

Source-identity return. The complete finite physical Master-Action K_U/D_f[U] family must be frozen or reconstructed. Its actual kinetic normalisation and finite edge weights must be used to calculate the physical C_{kin} and to establish direct identity with, or an admissible deformation into, the constructed local universality family. Until then the complete Master-Action D_f[U] must not be relabelled as source-returned. On the Euclidean reference carrier the constants

$\lambda\_max = 16$ and $d_+ = 4$ are already exact and $C\_kin \leq 4$ holds at canonical-reference grade, so this return is the narrower remaining constant-level debt.

Component-wise global return. On every fixed-index admissible background component A_adm,ν one ultimately requires

inf over U ∈ A_adm,ν of gap Hr[U] > 0

with index changes allowed only through excluded component boundaries where the Hermitian kernel crosses zero.

These are stronger returns. They do not reopen the free-pole and irrelevant-remainder structure already established for the constructed source-matched Hamiltonian-transfer regulator.

R1Q LOCAL ONE-LOOP FERMION UNIVERSALITY: CONDITIONAL PASS; FREE POLE AND IRRELEVANT-REMAINDER STRUCTURE ESTABLISHED, FULL GAUGED HAMILTONIAN-TRANSFER EXECUTION OPEN

STRICT COMPLETE MASTER-ACTION D_f[U] SOURCE IDENTITY: OPEN

D-SG6 LOCAL SOURCE-MATCHED CHIRAL REGULATOR: PASS

D-SG6 COMPLETE COMPONENT-WISE GLOBAL CERTIFICATE: OPEN

26. Falsifiers

F-CWGW1: Wilson-sign failure. For some $r > 0$, the interval $0 < \rho < 2r$ contains a doubler corner with negative Wilson mass. This would falsify Theorem 1.

F-CWGW2: Extra overlap zero. Within $0 < \rho < 2r$, D_ov develops another zero away from the physical branch.

F-CWGW3: GW failure. The constructed operator fails $\gamma_5 D + D\gamma_5 = \bar{a} D\gamma_5 D$.

F-CWGW4: Projector failure. $\hat{\gamma}_5^2 \neq 1$.

F-CWGW5: Continuum normalisation failure. The physical branch does not return $D\_ov = i\not{p} + O(ap^2)$.

F-CWGW6: Chiral-ledger mismatch. $\hat{P}_-$ does not approach the inherited VERSF weak-active chirality.

F-CWGW7: Shared-link failure. The finite chiral representative requires a gauge link different from the physical Master-Action U.

F-CWGW8: Index instability without gap closing. The overlap index changes under a continuous admissible deformation even though $H\rho$ never reaches zero.

F-CWGW9: Locality failure. The overlap representative remains nonlocal despite a uniform admissibility/gap condition.

F-CWGW10: ρ-as-physics insertion. ρ is fitted to a Standard Model observable instead of treated as regulator data within its topological phase.

F-CWGW11: Naive projector revival. Ordinary $P_L D_W P_L$ is used as though it were the exact finite chiral construction, ignoring the GW projector.

F-CWGW12: Primitive-time conflation. The Euclidean four-direction overlap reference is presented as proof that VERSF possesses four primitive symmetric spacetime lattice directions.

F-CWGW13: Beta-target selection. The window $0 < \rho < 2r$ or any regulator branch is chosen after consulting $(41/6, -19/6, -7)$.

F-CWGW14: D-SG6 overclaim. The source-explicit finite Master-Action $D_f[U]$ is declared derived when only a universality representative has been constructed.

F-CWGW15: Primitive-edge mismatch. If the chiral regulator is built from an edge difference inequivalent to the source-explicit Master-Action transport $\Psi t - Ue\ \Psi s$, the source-matched D-SG6 claim fails.

F-CWGW16: Roughness mismatch. If $(r/2)B_U^\dagger B_U$ does not return the inherited closure-Wilson mass function on the flat regular branch, the source-matched Wilson identification fails.

F-CWGW17: Max-gap failure. If some ρ in $0 < \rho < 2r$ has flat Hermitian gap larger than $\min(r, 1)$, or if $\rho = r$ fails to attain that value, Theorem 9B is false.

F-CWGW18: Local-gap-bound failure. If a faithful background satisfies the Theorem 9C norm bound but closes the Hermitian-kernel gap, the perturbative certificate is false.

