MFKR-1 | VERSF Standard Model Quantum Closure Series

The Finite Master-Action Fermion Kernel Reconstruction and Source-Identity Gate in VERSF

From the Primitive Shared-Link CAR/Fock Bilinear through Exact Finite Kinetic-Matrix Reconstruction, the Clifford-Linear Edge Reduction and K=7 Infrared Star-Moment Closure with Its Laplacian Square Rigidity and Quantitative Stability, the Source-Native Kinetic Lipschitz Bound and Its Exact Sharpness on Both Reference Carriers, the Record-Defined Sequential Generator, the Unique Quasi-Free Fock Lift, the Full Mixed-Tensor CAR/Fock Completion Certificate, and the Remaining Finite-Manifest, Post-Commitment and Attaching-Map Selections

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Status: Theorem-level source-reconstruction paper. Exact finite kinetic-operator construction; Clifford-linear edge reduction with $\varepsilon_{even} = 0$ structural on the inherited anti-adjoint branch, an input not required for the infrared closure; K=7 infrared star-moment closure at the source-derived refinement-geometry grade on the declared physical D = 3 branch, with $M_{IR} = I_3$ modulo the declared spin-frame equivalence and outright on the tangent-aligned positive-weight branch, hence a source-derived normalised spatial Dirac/Weyl principal symbol. The old involution/translation conflation stands as a source audit but is no longer load-bearing; the D = 3 selection is inherited, not derived here. The closure is rigid and quantitatively stable with constant one. Canonical sharpness $C_{kin}^{3,canon} = D_\Gamma^{3,canon} = 3$ is exact; the physical weighted degree, physical C_{kin} and saturation are finite-manifest outputs. Record-defined sequential generator unique in its class on the one-survival-macrostate lumping; unique quasi-free Fock lift of a supplied transfer; record-to-fermion transfer identification open; full mixed-tensor CAR/Fock compatibility pass; record/no-record semantics executed at the implemented F2F grain. The microscopic edge manifest remains required for finite-matrix instantiation, and the physical frame/post-commitment/attaching-map selections remain open

General Reader Summary

Earlier work showed that the machinery for putting matter particles on this framework's finite structure can be built from ingredients the framework already contains. One ingredient was always described as "to be filled in later": the table telling each particle how to hop from one point to the next.

Most of that ingredient was never missing. The founding rulebook already determines the table, and the paper writes out the assembly and proves it respects the required symmetries. The coefficients are far less free than they looked: only one shape is allowed, built from a direction and a single number per connection, and the unwanted even part vanishes once one inherited symmetry of the rulebook is granted; the low-energy result below does not depend on that symmetry. An audit of an older building block found it was asked to do two incompatible jobs at once, acting both as a mirror flip and as a small step in space, which no single operation can do; that diagnosis stands, but it is no longer an obstacle, because newer work in the programme builds genuine directed transport along the underlying network, proves it has a well-defined continuum limit, and uses it to construct the local frame and the geometry of emergent space itself. Comparing the particle rule directly with that independently derived geometry closes the low-energy check with no freedom at all: the mirror ambiguity is removed by the orientation the construction carries, a rotation is only a change of viewing frame, and on the most direct branch even that freedom disappears, so the correct low-energy particle equation needs no hand normalisation. The argument is also tighter than a lucky match: any hopping table whose energy rule comes out right must already be the correct table, apart from a harmless change of viewpoint, and a nearly right energy rule means a nearly right table by at least the same margin, so small errors cannot hide there. That checking procedure has now been run end to end on worked examples, including one deliberately detuned, and caught exactly what it should. The results hold in any number of dimensions, with three special in one respect: it is the smallest useful dimension where a mirror-image choice remains a real piece of physical data. What remains unpublished is the microscopic edge-by-edge manifest needed to print the finite regulator matrix and its finite-level constants. Sensitivity to background fields is bounded by a simple count of connections at a point, and that bound is the best possible.

The other half of the problem is sequential development, an ordered succession of opportunities for a new fact to be written. An earlier result looked discouraging: under mild conditions alone, many stepping rules are equally admissible. The record-making law supplies what is missing, and a later execution corrects what that law actually says. The distinction is not between one special channel and the rest, since the channel once read as "nothing happened" contributes to every kind of record. The retained ledger record decides: a handful of self-referring marks write nothing new, every other transition writes a record. Given the chance of writing one and the odds over record types, exactly one continuous stepping rule fits, and the earlier family is that same rule on differently

calibrated clocks. A first record is certain to arrive, after a little over one opportunity on average. Extending from one particle to many admits exactly one rule once a single-particle stepping rule is supplied; what has not yet been built is the bridge that turns the record-keeping rule into that single-particle stepping rule, and the two sides of that bridge are different kinds of object.

What remains is more specific, and one part is a genuine limit rather than unfinished work. The framework must pick the frame on which the record-making chance is measured, and supply the map from internal opportunities to real duration, since the bookkeeping counts do not match. It must also decide what happens to a record once written. Keeping, renewing, copying, erasing and overwriting are all still consistent with everything measured so far, so no further calculation of the present kind can settle it; only a choice made by the underlying rules can.

One compatibility worry closes completely. The fermion-completion structure hangs on a single primitive direction, so a check that looked as if it needed the full machinery collapses onto one already performed, and passes, while a stripped-down version fails outright. The remaining foundational question is whether the deepest dynamics singles these laws out rather than merely tolerating them.

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Central Result

$\Gamma_{kin}^{(0)} \Rightarrow K_U^{src} \Rightarrow C_{kin}^{src} \leq D_\Gamma$, with $D_\Gamma = \max_v \Sigma_{e\ni v} \|\Gamma_e\|$ attained exactly

exactly at the operator-form source grade, with the weighted-degree constant D_Γ proved sharp: on the canonical four-direction carrier the exact induced kinetic sensitivity equals $D_\Gamma = 4$, while the commuting class (containing every constant abelian background) has sharp supremum $\sqrt{d} \cdot \varepsilon_U$, attained by the uniform constant perturbation, with individual backgrounds free to respond below it.

On the inherited minimal anti-adjoint Clifford branch the edge coefficient reduces to

$\Gamma_e = (w_e/2)\, G(n_e)$, $\varepsilon_{even} = 0$ structurally, $D_\Gamma = (1/2) \max_v \Sigma_{e\ni v} w_e$,

so the source-identity manifest is not a table of matrices but the geometric map $e \mapsto (n_e, w_e)$, and the principal-symbol test becomes the star moment condition $\Sigma_e w_e n_e^i a_e^j = \delta^{ij}$. The older K=7 transport source correctly identified a three-axis structure but incorrectly asked exact antipodal involutions to act simultaneously as infinitesimal translations; that inconsistency remains an exact audit result, but later VERSF transport papers make it non-load-bearing. The K=7 commitment foam carries genuine oriented edge operators U_{ij}, inverse under path reversal, whose ordered path holonomies possess a refinement-Cauchy continuum limit T_γ, and the subsequent Lorentz-transport construction builds from them a refinement frame $e^a{}_\mu$ and the soldered continuum metric $g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$ with $\nabla_\mu e^a{}_\nu = 0$. On the declared physical $D = 3$ branch the three orthonormal spacelike legs of this frame give the substrate-derived second-order spatial kinetic symbol $|p|^2 = \Sigma p_i^2$, while the exact Clifford-linear identity proved here gives $K_{IR}(p)^2 = p^T(M^TM)p \cdot I$. Matching the first-order fermion square to the independently derived second-order geometry therefore forces $M^TM = I_3$. Orientation-preserving refinement excludes the reflection component, the surviving SO(3) freedom is exactly the common spin-frame equivalence of Theorem 6, and in the refinement spin frame

$M_{IR} = I_3$, $\varepsilon_{moment}^{IR} = 0$,

with the identity holding outright, without any quotient, on the tangent-aligned positive-weight branch. Thus the normalised spatial Dirac/Weyl principal symbol is source-derived at the refinement-geometry grade. The overall scale left in the substrate bilinear form does not reopen the directional normalisation: it rescales the common ruler, is absorbed when the refinement frame is orthonormalised, and cannot generate three independent coefficients. The global $D = 3$ branch itself remains an inherited/foundational dimensionality input; this paper does not claim to derive why the continuum has three spatial dimensions. The unpublished microscopic edge map remains required for finite-regulator matrix instantiation and finite-level constants, not for the infrared principal-symbol normalisation.

The closure is rigid and quantitatively stable. The Clifford-linear star symbol squares exactly to a scalar at every momentum, $K_I(q)^2 = |v(q)|^2 I$ with $v(q) = \Sigma_e w_e \sin(q \cdot a_e) n_e$, so a unit-coefficient transport Laplacian forces $M^T M = I$; the residual $O(3)$ freedom is exactly the declared spin-frame rotation and orientation choice, and it is empty on the tangent-aligned positive-weight branch, where $\|M - I\| \leq \|M^2 - I\|$ makes the closure stable under refinement error with constant one. The three-direction sensitivity constant is also attained on the canonical carrier: an explicit 2^3 Jordan-Wigner witness returns $\|\Delta K\| = 3\varepsilon_U$, so

$C_{kin}^{3,canon} = D_\Gamma = 3$ exactly, with equality on the fully instantiated physical K=7 carrier a finite-carrier execution question.

None of this is special to three dimensions except the one datum that should be: the scalar square, the Laplacian rigidity and the stability constant hold for arbitrary Clifford-linear stars in every dimension, the canonical d-direction carrier admits the exact saturation $C_{kin} = D_\Gamma = d$ in every dimension, and the orientation choice survives as independent source data precisely in odd dimension, where the Clifford volume element is scalar. And the certification pipeline the stability bound promises is executed, not merely available: with an a priori differencing budget computed from the manifest, four reference stars, one deliberately detuned and one deliberately misaligned, returned certified moment bounds at five probe scales with zero certificate violations.

The sequential composition form is constructed rather than assumed, and its event semantics are executed rather than premised. The distinction is record-defined: let $s(x)$ be the probability that one sequential opportunity leaves the history in one of the seven self-history marks, so that

$$\mathbf{h_F(x) = 1 - s(x)}$$

is the probability of writing a new retained record, and, with $p_a(x)$ the probability of retained ledger class $a \in \{1, \ldots, 84\}$,

$$\mathbf{q_a(x) = p_a(x)/h_F(x),\ \Sigma_a\, q_a(x) = 1.}$$

The first-record integrated intensity is then

$$\mathbf{\lambda_F(x) = -\log s(x) = -\log[1 - h_F(x)],}$$

generating, on the one-survival-macrostate lumping of the state-resolved data, the unique absorbing first-record semigroup $R(\delta) = e^{\delta Q}$ with $Q_{00} = -\lambda_F$, $Q_{0a} = \lambda_F q_a$ and $R(1) = P_{rec}$; the earlier admissible continuum is again a clock normalisation that the record-defined survival datum fixes. On the executed F2F carrier the no-record map has spectral radius

$\rho(NR) = 0.200305886206958 < 1$,

so a first record occurs almost surely from every initial internal state, with stationary mean first-record waiting count 1.234602249934960 sequential opportunities. Above any supplied one-particle transfer the quasi-free Fock lift is unique; the identification of the record semigroup with a fermionic one-particle transfer is a separately typed open map.

The last mixed CAR/Fock compatibility debt also closes. The declared RRHF/FCHI completion vertex carries a single primitive completion direction $c_H = (1/2)(1, 1, 1, 1)$ with $||c_H|| = 1$, so every declared nonzero completion tensor factorises as $T_f = c_H \otimes B_f$ and the normalised mode-1 residual is independent of the external CAR/Fock structure:

$\varepsilon_f^{(3)} = ||(I - P_{full})c_H|| / ||c_H|| = 1.149 \times 10^{-15}$, $\varepsilon_{\{f,D\}}^{(3)} = 1$.

The full physical chiral $D_f[U]$ is still not promoted to source-derived, but the kinetic infrared principal-symbol normalisation is no longer part of the residue. What remains on the kinetic side is the microscopic finite edge manifest needed for numerical finite-regulator instantiation and same-carrier constants; in parallel remain the physical record frame, the post-commitment operation selector, the attaching map to physical duration, fermionic positivity and primitive-source uniqueness, rather than the composition rule or the event semantics.

Abstract

The VERSF Master Substrate Action specifies the primitive fermionic matter transport on an oriented finite regulator by

$\Gamma_{kin}^{(0)} = \Sigma_{\{e:i\to j\}} \bar{\Psi}_j \gamma^e (\Psi_j - R_f(U_e)\Psi_i) + h.c.$

where the matter carrier is the occupied faithful Standard Model CAR/Fock representation and U_e is the same physical link used by the gauge and closure sectors. No independent matter-link normalisation is permitted. Earlier CWGW work used this source law to define the covariant incidence operator B_U, the positive roughness $L_U = B_U^\sharp B_U$, and the source-matched Wilson seed

$X_\rho[U] = K_U + (r/2)L_U - \rho I$,

but retained the fully instantiated finite K_U as a source-level execution debt. This paper reconstructs that operator exactly at finite regulator.

Let S and T be the source and target incidence pullbacks from the vertex fermion carrier to the oriented-edge carrier, let

$\mathcal{U}\_f = \bigoplus_{e \in E_n} R_f(U_e)$, $B_U = T - \mathcal{U}_f S$,

and let

$\Gamma_E = \bigoplus_{e \in E_n} \Gamma_e$

denote the edge Clifford/kinetic coefficient, with any frozen geometric factor absorbed into Γ_e. After canonicalisation of the positive one-particle fermion pairing, let $\sharp$ denote its adjoint. Then the primitive action has the exact matrix representation

$\Gamma_{kin}^{(0)} = \bar{\Psi} A_U \Psi + \text{h.c.}$, $A_U = T^\sharp \Gamma_E B_U$,

and hence the source kinetic Hessian is

$$\mathbf{K_U^{src} = T^\sharp \Gamma_E B_U + (T^\sharp \Gamma_E B_U)^\sharp.}$$

The finite block matrix is therefore fixed uniquely by the regulator incidence data, the source edge coefficients and the shared faithful links.

Gauge covariance follows exactly:

$K_{U^g}^{src} = G_f(g) K_U^{src} G_f(g)^{-1}$.

The link perturbation is also explicit:

$K_U^{src} - K_I^{src} = -T^\sharp\Gamma_E(\mathcal{U}_f - I)S - [T^\sharp\Gamma_E(\mathcal{U}_f - I)S]^\sharp$.

If

$\varepsilon_U = \max_e \|R_f(U_e) - I\|$,

then the block Schur bound gives

$$\mathbf{\|K_U^{src} - K_I^{src}\| \le D_\Gamma\, \varepsilon_U,\quad D_\Gamma = \max_v \Sigma_{e \ni v} \|\Gamma_e\|,}$$

so that

$$\mathbf{C_{kin}^{src} \le D_\Gamma.}$$

For the canonical d-direction nearest-neighbour comparison carrier, where each of the 2d incident edge blocks carries norm 1/2, this returns $D_\Gamma = d$, reproducing $C_{kin} \le 4$ on the four-dimensional Euclidean CWGW representative and $C_{kin} \le 3$ on the three-direction spatial comparison carrier.

The Lipschitz analysis is then sharpened in three exact steps. First, on any directional carrier satisfying the exact-stencil certificate the link perturbation factorises Clifford-diagonally,

$\Delta K = -\Sigma_\mu \Gamma_\mu N_\mu$, $N_\mu = Z_\mu S_\mu - S_\mu^\sharp Z_\mu^\sharp$, $N_\mu^\sharp = -N_\mu$, $\|N_\mu\| \le 2\varepsilon_U$,

with the internal perturbation Z_μ commuting with the spinorial coefficient. Squaring separates a definite quadrature term from pure commutator interference:

$$\|\Delta K\|^2 \le (1/4)\left[\, \|\Sigma_\mu c_\mu^2 N_\mu^2\| + \Sigma_{\mu<\nu} c_\mu c_\nu \|[N_\mu, N_\nu]\| \,\right],\ \Gamma_\mu = (c_\mu/2)G_\mu^{Cliff}.$$

Second, whenever the direction perturbations mutually commute (in particular for every constant abelian background), the commutators vanish and the response drops to the quadrature value

$$\|\Delta K\| \le \sqrt{d} \cdot \varepsilon_U \text{ (canonical weights } c_\mu = 1\text{)},$$

which is attained exactly by the uniform perturbation $\Delta_e = i\varepsilon$ at zero momentum. Third, the general weighted-degree bound is proved sharp: an explicit alternating-sign abelian perturbation on the 2^4 carrier realises pairwise anticommuting perturbation operators of Jordan-Wigner type and returns $\|\Delta K\| = 4\varepsilon_U$ exactly, so that

$$C_{kin}^{exact} = D_\Gamma = 4$$

on the canonical four-direction carrier, and genuine unitary U(1) link families approach the same ratio in the small-field limit. The constant D_Γ therefore cannot be improved without restricting the background class, and the commuting/anticommuting dichotomy identifies exactly which backgrounds pay the linear price and which only the quadrature price.

A further theorem identifies precisely what must be checked to match the source matrix to the inherited flat Dirac symbol. On a translation-invariant directional branch,

$$K_I(q) = \Sigma_\mu \left[\, \Gamma_\mu(1 - e^{-iq_\mu}) + \Gamma_\mu^\sharp(1 - e^{iq_\mu}) \,\right].$$

Writing

$$\Gamma_{\mu,\pm} = (1/2)(\Gamma_\mu \pm \Gamma_\mu^\sharp)$$

gives the exact decomposition

$$K_I(q) = 2\Sigma_\mu(1 - \cos q_\mu)\Gamma_{\mu,+} + 2i\Sigma_\mu \sin q_\mu\, \Gamma_{\mu,-},$$

hence

$$K_I(q) = 2i\Sigma_\mu q_\mu \Gamma_{\mu,-} + \Sigma_\mu q_\mu^2 \Gamma_{\mu,+} + O(q^3).$$

The normalised Clifford principal symbol therefore follows if and only if the anti-adjoint edge part satisfies

$$2\Gamma_{\mu,-} = G_\mu^{Cliff},$$

where G_μ^{Cliff} denotes the inherited normalised continuum Clifford generator in the declared Euclidean or Hamiltonian convention. Exact equality with the pure canonical sine stencil additionally requires

$$\Gamma_{\mu,+} = 0.$$

The first condition is the principal-symbol source-identity certificate; the second is the stronger exact-stencil certificate. An exact corner census shows that under the stencil certificate the source symbol vanishes at all 2^d momentum corners, so doubler lifting is entirely the responsibility of the Wilson roughness, never of the first-order source term.

The edge coefficient is then reduced, not merely tested. On the inherited minimal anti-adjoint Clifford branch the only compatible edge coefficient is

$$\Gamma_e = (w_e/2)\, G(n_e),\quad G(n_e) = \Sigma_i\, n_e^i\, G_i,$$

up to common spin-frame equivalence and orientation, with w_e a scalar geometric weight. Since $G(n_e)^\sharp = -G(n_e)$, the even part vanishes identically on that branch, so

$\varepsilon_{even} = 0$ structurally on the inherited anti-adjoint branch, and $\varepsilon_{Cliff} = 0$ on the canonical normalised three-channel branch,

with no numerical execution required. Because $\|G(n_e)\| = 1$ the weighted degree becomes purely scalar, $D_\Gamma = (1/2)\max_v \Sigma_{e\ni v}\, w_e$. For a general non-axis-aligned star the exact symbol collapses to $K_I(q) = i\Sigma_e\, w_e\, G(n_e) \sin(q\cdot a_e)$, and the principal-symbol test becomes the star moment certificate

$$M^{ij} = \Sigma_e\, w_e\, n_e^i\, a_e^j = \delta^{ij},$$

which for tangent-aligned edges is the discrete isotropy condition $\Sigma_e\, w_e|a_e|\, n_e^i\, n_e^j = \delta^{ij}$. An audit of the oldest K=7 transport source finds an inconsistency: its published channels are simultaneously exact involutions, $T^2 = I$, and translation generators through $(T - I)/\xi \to \partial_i$, which is impossible, since $T^2 = I$ forces the difference quotient's spectrum into $\{0, -2/\xi\}$. That audit stands as a source firewall, but it is no longer load-bearing: later VERSF transport papers construct genuine oriented edge transport U_{ij} on the K=7 commitment foam, prove the refinement-Cauchy continuum limit of its path holonomies T_γ, and build from them the refinement frame $e^a{}_\mu$ and soldered metric $g_{\mu\nu} = e^a{}_\mu\, e^b{}_\nu\, \eta_{ab}$. On the declared physical D = 3 branch the orthonormal spatial refinement frame gives the substrate-derived second-order kinetic symbol $\Sigma_i\, p_i^2$, and the exact Clifford-linear square $K_{IR}(p)^2 = p^T(M^TM)p \cdot I$ then forces $M^TM = I_3$. Orientation-preserving refinement excludes the reflection component, the surviving SO(3) freedom is exactly the declared spin-frame equivalence, and in the refinement spin frame

$M_{IR} = I_3$, $\varepsilon_{moment}^{IR} = 0$,

outright, without any quotient, on the tangent-aligned positive-weight branch. The normalised spatial Dirac/Weyl principal symbol is therefore source-derived at the refinement-geometry grade; the global D = 3 selection remains an inherited/foundational branch input, not a result of this paper. The remaining geometric publication obligation is narrower: publish the microscopic glued-edge map $e \mapsto (n_e,w_e)$ to instantiate the finite physical K_U, C_{kin}, λ_{max} and d_+. It is no longer load-bearing for the infrared principal-symbol normalisation. The closure is moreover rigid and stable. The Clifford-linear star symbol obeys the exact scalar square identity $K_I(q)^2 = |v(q)|^2 I$ at every momentum, with $v(q) = \Sigma_e\, w_e \sin(q\cdot a_e) n_e$, so matching the unit transport Laplacian at second order forces $M^TM = I$; the surviving O(3) freedom is exactly the spin-frame rotation and orientation choice already quotiented by the reduction, and on the tangent-aligned positive-weight branch it is empty, forcing M = I outright. Quantitatively, $\varepsilon_{moment} = \|M - I\| \le \|M^2 - I\|$ on that branch, so refinement error in the Laplacian coefficient controls the moment residual with constant one. The sharpness theorem transfers to three directions as well: an alternating-sign 2^3 Jordan-Wigner witness on the canonical periodic carrier returns $\|\Delta K\| = 3\varepsilon_U$ exactly, so $C_{kin} = D_\Gamma = 3$ as an induced norm there, with genuine U(1) links approaching the same ratio and the uniform constant-abelian background attaining the commuting-class ceiling $\sqrt{3}\,\varepsilon_U$; the exact physical K=7 weighted degree, physical C_{kin} and saturation of the canonical value 3 remain finite-manifest outputs. All of these statements except one are proved for Clifford-linear stars in arbitrary dimension d, with the exact saturation $C_{kin} = D_\Gamma = d$ admitted by the canonical d-direction carrier in every dimension; the exception is the orientation datum, which exists as independent source data exactly in odd d, where the Clifford volume element is central. The certified moment pipeline is executed on explicit reference stars with an a priori differencing budget, returning zero certificate violations across all manifests and scales, including a detuned star whose non-vanishing moment residual the certificate correctly brackets and a misaligned star for which it correctly refuses to certify more than the distance to the orthogonal set.

The sequential completion is treated with the same separation, and its event semantics are now executed rather than premised. The inherited recurrent heat-flow result remains a genuine class-scoped no-go: positivity, normalisation, reversibility, the stationary state and the symmetry carrier alone admit a continuum of sequential transfers. What supplies the missing physical typing is the executed record calculation, and it also corrects the typing previously assumed. The persistent ledger does not realise the earlier accepted-current split, because the channel formerly read as "no physical event" contributes to all 84 retained record classes and therefore cannot mean that. The implemented partition is instead

self-history ↔ no new record, nonself history ↔ record-producing,

which at the finest executed history-current grain is 7 no-record marks against 312 record-producing marks, the latter coarsening onto the 84 retained ledger classes. With survival $s = 1 - h_F$ and retained-record probabilities $p_a = h_F\, q_a$, setting

$$\boldsymbol{\lambda_F = -\log s = -\log(1 - h_F),\ Q_{00} = -\lambda_F,\ Q_{0a} = \lambda_F\, q_a,}$$

returns the unique absorbing first-record semigroup

$$\boldsymbol{R(\delta) = e^{\delta Q},\ R_{00}(\delta) = e^{-\lambda_F \delta},\ R_{0a}(\delta) = (1 - e^{-\lambda_F \delta})q_a,}$$

with $R(1) = P_{rec}$ exactly at one declared sequential opportunity, and the earlier admissible continuum is again a clock reparametrisation of this semigroup once the record-defined survival datum is supplied. The executed no-record map has spectral radius $\rho(NR) = 0.200305886206958 < 1$, so first entry into a retained record class is almost sure from every initial internal state and has finite mean waiting count, the stationary value being 1.234602249934960 sequential opportunities. These two numbers are differently typed and must not be conflated: $\rho(NR)$ certifies exhaustion, while λ_F is built from the state-resolved survival probability, and only for a state-independent no-record map do the scalar relations between them coincide.

Above the one-particle transfer, the vacuum-preserving, number-conserving quasi-free CAR/Fock lift is unique. The remaining sequential uncertainty therefore lies neither in composition nor in second quantisation.

Permanent persistence is a separate question with an exact answer about what the current data can and cannot decide. On the declared append-only renewal branch $\Phi_R = \Psi_A \otimes id_L$, Heisenberg invariance of the ledger projectors gives $\Phi_R^*(P_a) = P_a$ for every retained class and hence $\varepsilon_{leak} = 0$ exactly. But an admissible post-commitment overwrite continuation gives $\varepsilon_{leak} = 1$, and convex mixtures realise every value in between, so

$\varepsilon_{leak} \in [0, 1]$ is compatible with the already-fixed first-entry and record-copy data.

The first-entry evidence therefore does not determine persistence; source selection of the post-commitment operation is genuinely required, and no further leakage calculation can substitute for it.

Finally the last mixed CAR/Fock compatibility debt is removed. Because every declared RRHF/FCHI completion tensor carries the unique primitive leg $c_H = (1/2)(1, 1, 1, 1)$, the mode-1 residual factorises exactly, $T_f = c_H \otimes B_f$ with the external CAR/Fock structure cancelling from the normalised quotient. The full mixed-tensor residual therefore equals the primitive residual, $\varepsilon_f^{(3)} = 1.149 \times 10^{-15}$, while the six-defect-only residual is exactly unity, so the potential rows are load-bearing rather than cosmetic and D-HM29 passes at the declared conditional RRHF/FCHI source-stack grade.

The audited corpus does not publish the microscopic glued-regulator geometric map required to print the finite physical matrix and execute all finite-regulator constants. The K=7 coarse/infrared moment residual closes at the source-derived refinement-geometry grade of Section 14.3. Therefore the paper reaches the following grade:

FINITE MASTER-ACTION K_U: EXACTLY RECONSTRUCTED AS A SOURCE FUNCTIONAL

SOURCE-NATIVE C_kin BOUND: DERIVED AND PROVED SHARP

EDGE-CLIFFORD FORM: REDUCED TO $(w_e/2)G(n_e)$ WITH $\varepsilon_{even} = 0$ STRUCTURAL ON THE INHERITED ANTI-ADJOINT BRANCH; THE ANTI-ADJOINTNESS INPUT IS NOT REQUIRED FOR THE SOURCE-DERIVED IR PRINCIPAL-SYMBOL CLOSURE

K=7 COARSE/INFRARED STAR MOMENT: SOURCE-DERIVED REFINEMENT-GEOMETRY PASS ON THE DECLARED D = 3 BRANCH, $M^T M = I_3$, $M_{IR} = I_3$ MODULO THE DECLARED SPIN-FRAME EQUIVALENCE AND OUTRIGHT ON THE TANGENT-ALIGNED BRANCH

NORMALISED SPATIAL DIRAC PRINCIPAL SYMBOL: SOURCE-DERIVED INFRARED PASS AT THE REFINEMENT-GEOMETRY GRADE

MICROSCOPIC PHYSICAL GEOMETRIC EDGE MAP $e \mapsto (n_e, w_e)$: NOT YET PUBLISHED; REQUIRED FOR FINITE-MATRIX INSTANTIATION, NOT FOR THE IR PRINCIPAL SYMBOL

RECORD/NO-RECORD EVENT SEMANTICS: EXECUTED PASS AT THE IMPLEMENTED F2F GRAIN

RECORD-DEFINED SEQUENTIAL GENERATOR: UNIQUE IN THE ABSORBING FIRST-RECORD CLASS

RECORD FIRST-ENTRY COMPOSITION: CLOSED IN THE DECLARED ABSORBING CLASS

FERMIONIC FOCK LIFT OF A SUPPLIED ONE-PARTICLE TRANSFER: UNIQUE

RECORD-SEMIGROUP TO FERMIONIC ONE-PARTICLE-TRANSFER IDENTIFICATION: OPEN; REQUIRES EXPLICIT INTERTWINING MAP

APPEND-ONLY RENEWAL PERSISTENCE: ε_leak = 0 EXACTLY

PHYSICAL POST-COMMITMENT PERSISTENCE: NON-IDENTIFIABLE FROM CURRENT DATA, ε_leak ∈ [0, 1]

FULL MIXED-TENSOR CAR/FOCK COMPLETION DESCENT: PASS AT 1.149×10^{-15}

PHYSICAL FRAME, POST-COMMITMENT SELECTION AND ATTACHING MAP: OPEN

UNIQUE FULL CHIRAL D_f[U] SOURCE IDENTITY: OPEN, OBSTRUCTION NOW NAMED

The previous local one-loop universality result is therefore retained but not over-promoted. The remaining source debt is no longer an unspecified fermion operator: it is a finite publication-and-execution manifest.

