RCHS-1: The Twelve-Sector Current-Rigidity, Source-Generated Rank-Six Current-Stiffness and Current-Link Schur Reciprocity Theorem in VERSF

From Primitive Hazard Rigidity and Pre-Physical-Time Current to Exact C/W/Y Geometry, Native Current-Link Provenance, Positive Source Action and Finite-Refinement Current Persistence

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General Reader Summary

This programme tries to derive the structure and strength of particle physics from one primitive rule about how facts are committed, rather than by fitting parameters to experiment. A serious worry was that the theory secretly contained twelve hidden dials, one strength for each of its twelve matter/commitment sectors, or worse, twelve freely varying strength functions. On the same-grain attachment branch defined by AP1a/AP1b, no sector-dependent positive adjustment survives the primitive RUDM mark law; the relative freedom collapses to one common intensity, like the ticking rate of a clock. Counting how the one surviving law spreads over the twelve sectors produces a small table of how the three gauge directions, colour, weak and hypercharge, overlap on matter, reached by three structurally distinct derivations and protected against reshuffling by finer bookkeeping.

The paper also shows that this current exists before physical time. Part of it is a persistent circulation that no simple movement of facts from place to place could produce, and time is attached afterwards as a ruler that converts the already-existing current into a rate.

The headline result is stronger than the existence of a current strength alone. The same primitive block of the theory's machinery contains the current, its conjugate partner, the matched link (the seed of a gauge field) and an auxiliary bookkeeping phase, all priced by one common action. Read one way, with everything else allowed to settle, the block returns a definite positive stiffness for the current on all six physical directions; read the complementary way it returns a definite positive stiffness for the link. These are not two separate discoveries: an exact reciprocity law ties them together as two views of one parent, so the conversion between current strength and gauge strength is no longer a free choice. The auxiliary phase screens only one part in 265 of the current stiffness, and the survival holds for every admissible weighting of the machinery, with all unresolved details collapsing into one overall number. The same calculation also exposes a decision the physical theory must make: the simplest branch treats the three gauge directions democratically, but an earlier calculation assigns them to different modes of a wheel structure, and the reciprocity law proves both pictures cannot hold at once under the simplest common metric. That ambiguity is now a finite test rather than a hidden modelling choice.

Two further executions close long-standing worries. First, the previously advertised missing source ingredient turns out not to be one ordinary matrix at all: different sectors of the theory switch on at different depths, and once the bookkeeping respects that grading, the existing machinery already delivers the required descents, with the genuine gauge/current slice reconstructing exactly through the theory's own current defect, no hand-made embedding needed. Second, on any connected network with strictly positive physical conductances, a finite disturbance that is not perfectly uniform can never have its current switched off by uncertainty in those conductances: only the exactly uniform mode is invisible, so what remains undetermined is the current's magnitude and event law, never its existence.

A separate calculation advances one of the programme's long-standing topology debts: whether the theory's seven-state closure geometry carries an intrinsic sevenfold winding structure. The obvious routes fail, and provably so: neither the theory's natural circulation count nor any cyclic orbit of simple unit-incidence loops on the bare seven-state wheel can generate the required sevenfold character. But once two ingredients the theory already owns, the pairing of complementary fold relations and an oriented ordering around the boundary, are kept together, the smallest admissible attachment rule has essentially one order-seven branch, and its algebraic fingerprint is exactly sevenfold, collectively, with no seven appearing in any individual cell. On the bare wheel the resulting cell boundaries are pinched figure-eights rather than clean loops, but a minimal refinement of the hub, organised precisely by the three complementary pairs, turns all six boundaries into clean simple loops while preserving the sevenfold structure exactly, and the combined refined network cannot be embedded on a sphere without crossings and has minimum orientable embedding genus one. None of this yet proves nature installs that refinement; it reduces the vacuum-topology problem from an open search to one sharply constrained bridge between local fold boundaries and a single common refined carrier.

The bottom line is stated plainly. The twelve-dial ambiguity is gone, the overlap geometry is fixed, a positive current strength is derived, the gauge/current provenance slice is complete, finite non-uniform disturbances cannot lose their current, and the relative conversion from current to gauge strength is pinned by an exact reciprocity theorem. The specific numbers quoted are honest diagnostic values, not measured couplings, because the physical unit and the democratic-versus-mode-split decision are still open; a quarantined calibration still hints at the simple value one third under unproven identifications. No observed force strength is used or issued anywhere, and if the remaining calculations permanently fail, the approach must be rejected rather than fudged.

Contents

Abstract

The VERSF Standard-Model programme has reduced the gauge/current normalisation problem from a representation-dependent matrix choice to the physical normalisation of one source-generated current stiffness. The starting matter carrier is a 49-complex-dimensional tangent decomposed into twelve inequivalent gauge × generation commitment sectors with projectors P_A. At the RUDM primitive mark grain,

$h\_A(u \mid z, \rho) = h_0(u, \rho)\, w\_A(z)$, $w\_A(z) = \|P\_A z\|^2$.

The first theorem of this paper is a functional hazard-rigidity result: every positive same-grain intensity family preserving these conditional mark probabilities has the unique form

$\tilde{h}\_A(u, w, \rho) = c(u, w, \rho)\, w\_A$,

where c is one common positive scalar functional. The source therefore excludes every representation-dependent positive distortion at that event grain, not merely eleven unequal constants, and the rigidity is global: a single interior source ray, or equivalently a single record ledger, already suffices. Within the positive source-equivariant quadratic current class, $Q(J) = \Sigma\_A\, g\_A \|P\_A J\|^2$, same-grain descent forces g_A = g, hence

$Q(J) = g \|J\|^2$.

The corresponding factor scores s_F,A = 2 χ_F(A) give the exact C/W/Y Gram

H_CWY = (1/49) · [36 18 36 ; 18 26 26 ; 36 26 47],

positive definite, with the amplitude-log marked-rate information matrix M_JJ^event = 4 H_CWY and equal-factor marked-rate precision 1076/147. The same factor shape is independently returned by the MASD current-cycle geometry and the quadratic terminal-Born continuation. A microscopic refinement theorem proves that this Gram is unchanged if and only if the refinement is factor blind on supported marks; factor-sensitive refinement contributes an explicit positive-semidefinite conditional Fisher correction.

A common-parent audit turns the remaining same-source premise into finite operator tests. The RUDM law is equivalent to the primitive effect identities $E\_A = h_0 P\_A$. A scalar Gate-2 cost may alter the same event process only through the common intensity, while the Gate-2 canonical-wheel shortcut is excluded because its alternating zero-cost transport mode cannot be obtained as a faithful pullback of a positive-definite compressed information metric. The remaining ancestry object is the pre-compression source Jacobian L_λ and its parent-Hessian pullback.

A separate current audit establishes that the primitive MASD current is defined before physical time. At the neutral common background, $\delta J = \kappa(\delta\rho\_t - \delta\rho\_s)$, and the unavoidable cycle current lies in $\ker(B \otimes I_{49})$. A nonzero cycle current can therefore be divergence free with no independent density evolution, while exactly 3/5 of the squared physical CRTF current tangent lies in that irreducible cycle component. On the reversible K = 7 branch, $B C B^T = Q_7 = (1/31) L\_(W_7)$. The later σ-to-δ temporal attaching map converts this pre-existing current into current per physical time but does not set its interaction strength.

The central constructive result is obtained from the finite EPMD current-link-record source block. For each physical current-cycle direction, let p denote the retained current, q its conjugate current, θ the matched link coordinate and y the record phase. With positive primitive weights w_J, w_C, w_R, w_B and constitutive coefficient κ > 0, the quadratic Hessian is

$H_4(w)$ = [w_J 0 −κw_J/√2 0 ; 0 w_J +κw_J/√2 0 ; −κw_J/√2 +κw_J/√2 κ^2w_J + 4w_C + w_B −w_B ; 0 0 −w_B 4w_R + w_B].

On the unit residual-metric branch, eliminating q and θ while retaining p and y gives

H_pair^(J,y) = (1/11) · [10 −√2 ; −√2 53],

whose determinant is exactly 48/11 > 0. Eliminating y then gives

H_JJ^relaxed = (48/53) I_6.

The source therefore generates a positive rank-six current action after complete auxiliary relaxation.

The stronger result is that the current and matched link stiffnesses are not independent. Eliminating the conjugate current and record phase first gives the retained current-link block

H_Jθ^red = [w_J −κw_J/√2 ; −κw_J/√2 κ^2w_J/2 + K_θ],

where

K_θ = 4w_C + 4w_B w_R/(4w_R + w_B) > 0.

Relaxing the current gives the link stiffness K_θ; relaxing the link gives

K_J^relaxed = w_J K_θ / (K_θ + κ^2w_J/2),

equivalently

$$\mathbf{1/K\_J\hat{}relaxed = 1/w\_J + \kappa^2/(2K\_\theta).}$$

Expanding this relation reproduces exactly the positive-weight closed form

$$K\_J\hat{}relaxed = 8\, w\_J\, (\, w\_B\, w\_C + w\_B\, w\_R + 4\, w\_C\, w\_R\,) \,/\, (\, \kappa^2\, w\_B\, w\_J + 4\, \kappa^2\, w\_J\, w\_R + 8\, w\_B\, w\_C + 8\, w\_B\, w\_R + 32\, w\_C\, w\_R\,) > 0.$$

On the unit source branch,

$$\mathbf{K\_\theta = 24/5,\ K\_J\hat{}relaxed = 48/53.}$$

These are complementary reductions of one source block, not independent numerical confirmations.

The reciprocity theorem extends to the faithful matrix-valued carrier. After exact gauge/null quotienting, let $W\_J > 0$ be the bare current metric, $K\_\theta > 0$ the relaxed connection metric and $\mathcal{K}$ the constitutive current-connection map. Then

$$\mathbf{(\mathbb{K}\_J\hat{}relaxed)^{-1} = W\_J^{-1} + (1/2)\, \mathcal{K}\, K\_\theta^{-1}\, \mathcal{K}\dagger.}$$

This gives a direct physical source test,

$$\varepsilon\_dual = \|\, (\mathbb{K}\_J\hat{}relaxed)^{-1} - W\_J^{-1} - (1/2)\, \mathcal{K}\, K\_\theta^{-1}\, \mathcal{K}\dagger\, \|,$$

which must vanish on a faithful same-parent execution.

The scalar factor-democratic continuation is now regraded. The certified physical current embedding satisfies $E\_phys\dagger\, E\_phys = (49/50)\, H\_CWY \otimes I_2$, so whenever the faithful source returns $\mathbb{K}\_J\hat{}relaxed = K\_J\hat{}relaxed\, I_6$,

$$M\_JJ\hat{}source = (49/50)\, K\_J\hat{}relaxed\, H\_CWY \otimes I_2,$$

and on the unit diagnostic branch $M\_JJ\hat{}source,unit = (1176/1325)\, H\_CWY \otimes I_2$. However, the factor-democratic hypothesis is now a physical compatibility gate rather than an automatic consequence of positivity. The executed MASD wheel response has $(K_3, K_2, K\_q)\hat{}wheel = (24/5, 35/6, 3/2)$. For the common-weight source family these correspond to wheel eigenvalues (4, 5, 1) and, by current-link reciprocity, to current stiffnesses

(48/53, 35/38, 3/4).

Because $K\_J(\lambda)$ is strictly increasing in the wheel eigenvalue λ, distinct physical wheel assignments cannot simultaneously coexist with one common scalar current stiffness under a common positive separable metric. The physical W_prim, internal lift and auxiliary structure must decide which continuation survives.

The source-provenance obstruction is also sharpened. The earlier request for one representation-resolved derivative $d\Phi\_n$ is not type correct because the source sectors begin at different differential orders. The correctly typed object is a graded source jet. On that grading, the existing faithful stack returns machine-zero descent certificates for the gauge Hessian, full-action completion Hessian, record-phase tangent and shared-link CAR/Fock mixed derivative. The later representation-valued PFDMI source extension gives an even stronger native gauge/current result: the current-defect derivative on the 144 matrix-link directions and the differentiated native mark law both have rank 144, and the unique coupling on the reachable current-defect image reconstructs the mark derivative with relative residual 2.5840×10^{-15}. After the Wilson quotient, current-defect, mark and terminal responses all have rank 72, with physical mark reconstruction residual 1.8038×10^{-15}. Thus the old arbitrary selector-embedding interval is removed on the native gauge/current slice, although one unified primitive instrument generating every graded sector remains open.

The joint-endpoint preparation problem is reduced in parallel. The exact GSJB-to-PFDMI/MASD carrier reconciliation identifies the 49-complex-dimensional innovation with the projective tangent of the same 50-dimensional endpoint carrier, and common U(49) covariance reduces the endpoint cross-coherence to one scalar γ_e. For an arbitrary connected positive-conductance wheel with weighted Dirichlet generator L_K, every initial non-common innovation satisfies $\mathcal{D}$_K(s) = Tr[L_K e^(−2sL_K) Σ_0] > 0 at every finite refinement depth. For the maximally unresolved spatial law, every physical edge obeys 0 < γ_ij(s) < 1 at finite s > 0. Hence the remaining P2 question is numerical/event-grain selection, not zero-versus-nonzero current existence.

An independent Gate-T execution reduces the physical vacuum-topology problem. On the actual K7 wheel W_7, with six rim vertices, one hub and cycle lattice $Z_1(W_7; \mathbb{Z}) \cong \mathbb{Z}^6$, every integral two-cell attachment factors through a 6 × m integer winding matrix A. The source-returned first-cover circulation restricts to K(q) = Σ_i q_i; its kernel has quotient $\mathbb{Z}$, so it cannot generate order-seven torsion, and replacing K by K mod 7 remains incompatible with full-rank unit-incidence cells. The unordered six-relation Fold support incidence has rank four and likewise cannot determine the required attachment. When the source-returned complement pairing is combined with the conditional oriented C_6 successor S, so that $J = S^3$ in that ordering, however, the minimal local unit-incidence attachment family aI + bS + cJ, a, b, c ∈ {0, ±1}, has a unique order-seven branch up to overall orientation,

A_χ = J − I − S,

with det A_χ = 7 and SNF(A_χ) = diag(1, 1, 1, 1, 1, 7). Thus its conditional vacuum complex has $H_1 \cong \mathbb{Z}/7$ and $H^1(-; \mathbb{Z}_7) \cong \mathbb{Z}_7$. The bare-wheel boundaries are not embedded simple curves but $C_4 \vee C_3$ pinches. Replacing the hub by a contractible trivalent tree organised by the three Fold complement pairs leaves $\beta_1 = 6$, lifts the same attaching matrix unchanged, and turns all six cell boundaries into simple 13-edge cycles. The refined graph is non-planar but has minimum orientable genus one. These are exact mathematical and construction-level results; physical identification of the Fold relations with the vacuum cells, selection of the oriented branch, and installation of the refinement remain source-open.

The revised Standard-Model frontier is therefore more precise. The primitive source already fixes the relative current-link response inside the common finite-source action. What remains is to evaluate the faithful matrix-valued W_J, $\mathcal{K}$ and K_θ, test the reciprocity residual, determine whether factor democracy survives, and then attach the returned gauge/link stiffness to the continuum dimension-four Yang–Mills kinetic operator and current interaction. The separate dimension-six Yang–Mills corrections generated under coarse-graining remain differently typed downstream operators and must not be used to normalise the dimension-four gauge coupling. No measured gauge coupling, mass, mixing angle or phenomenological target enters this construction.

PART I: HAZARD RIGIDITY, QUADRATIC UNIQUENESS AND THE C/W/Y FACTOR THEOREM

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1. Programme Position

The current VERSF Standard-Model derivation is no longer blocked by the existence of the Standard-Model representation census, by the C/W/Y support directions, by the availability of an E_2 current tangent, or by the algebraic form of a candidate positive current metric. Those pieces have already been substantially reduced.

The present obstruction is narrower.

PMS-EX125 shows that a target-space operational metric cannot determine the physical current pullback without the current-to-operational-state derivative. PMS-EX126 sharpens the missing object to the primitive mark-score matrix

S_J = (s_C^e, s_W^e, s_Y^e),

with

M_JJ^phys = S_J Λ S_J^T

before any explicitly retained auxiliary shorting. The already-executed PFDMI current writer cannot return a pure common-radial magnitude score because it depends on the current only through its normalised ray.

Separately, the current-cycle programme establishes a six-dimensional $E_2 \otimes (C, W, Y)$ geometry and a quadratic marked construction with factor shape

H_CWY = (1/49) · [36 18 36 ; 18 26 26 ; 36 26 47],

but its absolute current-record strength remains one common open scale.

A recent Gate-2 matter-lift audit then reduced the most general source-symmetry-compatible positive matter operator from an arbitrary 49 × 49 positive matrix to the twelve-block form

G_∂ = Σ_{A=1}^{12} g_A P_A, g_A > 0,

where the P_A are the finest source-returned gauge × generation commitment projectors.

The questions addressed here are therefore:

> **Can the already-derived primitive commitment probability law itself eliminate the eleven relative coefficients g_A/g_B, and more, before any physical Standard-Model target is consulted? And can the remaining same-source attachment premise be turned from a verbal assumption into an exact, executable test?**

The answer to both is yes. The first is proved conditional on the attachment premise; the second is executed, with the honest verdict that one leg of the test does not yet pass on the published stack.

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2. Claim Grades and Firewalls

The results of this paper are separated into four grades.

Grade G1: exact finite-dimensional and information-geometric mathematics

These statements are unconditional once the supplied dimensions, projectors and factor signatures are fixed. They include:

- global identifiability of relative sector gains;
- the functional hazard-rigidity theorem and the uniqueness of its common scalar;
- the rank-eleven selector Jacobian and one-dimensional common-gain kernel;
- the quadratic source-equivariant classification $G = \Sigma_A g_A P_A$;
- the exact C/W/Y factor-Gram calculation with its spectral data;
- the microscopic mark-refinement decomposition, its positive-semidefinite correction theorem, and the factor-blind equivalence;
- the effect-equivalence theorem $E_A = h_0 P_A$, the common-parent variation identity, the block-unitary transfer characterisation, and the exact current-ray factorisation of the attachment audit;
- the current-link Schur reciprocity identity, its matrix Woodbury form, and the factor-democracy/wheel-mode compatibility theorem of Part VI.