F-CWGW19: Locality failure in a uniformly gapped finite-range family. If the overlap sign kernel remains nonlocal despite a uniform spectral gap and finite-range source seed, the admitted overlap functional-calculus bridge fails.

F-CWGW20: Global overclaim. A local source-matched gap neighbourhood must not be presented as a uniform theorem over all topological components before the frozen component-wise spectral certificate is executed.

F-CWGW21: Spectral-circle failure. Any eigenvalue of D_{ov} leaves the circle $|\lambda - \rho/a| = \rho/a$, or D_{ov} fails normality or γ_5-Hermiticity. This would falsify Theorem 2A.

F-CWGW22: Flat-index failure. The flat-sector overlap index is nonzero in the declared balanced trace convention, or $\mathrm{Tr}\,\mathrm{sgn}\,H\rho(q) \neq 0$ at some flat momentum. This would falsify Theorem 7A.

F-CWGW23: Locality-rate failure. On a domain with certified bounds δ and Λ, the overlap matrix elements decay slower than the Corollary 9C.2 rate μ up to the computable prefactor. This would falsify the quantitative locality certificate.

F-CWGW24: Reference-constant failure. On the declared four-dimensional Euclidean reference regulator, $\lambda\_max \neq 16$ or $d_+ \neq 4$, or the canonical nearest-neighbour covariant representative violates $\|K\_U - K\_I\| \leq 4\varepsilon\_U$. This would falsify Theorem 9D.

F-CWGW25: Carrier-mixing insertion. Constants evaluated on one carrier (for example the diamond value $\lambda\_max = 4$) are inserted into the gap bound of a different carrier to improve the admissibility radius. This violates Firewall FW-CWGW2.

F-CWGW26: Absolute-energy/local-kernel conflation. The positive operator $|K\_n|$ is used as though it were the local Weyl operator, despite $|K\_n|$ being spinor-scalar and therefore unable to reproduce the signed Clifford orientation $\alpha\cdot p$.

F-CWGW27: Particle/antiparticle doubling error. The charge-conjugate CAR sector is counted as an additional regulator-generated physical species rather than the independently typed antiparticle sector of the same relativistic field construction.

F-CWGW28: Weyl-normalisation failure. The Hamiltonian-transfer physical branch fails to return $D\_L = i\,\bar{\sigma}\cdot p + O[(h\_n + c\ a\_t)\ p^2]$ or $D\_R = i\ \sigma\cdot p + O[(h\_n + c\ a\_t)\ p^2]$ with unit leading coefficient.

F-CWGW29: Aspect-ratio overclaim. The two-cutoff remainder $O[(h\_n + c\ a\_t)\ p^2]$ is silently rewritten as $O(h\_n\ p^2)$ without either deriving or declaring a bounded refinement ratio $c\ a\_t/h\_n$.

F-CWGW30: UV-second-log failure. After subtracting the common Weyl pole, the regulator remainder produces an additional parity-even $F^2 \log p\_ext$ or $F^2 \log \mu$ term. This would falsify Theorem 9G and reopen the one-loop universality gate.

F-CWGW31: Source-identity overclaim. Closure of the constructed Hamiltonian-transfer universality family is presented as proof that the complete unpublished Master-Action D_f[U] has been source-returned.

F-CWGW32: Gauged-execution overclaim. The conditional R1Q result is presented as a full pass before the fully gauged operator D_HT[U] is written out, its transformation law and fermion-gauge vertices exhibited, and the one-loop power-counting assumptions demonstrated for it. This violates Firewall FW-CWGW4.

F-CWGW33: Gauge-chart conflation. The fixed-gauge ε_U neighbourhood of Theorem 9C is presented as a gauge-invariant physical admissibility neighbourhood without passing to the orbit radius ε_orb of Corollary 9C.3 or to a gauge-invariant plaquette admissibility condition.

F-CWGW34: Free-transfer extra zero. The free sequential kernel possesses a zero at some $(\omega, q) \neq (0, 0)$ within the fundamental frequency/momentum zones. This would falsify Theorem 9H and reopen the species census of the Hamiltonian-transfer branch.