1. Problem Statement

The VERSF Standard Model quantum-closure chain has reached

primitive matter transport → Wilson stiffness → local chiral regulator → Weyl one-loop universality.

CWGW-1 established a source-matched local regulator by defining

(B_UΨ)e = Ψ{t(e)} − R_f(U_e)Ψ_{s(e)}

from the primitive Master-Action transport, forming

L_U = B_U^♯B_U,

and using the seed

X_ρ[U] = K_U + (r/2)L_U − ρI.

That programme obtained the one-species phase, exact finite Ginsparg-Wilson chirality, the max-gap regulator value ρ = r, a nonzero faithful-background gap neighbourhood, and exponential locality.

The remaining caveat was consistently phrased as: the primitive kinetic operator is source-explicit, but the complete finite physical K_U/D_f[U] family has not been frozen and published.

The present paper separates three logically different questions hidden inside that sentence.

Q1: Operator reconstruction. Does the primitive Master action uniquely determine K_U as a finite matrix once its own finite source data are supplied?

Q2: Numerical source identity. Does the published source package contain enough finite data to instantiate every matrix entry and calculate the exact physical C_kin, flat spectrum and regulator constants?

Q3: Chiral completion identity. Does the primitive action uniquely select the full overlap/Ginsparg-Wilson or Hamiltonian-transfer chiral completion, rather than merely supplying a seed that belongs to the required universality class?

Q1: YES.

Q2: INFRARED PRINCIPAL-SYMBOL GATE CLOSED AT THE SOURCE-DERIVED REFINEMENT-GEOMETRY GRADE / FINITE MANIFEST STILL OPEN. The matrix freedom is eliminated on the inherited minimal Clifford branch and ε_even vanishes structurally. The older antipodal-involution construction cannot itself generate spatial derivatives, but later K=7 refinement-transport work independently constructs genuine oriented path transport, its continuum limit and the refinement-frame soldering that generates the continuum spatial metric. On the declared physical D = 3 branch, matching the exact Clifford-linear square to that substrate-derived spatial kinetic geometry forces $M^T M = I_3$; orientation preservation eliminates the reflection component, the remaining SO(3) freedom is precisely the declared spin-frame equivalence, and on the tangent-aligned positive-weight branch $M = I_3$ outright. Thus the normalised spatial Dirac/Weyl principal symbol is source-derived without an additional kinetic normalisation. What remains unpublished is the microscopic edge-by-edge map needed to instantiate the finite physical matrix and calculate the finite-regulator constants; the global D = 3 selection remains an inherited/foundational branch rather than a new dimensionality derivation of this paper.

Q3: PARTLY, AND THE REMAINDER IS RELOCATED. The bare positivity/reversibility premises admit a continuum, but the record-defined one-opportunity law generates a unique semigroup within the absorbing first-record class, and the Fock lift above it is unique. The event semantics are executed rather than premised at the implemented grain. What is not settled is the record-to-fermion intertwining map that selects a one-particle transfer A_f from the record semigroup, together with the source selection of the physical record frame, the post-commitment operation, and the attaching map from sequential opportunities to physical duration.

That distinction is the central purpose of MFKR-1.

2. Source-Locked Inputs

2.1 Finite regulator

The Master Action works at finite regulator on an oriented cellular complex

$\mathfrak{S}_n = (V_n, E_n, F_n, C_n)$,

with vertices, oriented edges, elementary faces and four-cells. Geometric primal and dual edge/face data supply the discrete geometric weights. Those weights are geometric regulator data, not independent couplings. The fermion reconstruction below therefore begins with a declared finite edge manifest rather than an assumed continuum lattice. Unless stated otherwise, edges satisfy $s(e) \neq t(e)$; a self-loop, if ever admitted, is counted with both of its incidences in every vertex sum below.

2.2 Primitive matter carrier

For each admitted physical fermion representation f, define the finite one-particle vertex carrier

$\mathcal{H}_{V,f}^{(n)} = \bigoplus_{v\in V_n} (\mathcal{S}_f \otimes R_f)_v$,

where $\mathcal{S}_f$ is the inherited spinorial carrier appropriate to the chiral/Dirac representation and R_f is the frozen faithful gauge representation of the corresponding matter address.

The full occupied matter carrier is the direct sum over the frozen matter ledger,

$\mathcal{H}_V^{matter} = \bigoplus f \, \mathcal{H}\{V,f\}^{(n)}$.

Its antisymmetric Fock completion is inherited from the substrate CAR/Fock construction.

2.3 Primitive shared-link kinetic law

$\Gamma_{kin}^{(0)} = \Sigma_{e:i\to j} \bar{\Psi}_j \gamma^e (\Psi_j - R_f(U_e)\Psi_i) + h.c.$

The physical link U_e is the same link used by the gauge and closure sectors. No independent matter-link normalisation may be attached to this term. Any canonicalisation must arise from the inherited positive matter pairing or kinetic metric and must be performed as a basis transformation, not as a new physical coupling.

2.4 Clifford structure

The earlier graph-Dirac programme establishes a first-order graph operator

$D_G = [[0, B^\sharp],[B, 0]]$

with

$D_G^2 = [[B^\sharp B, 0],[0, BB^\sharp]]$.

Requiring the first-order Hamiltonian to square to the relativistic second-order operator forces Clifford generators satisfying

$\alpha_i^2 = \beta^2 = I, \{\alpha_i, \alpha_j\} = 2\delta_{ij} I, \{\alpha_i, \beta\} = 0$.

Under the declared coarse-grained continuum identification,

$\alpha_i \to \gamma^0\gamma^i, \beta \to \gamma^0$.

The later refinement-transport programme supplies genuine oriented edge/path transport and constructs from its refinement limit a continuum soldering frame e^a_μ. On the declared physical D = 3 branch the refinement frame has three orthonormal spacelike legs. These, rather than the older antipodal-involution difference quotients, provide the physical spatial tangent directions consumed by the fermion principal-symbol calculation below.

2.5 Wilson roughness

$H_W = (r/2)\, \hbar c\, \xi\, ((D_G^{phys})^2 \otimes \beta)$, $r > 0$.

Its form is inherited; the absolute numerical value of r remains a separate entropy-calibration return. For the regulator analysis only the positivity $r > 0$ is required.

2.6 Sequential time firewall

The physical substrate does not possess a primitive fourth symmetric lattice direction. Temporal ordering arises through a succession of committed interfaces

$W_7^{(0)} \to W_7^{(1)} \to W_7^{(2)} \to \dots$

rather than by adding another class of static graph edges.

spatial finite kernel reconstruction ≠ complete temporal composition.

A four-direction Euclidean overlap operator may remain an admissible universality representative, but it must not be relabelled as literal substrate geometry.

2.7 Particle/antiparticle completion

The later charge-conjugation programme distinguishes anti-linear one-particle conjugation, unitary Fock-space charge conjugation, and optional Majorana self-conjugacy. For an ordinary Dirac carrier the particle and antiparticle CAR families are independent. Thus the presence of a charge-conjugate sector is not regulator doubling.

3. Exact Finite Incidence Construction

For one representation f, introduce the edge carrier

$\mathcal{H}_{E,f}^{(n)} = \bigoplus_{e \in E_n} (\mathcal{S}_f \otimes R_f)_e$.

Define source and target pullbacks

$S: \mathcal{H}_{V,f} \to \mathcal{H}_{E,f}$, $(S\Psi)e = \Psi\{s(e)\}$,

$T: \mathcal{H}_{V,f} \to \mathcal{H}_{E,f}$, $(T\Psi)e = \Psi\{t(e)\}$.

Define the block-diagonal faithful link operator

$\mathcal{U}_f = \bigoplus_{e \in E_n} R_f(U_e)$.

Then the primitive covariant edge difference is exactly

$B_U^{(f)} = T - \mathcal{U}_f S$.

No continuum assumption has been made. No lattice direction has been inserted. No gauge-field target has been used.

4. The Edge Clifford Operator

Define

$\Gamma_E = \bigoplus_{e \in E_n} \Gamma_e$.

Here Γ_e denotes the complete coefficient multiplying the edge difference in the primitive kinetic bilinear. If the finite action's geometric normalisation places a factor into the edge Clifford coefficient, it is part of Γ_e. No extra edge weight is to be added later merely to obtain a desired continuum normalisation.

After canonicalisation of the positive one-particle CAR/Fock pairing, let ♯ denote the corresponding adjoint,

$\langle \psi, A\varphi \rangle = \langle A^\sharp \psi, \varphi \rangle$.

This notation avoids conflating the positive Hilbert adjoint with a representation-dependent continuum Dirac-adjoint convention. Since Γ_e acts on the spinorial factor and $R_f(U_e)$ on the internal gauge factor of $\mathcal{S}_f \otimes R_f$, the two commute; this commutation is used repeatedly below.

5. Exact Master-Action Kinetic-Matrix Reconstruction

The non-Hermitian half of the primitive kinetic bilinear is

$\Sigma_e \bar{\Psi}_{t(e)} \Gamma_e (B_U\Psi)_e$.

Using the target pullback,

$\Sigma_e \bar{\Psi}_{t(e)} \Gamma_e (B_U\Psi)_e = \bar{\Psi}\, T^\sharp\, \Gamma_E\, B_U\, \Psi$.

Define

$A_U = T^\sharp\, \Gamma_E\, B_U$.

The full real quadratic form is therefore

$\Gamma_{kin}^{(0)} = \bar{\Psi}\, A_U\, \Psi + \bar{\Psi}\, A_U^\sharp\, \Psi$.

Theorem 1: Finite Master-Action Kinetic Reconstruction

[EXACT AS AN OPERATOR FUNCTIONAL OF THE FROZEN SOURCE DATA]

The primitive shared-link Master-Action kinetic bilinear determines the finite one-particle kinetic matrix

$$K_U^{src} = T^\sharp \Gamma_E B_U + (T^\sharp \Gamma_E B_U)^\sharp.$$

Once $(V_n, E_n, s, t, \Gamma_E, R_f, U)$ and the canonical one-particle pairing are frozen, no additional choice remains in K_U^{src}.

This is the first main return. The previously abstract finite K_U is not mathematically missing. It is the finite Hessian of the already-published source bilinear.

6. Explicit Block Matrix

Let $e: s \to t$. The contribution of this edge to the non-Hermitian half is

$(A_e)_{tt} = \Gamma_e$, $(A_e)_{ts} = -\Gamma_e R_f(U_e)$.

Taking the adjoint gives

$(A_e^\sharp)_{tt} = \Gamma_e^\sharp$, $(A_e^\sharp)_{st} = -R_f(U_e)^\sharp \Gamma_e^\sharp$.

Therefore the complete source matrix has the block structure

$$(K_U^{src})_{vv} = \Sigma_{e:\, t(e)=v} (\Gamma_e + \Gamma_e^\sharp),$$

$$(K_U^{src})_{t(e),s(e)} = -\Gamma_e R_f(U_e),$$

$$(K_U^{src})_{s(e),t(e)} = -R_f(U_e)^\sharp \Gamma_e^\sharp.$$

All other off-diagonal blocks are zero unless the corresponding vertices are joined by a primitive source edge. This provides a direct assembly algorithm. The finite matrix is sparse. Its sparsity graph is the physical matter-transport graph. It has no link that was not already present in the Master Action.

7. Same-Link Gauge Covariance

Let

$G_V = \bigoplus_v R_f(g_v)$

be a faithful local gauge transformation on the vertex carrier. On edges define its target action

$G_E = \bigoplus_e R_f(g_{t(e)})$.

The physical link transforms as

$R_f(U_e^g) = R_f(g_{t(e)}) R_f(U_e) R_f(g_{s(e)})^{-1}$.

Therefore

$B_{U^g} G_V = G_E B_U.$

The Clifford operator acts in spinorial space and commutes with the faithful internal gauge transformation. Consequently

$A_{U^g} G_V = G_V A_U,$

and hence

$K_{U^g}^{src} = G_V K_U^{src} G_V^{-1}$.

Theorem 2: Exact Source-Kernel Gauge Covariance

[EXACT]

The finite kinetic matrix reconstructed from the Master Action transforms covariantly under the same faithful physical link as the gauge and closure sectors. No second fermionic gauge connection is introduced.

8. No-Independent-Normalisation Theorem

Suppose one attempts to replace

$K_U^{src} \to z_f K_U^{src}$

for one matter sector while leaving the gauge transport fixed. This changes the coefficient multiplying the same physical U_e in the primitive transport law. Such a modification is not a basis change. It defines a different microscopic matter action.

A derived positive kinetic metric may of course be canonicalised. If

$\mathcal{K}_f > 0,$

one may transform

$\Psi \to \mathcal{K}_f^{1/2} \Psi$

together with the corresponding transformed operator and CAR pairing. That is a coordinate normalisation. It does not supply an independently adjustable matter-link coupling.

Theorem 3: Matter-Link Normalisation Firewall

[SOURCE-EXACT]

Once the primitive CAR/Fock pairing and representation are canonically fixed, the coefficient of the shared-link matter transport is part of the Master Action and cannot be rescaled independently of the physical gauge link.

Any remaining kinetic normalisation must be returned by the source metric and absorbed by canonical field coordinates, not selected phenomenologically.

9. Flat-Symbol Reconstruction

To compare with the prior Dirac and CWGW results, now restrict temporarily to a translationally regular directional carrier. This is a diagnostic branch, not a claim about primitive time.

Let a positive oriented edge in direction μ carry constant coefficient Γ_μ. For a Fourier mode

$$\Psi_x = e^{iq\cdot x}\,\Psi,$$

the non-Hermitian edge term returns

$$\Gamma_\mu(1 - e^{-iq_\mu}).$$

Including its ♯-adjoint gives

$$\mathbf{K_I(q) = \Sigma_\mu\,[\Gamma_\mu(1 - e^{-iq_\mu}) + \Gamma_\mu^\sharp(1 - e^{iq_\mu})].}$$

Define the adjoint-even and adjoint-odd parts

$$\Gamma_{\mu,+} = (1/2)(\Gamma_\mu + \Gamma_\mu^\sharp),$$

$$\Gamma_{\mu,-} = (1/2)(\Gamma_\mu - \Gamma_\mu^\sharp).$$

Then

$$\mathbf{K_I(q) = 2\Sigma_\mu(1 - \cos q_\mu)\Gamma_{\mu,+} + 2i\Sigma_\mu \sin q_\mu\, \Gamma_{\mu,-}.}$$

This identity is exact. Note that $i\Gamma_{\mu,-}$ and $\Gamma_{\mu,+}$ are ♯-self-adjoint, so $K_I(q)$ is ♯-self-adjoint at every momentum, as it must be.

10. Principal-Symbol Source-Identity Theorem

For small q,

$$1 - \cos q_\mu = (1/2)q_\mu^2 + O(q_\mu^4),\quad \sin q_\mu = q_\mu + O(q_\mu^3).$$

Hence

$$\mathbf{K_I(q) = 2i\Sigma_\mu\, q_\mu\, \Gamma_{\mu,-} + \Sigma_\mu\, q_\mu^2\, \Gamma_{\mu,+} + O(q^3).}$$

The first-order continuum physics depends only on the adjoint-odd part of the source edge coefficient. Let G_μ^{Cliff} denote the normalised inherited Clifford generator in the convention being used; in the positive canonical pairing G_μ^{Cliff} is ♯-anti-adjoint with $(G_\mu^{Cliff})^2 = -I$, so that the principal symbol $i\Sigma G_\mu^{Cliff} q_\mu$ is ♯-self-adjoint.

The normalised first-order symbol is reproduced precisely when

$$\mathbf{2\Gamma_{\mu,-} = G_\mu^{Cliff}.}$$

Theorem 4: Principal-Symbol Source-Identity Theorem

[EXACT NECESSARY-AND-SUFFICIENT FINITE CERTIFICATE]

On a declared regular flat carrier, the Master-Action kinetic matrix has the correctly normalised Dirac/Weyl principal symbol if and only if

$$\mathbf{2\Gamma_{\{\mu,-\}} = G_\mu^{Cliff}}$$

in every independent transport direction. The adjoint-even part $\Gamma_{\{\mu,+\}}$ does not modify the leading Weyl pole; it first enters at order q^2.

This converts "does the Master Action really produce the right kinetic operator?" into a finite matrix equality. No Standard Model target parameter appears in it.

11. Exact-Stencil Source-Identity Theorem

The stronger statement

$$K_I(q) = i\Sigma_\mu G_\mu^{Cliff} \sin q_\mu$$

at every momentum requires elimination of the even term. Therefore, in addition to

$$2\Gamma_{\{\mu,-\}} = G_\mu^{Cliff},$$

one requires

$$\mathbf{\Gamma_{\{\mu,+\}} = 0.}$$

Equivalently, $\Gamma_\mu^\sharp = -\Gamma_\mu$ in the corresponding convention.

Theorem 5: Exact Canonical-Stencil Identity Criterion

[EXACT NECESSARY-AND-SUFFICIENT TEST]

The Master-Action flat kinetic symbol equals the pure canonical sine stencil exactly if and only if the edge coefficients satisfy

$$\mathbf{\Gamma_{\{\mu,+\}} = 0,\ 2\Gamma_{\{\mu,-\}} = G_\mu^{Cliff}.}$$

If the first equality fails while the second holds, the continuum principal symbol remains correct but the source contains an additional local $O(q^2)$ operator which must be retained in the finite regulator rather than silently discarded.

Proposition 5.1: Corner Census of the Flat Source Symbol

[EXACT]

Let c be any of the 2^d momentum corners, with components $c_\mu \in \{0, \pi\}$. Then

$$\mathbf{K_I(c) = 4\ \Sigma_{\{\mu:\ c_\mu = \pi\}}\ \Gamma_{\{\mu,+\}}.}$$

Consequently, under the exact-stencil certificate $\Gamma_{\mu,+} = 0$ the source symbol vanishes at every corner: the first-order source term cannot lift any doubler, and doubler removal is entirely the responsibility of the Wilson roughness L_U, exactly as the CWGW seed structure assumes. Conversely, if $\varepsilon_{even} \neq 0$ the even part supplies an intrinsic source-level corner operator $4\Sigma\Gamma_{\mu,+}$, which must then be accounted for consistently with, not in place of, the closure-derived roughness.

12. Edge-Clifford Residuals

The source-identity test can therefore be published numerically using two residuals.

$\varepsilon_{Cliff} = \max_\mu \|2\Gamma_{\mu,-} - G_\mu^{Cliff}\|$.

This tests the normalised principal symbol.

$\varepsilon_{even} = \max_\mu \|\Gamma_{\mu,+}\|$.

This tests exact canonical-stencil purity.

$\varepsilon_{Cliff} = 0 \Rightarrow$ principal-symbol source identity PASS.

$\varepsilon_{Cliff} = 0$ and $\varepsilon_{even} = 0 \Rightarrow$ exact flat-stencil source identity PASS.

Neither residual should be evaluated after consulting measured fermion data. They are microscopic source checks. Section 14 shows that on the inherited minimal anti-adjoint Clifford branch ε_{even} is structurally zero, while ε_{Cliff} is zero on the canonical normalised branch. The general-star replacement is the geometric residual ε_{moment}. Corollary 7.2 below closes that replacement in the K=7 coarse/infrared sector by combining the exact Clifford-square rigidity proved here with the later substrate-derived refinement transport, refinement frame and soldered spatial metric. The old antipodal-involution derivative descent is not consumed. Beyond the infrared closure, only the finite microscopic star remains unpublished.

13. Present Corpus Status of the Edge-Clifford Test

The audited Master Action publishes the primitive coefficient as γ^e inside the kinetic law. The spinorial and graph-Dirac programmes independently specify the required Clifford algebra and the continuum directional generators.

What is not presently published in the audited finite source package is a complete regulator-level table

$\{ e, s(e), t(e), \Gamma_e, \Gamma_e^\sharp, \text{geometric normalisation} \}_{e\in E_n}$

for the physical fermionic regulator. Consequently:

ε_{Cliff}: DEFINED BUT NOT EXECUTABLE FROM THE CURRENT PUBLIC MANIFEST AS AN ARBITRARY-MATRIX TEST.

ε_{even}: DEFINED BUT NOT EXECUTABLE FROM THE CURRENT PUBLIC MANIFEST AS AN ARBITRARY-MATRIX TEST.

This is not an unspecified theoretical hole. It is a finite publication debt, and the next section shows that most of it dissolves once the inherited minimal Clifford structure is used rather than assuming an unconstrained coefficient per edge.

14. Edge-Clifford Reduction on the Minimal Clifford-Linear Branch

Sections 12 and 13 leave the edge coefficient Γ_e as an arbitrary matrix awaiting publication. That freedom is far larger than the inherited structure actually permits, and this section removes most of it.

14.1 The inherited constraint

Three inputs act together. First, the minimal graph-Dirac construction fixes the first-order Clifford architecture by requiring its square to reproduce the inherited second-order kinetic geometry; the Clifford algebra is forced, not chosen, and the spinorial carrier is minimal. Second, the later K=7 refinement-transport programme constructs genuine oriented path transport, its refinement-Cauchy continuum limit and the soldered continuum frame e^a_μ. Third, on the declared physical D = 3 branch the three spacelike legs of that refinement frame provide the orthonormal spatial tangent basis used below. The older antipodal involutions remain useful only as local opposition/axis data; their difference quotients $(T_i^{(\xi)} - I)/\xi$ are not consumed as physical derivative generators anywhere in the present proof.

Fix one spin frame with three normalised spatial generators

$G_i^\sharp = -G_i, \{G_i, G_j\} = -2\delta_{ij} I,$

and for an edge with normalised coarse spatial tangent $n_e = (n_e^1, n_e^2, n_e^3)$, $|n_e| = 1$, write

$G(n_e) = \Sigma_i n_e^i G_i.$

Theorem 6: Minimal Anti-Adjoint Clifford-Linear Edge Coefficient

[CONDITIONAL EXACT ON THE INHERITED MINIMAL GRAPH-DIRAC/CLIFFORD REALISATION]

On the inherited minimal anti-adjoint Clifford-linear branch, with the forced Clifford algebra on the minimal spinorial carrier, the only edge coefficient compatible with the inherited structure is, up to a common spin-frame unitary equivalence and an orientation choice,

$\Gamma_e = (w_e/2)\, G(n_e)$,

where $w_e > 0$ is a scalar geometric/canonical kinetic weight. The matrix freedom per edge is therefore replaced by the geometric data

$e \mapsto \{ n_e, w_e, \text{orientation} \}$.

Proposition 6A: Edge-Coefficient Classification within the Inherited Anti-Adjoint Class

[EXACT WITHIN THE DECLARED ANTI-ADJOINT CLASS; PER-EDGE ANTI-ADJOINTNESS IS AN INHERITED BRANCH INPUT, NOT DERIVED HERE; PROOF IN APPENDIX E.6]

The word "only" in Theorem 6 carries theorem weight within the declared class, by classification rather than assumption, and the class boundary must be stated honestly. Per-edge anti-adjointness $\Gamma_e^\sharp = -\Gamma_e$ is consumed as the defining property of the inherited minimal Clifford-linear branch; it is not derived from the square condition, and it cannot be: the coefficient $\Gamma_i = (1/2)G_i + i\eta G_i$ with $\eta \neq 0$ has nonzero even part $i\eta G_i$, assembles to $K(q) = \Sigma_i[\sin q_i + 2\eta(1 - \cos q_i)]\sigma_i$ whose square is still exactly scalar, and respects locality and spin covariance. The graph-Dirac square condition therefore excludes identity-direction contamination but not adjoint-even Clifford-vector contamination; the latter enters the symbol at $O(q^2)$, exactly the even order Theorem 4 names, so it cannot disturb the infrared Dirac pole, but it would modify the exact sine stencil and would share doubler lifting with the Wilson term. The microscopic derivation of per-edge anti-adjointness from the primitive source is registered in Appendix I. Within the anti-adjoint class, on the minimal spinorial carrier the operator space is spanned by the identity and the three Clifford generators; there are no higher grades. Write a general anti-adjoint edge coefficient as $\Gamma_e = i(b_e I + h_e\cdot\sigma)$ with real b_e and h_e. Then:

(i) the antisymmetrised assembly is $K(q) = 2i\Sigma_e \Gamma_e \sin(q\cdot a_e)$, so orientation oddness carries both anti-adjoint contributions into sine series and produces the self-adjoint symbol

$$K(q) = -b(q) I - v(q)\cdot\sigma,\ b(q) = 2\Sigma_e b_e \sin(q\cdot a_e),\ v(q) = 2\Sigma_e h_e \sin(q\cdot a_e);$$

(ii) the graph-Dirac square condition (the first-order operator squares to a scalar second-order kinetic operator) forces $K(q)^2 = (b(q)^2 + |v(q)|^2) I + 2\, b(q)\, (v(q)\cdot\sigma)$ to be scalar, i.e. $b(q)\, v(q) = 0$ at every momentum; since b and v are trigonometric polynomials, and each edge may be probed on stars where its sine mode is independent, this forces $b_e = 0$ for every edge;

(iii) locality and spin covariance then align the surviving vector part: the coefficient of an edge may depend only on that edge's geometric data, and equivariance under the spin implementation of frame rotations, $S\, G(n)\, S^\sharp = G(Rn)$, forces $h_e \parallel n_e$.

Hence $\Gamma_e = (w_e/2)\, G(n_e)$ with $w_e = 2|h_e| > 0$ and the sign absorbed into the orientation datum: within the inherited anti-adjoint class, locality, spin covariance, orientation oddness and the graph-Dirac square condition exclude any identity, commutant or (vacuously, on this carrier) higher-grade contribution. On larger carriers the same square-condition argument excludes identity-direction contamination, but the absence of higher grades is a property of the minimal carrier and the classification is stated at that grade, always within the declared anti-adjoint class.

Proposition 6B: Even-Part Firewall for the Infrared Principal Symbol

[EXACT; PROOF IN APPENDIX E.6; VERIFIED IN APPENDIX H]

The infrared closure does not inherit the open anti-adjointness input. On the minimal spin-covariant carrier, the allowed contamination is an adjoint-even Clifford-vector term aligned with the edge tangent,

$$\Gamma_e = (w_e/2)\, G(n_e) + i\, \eta_e\, G(n_e),$$

since the identity direction is excluded for both parities by the square condition and spin covariance aligns both parities with n_e. The even term contributes only (1 − cos) modes to the assembled symbol,

$$K(q) = i\, G\Big(\Sigma_e\, [w_e \sin(q\cdot a_e) + 2\eta_e(1 - \cos(q\cdot a_e))]\, n_e \Big),$$

which remains a pure Clifford vector, so the exact scalar square of Proposition 7.3(i) holds at every momentum and every η. The first-order symbol is $i\,G(Mq) + O(q^2)$ with the same moment tensor $M = \Sigma_e\, w_e\, n_e\, a_e^T$, built from Γ_- alone, and

$$K(q)^2 = q^T(M^TM)q \cdot I + O(q^3).$$

Consequently the principal-symbol moment tensor, the $M^TM = I_3$ refinement-geometry forcing of Corollary 7.2 and the normalised infrared Dirac/Weyl pole depend only on the adjoint-odd part Γ_-, at any value of the η_e. Per-edge anti-adjointness is required for exact sine-stencil purity and for the claim that the Wilson roughness alone lifts doublers; it is not required for the infrared principal-symbol closure. The source-derived infrared pass therefore stands independently of the open microscopic anti-adjointness derivation of Appendix I.

The following two properties are immediate and exact, since $G(n)^2 = -|n|^2 I = -I$ and $G(n)^\sharp = -G(n)$, so that $G(n)$ is ♯-anti-adjoint and ♯-unitary.

Corollary 6.1: Structural Vanishing of the Even Residual

[EXACT ON THE MINIMAL CLIFFORD-LINEAR BRANCH]

Since $\Gamma_e^\sharp = -\Gamma_e$ for every edge of the declared branch,

$\Gamma_{e,+} = (1/2)(\Gamma_e + \Gamma_e^\sharp) = 0$, hence $\boldsymbol{\varepsilon_{even} = 0}$

identically, with no numerical execution required. The vanishing is structural given the branch's defining anti-adjointness, which Proposition 6A shows is an inherited input rather than a consequence of the square condition: a microscopic source supplying an adjoint-even Clifford-vector component would give $\varepsilon_{even} \neq 0$ at $O(q^2)$, leaving the infrared Dirac pole intact by the even-part firewall of Proposition 6B but replacing the exact sine stencil and sharing doubler lifting between the even component and the Wilson roughness. The stencil-purity condition $\Gamma_{e,+} = 0$ of Theorem 5 is therefore satisfied structurally on this branch, and conditionally on the branch's anti-adjointness input at the microscopic grade. Full normalised principal-symbol identity additionally requires the directional moment normalisation of Theorem 7; Corollary 7.2 below closes that normalisation on the K=7 coarse/infrared transport branch. Once both conditions hold, Proposition 5.1 leaves doubler lifting entirely to the Wilson roughness, as the seed architecture requires.

Corollary 6.2: Vanishing of the Principal-Symbol Residual on the Canonical Branch

[EXACT ON THE CANONICAL NORMALISED THREE-CHANNEL BRANCH]

On the regular directional branch with canonical weights $w_i = 1$ and axis tangents $n_i = \hat{e}_i$,

$\Gamma_i = (1/2)G_i$, $2\Gamma_{i,-} = G_i$, hence $\boldsymbol{\varepsilon_{Cliff} = 0}$.

Therefore on the canonical homogeneous three-channel realisation both edge residuals vanish exactly:

$\boldsymbol{\varepsilon_{Cliff} = \varepsilon_{even} = 0}$.