Grade G2: inherited VERSF source law

The RUDM source law is consumed at its existing declared grade. The twelve commitment projectors are exact on the frozen 49-complex-dimensional tangent, while the unresolved Fubini–Study preparation law remains conditional on the inherited full-refinement/isosymmetry premise. RUDM also closes the relative component hazards while leaving their common absolute scale open.

The shared-interface record-localisation line supplies an additional inherited fact: under the declared GSJB-16 premises, one primitive relational fact is written on one common interface token before later copying, and combining the primitive interface locations with the twelve internal projectors yields the corresponding component-resolved primitive mark carrier. This materially narrows the carrier-typing problem but does not by itself derive the Fold/Gate-2-to-tangent map.

The SGPL marked-transfer architecture and the HMGBC history parent are likewise consumed at their declared grades: the event-resolved primitive Kraus density K_e = P_y P_(r(e)) e^(−a_t Ĥ/2) P_x and the parent Ĥ = −Δ̂ + V_loc + ⋯ with T̂ = e^(−a_t Ĥ).

Grade G3: same-source attachment gate, now decomposed

The earlier single premise AP1 was first split into carrier and intensity halves; the attachment audit of Part II sharpens the intensity half into three exact gates.

> **AP1a (primitive carrier compatibility).** The current/completion response relevant to this paper acts at the primitive shared-interface commitment stage, before post-copy proliferation, on the same component-resolved commitment carrier on which the twelve RUDM projectors classify primitive marks.

AP1a is **partly source supported** by the shared-interface record-localisation and twelve-projector results. It is not fully closed because the actual Fold/K_4-to-tangent intertwiner remains unreturned.

> **AP1b (intensity-descent identification).** The Gate-2/raw-current positive quadratic response changes the same primitive component-resolved mark intensities h_A(u | z, ρ) that enter the RUDM law, rather than defining a differently typed completion cost, later history weight, or post-copy maintenance observable.

AP1b is the load-bearing open source theorem. Part II **operationally reduces** it, within the adopted SGPL/HMGBC architecture, to a finite set of operator identities (Sections 26–27): common-parent ancestry (AP1b-I), mark-projector and realised-ray identity (AP1b-II), and a magnitude-bearing complete-fact event law (AP1b-III). A nonzero linear current-to-record amplitude is one sufficient realisation of the third gate, but the binary first-record audit of Part V shows it is not the only one: a source-selected nonzero quadratic curvature D^2_(J_cyc) Γ_F, equivalently D^2_(J_cyc) NR, is enough. The linear-amplitude route fails on the currently published stack; the RCCSJ binary-curvature route passes only as a positive existence witness with free gain; and Part III proves AP1b-I is itself not yet closed, with the wheel-metric shortcut exactly ruled out and the required deeper object named. The hazard-rigidity and quadratic-uniqueness results are exact consequences **if AP1b holds**; the paper does not silently identify a completion cost with a probability or intensity, and Part II shows no such identification is needed even in principle.

For compactness, when later sections say “the same-grain attachment passes,” they mean AP1a and AP1b (all three gates) together.

Grade G4: physical Standard-Model normalisation

The common absolute current-event strength, the source-derived microscopic history weighting, the physical value of λ_J, and the surviving Abelian closure portal remain open. The exact symbolic family λ_J = (2/(31 K_cyc^(2))) g_cyc^2, numerically

0.303446843055259 g_cyc^2 at the inherited K_cyc^(2), is now available, but the completed primitive source still does not select g_cyc; λ_J = 1/3 is retained only as a same-event saturated-birth consistency target at unit current normalisation.

No measured Standard-Model coupling, CKM entry, mass, mixing angle or target matrix is used anywhere in the selector, factor-Gram derivation, or attachment audit.

Part VII provenance and endpoint-dynamics grade

The source-provenance gate is now split more sharply. **P1a**, the native gauge/current graded provenance slice, passes on the PFDMI-EX2E source-extension branch: the 144 raw link directions and 72 Wilson-quotient physical directions factor through the actual current defect into the native marked response at machine precision. **P1b**, one unified primitive field-dependent instrument generating the gauge, record, completion and CAR/Fock graded jets together, remains open.

The joint-endpoint gate **P2** is also reduced. The internal covariance freedom collapses to the scalar endpoint coherence γ_e. On any connected MASD continuity branch with strictly positive edge conductances, every finite non-common spatial innovation has nonzero relative-current capacity. The unique physical γ_e, conductances, event grain, refinement depth/stopping rule and post-record Gibbs–Haar tilt remain open.

Part VIII Gate-T topology grade

Exact mathematics: $Z_1(W_7; \mathbb{Z}) \cong \mathbb{Z}^6$; the first-cover torsion no-go; the rank-four Fold-support obstruction; the classification of the minimal I, S, J attachment family; det A_χ = 7; SNF(A_χ) = diag(1, 1, 1, 1, 1, 7); the bare-wheel simple-loop no-go; the contractible hub desingularisation; preservation of $\beta_1 = 6$ and the Smith factor; and graph embedding genus one for the refined one-skeleton.

Conditional structural input: the C_6 successor ordering and its identification with the physical ordering of the six Fold relations.

Physically open: the Fold-to-vacuum carrier map, physical selection of S versus S^{-1}, installation of the refinement, and therefore the physical value of A_phys and the existence of order-seven torsion in K_vac.

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3. Inherited Twelve-Sector Commitment Geometry

The 49-complex-dimensional projective matter tangent decomposes into twelve inequivalent gauge × S_3 components:

Component	dim_ℂ
e_R^triv	1
e_R^std	2

Component	$\dim_{\mathbb{C}}$
N_R^{std}	2
L_L^{triv}	2
L_L^{std}	4
H	2
d_R^{triv}	3
d_R^{std}	6
u_R^{triv}	3
u_R^{std}	6
Q_L^{triv}	6
Q_L^{std}	12

so that

$$\Sigma_{A=1}^{12} d_A = 49.$$

Write

$$w_A(z) = \|P_A z\|^2, \Sigma_A w_A(z) = 1.$$

At the unresolved void, RUDM inherits the Fubini–Study law and therefore the exact Dirichlet disintegration

$$(w_1, \ldots, w_{12}) \sim \mathrm{Dirichlet}(d_1, \ldots, d_{12}).$$

Lemma 0 (almost-sure interior support)

Because every $d_A > 0$, the $\mathrm{Dirichlet}(d_1, \ldots, d_{12})$ law is absolutely continuous with respect to Lebesgue measure on the open simplex, and every face of the simplex has Lebesgue measure zero there. Hence

$w_A(z) > 0$ for every A, almost surely.

The source therefore supports strictly interior weight vectors with probability one, which is exactly the support condition consumed by the selector and rigidity theorems below. Moreover the Fubini–Study law has full support on the unit sphere of the carrier, a fact consumed by the effect-equivalence theorem of Part II. ■

The first-fact component probability is

$p_A = d_A / 49.$

After a component-resolved record ledger with counts n_A, the exact RUDM predictive law is

$p_A(\rho) = (d_A + n_A) / (49 + N), N = \Sigma_A n_A.$

The relative hazard form is

$h\_A(u \mid z, \rho) = h_0(u, \rho)\, w\_A(z)$,

with the common absolute opportunity/hazard scale h_0 left open.

This separation between relative component law and common event scale is central below.

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4. The Representation-Resolved Gate-2 Matter Family

The scalar Gate-2 completion primitive determines an endpoint-label sum-square structure at its declared grade. Once the matter tangent is resolved by the twelve physical projectors, the general positive source-symmetry-compatible internal quadratic weighting can be written

$G\_\partial = \Sigma\_A\, g\_A\, P\_A,\ g\_A > 0$.

The corresponding soft matter cost has the form

$Q\_\partial(z\_t, z\_s) = \langle z\_t + U z\_s, G\_\partial\, (z\_t + U z\_s) \rangle$

on the sum-square branch, or the corresponding sign-reversed form on the other branch.

Positivity implies that changing the g_A's does not alter the hard zero set when all $g\_A > 0$. It does, however, alter the off-shell representation-resolved response.

Therefore the hard Gate-2 zero condition is insufficient to choose the g_A's. A probability-level selector is required.

RUDM supplies such a selector, and more, if the same-grain attachment holds.

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5. Twelve-Sector Source-Law Selector and Hazard-Rigidity Theorems

Theorem 1 (conditional twelve-sector gain collapse, single-ray form)

Assume the same-grain intensity-descent identification AP1b. Let the component-resolved primitive mark intensity be modified by positive constants g_A:

$h\_A^\wedge(g) = h_0\, g\_A\, w\_A$.

Then the conditional mark probability given that an event occurs is

$q\_g(A \mid z, \text{event}) = g\_A\, w\_A / \Sigma\_B\, g\_B\, w\_B$.

The RUDM source law requires

$q(A \mid z, \text{event}) = w\_A$.

Let z be **any single ray** whose weight vector is strictly interior, $w_A(z) > 0$ for all A. Then

$q_g(\cdot \mid z, \text{event}) = q(\cdot \mid z, \text{event})$

if and only if

$\mathbf{g_1 = g_2 = \cdots = g_{12}}$**.**

By Lemma 0, source-supported rays are strictly interior almost surely, so the source law itself supplies the required test ray with probability one.

Proof

Set $q_g(A) = w_A$. Since $w_A > 0$, division gives

$g_A / \Sigma_B\, g_B w_B = 1$

for every A. The denominator is one scalar independent of A, so every g_A equals the same number g. Conversely, a common gain cancels from the conditional mark law. ■

Remark (global identifiability, not first-order identifiability)

Theorem 1 is exact, not a linearisation. The map from positive gain vectors modulo their common scale into conditional mark laws is injective on every interior ray. The rank-eleven Jacobian of Section 7 is only the infinitesimal shadow of this global statement.

Theorem 1A (functional primitive-hazard rigidity)

The constant-gain theorem can be strengthened to an arbitrary positive same-grain hazard family.

Fix opportunity variable u, record state ρ, and an interior component-weight vector

$w = (w_1, \ldots, w_{12}),\ w_A > 0,\ \Sigma_A w_A = 1.$

Let

$\breve{h}_A(u, w, \rho) > 0$

be any measurable positive component-resolved primitive hazard family. Suppose its conditional mark law agrees with the RUDM source law:

$\breve{h}_A(u, w, \rho) / \Sigma_B\, \breve{h}_B(u, w, \rho) = w_A$

for every A, for source-almost-every interior w.

Then there exists one positive common scalar functional

$c(u, w, \rho) > 0$

such that

$\breve{h}_A(u, w, \rho) = c(u, w, \rho)\, w_A$ for all A,

for source-almost-every interior w. The functional c is unique, and it is exactly the total intensity: $c = \Sigma_B \tilde{h}_B$. Conversely, every hazard family of this form has exactly the RUDM conditional mark law.

Proof

Define $c(u, w, \rho) = \Sigma_B \tilde{h}_B(u, w, \rho)$. Multiplying the assumed conditional probability equation by this common denominator gives $\tilde{h}_A = c\, w_A$ for every A. Uniqueness is immediate: summing $\tilde{h}_A = c'\, w_A$ over A gives $c' = \Sigma_B \tilde{h}_B = c$. The converse follows by cancellation of c. ■

Corollary 1A.1 (infinite-dimensional relative rigidity)

At the RUDM primitive mark grain, the source law is not merely insensitive to one common multiplicative constant. It is rigid against **all representation-dependent positive hazard distortions**: the excluded class is infinite dimensional, containing every A-dependent function of the opportunity, weight and record variables.

The surviving freedom is exactly one common scalar functional

$c(u, w, \rho)$,

which may depend on common opportunity, state or record variables but may not carry an A-dependent distortion without changing the source mark probabilities.

If the same-grain physical hazards are continuous on the simplex interior, source-almost-everywhere equality under the full-support Dirichlet/Fubini–Study law extends to equality throughout the interior.

Corollary 1.1 (identity form up to common scale)

For the constant quadratic block family of Section 4, AP1b implies

$\mathbf{G_{\partial} = g\, I_{49}}$.

Thus the twelve-sector source law removes the eleven representation-anisotropic directions without assuming full U(49) invariance of the physical interaction: the selector operates directly on the already-resolved physical component alphabet and consumes only one interior ray rather than an invariance postulate over the whole unitary group. The functional theorem shows that this is the finite-dimensional shadow of a stronger rigidity: the relative component law itself is fixed.

---

6. Marginal and Record-Conditioned Selector Theorems

The same conclusion follows without conditioning on a realised ray.

At first fact,

$p_A = d_A / 49.$

A gain-weighted model predicts

$p_A^{(g)} = g_A d_A / \Sigma_B g_B d_B.$

Because every $d_A > 0$, equality with the source law for all A again requires $g_A = g$. The marginal first-fact law alone is therefore already a complete selector.

The result also survives record conditioning, and in strengthened form: no accumulation of records is needed, and no ledger can hide a relative gain.

Theorem 2 (single-ledger selector)

Fix **any one** component-count ledger ρ with counts $n_A \geq 0$. The RUDM predictive law is

$p_A(\rho) = (d_A + n_A) / (49 + N).$

If a gain-weighted law

$p_A^{(g)}(\rho) = g_A (d_A + n_A) / \Sigma_B g_B (d_B + n_B)$

agrees with the RUDM law for every A at this single ledger, then

$g_A = g$ for all A.

Proof

Since every $d_A \geq 1$, every effective count $d_A + n_A$ is strictly positive whatever the ledger, so the twelve predictive probabilities are strictly positive. The argument of Theorem 1 applies verbatim with w_A replaced by $(d_A + n_A)/(49 + N)$. ■

Hence record accumulation cannot hide a relative representation gain at any stage of the ledger: the empty ledger, every finite ledger, and every conditional law each separately enforce the collapse. Agreement "for every ledger" is sufficient but far more than necessary.

---

7. Rank-Eleven Certificate

Parameterise

$g_A = e^{\gamma_A}$

around $g_A = 1$. The derivative of the first-fact probability vector is

$J_g = \mathrm{diag}(p) - p\, p^T$, $p_A = d_A / 49$.

This is the covariance matrix of the twelve-category source law.

Lemma 3 (exact kernel)

For any strictly interior probability vector p,

$\ker(\mathrm{diag}(p) - p\, p^T) = \mathrm{span}\{(1, 1, \ldots, 1)\}$.

Proof

If $(\mathrm{diag}(p) - p\, p^T)\, v = 0$ then $p_A v_A = p_A (p \cdot v)$ for every A. Since $p_A > 0$, $v_A = p \cdot v$ is one constant for all A, so v is proportional to the all-ones vector. Conversely $(\mathrm{diag}(p) - p\, p^T)(1, \ldots, 1)^T = p - p = 0$. With twelve categories the rank is therefore exactly eleven. ■

The frozen execution confirms

rank J_g = 11, nullity J_g = 1,

with the null vector the common gain direction (1, 1, …, 1) at numerical alignment

1.000000000000000.

Thus every one of the eleven relative gain directions changes the source probability law to first order, and Lemma 3 shows this is forced by interiority alone, independent of the particular dimension vector.

For the deliberately nonuniform, target-blind positive gain vector of the frozen certificate, the first-fact probability changes by total variation

TV = 0.048501275510,

and across 20,000 Fubini–Study/Dirichlet rays the mean conditional-mark total variation is

0.047797636485.

The same gain vector changes one explicit record-conditioned predictive law by a nonzero total variation.

Reproducible independent diagnostic

Because the certificate's gain vector is internal to the frozen package, this paper carries one fully reproducible diagnostic with a declared gain. Take the canonical target-blind nonuniform choice $g_A = d_A$. The perturbed first-fact law is $p_A^{(g)} = d_A^2 / 303$, and the exact first-fact total variation against the source law is

TV(g = d) = 1140/4949 = 0.230349565569,

verified by direct rational arithmetic. A seeded Monte Carlo over 20,000 Dirichlet(d) rays (generator seed 0) gives mean conditional-mark total variation 0.232893 under the same gain. Both diagnostics confirm, with a gain vector any reader can reconstruct, that nonuniform gains are macroscopically visible to the source law. The same declared choice reappears in the attachment audit of Part II as a per-sector transfer attenuation, where it produces the identical exact total variation, so the two diagnostics cross-check one another.

These tests are diagnostics, not the proof. The proof is algebraic, global and now functional (Theorems 1, 1A and 2); the numerical executions verify that the frozen implementation realises the theorems with the expected rank, null space and sensitivity.

---

8. Quadratic Current-Response Uniqueness and Magnitude-Bearing Mark Scores

The rigidity theorems fix the relative representation law at the primitive mark grain but leave one common magnitude. We now classify the complete positive quadratic response class before writing the mark scores.

Theorem 4 (source-equivariant quadratic classification)

Let the 49-complex-dimensional matter/current carrier decompose into the twelve inequivalent irreducible commitment sectors

$\mathcal{H} = \bigoplus_{A=1}^{12} \mathcal{H}_A$, $P_A : \mathcal{H} \to \mathcal{H}_A$.

Let

$Q(J) = \langle J, G J \rangle$

be a positive Hermitian quadratic functional that respects the source gauge × generation symmetry.

Because the twelve $\mathcal{H}_A$ are inequivalent irreducible source components, equivariance eliminates cross-irrep blocks and Schur's lemma makes the restriction of G to each $\mathcal{H}_A$ scalar. Therefore

$\mathbf{G = \Sigma_A\, g_A\, P_A,\ g_A \geq 0,}$

and

$\mathbf{Q(J) = \Sigma_A\, g_A\, \|P_A J\|^2.}$

If Q is strictly positive on every source sector, then $g_A > 0$ for every A. The decomposition is unique, because the P_A are the minimal central projections of the commutant of the source symmetry on $\mathcal{H}$, so the twelve scalars g_A are intrinsic to Q rather than a choice of presentation.

This classification is exact within the declared quadratic, source-equivariant class. No "minimality" prescription has been used.

Theorem 5 (unique positive quadratic same-grain current law up to common scale)

Assume AP1b: the sector contributions of the quadratic response enter the same primitive component-resolved mark intensities governed by RUDM.

Then Theorem 1, or equivalently Theorem 1A, forces

$g_A = g$ for all A.

Hence

$$\mathbf{G = g\, I_{49}}$$

and

$$\mathbf{Q(J) = g\, \|J\|^2.}$$

Equivalently, the unique component-additive positive quadratic intensity family has the form

$$\boldsymbol{\lambda_A = \lambda_0\, \|P_A J\|^2,}$$

with one common coefficient λ_0.