27. Formal Verdict

Object	Grade
Closure-graph Dirac first-order carrier	INHERITED / DERIVED AT PRIOR GRADE
Closure-Wilson mass structure	INHERITED / DERIVED UP TO r
Wilson coefficient sign $r > 0$	REQUIRED BY CLOSURE STIFFNESS BRANCH
Numerical value of r	OPEN SEPARATE ENTROPY CALIBRATION
Shared gauge/fermion faithful link U	SOURCE-EXPLICIT MASTER-ACTION RESULT
GW/overlap chiral family as admissible reference	INHERITED GLOBAL-QUANTUM ARCHITECTURE
Hermitian Wilson kernel $H\rho$	SOURCE-MATCHED REPRESENTATIVE CONSTRUCTED
$H\rho^2 = s^2 + (rW - \rho)^2$	EXACT
One-species interval $0 < \rho < 2r$	EXACT
Nonempty window for every $r > 0$	EXACT
Physical Wilson-mass sign	NEGATIVE
All doubler Wilson-mass signs in first window	POSITIVE
Overlap D_ov	EXPLICIT REFERENCE REPRESENTATIVE
Normality, γ_5-Hermiticity, spectral circle $\lvert\lambda - \rho/a\rvert = \rho/a$	EXACT
Physical zero $D_{ov}(0) = 0$	EXACT
Doubler corner value $2\rho/a$	EXACT
Extra light species	NONE ON FLAT REFERENCE BRANCH
GW relation	EXACT
$\hat{\gamma}_5 = -\mathrm{sgn}\, H\rho$	EXACT
$\hat{\gamma}_5^2 = 1$	EXACT
Finite chiral projectors	EXACT
$D\hat{\gamma}_5 = -\gamma_5 D$	EXACT
Continuum $D = i\not{p} + O(ap^2)$	EXACT ASYMPTOTIC RETURN

Object	Grade
$\hat{P}_- \to P\_L$	CONDITIONAL ON VERSF CHIRAL NAMING / PASS
Index = $-\frac{1}{2}$ Tr sgn $H\rho$	EXACT IN DECLARED CONVENTION
Flat-sector index zero	EXACT
Gap-protected index stability	EXACT
Existing VERSF index-one overlap witness	INHERITED EXECUTED GLOBAL RESULT
Shared-link overlap covariance	EXACT GIVEN COVARIANT WILSON SEED
Primitive Master CAR/Fock edge operator	SOURCE-EXPLICIT
Full-faithful CAR/Fock gauge jet	RANK 144 / 1.794×10^{-15} / PASS
Covariant source roughness $L\_U = B\_U^\dagger B\_U$	DERIVED
Source-matched Wilson seed $K\_U + (r/2)L\_U - \rho I$	CONDITIONAL EXACT
Max-gap overlap representative $\rho^\bigstar = r$	EXACT
Exact flat Hermitian gap $\delta^\bigstar = \min(r, 1)$	EXACT
Closed-form admissibility radius ε_crit	EXACT GIVEN (C_kin, λ_max, d_+)
Reference constants λ_max = 16, $d_+ = 4$	EXACT ON EUCLIDEAN REFERENCE CARRIER
Kinetic Lipschitz bound C_kin ≤ 4	CANONICAL-REFERENCE BOUND
Explicit parameter-only radius ε_safe(r)	CONDITIONAL EXACT AT REFERENCE GRADE
Carrier-mixing discipline (diamond λ_max = 4 excluded)	FIREWALLED
Nonzero full-dimensional faithful-gauge gap neighbourhood	CONDITIONAL EXACT; FIXED-GAUGE CHART, GAUGE-ORBIT FORM VIA SPECTRAL INVARIANCE
Whole-ledger orbit radius ε_orb^SM	DEFINED; SPECIES CERTIFICATES UNIFY AT ε_orb^SM < ε_crit, REPRESENTATION-UNIFORM ON REFERENCE CARRIER
Index stability in local source-matched neighbourhood	PASS
Exponential locality in uniformly gapped neighbourhood	PASS / STANDARD ADMITTED FUNCTIONAL CALCULUS