14.2 The general star: a moment certificate replaces a matrix table

Corollary 6.2 assumes axis-aligned edges. The physical regulator need not be axis aligned, and the reduction lets the general condition be stated exactly as a tensor identity rather than a matrix table. Let a vertex star consist of positively oriented edges with displacement vectors a_e and coefficients $\Gamma_e = (w_e/2)G(n_e)$. Since $\Gamma_e^\sharp = -\Gamma_e$, the exact translation-invariant symbol collapses to a single sine sum,

$$\mathbf{K_I(q) = i\, \Sigma_e\, w_e\, G(n_e) \sin(q \cdot a_e),}$$

with no even part at any momentum.

Theorem 7: Star Moment Certificate for the Principal Symbol

[EXACT NECESSARY-AND-SUFFICIENT GEOMETRIC CRITERION]

Define the star moment tensor

$$\mathbf{M^{ij} = \Sigma_e\, w_e\, n_e^i\, a_e^j.}$$

The Clifford-linear star reproduces the normalised Dirac/Weyl principal symbol $i\Sigma_i G_i q_i$ if and only if

$$\mathbf{M^{ij} = \delta^{ij}.}$$

If in addition the coarse tangent is the normalised edge displacement, $n_e = a_e/|a_e|$, the criterion becomes the discrete isotropy condition

$$\mathbf{\Sigma_e\, w_e\, |a_e|\, n_e^i\, n_e^j = \delta^{ij},}$$

and the moment tensor is automatically symmetric. A star whose moment tensor is not symmetric cannot satisfy the criterion in any spin frame, since the left side then mixes generators the continuum symbol does not contain.

This replaces ε_{Cliff} on a general star by a 3×3 numerical residual,

$$\varepsilon_{moment} = \|M - I\|,$$

computable from the geometric manifest alone, with $\varepsilon_{Cliff} = 0$ recovered as the axis-aligned special case. The condition is satisfiable by non-axis-aligned stars: the tetrahedral star $n_e \propto (1,1,1), (1,-1,-1), (-1,1,-1), (-1,-1,1)$ with $w_e|a_e| = 3/4$ returns $M = I$ exactly.

Corollary 7.1: Scalar Weighted Degree

[EXACT]

Because $\|G(n_e)\| = 1$ for every unit tangent, the weighted Clifford degree of Section 15 reduces to a purely scalar geometric quantity,

$$D_\Gamma = (1/2) \max_v \Sigma_{e\ni v} w_e,$$

so the kinetic sensitivity constant is fixed by the edge weights alone, with no matrix data at all. The canonical carrier with unit weights and 2d incident edges returns $D_\Gamma = d$, as before. The factorisation and sharpness results of Sections 16 and 17 apply verbatim on this branch, since $\Gamma_e = (w_e/2)G(n_e)$ is exactly the Clifford-linear class those theorems assume; the reduction therefore supplies their hypothesis rather than merely being consistent with it.

14.3 K=7 refinement transport, frame soldering and closure of the infrared moment certificate

The K=7 sector supplies more than the number of directions, but not in the way the oldest transport source presented it. This section separates an exact source audit, which stands, from the closure itself, which now rests on later and genuinely oriented source input.

Source audit (retained as a firewall). The original K=7 transport construction presented the same antipodal operators both as exact involutions, $T^2 = I$, and as infinitesimal translation generators through $D = (T - I)/\xi \to \partial$. Those two properties are incompatible: $T = I + \xi D$ and $T^2 = I$ force $2D + \xi D^2 = 0$, so the spectrum of D lies in $\{0, -2/\xi\}$ and no finite nonzero derivative limit exists, on any invariant subspace. The antipodal operators may therefore be used as opposition/axis selectors, but not as infinitesimal translations. That audit result is retained; its status is a historical source firewall rather than an open obstruction, because the transport object the earlier argument lacked is supplied independently by later VERSF work, without asking the involutions to become translations.

New source input: genuine oriented refinement transport. On the K=7 commitment foam, every oriented edge $e: v_i \to v_j$ carries an edge transport operator U_{ij} with inverse-path consistency $U_{ji} = U_{ij}^{-1}$. For an oriented path $\gamma = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$ the path transport is the ordered holonomy

$$T_\gamma = U_{i_{k-1}i_k} \cdots U_{i_2 i_3} U_{i_1 i_2}.$$

Closure-compatible refinement produces a refinement-Cauchy family $T_\gamma^{(n)}$ with sequence-independent continuum limit $T_\gamma = \lim_{n\to\infty} T_\gamma^{(n)}$. This is a genuine oriented path-transport construction; it is neither an antipodal involution nor a reinterpretation of one. The subsequent transport-group construction builds from these operators the continuum refinement frame $e^a_\mu(x): T_x\mathcal{M} \to V_{sub}$ as the refinement limit of the local substrate frames and solders the substrate bilinear form to the continuum tangent geometry by

$g_{\mu\nu}(x) = e^a_\mu(x)\, e^b_\nu(x)\, \eta_{ab}$, $\nabla_\mu e^a_\nu = 0$,

with the continuum transport fixed point orientation and time-orientation preserving. The old derivative descent is therefore unnecessary for the infrared moment theorem: the required comparison can be made directly to the substrate-derived continuum tangent geometry.

Dimensionality firewall. The present calculation is carried out on the declared physical branch $D = 3$. This section does not claim that the refinement-transport construction independently derives $D = 3$; that construction itself distinguishes the local $K=7$ cell architecture from the global continuum dimensionality and treats the physical $D = 3$ branch as an inherited/foundational substrate-embedding input. The closure proved below answers: given the declared physical $D = 3$ branch, what fermion normalisation does the source geometry force? It does not answer the separate foundational question of why the substrate selects $D = 3$ rather than another D.

Proposition 7.2A: Refinement-Frame Second-Order Kinetic Symbol

[SOURCE-DERIVED GEOMETRIC INPUT + STANDARD PRINCIPAL-SYMBOL MATHEMATICS]

At a continuum point choose the orthonormal refinement frame and let $\{e_1, e_2, e_3\}$ be its three spacelike legs on the $D = 3$ branch. In this frame $g(e_i, e_j) = \delta_{ij}$, so the principal symbol of the substrate-derived second-order spatial kinetic geometry is

$$\boldsymbol{\sigma_2(-\Delta_g)(p) = g^{ij} p_i p_j = \Sigma_{i=1}^3 p_i^2.}$$

No independent Euclidean metric or directional normalisation is inserted here: the orthonormal frame is the refinement frame of the soldered substrate metric. That a Laplace-type second-order principal symbol is $g^{ij}p_ip_j$ is standard differential geometry, not a new VERSF theorem.

Corollary 7.2: K=7 Infrared Star-Moment Closure

[SOURCE-DERIVED REFINEMENT-GEOMETRY PASS ON THE DECLARED D = 3 MINIMAL CLIFFORD BRANCH]

By Proposition 7.3(i) below, the Clifford-linear infrared star obeys

$$\mathbf{K\_IR(p)^2 = p^T(M^TM)p \cdot I}$$

at second order. By Proposition 7.2A, the independently substrate-derived spatial kinetic geometry has principal symbol p^Tp. The graph-Dirac/Clifford construction requires the first-order fermion operator to square to this second-order kinetic geometry, so for every p,

$$\mathbf{p^T(M^TM)p = p^Tp, \text{ hence } M^TM = I_3 \text{ and } M \in O(3).}$$

The refinement transport preserves cell orientation, excluding the orientation-reversing component on the physical branch, so $M \in SO(3)$.

Orientation intertwining. The orientation preserved by refinement transport and the orientation of the Clifford-frame map M are different typed objects until identified, and the identification is a declared soldering convention, stated here explicitly: the spatial Clifford index i is soldered to the i-th ordered spacelike leg of the refinement frame, so the ordered basis in which M is written is the refinement-frame basis itself. Under that declaration the orientation class of the refinement frame induces the orientation of the Clifford frame, $\det M = -1$ would assert that the edge-tangent assignment reverses the declared frame ordering, and the orientation-preserving fixed point of the transport construction excludes it. Absent that declaration, $\det M = -1$ is simply the freedom to re-declare the ordering, which is exactly the orientation datum of Proposition 7.3(iii). The tangent-aligned positive-weight branch is immune to this step entirely, since $M \succeq 0$ with $M^T M = I$ forces $M = I$ with no orientation input; that branch is the strongest form of the closure. By Proposition 7.3(iii), every $M \in SO(3)$ is implemented by the corresponding spin element and is precisely the common spin-frame unitary equivalence already quotiented by Theorem 6. Choosing that refinement spin frame therefore gives the canonical representative

$$\mathbf{M_{IR} = I_3,\ \varepsilon_{moment}^{IR} = 0,}$$

and consequently the spatial first-order source kernel has the correctly normalised Dirac/Weyl principal symbol without an independently chosen geometric normalisation:

$$\mathbf{K_{IR}(p) = i\Sigma_{i=1}^{3} G_i\, p_i + O(|p|^2)}$$

without assuming microscopic anti-adjointness, sharpening to

$$\mathbf{K_{IR}(p) = i\Sigma_{i=1}^{3} G_i\, p_i + O(|p|^3)}$$

on the anti-adjoint exact-sine branch, since by Proposition 6B the permitted adjoint-even term enters at $O(|p|^2)$ while a pure sine stencil has no even orders at all. The hierarchy is: the normalised infrared pole is source-derived independently of anti-adjointness; the absence of an $O(|p|^2)$ fermion correction is conditional on anti-adjointness; the exact sine stencil and the Wilson-only doubler claim are likewise conditional.

On the tangent-aligned positive-weight branch no spin-frame quotient is required: there $M = \Sigma_e w_e |a_e|\, n_e n_e^T$ is symmetric positive semidefinite and Proposition 7.3(iv) gives $M = I_3$ outright. On the canonical unit-weight six-link spatial reference star this also gives

$$\mathbf{D_\Gamma^{3,canon} = (1/2)\times 6 = 3.}$$

The soldered metric fixes the continuum second moment; it does not print the microscopic edge weights, so the physical weighted degree $D_\Gamma = (1/2)\max_v \Sigma_{e\ni v} w_e$, and with it the exact physical C_{kin}, remain finite-manifest outputs. By the even-part firewall of Proposition 6B, this closure consumes only the adjoint-odd edge part Γ_-: it does not rest on the open microscopic anti-adjointness input, which gates the exact sine stencil and the Wilson-only doubler claim but not the infrared principal symbol.

Overall-scale firewall. The substrate-derived bilinear form is unique up to one common overall scale. That surviving scale is not an independent directional coefficient: it rescales the common spatial ruler and is removed when the refinement frame is orthonormalised. It cannot generate (c_1, c_2, c_3) as three independent fermion normalisations and cannot reopen the moment identity.

Structural consequence. The K=7 infrared source-identity chain now reads

K=7 commitment foam → U_{ij} → T_γ → $e^a{}_\mu$ → $g_{\mu\nu}$ → $\sigma_2(-\Delta_g)$ → $M^T M = I$ → $M = I$ modulo the declared spin-frame equivalence → normalised Dirac/Weyl pole,

and the old chain, antipodal involution → infinitesimal translation, is neither needed nor consumed. This closes the coarse/infrared principal-symbol geometry. It does not print the finite microscopic Master-Action matrix: the glued-regulator map $e \mapsto (n_e, w_e)$ is still required to print the finite physical K_U and calculate its exact physical C_{kin}, λ_{max}, d_+, finite-star diagnostics and same-carrier gap data. That is a finite-instantiation debt, not a continuum-principal-symbol debt.

The closure is in fact rigid: the Laplacian identification alone determines the moment tensor up to precisely the freedoms Theorem 6 already declares, and with no freedom at all on the tangent-aligned branch. The mechanism is an exact scalar square identity for the Clifford-linear star symbol, valid at every momentum and not only in the infrared.

Proposition 7.3: Exact Scalar Square and Laplacian Rigidity of the Moment Tensor

[EXACT; PROOFS IN APPENDIX E.4]

Let a Clifford-linear star have symbol $K_I(q) = i\Sigma_e w_e G(n_e) \sin(q\cdot a_e)$ as in Theorem 7, and define the momentum-dependent geometric vector

$$\mathbf{v(q) = \Sigma_e\, w_e \sin(q\cdot a_e)\, n_e \in \mathbb{R}^3.}$$

Then:

(i) Exact scalar square. At every momentum, not only at leading order,

$$\mathbf{K_I(q)^2 = |v(q)|^2\, I,}$$

so the star symbol is ♯-self-adjoint with nonnegative scalar square and induced scalar dispersion $\omega(q)^2 = |v(q)|^2$. Expanding, $K_I(q)^2 = q^T(M^T M)q \cdot I + O(|q|^4)$, with M the moment tensor of Theorem 7.

(ii) Laplacian rigidity. The star reproduces the unit-coefficient transport Laplacian at second order, $q^T(M^T M)q = |q|^2$ for all q, if and only if

$$\mathbf{M^T M = I,}$$

that is, if and only if the moment tensor is orthogonal, $M \in O(3)$.

(iii) The residual freedom is exactly the declared freedom. If $M \in SO(3)$ the infrared symbol is $i\Sigma_j \tilde{G}_j q_j$ with $\tilde{G}_j = \Sigma_i M^{ij} G_i$. The rotated triple satisfies the same Clifford algebra, $\{\tilde{G}_j, \tilde{G}_k\} = -2\delta_{jk} I$, and there is a spin element S with $\tilde{G}_j = S\, G_j\, S^\sharp$: the rotation is inner and is exactly the common spin-frame unitary equivalence that Theorem 6 quotients by. The component det M = −1 is not inner in three dimensions: no unitary conjugation realises it, and it is exactly the orientation choice Theorem 6 declares. Hence, modulo the declared spin-frame and orientation freedom, the unit Laplacian alone forces M = I and the normalised Dirac/Weyl principal symbol.

(iv) Rigidity without any quotient on the tangent-aligned branch. If $n_e = a_e/|a_e|$ with $w_e > 0$, then $M = \Sigma_e\, w_e|a_e|\, n_e n_e^T$ is symmetric positive semidefinite, and the only symmetric positive semidefinite orthogonal matrix is the identity. On the tangent-aligned positive-weight branch the unit Laplacian therefore forces

M = I

outright, with no residual freedom of any kind.

Consequently the Corollary 7.2 closure does not depend on reading off the alignment $\hat{a}_i \to \hat{e}_i$ of the transport frame: the unit-coefficient Laplacian $\xi^2 L_\xi \Rightarrow -\Delta_{\mathbb{R}^3}$ by itself pins the moment tensor, and the transport frame merely names the canonical representative within the declared equivalence.

Corollary 7.4: Quantitative Stability of the Infrared Moment Closure

[EXACT; PROOF IN APPENDIX E.4]

Let $\varepsilon_{Lap} = \|M^T M - I\|$ measure the finite-ξ departure of the star's second-order coefficient tensor from the unit Laplacian. Then the distance from M to the orthogonal set obeys

$$\mathrm{dist}(M, O(3)) \le \varepsilon_{Lap},$$

and on the tangent-aligned positive-weight branch

$$\varepsilon_{moment} = \|M - I\| \le \|M^2 - I\| \,/\, (1 + \lambda_{min}(M)) \le \varepsilon_{Lap}.$$

The refinement-stability hypothesis of Corollary 7.2 is therefore quantitatively controlled rather than merely assumed: if the finite-ξ transport Laplacian coefficient tensor converges to the identity at some rate, the infrared moment residual converges to zero at least at the same rate, with constant one. A small Laplacian error cannot hide a large principal-symbol error.

Proposition 7.5: Dimension-General Rigidity and the Odd-Dimensional Orientation Obstruction

[EXACT; PROOF IN APPENDIX E.5]

Nothing in Proposition 7.3 or Corollary 7.4 except the orientation statement uses d = 3. For a Clifford-linear star in any dimension d ≥ 2, on a carrier with normalised anti-adjoint generators satisfying $\{G_i, G_j\} = -2\delta_{ij} I$:

(i) the exact scalar square $K_I(q)^2 = \|v(q)\|^2 I$ holds at all momenta;

(ii) a unit transport Laplacian forces $M^T M = I_d$, so $M \in O(d)$;

(iii) tangent-aligned positive weights force $M = I_d$ outright, and the quantitative stability bounds of Corollary 7.4 hold verbatim with constant one; these rigidity and stability statements are star-general. The canonical d-direction carrier additionally admits the exact saturation $C_{kin} = D_\Gamma = d$ in every dimension (Corollary 10.2); the numerical value d is specific to that canonical carrier, since a general weighted star need not have $D_\Gamma = d$;

(iv) the special orthogonal component is inner for every d, implemented by the spin element of the rotation; but the reflection component is inner if and only if d is even. The obstruction is the volume element $\omega = G_1 G_2 \cdots G_d$, which on the minimal carrier is a scalar exactly when d is odd; a reflection reverses ω while conjugation preserves scalars, so no unitary implements it. In even d the product of the remaining $d - 1$ generators implements the reflection explicitly.

The orientation choice recorded in Proposition 7.3 is therefore not an artefact of the proof but a structural feature of odd dimension: $d = 3$ is the lowest dimension above one in which the transport Laplacian pins the moment tensor while still leaving a physically meaningful mirror datum for the source manifest to declare. In even dimension that datum would be absorbed by an inner spin-frame change and would not exist as independent source data.

14.4 Resulting grade and residual finite-manifest debt

CANONICAL EDGE-CLIFFORD MANIFEST: CONDITIONAL EXACT ON THE INHERITED MINIMAL CLIFFORD REALISATION.

K=7 COARSE/INFRARED SECOND-ORDER MOMENT RIGIDITY: SOURCE-DERIVED REFINEMENT-GEOMETRY PASS ON THE DECLARED D = 3 BRANCH, $M^T M = I_3$.

K=7 COARSE/INFRARED STAR MOMENT: $M_{IR} = I_3$ MODULO THE DECLARED COMMON SPIN-FRAME EQUIVALENCE; $M_{IR} = I_3$ OUTRIGHT ON THE TANGENT-ALIGNED POSITIVE-WEIGHT BRANCH.

NORMALISED SPATIAL DIRAC/Weyl PRINCIPAL SYMBOL: SOURCE-DERIVED INFRARED PASS AT THE SAME REFINEMENT-GEOMETRY GRADE.

OLD INVOLUTION/TRANSLATION DESCENT: REJECTED AS A SOURCE IDENTIFICATION, BUT NO LONGER LOAD-BEARING FOR THE INFRARED CLOSURE.

LAPLACIAN SQUARE RIGIDITY AND STABILITY OF THE CLOSURE: EXACT.

The remaining geometric debt is now finite rather than infrared, and by Proposition 7.3 the closure is rigid rather than fortuitous, while by Corollary 7.4 it is stable: refinement error in the transport Laplacian bounds the moment residual with constant one. The audited package still does not publish the complete glued-regulator map assigning to every microscopic physical Master-Action edge its tangent n_e and scalar weight w_e. That map is needed to print the finite K_U and calculate its exact physical C_{kin}, λ_{max}, d_+, finite-star diagnostics and physical same-carrier gap quantities. It is not needed to establish the K=7 infrared moment identity or the normalised spatial principal symbol. The global $D = 3$ selection remains an upstream foundational inheritance and is not newly derived here.

remaining geometric execution: publish the microscopic (n_e,w_e) manifest for finite-regulator instantiation.

15. Exact Gauge-Link Perturbation of the Reconstructed Kernel

Let

$\Delta_e = R_f(U_e) - I$, $\varepsilon_U = \max_e \|\Delta_e\|$.

Since only the transported source term depends on U,

$K_U^{src} - K_I^{src} = -T^\sharp\Gamma_E(\mathcal{U}_f - I)S - [T^\sharp\Gamma_E(\mathcal{U}_f - I)S]^\sharp$.

At the block level an edge $e: s \to t$ contributes

$(\Delta K)_{ts} = -\Gamma_e \Delta_e$, $(\Delta K)_{st} = -\Delta_e^\sharp \Gamma_e^\sharp$.

Define the weighted incident Clifford degree

$\mathbf{D_\Gamma = \max_{v\in V_n} \Sigma_{e\ni v} \|\Gamma_e\|.}$

The block Schur estimate then gives

$\mathbf{\|K_U^{src} - K_I^{src}\| \leq D_\Gamma\, \varepsilon_U.}$

Theorem 8: Source-Native Kinetic Lipschitz Theorem

[EXACT FINITE BOUND GIVEN THE FROZEN EDGE NORM]

The finite kinetic matrix reconstructed from the Master Action satisfies

$\mathbf{C_{kin}^{src} \leq D_\Gamma = \max_v \Sigma_{e\ni v} \|\Gamma_e\|.}$

Therefore C_{kin} is not a new physical parameter. It is bounded by a finite weighted-degree calculation on the source regulator.

This is the second main result of the paper. The next two sections determine exactly how good this bound is.

16. Clifford-Diagonal Factorisation of the Link Perturbation

On a directional carrier satisfying the exact-stencil certificate of Theorem 5, the link perturbation admits a structure much finer than a row-sum estimate. Let the direction-μ coefficient be

$\Gamma_\mu = (c_\mu/2)\, G_\mu^{Cliff}$, $c_\mu > 0$, with G_μ^{Cliff} the ♯-anti-adjoint normalised Clifford generators, $(G_\mu^{Cliff})^2 = -I$, pairwise anticommuting. (The canonical carrier is $c_\mu = 1$ for all μ.) Let S_μ be the unitary direction-μ site shift and let Z_μ be the block-diagonal internal perturbation operator carrying Δ_e on the direction-μ edges, $\|Z_\mu\| \le \varepsilon_U$. Since Δ_e acts on the internal gauge factor and Γ_μ on the spinorial factor, the two commute, and the direction-μ part of the perturbation collects into a single ♯-anti-adjoint operator.

Theorem 9: Clifford-Diagonal Factorisation and Commutator-Refined Kinetic Bound

[EXACT ON THE $\varepsilon_{even} = 0$ DIRECTIONAL CLASS]

Define

$$N_\mu = Z_\mu S_\mu - S_\mu^\sharp Z_\mu^\sharp, \quad N_\mu^\sharp = -N_\mu, \quad \|N_\mu\| \le 2\varepsilon_U.$$

Then the exact link perturbation of the reconstructed kernel is

$$\Delta K = K_U^{src} - K_I^{src} = -\Sigma_\mu \Gamma_\mu N_\mu = -(1/2)\, \Sigma_\mu c_\mu G_\mu^{Cliff} N_\mu,$$

and its square separates into a definite quadrature part and pure commutator interference:

$$(\Delta K)^2 = (1/4)\left[\Sigma_\mu c_\mu^2 (-N_\mu^2) + \Sigma_{\mu<\nu} c_\mu c_\nu G_\mu^{Cliff} G_\nu^{Cliff} [N_\mu, N_\nu] \right],$$

where each $-N_\mu^2 \ge 0$ and each $G_\mu^{Cliff} G_\nu^{Cliff}$ is unitary. Consequently

$$\|\Delta K\|^2 \le (1/4)\left[\|\Sigma_\mu c_\mu^2 N_\mu^2\| + \Sigma_{\mu<\nu} c_\mu c_\nu \|[N_\mu, N_\nu]\|\right].$$

Corollary 9.1: Commuting-Fibre Quadrature Bound

[EXACT]

If the direction perturbation operators mutually commute, $[N_\mu, N_\nu] = 0$ for all μ, ν (in particular for every constant abelian background, where each N_μ is a function of the commuting shifts alone), then

$$\|\Delta K\| \le (\Sigma_\mu c_\mu^2)^{1/2} \varepsilon_U,$$

which on the canonical d-direction carrier reads

$$\|\Delta K\| \le \sqrt{d} \cdot \varepsilon_U.$$

For $d = 4$ this is exactly half the weighted-degree bound: smooth commuting backgrounds disturb the kinetic matrix at the quadrature rate $2\varepsilon_U$, not the worst-case rate $4\varepsilon_U$.

Corollary 9.2: Recovery of the Weighted-Degree Bound

[EXACT]

Using $\|N_\mu^2\| \le 4\varepsilon_U^2$ and $\|[N_\mu, N_\nu]\| \le 8\varepsilon_U^2$, the refined estimate returns

$$\|\Delta K\|^2 \le (1/4)\left[4\varepsilon_U^2 \Sigma_\mu c_\mu^2 + 8\varepsilon_U^2 \Sigma_{\mu<\nu} c_\mu c_\nu \right] = \varepsilon_U^2 (\Sigma_\mu c_\mu)^2,$$

that is $\|\Delta K\| \le (\Sigma_\mu c_\mu)\, \varepsilon_U = D_\Gamma\, \varepsilon_U$. The refined bound therefore interpolates continuously between the quadrature value and the Schur value, with the excess above quadrature controlled entirely by the non-commutativity of the direction perturbations. On any given background the refined bound is computable from the manifest and is never weaker than Theorem 8.

If $\varepsilon_{even} \ne 0$ the perturbation splits as $\Delta K = \Delta K_- + \Delta K_+$ along the adjoint-odd and adjoint-even parts of Γ_μ; Theorem 9 applies verbatim to ΔK_-, and ΔK_+ is separately controlled by the Schur estimate with the weights $\|\Gamma_{\mu,+}\|$.

17. Sharpness of the Weighted-Degree Bound

The natural question is whether Theorem 8 wastes anything. It does not.

Theorem 10: Exact Sharpness of D_Γ on the Canonical Carrier

[EXACT; EXPLICIT FINITE WITNESS]

On the canonical four-direction nearest-neighbour carrier ($c_\mu = 1$, $2\Gamma_{\mu,-} = G_\mu^{Cliff}$, $\Gamma_{\mu,+} = 0$):

(i) Commuting class, exact value. The supremum of $\|\Delta K\|/\varepsilon_U$ over constant abelian perturbations equals $\sqrt{d}$ exactly. The uniform perturbation $\Delta_e = i\varepsilon$ on every edge gives $\Delta K(q) = -\varepsilon\, \Sigma_\mu\, \alpha_\mu \cos q_\mu$ (with $G_\mu^{Cliff} = -i\alpha_\mu$, α_μ Hermitian anticommuting), whose norm at $q = 0$ is $\varepsilon\|\Sigma_\mu\, \alpha_\mu\| = \sqrt{d} \cdot \varepsilon$, matching the Corollary 9.1 upper bound.

(ii) Full induced norm, exact value. The exact induced-norm constant of Section 19 satisfies

$C_{kin}^{exact} = D_\Gamma = 4$.

Witness: on the 2^4 periodic carrier, take the abelian perturbation

$\Delta_e = i\varepsilon\, f_\mu(x)$ on the direction-μ edge $x \to x+\hat{\mu}$, $f_\mu(x) = (-1)^{x_1+\ldots+x_{\mu-1}}$,

so that every edge carries $\|\Delta_e\| = \varepsilon$ exactly. Then each direction operator becomes $N_\mu = 2i\varepsilon\, A_\mu$, where

$A_\mu = \sigma_3^{\otimes(\mu-1)} \otimes \sigma_1 \otimes I^{\otimes(4-\mu)}$

are Hermitian unitary and pairwise anticommuting (a Jordan-Wigner family on the 16 sites). Hence

$\Delta K = -\varepsilon\, \Sigma_\mu\, \alpha_\mu \otimes A_\mu$, $(\Sigma_\mu\, \alpha_\mu \otimes A_\mu)^2 = 4\cdot I + 2\, \Sigma_{\mu<\nu}\, Q_{\mu\nu}$, $Q_{\mu\nu} = (\alpha_\mu\alpha_\nu) \otimes (A_\mu A_\nu)$,

and the six operators $Q_{\mu\nu}$ are pairwise commuting involutions generated by Q_{12}, Q_{23}, Q_{34}, with a nonempty common +1 eigenspace. On that eigenspace $(\Delta K)^2 = 16\varepsilon^2$, so

$\|\Delta K\| = 4\varepsilon = D_\Gamma\, \varepsilon_U$ exactly.

(iii) Unitary realisability. The witness directions are purely imaginary, so genuine U(1) link families $U_e = \exp(i\theta f_\mu(x))$ realise the same alternating structure with $\Delta_e = e^{\pm i\theta} - 1$. For these strictly unitary backgrounds,

$\|\Delta K\| / (D_\Gamma \varepsilon_U) \to 1$ as $\theta \to 0$,

so no constant smaller than D_Γ can hold even when the perturbation class is restricted to exact unitary links.

Consequences. The weighted-degree constant of Theorem 8 is not an estimate with slack: it is the exact worst-case source sensitivity of the canonical carrier, and the reference value $C_{kin} \le 4$ inherited by CWGW is in fact $C_{kin}^{exact} = 4$ as an induced norm. At the same time the commuting/anticommuting dichotomy is sharp at both ends: the commuting class has sharp supremum $\sqrt{d}\, \varepsilon_U$, attained by the uniform constant-abelian background, the unrestricted class has sharp supremum $d\, \varepsilon_U$, attained by the alternating Jordan-Wigner witness, and every background responds at or below the ceiling of its class, with its refined upper bound controlled by its direction commutators through Theorem 9. Individual backgrounds may respond below either sharp ceiling; a commuting perturbation supported in a single direction, for example, responds at ε_U. No admissibility radius built from D_Γ can therefore be enlarged without restricting the background class; on the commuting class it can, and Section 22 records the exact improvement.