Thus the quadratic raw-current intensity used below is not merely the simplest admissible candidate. **Within the positive source-equivariant quadratic same-grain class, it is unique up to the one common magnitude.**

Physical factor perturbations

Let $F \in \{C, W, Y\}$. Define the support signature

$\chi_F(A) \in \{0, 1\}$

according to whether component A carries factor F.

A radial amplitude perturbation acts as

$P_A J \mapsto e^{(\varepsilon_F \chi_F(A))} P_A J.$

Therefore

$\lambda_A \mapsto e^{(2 \varepsilon_F \chi_F(A))} \lambda_A,$

and the primitive intensity score is

$s_{F,A} = \partial \log \lambda_A / \partial \varepsilon_F |_{(\varepsilon=0)} = 2 \chi_F(A)$.

This score is magnitude bearing. The common-radial change is not removed by projective normalisation because the total event intensity changes. It is precisely the information the existing projective PFDMI current writer discards, and it is why the present construction can supply a score support where that instrument cannot.

The resulting coarse score matrix is

$S_J^{coarse} = 2 \chi^T$.

Lemma 5.1 (radial-coordinate convention invariance)

The factor of two reflects the choice of amplitude-log radial coordinate. Any smooth scalar reparameterisation of the common radial coordinate rescales every factor-score row by one common Jacobian factor at the reference point. It therefore multiplies the factor Gram by one positive scalar while leaving the factor shape, entry ratios, rank and eigenvectors unchanged.

No radial-coordinate convention can change the C/W/Y factor shape derived in Section 9. ■

---

9. Exact C/W/Y Factor-Gram Closure and the Refinement Theorem

The twelve finest components carry the factor signatures

Class	(C, W, Y)
N_R	(0, 0, 0)
e_R	(0, 0, 1)
L_L / H	(0, 1, 1)
d_R / u_R	(1, 0, 1)
Q_L	(1, 1, 1)

Weighting the twelve finest components by their exact first-fact source probabilities $d_A/49$ gives

$H_{CWY} = \Sigma_A (d_A/49) \chi(A) \chi(A)^T$.

Direct evaluation returns

$$49 H_{CWY} = [36\ 18\ 36 ; 18\ 26\ 26 ; 36\ 26\ 47]$$

and therefore

$$H_{CWY} = (1/49) \cdot [36\ 18\ 36 ; 18\ 26\ 26 ; 36\ 26\ 47].$$

Each integer entry is an exact representation count: the diagonal entries 36, 26 and 47 are the total complex dimensions carrying C, W and Y respectively, and each off-diagonal entry is the total dimension carrying both factors, so (C, W) = $\dim(Q_L)$ = 18, while (C, Y) = 36 and (W, Y) = 26 because every coloured and every weak component is hypercharged.

This is exactly the factor matrix previously obtained from two structurally distinct constructions:

1. the unavoidable representation-resolved MASD current-cycle geometry; and
2. the quadratic terminal-Born current-to-terminal construction.

The equality is structural. No downstream Standard-Model target enters it.

Exact spectral data

The integer Gram is positive definite, with leading principal minors

36, 612, 4428,

all strictly positive, trace 109 and determinant 4428. Positive definiteness means the three factor directions are informationally independent at event grain: no linear combination of C, W and Y radial perturbations is invisible to the realised-event marked-rate law.

Using the amplitude-log scores s_F,A = 2 χ_F(A), the primitive marked-rate intensity-score Gram, the factor information per unit realised-event rate, is

M_JJ^event = S_J^coarse diag(p_A) $(S\_J\text{\^{}coarse})^T$ = 4 H_CWY.

Hence

M_JJ^event = (4/49) · [36 18 36 ; 18 26 26 ; 36 26 47].

The machine residual against the exact matrix is

1.132×10^{-15},

and this paper independently reconstructs the matrix from the component ledger with exact integer arithmetic, so the residual is a floating-point artefact of the frozen package only. Its eigenvalues are

(0.314682300680, 1.010864036230, 7.572412846763),

reverified to fifteen digits from the exact rational matrix. The equal-factor unit direction

$f = (1, 1, 1)^T / \sqrt{3}$

has marked-rate precision

f^T M_JJ^event f = 1076/147 = 7.319727891156,

stated exactly: f^T (49 H_CWY) f is one third of the total entry sum 269, and the amplitude-log factor four gives 4 · 269/147 = 1076/147.

The overall factor four is not a new physical number. It is the universal conversion between the chosen amplitude-log radial coordinate and an intensity-log coordinate (Lemma 5.1). The physically relevant result here is the **factor shape and relative overlap structure**, not the common scalar normalisation.

Typing note (uncentred rate Gram versus conditional-mark Fisher matrix)

M_{JJ}^{event} is an **uncentred intensity-score Gram of the realised-event rate process**. It is not the Fisher matrix of the conditional categorical mark distribution q_A. For the latter, the score must be centred because the probabilities sum to one:

$s_{F,A}^{cond} = 2\,(\chi_F(A) - \mu_F)$, $\mu = (1/49)\,(36, 26, 47)^T$,

where μ_F is the p-mean of the factor signature, so the conditional-mark Fisher matrix is

$\mathbf{F_{mark} = 4\,(H_{CWY} - \mu\,\mu^T)}$**,**

with the exact centred matrix

$H_{CWY} - \mu\,\mu^T = (1/2401) \cdot [\,468\ -54\ 72\ ;\ -54\ 598\ 52\ ;\ 72\ 52\ 94\,]$

and equal-factor conditional-mark precision

$f^T F_{mark}\, f = 5200/7203 = 0.721921421630$,

not 1076/147. The two objects answer different questions. The uncentred Gram is the physically relevant one here because, as Section 35 makes precise, a change in the **total occurrence rate is itself physical information** in the binary event-rate/waiting-rate process; conditioning it away is exactly what the projective PFDMI writer does and exactly what a magnitude-bearing law must not do. The centred matrix F_{mark} is what an instrument restricted to the conditional mark alphabet could at most recover. Both are stated so the typing can never be silently conflated.

Theorem 6 (microscopic mark-refinement decomposition)

The event-grain factor matrix above is computed on the twelve coarse primitive commitment components. We now ask whether a finer microscopic realised-event mark alphabet can alter it.

Let coarse component A split into microscopic marks (A, α) with intensities

$\lambda_{A\alpha} = \lambda_A\, r_{A\alpha}$, $r_{A\alpha} \geq 0$, $\Sigma_\alpha\, r_{A\alpha} = 1$.

Let

$s_{F,A} = \partial_F \log \lambda_A$

be the coarse factor score and define the refinement score

$u_{F,A\alpha} = \partial_F \log r_{A\alpha}$.

Then

$s_{F,A\alpha} = s_{F,A} + u_{F,A\alpha}$.

Because $\Sigma_\alpha\, r_{A\alpha} = 1$,

$\Sigma_\alpha\, r_{A\alpha}\, u_{F,A\alpha} = \Sigma_\alpha\, \partial_F\, r_{A\alpha} = 0$.

Therefore the microscopic marked-rate information matrix decomposes exactly as

$$\mathbf{M_{FG}^{micro} = M_{FG}^{coarse} + \Sigma_A \lambda_A I_{FG}^{ref}(A),}$$

where

$I_{FG}^{ref}(A) = \Sigma_\alpha r_{A\alpha} u_{F,A\alpha} u_{G,A\alpha}$

is the conditional Fisher matrix of the microscopic splitting inside component A.

Each $I^{ref}(A)$ is positive semidefinite. The decomposition is the exact chain rule of Fisher information for the two-stage mark (A, α): total information equals coarse information plus the intensity-weighted conditional information of the splitting.

Corollary 6.1 (factor-blind refinement equivalence)

The following are equivalent for a refinement family that is smooth in the factor parameters:

1. the microscopic splitting is factor-radial blind on supported marks: $\partial_F r_{A\alpha} = 0$ for every F and every (A, α) with $\lambda_A r_{A\alpha} > 0$;
2. $I^{ref}(A) = 0$ for every supported A;
3. $\mathbf{M^{micro} = M^{coarse}}$.

That (1) implies (2) implies (3) is immediate from the decomposition. Conversely, if $M^{micro} = M^{coarse}$ then $\Sigma_A \lambda_A I^{ref}(A) = 0$; each term is positive semidefinite, so every supported $I^{ref}(A)$ vanishes, and a vanishing conditional Fisher matrix forces $u_{F,A\alpha} = 0$ on the support of r_A, that is, factor blindness. ■

Thus any deterministic copying, bookkeeping refinement or factor-blind subdivision of a primitive mark leaves the C/W/Y factor Gram exactly unchanged, and **nothing else does**: invariance of the Gram is not merely a sufficient-condition safe harbour but a complete characterisation.

Corollary 6.2 (factor-sensitive refinement can only add information)

If the refinement is factor sensitive, then

$$\mathbf{M^{micro} - M^{coarse} \succeq 0.}$$

The coarse factor matrix is therefore not arbitrarily deformable by microscopic refinement. A change in shape must be traced to a **specific factor-sensitive microscopic splitting mechanism**, and its contribution is an explicit positive-semidefinite conditional Fisher term. In particular no refinement, however constructed, can reduce the quadratic information in any fixed factor direction, although a factor-sensitive term can change the eigenvectors.

This theorem materially narrows the coarse-to-microscopic caveat in PMS-EX126. It also fits the inherited shared-interface record-localisation result: downstream deterministic copying of one primitive record cannot alter the primitive factor Gram unless the copy process itself acquires new factor-sensitive physics.

---

10. Triple Convergence of the Factor Shape

The present result produces a useful convergence which did not exist at the beginning of the current-normalisation programme.

The same matrix

$H\_CWY = (1/49) \cdot [\ 36\ 18\ 36\ ;\ 18\ 26\ 26\ ;\ 36\ 26\ 47\ ]$

now appears from three conceptually different routes.

Route A: representation/current action geometry

The unavoidable cycle part of the physical current tangent carries the exact C/W/Y support geometry and yields the same uncentred factor Gram.

Route B: quadratic terminal-Born successor

The minimal positive terminal-resolved quadratic current intensity yields the same factor matrix when the invariant terminal populations are used.

Route C: primitive commitment-mark score law

The present twelve-sector RUDM rigidity plus the unique quadratic magnitude-bearing current intensity yields the same matrix directly from the source component alphabet and mark-score support.

The three routes consume different corpus inputs: Route A consumes the current-cycle geometry, Route B the terminal populations, Route C only the commitment ledger and the quadratic intensity law. Their agreement is therefore not a single calculation repeated three times; it is one integer matrix reached from three distinct places in the source architecture. And Route C now carries a uniqueness theorem rather than a minimality choice: within its declared class there was no other quadratic law whose Gram could have been computed.

These routes do not yet prove the physical common scale. They do, however, make the relative factor orientation substantially harder to regard as an arbitrary modelling choice.

The remaining freedom is scalar at this grade.

---

11. Relation to M_JJ^phys and the Existing Standard-Model Chain

PMS-EX126 gives the physical architecture

M_JJ^phys = S_J Λ S_J^T.

The present paper supplies, under the positive quadratic same-grain theorem, the coarse primitive score support

S_J^coarse = 2 χ^T.

Theorem 6 then proves that a microscopic refinement preserves the corresponding factor Gram exactly if and only if it is factor blind, while a factor-sensitive refinement must enter as a named positive-semidefinite correction.

The physical history metric still requires the correct history-weighted primitive intensity measure, the microscopic mark descent, the absolute common event scale, and the physical two-quadrature spatial weighting.

Corollary 6.3 (direct current-metric reduction)

Assume:

1. AP1a and AP1b;
2. the microscopic mark refinement is factor blind at the retained quadratic order, or its factor-sensitive correction is separately derived and shown absent;
3. the two E_2 quadratures are related by the required D_6 covariance;
4. the history weighting of the factor scores is common across C/W/Y at the retained grain.

Then all remaining direct current-metric freedom is scalar and

M_JJ^direct = Λ_common · H_CWY ⊗ I_2,

up to the universal radial-coordinate convention.

This is the strongest direct result that follows from the present source law alone. It does not determine Λ_common.

Corollary 6.4 (handoff to the inherited current-cycle normalisation)

The independently derived current-cycle/terminal-Born continuation supplies the spatial factor

K_cyc^(2) = 0.212610974570295

and defines

M_0 = K_cyc^(2) H_CWY = [0.156203981317 0.078101990658 0.156203981317 ; 0.078101990658 0.112813986507 0.112813986507 ; 0.156203981317 0.112813986507 0.203932975608], each entry verified here as $K\_cyc^{(2)} \cdot (49\ H\_CWY)\_FG / 49$ to better than 5×10^{-13}.

Within that inherited quadratic continuation, all universal radial-coordinate and history-normalisation factors can be absorbed into one nonnegative source coefficient λ_J, giving

$M\_JJ^{quad} = \lambda\_J\ M_0 \otimes I_2$.

The present paper therefore supplies a source-level explanation for why the **relative C/W/Y factor freedom has collapsed before λ_J is fixed**.

For

$f = (1, 1, 1)^T / \sqrt{3}$,

the inherited direct axle precision remains

$f^T M_0 f = 0.389063620132$,

with the exact form $f^T M_0 f = K\_cyc^{(2)} \cdot 269/147$, since $f^T H\_CWY f = 269/147$ exactly.

No part of this corollary sets λ_J = 1, derives the physical history scale, or licenses the canonical wheel magnitude as physical. It merely makes explicit how RCHS-1 hands its factor-rigidity result into the already-reduced Standard-Model current chain.

The current paper therefore changes the outstanding problem from a representation-dependent matrix choice to the derivation of one common source scale and the surviving Abelian shorting.

---

12. The Common-Scale Boundary

The rigidity theorems deliberately leave one common gain g, and in the functional case one common scalar functional c.

That is not a defect of the proof. It is exactly what a conditional mark distribution cannot see.

If

$h\_A = h_0\ g\ w\_A$,

then g can be absorbed into the common absolute event rate. All conditional component probabilities remain unchanged. Lemma 3 records the same fact infinitesimally: the common-gain direction is the entire kernel of the selector Jacobian, so the invisibility of g is exact, structural, and confined to precisely one dimension. Theorem 1A records it functionally: the surviving object c(u, w, ρ) is a full scalar functional, but it is common to all twelve components. The attachment audit of Part II then exhibits this surviving functional concretely: a scalar cost in the common Euclidean parent produces exactly $c = h_0\ e^{(-a\_t\ Q)}$, and a current magnitude entering through the realised ray produces exactly $c = \lambda_0\ r^2$.

This is consistent with the RUDM result that relative component hazards close while the common absolute hazard remains open.

Therefore

> **RUDM selects representation ratios, not the absolute current-event clock/intensity scale.**

The remaining absolute current-normalisation problem is no longer twelve-dimensional, and no longer infinite dimensional in the relative directions. It is one common scalar functional at this grain.

---

13. Why This Is Not Yet a Derivation of λ_J or of the Physical Gauge Couplings

The paper now derives more than a relative matrix: it derives a nonzero rank-six finite-source current action. But three typed steps still separate that result from a physical Standard-Model coupling.

First, the unit residual metric used for the exact 48/53 result is a diagnostic source metric. The general theorem proves positivity for every positive weighting, but the physical W_prim has not yet selected the numerical value of $K_J^{relaxed}$.

Second, the finite source-action Hessian must be pushed into the continuum Lorentzian current/field variables. The required map must establish that the discrete/interface current and gauge connection produce the continuum action in the appropriate gauge-invariant form, schematically

$$S_{gauge}[A] + S_{int}[A, j] = -(1/4) \int Z_{ab} F^a{}_{\mu\nu} F^{b\,\mu\nu} + \int A^a{}_\mu j_a{}^\mu + \cdots .$$

The current source stiffness cannot simply be renamed Z_{ab}; the continuum attaching map must return the relation.

Third, canonical field normalisation must then convert the source-returned kinetic/current coefficients into the physical gauge couplings. Only after that step can the surviving Abelian portal be executed and g_s, g, g', or any equivalent physical normalisation, be issued.

The inherited λ_J belongs to the information/history continuation,

$$M_{JJ}^{quad} = \lambda_J K_{cyc}^{(2)} H_{CWY} \otimes I_2.$$

The newly derived source-action coefficient belongs to

$$M_{JJ}^{source} = (49/50) K_J^{relaxed} H_{CWY} \otimes I_2.$$

Their equality would itself be a physical theorem. It may not be assumed merely because their relative shapes agree.

Accordingly,

source current stiffness: derived,

but

$\lambda_J^{physical}$, g_s, g, g': not yet derived.

The crucial improvement is that the remaining task is now a **normalisation and attaching-map problem**, not an existence problem for the current action.

---

14. Same-Source Attachment Decision Tree

The revised paper separates the physical branches more sharply.

Branch A: primitive mark-intensity attachment passes

If AP1a and AP1b are proven, the source current action is also the primitive magnitude-bearing RUDM event law. Hazard rigidity then fixes the relative mark intensities and the common scale must be returned by the same source event/history law.

Branch B: present published binary-occurrence architecture

The current source action exists, but the factorised F2F history selector does not read it as a change in whether a fact occurs. The all-orders binary occurrence derivative is zero while current-dependent record identity and phase remain nonzero. This branch is now the executed status of the published first-record architecture.

Branch C: primitive carrier attachment fails

If the Gate-2/raw-current construction is ultimately shown to inhabit a different primitive carrier from the RUDM marks, the RUDM rigidity theorem remains valid for the mark law but cannot be promoted to that Gate-2 source. This does not undo the separately derived EPMD current stiffness.

Branch D: continuum action attachment closes first

This is now the preferred constructive route. The source already returns

$M_{JJ}^{source} = (49/50)\, K_J^{relaxed}\, H_{CWY} \otimes I_2$.

If the physical primitive metric selects $K_J^{relaxed}$ and the discrete-to-continuum attaching map carries this action into the gauge kinetic/current interaction, then the physical coupling normalisation can be derived without requiring current to alter binary fact occurrence.

Thus every branch has a clear consequence:

mark attachment pass ⇒ current stiffness also controls primitive event intensity;

binary selector blind ⇒ event occurrence is not the readout, but the source current action survives;

continuum attachment pass ⇒ physical gauge normalisation can proceed directly from the source action.

The paper no longer treats failure of the binary branch as failure of the current-strength programme itself. No branch licenses fitting the eleven relative coefficients to Standard-Model data.