Object	Grade
Quantitative locality rate $\mu = (1/(2R\_seed)) \ln[(\Lambda+\delta)/(\Lambda-\delta)]$	PASS AT ADMITTED GRADE
Independent particle/antiparticle CAR sectors	INHERITED COMPLETION / PASS
Positive Fock excitation spectrum versus signed local kernel	SEPARATED / PASS
Hamiltonian-transfer local quadratic kernel	CONSTRUCTED AT STANDARD COHERENT-STATE GRADE
$D\_L = i \bar{\sigma}\cdot p + O[(h\_n + c\, a\_t)\, p^2]$	CONDITIONAL EXACT ASYMPTOTIC RETURN
$D\_R = i \sigma\cdot p + O[(h\_n + c\, a\_t)\, p^2]$	CONDITIONAL EXACT ASYMPTOTIC RETURN
Unit Weyl kinetic coefficient	EXACT IN THE CONSTRUCTED LOW-MOMENTUM EXPANSION
Free-transfer zero census $D\_HT(\omega, q) = 0 \Leftrightarrow (\omega, q) = (0, 0)$	EXACT
Single-cutoff $O(h\_n\, p^2)$ remainder	CONDITIONAL ON BOUNDED c a_t/h_n
UV-local regulator remainder	PASS AT LOCAL/PERTURBATIVE GRADE
Additional regulator-generated F^2 infrared logarithm	NONE AT FREE-POLE GRADE / CONDITIONAL ON GAUGED EXECUTION
Gauged Hamiltonian-transfer operator D_HT[U] with vertices	OPEN
R1Q local one-loop fermion universality	CONDITIONAL PASS: FREE POLE AND IRRELEVANT REMAINDER ESTABLISHED; FULL GAUGED EXECUTION OPEN
Complete physical Master-Action D_f[U] identity	OPEN / MUST NOT BE OVERCLAIMED
Flat/principal-symbol D-SG6 match	PASS; SUPERSEDED BY LOCAL SOURCE-MATCHED PASS
Uniform all-background overlap locality	LOCAL GAPPED NEIGHBOURHOOD PASS; COMPLETE COMPONENT-WISE CERTIFICATE OPEN
Strict source-explicit MASD D_f[U]	PRIMITIVE KINETIC OPERATOR SOURCE-EXPLICIT; COMPLETE CHIRAL KERNEL FAMILY NOT YET FROZEN/PUBLISHED
Overlap-reference Weyl coefficient	2T(R)/3 / CLOSED AT DECLARED GRADE
Hamiltonian-transfer Weyl coefficient	2T(R)/3 / CONDITIONAL, PENDING GAUGED EXECUTION

Object	Grade
SM beta coefficients	(41/6, −19/6, −7) / CONDITIONAL EXACT
Full determinant-line/RP nonperturbative completion	SEPARATE STRONGER GLOBAL GATE
D-SG6 local source-matched chiral regulator	PASS
D-SG6 complete component-wise global certificate	OPEN

28. Scientific Conclusion

The VERSF fermion-regulator programme has moved beyond a flat overlap reference calculation.

The decisive source-level fact is that the Master Substrate Action already specifies the primitive finite fermion transport. Matter is carried across an oriented edge by the same faithful link used by gauge and closure transport, through $\Psi_{t(e)} - U_e \Psi_{s(e)}$. The full-faithful execution differentiates that exact shared-link structure and returns a rank-144 CAR/Fock gauge jet with machine-zero residual at the declared grade.

This source-explicit difference provides the bridge to the Wilson construction. Its positive quadratic roughness $B_U^\dagger B_U$ is local, gauge covariant and reduces on the flat regular regulator to twice the standard Wilson function. The inherited closure-entropy coefficient therefore produces the source-matched mass term $(r/2)B_U^\dagger B_U$ rather than an independently chosen lattice regulator.

Inside the one-species phase $0 < \rho < 2r$, the calculation removes avoidable regulator freedom. The midpoint $\rho = r$ is a max-gap representative, and its exact flat Hermitian-kernel gap is $\min(r, 1)$. This result is target blind and depends only on the positive closure-Wilson stiffness already present in the VERSF fermion programme. The overlap representative it seeds is normal and γ_5-Hermitian, its spectrum confined exactly to the Ginsparg-Wilson circle, and its flat-sector index vanishes exactly, so the trivial and nontrivial topological sectors of the architecture are cleanly separated by mandatory spectral flow.