Corollary 10.1: Canonical Three-Direction Sharpness

[EXACT ON THE CANONICAL THREE-DIRECTION CARRIER; PHYSICAL K=7 IR EQUALITY A FINITE-CARRIER EXECUTION QUESTION]

The same construction saturates the three-direction constant. On the canonical 2^3 periodic three-direction carrier (unit weights, six incident half-normalised links, $D_\Gamma = 3$), the alternating-sign abelian perturbation $\Delta_e = i\varepsilon f_i(x)$ on the direction-i edge, $f_i(x) = (-1)^{x_1+\ldots+x_{i-1}}$, gives three pairwise anticommuting Jordan-Wigner direction operators and returns

$\|\Delta K\| = 3\varepsilon = D_\Gamma \varepsilon_U$ exactly, hence $C_{kin}^{3,canon} = D_\Gamma = 3$.

Genuine U(1) link families with the same sign pattern approach the ratio one in the small-field limit (0.99995 at $\theta = 0.02$), and the uniform constant-abelian background attains the commuting-class ceiling $\sqrt{3} \cdot \varepsilon_U$, so the commuting/anticommuting dichotomy of Theorem 9 is realised at both ends in three directions as well as four; individual backgrounds in either class may respond below their ceiling. What this establishes unconditionally is the exact induced norm on the canonical three-direction carrier. The refinement-geometry result identifies the physical K=7 infrared principal-symbol geometry with the same normalised three-dimensional spatial frame, so the previous principal-symbol conditionality is removed. It does not, by itself, prove that the fully instantiated physical finite K=7 regulator admits the particular global alternating-sign background used by the saturation witness, and, because the physical weighted degree $D_\Gamma = (1/2)\max_v \Sigma_{e\ni v} w_e$ depends on the microscopic edge weights the soldered metric does not print, it does not by itself fix the physical bound either. The exact physical D_Γ, the exact physical C_{kin}

and the saturation of the canonical value 3 therefore all remain finite-manifest outputs, awaiting the edge/gluing manifest or an independent proof that the physical carrier admits the saturation background. This residual is separate from, and does not weaken, the closed infrared Dirac/Weyl principal-symbol normalisation.

Corollary 10.2: Dimension-General Sharpness

[EXACT; VERIFIED AT $d = 2, 3, 4$]

The witness construction is dimension-blind. On the 2^d periodic carrier with the axis star in any dimension d, the alternating-sign perturbation $\Delta_e = i\varepsilon f_i(x)$, $f_i(x) = (-1)^{x_1+\ldots+x_{i-1}}$, produces d pairwise anticommuting Jordan-Wigner direction operators and returns $\|\Delta K\| = d\cdot\varepsilon$ exactly, so the canonical d-direction carrier admits the exact saturation $C_{kin} = D_\Gamma = d$ in every dimension, and the uniform constant-abelian background attains the commuting-class ceiling $\sqrt{d} \cdot \varepsilon$. Machine verification returns the ratio one to the last digit at $d = 2$ and $d = 4$ alongside the $d = 3$ case of Corollary 10.1. The canonical three-direction constant is therefore an instance of a fully general exact law, not a low-dimensional accident.

18. Reference Checks

18.1 Four-direction Euclidean overlap reference

For the canonical nearest-neighbour Euclidean representative, each vertex has $2d = 8$ incident links for $d = 4$, and each oriented kinetic contribution carries the canonical norm 1/2:

$D_\Gamma = 8 \times (1/2) = 4.$

Hence

$C_{kin}^{ref} \le 4$,

and by Theorem 10 this is an equality of induced norms,

$C_{kin}^{exact, ref} = 4$.

This reproduces the CWGW reference constant without treating C_{kin} as an independent assumption, and shows that constant to be unimprovable on the unrestricted background class.

18.2 K=7 three-direction infrared spatial branch

The K=7 refinement-geometry closure upgrades the regular three-direction result to the physical coarse/infrared source branch at the principal-symbol level. In the orthonormal spatial refinement frame $M_{IR}^T M_{IR} = I_3$, and, modulo the common spin-frame equivalence already declared by Theorem 6, $M_{IR} = I_3$ with $\varepsilon_{moment}^{IR} = 0$; on the tangent-aligned positive-weight branch the identity is literal rather than quotient-relative. The sharpness theorem also applies exactly to the canonical three-direction comparison carrier: the six canonical half-normalised spatial contributions give

$$D_\Gamma^{3,\mathrm{canon}} = 6 \times (1/2) = 3,\ C_{\mathrm{kin}}^{3,\mathrm{canon}} = 3,$$

the latter attained by the explicit 2^3 Jordan-Wigner witness of Corollary 10.1. This is an exact reference-carrier result. The refinement-geometry theorem independently closes the physical K=7 infrared principal-symbol normalisation through $M_{IR}^T M_{IR} = I_3$, but it does not, without the microscopic glued-edge manifest, determine the complete physical weighted-degree sum $(1/2)\max_v \Sigma_{e\ni v} w_e$. Therefore the exact physical D_Γ, the exact physical C_{kin} and the saturation of the canonical value 3 remain finite-manifest outputs. This residual is separate from, and does not weaken, the closed infrared Dirac/Weyl principal-symbol normalisation.

18.3 Executed finite-star certification pipeline

[EXECUTED; ZERO CERTIFICATE VIOLATIONS]

The certified moment pipeline promised by the stability corollary is here run end to end on explicit finite stars, so that the future physical-manifest execution inherits a demonstrated procedure rather than an available one. The observable is the scalar square symbol $k(q) = \|v(q)\|^2$ alone; the pipeline never reads the moment tensor directly.

Procedure. At probe scale h, the second-order coefficient tensor Q_h is measured by symmetric central differences of k at step h, so $Q_h = M^T M + O(h^2)$. Because k is a finite cosine polynomial with explicitly known support, the differencing error carries a computable a priori budget:

$$\|Q_h - M^T M\| \le \tau(h) = C_4 h^2/3,\ C_4 = \Sigma_{e,e'} (1/2)|w_e w_{e'} (n_e\cdot n_{e'})| (\|a_e - a_{e'}\|^4 + \|a_e + a_{e'}\|^4),$$

proved in Appendix E.5 and validated over 400 random stars at three scales with worst observed ratio 0.417 of the budget. The certificate is then, on the tangent-aligned positive-weight branch,

$$\varepsilon_{\mathrm{moment}} = \|M - I\| \le \|Q_h - I\| + \tau(h),$$

and off that branch the same quantity certifies dist(M, O(3)) only.

Reference stars. Four manifests were run at h = 0.2, 0.1, 0.05, 0.025, 0.0125: the K=7 infrared axis star S1; the tetrahedral star S2 of Appendix E.3 (weights 3/4); a deliberately detuned star S3 with weight perturbations (+0.05, −0.03, +0.04), true $\varepsilon_{\mathrm{moment}} = 0.05$; and a rotated-tangent star S4 (rotation angle 0.4, tangent alignment broken) as the off-branch negative control.

h	S1 certified $\varepsilon_{\mathrm{moment}} \le$	S2 certified $\varepsilon_{\mathrm{moment}} \le$	S3 certified $\varepsilon_{\mathrm{moment}} \le$ (true 0.05)	S4 certified dist(M, O(3)) ≤ (true $\|M - I\|$ = 0.397)
0.2	3.72×10^{-1}	3.91×10^{-1}	4.42×10^{-1}	3.72×10^{-1}

h	S1 certified ε_moment ≤	S2 certified ε_moment ≤	S3 certified ε_moment ≤ (true 0.05)	S4 certified dist(M, O(3)) ≤ (true ‖M − I‖ = 0.397)
0.1	9.33×10^{-2}	9.78×10^{-2}	1.71×10^{-1}	9.33×10^{-2}
0.05	2.33×10^{-2}	2.44×10^{-2}	1.20×10^{-1}	2.33×10^{-2}
0.025	5.83×10^{-3}	6.11×10^{-3}	1.07×10^{-1}	5.83×10^{-3}
0.0125	1.46×10^{-3}	1.53×10^{-3}	1.04×10^{-1}	1.46×10^{-3}

Findings. The certificate held at every scale for every star with zero violations. For the exact stars S1 and S2 the certified bound converges to zero quadratically, as the budget predicts. For the detuned star S3 the certified bound converges to $\|M^2 - I\| = 0.1025$ from above and never crosses the true value 0.05; the factor-two headroom is exactly the constant-one conservatism that the refined bound with $1/(1 + \lambda_{min})$ would remove, and the pipeline correctly reports a non-vanishing moment residual that no refinement removes. For the rotated star S4 the Laplacian defect converges to zero while the true moment error stays at 0.397: the pipeline correctly certifies only the distance to O(3), identifies the residual as the declared frame freedom, and any claim of M = I from this data alone is the misuse named in F-MFKR40. The truncation budget is not decorative: at h = 0.1 the raw defect for S3 alone dips below the true error and only the budgeted certificate remains valid, which is the misuse named in F-MFKR43. This spatial constant must still not be inserted into a gap formula whose other constants were evaluated on the four-direction Euclidean overlap carrier; carrier mixing remains forbidden.

19. Exact Induced-Norm Definition of C_kin

The weighted-degree result is a rigorous bound, sharp on the canonical carrier. Once the full physical edge manifest is available, the exact finite constant can be calculated on the physical regulator as well.

Define the real-linear link-perturbation map

$\mathcal{L}_K: \{\Delta_e\} \mapsto -T^\sharp\Gamma_E \, \Delta \, S - (T^\sharp\Gamma_E \, \Delta \, S)^\sharp.$

Then

$$\boldsymbol{C_{kin}^{exact} = \sup_{\Delta\neq0} \|\mathcal{L}_K(\Delta)\| \, / \, \max_e \|\Delta_e\|.}$$

This is a finite-dimensional induced operator norm. It can be computed numerically once and published with the source manifest. No fit is involved. The weighted-degree theorem guarantees

$C_{kin}^{exact} \leq D_\Gamma,$

and Theorem 10 shows that on the canonical carrier the two coincide. On the physical regulator the gap, if any, between C_{kin}^{exact} and D_Γ is itself a deterministic output of the manifest.

20. Same-Carrier Wilson Roughness

The source edge difference is

$B\_U = T - \mathcal{U}\_f\, S.$

Using the inherited physical one-particle inner products, define

$L\_U = B\_U^\sharp\, B\_U \geq 0.$

All geometric weights enter through the frozen carrier metrics and their adjoints. No independent Wilson link is introduced. The source-matched Wilson seed is consequently

$X\_\rho^{src}[U] = K\_U^{src} + (r/2)L\_U - \rho I.$

Theorem 11: Fully Reconstructed Source-Wilson Seed

[EXACT AS AN OPERATOR FUNCTIONAL; CONDITIONAL ON THE INHERITED $r > 0$ CLOSURE BRANCH]

Once the finite source manifest is frozen, both the first-order kinetic leg and the positive Wilson roughness are explicit functions of the same physical faithful link. Hence $X\_\rho^{src}[U]$ contains no independently selected matter transport.

21. Physical-Regulator Constants

Once one declared finite source regulator is frozen, the remaining constants are direct matrix outputs:

$d_+^{(n)} = \max\_v \#\{e: s(e) = v\},$

$\lambda\_\max^{(n)} = \|B\_I^\sharp\, B\_I\|,$

$D\_\Gamma^{(n)} = \max\_v \Sigma\_\{e\ni v\}\ \|\Gamma\_e\|,$

$C\_kin^{(n)} = \|\mathcal{L}\_K\|\_\{\infty\to 2\}.$

The first three require no gauge-background scan. They are properties of the flat source manifest. The fourth is an induced norm of an explicitly known finite linear map, with the exact reference value already returned in Theorem 10.

Thus the prior instruction “freeze K_U and calculate C_kin” can now be sharpened to “publish the edge manifest and execute four finite matrix routines”.

22. Same-Carrier Admissibility Radius

Let $H\_r^{(n)}[U]$ be the Hermitian chiral seed constructed from the same finite carrier and let

$\delta\_n = \mathrm{gap}\ H\_r^{(n)}[I].$

Do not import δ_n from a different regulator. Using $C\_kin^{(n)} \le D\_\Gamma^{(n)}$, the previous perturbation argument gives the same-carrier sufficient condition

$[D\_\Gamma^{(n)} + r\sqrt{(\lambda\_max^{(n)} d_+^{(n)})}]\ \varepsilon\_U + (r\ d_+^{(n)}/2)\ \varepsilon\_U^2 < \delta\_n.$

Define

$A\_n = D\_\Gamma^{(n)} + r\sqrt{(\lambda\_max^{(n)} d_+^{(n)})}.$

Then a completely manifest-level sufficient radius is

$$\boldsymbol{\varepsilon\_src^{(n)} = 2\delta\_n / [A\_n + \sqrt{(A\_n^2 + 2r\ d_+^{(n)}\ \delta\_n)}].}$$

Theorem 12: Manifest-Level Source Admissibility Radius

[EXACT GIVEN SAME-CARRIER INPUTS]

Once the finite edge manifest and the same-carrier flat chiral gap δ_n are frozen, the Master-Action source data alone provide a nonzero sufficient faithful-link radius with no phenomenological constant.

On the four-direction Euclidean comparison carrier, $D\_\Gamma = 4$, $\lambda\_max = 16$, $d_+ = 4$ and $\delta = \min(r, 1)$, and the formula reduces exactly to the earlier CWGW safe-radius expression. This is an internal consistency check, not a new physical identification of Euclidean time with substrate time.

Remark 12.1: Commuting-class improvement. On backgrounds whose direction perturbations mutually commute (Corollary 9.1), the kinetic leg of A_n improves from $D\_\Gamma = d$ to $\sqrt{d}$, so the sufficient radius on that restricted class is the same closed form with A_n replaced by $A\_n' = \sqrt{d} + r\sqrt{(\lambda\_max\ d_+)}$. By Theorem 10 this improvement must never be applied outside the commuting class; doing so is falsifier F-MFKR17.

23. What the Reconstruction Does Not Yet Prove

23.1 It does not print the numerical physical matrix

The formula for K_U^src is exact. But a matrix formula and an instantiated matrix are different deliverables. To print the numerical operator one still needs the finite manifest

$\mathfrak{M}\_\{f,n\} = \{V\_n, E\_n, s, t, (n\_e, w\_e), \text{carrier metrics}, R\_f, U\_e\}.$

After the reduction of Section 14 the coefficient entry is geometric rather than matrix valued, which shrinks the debt substantially but does not discharge it: the glued-regulator map assigning each physical edge its tangent and weight is not currently published in the audited package.

23.2 It does not print the exact microscopic finite-star stencil

The continuum principal-symbol question is closed at the declared K=7 refinement-geometry grade. Corollary 7.2 gives $M\_IR^T M\_IR = I_3$ from the substrate-derived spatial metric; orientation preservation and the declared spin-frame quotient then give the canonical representative $M_{IR} = I_3$ with $\varepsilon_{moment}^{IR} = 0$, while $\varepsilon_{even} = 0$ holds structurally, and on the tangent-aligned positive-weight branch $M_{IR} = I_3$ outright. Therefore the normalised spatial Dirac/Weyl principal symbol no longer depends on an unexecuted geometric assumption. What is still not published is the edge-by-edge finite physical star. Its finite-level moment residual, exact C_{kin} and other same-carrier constants remain deterministic manifest outputs needed for the instantiated finite K_U, not for the already-closed infrared principal symbol.

23.3 It does not uniquely select the complete chiral functional calculus

Once $X_\rho^{src}[U]$ is given, one admissible chiral completion is

$$D_{ov}^{src} = (\rho/a)\left[I + X_\rho^{src} (X_\rho^{src\sharp} X_\rho^{src})^{-1/2} \right].$$

This operator uses only the source seed and the same physical gauge link. Its gauge covariance follows automatically. But the overlap functional calculus is an admitted regulator construction from the global quantum architecture. The primitive kinetic action by itself does not prove overlap is the unique possible chiral completion.

source-functional overlap construction: PASS.

source-unique chiral completion: OPEN.

Section 24 sharpens the sequential half of this statement considerably in both directions.

24. Sequential Transfer: Class-Scoped Non-Uniqueness, the Record-Defined Generator, and the Unique Fock Lift

Section 23.3 left the chiral/sequential completion open. The audited corpus permits a much sharper treatment: a class-scoped no-go, an explicit construction that escapes it using data the no-go did not have, and a uniqueness theorem downstream. The three must be kept distinct, because the no-go is not refuted by the construction; it is evaded by supplying more.

24.1 The no-go, stated with its class

Notation firewall for this section: δ denotes the semigroup Fold-fraction parameter introduced below, with $\delta = 1$ one Fold. It is not the regulator refinement parameter used in the hazard-scaling statements of §24.4. Conflating the two is falsifier F-MFKR32.

The recurrent heat-flow uniqueness programme (RHFU-1) tests exactly the required question and returns a no-go. It exhibits an explicit one-parameter continuum of admissible transfers,

$$T_s = \exp[-s(I - T_{OS})], \quad s > 0,$$

whose members share positivity, Markov normalisation, reversibility, the stationary state and the same symmetry carrier, while producing different excitation spectra and history rates.

Theorem 13: Class-Scoped Sequential Non-Uniqueness

[INHERITED EXPLICIT NO-GO, WITH ITS HYPOTHESIS CLASS MADE EXPLICIT]

If the only conditions imposed on the sequential transfer are positivity, Markov normalisation, reversibility, the stationary state and the symmetry carrier, then the substrate does not uniquely select it: the family above is admissible throughout. The implication

Master/history law ⇒ unique sequential transfer

is therefore unavailable *from those premises*, and no further calculation on those premises alone will restore it.

The scope matters, and the next subsection uses it. The no-go says a selection needs more input, not that no input can select. What Section 24.2 supplies is exactly the kind of input the family is blind to: the observed one-step survival probability and the terminal conditional law.

24.2 The record-defined first-entry law generates a unique semigroup

The correct physical distinction at the implemented F2F/PFDMI grain is defined by the retained record, not by the microscopic current-channel label. This is an executed correction to the earlier reading, not a reinterpretation of it: the channel previously proposed as “no physical event” contributes to all 84 retained record classes, so it cannot carry that meaning.

The executed record architecture gives

no new record ↔ the seven self-history marks, record-producing event ↔ every nonself history transition,

which at the finest executed history-current grain is 7 + 312 = 319 marks, the 312 record-producing marks coarsening onto the 84 retained ledger classes.

Let s(x) be the total self-history probability from source state x, and define h_F(x) = 1 − s(x). For retained ledger class a let p_a(x) be its one-opportunity probability, and define q_a(x) = p_a(x)/h_F(x) whenever h_F(x) > 0. The generator below is stated on the lumped one-survival-macrostate description, with scalar s, h_F and q_a; the reduction from the state-resolved data (s(x), q_a(x)) to a single survival macrostate is a lumpability hypothesis on the F2F coarse-graining, made explicit in the theorem grade and registered in Appendix I, not a proved consequence of the executed no-record dynamics. On the lumped description one sequential first-record opportunity is

P_rec(0→0) = s = 1 − h_F, P_rec(0→a) = h_F q_a,

with terminal first-record states absorbing for this first-entry process.

Theorem 14: Record-Defined Sequential Generator and Its Uniqueness

[EXACT IN THE ABSORBING FIRST-RECORD SEMIGROUP CLASS, CONDITIONAL ON THE IMPLEMENTED RECORD ARCHITECTURE AND ON THE ONE-SURVIVAL-MACROSTATE LUMPING OF THE STATE-RESOLVED NO-RECORD DYNAMICS]

Define

$$\lambda_F = -\log s = -\log(1 - h_F),\ Q_{00} = -\lambda_F,\ Q_{0a} = \lambda_F q_a,\ Q_{aa} = 0,$$

all other entries zero. Then Q has zero row sums and nonnegative off-diagonal entries, so $R(\delta) = e^{\delta Q}$ is a valid Markov semigroup with

$$R_{00}(\delta) = e^{-\lambda_F \delta},\ R_{0a}(\delta) = (1 - e^{-\lambda_F \delta})q_a,$$

$R(0) = I$, exact composition $R(\delta_1 + \delta_2) = R(\delta_1)R(\delta_2)$, generator $Q = \partial R(\delta)/\partial\delta|_{\delta=0}$, and

$$R(1) = P_{rec}$$

at one declared sequential opportunity. The generator is unique in the class with one survival state, absorbing first-record terminals, time-homogeneous composition and fixed terminal conditional law: matching the survival entry forces $e^{-\lambda_F} = s$, whose unique positive solution is $\lambda_F = -\log s$, and matching the terminal row forces the q_a.

The earlier accepted-current hazard must not be substituted for h_F. The execution shows that microscopic current subchannels are routes to a record, not the criterion for whether a record is written.

Two remarks keep the construction honest. First, the embedding is not automatic: a Markov matrix need not admit any real generator, as the two-state exchange matrix with $p > 1/2$ shows, since a real generator forces positive determinant. The record law is embeddable because of its absorbing first-record structure. Second, λ_F is an integrated intensity fixed by the supplied survival probability, not a fitted rate.

Executed first-entry return. The no-record map satisfies

$$\rho(NR) = 0.200305886206958 < 1,$$

so $\Sigma_k NR^k$ converges and the expected waiting count $(I - NR)^{-1}\mathbb{1}$ is finite at every internal state. Hence

$$P(\text{eventually write a first record}) = 1$$

for every initial internal state, with stationary mean waiting count

$$E_\pi[N_{fact}] = 1.234602249934960$$

sequential opportunities. The record/no-record classification and first-entry exhaustion are therefore executed results at the implemented record-architecture grade.

Typing firewall. $\rho(NR)$ and λ_F are differently typed and must not be substituted for one another. The spectral radius certifies exhaustion and controls the mean waiting count through $(I - NR)^{-1}$; the intensity λ_F is built from the state-resolved survival probability $s(x)$ entering the generator at that state. The scalar identity $E\_\pi[N] = 1/(1 - \rho)$ holds only when the no-record map has state-independent survival, and the two executed numbers above are not related by it, which is exactly what a state-dependent no-record map produces. Writing $\lambda\_F = -\log \rho(NR)$ is falsifier F-MFKR36.

Lumping firewall. The same state dependence bounds what the scalar generator can claim. If a no-record transition can carry internal state x to a different internal state x′, the transient sector cannot in general be represented by one survival scalar while preserving the dynamics. Two honest routes exist: prove that the F2F coarse-graining is strongly lumpable onto a single survival macrostate, so that the scalar (s, q_a) description is exact and not merely stationary-averaged; or construct the full first-entry transfer in block form,

$$\mathbf{P = [[NR, B], [0, I]],}$$

on the transient no-record state space joined to the 84 absorbing record classes, and ask whether that executed matrix admits a continuous-time generator and whether the source law selects it uniquely. The second route is the stronger closure test, precisely because it removes the lumping hypothesis instead of certifying it. Both are registered in Appendix I; until one is executed, Theorem 14 is a theorem about the lumped description, exact in its stated class, and the executed $\rho(NR)$ results above already live at the state-resolved grain and are unaffected.

Corollary 14.1: The No-Go Family Is a Clock Reparametrisation, Fixed by the Survival Datum

[EXACT]

Write J for the jump matrix of the record law, $J_{0a} = q\_a$, $J_{00} = 0$, $J\_aa = 1$. Then

$I - P\_rec = h\_F(I - J)$, $Q = -\lambda\_F(I - J)$,

so the admissible family of Theorem 13 built on this base is exactly a time reparametrisation of the single semigroup constructed above,

$$\mathbf{T\_\sigma = \exp[-\sigma(I - P\_rec)] = R(\sigma h\_F/\lambda\_F),}$$

and the record-defined survival probability selects one member,

$$\mathbf{\sigma\bigstar = \lambda\_F/h\_F,\ T\_\{\sigma\bigstar\} = P\_rec.}$$

On the record-defined carrier the earlier continuum is therefore not a dynamical ambiguity at all: it is an unfixed clock normalisation, and the survival datum fixes it. This is why Theorems 13 and 14 are consistent rather than contradictory, and it identifies precisely which datum did the work.

Corollary 14.2: Corrected First-Record Intensity

[EXACT SURVIVAL FORM; CORRECTED INTENSITY IDENTIFICATION]

The exponential survival structure previously proposed for first-birth statistics is correct,

$p_0 = e^{-\Gamma_F}$, $p_a = (1 - e^{-\Gamma_F})q_a$,

but the integrated first-record intensity must be identified as

$$\Gamma_F = -\log s = -\log(1 - h_F),$$

where h_F is the probability of a nonself, ledger-producing history transition. Neither the earlier accepted-current hazard nor a raw-current expression of the form $\nu_J\|j\|^4R$ is the first-record intensity; these are differently typed physical objects and scale differently in the regulator refinement parameter, so the substitution is not a normalisation choice. Using either in place of Γ_F is falsifier F-MFKR31.

Proposition 14.3: Post-Commitment Persistence Interval

[EXACT IDENTIFIABILITY RESULT]

Let $P_a = I_A \otimes |a\rangle\langle a|$ be the projector onto retained ledger class a. On the append-only renewal branch let $\Phi_R = \Psi_A \otimes \mathrm{id}_L$ with Ψ_A any trace-preserving channel on the reopened active carrier. Since a trace-preserving channel has a unital Heisenberg adjoint,

$$\Phi_R^*(P_a) = \Psi_A^*(I_A) \otimes |a\rangle\langle a| = P_a,$$

and therefore $\varepsilon_{a\to b} = 0$ for $a \neq b$: the record-preserving renewal branch passes exactly.

However, choose inequivalent classes $a \neq b$ and let U_{ab} interchange their ledger basis vectors. The post-commitment overwrite channel

$$\Phi_O(\rho) = (I_A \otimes U_{ab})\, \rho\, (I_A \otimes U_{ab}^\sharp)$$

gives $\varepsilon_{a\to b} = 1$, and for the convex family $\Phi_p = (1 - p)\Phi_R + p\Phi_O$ with $0 \le p \le 1$ one obtains $\varepsilon_{a\to b} = p$. Hence

$$\varepsilon_{leak} \in [0, 1]$$

is compatible with the already-fixed first-entry and record-copy data, so those data do not determine persistence. The physical persistence problem is therefore not another leakage calculation: it is the upstream selection of the post-commitment operation among retain, renew, copy, proliferate, erase and overwrite.

24.3 The downstream Fock ambiguity closes completely

Whatever fixes the one-particle map, nothing downstream of it is free.

Let $A_f: \mathcal{H}_f \to \mathcal{H}_f$ be the supplied one-particle transfer for representation f, with the conjugate transfer A_{f^c} fixed by the inherited charge-conjugation architecture, which requires independent particle and antiparticle CAR sectors exchanged by a unitary Fock-space charge conjugation. On antisymmetric Fock space define

$\Gamma(A_f) = \bigoplus_{N=0}^{\dim \mathcal{H}_f} \wedge^N A_f,$

so that the particle/antiparticle transfer is

$$\mathbb{T}_f = \Gamma(A_f) \otimes \Gamma(A_{f^c}).$$

Theorem 15: Uniqueness of the Quasi-Free Fock Lift

[EXACT IN THE DECLARED CLASS]

Require the lift to preserve the Fock vacuum, be number preserving on the kinetic sector, act quasi-freely on the CAR generators, and induce A_f on every one-particle creation operator. The inducing condition is stated as the intertwining relation

$$\mathfrak{A}\, a^\sharp(\psi) = a^\sharp(A_f\psi)\, \mathfrak{A},$$

which requires no invertibility of a general supplied A_f. A terminology note keeps this precise: for non-unitary $A_f = e^{-a_t\Omega_f}$ this intertwining is not a CAR ∗-automorphism, which would require A_f unitary or isometric; "quasi-free" here means the unique vacuum-preserving, number-preserving exterior second-quantised transfer, not an automorphism of the CAR algebra. Then on an N-particle state the lift must act as

$\psi_1 \wedge \ldots \wedge \psi_N \mapsto A_f\psi_1 \wedge A_f\psi_2 \wedge \ldots \wedge A_f\psi_N,$

so the lift is exactly $\Gamma(A_f)$ and is unique. If moreover

$A_f = e^{-a_t \Omega_f},$

standard second quantisation gives $\Gamma(A_f) = e^{-a_t\, d\Gamma(\Omega_f)}$, and therefore

$$\mathbb{T}_f = \exp\{ -a_t [d\Gamma(\Omega_f) + d\Gamma(\Omega_{f^c})] \}.$$

Corollary 15.1: Generator Uniqueness

[INHERITED EXACT]

If a strictly positive self-adjoint transfer A and a step a_t are supplied, its self-adjoint generator is uniquely

$$\Omega = -(1/a_t) \operatorname{Log} A.$$

Corollary 15.2: Exact Localisation of the Remaining Freedom

[EXACT]

The chain

$A_f(\delta) \to \Omega_f \to \Gamma(A_f(\delta)) \to \mathbb{T}_f(\delta) = \exp\{-\delta[d\Gamma(\Omega_f) + d\Gamma(\Omega_{f^c})]\}$

contains no residual choice in the declared quasi-free class, at any Fold fraction δ. Consequently no part of the remaining sequential ambiguity lives in the second quantisation, the generator extraction, or the antiparticle sector. One typed gap must, however, be stated plainly. Theorem 14 constructs R(δ) on the classical retained-record/first-entry state space; Theorem 15 lifts a supplied one-particle transfer $A_f(\delta)$ acting on the fermionic one-particle Hilbert carrier. These are differently typed objects, and no theorem in this paper defines or derives the arrow $R(\delta) \to A_f(\delta)$. What is settled is therefore the composition form within each typed sector separately: the record first-entry composition is closed in its declared absorbing class, and the fermionic composition above any supplied A_f is closed in the quasi-free class. The identification of the record semigroup with, or its selection of, a fermionic one-particle transfer requires an explicit intertwining map that has not been constructed, and is registered in Appendix I as an open execution alongside the physical hazard and attaching-map selections.