---

15. Why the Existing Block-Fold RG Does Not Replace This Result

The primitive-source uniqueness audit proves that the previously used minimal Fold action is underselected. In addition to the separate σ and ω nearest-neighbour terms, a mixed invariant

$J_{\sigma\omega}\,(1 - \sigma_i\sigma_j)(1 - \omega_i\omega_j)$

is allowed and changes physical outcome distributions.

The same audit also identifies higher Wilson harmonics and nonlinear completion-amplitude counterfamilies. Therefore a genuine whole-action uniqueness theorem requires an enlarged block-Fold RG basis.

The published block-Fold calculation predates that audit and uses the older truncated action. It is useful as an RG prototype but cannot be invoked to set the additional couplings to zero by derivation.

The present paper therefore does not manufacture a twelve-sector RG flow.

Instead, it uses an already-derived primitive probability law to eliminate the eleven relative matter gains, and every representation-dependent hazard distortion, **at the mark grain where that law has authority**.

An enlarged action-level RG remains relevant to whole-action uniqueness and to the common scale, but it is no longer needed merely to choose the eleven ratios g_A/g_B if the same-grain attachment passes.

---

16. Relation to the Hard Gate-2 Matter Sign

A later current audit warns that the hard internal Gate-2 sign should not be used as a physical normalisation theorem because the source does not yet select the internal orientation action.

The present paper does not depend on selecting

$z_t = -U z_s$.

The twelve-sector rigidity is probability based and operates on component weights and primitive mark intensities.

Therefore the central results

$g_1 = \cdots = g_{12}$ and $\tilde{h}_A = c\, w_A$

survive even if the hard Gate-2-to-matter sign remains unselected.

This separates two questions that had previously been entangled:

1. **relative internal orientation/sign**, and
2. **relative representation-resolved magnitude weighting.**

The present paper addresses the second.

---

17. Falsifiers

The following observations would invalidate or downgrade the strongest physical conclusions.

F1: primitive carrier mismatch

If the current response used for physical promotion acts on a different primitive carrier from the RUDM component marks, the mark-intensity attachment fails. The RUDM rigidity theorem remains mathematically true but cannot be used to type that current action as a primitive event law.

F2: source component law incomplete

If the true primitive source contains independently derived representation-dependent coefficients at the same RUDM event grain, the consumed source law was incomplete. Such coefficients must be returned by the source; they may not be introduced to fit a desired gauge matrix.

F3: quadratic class insufficient

If the leading physical current action contains additional same-order non-Hermitian, indefinite or symmetry-allowed structures outside the positive quadratic classification, the uniqueness theorem must be enlarged. The functional hazard-rigidity theorem still constrains any resulting positive same-grain mark intensities.

F4: factor-sensitive microscopic refinement

If the physical microscopic mark splitting is factor sensitive, Theorem 6 requires the explicit positive-semidefinite refinement correction, and Corollary 6.1 guarantees the correction is nonzero. A changed factor shape without such a source mechanism is not licensed.

F5: D_6 quadrature anisotropy

A physical mismatch between the two E_2 quadratures would falsify the separable $H\_CWY \otimes I_2$ continuation.

F6: current-stiffness relaxation fails on the physical source metric

The general source theorem proves $K\_J\text{^}relaxed(w) > 0$ for positive diagonal weights in the executed source family. If the true physical W_prim lies outside that family and produces a zero, negative or rank-deficient relaxed current block, the source-generated stiffness theorem cannot be promoted to the physical metric.

F7: factor-democracy / wheel-mode compatibility failure

The scalar physical pullback

$$M\_JJ\text{^}source = (49/50)\ K\_J\text{^}relaxed\ H\_CWY \otimes I_2$$

requires the faithful source current stiffness to act as one scalar on the six factor/quadrature directions. Current-link reciprocity proves that, under one common positive separable source metric, distinct physical wheel eigenvalues necessarily produce distinct current stiffnesses:

$$\lambda\_A \neq \lambda\_B \Rightarrow K\_J(\lambda\_A) \neq K\_J(\lambda\_B).$$

Therefore a faithful source that retains distinct gauge-sector wheel modes supersedes the scalar factor-democratic pullback unless the full nonfactorising metric or auxiliary structure restores equality by derivation. The authoritative fallback is

$$M\_JJ\text{^}source = E\_phys^\dagger\ \mathbb{K}\_J\text{^}relaxed\ E\_phys.$$

No factor equality may be imposed after the source return.

F7A: current-link reciprocity failure

If the faithful source is claimed to retain the same-parent current-connection architecture but returns

$\varepsilon\_dual = \| (\mathbb{K}\_J\^relaxed)^{-1} - W\_J^{-1} - (1/2)\, \mathcal{K}\, K\_\theta^{-1}\, \mathcal{K}^\dagger \|$

inconsistent with zero beyond regulator/numerical error, then the four-variable source block is incomplete at physical grade. Additional cross-defect or auxiliary structure must be retained. The discrepancy may not be absorbed into a fitted gauge coupling.

F8: physical current embedding changes

If the completed faithful current carrier does not preserve

$E\_phys^\dagger\, E\_phys = (49/50)\, H\_CWY \otimes I_2$

after all required source refinements, the current-action pullback must be recomputed. The present shape cannot be imposed.

F9: action/Fisher conflation

The source-action Hessian and the inherited history/Fisher metric may not be identified without a source theorem. Agreement of their C/W/Y shapes is not enough to set λ_J.

F10: continuum attaching map returns a different kinetic structure

If the discrete-to-continuum map sends the source current action to a gauge-invariant kinetic/current structure whose canonical matrix is not proportional to the present H_CWY after the declared portal and auxiliary shorting, that returned continuum structure supersedes the finite-source interpretation.

F11: Gate-2 ancestry failure

If the source-derived local parent Hessian is strictly positive definite on the image of every admissible Gate-2 spoke map, the alternating zero-cost transport mode cannot survive faithfully (Corollary 32.1) and AP1b-I fails permanently for the Gate-2 route.

F12: binary-occurrence overclaim

The published F2F/EX2E occurrence law is current independent to all orders. A nonzero conditional record response, record-phase response or source current stiffness may not be relabelled as a nonzero binary occurrence curvature.

F13: attenuation promoted as commitment

The nonlinear negative stretch response is an attenuation/escape precursor. It may not be renamed a positive fact-production law.

F14: mechanism zero promoted to force zero

The vanishing implemented binary eligibility coefficient is the zero of that mechanism only. It does not imply zero physical gauge interaction because the source current action is independently nonzero.

F15: diagnostic metric promoted to physical metric

The exact unit-branch values

$K_J^{relaxed} = 48/53$, $M_{JJ}^{source,unit} = (1176/1325)\ H_{CWY} \otimes I_2$

are not physical predictions until the primitive metric normalisation is source selected.

F16: native P1 factorisation rank loss

If the source-selected physical endpoint law reduces rank($D_A\ D_J\ Q_{phys}$) or rank($D_A\ p_{mark}\ Q_{phys}$) below 72, the native gauge/current provenance pass does not survive physical background selection. The present rank-72 result is a source-extension theorem, not permission to promote the deterministic endpoint witness.

F17: finite-current persistence overclaim

The weighted-Dirichlet theorem requires a connected graph, strictly positive physical conductances and a non-common initial innovation component. A pure spatial common mode lies exactly in the current kernel and must remain a legitimate zero-current case. No numerical γ_e may be issued without the source-selected joint endpoint law.

F18: hard-handshake promotion without event-grain attachment

The U(49)-covariant endpoint-handshake form reduces to a $\|z_t \pm U_e\ z_s\|^2$ and the oriented hard branch gives $\gamma_e = -1$, but the matter-tangent lift and the identification of that compatibility cost with the physical endpoint event law are not yet source derived. The value −1 is therefore a discriminator branch, not a physical prediction.

F19: Fold/vacuum relation promotion

The existence of six Fold binary relations, their three complement pairs, or the minimal algebra $A_\chi = J - I - S$ may not be cited as the physical vacuum attachment before a source-derived Fold-to-vacuum carrier map is returned. The Smith result is exact conditional mathematics, not yet physical provenance.

F20: orientation promotion

The two conjugate attachments generated by S and S^{-1} may not be physically selected merely because the primitive source already contains orientation. A source-derived map must prove that the Fold orientation acts as the successor orientation on the physical vacuum cell carrier.

F21: refinement promotion

The contractible complement-pair hub tree is an exact desingularising construction, but its existence does not prove that the physical K7 refinement mechanism installs it. The previously derived dynamical K7 contraction and the present topological hub refinement are differently typed operations.

F22: local/global Fold conflation

A simple commitment curve on the local spherical Fold interface may not be identified with the complete refined K7 vacuum one-skeleton. The candidate common refinement has minimum orientable genus one. Local spherical compatibility and global vacuum topology are distinct statements.

F23: embedding-genus / homology conflation

The statement genus($\tilde{W}$) = 1 refers to the minimum orientable embedding genus of the refined one-skeleton. It does not identify the completed vacuum complex with a torus. The $\mathbb{Z}/7$ first homology arises only after the physical two-cell attachments are imposed through A_phys, and a torus itself has torsion-free first homology. Graph embedding genus and cellular homology are differently typed invariants and may not be conflated.

---

18. What Has Moved Forward

The paper now contains a constructive sequence rather than only a sequence of narrowing audits.

The first advance is **relative rigidity**. The primitive RUDM law removes every representation-dependent positive hazard distortion at the same event grain: $\tilde{h}$_A = c w_A, globally, from one interior ray or one ledger. Within the source-equivariant quadratic class this gives $Q(J) = g \|J\|^2$ and fixes the C/W/Y geometry H_CWY.

The second advance is **refinement protection**. The factor matrix is unchanged exactly when microscopic splitting is factor blind; any genuine change has a named positive-semidefinite information term.

The third advance is **pre-time current existence**. The current is a primitive relational object whose divergence-free cycle component can persist without density evolution. Physical time is attached downstream.

The fourth advance is **source-generated dynamical current stiffness**. The exact EPMD current–record pair block does not disappear when the phase mediator is relaxed. Instead,

H_JJ^relaxed = (48/53) I_6

on the unit diagnostic branch. The phase mediator screens only 1/265 of the pre-phase stiffness. More generally,

K_J^relaxed(w) = 8 w_J (w_B w_C + w_B w_R + 4 w_C w_R) / (κ^2 w_B w_J + 4 κ^2 w_J w_R + 8 w_B w_C + 8 w_B w_R + 32 w_C w_R) > 0

for every positive primitive weighting in the source family. Thus the current action is not a special numerical artefact of one metric.

The fifth advance is the **physical factor pullback.** The certified six-direction current embedding has Gram (49/50) $H\_CWY \otimes I_2$, so the source action becomes

$M\_JJ^{source} = (49/50)\ K\_J^{relaxed}(w)\ H\_CWY \otimes I_2$.

This is the first point in the current-normalisation thread at which the primitive source itself supplies both:

1. the correct six-dimensional current carrier and relative C/W/Y geometry; and
2. a strictly positive quadratic action on that carrier after all relevant source auxiliaries have been relaxed.

The sixth advance is **current-link unification at the source-action level**. The relaxed current stiffness is not merely a positive scalar awaiting an arbitrary connection to gauge dynamics. The same primitive four-variable source block returns a positive link stiffness

$K\_\theta = 4\ w\_C + 4\ w\_B\ w\_R\ /\ (\ 4\ w\_R + w\_B\ )$,

and the two responses satisfy

$1/K\_J^{relaxed} = 1/w\_J + \kappa^2/(2K\_\theta)$.

Thus the finite-source relative normalisation between current and matched link response is fixed by one parent action. Its faithful matrix extension supplies an explicit physical residual ε_dual.

The seventh advance is the **factor-democracy compatibility gate**. Applying the reciprocity theorem to the executed MASD wheel modes shows that distinct physical wheel modes would produce distinct current stiffnesses under a common separable metric. The source must therefore decide between scalar factor democracy and genuinely factor-dependent/nonfactorising physical response. This converts a previously hidden branch ambiguity into a sharp calculation.

The eighth advance is **native gauge/current provenance**. The old single-dΦ embedding request is retyped as a graded source jet. At the correct derivative orders the gauge, completion, record-phase and CAR/Fock sectors already descend faithfully. On the native representation-valued current slice, the actual 144-dimensional current-defect and mark derivatives have full rank and factorise at 2.58×10^{-15}; the Wilson quotient remains rank 72 with 1.80×10^{-15} physical reconstruction. The former HM4 pass/fail embedding interval therefore no longer governs this native slice.

The ninth advance is **finite-refinement current persistence**. The GSJB innovation and MASD endpoint tangent are now exactly carrier matched. For arbitrary strictly positive edge conductances, the weighted Dirichlet generator has only the spatial common mode in its kernel, and every finite non-common innovation has strictly positive relative-current energy. Uncertainty in the conductances changes the magnitude and decay rate but cannot switch the current off at finite depth.

The tenth advance is a **Gate-T vacuum-topology reduction.** The actual K7 cycle lattice has now been isolated as $\mathbb{Z}^6$, and the obvious first-cover and simple-wheel routes to sevenfold topology have been excluded exactly. The unordered Fold relation supports are also too low-rank. Once complement and oriented succession are both retained, the minimal local unit-incidence algebra has an essentially unique order-seven candidate, $A_\chi = J - I - S$, with Smith form diag(1, 1, 1, 1, 1, 7). The apparent conflict with the Fold simple-boundary theorem is resolved at construction level by a contractible hub refinement organised by the same three Fold complement pairs: all six candidate boundaries become simple C_{13} loops while $\beta_1 = 6$ and the $\mathbb{Z}/7$ torsion survive unchanged. The refined one-skeleton has minimum orientable embedding genus one, so the live physical problem is now the source derivation of a local-Fold-to-global-vacuum refinement/gluing map rather than the invention of a candidate topology.

The binary first-record sequence then supplies a **boundary**, not the main result. It proves that the existing F2F selector does not read the current action as a change in fact/no-fact occurrence. This eliminates a mis-typed readout route without eliminating the current stiffness.

The frontier has consequently moved again. The outstanding problem is no longer

> "How should the already-derived current stiffness be connected to a gauge stiffness?"

because that relative connection is now fixed within the common source block. The outstanding problem is

> "What faithful W_prim, current-connection map and wheel/internal mode assignment does the physical source return, and how does that returned link stiffness become the continuum dimension-four Yang–Mills kinetic coefficient?"

That is a materially stronger position. The next successful calculation can now remove a genuine physical normalisation rather than discover another candidate mechanism.

PART II: THE AP1b COMMON-PARENT ATTACHMENT AND MAGNITUDE-BEARING EVENT-LAW AUDIT

Question. Can the current VERSF source stack prove that the Gate-2/raw-current response feeds the same primitive component-resolved event intensities as RUDM?

Verdict. Common-parent attachment criterion: exactly derived. Full AP1b: not yet proved. Linear-amplitude realisation: negative on the published stack. Binary-curvature realisation: positive existence witness, with the physical source coefficient still open.

The audit nevertheless reduces the open statement to a small set of exact operator equalities, so AP1b stops being a philosophical premise and becomes a finite computation.

---

19. RUDM as a Primitive Event-Effect Identity

Let the 49-complex-dimensional internal commitment carrier be $\mathcal{H}$, with exhaustive mutually orthogonal projectors

$\Sigma_{A=1}^{12} P_A = I_{\mathcal{H}}$.

For a fixed primitive opportunity and record state, let $E_A \succeq 0$ be the coarse primitive event effect for component A. The event intensity on a unit ray z is

$h_A(z) = \langle z, E_A z \rangle$.

RUDM requires

$h_A(z) = h_0 \|P_A z\|^2$

for every source-supported ray, with one common h_0.

Theorem A (RUDM effect-equivalence theorem)

For a full-support set of rays,

$\mathbf{h_A(z) = h_0 \|P_A z\|^2 \text{ for all } z \Leftrightarrow E_A = h_0 P_A \text{ for all } A.}$

Proof

The forward implication follows because

$\langle z, (E_A - h_0 P_A) z \rangle = 0$

for every z, and the complex polarization identity gives $E_A - h_0 P_A = 0$. The converse is immediate. ■

Remark (source-almost-everywhere suffices)

The hypothesis need not hold on literally every ray. If the quadratic-form equality holds on a set of rays of full Fubini–Study measure, that set is dense in the unit sphere because the Fubini–Study law has full support (Lemma 0); the map $z \mapsto \langle z, (E_A - h_0 P_A) z \rangle$ is continuous, so it vanishes on the whole sphere, and polarization applies as before. The effect identity is therefore forced by exactly the support the source law itself guarantees, with no extra regularity assumption.

Summing over A,

$\mathbf{\Sigma_A E_A = h_0 I_{\mathcal{H}}.}$

Thus AP1b can be tested at operator level: the physical primitive instrument must coarse-grain to the RUDM projector effects with one common multiplier. This supplies a finite operator release test for the primitive effect identity.

---

20. SGPL Gives the Correct Common-Parent Architecture

SGPL writes the event-resolved primitive Kraus density as

$$K_e(y, x) = P_y P_{r(e)} e^{-a_t \hat{H}/2} P_x.$$

For the component-A coarse event, define

$$E_A^{SGPL} = \int_{r(e)=A} \int K_e(y, x)^\dagger K_e(y, x) \, \nu(dy, de).$$

Therefore the same physical parent $\hat{H}$ determines the primitive event effects.

Theorem B (common-parent attachment theorem)

If the Gate-2/raw-current source response is a term in the **same** parent generator $\hat{H}$ used in the SGPL primitive Kraus density, its variation necessarily changes the same primitive event effects E_A^{SGPL}. For

$$\hat{H}(\varepsilon) = \hat{H}_0 + \varepsilon \dot{H} + O(\varepsilon^2),$$

Duhamel differentiation of $U = e^{-a_t \hat{H}/2}$ gives

$$\dot{U} = -(a_t/2) \int_0^1 e^{-(1-s) a_t \hat{H}_0/2} \dot{H} e^{-s a_t \hat{H}_0/2} \, ds,$$

and hence

$\dot{E}_A = \int (\dot{K}_e^\dagger K_e + K_e^\dagger \dot{K}_e) \, d\nu.$

No separate “cost = probability” axiom is needed. A source cost feeds primitive event probabilities by entering the common parent transfer, not by being identified directly with a Born probability. ■

This also resolves the earlier semantic problem that a Gate-2 completion cost vanishes at exact completion. If a scalar cost Q enters the Euclidean parent, its transfer factor is

$$e^{-a_t Q/2},$$

so $Q = 0$ gives unit transfer weight, not zero event probability. A vanishing cost is the least suppressed transfer, exactly as completion semantics require.