The calculation can then leave the flat background. Weyl spectral perturbation and the explicit source edge operator give a finite faithful-link neighbourhood in which the Hermitian overlap seed cannot close its gap, and on the Euclidean reference carrier the radius is now fully explicit: with $\lambda_{max} = 16$, $d_+ = 4$ and $C_{kin} \le 4$, every background with $\varepsilon_U < \varepsilon_{safe}(r) = 2 \min(r, 1)/[4 + 8r + \sqrt{(4 + 8r)^2 + 8r \min(r, 1)}]$ is guaranteed safe, a parameter-only formula with no fitted quantity. Because the full-faithful CAR/Fock derivative spans all 144 local gauge directions, this is a full-dimensional local physical gauge-background result, not a statement restricted to one simple factor or a hand-selected perturbation.

Within every such gapped neighbourhood the overlap index is fixed, the Ginsparg-Wilson relation remains exact and the overlap kernel is exponentially local at an explicit computable rate. The existing index-one overlap witness in the global gauge-measure programme supplies a nontrivial topological component of the same architecture.

The regulator-provenance chain can therefore be written:

Master shared-link CAR/Fock transport → B_U → $L_U = B_U^\dagger B_U$ → closure-Wilson mass → $\rho^\bigstar = r$ → $\delta^\bigstar = \min(r, 1)$ → local faithful-background gap → exact GW chirality/index → Weyl one-loop coefficient

This materially changes the local quantum-running grade. The source-matched chiral regulator is no longer only a Euclidean reference construction. The primitive shared-link transport supplies its kinetic provenance, $B_U^\dagger B_U$ supplies the covariant Wilson roughness, the overlap representative supplies exact finite chirality and local index control, and the genuine VERSF three-space-plus-sequential-transfer architecture now independently returns the correctly normalised Weyl pole.

The particle/antiparticle completion is essential to this result. Positive physical excitation energy belongs to the second-quantised Fock Hamiltonian; it does not require replacing the signed local Clifford operator by its absolute value. Retaining that sign information gives the sequential local kernel

$$D_n^{HT} = [I - e^{(-a_t K_n)} e^{(i\omega a_t)}] / a_t$$

whose continuum chiral blocks have unit Weyl normalisation and regulator corrections beginning only at $O[(h_n + c\, a_t)\, p^2]$.

The remaining regulator terms also cannot alter the one-loop logarithmic coefficient, at the grade so far established. Around the unique Weyl pole they are suppressed by positive powers of the regulator scale and are infrared-integrable after subtraction of the common Weyl singularity. Away from that pole the spectrum is gapped and locality makes the determinant analytic in external momentum. The regulator difference therefore consists only of finite local gauge-coupling matching and higher-derivative operators, and generates no second parity-even F^2 logarithm. This structure is demonstrated at the free-pole and irrelevant-remainder level; its demonstration for the fully gauged operator $D_{HT}[U]$, with its transformation law and fermion-gauge vertices written out, is the step that remains.

The local fermionic one-loop chain can therefore be written

Master shared-link CAR/Fock transport → source-matched Wilson operator → unique physical branch → particle/antiparticle Fock completion → signed sequential local kernel → unit-normalised Weyl pole → UV-local regulator remainder → $b_{Weyl}(R) = (2/3)T(R)$

Thus the local R1Q fermion-universality gate is placed at conditional-pass grade for the constructed source-matched Hamiltonian-transfer family.

Three stronger problems remain and must not be conflated with that result. The fully gauged sequential operator D_HT[U] must be written out and its one-loop power-counting assumptions demonstrated directly; this is the narrowest item and converts the R1Q verdict to a full pass. The complete physical Master-Action K_U/D_f[U] family still has to be frozen or reconstructed to establish strict source identity and its true finite constants. Separately, the D-SG6 programme still requires a component-wise positive gap certificate over every admitted fixed-index faithful-group background component, together with the stronger determinant-line and reflection-positivity structure required for nonperturbative completion.

The one-loop Standard Model result remains

$(b_Y, b_2, b_3) = (41/6, -19/6, -7)$

PRIMITIVE SHARED-LINK FERMION TRANSPORT: SOURCE-EXPLICIT.

FULL FAITHFUL CAR/FOCK GAUGE TANGENT: 144-DIMENSIONAL EXECUTED PASS.

SOURCE-MATCHED COVARIANT WILSON ROUGHNESS: DERIVED.

ONE-SPECIES OVERLAP WINDOW $0 < \rho < 2r$: EXACT.

SPECTRAL CIRCLE AND FLAT-SECTOR INDEX ZERO: EXACT.

MAX-GAP REPRESENTATIVE $\rho\star = r$ AND $\delta\star = \min(r, 1)$: EXACT.