24.4 What is closed and what remains open

The record-production semantics are now executed at the implemented F2F/PFDMI grain, and the earlier reading is rejected rather than merely superseded: the channel formerly identified with “no physical event” contributes to all retained record classes and therefore cannot carry that meaning. The correct implemented distinction is self-history ↔ no new record, nonself history ↔ new record. Accordingly:

RECORD/NO-RECORD EVENT SEMANTICS: PASS AT THE IMPLEMENTED F2F RECORD-ARCHITECTURE GRADE.

FIRST-ENTRY EXHAUSTION: PASS.

FIRST-ENTRY LABEL COPY INTO LEDGER: PASS.

PERMANENT PERSISTENCE UNDER EVERY LATER ADMISSIBLE CHANNEL: NOT YET SOURCE-SELECTED.

Four upstream questions remain, and none of them is about composition within a typed sector. The first is the typed identification itself.

Record-to-fermion intertwining. Theorem 14’s semigroup acts on the classical retained-record state space; Theorem 15’s lift acts above a supplied fermionic one-particle transfer. The map that selects $A_f(\delta)$ from the record dynamics is not constructed in this paper, and it is the missing middle arrow of the sequential chain.

Physical frame and hazard selection. For a supplied record frame the fact probability h_F and hence $\lambda_F = -\log(1 - h_F)$ are fixed. The primitive dynamics must still select the physical frame or endpoint witness, rather than that choice being supplied externally.

Post-commitment operation selection. The append-only renewal branch has exactly $\varepsilon_{leak} = 0$, but by Proposition 14.3 the current pre-commitment source data admit post-commitment extensions realising every value $0 \le \varepsilon_{leak} \le 1$. The primitive action must decide among retention, renewal, copying, proliferation, erasure and overwrite. This is a selection problem, not a computation that has yet to be run.

Sequential-opportunity to physical-time attaching map. The sequential parameter δ remains an opportunity fraction, not a physical duration. The history-mark structure and the closure traversal structure are not one-to-one, so the attaching map must be derived before λ_F can be read as a dimensionful physical rate.

Beside these sits the independent fermionic positivity gate: bosonic cone positivity does not prove positivity of the fermion-including transfer.

25. Sequential-Time Source Identity

There is an additional structural reason not to claim complete $D_f[U]$ identity yet. The strict microscopic ontology reconstructs temporal direction from sequential interfaces,

$W_7^{(m)} \to W_7^{(m+1)}$,

not from an additional set of primitive graph edges. Therefore the finite source kernel naturally factorises conceptually as

within-interface fermion operator + sequential-interface composition.

The present paper reconstructs the first object exactly at the source-functional level. The previous Hamiltonian-transfer calculation constructed an admissible sequential local kernel whose low-energy chiral block returns the unit-normalised Weyl pole and whose regulator remainder produces no second parity-even gauge logarithm at the free-pole grade, with promotion to the gauged determinant conditional on the declared locality, covariance and regularity conditions of that programme.

By Theorem 13 the bare positivity/reversibility premises do not select the transfer. By Theorem 14 the record-defined one-opportunity law does select it uniquely within the absorbing first-record class, and by Corollary 14.1 the earlier continuum on that carrier is a clock normalisation that the record-defined survival datum fixes. By Theorem 15 the lift to the fermionic Fock sector is unique once the one-particle map is supplied. What survives is not the composition rule within either typed sector or the event semantics, but the selection of the physical record frame, the record-to-fermion intertwining map, the post-commitment operation and the sequential-opportunity to physical-time attaching map.

Theorem 16: Sequential Source-Identity Firewall

[STRUCTURAL FIREWALL, WITH THE RESIDUAL OBSTRUCTION NAMED]

The finite spatial/shared-link kinetic matrix may be reconstructed from the Master Action without adding primitive time edges. A complete physical 3+1 fermion kernel additionally requires a sequential-interface fermionic composition map. Its form is constructed and unique within each typed sector, the record first-entry semigroup on its lumped class and the Fock composition above any supplied one-particle transfer, but the record dynamics has not yet been connected to A_f: the record-to-fermion intertwining map is open, and the physical record frame, the post-commitment operation and the sequential-opportunity to physical-time attaching map are not yet source-selected. The Hamiltonian-transfer completion therefore remains an admissible source-matched construction rather than a unique microscopic identity theorem, and δ remains an opportunity-fraction parameter rather than a physical duration until the attaching map is fixed.

26. Particle/Antiparticle Compatibility

The signed local kinetic operator may possess positive- and negative-frequency branches. This is not a failure of positive physical excitation energy. The completed CAR/Fock architecture contains independent particle and antiparticle families, and unitary Fock-space charge conjugation exchanges them.

After standard particle-hole reinterpretation the excitation Hamiltonian is nonnegative even though the local first-order operator retains its signed Clifford orientation. Therefore

$K_U \nrightarrow |K_U|$

inside the local fermion bilinear. Replacing K_U by its absolute value would erase the Clifford orientation required for the Weyl principal symbol.

Corollary 16.1: Signed-Kernel Preservation

[PASS ON INHERITED PARTICLE/ANTIPARTICLE COMPLETION]

Positive physical excitation energy does not require the finite local source kernel to be a positive operator. The sign information in K_U^{src} is physical first-order spinorial information and must survive into the local fermion action.

27. Source Identity Versus Universality Identity

Three grades should now be kept separate.

Grade I: operator-form source identity

$K_U = K_U^{src}$ as a finite functional of the source manifest. Status: PASS.

Grade II: instantiated kinetic source identity

The spatial infrared principal-symbol identity is now source-derived: on the K=7 coarse transport branch $M_{IR} = I_3$ and $\varepsilon_{moment}^{IR} = 0$, with $\varepsilon_{even} = 0$ structural on the inherited anti-adjoint branch and not required for the infrared closure. Status: IR PRINCIPAL-SYMBOL PASS / FINITE INSTANTIATION PARTIALLY DISCHARGED. What remains is publication of the microscopic geometric map $e \mapsto (n_e,w_e)$ to print the finite K_U and calculate exact finite-regulator C_{kin}, λ_{max}, d_+ and finite-star diagnostics.

Grade III: complete chiral/sequential source identity

The complete physical source dynamics uniquely selects the finite sequential chiral family $D_f[U]$, not merely its universality class.

Status: SPLIT AND SHARPLY LOCALISED.

Closed at their declared grades: the record/no-record partition at the implemented F2F grain; almost-sure first entry into a retained record class; exact copying of the first-entry label into the ledger; the unique absorbing first-record semigroup generated by the record-defined one-opportunity law; the unique quasi-free Fock lift of a supplied one-particle transfer; the unique self-adjoint generator of a supplied strictly positive transfer; exact persistence on the append-only renewal branch; and full mixed-tensor CAR/Fock completion compatibility on the declared conditional RRHF/FCHI source stack.

Open, and all upstream: the physical frame selecting the record hazard; the primitive post-commitment operation selector deciding preservation against erasure, overwrite, copying or proliferation; the physical terminal record quotient and readout where required to promote implemented ledger labels to physical equivalence classes; the sequential-opportunity to physical-time attaching map; fermionic transfer positivity; the glued-regulator geometric edge map; primitive-source uniqueness of the declared record, completion and CAR/Fock laws; and unique selection of the final chiral functional completion.

CWGW local one-loop universality does not require Grade III. Complete nonperturbative Standard Model quantisation ultimately does.

28. Source-Identity Gate Theorem

Theorem 17: Finite Master-Action Source-Identity Gate

[MIXED GRADE: EXACT RECONSTRUCTION / CONDITIONAL PHYSICAL IDENTITY]

Given a declared finite VERSF regulator, its primitive CAR/Fock pairing, its edge Clifford coefficients and its faithful physical links, the Master Action uniquely determines the finite kinetic matrix.

The matrix is exactly gauge covariant under the same physical link and carries no independent matter-link normalisation.

Its normalised continuum principal-symbol identity is decided by ε_{Cliff}; exact canonical-stencil identity is decided additionally by ε_{even}.

Its faithful-link sensitivity obeys $C_{kin}^{src} \leq D_\Gamma$, the bound is exactly attained on the canonical carrier, and the refinement of any given background's kinetic upper bound beyond the commuting-class estimate $\sqrt{\Sigma c_\mu^2}\, \varepsilon_U$ is controlled by the commutators of its direction perturbations. Once the manifest is frozen, C_{kin}, λ_{max}, d_+, the flat gap and the same-carrier admissibility radius are finite deterministic calculations.

On the inherited minimal anti-adjoint Clifford branch the coefficient itself is determined up to a tangent and a scalar weight, the even residual vanishes identically on that branch, and the principal-symbol test becomes a 3×3 star moment condition. The later K=7 refinement-transport programme independently constructs oriented path transport, its refinement-

Cauchy limit, the refinement frame and the soldered substrate metric. On the declared physical D = 3 branch the spatial second-order principal symbol in the orthonormal refinement frame is $|p|^2$; combined with the exact Clifford square this forces $M^T M = I_3$, orientation-preserving refinement excludes the reflection component, the surviving SO(3) freedom is exactly the common spin-frame equivalence already declared by the Clifford reduction, and the canonical infrared representative satisfies $M_{IR} = I_3$ with $\varepsilon_{moment}^{IR} = 0$, outright on the tangent-aligned positive-weight branch. The normalised spatial Dirac/Weyl principal symbol is therefore source-derived at the refinement-geometry grade without consuming the older involution-as-translation descent.

Corollary 17.1: Full Mixed-Tensor CAR/Fock Completion Descent

[EXACT ON THE DECLARED RRHF/FCHI SINGLE-PRIMITIVE-LEG COMPLETION STRUCTURE]

The declared RRHF/FCHI completion vertex has exactly one primitive completion direction,

$c_H = (1/2)(1, 1, 1, 1)$ on $(\varphi_H, X_{H,0}, X_{H,1}, X_{H,2})$,

so every nonzero mixed completion tensor has mode-1 source support inside $\mathrm{span}\{c_H\}$ and may be written $T_f = c_H \otimes B_f$, where B_f carries the full representation, generation, chirality and branch dependent left/right CAR/Fock structure. The mixed-jet quotient acts only on the primitive leg,

$T_f \times_1 P_{full} = (P_{full}\, c_H) \otimes B_f$, $T_f - T_f \times_1 P_{full} = [(I - P_{full})c_H] \otimes B_f$,

and multiplicativity of the Hilbert/Frobenius norm on tensor products, $\|a \otimes B\|_F = \|a\|\, \|B\|_F$, cancels the external factor from the normalised residual:

$\varepsilon_f^{(3)} = \|(I - P_{full})c_H\|\, \|B_f\|_F / (\|c_H\|\, \|B_f\|_F) = \|(I - P_{full})c_H\| / \|c_H\|$.

Since $\|c_H\| = 1$ and the executed full-action result is $\|(I - P_{full})c_H\| = 1.149 \times 10^{-15}$, every declared nonzero charged or neutral completion tensor satisfies $\varepsilon_f^{(3)} = 1.149 \times 10^{-15}$. Complex conjugation on the conjugate branch preserves the norm, so the conjugate-branch residual equals the primary one. Against the six-defect quotient, $\|(I - P_D)c_H\| = 1$ and hence $\varepsilon_{\{f,D\}}^{(3)} = 1$ exactly. Therefore

DEFECT-ONLY CAR/FOCK COMPLETION DESCENT: FAIL EXACTLY,

FULL-ACTION CAR/FOCK COMPLETION DESCENT: PASS AT MACHINE ZERO,

and D-HM29 full mixed-tensor compatibility passes at the declared conditional RRHF/FCHI source-stack grade. The potential rows are load-bearing rather than cosmetic: they are precisely what turns complete failure into full-action descent. An explicit per-sector tensor-array publication remains desirable as a reproducibility return, but it is no longer mathematically load-bearing for the normalised mode-1 residual.

The strongest statement $D_f^{physical}[U] = D_f^{constructed}[U]$ is not proved, but the kinetic reason is now narrower. The normalised spatial infrared principal symbol is already source-derived; the unpublished glued-regulator map $e \mapsto (n_e, w_e)$ is required for finite-matrix instantiation and finite-regulator constants, not to normalise the continuum pole. The sequential reason has also changed character: the composition form is constructed and unique within each typed sector, so what is missing is the source selection of the physical record frame, the post-commitment operation, the record-to-fermion intertwining map and the attaching map that makes the ordered rule a physical-duration law. Thus the remaining source-identity debt consists of finite instantiation plus named upstream selections, rather than an unnormalised fermion principal symbol.

29. Consequence for CWGW and One-Loop Running

CWGW-1 already established at the declared local grade:

one physical species; a source-matched shared-link Wilson seed; exact finite Ginsparg-Wilson chirality for the overlap representative; local faithful-background gap protection; exponential locality; the correctly normalised Weyl continuum pole; a UV-local regulator remainder; and, at the free-pole grade with gauged execution open, no second parity-even F^2 infrared logarithm.

MFKR-1 adds:

the primitive K_U is explicitly reconstructible from the Master Action.

It also closes the spatial principal-symbol normalisation on the K=7 infrared transport branch using the later refinement-geometry construction:

K=7 edge transport U_ij → T_γ → e^a_μ → g_μν → $M^T M = I_3$ → M_IR = I_3 mod spin frame → ε_moment^IR = 0 → K_IR(p) = iG^ip_i + $O(|p|^2)$, sharpening to $O(|p|^3)$ on the anti-adjoint exact-sine branch.

The older antipodal-involution derivative descent is not used.

This removes the previously separate geometric normalisation assumption from the low-energy spatial Weyl/Dirac pole. The microscopic edge manifest remains necessary for the finite regulator, but not for the continuum principal-symbol coefficient.

It also replaces the abstract statement

||K_U − K_I|| ≤ C_kin ε_U

with the source-native estimate

||K_U − K_I|| ≤ D_Γ ε_U,

now known to be exactly sharp on the reference carrier, so that CWGW's constant C_kin ≤ 4 was neither an assumption nor a loose estimate: it is the exact induced sensitivity of the reference kinetic operator. The sharpness result cuts both ways. It certifies that no CWGW admissibility radius built from D_Γ was needlessly small, and it forbids any future tightening of those radii except by explicit restriction to commuting-class backgrounds, whose sharp ceiling is the quadrature value √d instead.

The later full-faithful source programme also removes the final mixed-jet compatibility caveat on the declared RRHF/FCHI completion structure. Five source sectors had already passed at their first nonzero differential order, while the CAR/Fock completion sector had been graded only as a primitive-leg pass. Because that sector has a unique primitive completion direction, the full third-order tensor residual factorises exactly and equals the primitive residual, so the entire declared graded compatibility stack passes its quotient test at the corresponding conditional source-law grade. This is a compatibility result, not a primitive-selection theorem: it shows that the declared record, gauge, completion and CAR/Fock source laws descend consistently through the full faithful action, not that the deepest dynamics uniquely generates them. That remaining question is the primitive-source uniqueness gate.

Therefore the local one-loop result remains

$b_{Weyl}(R) = (2/3)T(R)$.

On the frozen three-generation matter census,

$b_3^{ferm} = 4$, $b_2^{ferm} = 4$, $b_Y^{ferm} = 20/3$.

Adding gauge, ghost and scalar contributions retains

$(b_Y, b_2, b_3) = (41/6, -19/6, -7)$

at the previously declared conditional Standard Model one-loop grade. MFKR-1 does not derive these coefficients by fitting its source-identity residuals to them. The coefficients are downstream checks.

30. What Has Actually Advanced

Before MFKR-1 the strict statement was:

primitive kinetic operator source-explicit; complete finite K_U not frozen.

After MFKR-1 the statement becomes:

primitive finite K_U reconstructed exactly as a sparse source functional, with its link sensitivity bounded, refined, and proved sharp.

The unexecuted kinetic part is reduced again. The K=7 infrared branch closes the general-star moment condition at the continuum pole through the source-derived refinement geometry,

$U_{ij} \to T_\gamma \to e^a{}_\mu \to g_{\mu\nu} \to M^T M = I \to M_{IR} = I_3$ modulo the declared spin-frame equivalence $\to \varepsilon_{moment}^{IR} = 0 \to$ normalised spatial Dirac principal symbol,

so the remaining coefficient debt is only

publish the microscopic geometric map $e \mapsto (n_e, w_e) \to$ instantiate numerical finite K_U and same-carrier constants.

The infrared symbol no longer waits on either the microscopic edge manifest or a reconstructed oriented shift derived from the old antipodal involutions. The latter remain mathematically incapable of serving as infinitesimal translations, but later VERSF work supplies a different and genuinely oriented refinement transport, so that old inconsistency is now a source firewall rather than an open physical gate. A separate foundational debt remains upstream: the physical $D = 3$ global branch is inherited by the refinement-transport construction and is not newly derived here. The closure is also protected: by Proposition 7.3 the closure is rigid up to the declared spin-frame and orientation freedom, with no freedom at all on the tangent-aligned branch, by Corollary 7.4 it is stable with constant one against refinement error, and by Corollary 10.1 the three-direction constant is attained exactly on the canonical carrier, with the physical weighted degree, physical C_{kin} and saturation remaining finite-manifest outputs. By Proposition 7.5 and Corollary 10.2 all of this is dimension-general apart from the orientation datum, which is a structural feature of odd dimension, and by Section 18.3 the certification pipeline that will process the physical manifest is already executed and validated on reference stars, with its differencing budget proved and empirically margined.

And the stronger chiral identity problem separates cleanly. The chain

$K_U^{src} \to X_\rho^{src} \to$ admissible chiral family

is already constructed; and the sequential chain is constructed and unique in two typed segments,

record law $(h_F, q) \to \lambda_F = -\log(1 - h_F) \to Q \to R(\delta) = e^{\delta Q}$

on the classical record state space, and

$A_f(\delta) \to \Gamma(A_f(\delta)) \to \mathbb{T}_f(\delta)$

on the fermionic carrier above any supplied one-particle transfer. The arrow joining them, $R(\delta) \to A_f(\delta)$, is an identification between differently typed objects that no theorem here constructs; it is a named open execution, not a composition question. What is not yet returned is therefore this intertwining map together with the upstream selections:

source → (physical record frame, post-commitment operation, record-to-fermion intertwining, sequential-opportunity to physical-time attaching map).

The problem has therefore not merely narrowed, it has changed type twice: from a missing operator, to a missing derivation, to a short list of named physical selections and one named identification map.

31. Minimal Physical Source Manifest

A reproducible physical execution should publish, for every regulator level n:

$\mathfrak{M}_{f,n}$ = {V_n, E_n, F_n, C_n; s(e), t(e); vertex/edge carrier metrics; (n_e, w_e) with orientation; $R_f(U_e)$; canonicalisation maps; r_n; $a_{t,n}$; the record-defined pair

$(h_{F,n}, q)$ with its declared record frame, fixing the record first-entry semigroup $R(\delta) = e^{\delta Q}$ on its lumped class; the fermionic one-particle transfer A_f is a separate entry pending the record-to-fermion intertwining map}.

The coefficient entry is geometric, not matrix valued, by Theorem 6. The sequential entry is the record-defined pair (h_F, q) together with the declared record frame, from which the small-step family $R(\delta) = e^{\delta Q}$ and its generator follow uniquely by Theorem 14; a bare transfer at one fixed step size is the underdetermining input the class-scoped no-go rules out.

From this one manifest a deterministic execution should return

K_U, L_U, $D_\Gamma = (1/2)\max_v \Sigma_{e\ni v} w_e$, C_{kin}, λ_{max}, d_+, M^{ij}, ε_{moment}, ε_{Cliff}, ε_{even}, $\lambda_F = -\log(1 - h_F)$ with the composition residual $\|R(\delta_1)R(\delta_2) - R(\delta_1+\delta_2)\|$ and the one-opportunity residual $\|e^Q - P_{rec}\|$, together with $\rho(NR)$ and the leakage ε_{leak} of the selected post-commitment operation,

followed, once the chiral completion is fixed, by

δ_n, $\varepsilon_{src}^{(n)}$, index D_f, locality rate.

No measured Standard Model quantity belongs in this manifest.

32. Falsifiers

F-MFKR1: Matrix reconstruction failure. Direct differentiation of the published primitive kinetic bilinear produces a finite matrix different from $T^\sharp\Gamma_E B_U + (T^\sharp\Gamma_E B_U)^\sharp$.

F-MFKR2: Hidden-link failure. The reconstructed physical kinetic matrix requires a matter gauge link not present in the Master Action.

F-MFKR3: Independent-normalisation failure. A free matter-link multiplier survives after canonicalisation of the source-derived kinetic metric.

F-MFKR4: Gauge covariance failure. For some allowed faithful gauge transformation, $K_{U^g}^{src} \neq G K_U^{src} G^{-1}$.

F-MFKR5: Principal-symbol failure. The executed physical source manifest returns $\varepsilon_{Cliff} \neq 0$.

F-MFKR6: Exact-stencil failure. The source manifest returns $\varepsilon_{Cliff} = 0$ but $\varepsilon_{even} \neq 0$. This does not by itself falsify the Weyl continuum pole, but it falsifies exact identity with the pure minimal sine stencil and requires the additional source-derived $O(q^2)$ operator to be retained.

F-MFKR7: Kinetic-bound failure. A link background satisfies $\|K_U - K_I\| > D_\Gamma \varepsilon_U$.

F-MFKR8: Reference recovery failure. The canonical four-direction comparison carrier with half-normalised nearest-neighbour edge blocks does not return $D_\Gamma = 4$.

F-MFKR9: Carrier mixing. D_Γ, C_{kin}, λ_{max}, d_+ or δ are imported from different regulator carriers to obtain a stronger gap radius.

F-MFKR10: Primitive-time conflation. The finite source matrix is extended by invented temporal graph edges even though sequential time is represented by interface succession.

F-MFKR11: Absolute-value conflation. The local signed kinetic matrix K_U is replaced by $|K_U|$, erasing the Clifford orientation required for the Weyl principal symbol.

F-MFKR12: Antiparticle doubling error. The charge-conjugate CAR sector is counted as an unwanted regulator species.

F-MFKR13: Chiral-uniqueness overclaim. The overlap/Ginsparg-Wilson functional completion is declared uniquely derived from the primitive kinetic bilinear without an independent source-selection theorem.

F-MFKR14: Source-publication overclaim. The symbolic operator reconstruction is reported as an executed numerical physical kernel before the edge manifest is published.

F-MFKR15: Beta-target selection. Any edge coefficient, normalisation, chiral completion or branch is chosen by consulting the target one-loop Standard Model coefficients.

F-MFKR16: Factorisation failure. On an $\varepsilon_{even} = 0$ directional carrier, the link perturbation fails to equal the Clifford-diagonal form $-\Sigma\, \Gamma_\mu N_\mu$, or a background violates the commutator-refined bound of Theorem 9.

F-MFKR17: Quadrature misuse. The commuting-class value $\sqrt{(\Sigma c_\mu^2)}\, \varepsilon_U$ is applied to a background whose direction perturbations do not commute, or is used to shrink the general admissibility radius of Theorem 12.

F-MFKR18: Sharpness-witness failure. The alternating-sign 2^4 abelian witness of Theorem 10 fails to return $\|\Delta K\| = D_\Gamma\, \varepsilon_U$, or genuine unitary links fail to approach that ratio in the small-field limit.

F-MFKR19: Corner misattribution. Under $\varepsilon_{even} = 0$, doubler lifting is attributed to the first-order source term rather than to the Wilson roughness, contradicting Proposition 5.1.

F-MFKR20: Clifford-linearity failure. The physical manifest requires an edge coefficient outside the class $(w_e/2)G(n_e)$ while still being declared the inherited minimal graph-Dirac/Clifford realisation, or a coefficient in that class fails $\Gamma_e^\sharp = -\Gamma_e$.

F-MFKR21: Structural-residual failure. A branch declared minimal Clifford-linear returns $\varepsilon_{even} \neq 0$, or the canonical normalised three-channel branch returns $\varepsilon_{Cliff} \neq 0$.

F-MFKR22: Moment-certificate failure or grade conflation. On any finite star where principal-symbol identity is claimed directly, $M^{ij} = \Sigma_e\, w_e\, n_e^i\, a_e^j$ must equal δ^{ij}; at the K=7 infrared grade the required return is $M_{IR} = I_3$ from the transport theorem. A non-symmetric or non-identity moment may not be promoted, and the infrared pass may not be misquoted as publication of every microscopic finite-star coefficient.

F-MFKR23: Scalar-degree failure. The reduced weighted degree $(1/2)\max_v \Sigma_{e\ni v} w_e$ disagrees with the matrix-level D_Γ on a Clifford-linear manifest.

F-MFKR24: Sequential-uniqueness overclaim. The sequential transfer is declared uniquely selected from bare positivity/reversibility premises, without the record-defined data, despite the explicit admissible continuum.

F-MFKR25: Fock-lift ambiguity. A second vacuum-preserving, number-conserving, quasi-free CAR/Fock lift inducing the same one-particle transfer A_f is exhibited, or $\Gamma(e^{-a_t\Omega}) \neq e^{-a_t\, d\Gamma(\Omega)}$ on the declared carrier.

F-MFKR26: Generator ambiguity. A strictly positive self-adjoint supplied transfer admits a self-adjoint generator other than $-(1/a_t)\mathrm{Log}\, A$ at the declared step.

F-MFKR27: Positivity import. Bosonic cone positivity is used as though it established positivity of the fermion-including transfer, without a real nonnegative fermion determinant or a reflection/transfer positivity certificate.

F-MFKR28: Single-step closure attempt. The composition form is declared closed by supplying a transfer at one fixed step size, without the record-defined law (h_F, q) or an equivalent generator-determining datum.

F-MFKR29: Generator failure. The constructed Q fails $e^Q = P_{rec}$, or $\lambda_F \neq -\log(1 - h_F)$, or the closed forms $R_{00}(\delta) = e^{-\lambda_F \delta}$ and $R_{0a}(\delta) = (1 - e^{-\lambda_F \delta})q_a$ fail at some δ, or Q has a negative off-diagonal entry or nonzero row sum.

F-MFKR30: Class-scope violation. The class-scoped no-go is reported as refuted rather than evaded by additional supplied data, or the record-defined selection is applied outside the absorbing first-record class where its uniqueness argument does not run.

F-MFKR31: Raw-current clock import. A raw-current expression of the form $\nu_J\|j\|^4 R$ is used as the first-birth intensity in place of $-\log(1 - h)$, or the two are treated as the same object despite their different refinement scaling.

F-MFKR32: Parameter conflation. The semigroup Fold-fraction parameter δ is conflated with the regulator refinement parameter used in hazard-scaling statements, or a dimensionful physical rate is read off from λ before the attaching map is fixed.

F-MFKR33: Sequential closure overclaim. The sequential problem is declared closed despite the open physical frame selection, the unselected post-commitment operation, or the unfixed attaching map; or one sequential opportunity is identified with one geometric traversal despite the mark/traversal mismatch.

F-MFKR34: Embeddability assumption. A supplied one-step law is assumed to possess a Markov generator without checking, contrary to the fact that embeddability can fail.

F-MFKR35: Event-semantics regression. The record/no-record partition is replaced by a current-channel partition, or a channel contributing to retained record classes is again read as “no physical event”, contradicting the executed F2F classification.

F-MFKR36: Exhaustion/intensity conflation. $\rho(NR)$ is substituted for the survival probability in $\lambda_F = -\log s$, or the scalar relation $E_\pi[N] = 1/(1 - \rho)$ is applied to a no-record map with state-dependent survival.

F-MFKR37: Persistence overclaim. Exact persistence is asserted for the physical dynamics on the strength of the append-only renewal branch alone, despite the proved non-identifiability interval $\varepsilon_{leak} \in [0, 1]$; or the post-commitment question is treated as a leakage computation rather than a selection.

F-MFKR38: Completion-descent misattribution. The full mixed-tensor residual is claimed to differ from the primitive residual on a completion tensor with a single primitive leg, or the defect-only quotient is reported as passing, or the potential rows are treated as cosmetic.

F-MFKR39: Compatibility-for-selection substitution. The mixed-jet compatibility pass is presented as showing that the deepest dynamics uniquely generates the declared record, gauge, completion and CAR/Fock laws, rather than that those laws descend consistently through the full action.

F-MFKR40: Rigidity-scope violation. The Laplacian square identity is used to assert $M = I$ without either restricting to the tangent-aligned positive-weight branch or quotienting by the declared spin-frame and orientation freedom; or an orthogonal non-identity moment tensor is reported as a principal-symbol failure when it is the declared frame freedom, or as a pass without checking orthogonality.

F-MFKR41: Orientation-component conflation. A moment tensor with $\det M = -1$ is absorbed as an innocuous spin-frame rotation. The parity component is not inner in three dimensions and flips the declared orientation labels; treating it as a rotation misreports the orientation data of the source manifest.

F-MFKR42: Stability misuse. The bound $\|M - I\| \le \|M^2 - I\|$ is applied off the tangent-aligned positive-weight branch, or the general statement $\mathrm{dist}(M, O(3)) \le \|M^T M - I\|$ is quoted as if it bounded $\|M - I\|$ itself without the alignment hypothesis.

F-MFKR43: Unbudgeted certificate. A finite-star execution reports the raw measured Laplacian defect $\|Q_h - I\|$ as a certified moment bound without adding the a priori differencing budget $\tau(h)$, or quotes a budget not computed from the manifest. Section 18.3 exhibits a manifest and a scale at which the raw defect alone lies below the true moment error, so the omission is not a harmless simplification.

F-MFKR44: Dimension transfer of the orientation datum. The odd-dimensional orientation obstruction of Proposition 7.5 is cited in even dimension, where the reflection is inner and no independent orientation datum exists; or the $d = 3$ orientation choice is presented as a generic feature of Clifford-linear reductions rather than a structural property of odd carriers.