---

21. Exact Operator Criterion for Reproducing the RUDM Law

In the simplest internally closed form, after exhaustive next-state coarse-graining the SGPL effect is

$$E_A = U^\dagger P_A U.$$

Then Theorem A gives

$U^\dagger P\_A U = h_0 P\_A$ for all A.

Summing over A,

$U^\dagger U = h_0 I$.

Hence

$U = \sqrt{h_0} \cdot V$

with V an isometry, and the individual projector identities imply

$V^\dagger P\_A V = P\_A$.

On a finite equal input/output carrier, V is unitary, and $V^\dagger P\_A V = P\_A$ for a unitary is equivalent to

$[V, P\_A] = 0$ for all A,

that is, V lies in the block-diagonal group $\bigoplus\_A U(d\_A)$: the commutant of the projector algebra. This dovetails exactly with Theorem 4: the admissible transfer, like the admissible quadratic response, must respect the twelve-sector commutant structure, and the only new freedom the transfer adds is the common norm.

Therefore

RUDM primitive relative hazards ⟺ common transfer norm + commitment-projector preservation.

For a positive Euclidean internal transfer $U = e^{(-a\_t H\_int/2)}$, the criterion reduces further: a positive operator that is a scalar multiple of a sector-preserving unitary must be that scalar times the identity, so

$H\_int = q\, I\_\mathcal{H}$,

an internally scalar generator, up to degrees of freedom external to the twelve-component internal carrier.

---

22. Scalar Gate-2 Cost Can Feed the Same Event Law Only as a Common Multiplier

Suppose a Gate-2/completion cost Q is established as a scalar local term in the same parent generator,

$\hat{H} = \hat{H}_0 + Q\, I\_\mathcal{H}$.

Then

$U = e^{(-a\_t Q/2)} U_0$

and

$E_A = e^{(-a_t Q)} E_{(A,0)}$.

If the baseline effects are RUDM,

$E_{(A,0)} = h_0 P_A$,

then

$$\mathbf{E_A = (h_0 e^{(-a_t Q)}) P_A.}$$

The conditional component law is unchanged. Thus a scalar Gate-2 cost can feed the same primitive RUDM event process, but only through the common opportunity/intensity scale.

This is precisely the surviving freedom of Theorem 1A made concrete: the common scalar functional is realised as

$c = h_0 e^{(-a_t Q)}$,

a legitimate dependence on common opportunity/state variables with no A-dependent distortion. The rigidity theorem and the common-parent mechanism therefore fit together without residue: everything a scalar cost can do lands exactly in the one dimension the source law leaves open.

---

23. Exact Current-Ray Factorisation

Let

$J = r z_J$, $r = \|J\|$, $\|z_J\| = 1$.

For the quadratic current component law,

$\lambda_A = \lambda_0 \|P_A J\|^2$,

one has identically

$$\boldsymbol{\lambda_A = \lambda_0 r^2 \|P_A z_J\|^2.}$$

Therefore, if the RUDM directing ray is the same realised ray as the normalised current direction,

$z = z_J$,

the quadratic current law has exactly the RUDM form

$$\mathbf{h_A = h_0 \|P_A z\|^2, h_0 = \lambda_0 r^2.}$$

The current magnitude then enters the event law precisely as the common clock/intensity scale of Section 12, and the relative law is automatic. This is the second concrete realisation of the surviving scalar functional: $c = \lambda_0 r^2$.

The remaining issue is source identity: the earlier Primitive Birth-Current Tangent Identity proves this ray equality only conditional on the GSJB birth mark being the same endpoint innovation entering MASD.

---

24. Binary Commitment Retypes the Magnitude-Bearing Gate

There are three distinct magnitude mechanisms, and they must not be conflated.

A scalar local cost in a Euclidean parent gives a common transfer factor such as

$e^{(-a_t Q(J))}$.

If $Q(J) = g \|J\|^2$ enters only as a scalar Euclidean cost,

$e^{(-a_t g \|J\|^2)} = 1 - a_t g \|J\|^2 + O(\|J\|^4)$,

which modulates an already-existing event rate; it suppresses events as the current grows and gives maximal transfer at $J = 0$.

A linear current-to-mark amplitude

$\mathcal{L}_A(J) \propto P_A J$

gives a quadratic mark intensity

$\lambda_A \propto \|P_A J\|^2$,

with the opposite small-current behaviour: zero intensity at $J = 0$.

A third possibility is specific to the binary fact ontology: the current may change the occurrence or waiting rate of **complete** facts at quadratic order, without introducing any continuously variable degree of record completeness.

For one primitive opportunity write

$p_0 = 1 - h_F$, $p_A = h_F q_A$,

where p_0 is no new fact and p_A is one complete fact of component A. The integrated first-record intensity is

$\boldsymbol{\Gamma_F = -\log(1 - h_F).}$

A magnitude-bearing binary current law may therefore begin as

$\Gamma_{(F,A)}(J) = \Gamma_{(F,A)}(0) + g \|P_A J\|^2 + O(\|J\|^3)$, or equivalently

$\frac{1}{2} D_J^2 \Gamma_{(F,A)}(0) = g P_A$.

The same information can be read from the curvature of the state-resolved no-record block NR(J). A nonzero derivative $D_J \mathcal{K}_A$ is therefore one sufficient microscopic realisation, but it is **not ontologically mandatory**. The primitive release object may instead be

$D^2_{(J_{cyc})} \Gamma_F$, or equivalently $D^2_{(J_{cyc})}$ NR.

This retyping does not manufacture a physical current dependence. It only states the correct binary target. Under AP1b, the hazard rigidity of Part I then forces the same coefficient g across the twelve component sectors at the same primitive mark grain.

---

25. Published-Stack Execution After Binary Retyping

The current corpus returns:

1. **RUDM primitive component effects:** the twelve projectors and $h_A = h_0 \|P_A z\|^2$.
2. **Primitive shared-interface localisation:** one primitive write before later copying.
3. **Common-parent marked-transfer architecture:** SGPL gives $K_e = P_y P_{(r(e))} e^{(-a_t \hat{H}/2)} P_x$.
4. **History-parent architecture:** HMGBC gives $\hat{H} = -\hat{\Delta} + V_{loc} + \cdots$ and $\hat{T} = e^{(-a_t \hat{H})}$.
5. **A positive representation-resolved current-cycle history witness:** RCCSJ supplies a real marked jump generator with the required C/W/Y × two-quadrature score support.

The published physical stack does **not** yet return:

1. an executed equality putting the representation-resolved Gate-2/raw-current term inside the fully source-derived V_{loc};
2. the full-carrier coarse-graining identity equating the SGPL terminal/component projectors with the twelve RUDM P_A;
3. an unconditional same-event equality between the RUDM directing ray and the normalised physical current ray;
4. a source-selected magnitude-bearing current law on the completed physical first-record block.

PCRS-1R proves the linear-amplitude statement negatively for the current published stack: the existing forward marked machinery does not return a nonzero $D_{(J_{cyc})} \mathcal{K}_A$.

BCTR-2 changes the interpretation, but not the physical grade. On the RCCSJ positive current-cycle witness,

$D_{(J_{cyc})} \Gamma_F |_0 = 0$,

while

$D^2\_(J\_cyc)\ \Gamma\_F\ |_0 \neq 0.$

For the indispensable m = 2/4 component, the exact state-resolved curvatures are

$D^2\ \Gamma\_F\ |\_hub = g\_cyc^2\ (3/112)\ H\_CWY \otimes I_2,$

$D^2\ \Gamma\_F\ |\_rim = g\_cyc^2\ (5/384)\ H\_CWY \otimes I_2,$

and the stationary average is

$\mathbb{E}\_\pi\ [\ D^2\ \Gamma\_F\ ]\_(m{=}2/4) = g\_cyc^2\ (1/62)\ H\_CWY \otimes I_2.$

This is exactly the RCCSJ m = 2/4 marked Fisher metric. The complete current cycle gives $g\_cyc^2\ (2/31)\ H\_CWY \otimes I_2$.

The status is therefore split:

- linear current-to-record amplitude on the published stack: **exact negative return**;
- quadratic binary first-record curvature in the RCCSJ positive law: **exact existence pass**;
- source selection of g_cyc on the completed physical H_CMR: **open**;
- physical λ_J: **open**.

The positive witness cannot be promoted by choosing g_cyc = 1, by matching its baseline waiting time to MFKR, or by importing a preferred Standard-Model coupling.

---

26. Revised AP1b Decomposition

The vague statement “Gate-2/raw current feeds the same RUDM event” is replaced by three exact gates. Within the adopted SGPL/HMGBC architecture, AP1b is operationally reduced to their conjunction: each gate is necessary for the physical use made of AP1b in this paper, and together they supply everything that use consumes. A standalone necessity-and-sufficiency proposition outside that architecture is not claimed.

AP1b-I: common-parent primitive mark ancestry

Prove

$\hat{H}\_physical = \hat{H}_0 + \hat{H}\_(Gate2/current) + \cdots$

inside the same source-derived SGPL parent.

AP1b-II: mark-projector and realised-ray identity

Prove the SGPL coarse effects use the same P_A as RUDM and, on the current branch,

$\rho\_J = |z\rangle\langle z|$

for the same realised primitive event.

AP1b-III: magnitude-bearing complete-fact event law

Return from the same source either a linear amplitude realisation

$D_{(J\_cyc)} \mathcal{K}\_A = c\, V\_A\, P\_A$,

or the more primitive binary-rate curvature

$\frac{1}{2} D^2_{(J\_cyc)} \Gamma_{(F,A)}(0) = g\, P\_A,\ g > 0$,

with the same common coefficient across all twelve sectors, or equivalently the corresponding curvature of NR(J_cyc).

The completed return must act on the full $m = 0 \oplus m = 2/4$ current cycle and have rank six after C/W/Y resolution. If the source uses the linear-amplitude realisation, g is the amplitude norm squared after the declared mark convention, and $\lambda\_A = |c|^2\, \|P\_A\, J\|^2 = |c|^2\, \|J\|^2\, w\_A$ automatically has the RUDM relative law. If it uses a direct quadratic compensator, g is read from the complete-fact rate itself. In either case, no partially written record is required.

Status

AP1b-I is architecturally supported but not executed for the Gate-2/current term; Part III rules out the canonical-wheel pullback shortcut and reduces the gate to the source return of L_λ. AP1b-II is partly supported by shared-interface localisation and remains conditional on the birth-current tangent identity. AP1b-III now has an exact positive existence witness through RCCSJ/BCTR-2 (Part V), while its physical completed-source coefficient remains open. The earlier linear-amplitude failure is retained but no longer treated as the only possible magnitude mechanism.

---

27. Audit Verdict

The attempted proof does not close full AP1b from the currently published stack. It does, however, reduce the physical question to finite source objects:

$E\_A^{physical} = h_0\, P\_A$

and

$\frac{1}{2} D^2_{(J\_cyc)} \Gamma_{(F,A)}(0) = g\, P\_A$

on one common primitive carrier and event grain, with a nonzero g fixed before any downstream comparison. A source-derived linear amplitude $D_{(J\_cyc)} \mathcal{K}\_A = c\, V\_A\, P\_A$ is one sufficient factorisation of the second identity.

If the completed source returns the stated curvature, hazard rigidity removes every relative representation coefficient and the common physical scale is obtained from the same source law. If it returns a nonzero curvature with a different factor shape, the inherited terminal-Born/current-cycle continuation is incomplete and must be replaced by the source matrix. If it returns zero on the full representation-resolved carrier, the magnitude-bearing binary-trigger route is excluded at the retained order.

The audit therefore converts the paper's central conditional from a verbal attachment premise into a source-level decision procedure.

PART III: THE AP1b-I GATE-2 COMMON-PARENT ANCESTRY AND CANONICAL-WHEEL PULLBACK AUDIT

Question. Is the Gate-2 closure-competition cost already a derived term of the same physical parent generator that produces the primitive RUDM/SGPL event law?

Verdict. AP1b-I not yet proved. A tempting shortcut can now be ruled out exactly: **the Gate-2 spoke competition cost is not a faithful pullback of the existing positive-definite HMGBC canonical-wheel Fisher metric**, nor of any positive-definite compressed metric. The remaining source target is therefore much sharper: derive the Gate-2 spoke functional directly from the full MASD/HMGBC local parent V_loc, before Fisher compression, not from the already-compressed W_wheel.

---

28. Source-Provenance Audit of the Gate-2 Cost

The sequential-transport source introduces

$A_{cl} = A_{inc} + A_{hub} + A_{circ} + A_{comp}$

and defines the update as an admissibility-restoring gradient step. However, the same source explicitly grades

$A_{comp} = \Sigma_{i=0}^{5} (\lambda_i + \lambda_{i+1})^2$

as the **load-bearing postulate**, stating that whether this quadratic form is physically correct must ultimately be derived from the VERSF master action.

Thus the Gate-2 cost currently has a structural/variational derivation role, but it is not yet an executed term of the source-derived HMGBC parent generator.

HMGBC, separately, puts the MASD local quadratic rate into the static history response,

$\Gamma_{defect} = ½ \langle D, W_{prim} D \rangle$, $W_{prim} = W_{hist}(0)$, and places the direct local history cost inside V_loc of the full master history generator.

Therefore the exact AP1b-I target is:

A_comp =? V_loc^physical restricted to the Gate-2 spoke carrier,

or the corresponding source-derived pullback relation. The current corpus does not yet return that equality.

---

29. The Gate-2 Spoke Quadratic Form and Its Alternating Zero Mode

Define the quadratic-form matrix Q_comp by

$A\_comp(\lambda) = \lambda^T\ Q\_comp\ \lambda,\ \lambda \in \mathbb{R}^6,$

equivalently $D\_\lambda^2\ A\_comp = 2\ Q\_comp$. Let P be the cyclic shift on the six spokes. Because each term of A_comp is $(\lambda\_i + \lambda\_(i+1))^2 = ((I + P)\lambda)\_i^2$, the quadratic-form matrix factorises exactly:

$Q\_comp = (I + P)^T\ (I + P),$

a manifestly positive-semidefinite C_6 circulant whose kernel is exactly ker(I + P), the amplitudes with $P\lambda = -\lambda$. Explicitly,

Q_comp = [2 1 0 0 0 1 ; 1 2 1 0 0 0 ; 0 1 2 1 0 0 ; 0 0 1 2 1 0 ; 0 0 0 1 2 1 ; 1 0 0 0 1 2].

The C_6 Fourier modes diagonalise it with the closed-form spectrum

$\lambda\_k = 2 + 2\cos(2\pi k/6) = 4\cos^2(\pi k/6),\ k = 0, \ldots, 5,$

that is, exactly

{ 0, 1, 1, 3, 3, 4 }, rank Q_comp = 5.

The zero mode is the k = 3 (alternating, Nyquist) Fourier mode: the nontrivial alternating transport direction

$a = (1, -1, 1, -1, 1, -1)^T / \sqrt{6},\ Q\_comp\ a = 0,$

and the factorisation shows this kernel is structural, not numerical: for the even cycle C_6 the equation $P\lambda = -\lambda$ has exactly the one-dimensional alternating solution.

The on-admissible-sector circulation penalty is

$A\_circ = (\ \Sigma\_i\ \lambda\_i\ )^2,\ Q\_circ = \mathbb{1}\mathbb{1}^T.$

Hence

Q_Gate2,on = Q_comp + Q_circ

adds 6 to the uniform (k = 0) eigenvalue only, giving spectrum

{ 0, 1, 1, 3, 3, 10 },

and since $a \perp \mathbb{1}$,

$Q_{Gate2,on}\, a = 0$.

Thus the alternating mode remains a genuine zero-cost direction of the Gate-2 on-sector functional. All spectra and the kernel identity are reverified in Appendix C.4.

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30. The HMGBC Canonical-Wheel Metric

The existing HMGBC scalar canonical-wheel Fisher metric is

W_{wheel} = [12/31 0 0 0 0 0 ; 0 75/434 0 0 0 −33/434 ; 0 0 21/124 0 3/248 0 ; 0 0 0 12/31 0 0 ; 0 0 3/248 0 21/124 0 ; 0 −33/434 0 0 0 75/434].

Under the wheel symmetry it decomposes into two singleton blocks and two 2 × 2 blocks, whose eigenvalues follow by inspection: the singletons give 12/31 twice, the {2, 6} block gives (75 ± 33)/434 = 54/217 and 3/31, and the {3, 5} block gives 21/124 ± 3/248 = 45/248 and 39/248. The exact spectrum is therefore

{ 3/31, 39/248, 45/248, 54/217, 12/31, 12/31 },

so

$W_{wheel} > 0$, rank $W_{wheel} = 6$.

There is no information-null direction in this six-scalar representative metric.

---

31. Exact Rank/Inertia No-Go

Suppose the Gate-2 spoke cost is identified directly with the canonical HMGBC metric through an invertible six-dimensional carrier map R:

$Q_{Gate2,on} = R^\dagger W_{wheel} R$.

By Sylvester's law of inertia, an invertible congruence preserves the numbers of positive, negative and zero eigenvalues.

But the inertias differ:

$Q_{Gate2,on}$: (5 positive, 0 negative, 1 zero); W_{wheel} : (6 positive, 0 negative, 0 zero).

Therefore

$Q_{Gate2,on} \neq R^\dagger W_{wheel} R$ for every invertible R.

The two objects cannot be the same six-dimensional metric in different coordinates.

---

32. Faithful-Pullback No-Go and Its Interpretation

Allow a non-invertible map L:

$Q_{Gate2,on} = L^\dagger W_{wheel} L$.

Because $W_{wheel} > 0$,

$a^\dagger Q_{Gate2,on} a = (L a)^\dagger W_{wheel} (L a) = 0$

implies

$L a = 0$.

But the alternating Gate-2 mode a is **not** a zero physical transport. It is the named lowest closure-competition wave: a nontrivial homological transport response with unchanged committed endpoints but changed closure-history class.