EXPLICIT REFERENCE CONSTANTS $\lambda_{max} = 16$, $d_+ = 4$, $C_{kin} \leq 4$ AND SAFE RADIUS $\varepsilon_{safe}(r)$: PASS AT DECLARED GRADES.

LOCAL FAITHFUL-GAUGE GAP, INDEX AND OVERLAP LOCALITY: PASS AT DECLARED GRADES.

HAMILTONIAN-TRANSFER NORMALISED WEYL POLE: PASS.

FREE-TRANSFER ZERO CENSUS, SINGLE ZERO AT $(\omega, q) = (0, 0)$: EXACT.

UV-LOCAL REGULATOR REMAINDER / NO SECOND F^2 LOG: PASS AT FREE-POLE GRADE.

GAUGED HAMILTONIAN-TRANSFER OPERATOR D_HT[U]: OPEN.

R1Q LOCAL ONE-LOOP FERMION UNIVERSALITY: CONDITIONAL PASS; FREE POLE AND IRRELEVANT-REMAINDER STRUCTURE ESTABLISHED, FULL GAUGED EXECUTION OPEN.

COMPLETE MASTER-ACTION D_f[U] SOURCE IDENTITY: OPEN.

D-SG6 LOCAL SOURCE-MATCHED CHIRAL REGULATOR: PASS.

D-SG6 COMPLETE COMPONENT-WISE GLOBAL CERTIFICATE: OPEN.

Appendix A: Corner Census

In four Euclidean regulator directions there are $2^4 = 16$ corners. The number with n_π components equal to π is $C(4, n_\pi)$, so the multiplicities are

1, 4, 6, 4, 1

Their Wilson masses are

$-\rho$, $2r - \rho$, $4r - \rho$, $6r - \rho$, $8r - \rho$

In the first phase $0 < \rho < 2r$, the sign pattern is

−, +, +, +, +

Exactly one corner therefore supplies the light overlap branch.

Appendix B: Proof that an Overlap Zero Requires the Physical Corner

$D_{ov} = (\rho/a)(1 + V)$. A zero mode requires V to possess an eigenvalue −1, not that V equal the scalar −1. The eigenvalues can be listed exactly. At momentum q, $X = i\gamma\cdot s + M$ is normal, because $(\gamma\cdot s)^2 = s^2$ is a scalar; $\gamma\cdot s$ is Hermitian with eigenvalues $\pm|s|$, each of multiplicity two, so the eigenvalues of X are $M \pm i|s|$ and the eigenvalues of $V = X/\sqrt{(X^\dagger X)}$ are

$$\boldsymbol{\lambda_\pm(V) = (M \pm i|s|) / \sqrt{(M^2 + s^2)}}$$

The condition $\lambda = -1$ forces $|s| = 0$ and $M < 0$. Since $s_\mu = \sin q_\mu$, the first condition means every momentum component satisfies $\sin q_\mu = 0$, so q is a Brillouin corner, where

$M = 2r\, n_\pi - \rho$

In $0 < \rho < 2r$, only $n_\pi = 0$ gives $M < 0$. Hence the unique overlap zero is the physical corner.

Appendix C: Modified Chirality Algebra

$D = \bar{a}^{(-1)}(1 + V)$ gives $1 - \bar{a}D = -V$. Therefore $\hat{\gamma}_5 = -\gamma_5 V$. Since $V = \gamma_5\varepsilon$ with $\varepsilon = \mathrm{sgn}\, H$,

$\hat{\gamma}_5 = -\varepsilon$

Thus $\hat{\gamma}_5^2 = \varepsilon^2 = 1$, and the finite chiral projectors are exact. The spectral-circle identity of Theorem 2A follows from the same algebra: $D^\dagger D = \bar{a}^{-2}(1 + V^\dagger)(1 + V) = \bar{a}^{-2}(2 + V + V^\dagger) = \bar{a}^{-1}(D + D^\dagger)$.

Appendix D: Index Stability and the Flat Trace

Let $\lambda_j(t)$ be the eigenvalues of a continuous family of Hermitian Wilson kernels $H(t)$. The index is $-\tfrac{1}{2} \Sigma_j \operatorname{sgn} \lambda_j$. As long as $\lambda_j(t) \neq 0$ for every j, t, each sign remains constant. Therefore the index cannot change. An index jump occurs only through spectral flow across zero.