F-MFKR45: Transport-object conflation. An exact antipodal involution satisfying $T^2 = I$ is used as an infinitesimal translation through $(T - I)/\xi \to \partial$, or the old involution is identified with the later oriented edge/path transport U_{ij}, T_γ. The identity $2D + \xi D^2 = 0$ remains an exact no-go against the former, with spectrum $\{0, -2/\xi\}$ on every invariant subspace.

Conversely, that old no-go may not be cited as an obstruction to Corollary 7.2 once the independent refinement-transport and soldered-frame construction is used.

F-MFKR46: Dimensionality/axis conflation. The source-derived refinement-frame result is reported as a new derivation of D = 3, or the three antipodal D_6 axes of the PFDMI/W7 carrier are silently identified with the three spacelike refinement-frame legs without a typed map. Section 14.3 requires neither overclaim: it is evaluated on the declared physical D = 3 branch and uses the substrate-derived soldered refinement frame directly. The foundational selection of D = 3 remains separately typed.

F-MFKR47: Record/fermion type conflation. The record semigroup R(δ), which acts on the classical retained-record/first-entry state space, is treated as if it acted on, generated, or determined the fermionic one-particle transfer A_f(δ) without an explicit intertwining map; or Theorems 14 and 15 are cited jointly as a construction of the chain R(δ) → A_f(δ) → Γ(A_f) → 𝕋_f when the middle arrow has no theorem.

F-MFKR48: Unlicensed survival lumping. The scalar one-survival-state generator of Theorem 14 is quoted for the state-resolved no-record dynamics without either a strong-lumpability certificate for the F2F coarse-graining or the executed block-generator construction on the transient space joined to the 84 absorbing record classes; the executed no-record map has state-dependent survival, which is exactly why ρ(NR) and the scalar survival probability are not interchangeable.

F-MFKR49: Anti-adjointness bootstrapping. Per-edge anti-adjointness Γ_e^♯ = −Γ_e is presented as a consequence of the graph-Dirac square condition, locality or spin covariance, rather than as the inherited definition of the minimal Clifford-linear branch. The contamination Γ_i = (1/2)G_i + iηG_i defeats any such derivation: its even part is nonzero, its assembled square is exactly scalar, and its effect appears only at $O(q^2)$. Equivalently, ε_even = 0 or the exact sine stencil is quoted at the microscopic grade without the anti-adjointness input being either inherited from a prior source theorem or derived.

33. Formal Verdict

Object	MFKR-1 grade
Primitive CAR/Fock edge transport	SOURCE-EXPLICIT
Same physical faithful link in matter and gauge sectors	SOURCE-EXPLICIT
Finite regulator incidence structure	INHERITED / SOURCE-DEFINED
Finite source operator B_U = T − $\mathcal{U}$S	EXACT
Finite edge Clifford operator Γ_E	SOURCE-TYPED; REDUCED TO (w_e/2)G(n_e) ON THE INHERITED ANTI-ADJOINT MINIMAL BRANCH; GEOMETRIC EDGE MAP NOT PUBLISHED

Object	MFKR-1 grade
Exact finite kinetic matrix K_U^src	NEW / EXACT OPERATOR-FUNCTIONAL RECONSTRUCTION
Sparse vertex-block assembly formula	NEW / EXACT
Gauge covariance of K_U^src	EXACT
Independent matter-link normalisation	FORBIDDEN
Flat-symbol even/odd decomposition	NEW / EXACT
Principal-symbol identity criterion	NEW / EXACT NECESSARY AND SUFFICIENT TEST
Exact canonical-stencil criterion	NEW / EXACT NECESSARY AND SUFFICIENT TEST
Corner census K_I(c) = 4Σ_{μ∈c}Γ_{μ,+}	NEW / EXACT
Minimal Clifford-linear edge form Γ_e = (w_e/2)G(n_e)	NEW / CONDITIONAL EXACT ON THE INHERITED MINIMAL REALISATION
ε_even	ZERO STRUCTURALLY ON THE INHERITED ANTI-ADJOINT MINIMAL BRANCH; INPUT NOT REQUIRED FOR THE IR CLOSURE
ε_Cliff	ZERO ON THE CANONICAL NORMALISED THREE-CHANNEL BRANCH
Star moment certificate M^{ij} = δ^{ij}	NEW / EXACT NECESSARY AND SUFFICIENT GEOMETRIC CRITERION
K=7 old antipodal involution derivative descent	**REJECTED AS A TRANSLATION IDENTIFICATION; RETAINED AS SOURCE-AUDIT FIREWALL**
Later K=7 oriented edge/path refinement transport U_ij → T_γ	**SOURCE-DERIVED / REFINEMENT-CAUCHY CONTINUUM LIMIT**
Refinement frame and soldered metric g = e e η	**SOURCE-DERIVED AT DECLARED TRANSPORT-GEOMETRY GRADE**
Global D = 3 physical branch	**INHERITED / FOUNDATIONAL; NOT RE-DERIVED BY MFKR-1**
K=7 coarse/IR moment square $M^T M$	**PASS: I_3 ON DECLARED D = 3 REFINEMENT-GEOMETRY BRANCH**
Even-part firewall for the IR principal symbol	**NEW / EXACT: CLOSURE CONSUMES Γ_– ONLY; ANTI-ADJOINTNESS INPUT GATES STENCIL PURITY, NOT THE POLE**

Object	MFKR-1 grade
K=7 coarse/IR star moment M_IR	**I_3 MODULO DECLARED COMMON SPIN-FRAME EQUIVALENCE; I_3 OUTRIGHT ON TANGENT-ALIGNED POSITIVE-WEIGHT BRANCH**
Normalised spatial Dirac/Weyl principal symbol	**SOURCE-DERIVED IR PASS AT REFINEMENT-GEOMETRY GRADE**
Exact scalar square of the Clifford-linear star symbol	NEW / EXACT AT ALL MOMENTA: $K\_I(q)^2 = \|v(q)\|^2 I$
Laplacian square rigidity of the moment tensor	NEW / EXACT: $M^T M = I$ FORCED; M = I OUTRIGHT ON THE TANGENT-ALIGNED POSITIVE-WEIGHT BRANCH
Residual freedom off tangent alignment	EXACTLY THE DECLARED SPIN-FRAME ROTATION AND ORIENTATION CHOICE
Quantitative stability of the moment closure	NEW / EXACT: $\|M − I\| \le \|M^2 − I\|$ WITH CONSTANT ONE
Canonical three-direction weighted degree	**EXACT: D_Γ^{3,canon} = 3**
Canonical three-direction kinetic sharpness	**EXACT: C_kin^{3,canon} = 3 ATTAINED BY EXPLICIT 2^3 WITNESS**
Physical K=7 weighted degree D_Γ	**FINITE-MANIFEST OUTPUT; NOT YET INSTANTIATED**
Physical K=7 C_kin	**FINITE-MANIFEST OUTPUT; NOT YET INSTANTIATED**
Dimension generality of the rigidity, stability and sharpness package	NEW / PROVED FOR ALL d; RIGIDITY NUMERICALLY CHECKED AT d = 2, 3, 4, 5, SHARPNESS AT d = 2, 3, 4
Orientation datum	NEW / STRUCTURAL: INDEPENDENT SOURCE DATA EXACTLY IN ODD d, VOLUME-ELEMENT OBSTRUCTION
Finite-star certification pipeline	NEW / EXECUTED: BUDGETED CERTIFICATE, FOUR REFERENCE STARS, FIVE SCALES, ZERO VIOLATIONS
Finite microscopic physical-star ε_moment	DETERMINISTIC MANIFEST OUTPUT; FINITE EXECUTION OPEN, NO LONGER LOAD-BEARING FOR IR POLE
Scalar weighted degree D_Γ = (1/2)Σ_{e∋v} w_e	NEW / EXACT ON THE CLIFFORD-LINEAR BRANCH
Canonical three-direction IR comparison carrier D_Γ^{3,canon}	**EXACT: 3; PHYSICAL K=7 D_Γ IS A FINITE-MANIFEST OUTPUT**
Physical geometric edge map e ↦ (n_e, w_e)	OPEN ONLY FOR FINITE-MATRIX / FINITE-CONSTANT INSTANTIATION
Weighted Clifford degree D_Γ	NEW / EXACT FINITE SOURCE QUANTITY
C_kin^src ≤ D_Γ	NEW / DERIVED

Object	MFKR-1 grade
Clifford-diagonal perturbation factorisation	NEW / EXACT ON THE $\varepsilon_{even} = 0$ CLASS
Commutator-refined kinetic bound	NEW / EXACT
Commuting-class sharp supremum $\sqrt{d}\, \varepsilon_U$	NEW / EXACT CEILING, ATTAINED BY THE UNIFORM CONSTANT-ABELIAN BACKGROUND
Sharpness $C_{kin}^{exact} = D_\Gamma = 4$ (canonical carrier)	NEW / EXACT, EXPLICIT 2^4 WITNESS
Unitary-link saturation in the small-field limit	NEW / EXACT LIMIT, NUMERICALLY WITNESSED
Canonical Euclidean recovery $D_\Gamma = 4$	PASS AT REFERENCE GRADE, NOW EXACT
Three-direction reference $D_\Gamma = 3$	PASS AT REFERENCE GRADE
Exact physical C_{kin}	FINITE COMPUTATION DEFINED; REFERENCE VALUE 4 RETURNED; MANIFEST EXECUTION OPEN
Source roughness $L_U = B_U^{\sharp} B_U$	DERIVED
Source Wilson seed X_ρ^{src}	CONDITIONAL EXACT ON $r > 0$
Manifest-level admissibility formula	NEW / EXACT GIVEN SAME-CARRIER INPUTS
Commuting-class radius improvement	NEW / EXACT ON THE RESTRICTED CLASS ONLY
Numerical physical admissibility radius	OPEN PENDING MANIFEST + SAME-CARRIER GAP
Overlap completion from source seed	ADMISSIBLE SOURCE-FUNCTIONAL CONSTRUCTION
Unique overlap selection by primitive action	OPEN
Quasi-free Fock lift of a supplied one-particle transfer	NEW / UNIQUE IN THE DECLARED CLASS
Identity $\Gamma(e^{-a_t \Omega}) = e^{-a_t\, d\Gamma(\Omega)}$	EXACT
Generator of a supplied strictly positive transfer	INHERITED UNIQUE: $\Omega = -(1/a_t) \mathrm{Log}\, A$
Localisation of residual sequential freedom to A_f	NEW / EXACT

Object	MFKR-1 grade
Class-scoped no-go on bare positivity/reversibility premises	INHERITED EXACT, HYPOTHESIS CLASS NOW EXPLICIT
Record-defined generator Q, $R(\delta) = e^{\delta Q}$, $R(1) = P_{rec}$	NEW / EXACT AND UNIQUE IN THE ABSORBING FIRST-RECORD CLASS
Earlier continuum as a fixed clock normalisation, $\sigma\star = \lambda_F/h_F$	NEW / EXACT
Embeddability of the record law	ESTABLISHED FROM ITS STRUCTURE, NOT ASSUMED
Corrected first-record intensity $\Gamma_F = -\log(1 - h_F)$	NEW / SURVIVAL FORM RETAINED, INTENSITY CORRECTED
$\rho(NR)$ versus λ_F typing	DISTINCT OBJECTS; SCALAR IDENTITY ONLY FOR STATE-INDEPENDENT SURVIVAL
Record first-entry composition	CLOSED IN THE DECLARED ABSORBING CLASS, ON THE ONE-SURVIVAL-MACROSTATE LUMPING
Record-semigroup to fermionic one-particle-transfer identification	**OPEN; REQUIRES EXPLICIT INTERTWINING MAP**
Physical hazard and frame selection	OPEN
Record/no-record semantics at implemented F2F grain	EXECUTED PASS: SELF-HISTORY = NO NEW RECORD, NONSELF HISTORY = RECORD-PRODUCING
Earlier accepted-current persistent-fact partition	REFUTED BY THE F2F RECORD EXECUTION
F2F first-entry exhaustion	PASS: $\rho(NR) = 0.200305886206958 < 1$
F2F first-entry record copy	PASS
Stationary first-record waiting count	1.234602249934960 SEQUENTIAL OPPORTUNITIES
Record-preserving renewal branch	EXACT PASS
Cross-record leakage on the renewal branch	$\varepsilon_{leak} = 0$
Post-commitment leakage from current pre-commitment data	NON-IDENTIFIABLE: FULL INTERVAL [0, 1]
Physical post-commitment operation selector	OPEN: RETAIN / RENEW / COPY / PROLIFERATE / ERASE / OVERWRITE

Object	MFKR-1 grade
CAR/Fock completion primitive leg	PASS ON FULL ACTION: 1.149×10^{-15}
CAR/Fock completion full mixed-tensor residual	PASS BY EXACT TENSOR-FACTOR REDUCTION: 1.149×10^{-15}
CAR/Fock completion defect-only residual	FAIL EXACTLY: 1
D-HM29 full mixed-tensor compatibility	PASS AT DECLARED CONDITIONAL RRHF/FCHI SOURCE-STACK GRADE
Primitive-source uniqueness of the declared record/completion laws	OPEN: UPSTREAM SELECTION GATE
Sequential-opportunity to physical-time attaching map	OPEN: MARK AND TRAVERSAL STRUCTURES NOT ONE-TO-ONE
Fermionic transfer positivity	OPEN; BOSONIC CONE POSITIVITY INSUFFICIENT
Sequential fermionic composition above a supplied transfer	FORM CONSTRUCTED AND UNIQUE IN DECLARED CLASSES; PHYSICAL SELECTION AND RECORD-TO-FERMION INTERTWINING OPEN
Normalised Weyl pole	PRIOR LOCAL PASS / RETAINED
UV-local no-second-log result	PRIOR FREE-POLE-GRADE PASS / RETAINED
R1Q local one-loop fermion universality	CONDITIONAL PASS AT CONSTRUCTED SOURCE-MATCHED FREE-POLE GRADE; GAUGED EXECUTION OPEN
Complete physical D_f[U] source identity	OPEN: GEOMETRIC EDGE MAP + PHYSICAL FRAME/HAZARD + RECORD-TO-FERMION INTERTWINING + POST-COMMITMENT SELECTION + ATTACHING MAP + FERMIONIC POSITIVITY + PRIMITIVE-SOURCE UNIQUENESS + FINAL CHIRAL SELECTION
D-SG6 local source-matched regulator	PASS
D-SG6 component-wise global certificate	OPEN

34. Scientific Conclusion

The principal result of this paper is that the finite Master-Action fermion kinetic operator was less missing than the previous ledger implied.

The primitive action already contains enough structure to determine it:

K_U^src = T^♯Γ_E(T − 𝓤_f S) + [T^♯Γ_E(T − 𝓤_f S)]^♯.

This is the complete finite kinetic matrix as a function of the source data. It is sparse, local and exactly gauge covariant. No independent matter gauge link and no separate matter-link normalisation survive.

The reconstruction also removes the ambiguity surrounding C_{kin}, and does so completely at the reference grade. The link variation of the source matrix is explicit, and its norm is bounded by one finite weighted graph quantity,

$$C_{kin}^{src} \le D_\Gamma = \max_v \Sigma_{e\ni v} \|\Gamma_e\|.$$

The earlier value $C_{kin} \le 4$ on the canonical four-direction overlap carrier is recovered immediately as the special case of eight incident half-normalised edge contributions, and then proved to be exactly attained: an explicit alternating-sign abelian background on sixteen sites realises pairwise anticommuting perturbation operators and returns $\|\Delta K\| = 4\varepsilon_U$ to the last digit, with genuine unitary links approaching the same ratio. So the constant 4 is not an arbitrary continuum estimate, and not merely a bound: it is the exact induced sensitivity of the reference kinetic operator. Underneath it sits an exact structure theorem. On the certified stencil class the perturbation is Clifford-diagonal, its square separates a definite quadrature part from pure commutator interference, and the two extremes are both realised: the commuting class has sharp supremum $\sqrt{d}\,\varepsilon_U$, attained by the uniform constant-abelian background; the unrestricted class has sharp supremum $d\,\varepsilon_U$, attained by the alternating Jordan-Wigner witness; individual backgrounds may respond below either sharp ceiling, with their refined upper bounds controlled by their direction commutators.

A second exact result isolates the source-identity test itself. The flat source matrix decomposes into an adjoint-even $2(1 - \cos q)\Gamma_+$ term and an adjoint-odd $2i \sin q\, \Gamma_-$ term. The anti-adjoint edge component controls the first-order Dirac/Weyl pole. The normalised principal symbol is present exactly when

$$2\Gamma_{\mu,-} = G_\mu^{Cliff}.$$

The adjoint-even component controls the additional finite $O(q^2)$ source term. Exact identity with the minimal sine stencil requires it to vanish, and the corner census shows that under that certificate the source symbol vanishes at all sixteen momentum corners, leaving doubler removal entirely to the Wilson roughness, exactly as the seed architecture requires.

The remaining kinetic source question then shrinks again, because the edge coefficient is not free. On the inherited minimal anti-adjoint Clifford branch the only compatible coefficient is $(w_e/2)G(n_e)$, so an arbitrary matrix per edge is replaced by a tangent and a scalar weight. On that branch the coefficient is anti-adjoint by definition, so ε_{even} vanishes identically there, an input Proposition 6B shows the infrared pole does not need. For a general star the whole principal-symbol question compresses into one 3×3 identity, $M^{ij} = \Sigma_e w_e n_e^i a_e^j = \delta^{ij}$.

The latest transport papers close that identity in the infrared without relying on the older antipodal involutions as translations. Genuine oriented edge transport U_{ij} generates path holonomy T_γ; closure-compatible refinement gives its sequence-independent continuum limit; and the refinement-frame construction solders the substrate bilinear form directly into the continuum metric, $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$. On the declared physical $D = 3$ branch the three spacelike legs of the orthonormal refinement frame carry the unit spatial second-order principal symbol, and the exact Clifford-linear square then gives $p^T(M^TM)p = p^Tp$ for every p, so $M^TM = I_3$. Orientation-preserving refinement removes the reflection component, and the surviving SO(3) transformation is exactly the common spin-frame equivalence already declared by the minimal Clifford reduction, giving the canonical representative

$M_{IR} = I_3$, $\varepsilon_{moment}^{IR} = 0$, $K_{IR}(p) = iG^ip_i + O(|p|^2)$, sharpening to $O(|p|^3)$ on the anti-adjoint exact-sine branch,

with $M = I_3$ outright on the tangent-aligned positive-weight branch. The surviving overall scale of the substrate bilinear form is a common ruler scale, not an independent directional fermion normalisation, and disappears from the dimensionless moment identity in the orthonormal refinement frame. The global $D = 3$ dimensionality of the physical branch remains a separate foundational inheritance, not a new result claimed here.

The normalised spatial Dirac principal symbol is consequently no longer an independently supplied geometric normalisation, and no longer conditional on a future construction of oriented shifts from the old antipodal involutions. Those involutions still cannot generate translations; the point is that the later refinement programme supplies genuine oriented transport independently, so the old inconsistency stands as a source firewall while the closure itself rests on the soldered refinement geometry. And it could not have come out otherwise: the star symbol squares exactly to a scalar at every momentum, so the unit transport Laplacian pins the moment tensor up to the spin-frame and orientation freedom the reduction already declares, pins it completely on the tangent-aligned branch, and controls its residual by the Laplacian error with constant one. The three-direction sensitivity constant is attained by an explicit witness on the canonical carrier, which therefore carries the same exact-sharpness status as the four-direction reference carrier; the exact physical weighted degree, physical C_{kin} and saturation of the canonical value 3 remain finite-manifest outputs, separate from the closed moment identity. These are not three-dimensional accidents: every statement except one holds for Clifford-linear stars in arbitrary dimension with the sharpness constant d attained, and the exception is the right one, since the orientation datum exists as independent source data exactly where the Clifford volume element is central, in odd dimension. The remaining finite-manifest work is purely data entry into a demonstrated procedure: the certified pipeline has been run end to end on reference stars, including detuned and misaligned controls, with an a priori error budget and zero certificate violations. The remaining problem is therefore no longer that the fermion operator is unknown. The finite microscopic manifest is an instantiation problem; the interesting outstanding foundational issue is the sequential sector, specifically how the record dynamics generates or selects the fermionic one-particle evolution across the typed gap named above, and how sequential opportunities attach to physical time. The audited source package still does not publish the microscopic glued-regulator map giving every physical edge its tangent and weight, so the paper does not claim the fully instantiated finite K_U or exact finite-regulator constants. That remaining debt is a finite manifest publication/execution, not an infrared principal-symbol debt.

The same discipline applies to the chiral completion. The source-reconstructed K_U, its covariant incidence B_U, and its positive roughness $B_U^\sharp B_U$ supply an explicit source-matched Wilson seed. The overlap/Ginsparg-Wilson construction can therefore be built without introducing a second gauge connection and already has the required local one-loop universality properties at the declared conditional grade. But the primitive kinetic action alone has not been shown to select that functional calculus uniquely.

Nor may sequential time be replaced by a fourth primitive graph direction merely to simplify the finite matrix. The complete physical fermion family must respect the ordered-interface ontology of emergent time.

The later record execution makes the sequential question sharper than the earlier ledger allowed, and it corrects it. The fact/no-fact classification is no longer a premise inherited from the current-channel labelling; the retained record decides the partition. The channel previously read as “no physical event” contributes to all 84 retained record classes and therefore cannot mean that. At the executed history-current grain the seven self-history marks create no new record and every nonself transition is record-producing, giving 7 no-record marks against 312 record-producing ones. The correct first-record hazard is accordingly $h_F = 1 - s$, with s the self-history survival probability, and its integrated sequential intensity is $\lambda_F = -\log s$. The resulting absorbing first-record semigroup is unique in its declared class on the one-survival-macrostate lumping, the earlier admissible continuum is a clock reparametrisation of it fixed by the survival datum, and the quasi-free CAR/Fock lift of a supplied one-particle transfer is unique. Record production is therefore no longer open, and sequential composition is closed within each typed sector; what remains between them is the record-to-fermion intertwining identification, an explicit map, not a composition rule. The exhaustion result is separately typed: $\rho(NR) = 0.200305886206958 < 1$ makes first entry almost sure with finite expected waiting count, but it is not the intensity, and only a state-independent no-record map would let the two be related by the scalar formula.

Permanent persistence remains a distinct question, and the paper can now say exactly why the existing data cannot settle it. On the append-only renewal branch $\Phi_R^*(P_a) = P_a$ and hence $\varepsilon_{leak} = 0$ exactly. But an equally admissible post-commitment overwrite extension gives $\varepsilon_{leak} = 1$, and convex combinations give every value between. So the unresolved question is not whether the record is initially produced, nor how sequential evolution composes, but which post-commitment operation the primitive source selects. No further leakage calculation can answer it; only a selection can.

A final compatibility debt also disappears. The declared completion tensors carry only the primitive direction $c_H = (1/2)(1, 1, 1, 1)$. Because the mixed-jet quotient acts solely on that leg, the external CAR/Fock structure cancels from the normalised residual, so the full third-order residual equals the already-measured primitive residual, 1.149×10^{-15}, for every declared nonzero charged or neutral completion tensor, while the six-defect-only residual is exactly unity. D-HM29 is therefore closed at the declared conditional source-stack grade, and the potential rows are shown to be load-bearing rather than cosmetic.

The remaining foundational question has consequently moved upstream again. It is no longer whether the declared record, completion and CAR/Fock laws are compatible with the full faithful action, since they are. It is whether the most primitive dynamics uniquely generates those laws rather than merely admitting them as compatible constructions.

The source-provenance chain can now be written more precisely:

$\Gamma\_kin^{(0)}$ → B_U → K_U^src → L_U = B_U^♯B_U → X_ρ^src → local admissible chiral family → Weyl one-loop universality.

The unresolved chain is shorter and now has two distinct kinds of link:

K=7 IR geometry → M_IR = I_3 → normalised spatial Dirac pole [CLOSED], while publish microscopic (n_e,w_e) → finite K_U and C_kin^exact, λ_max, d_+ [FINITE EXECUTION], then source-select the physical record frame → h_F → R(δ) = e^{δQ},

followed by the open typed identification

R(δ) ⇢ A_f(δ) [RECORD-TO-FERMION INTERTWINING, OPEN],

and then

source-select the post-commitment operation → ε_leak → attach sequential opportunities to physical duration → fermionic positivity → D_f^physical[U].

In parallel the foundational provenance branch is

deepest primitive dynamics ⟹? the declared record, gauge, completion and CAR/Fock source laws,

whose mixed-jet compatibility with the full action is now passed and whose unique primitive selection remains open. The first link of each chain is an execution, the selection links are named physical choices, and every link after them is now a theorem.

That is the correct post-MFKR status.

FINITE MASTER-ACTION KINETIC OPERATOR: EXACTLY RECONSTRUCTED AS A SOURCE FUNCTIONAL.

SAME PHYSICAL FERMION/GAUGE LINK: EXACT.

MINIMAL CLIFFORD-LINEAR EDGE FORM: CONDITIONAL EXACT.

STAR MOMENT SOURCE-IDENTITY CERTIFICATE: EXACT.

K=7 COARSE/INFRARED MOMENT SQUARE: SOURCE-DERIVED REFINEMENT-GEOMETRY PASS ON THE DECLARED D = 3 BRANCH, $M^TM = I_3$.

K=7 COARSE/INFRARED STAR MOMENT: M_IR = I_3 MODULO THE DECLARED COMMON SPIN-FRAME EQUIVALENCE; EXACTLY I_3 ON THE TANGENT-ALIGNED POSITIVE-WEIGHT BRANCH.

OLD ANTIPODAL INVOLUTION/TRANSLATION IDENTIFICATION: EXACTLY REJECTED, BUT NO LONGER LOAD-BEARING FOR THE INFRARED CLOSURE.

GLOBAL D = 3 SELECTION: INHERITED / FOUNDATIONAL; NOT RE-DERIVED HERE.

LAPLACIAN SQUARE RIGIDITY AND STABILITY OF THE MOMENT CLOSURE: EXACT, AND DIMENSION-GENERAL WITH THE ORIENTATION DATUM STRUCTURAL TO ODD DIMENSION.

FINITE-STAR CERTIFICATION PIPELINE: EXECUTED ON REFERENCE STARS WITH A PRIORI BUDGET AND ZERO CERTIFICATE VIOLATIONS.

NORMALISED SPATIAL DIRAC/Weyl PRINCIPAL SYMBOL: SOURCE-DERIVED INFRARED PASS AT THE REFINEMENT-GEOMETRY GRADE.

ε_even = 0 STRUCTURAL ON THE DECLARED ANTI-ADJOINT BRANCH, WITH THE PER-EDGE ANTI-ADJOINTNESS INPUT INHERITED, NOT DERIVED; BY THE EVEN-PART FIREWALL THIS INPUT DOES NOT GATE THE INFRARED PRINCIPAL SYMBOL; ε_Cliff = 0 ON THE CANONICAL BRANCH.

SOURCE-NATIVE KINETIC BOUND AND ITS SHARPNESS: EXACT ON BOTH CANONICAL REFERENCE CARRIERS, C_kin^exact = D_Γ = 4 AND C_kin^{3,canon} = D_Γ^{3,canon} = 3; PHYSICAL K=7 WEIGHTED DEGREE, C_kin AND SATURATION: FINITE-MANIFEST OUTPUTS, NOT YET INSTANTIATED.

RECORD/NO-RECORD EVENT SEMANTICS: EXECUTED PASS AT THE IMPLEMENTED F2F GRADE.

EARLIER ACCEPTED-CURRENT PERSISTENT-FACT PARTITION: REFUTED.

FIRST RECORD OCCURS ALMOST SURELY: PASS, ρ(NR) = 0.200305886206958.

FIRST-ENTRY RECORD COPY: PASS.

RECORD-DEFINED FIRST-ENTRY GENERATOR: UNIQUE IN THE DECLARED ABSORBING CLASS, ON THE ONE-SURVIVAL-MACROSTATE LUMPING.

QUASI-FREE CAR/FOCK LIFT OF A SUPPLIED ONE-PARTICLE TRANSFER: UNIQUE.

RECORD-SEMIGROUP TO FERMIONIC ONE-PARTICLE-TRANSFER IDENTIFICATION: OPEN; REQUIRES EXPLICIT INTERTWINING MAP.

APPEND-ONLY RENEWAL PERSISTENCE: ε_leak = 0 EXACTLY.

PHYSICAL POST-COMMITMENT PERSISTENCE FROM CURRENT SOURCE DATA: NON-IDENTIFIABLE, ε_leak ∈ [0, 1].

POST-COMMITMENT OPERATION SELECTION: OPEN.

CAR/FOCK FULL MIXED-TENSOR COMPLETION DESCENT: PASS, $\varepsilon^{(3)} = 1.149 \times 10^{-15}$.

D-HM29 FULL MIXED-TENSOR COMPATIBILITY: PASS AT DECLARED CONDITIONAL SOURCE-STACK GRADE.

PRIMITIVE-SOURCE UNIQUENESS OF THE DECLARED RECORD/COMPLETION LAWS: OPEN.

MICROSCOPIC PHYSICAL GEOMETRIC EDGE MAP: OPEN FOR FINITE-MATRIX / FINITE-CONSTANT INSTANTIATION ONLY; NOT AN IR PRINCIPAL-SYMBOL OBSTRUCTION.

PHYSICAL RECORD FRAME/HAZARD SELECTION: OPEN.

SEQUENTIAL-OPPORTUNITY TO PHYSICAL-TIME ATTACHING MAP: OPEN.

FERMIONIC TRANSFER POSITIVITY: OPEN.

FULL PHYSICAL SEQUENTIAL CHIRAL D_f[U] SOURCE IDENTITY: OPEN, WITH THE REMAINING OBSTRUCTIONS NOW EXPLICIT.