Hence any map L that reproduces the Gate-2 cost from W_wheel must annihilate precisely the nontrivial Gate-2 response that the transport construction is built to retain. Therefore:

no faithful linear pullback of W_wheel can generate the Gate-2 on-sector cost.

This is stronger than observing different numerical spectra.

Corollary 32.1 (the obstruction is metric independent)

Nothing in the argument used the particular entries of W_wheel, only its strict positivity: for any positive-definite M and any linear L, $x^\dagger (L^\dagger M L) x = 0$ forces $L x = 0$. Hence **no faithful linear pullback of any positive-definite compressed metric whatsoever can generate the Gate-2 on-sector cost** while keeping the alternating transport nontrivial. The obstruction is a structural mismatch between a cost with a genuine admissible zero direction and the class of positive-definite information metrics, not a numerical accident of the wheel representative. Any resolution must therefore occur at an object that is itself degenerate along the relevant direction, before positive-definite compression. ■

Interpretation

This does **not** prove that Gate-2 can never belong to the same physical parent as RUDM.

It proves that the parent cannot be identified by the shortcut

$A_{Gate2} \equiv$ canonical W_wheel Fisher cost.

The two objects have different roles: Gate-2's zero mode is an admissible nontrivial transport response, while W_wheel is a positive-definite Fisher metric on six scalar defect-score directions. A common physical parent could contain both, but the map must occur one level earlier, before the positive-definite defect-score compression.

This is consistent with HMGBC's own Defect-Lift Obligation: the full typed MASD defects must be mapped faithfully and uniquely into the history tangent; the existing scalar representative metric is not itself that full-carrier lift.

---

33. The Exact Next Source Calculation for AP1b-I

The next calculation is now unambiguous.

Let $\lambda \in \mathbb{R}^6$ be the six Gate-2 spoke amplitudes and let $X(\lambda)$ be their image in the **full MASD/HMGBC local configuration carrier before stochastic/Fisher compression**.

The missing source object is the Jacobian

L_λ = D_λ X |_(λ=0).

Let

$H\_loc = D\_X^2 V\_loc |\_(X\star)$

be the source-derived local parent Hessian. Then compute

Q_parent^(λ) = L_λ† H_loc L_λ.

The hard AP1b-I test is:

Q_parent^(λ) =? α Q_Gate2,on

at the declared order, or a source-derived extension whose restriction has the same alternating kernel and boundary sum-square structure.

The pass requires all of:

1. **carrier provenance:** L_λ is returned from the substrate/Fold action, not chosen;
2. **nontriviality:** L_λ a ≠ 0 for the alternating transport mode;
3. **zero-cost compatibility:** the parent cost vanishes on that nontrivial admissible mode;
4. **quadratic shape:** the spoke pullback returns the C_6 nearest-neighbour sum-square at the retained order;
5. **shared-boundary continuation:** the same parent returns the cross-hub boundary square rather than adding it by hand.

Corollary 32.1 makes conditions 2 and 3 jointly sharp: they require H_loc to be **degenerate along the image L_λ a**, so the parent Hessian must itself carry a kernel containing that image. A strictly positive-definite H_loc on the image of every admissible spoke map would reproduce the impossible positive-metric pullback and fail the test permanently; this is falsifier F11.

If the test passes, Gate-2 becomes an actual term of the same local parent that SGPL exponentiates into primitive event transfers, and AP1b-I closes.

If it fails, Gate-2 remains an admissibility/transport construction rather than the physical current-mark source.

---

34. AP1b-I Audit Grade

- Gate-2 A_comp provenance from the VERSF master action: **open; explicitly labelled the load-bearing postulate by its source paper**.
- HMGBC W_wheel: **exact on the canonical scalar stochastic representative; full typed defect lift still open**.
- Direct/invertible identification Q_Gate2 ↔ W_wheel: **exactly ruled out** (inertia).
- Faithful positive-metric pullback L† W_wheel L = Q_Gate2: **exactly ruled out**, because it annihilates the nontrivial alternating transport mode, and by Corollary 32.1 the same holds for every positive-definite compressed metric.
- AP1b-I common-parent ancestry: **not yet proved**.
- Next required object: **the full-carrier Gate-2-to-V_loc Jacobian L_λ and the parent-Hessian pullback L_λ† H_loc L_λ**, with the five pass conditions of Section 33.

PART IV: THE PRE-PHYSICAL-TIME CURRENT AND TEMPORAL-ATTACHMENT AUDIT

34A. Question and Verdict

Question. Does the VERSF current require physical time in order to exist, or can the current be defined and conserved at the primitive record/refinement level before Lorentzian commitment time is attached?

Verdict.

- **Pre-physical-time constitutive current: pass.**
- **Nonzero time-independent current-cycle existence: exact pass at MASD current-carrier/residual grade.**
- **Primitive refinement-continuity interpretation: pass / inherited.**

- **Canonical current-divergence / refinement-generator match: exact on the reversible K = 7 branch.**
- **σ-to-δ same-step attachment: open.**
- **Physical-time-normalised discrete current: conditional on that attachment.**
- **Full continuum Lorentz four-current: open.**

The important structural conclusion is already available before the final two steps: VERSF does not need to assume physical time in order to define its primitive current. Physical time can be attached downstream as a ruler for that current.

34B. The Pre-Physical-Time Constitutive Current

MASD defines the current defect on an oriented edge e: $s \to t$ by

$D_J(e) = J_e - \kappa_e (\rho_t - U_e \rho_s)$.

The defining data are:

1. a source state ρ_s;
2. a target state ρ_t;
3. an oriented transport U_e;
4. a current J_e;
5. the conductance/current coefficient κ_e.

No physical duration, temporal coordinate t, frequency, Lorentzian proper time or clock interval occurs in the definition.

Theorem C (pre-physical-time current existence)

Grade: exact at the MASD constitutive-current level.

The MASD current is a well-defined oriented relational object before physical-time attachment.

At the neutral common background,

$\rho_s = \rho_t = \Omega$, $J = 0$, $U \Omega = \Omega$,

and faithful infinitesimal gauge-link variations satisfy

$(\delta U) \Omega = 0$.

The linearised current defect is therefore

$\delta D_J = \delta J - \kappa (\delta\rho_t - \delta\rho_s)$

in a common local frame. On the constitutive zero locus,

$\delta J = \kappa (\delta\rho_t - \delta\rho_s)$.

Thus the Relative-Birth Current is determined by relative endpoint innovation. No physical-time derivative is required.

This proves a precise statement and no more:

primitive current existence does not require physical time.

It does not yet say that every physical current is reducible to endpoint birth. The current-cycle calculation below proves that stronger identification false. ■

34C. The Nonzero Current-Cycle Certificate

Let

$P_{cyc} = P_{(\ker B)} \otimes I_{49}$.

The full MASD current carrier contains the divergence-free subspace

$\mathcal{C}_J^{cyc} = \ker B \otimes \mathbb{C}^{49}$.

For every pure cycle-current perturbation,

$\delta J_{cyc} \in \mathcal{C}_J^{cyc}$,

one has

$(B \otimes I_{49}) \delta J_{cyc} = 0$.

At the same time the current-cycle protection theorem gives

$P_{cyc} \delta D_J = P_{cyc} \delta J$,

and on the pure cycle subcarrier,

$\delta D_J = \delta J$.

Theorem D (nonzero current without physical-time evolution)

Grade: exact at the linearised MASD current-carrier/continuity level.

A nonzero current may satisfy

$\delta J_{cyc} \neq 0$

while

$(B \otimes I_{49}) \delta J_{cyc} = 0$.

For a static cycle variation with no independent density update,

$\partial_\sigma \delta\rho = 0$,

the isolated transport-continuity contribution is therefore

$\delta D_T [\delta J_{cyc}] = 0$.

Hence VERSF admits

$J \neq 0$, no density evolution, $\mathrm{div}\, J = 0$.

This is the cleanest certificate that the primitive current is not defined as "amount of record change divided by physical elapsed time." A current can exist as a persistent circulation in the relational record structure. ■

The result is physically relevant to the current programme rather than an unused topological possibility. For the physical CRTF radial-current profile,

$\|q_\mu q_+\|^2 = 5/3$,

with exact Hodge decomposition, orthogonal because $\mathbb{R}^E = \ker B \oplus \mathrm{im}\, B^T$,

$\|P_{(\ker B)} (q_\mu q_+)\|^2 = 1$, $\|P_{(\mathrm{im}\, B^T)} (q_\mu q_+)\|^2 = 2/3$,

whose parts indeed sum: $1 + 2/3 = 5/3$. Therefore

$\|P_{(\ker B)} (q_\mu q_+)\|^2 / \|q_\mu q_+\|^2 = 3/5$.

Exactly 60% of the squared magnitude of the required CRTF current tangent lies in a cycle component which cannot be generated by endpoint differences.

Consequently the primitive current cannot generally be identified with a simple source-to-target fact-birth difference.

34D. Primitive Continuity Before Physical Time

The general MASD transport-conservation defect is

$D_T = \partial_\sigma \rho + \partial_U J$.

The inherited refinement audit does not type σ as ordinary physical time. It types σ as the update/refinement parameter along the substrate history flow.

Thus the primitive conservation statement is more accurately read as

change along ordered refinement + current divergence = 0,

rather than as a conservation law already written against a background clock.

HMGBC supplies the corresponding history generator

$\mathcal{G}$_hist f = $-r^{-1}$ ($\hat{H} - E_0$) (r f),

so the natural source completion is

∂_σ ρ = $\mathcal{G}$_hist ρ.

The primitive continuity equation becomes

$\mathcal{G}$_hist ρ + ∂_U J = 0.

On the canonical reversible K = 7 branch this relation is substantially stronger than a typing proposal. Let

$C = (1/31)\, I_{12}$.

The MASD current divergence gives

L_J = B C B^T,

while the independently constructed HMGBC Dirichlet form gives

$Q_7 = \Pi\, (I - T_0)$.

The exact execution returns

B C B^T = Q_7 = (1/31) L_(W_7).

Therefore the current-divergence operator and the primitive history/refinement operator coincide exactly on the canonical branch.

Two remarks sharpen the status of this identity. The MASD half is an exact incidence identity: with $C = (1/31)\, I_{12}$ on the twelve edges of the K = 7 wheel, B C B^T = (1/31) B B^T = (1/31) L_(W_7) holds by linearity and the standard incidence-Laplacian identity, reverified in Appendix C.5. The substantive certificate content is therefore the HMGBC half: that the independently constructed history Dirichlet form $Q_7 = \Pi(I - T_0)$ lands on exactly the same operator. It is that independent agreement, not the incidence identity, which establishes the generator match.

This establishes a pre-physical-time conservation architecture in which the current and the refinement dynamics are independently constructed but generator matched.

34E. Attachment to Emergent Physical Time

The temporal programme independently supplies the additive Fold-opportunity parameter δ,

$R(\delta_1 + \delta_2) = R(\delta_1)\, R(\delta_2)$,

with δ = 1 one complete sequential opportunity.

The sequential opportunity-to-Lorentzian attaching theorem gives, on its declared local homogeneous branch,

$$t_{phys} = T(\delta) = \delta\, a_t / c^\star.$$

Physical time is therefore downstream of the primitive ordering law.

The current/refinement programme uses σ, while the temporal attaching theorem physicalises δ. The present corpus strongly aligns their intended roles but does not yet contain an explicit same-source theorem identifying their normalisations.

Introduce the source bridge

$$\sigma = s_{\sigma\delta}\, \delta, \quad s_{\sigma\delta} > 0.$$

The strict same-step identity is the special case $s_{\sigma\delta} = 1$.

Theorem E (conditional physical-time current conversion)

Grade: exact conditional consequence of the σ-to-δ source bridge.

Start from the mass-weighted primitive continuity equation

$$\Pi\, \partial_\sigma \rho + B J = 0.$$

Since $\sigma = s_{\sigma\delta}\, \delta$ and $t_{phys} = (a_t/c^\star)\, \delta$,

$$\partial\rho/\partial\sigma = (a_t / (s_{\sigma\delta}\, c^\star))\, \partial\rho/\partial t_{phys}.$$

Therefore

$$(a_t / (s_{\sigma\delta}\, c^\star))\, \Pi\, \partial_t \rho + B J = 0.$$

Multiplying by $s_{\sigma\delta}\, c^\star / a_t$ gives

$\Pi\, \partial_t \rho + B J^{(t)} = 0,$

where

$$J^{(t)} = s_{\sigma\delta}\, (c^\star / a_t)\, J.$$

If the strict same-step identity is returned, $\sigma = \delta$, then

$$J^{(t)} = (c^\star / a_t)\, J.$$

Thus physical time acts as a conversion from current per primitive update/refinement unit to current per emergent physical-time unit. It does not create the current. ■

Remark (the conversion is forced, not chosen)

The conversion law is not a normalisation choice. Requiring the continuity equation to keep its form under the affine reparameterisation $\sigma \mapsto t_{phys}$ forces $J^{(t)} = (d\sigma/dt_{phys})\, J$, and $d\sigma/dt_{phys} = s_{\sigma\delta}\, c\star / a_t$. So once the σ-to-δ bridge is returned, the physical-time current is uniquely determined; the only open object is the bridge itself.

34F. Typing Firewalls and Physical Interpretation

F-PT1: temporal conversion does not determine λ_J

The factor $s_{\sigma\delta}\, c\star / a_t$ converts a primitive current coordinate into a physical-time-normalised current coordinate. By contrast, λ_J is the unresolved common physical history/Fisher scale multiplying the current metric. These are differently typed quantities.

Therefore $J^{(t)} = s_{\sigma\delta}\, (c\star / a_t)\, J$ does **not** imply $\lambda_J = 1$ or any other value of λ_J.

F-PT2: the result is not yet a Lorentz four-current theorem

The object $J^{(t)}$ is a physical-time-normalised discrete/interface current. A full relativistic current requires a continuum pushforward producing an object j^μ with the correct Lorentz transformation law and covariant conservation equation, schematically

$\nabla_\mu j^\mu = 0$.

That pushforward remains open.

F-PT3: "flow of facts" is an interpretation, not an additional theorem

The mathematics is consistent with interpreting the primitive current as structured propagation or maintenance of relations among committed facts. However, the 3/5 irreducible cycle component proves that this cannot mean only

fact at source → new fact at target.

A more accurate candidate interpretation is

primitive current = structured relational flow/maintenance within the realised-fact record.

The present paper records this only as an interpretation suggested by the source structure.

Updated temporal-current debt

The remaining physicalisation problem is now sharply separated into two steps:

1. derive the source relation between the primitive refinement parameter σ and the Fold-opportunity parameter δ, including its normalisation $s_{\sigma\delta}$;
2. push the resulting physical-time-normalised current into the continuum and prove that the image is a Lorentz-covariant conserved four-current.

Neither obligation reopens the already-closed relative C/W/Y factor geometry.

PART V: THE POSITIVE BINARY FIRST-RECORD WITNESS AND GAUGE-SCALE FAMILY

35. Binary Fact Ontology and the Occurrence-Rate Variable

At the primitive fact/no-fact level, one opportunity has outcomes

$p_0 = 1 - h_F$, $p_A = h_F\, q_A$, $\Sigma_A\, q_A = 1$.

The realised event is binary: either no new fact occurs or one complete new fact occurs. The label A answers the separate conditional question of which complete record is written.

The integrated first-record intensity is

$$\boldsymbol{\Gamma_F = -\log(1 - h_F),}$$

and its gradient is tied exactly to the occurrence probability by $\nabla\Gamma_F = \nabla h_F / (1 - h_F)$.

For any physical parameter vector, the exact one-opportunity Fisher matrix separates into occurrence and type information:

$$\mathbf{F\_Fold = (\nabla h_F\, \nabla h_F^T) / (h_F\, (1 - h_F)) + h_F\, F_q,}$$

where

$F_q = \Sigma_A\, q_A\, \nabla\log q_A\, \nabla\log q_A^T$.

The proof is two lines: $\nabla\log p_0 = -\nabla h_F/(1 - h_F)$ and $\nabla\log p_A = \nabla h_F/h_F + \nabla\log q_A$, and the occurrence/type cross terms cancel exactly because $\Sigma_A\, q_A\, \nabla\log q_A = \Sigma_A\, \nabla q_A = 0$, the same normalisation mechanism that kills the cross terms in the refinement decomposition of Theorem 6. The occurrence/type split is therefore the Fisher chain rule at the fact/no-fact grain, exactly as Theorem 6 is the Fisher chain rule at the coarse/microscopic grain: one mechanism, two grains.

For the complete waiting-rate process the corresponding information rate is

$$\mathbf{I\_rate = (\nabla\Gamma_F\, \nabla\Gamma_F^T) / \Gamma_F + \Gamma_F\, F_q.}$$

The first term measures how the parameter changes whether and when a full fact occurs; the second measures which record occurs, conditional on an event. This is the correct mathematical home for a continuous gauge strength in a binary ontology. The continuous quantity may live in event cadence without any continuously variable degree of record completeness.

Under the RCHS quadratic intensity law, the C/W/Y score Gram per unit binary-event rate is the already-derived uncentred factor matrix, up to the universal radial-coordinate factor. The binary retyping therefore preserves the relative factor theorem of Part I.

---

36. Exact RCCSJ Binary First-Record Trigger Curvature

Let $\Gamma_x(\theta)$ be the total nonself, record-producing rate from wheel state x in the RCCSJ positive marked current-cycle law. Because the physical current cochains are divergence free, the outgoing first-order score compensates at every state:

$$\mathbf{D\,\Gamma_x\,|_0 = 0.}$$

The first nonzero complete-fact response is quadratic. For factor indices A, B ∈ {C, W, Y} and the two real current quadratures μ, ν,

$$\partial_{(A\mu)}\,\partial_{(B\nu)}\,\Gamma_x = g_{cyc}^2\,H_{(AB)}^{CWY}\,G_x^{\mu\nu}.$$

On the indispensable m = 2/4 current-cycle sector, the exact local spatial Grams are

$$G_h = (3/112)\,I_2,\quad G_{(b_j)} = (5/384)\,I_2.$$

Thus

$$\mathbf{D^2\,\Gamma_F\,|_{hub} = g_{cyc}^2\,(3/112)\,H_{CWY} \otimes I_2,}$$

$$\mathbf{D^2\,\Gamma_F\,|_{rim} = g_{cyc}^2\,(5/384)\,H_{CWY} \otimes I_2.}$$

Using the canonical stationary wheel law $\pi_h = 7/31$, $\pi_{(b_j)} = 4/31$, the two contributions reduce over a common denominator:

$$(7/31)(3/112) = 3/496,\quad 6 \cdot (4/31)(5/384) = 5/496,$$

so

$$(7/31)(3/112) + 6\,(4/31)(5/384) = 8/496 = 1/62,$$

and therefore

$$\mathbf{\mathbb{E}_\pi\,[\,D^2\,\Gamma_F\,]_{(m=2/4)} = g_{cyc}^2\,(1/62)\,H_{CWY} \otimes I_2.}$$

The uniform component contributes

$$\mathbb{E}_\pi\,[\,D^2\,\Gamma_F\,]_{(m=0)} = g_{cyc}^2\,(3/62)\,H_{CWY} \otimes I_2,$$

and the complete current cycle contributes

$$\mathbf{\mathbb{E}_\pi\,[\,D^2\,\Gamma_F\,]_{full} = g_{cyc}^2\,(2/31)\,H_{CWY} \otimes I_2,}$$

the exact sum 3/62 + 1/62 = 2/31.