For the flat trace of Theorem 7A: at each momentum, $\operatorname{Tr} H\rho(q) = 0$ because $\operatorname{Tr}(\gamma_5\gamma\mu) = 0$ and $\operatorname{Tr} \gamma_5 = 0$, while $H\rho(q)^2$ is the scalar $\Sigma\mu \sin^2 q\mu + M\rho(q)^2$. A traceless 4×4 Hermitian matrix whose square is a positive scalar c^2 has eigenvalues $+c, +c, -c, -c$, so $\operatorname{Tr} \operatorname{sgn} H\rho(q) = 0$ pointwise and the flat index vanishes identically.

Appendix E: Max-Gap Case Analysis

For completeness the two equality loci of Theorem 9B are recorded.

For $r \geq 1$ the flat gap 1 is attained on the set of momenta with exactly one component q satisfying $1 - \cos q = 1$ (that is, $q = \pm\pi/2$) and all other components zero: there $S = Q = 1$, the Wilson mass vanishes and $Hr^2 = 1$. For $r > 1$ this locus is exactly the minimising set; for $r = 1$ it extends along the single-component curve.

For $0 < r \leq 1$ the flat gap r is attained at both the physical corner ($S = 0$, $Hr^2 = \rho^2 = r^2$) and each nearest doubler corner (single component with $x = 2$, $Hr^2 = (2r - \rho)^2 = r^2$): the midpoint balances the two corner masses exactly, which is the structural reason the maximum sits at $\rho = r$.

The three upper-bound witnesses (physical corner, nearest doubler corner, mid-edge zero-mass momentum) show that no ρ in the window can exceed $\min(r, 1)$, so no further regulator optimisation is available on the flat branch.

Appendix F: Remaining Execution

The following steps are discharged at the grades declared above:

- source-explicit primitive shared-link kinetic difference identified;
- covariant positive roughness $B_U^\dagger B_U$ constructed;
- source-matched Wilson seed constructed;
- max-gap choice $\rho = r$ derived;
- exact flat gap $\min(r, 1)$ derived;
- flat-sector index zero and spectral-circle confinement derived;
- nonzero faithful-background gap radius derived in closed form;
- reference constants $\lambda_{max} = 16$ and $d_+ = 4$ evaluated exactly, $C_{kin} \leq 4$ bounded at canonical-reference grade, and the explicit parameter-only radius $\varepsilon_{safe}(r)$ derived;

- local index stability and overlap locality established, with explicit rate;
- particle/antiparticle CAR-sector typing consumed from the charge-conjugation completion programme;
- positive physical Fock excitation energy separated from the signed local Clifford operator;
- sequential Hamiltonian-transfer quadratic kernel constructed;
- unit-normalised left- and right-Weyl continuum poles derived;
- exact free-transfer zero census executed, single zero at $(\omega, q) = (0, 0)$;
- two-cutoff regulator remainder $O[(h_n + c\, a_t)\, p^2]$ derived;
- UV determinant split into unique-pole scaling region and gapped analytic regulator region executed;
- regulator-dependent remainder proved incapable of generating a second parity-even F^2 infrared logarithm at the free-pole/irrelevant-remainder grade;
- R1Q local one-loop fermion universality placed at conditional-pass grade.

The remaining strict execution is now the gauged Hamiltonian-transfer operator, source-identification and the global certificate, rather than local one-loop coefficient closure:

1. write out the fully gauged Hamiltonian-transfer operator D_HT[U], with its background-gauge transformation law and fermion-gauge vertices, and demonstrate the locality, continuum-limit and regularity assumptions of the one-loop power counting for it directly, converting the R1Q verdict from conditional to full;
2. freeze or reconstruct the complete physical finite Master-Action K_U/D_f[U] family;
3. calculate its actual kinetic Lipschitz constant C_kin and other finite-carrier constants rather than relying on comparison-representative bounds;
4. prove direct identity with, or an admissible regulator deformation into, the constructed source-matched Hamiltonian-transfer/overlap universality family;
5. freeze the admissible faithful-group background class and its topological component labels, preferably through a gauge-invariant plaquette admissibility condition superseding the fixed-gauge chart radius;
6. calculate or bound inf gap Hr[U] on each fixed-index component;
7. verify quantitative locality across refinement and confirm the known index-one witness in the same frozen kernel family;
8. execute a blind finite determinant calculation on the frozen source kernel as a validation of the analytically fixed one-loop coefficient, rather than as the mechanism used to select that coefficient;
9. continue the stronger determinant-line, global-anomaly and reflection-positivity programme required for complete nonperturbative quantisation.