R1Q LOCAL ONE-LOOP FERMION UNIVERSALITY: CONDITIONAL PASS AT CONSTRUCTED SOURCE-MATCHED FREE-POLE GRADE.

D-SG6 LOCAL SOURCE-MATCHED CHIRAL REGULATOR: PASS.

D-SG6 COMPONENT-WISE GLOBAL CERTIFICATE: OPEN.

Appendix A: Direct Matrix Assembly Algorithm

For each fermion representation f:

1. Initialise the finite vertex matrix $K_f = 0$.
2. For every oriented edge $e: s \to t$, read Γ_e and $R_f(U_e)$ from the frozen manifest.
3. Add to the target diagonal block: $(K_f)_{tt} \mathrel{+}= \Gamma_e + \Gamma_e^\sharp$.
4. Add to the target-source block: $(K_f)_{ts} \mathrel{+}= -\Gamma_e R_f(U_e)$.
5. Add the adjoint source-target block: $(K_f)_{st} \mathrel{+}= -R_f(U_e)^\sharp \Gamma_e^\sharp$.
6. Repeat for all edges.

The resulting sparse matrix is K_U^{src}. The construction should be checked by direct equality with the Hessian of the quadratic source action.

Appendix B: Flat-Symbol Algebra

Starting from

$K(q) = \Gamma(1 - e^{-iq}) + \Gamma^\sharp(1 - e^{iq})$,

write $e^{\pm iq} = \cos q \pm i \sin q$. Then

$K(q) = (\Gamma + \Gamma^\sharp)(1 - \cos q) + i(\Gamma - \Gamma^\sharp) \sin q.$

With $\Gamma_\pm = (1/2)(\Gamma \pm \Gamma^\sharp)$, this becomes

$K(q) = 2\Gamma_+(1 - \cos q) + 2i\Gamma_- \sin q.$

Therefore

$K(q) = 2i\Gamma_- q + \Gamma_+ q^2 + O(q^3),$

and the source principal symbol is

$\sigma_1(K)(p) = 2i\Gamma_- p.$

At a corner component $q = \pi$: $1 - \cos \pi = 2$ and $\sin \pi = 0$, giving the contribution $4\Gamma_+$ per π-direction and 0 per 0-direction, which is Proposition 5.1.

Appendix C: Proof of the Kinetic Lipschitz Bound

Let $\Delta_e = R_f(U_e) - I$ with $\|\Delta_e\| \le \varepsilon_U$. The link-dependent off-diagonal block associated with $e: s \to t$ is $-\Gamma_e R_f(U_e)$. Relative to the flat link this changes by $-\Gamma_e \Delta_e$, with norm $\|\Gamma_e \Delta_e\| \le \|\Gamma_e\| \varepsilon_U$. The adjoint block has the same norm.

For one vertex v, the sum of norms of all changed blocks in its matrix row is therefore at most

$\varepsilon_U \Sigma_{e\ni v} \|\Gamma_e\|.$

The block Schur test for the self-adjoint canonicalised matrix gives

$\|\Delta K\| \le \varepsilon_U \max_v \Sigma_{e\ni v} \|\Gamma_e\|.$

Hence

$\boldsymbol{\|\Delta K\| \le D_\Gamma \varepsilon_U.}$

Appendix D: Proofs of the Factorisation, Refined Bound, and Sharpness

D.1 Factorisation

On the $\varepsilon_{even} = 0$ directional carrier with $\Gamma_\mu = (c_\mu/2)G_\mu^{Cliff}$, the direction-$\mu$ perturbation blocks are (ΔK)*{ts} = $-\Gamma_\mu \Delta_e$ and (ΔK)*{st} = $-\Delta_e^\sharp \Gamma_\mu^\sharp = +\Delta_e^\sharp \Gamma_\mu$. Collecting the direction-μ edges into the block-diagonal internal operator Z_μ and the unitary shift S_μ, and using $[\Gamma_\mu, Z_\mu] = 0$,

$\Delta K = \Sigma_\mu (-\Gamma_\mu Z_\mu S_\mu + S_\mu^\sharp Z_\mu^\sharp \Gamma_\mu) = -\Sigma_\mu \Gamma_\mu (Z_\mu S_\mu - S_\mu^\sharp Z_\mu^\sharp) = -\Sigma_\mu \Gamma_\mu N_\mu,$

with $N_\mu^\sharp = -N_\mu$ and $\|N_\mu\| \le \|Z_\mu S_\mu\| + \|S_\mu^\sharp Z_\mu^\sharp\| \le 2\varepsilon_U$.

D.2 Refined bound

Since the G_μ^{Cliff} pairwise anticommute, square to $-I$, and commute with every N_ν,

$(\Delta K)^2 = (1/4)\, \Sigma_{\mu\nu}\, c_\mu c_\nu G_\mu G_\nu N_\mu N_\nu = (1/4)[\, \Sigma_\mu c_\mu^2 (-N_\mu^2) + \Sigma_{\mu<\nu} c_\mu c_\nu G_\mu G_\nu [N_\mu, N_\nu]\,]$.

Each $-N_\mu^2$ is positive; each $G_\mu G_\nu$ is unitary. Taking norms yields Theorem 9. If all commutators vanish, $\|\Delta K\|^2 \le (1/4)\, \Sigma\, c_\mu^2 \|N_\mu\|^2 \le \varepsilon_U^2\, \Sigma\, c_\mu^2$, Corollary 9.1. The bounds $\|N_\mu^2\| \le 4\varepsilon_U^2$ and $\|[N_\mu, N_\nu]\| \le 8\varepsilon_U^2$ give $(\Sigma c_\mu)^2\, \varepsilon_U^2$, Corollary 9.2.

D.3 Constant-abelian value

For $\Delta_e = i\varepsilon$ on every edge (canonical weights), $N_\mu = i\varepsilon(S_\mu + S_\mu^\sharp)$, all mutually commuting, and in Fourier variables $\Delta K(q) = -\varepsilon\, \Sigma_\mu\, \alpha_\mu \cos q_\mu$ with $G_\mu = -i\alpha_\mu$. Its norm is $\varepsilon(\Sigma_\mu \cos^2 q_\mu)^{1/2}$, maximised at $q = 0$ with value $\sqrt{d}\, \varepsilon$, using $(\Sigma\alpha_\mu)^2 = d\cdot I$. This meets the Corollary 9.1 bound, so the commuting-class supremum is exactly $\sqrt{d}\, \varepsilon_U$.

D.4 Jordan-Wigner saturation witness

On the 2^4 periodic carrier, put $\Delta_e = i\varepsilon\, f_\mu(x)$ with $f_\mu(x) = (-1)^{x_1+\ldots+x_{\mu-1}}$ on the direction-μ edge leaving x. Within each direction-μ two-site fibre both oriented edges carry the same value $i\varepsilon f_\mu$, and the two-site computation gives the fibre operator $2i\varepsilon f_\mu \sigma_1$. The sign f_μ, read as a diagonal operator on the earlier coordinates, is $\sigma_3^{\otimes(\mu-1)}$. Hence

$N_\mu = 2i\varepsilon A_\mu$, $A_\mu = \sigma_3^{\otimes(\mu-1)} \otimes \sigma_1 \otimes I^{\otimes(4-\mu)}$,

a Jordan-Wigner family: Hermitian, unitary, pairwise anticommuting. Then

$\Delta K = -\varepsilon\, \Sigma_\mu\, \alpha_\mu \otimes A_\mu$, $(\Sigma_\mu\, \alpha_\mu \otimes A_\mu)^2 = 4\cdot I + 2\, \Sigma_{\mu<\nu} Q_{\mu\nu}$, $Q_{\mu\nu} = (\alpha_\mu \alpha_\nu) \otimes (A_\mu A_\nu)$.

Each $Q_{\mu\nu}$ is an involution (both factors square to $-I$), the six pairwise commute (transposing either pair produces the same sign in both tensor factors), and $Q_{13} = Q_{12}Q_{23}$, $Q_{24} = Q_{23}Q_{34}$, $Q_{14} = Q_{12}Q_{23}Q_{34}$, so the family is generated by the three independent commuting involutions Q_{12}, Q_{23}, Q_{34}. Their common $+1$ eigenspace in the 64-dimensional carrier has dimension 8, and on it $(\Delta K)^2 = \varepsilon^2(4 + 2\times 6) = 16\varepsilon^2$. Hence $\|\Delta K\| = 4\varepsilon$ exactly, with $\max_e \|\Delta_e\| = \varepsilon$ exactly, so $C_{kin}^{exact} \ge 4$. Theorem 8 gives $C_{kin}^{exact} \le D_\Gamma = 4$, so equality holds.

D.5 Unitary realisability

Replace $i\varepsilon f_\mu(x)$ by $e^{i\theta f_\mu(x)} - 1$ with genuine $U(1)$ links, so $\varepsilon_U = |e^{i\theta} - 1| = 2 \sin(\theta/2)$. Since $e^{i\theta f} - 1 = i\theta f + O(\theta^2)$, the perturbation reproduces the witness structure to leading order and $\|\Delta K\|/(D_\Gamma\, \varepsilon_U) \to 1$ as $\theta \to 0$. The witness ratio is therefore approached arbitrarily closely inside the strictly unitary background class.

Appendix E: Clifford-Linear Reduction, Corner Behaviour, and the Moment Certificate

E.1 Anti-adjointness and unit norm

With $G_i^\sharp = -G_i$ and $\{G_i, G_j\} = -2\delta_{ij} I$, for any unit tangent n,

$$G(n)^2 = \Sigma_{ij}\, n^i n^j G_i G_j = (1/2)\Sigma_{ij}\, n^i n^j \{G_i, G_j\} = -|n|^2 I = -I,$$

and $G(n)^\sharp = -G(n)$. Hence $G(n)^\sharp G(n) = I$, so $G(n)$ is $\sharp$-unitary with $\|G(n)\| = 1$, and

$$\Gamma_e = (w_e/2)G(n_e) \Rightarrow \Gamma_e^\sharp = -\Gamma_e \Rightarrow \Gamma_{e,+} = 0,\ \|\Gamma_e\| = w_e/2.$$

This gives Corollary 6.1 and Corollary 7.1 at once. On the canonical branch $n_i = \hat{e}_i$, $w_i = 1$ one has $\Gamma_i = (1/2)G_i$ and $2\Gamma_{i,-} = G_i$, which is Corollary 6.2.

E.2 The star symbol collapses to a sine sum

For a translation-invariant star with displacements a_e and anti-adjoint coefficients, the general expression of Section 9 gives

$$K_I(q) = \Sigma_e \left[\Gamma_e(1 - e^{-iq\cdot a_e}) + \Gamma_e^\sharp(1 - e^{iq\cdot a_e})\right] = \Sigma_e\, \Gamma_e(e^{iq\cdot a_e} - e^{-iq\cdot a_e}) = 2i\, \Sigma_e\, \Gamma_e \sin(q\cdot a_e),$$

that is $K_I(q) = i\, \Sigma_e\, w_e\, G(n_e) \sin(q\cdot a_e)$. Every even term is absent at every momentum, not only to leading order, so by Proposition 5.1 the symbol vanishes at all corners.

E.3 The moment certificate

Expanding $\sin(q\cdot a_e) = q\cdot a_e + O(|q|^3)$,

$$K_I(q) = i\, \Sigma_e\, w_e\, G(n_e)(q\cdot a_e) + O(|q|^3) = i\, \Sigma_i\, G_i \left[\, \Sigma_j\, M^{ij} q_j \,\right] + O(|q|^3),\quad M^{ij} = \Sigma_e\, w_e\, n_e^i a_e^j.$$

Since the G_i are linearly independent, the leading term equals the normalised symbol $i\Sigma_i G_i q_i$ for all q if and only if $M^{ij} = \delta^{ij}$, which is Theorem 7. If $n_e = a_e/|a_e|$ then $M^{ij} = \Sigma_e\, w_e|a_e|\, n_e^i n_e^j$ is manifestly symmetric, and the criterion is the discrete isotropy condition. A star with non-symmetric M cannot meet the criterion, since δ^{ij} is symmetric and no spin-frame change alters that.

Worked non-axis-aligned example: the four tetrahedral tangents $n \propto (1,1,1), (1,-1,-1), (-1,1,-1), (-1,-1,1)$ satisfy $\Sigma_e\, n_e n_e^T = (4/3)I$, so weights $w_e|a_e| = 3/4$ return $M = I$ exactly, and the resulting symbol matches $i\Sigma G_i q_i$.

E.4 Exact scalar square, Laplacian rigidity, and stability

For Proposition 7.3(i), $\sharp$-anti-adjointness of each $G(n_e)$ gives $K_I(q)^\sharp = K_I(q)$, and

$K_I(q)^2 = -\Sigma_{e,e'} w_e w_{e'} \sin(q\cdot a_e) \sin(q\cdot a_{e'}) G(n_e)G(n_{e'}) = -(1/2)\Sigma_{e,e'} w_e w_{e'} \sin(q\cdot a_e) \sin(q\cdot a_{e'}) \{G(n_e), G(n_{e'})\} = \Sigma_{e,e'} w_e w_{e'} (n_e\cdot n_{e'}) \sin(q\cdot a_e) \sin(q\cdot a_{e'}) I = |v(q)|^2 I$,

using $\{G(n), G(n')\} = -2(n\cdot n')I$. Expanding $\sin(q\cdot a_e) = q\cdot a_e + O(|q|^3)$ gives $v(q) = Mq + O(|q|^3)$, hence $K_I(q)^2 = q^T(M^TM)q \cdot I + O(|q|^4)$. The second-order coefficient tensor equals the unit Laplacian symbol for all q if and only if $M^TM = I$, which is (ii).

For (iii), write $\tilde{G}_j = \Sigma_i M^{ij}G_i$. *Then* $\{\tilde{G}_j, \tilde{G}_k\} = \Sigma_{i,i'} M^{ij}M^{i'k}\{G_i, G_{i'}\} = -2(M^TM)_{jk}I = -2\delta_{jk}I$, so the rotated triple is again a normalised anti-adjoint Clifford triple. For $M \in SO(3)$ the map $G_j \mapsto \tilde{G}_j$ is the spin representation of the rotation and is implemented by conjugation with an explicit spin element S; on the minimal carrier this is the standard two-to-one spin covering of the rotation group. For $\det M = -1$ no unitary conjugation exists: composing with a rotation reduces the question to $G_3 \mapsto -G_3$ with G_1 and G_2 fixed, and on the minimal carrier a unitary commuting with G_1 and G_2 is a multiple of the identity, which fixes G_3 rather than negating it. The reflection component is therefore exactly the declared orientation choice.

For (iv), tangent alignment makes $M = \Sigma_e w_e|a_e| n_e n_e^T$ a nonnegative combination of rank-one projectors, hence symmetric positive semidefinite. Orthogonality forces its eigenvalues into $\{-1, +1\}$; positivity leaves only $+1$; so $M = I$.

For Corollary 7.4, let $M = UP$ be the polar decomposition with U orthogonal and $P = (M^TM)^{1/2} \succeq 0$. Then $\mathrm{dist}(M, O(3)) \le \|M - U\| = \|P - I\|$, and for any symmetric positive semidefinite P with eigenvalues p_k,

$|p_k - 1| \le |p_k - 1|(p_k + 1) = |p_k^2 - 1|$,

so $\|P - I\| \le \|P^2 - I\| = \|M^TM - I\|$. On the tangent-aligned positive-weight branch M itself is symmetric positive semidefinite, and the same eigenvalue estimate refines to $|m_k - 1| = |m_k^2 - 1|/(m_k + 1) \le \|M^2 - I\|/(1 + \lambda_{\min}(M))$.

E.5 Dimension-general statements and the certification budget

Proposition 7.5(i) and (ii): the anticommutator collapse in E.4 uses only $\{G(n), G(n')\} = -2(n\cdot n')I$, which holds for normalised anti-adjoint generators in any dimension, so $K_I(q)^2 = \|v(q)\|^2 I$ and the forcing $M^TM = I_d \Leftrightarrow$ unit Laplacian are dimension-blind, as are the polar-decomposition stability bounds of Corollary 7.4 and the positive-semidefinite argument for (iii).

Proposition 7.5(iv): let $\omega = G_1G_2\cdots G_d$. The rotated generators $\tilde{G}_j = \Sigma_i M^{ij}G_i$ satisfy $\tilde{G}_1\cdots\tilde{G}_d = (\det M)\, \omega$ for any $M \in O(d)$. On the minimal carrier, ω commutes with every G_i when d is odd (moving G_i through the $d - 1$ other factors of ω contributes $(-1)^{d-1} = +1$), so ω is central and hence a scalar multiple of the identity by irreducibility. A unitary conjugation fixes every scalar, but a reflection sends ω to −ω; the two are incompatible, so no unitary implements a reflection in odd d. When d is even, ω anticommutes with each G_i and is traceless rather than scalar, the obstruction vanishes, and the product $U = G_1\cdots\hat{G}_j\cdots G_d$ of the other $d - 1$ generators explicitly implements the reflection of G_j: moving G_j through $d - 1$ factors gives $(-1)^{d-1} = -1$, while each G_i with $i \ne j$ passes through $d - 2$ sign flips and one self-commutation, giving $(-1)^{d-2} = +1$. The special orthogonal part is implemented in every dimension by the spin element of the rotation, $\exp((\theta/2)G_iG_j)$ for the plane rotations that generate it.

Corollary 10.2: the alternating-sign witness produces d direction operators whose pairwise anticommutation is the Jordan-Wigner relation in each coordinate plane; the norm additivity $\|\Delta K\| = d\cdot\varepsilon$ then follows exactly as in the $d = 3$ and $d = 4$ proofs of Appendix D, with the constant abelian case attaining the commuting-class ceiling $\sqrt{d} \cdot \varepsilon$.

Certification budget (Section 18.3): write $k(q) = \Sigma_m c_m \cos(q\cdot b_m)$ with $c_m = \pm(1/2) w_e w_{e'}(n_e\cdot n_{e'})$ and $b_m = a_e \mp a_{e'}$, a finite cosine polynomial. The symmetric second difference of a smooth function at step h differs from the exact second derivative by at most $(h^2/12)$ times a fourth directional derivative bound on the probe segment; for the mixed four-point stencil used in Section 18.3 the combined constant is at most $h^2/3$ times

$C_4 = \Sigma_m |c_m|\ \|b_m\|^4$,

since every fourth directional derivative of $\cos(q\cdot b_m)$ along a unit direction is bounded by $\|b_m\|^4$. The empirical worst case over 400 random stars at three scales reached 0.417 of this budget, confirming both validity and a safety margin of roughly 2.4.

E.6 Edge-coefficient classification on the minimal carrier

Proof of Proposition 6A. The minimal carrier is two-dimensional, so $\mathrm{End}(\mathbb{C}^2) = \mathrm{span}\{I, \sigma_1, \sigma_2, \sigma_3\}$ and every operator is scalar plus vector; there is no higher grade to exclude. Anti-adjointness is consumed as the inherited class definition, not derived (the demonstration that it cannot be derived belongs to Proposition 6B below). Within the anti-adjoint class, $\Gamma^\sharp = -\Gamma$ forces $\Gamma = i(bI + h\cdot\sigma)$ with $b \in \mathbb{R}$, $h \in \mathbb{R}^3$. Orientation oddness ties the coefficient data to the oriented tangent, and the antisymmetrised star assembly is $K(q) = 2i\Sigma_e \Gamma_e \sin(q\cdot a_e)$, so both anti-adjoint contributions enter through sine modes and the symbol is self-adjoint,

$K(q) = -b(q) I - v(q)\cdot\sigma$, $b(q) = 2\Sigma_e b_e \sin(q\cdot a_e)$, $v(q) = 2\Sigma_e h_e \sin(q\cdot a_e)$,

with v expressed through the not-yet-aligned vector parts h_e. Direct squaring gives

$K(q)^2 = (b(q)^2 + |v(q)|^2) I + 2\, b(q)\, (v(q)\cdot\sigma)$,

whose traceless part vanishes iff $b(q)\, v(q) = 0$. Both are trigonometric polynomials; on any star where an edge's sine mode is linearly independent of the others (a locality-admissible probe, since edge coefficients may not depend on which other edges are present), $b(q)\, v(q) \equiv 0$ with $v \not\equiv 0$ forces $b(q) \equiv 0$ and hence $b_e = 0$ edge by edge. Spin covariance $S\, G(n)\, S^\sharp = G(Rn)$ plus locality (the coefficient depends only on the edge's own geometric data, whose only vector is n_e) then forces $h_e = (w_e/2)\, n_e$ with $w_e = 2|h_e| > 0$ and the sign carried by the orientation datum; only at this point does $v(q)$ collapse to the familiar $\Sigma_e w_e \sin(q\cdot a_e)\, n_e$. Hence $\Gamma_e = (w_e/2)\, G(n_e)$ within the inherited anti-adjoint class.

Proof of Proposition 6B. Now drop the anti-adjointness restriction and allow the separate adjoint-even vector contamination $i\, \eta_e\, G(n_e)$; spin covariance aligns the even vector part with n_e exactly as it aligns the odd part, and the identity direction is excluded for both parities by the cross-term argument above, since an identity mode of either parity produces a first-grade cross term with the vector part. This contamination is exactly why anti-adjointness cannot be derived from the square condition: its antisymmetrised assembly contributes $2\eta_e(1 - \cos(q\cdot a_e))$ modes, not sine modes, so the assembled symbol is $i\,G(B(q))$ with $B(q) = \Sigma_e\,[w_e \sin(q\cdot a_e) + 2\eta_e(1 - \cos(q\cdot a_e))]\,n_e$, still a pure Clifford vector, and $K(q)^2 = \|B(q)\|^2\,I$ at every momentum and every η; verified numerically to 4.6×10^{-16} at $\eta = 0.3$ with even-part norm 0.42. Expanding, $\sin(q\cdot a) = q\cdot a + O(|q|^3)$ while $1 - \cos(q\cdot a) = (q\cdot a)^2/2 + O(|q|^4)$, so $B(q) = Mq + O(q^2)$ with $M = \Sigma_e\, w_e\, n_e\, a_e^T$ built from the odd data alone, and $K(q)^2 = q^T(M^TM)q \cdot I + O(q^3)$. The linear moment is untouched by the contamination, and every statement of Corollary 7.2 that consumes only the second-order coefficient tensor is η-independent.

Appendix F: The Sequential Semigroup and the Uniqueness of the Quasi-Free Fock Lift

F.1 Uniqueness of the Fock lift

Let A act on the one-particle carrier $\mathcal{H}$. Suppose $\mathfrak{A}$ on the antisymmetric Fock space preserves the vacuum, preserves particle number on the kinetic sector, acts quasi-freely on the CAR generators (in the exterior second-quantised sense of Theorem 15's terminology note, not as a CAR $*$-automorphism when A is non-unitary), and satisfies the intertwining relation $\mathfrak{A}\, a^\sharp(\psi) = a^\sharp(A\psi)\,\mathfrak{A}$ on one-particle creation operators, which needs no invertibility of A. Writing an N-particle state as $a^\sharp(\psi_1)\cdots a^\sharp(\psi_N)\Omega$ and moving $\mathfrak{A}$ through each creation operator by the intertwining relation, vacuum preservation gives

$$\mathfrak{A}\,(\psi_1 \wedge \ldots \wedge \psi_N) = A\psi_1 \wedge A\psi_2 \wedge \ldots \wedge A\psi_N,$$

which is $\wedge^N A$ on each sector. Number preservation forbids mixing sectors, so $\mathfrak{A} = \bigoplus_N \wedge^N A = \Gamma(A)$ with no residual freedom. This is Theorem 15. Multiplicativity $\Gamma(AB) = \Gamma(A)\Gamma(B)$ follows from the same sector formula.

If $A = e^{-a_t\Omega}$ with Ω self-adjoint, then $\Gamma(A)$ is the one-parameter group generated by the second quantisation $d\Gamma(\Omega)$, giving $\Gamma(e^{-a_t\Omega}) = e^{-a_t\, d\Gamma(\Omega)}$ and hence

$$\mathbb{T}_f = \Gamma(A_f) \otimes \Gamma(A_{f^c}) = \exp\{-a_t[d\Gamma(\Omega_f) + d\Gamma(\Omega_{f^c})]\}.$$

For Corollary 15.1, a strictly positive self-adjoint A has a unique self-adjoint logarithm by the spectral theorem, so $\Omega = -(1/a_t)\operatorname{Log} A$ is the only self-adjoint generator at that step.

Nothing in this argument selects A itself, which is the content of Corollary 15.2; the selection is supplied instead by the marked law treated next.

F.2 The record-defined generator, its uniqueness, and the clock reparametrisation

Generator legitimacy. With $\lambda_F = -\log(1 - h_F) > 0$ for $0 < h_F < 1$, the matrix Q has row sum $-\lambda_F + \lambda_F \Sigma_a q_a = 0$ and nonnegative off-diagonal entries $\lambda_F q_a$, so it generates a Markov semigroup. Terminal rows vanish, so first-record terminals are absorbing.

Closed form. On the survival state, $d/d\delta\, R_{00} = -\lambda_F R_{00}$ gives $R_{00}(\delta) = e^{-\lambda_F \delta}$, and

$R_{0a}(\delta) = \int_0^\delta e^{-\lambda_F u} \lambda_F q_a\, du = (1 - e^{-\lambda_F \delta})q_a$.

At $\delta = 1$ this returns $R_{00}(1) = e^{-\lambda_F} = s = 1 - h_F$ and $R_{0a}(1) = h_F q_a$, that is $R(1) = P_{rec}$. Composition and $R(0) = I$ are automatic for a matrix exponential, and $Q = \partial R(\delta)/\partial\delta|_{\delta=0}$, so the generator is derived rather than posited.

Uniqueness in the class. Let G be any generator with one survival state, absorbing terminals, time-homogeneous composition and a constant terminal conditional law, so $G_{00} = -a$ and $G_{0a} = a\, p_a$ for some $a > 0$ and probability vector p. Then e^G has survival entry e^{-a} and terminal entries $(1 - e^{-a})p_a$. Matching the supplied law forces $e^{-a} = s$, whose unique positive root is $a = \lambda_F$, and then $(1 - e^{-\lambda_F})p_a = h_F q_a$ forces $p = q$. Hence $G = Q$.

Embeddability is a real hypothesis. Not every Markov matrix has a real generator: for the two-state exchange matrix with off-diagonal probability $p > 1/2$ the determinant is $1 - 2p < 0$, while any real generator gives $\det e^G = e^{\mathrm{tr}\, G} > 0$. The record law is embeddable because of its absorbing first-record structure, and the theorem should be read as a statement about that structure rather than as a general fact.

Relation to the admissible family. Let J denote the jump matrix, $J_{0a} = q_a$, $J_{00} = 0$, $J_{aa} = 1$. Then

$(I - P_{rec})_{00} = h_F$, $(I - P_{rec})_{0a} = -h_F q_a$, terminal rows zero,

so $I - P_{rec} = h_F(I - J)$, while $Q = -\lambda_F(I - J)$. Therefore

$T_\sigma = \exp[-\sigma(I - P_{rec})] = \exp[-\sigma h_F(I - J)] = R(\sigma h_F/\lambda_F)$,

so the whole family is one semigroup in a rescaled clock, and $\sigma\bigstar = \lambda_F/h_F$ returns $T_{\sigma\bigstar} = P_{rec}$. The record-defined survival probability fixes the clock; nothing else in the family is dynamically distinct on this carrier.

Exhaustion and its typing. For the no-record map NR with $\rho(NR) < 1$, $\Sigma_k NR^k$ converges and the expected first-record waiting count is $(I - NR)^{-1}\mathbb{1}$, finite at every internal state, so first entry is almost sure. The stationary mean is $\pi(I - NR)^{-1}\mathbb{1}$. The scalar identity $\pi(I - NR)^{-1}\mathbb{1} = 1/(1 - \rho)$ holds when the no-record map has state-independent survival and fails otherwise, so $\rho(NR)$ may not be substituted for s in $\lambda_F = -\log s$.

F.3 The post-commitment persistence interval

With $P_a = I_A \otimes |a\rangle\langle a|$ and $\Phi_R = \Psi_A \otimes id_L$ for a trace-preserving Ψ_A, the Heisenberg adjoint is unital on the active factor, so $\Phi_R^*(P_a) = \Psi_A^*(I_A) \otimes |a\rangle\langle a| = P_a$ and every cross-record leakage vanishes. For the overwrite channel $\Phi_O(\rho) = (I_A \otimes U_{ab})\rho(I_A \otimes U_{ab}^\sharp)$ with U_{ab} interchanging the ledger basis vectors of $a \neq b$, $\Phi_O^*(P_b) = P_a$, giving $\varepsilon_{a\to b} = 1$. Both are completely positive and trace preserving, and the set of such channels is convex, so $\Phi_p = (1 - p)\Phi_R + p\Phi_O$ is admissible for every $p \in [0, 1]$ and returns $\varepsilon_{a\to b} = p$ by linearity of the Heisenberg adjoint. Hence the whole interval is realised by admissible post-commitment extensions of the same pre-commitment data, which is the identifiability statement of Proposition 14.3.

F.4 Tensor-factor reduction of the completion residual

Let $T = c \otimes B$ with $c \neq 0$ in the mode-1 factor, and let P act on mode 1 alone. Then $T \times_1 P = (Pc) \otimes B$ and $T - T \times_1 P = [(I - P)c] \otimes B$. Multiplicativity of the Frobenius norm on tensor products, $\|a \otimes B\|_F = \|a\| \, \|B\|_F$, gives

$$\varepsilon^{(3)} = \|(I - P)c\| \, \|B\|_F / (\|c\| \, \|B\|_F) = \|(I - P)c\| / \|c\|,$$

independently of B. Applied to the declared completion tensors with $c = c_H$, this is Corollary 17.1: the external CAR/Fock structure cancels exactly, so the full mixed-tensor residual equals the primitive-leg residual for every declared nonzero completion tensor, and the defect-only quotient, which annihilates c_H, returns exactly 1.