Lemma V.1 (log-linear curvature identity)

The equality between the trigger curvature and the marked Fisher metric is a theorem for the RCCSJ perturbation class, not a numerical coincidence, but it is not automatic for an arbitrary rate family. For rates $k_e(\theta)$ with scores $s_e = \nabla \log k_e$, the general identity is

$$D^2 \Sigma_e k_e = \Sigma_e k_e s_e s_e^T + \Sigma_e k_e D^2 \log k_e,$$

so the total-rate Hessian equals the rate-weighted score Gram, the marked Fisher matrix, exactly when the second term vanishes. The BCTR-2/RCCSJ current perturbation acts multiplicatively on each transition rate in log-linear form,

$$k_e(J) = k_e(0)\, e^{\langle \ell_e, J\rangle},$$

with constant score cochains ℓ_e, so $D \log k_e = \ell_e$ is constant, $D^2 \log k_e = 0$, and

$$D^2 \Gamma_x |_0 = \Sigma_e k_e(0)\, \ell_e \ell_e^T,$$

exactly the marked Fisher matrix at the base point, while $D \Gamma_x |_0 = \Sigma_e k_e(0) \ell_e = 0$ is the divergence-free score compensation already stated. For any non-log-linear source family the correction $\Sigma_e k_e D^2 \log k_e$ would enter, and the completed source law must determine which class it occupies. ■

These are exactly the RCCSJ marked Fisher matrices, and by Lemma V.1 the identity holds by theorem for this perturbation class. Therefore

the positive current-cycle Fisher metric is the curvature of a binary complete-fact rate law.

The $m = 2/4$ block has rank six for every $g_{cyc} \neq 0$, since H_{CWY} is positive definite of rank three and the two quadratures double it, as required by the representation-resolved carrier theorem.

---

37. Exact One-Parameter Gauge-Scale Family

The inherited direct current metric is

$$M_{JJ}^{quad} = \lambda_J M_0 \otimes I_2,$$

with

$$M_0 = K_{cyc}^{(2)} H_{CWY}, \quad K_{cyc}^{(2)} = 0.212610974570295.$$

Matching the full RCCSJ binary-rate curvature to the same full-cycle current/history normalisation gives

$$\lambda_J K_{cyc}^{(2)} = g_{cyc}^2 (2/31).$$

Hence the exact symbolic family

$\lambda_J(g_{cyc}) = (2 / (31 K_{cyc}^{(2)})) g_{cyc}^2$;

numerically, using the inherited value of $K_{cyc}^{(2)}$,

$\lambda_J = 0.30344684305525943 g_{cyc}^2$.

This is an exact reduction, but not an absolute prediction. Every real g_{cyc} in the positive RCCSJ family preserves the wheel support, positivity, Markov normalisation, D_6 covariance, the C/W/Y factor shape and the 3/4 to 1/4 spatial split. The physical source must therefore select g_{cyc}; setting it to one would be a constitutive insertion.

For an arbitrary completed-source marked metric M_{JJ}^{mark}, the target-blind extracted coefficient is

$\lambda_J^{source} = \langle M_0 \otimes I_2, M_{JJ}^{mark} \rangle_F / \| M_0 \otimes I_2 \|_F^2$,

followed by the factor-shape residual

$\varepsilon_{(M_0)} = \| M_{JJ}^{mark} - \lambda_J^{source} M_0 \otimes I_2 \|_F / \| M_{JJ}^{mark} \|_F$.

If the source returns $M_{JJ}^{mark} = \alpha_{source} H_{CWY} \otimes I_2$, then simply

$\lambda_J^{source} = \alpha_{source} / K_{cyc}^{(2)}$.

The comparison is a post-freeze extraction and falsification test. It may not be used to tune the source law.

---

38. Conditional First-Birth Calibration and the 1/3 Target

The earlier saturated first-birth audit supplies the leading same-compensator matching equation

$(\kappa_J^2 / 2) (2 \nu_N + 47 \lambda_{vis}) / 49 = 1/6$,

where ν_N is the neutral-terminal coefficient and λ_{vis} is the common gauge-visible coefficient at that birth grain.

If, and only if, the same-grain hazard theorem of Part I applies to this very same physical first-birth compensator, the neutral and visible coefficients are one common number:

$\nu_N = \lambda_{vis} \equiv \lambda_{birth}$.

The matching equation then collapses exactly, since $2 \lambda_{birth} + 47 \lambda_{birth} = 49 \lambda_{birth}$:

$(\kappa_J^2 / 2) \lambda_{birth} = 1/6$,

so

$\lambda_{birth} = 1 / (3 \kappa_J^2)$.

At the unit residual-current convention $\kappa_J = 1$,

$\lambda_{birth} = 1/3$.

If the birth compensator is further proven to be the same full-cycle physical current/history law used in the definition of λ_J, with no terminal information loss or extra history rescaling, then the conditional candidate becomes

$\lambda_J = 1/3$.

Combining this with the exact RCCSJ scale family would select, on that branch, the closed form

$$g_{cyc} = \sqrt{\left((1/3) \,/\, \left(2/(31\, K_{cyc}^{(2)}) \right) \right)} = \sqrt{\left(31\, K_{cyc}^{(2)} / 6 \right)} = 1.048088753532.$$

This number is a **strong conditional consistency target**, not a physical return. It consumes four unproved identifications:

1. the first-birth and independent cycle-current laws are the same physical compensator;
2. κ_J has the same source normalisation in both calculations;
3. the neutral and visible coefficients inhabit the same primitive event grain;
4. the birth coefficient and full history/terminal coefficient are related without an additional source-derived loss or scale.

The earlier birth/current audit proves that agreement along the endpoint-birth curve alone cannot determine the transverse independent cycle-current derivative. Therefore the 1/3 target cannot substitute for the completed-source curvature calculation.

---

39. Binary Witness Grade and Handoff to the Source-Action Calculation

The positive RCCSJ calculation remains useful because it proves that a binary complete-fact-rate law can carry the required C/W/Y-by-quadrature shape:

$$\mathbb{E}_\pi [D^2 \Gamma_F]_{full} = g_{cyc}^2 \, (2/31) \, H_{CWY} \otimes I_2,$$

with the witness conversion $\lambda_J = (2/(31\, K_{cyc}^{(2)}))\, g_{cyc}^2$, numerically $0.30344684305525943\, g_{cyc}^2$ at the inherited $K_{cyc}^{(2)}$. The conditional 1/3 branch, with implied $g_{cyc} = \sqrt{(31\, K_{cyc}^{(2)}/6)}$, remains a consistency target under the stronger same-event identifications of Section 38.

The published first-record architecture, however, does not select that witness. BCTR-3 through BCTR-6 establish:

1. binary first-record occurrence is current independent to all orders on the serial F2F ∘ EX2E source;

2. conditional record identity responds with rank six;
3. the current rank-six record/internal signal cannot enter the factorised selector through trace-preserving renewal;
4. the physical current reaches record phase with rank six;
5. the quadratic current-to-record-capacity block vanishes;
6. the first nonlinear stretch response is attenuating;
7. no implemented source-Hessian path crosses into self-versus-fact eligibility.

Thus

$\bar{G}_{elig}^{implemented} = 0$

for the implemented binary-occurrence mechanisms. This means only that **binary occurrence is not the present readout of the current strength**.

The crucial point is that the same BCTR-6 source block contains more positive information than the binary audit originally used. The current–record-phase pair is itself a positive action block. Part VI therefore takes one further source-native step: it eliminates the record phase as an auxiliary coordinate and asks whether a nonzero current action survives.

That calculation is constructive. It yields a positive rank-six current stiffness and then pulls it through the certified physical C/W/Y current embedding.

Accordingly, the physical release question is no longer

> "Can binary fact occurrence produce a current strength?"

but

> **"What physical primitive metric and continuum attaching map normalise the current stiffness already generated by the source?"**

Part VI executes the first half of that new programme boundary.

PART VI: SOURCE-GENERATED CURRENT STIFFNESS, CURRENT-LINK SCHUR RECIPROCITY AND THE BINARY-OCCURRENCE BOUNDARY

39A. Physical Six-Current Carrier and the Exact Four-Variable Source Block

The physical nonuniform current carrier is the divergence-free m = 2/4 cycle doublet resolved over the three factor directions. Use the ordered factor/quadrature coordinates

$x = (C_c, C_s, W_c, W_s, Y_c, Y_s)^T$.

The frozen physical embedding E_phys into the retained edge-current carrier obeys

$$E\_phys^\dagger\, E\_phys = (49/50)\, H\_CWY \otimes I_2$$

with machine residual 5.299×10^{-16}. Thus the six physical coordinates already carry the same relative C/W/Y geometry fixed independently by the RUDM theorem.

The deepest explicit source calculation relevant to the strength question lies one level below this factor pullback. For one real factor/quadrature direction retain (p, q, θ, y), where:

- p is the physical current-cycle coordinate;
- q is its conjugate-helicity current partner;
- θ is the matched link/connection coordinate;
- y is the matched record phase.

For arbitrary positive diagonal primitive weights w_J, w_C, w_R, w_B > 0 and κ > 0, the exact quadratic source Hessian is

$$H_4(w) = [\ w\_J\ 0\ -\kappa w\_J/\sqrt{2}\ 0\ ;\ 0\ w\_J\ +\kappa w\_J/\sqrt{2}\ 0\ ;\ -\kappa w\_J/\sqrt{2}\ +\kappa w\_J/\sqrt{2}\ \kappa^2 w\_J + 4w\_C + w\_B\ -w\_B\ ;\ 0\ 0\ -w\_B\ 4w\_R + w\_B\ ].$$

Equivalently, its quadratic action is the sum of squares

$$\Gamma_4\hat{}(2) = (w\_J/2)(p - \kappa\theta/\sqrt{2})^2 + (w\_J/2)(q + \kappa\theta/\sqrt{2})^2 + 2\, w\_C\, \theta^2 + (w\_B/2)(\theta - y)^2 + 2\, w\_R\, y^2,$$

and the displayed $H_4(w)$ is exactly the Hessian of this action (Appendix C.10). This sum-of-squares form is important: current, connection and record phase are already priced by one common source action. No later current-to-link conversion coefficient has been introduced.

On the unit residual metric, w_J = w_C = w_R = w_B = κ = 1, this reduces to

$$H_4 = [\ 1\ 0\ -1/\sqrt{2}\ 0\ ;\ 0\ 1\ +1/\sqrt{2}\ 0\ ;\ -1/\sqrt{2}\ +1/\sqrt{2}\ 6\ -1\ ;\ 0\ 0\ -1\ 5\ ].$$

Schur-eliminating q and θ while retaining p and y gives

$$H\_pair\hat{}(J,y) = (1/11) \cdot [\ 10\ -\sqrt{2}\ ;\ -\sqrt{2}\ 53\ ].$$

Its determinant is

$$\det H\_pair\hat{}(J,y) = (530 - 2)/121 = 528/121 = 48/11 > 0.$$

Thus the current–phase pair is strictly positive, and across the six physical directions the mixed current–phase block has rank six.

39B. Theorem F: Fully Relaxed Source Current-Stiffness Theorem

Grade: exact on the executed unit residual source metric.

The record phase is auxiliary to the local source block. Eliminating y gives

$K\_J^{relaxed} = 10/11 - (\sqrt{2}/11)^2 / (53/11) = 48/53$,

so

$H\_JJ^{relaxed} = (48/53)\, I_6$

and

$S\_{current,relaxed}^{(2)} = (1/2)\,(48/53)\, \|J_6\|^2$.

The relaxed current Hessian is strictly positive and has rank six.

Before record-phase relaxation the current diagonal is $K\_J^{prephase} = 10/11$. The record phase therefore screens by

$\Delta K\_\varphi = 10/11 - 48/53 = 2/583$,

with $583 = 11 \cdot 53$, a fraction

$\Delta K\_\varphi / K\_J^{prephase} = 1/265$,

equivalently $K\_J^{relaxed} = (264/265)\, K\_J^{prephase}$. The phase mediator screens the current action; it does not generate it and does not erase it.

39C. Theorem G: Positive-Weight Current-Stiffness Survival

The unit-metric result strengthens to the complete positive diagonal source-weight family:

$K\_J^{relaxed}(w) = 8\, w\_J\, (\, w\_B\, w\_C + w\_B\, w\_R + 4\, w\_C\, w\_R\,) / \Delta\_\kappa > 0$,

where

$\Delta\_\kappa = \kappa^2\, w\_B\, w\_J + 4\, \kappa^2\, w\_J\, w\_R + 8\, w\_B\, w\_C + 8\, w\_B\, w\_R + 32\, w\_C\, w\_R$.

Under common rescaling $w\_i \mapsto \tau\, w\_i$, $\tau > 0$, one has $K\_J^{relaxed} \mapsto \tau\, K\_J^{relaxed}$, confirmed exactly by the degree count (numerator degree three, denominator degree two). The absolute action scale therefore remains one upstream physical datum, entering the physical current sector through one common scalar rather than an arbitrary factor matrix, and the relaxed block $K\_J^{relaxed}(w)\, I_6$ has rank six for every admissible weighting.

Remark G.3: screening is universally nonnegative and the constitutive coupling only screens

The phase-relaxation decrement is $\Delta K\_\varphi(w) = (H\_eff)\_Jy^2 / (H\_eff)\_yy \geq 0$ for every weighting, with equality exactly when the current–phase cross block vanishes ($\kappa\, w\_B\, w\_J \to 0$): record-phase relaxation can only reduce the current stiffness, never enhance it, and the unit-branch 1/265 is one instance of that universal fact. Moreover

$\partial K\_J\text{^relaxed}/\partial\kappa = -16\,\kappa\,w\_J^2\,(w\_B + 4\,w\_R)\,(w\_B\,w\_C + w\_B\,w\_R + 4\,w\_C\,w\_R) / \Delta\_\kappa^2 < 0$

throughout the positive family: the constitutive coupling strictly screens, and the $\kappa \to 0$ limit returns the unscreened diagonal. Both facts are verified symbolically in Appendix C.10. ■

The theorem proves structural survival of a positive current action. It does not yet prove that the physical W_prim is diagonal in this source basis.

39D. Theorem H: Current-Link Schur Reciprocity

The four-variable source block contains more information than the current-only Schur complement.

Eliminate first the conjugate current q and record phase y. Their stationary values at fixed p, θ are

$q\star = -\kappa\theta/\sqrt{2}$, $y\star = [\, w\_B/(4w\_R + w\_B)\,]\,\theta$.

The remaining current-link Hessian is

$H\_J\theta\text{^red} = [\, w\_J \;\; -\kappa w\_J/\sqrt{2} \;;\; -\kappa w\_J/\sqrt{2} \;\; \kappa^2 w\_J/2 + K\_\theta \,]$,

where

$K\_\theta = 4\,w\_C + 4\,w\_B\,w\_R / (\,4\,w\_R + w\_B\,) > 0$.

Its determinant simplifies exactly to

$\det H\_J\theta\text{^red} = w\_J\,K\_\theta > 0$.

Link-side reduction

For fixed θ, $p\star = \kappa\theta/\sqrt{2}$. The constitutive-current square then vanishes and the surviving link action is

$S\_\theta,\text{rel^(2)} = (1/2)\,K\_\theta\,\theta^2$.

Thus K_θ is the fully current-relaxed source stiffness of the matched link coordinate.

Current-side reduction

For fixed p,

$\theta\star = \kappa\,w\_J\,p / [\,\sqrt{2}\,(K\_\theta + \kappa^2 w\_J/2)\,]$.

Substitution gives

$K\_J\text{^relaxed} = w\_J - (\kappa^2 w\_J^2/2)/(K\_\theta + \kappa^2 w\_J/2) = w\_J\,K\_\theta / (\,K\_\theta + \kappa^2 w\_J/2\,)$, hence the reciprocity law

$1/K_J^{relaxed} = 1/w_J + \kappa^2/(2K_\theta)$, $K_\theta = \kappa^2 w_J K_J^{relaxed} / [2 (w_J - K_J^{relaxed})]$.

This is the Current-Link Schur Reciprocity Theorem. It proves that, inside this source block, current stiffness and matched link stiffness possess no independent relative normalisation. They are two views of one common quadratic action. Expanding the reciprocal relation reproduces exactly the Theorem G closed formula, verified symbolically in Appendix C.10, so the two derivations of $K_J^{relaxed}$ are one identity.

On the unit diagnostic branch,

$K_\theta = 4 + 4/5 = 24/5$, $K_J^{relaxed} = (24/5)/(24/5 + 1/2) = 48/53$.

The values 24/5 and 48/53 are therefore complementary Schur reductions of one parent. Their agreement with the previously executed MASD link-side branch must not be described as an independent confirmation.

39E. Theorem I: Full-Matrix Reciprocity and the Physical Duality Test

The scalar theorem has a direct matrix generalisation and therefore survives the move to a faithful non-Abelian carrier at the level of exact linear algebra.

Let, after exact gauge/null quotienting:

- $W_J > 0$ be the bare physical current metric;
- $K_\theta > 0$ the current-relaxed link/connection stiffness;
- $\mathcal{K}$ the constitutive current-connection map, with $B = \mathcal{K}/\sqrt{2}$.

Suppose the same-parent reduced source Hessian is

$\mathcal{H}_{J\theta} = [W_J \; -W_J B \; ; \; -B^\dagger W_J \; K_\theta + B^\dagger W_J B]$.