R1Q LOCAL ONE-LOOP FERMION UNIVERSALITY: CONDITIONAL PASS; FULL GAUGED EXECUTION OPEN

STRICT COMPLETE MASTER-ACTION D_f[U] SOURCE IDENTITY: OPEN

D-SG6 LOCAL SOURCE-MATCHED REGULATOR: PASS

D-SG6 COMPONENT-WISE GLOBAL ADMISSIBILITY/GAP CERTIFICATE: OPEN

Internal VERSF Sources Consumed

1. From Schrödinger to Dirac: graph Dirac operator, closure-derived Wilson stiffness, doubler lifting and continuum Dirac limit.
2. Spinorial Closure Transport and Fermionic Source-Carriers in VERSF: Clifford structure, chirality and spinorial gauge coupling.
3. The Fermionic Fock-Space Reconstruction in VERSF: positive Fock/CAR matter carrier.
4. The Master Substrate Action and Constitutive programme: shared physical gauge link and fermionic kinetic kernel.
5. The Electroweak Representation and Connection Closure in VERSF: $P_W = P_L$ at the declared grade.
6. The Weak-Orientation Admissibility Theorem in VERSF: orientation and mirror-sector discipline.
7. The Chiral Matter Representation Theorem in VERSF: frozen chiral matter ledger.
8. The Generation and Species Census Theorem in VERSF: three-generation census.
9. The Quantum Gauge Completion of the VERSF Standard Model: anomaly, background-field running and regulator-independence architecture.
10. The Substrate Cohomology, Global Gauge Measure and Strong-Phase programme: finite chiral Dirac family requirements, overlap/Ginsparg-Wilson reference implementation, determinant-line architecture and index-one witness.
11. The Route-M Off-Shell Integrability, Current-Source Closure and Direct Physical Running Theorem in VERSF: one-loop coefficient synthesis and explicit shared-link fermion universality debt.
12. The Multi-Regulator Full-Faithful Source Assembly: exact faithful per-edge matrix transport, executed 45-dimensional matter representation, full-rank 144 CAR/Fock gauge jet and conditional source-law compatibility certificates.
13. The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF: corrected independent particle/antiparticle CAR families, charge-conjugate carrier typing, Fock-space charge conjugation and the non-doubling firewall used to separate physical antiparticle completion from regulator-generated species doubling.

Imported mathematics, not claimed as new VERSF output: overlap/Ginsparg-Wilson functional calculus, spectral sign operators, standard lattice Wilson-kernel topology, Weyl spectral perturbation bounds, Chebyshev polynomial approximation of the inverse square root, and standard locality consequences of a finite-range uniformly gapped overlap seed. The new VERSF-specific returns of this paper are the source-matched identification B_U from the primitive Master kinetic action, the covariant roughness $L_U = B_U^\dagger B_U$, the max-gap representative $\rho = r$, the exact gap $\delta\bigstar = \min(r, 1)$, the closed-form admissibility radius ε_{crit}, the spectral-circle and flat-index-zero certificates for the VERSF-seeded representative, the explicit nonzero faithful-gauge neighbourhood preserving gap, index and locality, and the evaluated reference constants $\lambda_{max} = 16$, $d_+ = 4$, $C_{kin} \leq 4$ with the parameter-only safe radius $\varepsilon_{safe}(r)$ under the carrier-mixing firewall.

Additional imported mathematics used in the Hamiltonian-transfer extension, and not claimed as new VERSF output, consists of standard fermionic coherent-state reconstruction of a quadratic transfer kernel, particle-hole reinterpretation of negative-frequency one-particle modes, the trace-log expansion of determinant ratios, Wilsonian separation of scaling and gapped regulator regions, and the standard analyticity/local-effective-action consequence of integrating out a local gapped sector. The new VERSF-specific return is the application of those tools to the already-derived shared-link Wilson structure, sequential-transfer ontology, completed CAR/Fock particle/antiparticle carrier and frozen chiral matter ledger, yielding the unit-normalised Hamiltonian-transfer Weyl pole and the local no-second-log regulator certificate.