Appendix G: Required Reproducibility Returns

A numerical source-identity execution should publish at minimum $N_V = |V_n|$, $N_E = |E_n|$, the ordered source and target arrays $s: E_n \to V_n$ and $t: E_n \to V_n$, the edge tangents n_e and scalar weights w_e with their orientation convention (from which $\Gamma_e = (w_e/2)G(n_e)$ follows), the edge displacements a_e, the representation matrices $R_f(U_e)$, and the positive carrier metrics used to define every norm. If any edge coefficient falls outside the Clifford-linear class, that coefficient must be published as a matrix together with a statement of why the minimal realisation does not apply.

It should then report:

$$\varepsilon_{Hess} = \|K_{assembled} - K_{direct\ Hessian}\| / \|K_{direct\ Hessian}\|,$$

$$\varepsilon_{gauge} = \max_g \|K[U^g] - G\, K[U]\, G^{-1}\| / \|K[U]\|,$$

the star moment tensor M^{ij} and its residual $\varepsilon_{moment} = \|M - I\|$, ε_{Cliff}, ε_{even}, D_Γ, C_{kin}^{exact}, λ_{max}, d_+,

together with the commutator diagnostics $\|[N_\mu, N_\nu]\|$ of the declared background class where the directional factorisation applies. For the sequential sector it should publish the small-step family $R_n(\delta)$, the verified semigroup residual $\|R_n(\delta_1)R_n(\delta_2) - R_n(\delta_1+\delta_2)\|$, the extracted generator, and the fermionic positivity certificate used.

The calculation should be run before comparison with any Standard Model observable.

Appendix H: Numerical Verification of This Paper's Exact Claims

All checks below were executed with explicit matrices; residuals are relative spectral norms.

1. Assembly identity (Theorem 1, Appendix A): on a random 6-vertex, 14-edge oriented multigraph with random 4×4 spinor coefficients and random SU(2) links, the block-assembled matrix equals $T^\sharp\Gamma_E B_U + (T^\sharp\Gamma_E B_U)^\sharp$ with residual 9.5×10^{-17}, and the quadratic form of the assembled matrix equals the primitive bilinear on random field configurations to the same precision.
2. Gauge covariance (Theorem 2): under random local SU(2) transformations, $\|K_{U^g} - G K_U G^{-1}\|/\|K_U\| = 1.8\times10^{-16}$.
3. Perturbation identity (Section 15): the exact expression for $K_U - K_I$ holds with residual 1.1×10^{-16}, and the Schur bound $\|K_U - K_I\| \le D_\Gamma \varepsilon_U$ holds on the random-graph background.
4. Flat symbol (Sections 9 to 11): the even/odd decomposition and the canonical sine-stencil recovery hold exactly at random momenta; the corner census of Proposition 5.1 holds at all sixteen corners, including with an injected even part.
5. Factorisation (Theorem 9): on a 2^4 carrier with random U(1) perturbations of size $\varepsilon = 0.1$, the spectrum of the assembled ΔK matches the Clifford-diagonal form $-\Sigma\, \Gamma_\mu N_\mu$ to 4.4×10^{-16}; the commutator-refined bound returned 0.3072, strictly inside the Schur value 0.4000, and above the observed norm 0.2198.
6. Constant-abelian value (Corollary 9.1, Theorem 10(i)): $\Delta_e = i\varepsilon$ on a 4^4 carrier returned $\|\Delta K\| = 0.20000000 = \sqrt{4} \cdot \varepsilon$ at $\varepsilon = 0.1$.
7. Sharpness witness (Theorem 10(ii)): the alternating-sign 2^4 witness returned $\|\Delta K\|/(D_\Gamma \varepsilon_U) = 1.0000000000$.
8. Unitary saturation (Theorem 10(iii)): genuine U(1) links with $\theta = 0.5, 0.1, 0.02$ returned ratios 0.9689, 0.9988, 0.99995.
9. Clifford-linear reduction (Theorem 6, Corollaries 6.1 and 6.2): for random unit tangents and weights, $G(n)^\sharp = -G(n)$ and $G(n)^2 = -I$ to 10^{-14}, $\|\Gamma_e\| = w_e/2$ exactly, the even part vanishes identically on the anti-adjoint branch, and the canonical branch returns $\varepsilon_{Cliff} = \varepsilon_{even} = 0$ exactly.
10. Moment certificate (Theorem 7): the tetrahedral star with $w_e|a_e| = 3/4$ returns $M = I$ exactly and reproduces the normalised symbol with relative residual $\sim2\times10^{-9}$ at $|q| \sim 10^{-4}$ (consistent with the $O(|q|^2)$ truncation); scaling the weights by 0.9 produces exactly the predicted 10% symbol error, confirming the criterion is necessary as well as sufficient.
11. K=7 infrared moment closure (Corollary 7.2): the proof no longer treats the old antipodal-involution derivative descent as a hypothesis. On the declared $D = 3$ refinement-geometry branch, the substrate-derived orthonormal spatial frame gives second-order principal symbol $\Sigma_i\, p_i^2$; combining this with the exact Clifford-square identity gives $M^TM = I_3$, orientation-preserving refinement removes the

reflection component, and the remaining SO(3) freedom is exactly the declared common spin-frame equivalence, so the canonical refinement-frame representative returns $M\_IR^T M\_IR = I_3$, ε_moment^IR = 0 and $K\_IR^2 = |p|^2 I$, with $M = I_3$ outright on the tangent-aligned positive-weight branch. The calculation is structural rather than fitted to a fermion target. The old involution/translation incompatibility remains independently verified in item 29 but is no longer a hypothesis of this closure.

12. Fock lift (Theorem 15, Corollary 15.1): on a three-mode antisymmetric Fock space, ⨁_N ∧^N e^{−a_tΩ} equals e^{−a_t dΓ(Ω)} with residual 9.5×10^{-17}; multiplicativity Γ(AB) = Γ(A)Γ(B) holds to 9.5×10^{-17}; the vacuum is preserved, the one-particle block reproduces A to 3×10^{-17}, and Ω is recovered from −(1/a_t)Log A to 1.2×10^{-15}.
13. Class-scoped non-uniqueness (Theorem 13): for a reversible five-state base transfer, every member of T_s = exp[−s(I − T_OS)] with s ∈ {0.25, 0.5, 1, 2, 4} was verified to be Markov normalised, positive, reversible with respect to the same stationary state, while returning strictly different spectral gaps (0.204, 0.367, 0.599, 0.839, 0.974). The continuum is genuine, not an artefact of relaxing a stated property.
14. Absorbing first-record generator (Theorem 14): over random (h_F, q) with h_F ∈ (0.05, 0.9) and two to four terminal records, Q has exact zero row sums and nonnegative off-diagonal entries, ||e^Q − P_rec|| ≤ 7×10^{-16}, the closed forms for $R_{00}(\delta)$ and $R_{0a}(\delta)$ hold to 10^{-13} at δ ∈ {0.13, 0.5, 1, 2.7}, composition $R(\delta_1+\delta_2) = R(\delta_1)R(\delta_2)$ holds to 10^{-13}, and the numerical derivative at zero returns Q. The principal real matrix logarithm of P_rec coincides with Q to 4×10^{-16}.
15. Clock reparametrisation (Corollary 14.1): I − P_rec = h_F(I − J) holds exactly, T_σ = R(σh_F/λ_F) holds to 10^{-13} across σ ∈ {0.3, 1, 4}, and σ★ = λ_F/h_F returns T_{σ★} = P_rec to 10^{-16} on every random instance tested.
16. Embeddability caveat: the two-state exchange matrix with p = 0.7 and p = 0.9 has determinant 1 − 2p < 0 and therefore admits no real generator, confirming that the embedding in Theorem 14 is a consequence of the absorbing first-record structure rather than a general property of Markov matrices.
17. Uniqueness sensitivity (Theorem 14): the survival equation e^{−λ_F} = s has a single positive root, recovered to twelve digits, and perturbing λ_F or the q_a by a few percent breaks the one-opportunity match at the percent level, so both supplied data are genuinely constraining.
18. Exhaustion typing (Theorem 14, executed return): for a substochastic no-record map with ρ < 1 the series Σ_k NR^k converges and $(I - NR)^{-1}\mathbb{1}$ is finite at every state. With state-independent survival the identity E_π[N] = 1/(1 − ρ) holds exactly (checked at ρ = 0.200305886206958, giving 1.250478130015093); with state-dependent survival the two quantities differ, which is consistent with the executed pair (0.200305886206958, 1.234602249934960) and confirms that ρ(NR) is not the intensity.
19. Persistence interval (Proposition 14.3): for random trace-preserving Ψ_A on a three-dimensional active carrier and a four-class ledger, ||Φ_R^∗(P_a) − P_a|| ≤ 2.7×10^{-15} for every class; the overwrite channel returns ε_{a→b} = 1 exactly, the

renewal branch 0 exactly, and the convex family Φ_p returns $\varepsilon_{a\to b} = p$ at $p = 0$, 0.25, 0.5, 0.75, 1 to twelve digits, so the full interval is realised.

20. Completion descent (Corollary 17.1): with $c_H = (1/2)(1,1,1,1)$ and random external factors B_f of varying shape, the normalised mode-1 residual of $T_f = c_H \otimes B_f$ equals the primitive residual $\|(I - P)c_H\|/\|c_H\|$ to 10^{-12} independently of B_f, returning machine zero against the full-action quotient and exactly 1 against the defect-only quotient.
21. Admissibility radius (Theorem 12): the closed form ε_{src} solves the defining quadratic identically, and with $D_\Gamma = 4$, $\lambda_{max} = 16$, $d_+ = 4$, $\delta = \min(r, 1)$ it reduces exactly to the CWGW safe-radius expression, returning 0.082207 at the illustration point $r = 1$ ($r = 1$ is an illustration, not a VERSF prediction).
22. Exact scalar square (Proposition 7.3(i)): on a random eight-edge Clifford-linear star with random unit tangents, displacements and positive weights, $K_I(q)^2 = |v(q)|^2 I$ holds at twenty random momenta with residual 1.8×10^{-15}; the small-momentum quadratic form equals $q^T(M^TM)q$ to 3×10^{-10} under 10^{-5} finite differencing; and the axis star returns $K^2 = (\Sigma_i \sin^2 q_i)I$ exactly, fixing the positive sign of the scalar square.
23. Laplacian rigidity (Proposition 7.3(ii) and (iii)): the rotated star with $n_e = R\hat{e}_e$, $a_e = \hat{e}_e$ for a random rotation R returns $\|M^TM - I\| = 2.2\times10^{-16}$ while $\|M - I\| = 0.686$, so the Laplacian test alone does not force the identity off the tangent-aligned branch; the rotated generators satisfy the Clifford algebra to 4.6×10^{-16} and equal $S G_j S^\sharp$ for the explicit spin element of the same rotation to 2.3×10^{-16}, confirming that the residual freedom is the declared spin-frame rotation.
24. Stability (Corollary 7.4): over 2000 random symmetric positive semidefinite moment tensors, $\|M - I\| \le \|M^2 - I\|/(1 + \lambda_{min}(M))$ held with zero violations at machine precision; over 2000 random general 3×3 tensors, the polar distance to the orthogonal set obeyed $\|P - I\| \le \|M^TM - I\|$ with zero violations.
25. Three-direction sharpness (Corollary 10.1): the alternating-sign 2^3 Jordan-Wigner witness with $\|\Delta_e\| = \varepsilon$ returned $\|\Delta K\|/(3\varepsilon) = 1.0000000000000002$; genuine U(1) links with the same sign pattern at $\theta = 0.02$ returned ratio 0.99995; and the constant abelian perturbation (the uniform one) returned exactly $\sqrt{3}\cdot\varepsilon$, attaining the commuting-class ceiling of Corollary 9.1 in three directions.
26. Executed certification pipeline (Section 18.3): on the axis, tetrahedral, detuned and rotated reference stars at five probe scales $h = 0.2$ to 0.0125, the budgeted certificate $\|M - I\| \le \|Q_h - I\| + C_4h^2/3$ (or the O(3)-distance version off the aligned branch) held with zero violations; the exact stars converged quadratically to certified bounds 1.46×10^{-3} and 1.53×10^{-3} at the finest scale; the detuned star's certified bound converged to 0.104 above its true error 0.05; and the budget lemma was validated over 400 random stars at three scales with worst ratio 0.417.
27. Dimension-general rigidity (Proposition 7.5): explicit Clifford carriers at $d = 2, 4$ (carrier $\mathbb{C}^4$) and $d = 5$ returned the exact scalar square on random seven-edge stars to 3.6×10^{-15}; rotated generator triples satisfied the Clifford algebra to 1.8×10^{-15} and equalled the conjugation by $\exp((\theta/2)G_1G_2)$ to 2.2×10^{-16}; the volume element was scalar at $d = 5$ and traceless at $d = 2, 4$; and the product of the remaining generators

negated the last generator exactly at $d = 2, 4$ while fixing it at $d = 5$, confirming the parity of the obstruction.

28. Dimension-general sharpness (Corollary 10.2): the alternating-sign witness on the 2^2 and 2^4 carriers returned $\|\Delta K\|/(d\cdot\varepsilon) = 1.0000000000000007$ and 1.0000000000000002 respectively, with the uniform constant-abelian background attaining its class ceiling $\sqrt{d} \cdot \varepsilon$ in both cases.
29. Involution-translation incompatibility (Section 14.3 source note): for random 6×6 involutions at $\xi = 0.1, 0.01, 0.001$ the difference quotient $D = (T - I)/\xi$ returned spectrum concentrated at exactly 0 and $-2/\xi$ (to 2×10^{-13}) and the identity $2D + \xi D^2 = 0$ to 1.3×10^{-12}; by contrast the genuine cyclic shift on 64 sites satisfies $S^2 \neq I$ and returns the standard symmetric-difference and discrete-Laplacian symbols. This confirms the historical source-audit result that the old antipodal involution cannot itself be the infinitesimal translation; the refinement-transport/soldered-frame closure of Corollary 7.2 does not make that identification and is therefore unaffected by the no-go.
30. Edge-coefficient classification (Proposition 6A): contaminating one edge of a random six-edge star with an identity component $i\, b_0\, I$ breaks the exact scalar square, with non-scalar residual equal to the predicted cross term $2|b(q)||v(q)|$ to machine precision (residual 3.1×10^{-1} at $b_0 = 0.05$ and 1.13 at $b_0 = 0.2$, against 4.7×10^{-16} at $b_0 = 0$), confirming that the graph-Dirac square condition excludes identity-direction contamination edge by edge.
31. Anti-adjointness is not square-derivable (Proposition 6A scope): the adjoint-even contamination $\Gamma_i = (1/2)G_i + i\eta G_i$ at $\eta = 0.3$ assembles to $K(q) = \Sigma[\sin q_i + 2\eta(1 - \cos q_i)]\sigma_i$ with even-part norm 0.42, yet $K(q)^2$ is scalar to 4.6×10^{-16} at random momenta, and the even term enters the symbol at $O(q^2)$ (measured quadratic coefficient of order η). The square condition, locality and spin covariance therefore cannot exclude the adjoint-even Clifford-vector direction, confirming that per-edge anti-adjointness is an inherited input.
32. Even-part firewall (Proposition 6B): on a random six-edge star with independent per-edge contaminations $\eta_e \in [-0.4, 0.4]$, the assembled symbol kept the exact scalar square (9.4×10^{-16} at random momenta), the measured second-order coefficient of K^2 equalled $M^T M$ built from the adjoint-odd data alone to 1.4×10^{-7} under 10^{-4} differencing, and the first-order symbol equalled $iG(Mq)$ with the contamination entering only at $O(q^2)$ (quadratic residual coefficient of order one). The moment tensor and the $M^T M = I$ forcing are therefore η-blind, as the firewall states.

Appendix I: Remaining Execution

The following are discharged by MFKR-1 at their declared grades:

exact finite incidence reconstruction of B_U; exact finite source-functional reconstruction of K_U; exact sparse block assembly; same-link gauge covariance; no-independent-matter-normalisation theorem; exact flat even/odd edge-Clifford decomposition; exact principal-symbol identity criterion; exact canonical-stencil identity criterion; exact corner census;

source-native weighted Clifford degree D_Γ; source-native bound $C_{kin} \le D_\Gamma$; the Clifford-diagonal factorisation and commutator-refined bound; the exact commuting-class quadrature value; the sharpness theorem $C_{kin}^{exact} = D_\Gamma = 4$ with explicit witness; recovery of the previous Euclidean-reference $C_{kin} \le 4$ as an exact value; deterministic finite definition of the exact physical C_{kin}; manifest-level same-carrier admissibility-radius formula with its commuting-class refinement.

To this are added the Clifford-linear edge reduction with its even residual vanishing structurally on the inherited anti-adjoint branch and its scalar weighted degree; the star moment certificate; the K=7 coarse/infrared execution $M^T M = I_3$ with $M_{IR} = I_3$ modulo the declared spin-frame equivalence at the source-derived refinement-geometry grade, outright on the tangent-aligned positive-weight branch; the exact scalar square identity and Laplacian square rigidity of the moment tensor, with its quantitative stability at constant one and its dimension-general form with the odd-dimensional orientation obstruction; the executed finite-star certification pipeline with its a priori differencing budget; the source-derived normalised spatial Dirac/Weyl principal symbol, with the canonical weighted degree $D_\Gamma^{3,canon} = 3$ and its sharpness witness exact in every dimension; the uniqueness of the quasi-free Fock lift and of the generator of a supplied transfer; the exact localisation of the residual sequential freedom to the one-particle selection; the record-defined sequential generator with its uniqueness in the absorbing first-record class; the identification of the earlier admissible continuum as a clock normalisation fixed by the survival datum; the correction of the first-record intensity to $-\log(1 - h_F)$; the executed record/no-record classification and the refutation of the earlier accepted-current partition; the 7 self-history versus 312 record-producing mark classification; first-entry exhaustion at $\rho(NR) = 0.200305886206958$ with exact first-entry record copy; exact $\varepsilon_{leak} = 0$ on the record-preserving renewal branch together with the exact non-identifiability interval [0, 1] before operation selection; and the full mixed-tensor CAR/Fock completion residual reduction with its D-HM29 pass on the declared conditional source stack.

The remaining strict execution is:

1. publish or derive the glued-regulator geometric map $e \mapsto (n_e, w_e)$ with orientation on the microscopic physical fermionic regulator, solely for finite-matrix and finite-constant instantiation;
2. evaluate the physical finite-star moment tensor as a regulator-level reproducibility diagnostic and verify its convergence to the source-derived infrared equivalence class $M^T M = I_3$, and to $M = I_3$ on the tangent-aligned positive-weight branch, using the Section 18.3 budgeted certification pipeline ($\|M - I\| \le \|Q_h - I\| + C_4 h^2/3$); the pipeline itself is executed and validated on reference stars, so only the physical (n_e, w_e) data remains;
3. verify that no microscopic physical edge falls outside the minimal Clifford-linear class, or publish the exception explicitly;
4. derive per-edge anti-adjointness $\Gamma_e^\sharp = -\Gamma_e$ (equivalently the microscopic oddness principle) from the primitive source, or identify the prior VERSF theorem that proves it, so that the inherited input of Proposition 6A and the structural ε_{even}

= 0 of Corollary 6.1 become source-derived rather than declared; by Proposition 6B this item gates the exact sine stencil and the Wilson-only doubler claim, not the infrared principal-symbol closure;

5. calculate C_{kin}^{exact} on the physical regulator, together with the commutator diagnostics of the physically relevant background class, and determine whether the finite physical K=7 carrier actually attains the canonical three-direction value 3;
6. calculate the same-carrier λ_{max}, d_+ and flat chiral gap;
7. instantiate the physical safe radius without carrier mixing;
8. source-select the physical record frame or endpoint witness on which h_F is evaluated;
9. execute the physical terminal record quotient and readout where required to promote implemented ledger labels to physical equivalence classes;
10. construct the record-to-fermion intertwining map: define or derive the arrow $R(\delta) \to A_f(\delta)$ identifying, or selecting from, the classical record semigroup a fermionic one-particle transfer on the Hilbert carrier; Theorems 14 and 15 close their own typed sectors, and this map is the missing middle arrow of the sequential chain;
11. resolve the survival-lumping question: either prove that the F2F coarse-graining is strongly lumpable onto one survival macrostate, certifying the scalar (s, q_a) generator of Theorem 14 as exact rather than stationary-averaged, or construct the full block first-entry transfer $P = [[NR, B], [0, I]]$ on the transient no-record space joined to the 84 absorbing record classes and test whether it admits a continuous-time generator uniquely selected by the source law, the stronger closure test;
12. derive the post-commitment atomic operation selector from the primitive action, distinguishing retain-final, renew-only, copy, proliferation, erase/reset and overwrite;
13. once that selector is returned, issue the physical persistence result; the renewal branch already has $\varepsilon_{leak} = 0$;
14. derive the sequential-opportunity to physical-time attaching map, so that δ acquires a duration and λ_F a dimensionful rate;
15. establish fermionic transfer positivity by a real nonnegative fermion determinant or a reflection/transfer positivity certificate, since bosonic cone positivity does not suffice;
16. establish primitive-source uniqueness: show whether the deepest dynamics uniquely generates the declared record action, representation ledger and CAR/Fock completion amplitudes;
17. prove exact identity with, or an admissible source-selected deformation into, the certified chiral family;
18. then proceed to the component-wise global gap/index theorem.

The previous requirement to construct new oriented shifts S_i from the antipodal involutions is removed from this list. The involutions remain axis/opposition data only; the later refinement-transport programme supplies the genuine oriented transport independently.

Internal VERSF Sources Consumed

1. The VERSF Master Substrate Action and Constitutive Programme: finite regulator, primitive shared-link CAR/Fock kinetic action, no independent matter-link normalisation, terminal matter-address and kinetic-normalisation structure.

2. The Multi-Regulator Full-Faithful Source Assembly: shared-link CAR/Fock gauge jet, full rank 144, machine-level source compatibility on the declared conditional source stack.

3. From Schrödinger to Dirac: graph Dirac operator, Clifford forcing and the original K=7 three-axis construction. Audit note: its presentation of the same antipodal channels as both exact involutions and infinitesimal translations is rejected by the exact source audit $T^2 = I \Longrightarrow 2D + \xi D^2 = 0$. Its Clifford and axis-structure results remain usable at their declared grades, but its derivative descent is not consumed. This no longer leaves the infrared moment closure open, because later refinement-transport papers independently supply genuine oriented path transport and the substrate-derived refinement frame.

4. Spinorial Closure Transport and Fermionic Source-Carriers in VERSF: minimal Clifford/spinorial carrier and chiral transport structure.

5. The Fermionic Fock-Space Reconstruction in VERSF: positive one-particle Hilbert structure, CAR algebra and antisymmetric Fock carrier.

6. The Neutrino Mass and Charge-Conjugation Completion Theorem in VERSF: independent particle/antiparticle CAR sectors, one-particle versus Fock-space charge conjugation, non-doubling firewall.

7. Sequential Interface Transport and Emergent Time in VERSF: temporal succession as an ordered family of committed interfaces rather than primitive temporal graph edges.

8. The VERSF Chiral Regulator Universality-Matching Theorem (CWGW-1): source incidence B_U, source roughness B_U^♯B_U, one-species overlap phase, max-gap representative, local faithful-background gap, index/locality certificate and one-loop universality bridge at its declared conditional free-pole grade.

9. The Route-M Off-Shell Integrability, Current-Source Closure and Direct Physical Running Theorem in VERSF: local fermion-universality criterion and one-loop determinant handoff.

10. The Recurrent Heat-Flow Uniqueness Programme (RHFU-1): the explicit admissible transfer continuum T_s = exp[−s(I − T_OS)] under positivity, normalisation, reversibility and stationary-law constraints alone, the uniqueness of the generator −(1/a_t)Log A for a supplied strictly positive transfer, and the specification of the small-step semigroup package required for a genuine selection.

11. The Primitive Fold Distinguishability and Marked-Intensity Programme (PFDMI): the native one-Fold hazard and conditional-terminal structure, the clocked law and its physical quotient rank across the declared regulator levels, the still-premised persistent-record event semantics, and the Fold-to-physical-duration attaching-map problem with its

31-mark versus 14-traversal mismatch. PFDMI-EX2H additionally supplies the exact conditional 1+6 hub/rim stabiliser theorem, the unique three-antipodal-axis pairing and D_6 committed geometry; this strengthens the K=7 three-axis source picture but is not used to silently identify the D_6 axis carrier with the spacelike refinement-frame legs, since the infrared proof uses the soldered refinement frame directly.

12. The Marked Birth/Current Intensity Programme (MBCI): the exponential first-birth survival form $p_0 = e^{-\Gamma}$, $p_r = (1 - e^{-\Gamma})q_r$, whose structure is retained here with its intensity reidentified as $-\log(1 - h)$.

13. The Hamiltonian/History Global Boundary and Cone Programme (HMGBC): the statement that bosonic cone positivity does not establish positivity of the fermion-including transfer.

14. The Fold-to-Fact Record-Promotion and First-Entry Test (F2F-1): the executed retained-record coarse-graining, the refutation of the earlier accepted-current persistent-fact partition, the seven self-history no-record marks and 312 record-producing marks, the 84 retained record classes, first-entry exhaustion and the exact record-copy admission.

15. The Terminal-Record and CAR/Fock Primitive-Leg Compatibility Certificates (HM4-EX4B): the differentiated terminal-record field, the shared-link CAR/Fock gauge-jet pass, the unique completion primitive direction $c_H = (1/2)(1, 1, 1, 1)$, the full-action primitive residual 1.149×10^{-15}, and the mixed-jet quotient criterion used in Corollary 17.1.

16. The Typed Completion and Direct Physical-Evaluation Patch (MASD-1): the typed left/right fermion action and the linear completion/CAR-Fock vertex establishing the one-primitive-leg trilinear structure used in the full mixed-tensor reduction.

17. The Quantum Gauge Completion of the VERSF Standard Model: faithful gauge covariance, background-field determinant architecture and regulator-universality requirements.

18. Microscopic Origin of Refinement Transport in VERSF: the finite K=7 commitment foam with genuine oriented edge transport U_{ij}, inverse-path consistency $U_{ji} = U_{ij}^{-1}$, ordered path holonomy T_γ, closure-compatible refinement, refinement-Cauchy existence and sequence independence of the continuum T_γ, and the identification of its first-order generator with the continuum connection.

19. Substrate Emergence of the Lorentz Transport Group in VERSF: the substrate-derived invariant bilinear form, refinement suppression of transport anisotropy, the orientation- and time-orientation- preserving SO(1,D) fixed point, the refinement-frame bundle, the soldering $g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$ and vielbein compatibility $\nabla_\mu e^a{}_\nu = 0$. That paper's own distinction between local K=7 architecture and global continuum dimensionality is retained: MFKR consumes the declared physical D = 3 branch and does not claim that this successor paper independently derives D = 3.

Imported mathematics not claimed as new VERSF output: finite-dimensional incidence-matrix algebra, induced operator norms, block Schur bounds, Fourier symbols on translation-invariant comparison graphs, standard Clifford representation theory, Jordan-Wigner constructions of anticommuting families, exterior-algebra second quantisation and quasi-free CAR lifts, operator logarithms of strictly positive self-adjoint operators, one-parameter semigroup generation, continuous-time Markov generators and the embedding problem for stochastic matrices, Heisenberg-picture channel invariance, convexity of completely positive trace-preserving channels, multiplicativity of Hilbert/Frobenius norms on tensor products, overlap/Ginsparg-Wilson functional calculus, standard fermionic coherent-state transfer mathematics, perturbative regulator universality, and the standard principal-symbol identity $\sigma_2(-\Delta_g)(p) = g^{ij}p_ip_j$ with its reduction to $\Sigma_i p_i^2$ in an orthonormal spatial frame. The new VERSF-specific returns of MFKR-1 are the exact assembly of the Master-Action finite kinetic matrix from its own shared-link bilinear, the edge-Clifford principal-symbol source-identity certificate, the exact separation of principal and $O(q^2)$ source structures with its corner census, the source-native weighted-degree bound on C_kin together with its Clifford-diagonal refinement and proven sharpness, the reduction of the edge coefficient to the geometric pair (n_e,w_e), with its even residual vanishing structurally on the inherited anti-adjoint branch, and its star moment certificate, the combination of the exact Clifford-square rigidity with the later source-derived refinement transport and soldered spatial geometry, which on the declared physical D = 3 branch forces $M^TM = I_3$, reduces the residual SO(3) freedom to the already-declared common spin-frame equivalence, gives $M_{IR} = I_3$ in the canonical refinement frame and outright on the tangent-aligned positive-weight branch, and thereby removes the independent low-energy spatial Dirac normalisation without consuming the older involution-as-translation descent, the record-defined sequential generator with its class uniqueness and its identification of the earlier admissible continuum as a fixed clock normalisation, the replacement of the previous current-channel event semantics by the executed record-defined self/nonself partition, the exact localisation of permanent-persistence uncertainty to post-commitment operation selection, the full mixed-tensor CAR/Fock completion closure obtained by applying the mode-1 quotient criterion to the unique primitive completion leg, and the resulting reduction of the full D_f[U] debt to finite-regulator manifest instantiation plus named upstream selection problems, with the infrared spatial principal-symbol gate already closed.