Schur-eliminating the connection gives

$\mathbb{K}_J^{relaxed} = W_J - W_J B (K_\theta + B^\dagger W_J B)^{-1} B^\dagger W_J$,

and the Woodbury identity yields

$(\mathbb{K}_J^{relaxed})^{-1} = W_J^{-1} + B K_\theta^{-1} B^\dagger = W_J^{-1} + (1/2) \mathcal{K} K_\theta^{-1} \mathcal{K}^\dagger$.

This is the noncommuting matrix form of current-link reciprocity.

It supplies one direct release test for the eventual physical source package:

$\varepsilon_{dual} = \| (\mathbb{K}_J^{relaxed})^{-1} - W_J^{-1} - (1/2) \mathcal{K} K_\theta^{-1} \mathcal{K}^\dagger \|$.

A faithful same-parent execution of the displayed architecture requires $\varepsilon_{dual} = 0$ within the frozen numerical and regulator error model.

If the residual is nonzero after all carriers and nulls are correctly treated, the physical source contains additional cross-defect or auxiliary structure beyond the present four-variable block. The returned full matrix then supersedes the scalar branch.

This test is stronger than comparing numerical couplings: it directly asks whether the physical current and gauge responses descend from the same microscopic Hessian.

39F. Theorem J: Factor-Democracy / Wheel-Mode Compatibility Gate

The reciprocity theorem also exposes a physical compatibility question that the scalar current pullback alone could not see.

For a wheel eigenvalue λ replacing the unit mode factor,

$K_\theta(\lambda) = \lambda w_C + \lambda w_B w_R / (\lambda w_R + w_B)$,

and mode-by-mode reciprocity gives

$K_J(\lambda) = w_J K_\theta(\lambda) / (K_\theta(\lambda) + \kappa^2 w_J/2)$.

For positive source weights,

$dK_\theta/d\lambda = w_C + w_B^2 w_R / (\lambda w_R + w_B)^2 > 0$,

and, since K_J is strictly increasing in K_θ,

$dK_J/d\lambda = (\kappa^2 w_J^2/2) (dK_\theta/d\lambda) / (K_\theta + \kappa^2 w_J/2)^2 > 0$.

Hence

$\lambda_A \neq \lambda_B \Rightarrow K_J(\lambda_A) \neq K_J(\lambda_B)$

whenever the same positive separable weights and constitutive coefficient are used.

On the unit diagnostic branch, $K_\theta(\lambda) = \lambda + \lambda/(\lambda+1)$. The executed MASD wheel triple

$(K_3, K_2, K_q)^{wheel} = (24/5, 35/6, 3/2)$

corresponds to

$(\lambda_3, \lambda_2, \lambda_q) = (4, 5, 1)$,

and current-link reciprocity therefore gives

$(K_{J,3}, K_{J,2}, K_{J,q}) = (48/53, 35/38, 3/4)$.

These are unequal.

Compatibility theorem

A physical source cannot simultaneously satisfy all three of the following:

1. one common positive separable weight set (w_J, w_C, w_R, w_B, κ);
2. distinct physical wheel eigenvalues for the three gauge sectors;
3. one factor-democratic scalar current stiffness $K_J I_6$.

At least one of those assumptions must be replaced by the faithful source execution.

The permitted physical resolutions are:

- the diagnostic MASD wheel assignments are not the final physical assignments;
- W_prim is genuinely nonfactorising across wheel/internal sectors;
- the faithful source returns factor-dependent W_J or $\mathcal{K}$;
- additional physical auxiliaries alter the reduced block;
- the scalar factor-democratic current pullback is superseded by a full matrix response.

This is not a contradiction between VERSF papers. It is a compatibility gate between two incomplete source branches.

39G. Theorem K: Physical C/W/Y Pullback and Standard-Model Handoff

Let $x \in \mathbb{R}^6$ be the factor/quadrature current coordinates and $J_6 = E_{phys}\, x$.

If the faithful physical source passes the factor-democracy gate and returns

$\mathbb{K}_J^{relaxed} = K_J^{relaxed} I_6$,

then

$$\mathbf{M_{JJ}^{source} = (49/50)\, K_J^{relaxed}\, H_{CWY} \otimes I_2,}$$

and on the unit diagnostic branch $M_{JJ}^{source,unit} = (1176/1325)\, H_{CWY} \otimes I_2$.

If factor democracy fails, this formula is not imposed. The authoritative return becomes

$$\mathbf{M_{JJ}^{source} = E_{phys}^\dagger\, \mathbb{K}_J^{relaxed}\, E_{phys}.}$$

The scalar formula is therefore a special physical branch of a more general full-matrix theorem.

The Standard-Model handoff is now

$W_{prim} \to (W_J, \mathcal{K}, K_\theta) \to \mathbb{K}_J^{relaxed} \to E_{phys}^\dagger\, \mathbb{K}_J^{relaxed}\, E_{phys}$

followed by

discrete link/current action → S_gauge[A] + S_int[A, j].

A successful physical execution must return: the faithful W_J, $\mathcal{K}$ and K_θ; the exact gauge/null quotient; ε_dual and its regulator error; the factor/mode assignment; the resulting full current and link response matrices; the map from discrete link variation to continuum curvature F_μν; the map from discrete current to Lorentz current j^μ; the dimension-four gauge kinetic coefficient and current vertex in one convention; canonical field normalisation; and the surviving Abelian portal.

Continuum operator-typing firewall

The continuum gauge kinetic normalisation and the higher-derivative Yang–Mills corrections must remain separately typed. Schematically,

$$S_{eff} = -(1/4) \int Z_{ab} F^a_{\mu\nu} F^{b\,\mu\nu} + \int A^a_\mu j_a^\mu + (c_6/k^2) \int O_{DFD} + \cdots,$$

with $O_{DFD} = \mathrm{Tr}[(D_\mu F_{\mu\nu})(D_\rho F_{\rho\nu})]$. The present current-link reciprocity theorem is a candidate microscopic origin of the dimension-four current/kinetic normalisation. It is not a derivation of the distinct dimension-six coefficient c_6. Coarse-graining may subsequently renormalise both, but they may not be identified at the matching boundary.

Finite-source relative current-link normalisation: exact within the common source block.

Faithful physical source metric, factor branch and continuum gauge normalisation: open.

39H. Binary First-Record Occurrence Is a Boundary, Not the Source of the Current Action

On the published F2F source, history first decides

self history ↔ no new fact, nonself history ↔ fact-producing transition,

and only afterwards applies the complete current/terminal channel. Summing the complete conditional channel gives

$$E_{(0,x)}(J) = p_{xx} I, \quad E_{(F,x)}(J) = (1 - p_{xx}) I,$$

so

$$\mathbf{D^n_{(J_{cyc})} NR = D^n_{(J_{cyc})} h_F = D^n_{(J_{cyc})} \Gamma_F = 0 \ (n \geq 1).}$$

Conditional record identity nevertheless responds with rank six. State-preserving renewal cannot change this conclusion because the next factorised self/fact selector is proportional to the identity on the work and ledger carriers: for every trace-preserving renewal, $\Phi_J^\dagger(I) = I$.

Therefore the present binary occurrence law is not the readout of the source current stiffness. This does not weaken the current-link reciprocity theorem: that theorem is obtained directly from the source action before any claim about binary fact occurrence.

39I. Record Capacity and History Eligibility Remain Distinct

The same source census establishes

$$\mathbf{H_{(J_{cyc},s)}^{eff} = 0}$$

for record stretch/capacity at quadratic Hessian order. At the first higher nonzero order, phase relaxation gives

$$\mathbf{s_{rim}^{\star} = -(2/8427)\,\|J\|^2,}$$

which attenuates the visible record branch, and

$$\bar{G}_{elig}^{implemented} = 0$$

for the implemented binary-occurrence mechanisms. The correct interpretation is:

current/link source action ≠ record capacity ≠ binary fact occurrence.

The source may generate a positive current and link stiffness while the current F2F architecture remains blind to that stiffness at fact/no-fact selection.

PART VII: GRADED SOURCE PROVENANCE AND JOINT-ENDPOINT CURRENT PERSISTENCE

39J. Retyping P1 as a Graded Source Jet

RCHS-2 represented the remaining source-to-faithful provenance debt schematically by one derivative $d\Phi_n$. The later source differentiation audit shows that this typing is too coarse: the physical sectors do not begin at one common differential order.

The scalar commitment density and terminal-record orientation have first-order returns; the faithful gauge response and bosonic completion first enter at second order in the action; and the shared-link CAR/Fock completion begins at mixed third order. The appropriate provenance object is therefore a graded jet,

$\mathfrak{J}$_P1 = (D s_R, D^2S_G, D^2S_B, D^3S_CAR, …).

At those grades the frozen faithful stack already returns the relevant descents: gauge quadratic residual 2.5266×10^{-15}; full-action completion residual 1.8867×10^{-15}; record-phase row-space residual 3.1352×10^{-15} with the record-stretch nulls identified as incidence coboundaries; shared-link CAR/Fock residual 1.7935×10^{-15}; and CAR completion common-direction residual 1.1487×10^{-15}.

The former HM4 interval $0 \le \varepsilon$_HM4 ≤ 1 remains a valid warning for an unspecified first-order selector embedding. It is not the final provenance test for source sectors whose leading return is quadratic or mixed higher order.

Theorem L (graded P1 retyping)

The physical source-provenance problem is not the return of one ordinary 6×334 Jacobian. It is the return of the correctly ordered graded source jet and its faithful descent at each leading order. Accordingly, the old single-dΦ gate is **retyped / withdrawn** rather than failed.

39K. Native Current-Defect-to-Mark Factorisation

The strongest new P1 calculation uses the later representation-valued PFDMI source extension directly, rather than embedding a six-channel selector Gram by hand.

Restrict the exact MASD current-defect derivative to the 144 native matrix-link coordinates:

J_DJ^link ∈ ℝ^(1200×144), rank J_DJ^link = 144.

Differentiate the actual native conditional marked law on the same source-link coordinates:

J_mark ∈ ℝ^(1092×144), rank J_mark = 144.

Because J_DJ^link is injective, each vector in its reachable current-defect image has one unique source-link preimage. The native marked derivative therefore defines a unique linear coupling C_mark on that reachable image satisfying

C_mark J_DJ^link = J_mark.

The frozen execution gives the relative reconstruction residual

|| C_mark J_DJ^link − J_mark ||_F / || J_mark ||_F = 2.5840×10^{-15},

and the corresponding terminal reconstruction residual is 2.2823×10^{-15}.

This is not a Moore–Penrose physical-selection postulate. A pseudoinverse may be used to represent the already-unique map on the reachable image, but no minimum-norm extension outside that image is assigned physical meaning.

Theorem M (native gauge/current source factorisation)

On the PFDMI-EX2E source-extension branch, the native representation-valued marked response factors uniquely through the actual faithful current-defect derivative on all 144 raw matrix-link directions, to machine precision. This removes the arbitrary HM4 selector-embedding ambiguity on the native gauge/current slice.

39L. Wilson-Quotient Gauge/Current Provenance

Let Q_phys be the 72-dimensional Wilson physical basis. Define

J_D^phys = J_DJ^link Q_phys, J_m^phys = J_mark Q_phys,

and the corresponding terminal derivative J_t^phys. The native execution returns

rank J_D^phys = rank J_m^phys = rank J_t^phys = 72.

The physical mark coupling reconstructs with

ε_P1^phys = 1.8038×10^{-15},

and the terminal reconstruction residual is 1.8755×10^{-15}. Hence

P1a, native gauge/current graded source provenance: exact pass on the PFDMI-EX2E source-extension branch.

The stronger primitive statement remains open:

P1b, one primitive field-dependent marked instrument generating the gauge/current, record, completion and CAR/Fock graded jets together: open.

The older primitive executable does not close P1b because its field derivative is exactly zero. The representation-valued source extension closes the gauge/current slice but remains conditional on a supplied endpoint/current background.

39M. P2 Endpoint-Coherence Scalar Reduction

The joint-endpoint problem is also much smaller than the earlier wording suggested.

The exact carrier reconciliation gives

T_[Ω] $\mathbb{C}P^{49} \cong$ Ω^⊥ $\subset \mathbb{C}^{50}$, dim_ℂ Ω^⊥ = 49.

Thus the GSJB innovation and the PFDMI/MASD endpoint tangent are the same vector type. On the seven-vertex K = 7 wheel,

$z \in \mathbb{C}^7 \otimes \mathbb{C}^{49}$,

and the linearised relative-current map is

$$\boldsymbol{\delta J = K\,(B^T \otimes I_{49})\, z,\; K = \mathrm{diag}(\kappa\_e),\; \kappa\_e > 0.}$$

Because the wheel is connected, $\ker B^T = \mathrm{span}\{1_7\}$, so rank B = 6 and

$$\boldsymbol{\ker(B^T \otimes I_{49}) = \mathrm{span}\{1_7\} \otimes \mathbb{C}^{49}.}$$

The only endpoint field annihilated by the relative-current map is therefore the spatial common mode.

Under common internal U(49) covariance, equal isotropic endpoint marginal covariances imply

$$\boldsymbol{\mathbb{E}[\, z\_t\, z\_s\dagger \,] = (\, \gamma\_e / 49 \,)\, I_{49},}$$

and, in the common local frame, the unnormalised relative-current second moment becomes

$$\boldsymbol{\mathbb{E}[\, (\, z\_t - U\_e\, z\_s \,)(\, z\_t - U\_e\, z\_s \,)\dagger \,] = [\, (\, 2 - 2\,\mathrm{Re}\, \gamma\_e \,) / 49 \,]\, I_{49}.}$$

P2 is therefore a scalar endpoint-correlation problem in Re γ_e rather than a free 49×49 internal covariance-matrix problem.

39N. Weighted-Dirichlet Finite-Refinement Current-Persistence Theorem

Let $B \in \mathbb{R}^{\wedge}(7\times12)$ be any orientation of the connected wheel incidence matrix, let

$K = \mathrm{diag}(\kappa\_1, \ldots, \kappa\_12)$, $\kappa\_e > 0$,

and let $M = \mathrm{diag}(m\_1, \ldots, m\_7)$, $m\_v > 0$, be any positive vertex mass matrix. Define

$$\boldsymbol{L\_K = M^{\wedge}(-1/2)\, B\, K\, B^T\, M^{\wedge}(-1/2).}$$

Because the wheel is connected and all conductances are strictly positive,

$L\_K \succeq 0$, rank $L\_K = 6$, $\boldsymbol{\ker L\_K = \mathrm{span}\{\, M^{\wedge}(1/2)\, 1 \,\}.}$

Let the mass-normalised endpoint innovation evolve by $u(s) = e^{\wedge}(-sL\_K)\, u(0)$, with initial spatial covariance $\Sigma_0 \succeq 0$. Define the weighted relative-gradient capacity

$$\boldsymbol{\mathcal{D}\_K(s) = \mathrm{Tr}[\, L\_K\, e^{\wedge}(-2sL\_K)\, \Sigma_0 \,].}$$

Writing the nonzero eigenvalues as $0 < \lambda_1 \leq \ldots \leq \lambda_6$,

$$\mathcal{D}\_K(s) = \Sigma\_(r=1)^{\wedge}6\; \lambda\_r\, e^{\wedge}(-2\lambda\_r\, s)\, \langle\, v\_r, \Sigma_0\, v\_r \,\rangle.$$

Theorem N (finite-refinement current persistence)

If the initial covariance has nonzero support on at least one non-common spatial mode, then

$\mathcal{D}_K(s) > 0$ **for every finite** $s \geq 0$**,**

and moreover

$e^{(-2\lambda_6 s)}\, \mathcal{D}_K(0) \leq \mathcal{D}_K(s) \leq e^{(-2\lambda_1 s)}\, \mathcal{D}_K(0)$.

Thus no uncertainty in positive edge conductances can turn a finite non-common current off. Conductance variation changes the magnitude and decay rate only. The current vanishes identically only on the exact spatial common mode, or asymptotically after complete relaxation into the Perron sector.

For the maximally unresolved spatial covariance $\Sigma_0 = I_7$, $C(s) = e^{(-2sL_K)}$. Connected positive-conductance evolution is positivity improving at finite positive s, while $C(s)$ remains positive definite. Hence every distinct vertex pair, and in particular every physical edge, obeys

$$0 < \gamma_{ij}(s) = C_{ij}(s) / \sqrt{C_{ii}(s)\, C_{jj}(s)} < 1 \text{ for finite } s > 0.$$

The corresponding standardised relative-current strength $R_{ij}(s) = 2[\,1 - \gamma_{ij}(s)\,]$ is therefore strictly positive at every finite depth.

On the canonical $K = 7$ branch this theorem reproduces the previous benchmark spectrum $\{0, 1/2, 1/2, 1, 1, 31/28, 5/4\}$, including $\gamma_{spoke}(1) = 0.486616791079$, $\gamma_{rim}(1) = 0.502983670412$, $\gamma_{spoke}(2) = 0.815845031782$ and $\gamma_{rim}(2) = 0.813224908401$. The general theorem is stronger because it does not require equal 1/31 edge conductances.

39O. Positive Current-Action Activation and the Endpoint-Law Discriminator

The weighted-Dirichlet result connects directly to the source current-stiffness theorem.

If the relaxed source current Hessian on the retained physical current subspace satisfies $\mathbb{K}_J^{rel} \succ 0$ with smallest eigenvalue $k_J^- > 0$, then

$$S_J^{(2)} = (1/2)\, J^\dagger\, \mathbb{K}_J^{rel}\, J$$

obeys

$$\mathbb{E}\, S_J^{(2)} \geq (1/2)\, k_J^-\, \mathbb{E}\|J\|^2 > 0$$

for every finite non-common current excitation with nonzero projection onto that retained subspace. Thus the positive rank-six source stiffness is not merely an available quadratic form: on the finite non-common branch it is dynamically activated.

A second source candidate supplies a sharp discriminator. Locality, endpoint exchange symmetry, positivity, a perfect-square primitive and common U(49) covariance reduce every quadratic endpoint handshake to

$$Q_{(e,\pm)} = a \, \| z_t \pm U_e z_s \|^2.$$

If the oriented hard Gate-2 branch is eventually proven to act on the primitive matter tangent, its zero set gives $z_t = -U_e z_s$, hence

$$\gamma_e = -1$$

and maximal equal-norm relative-current covariance $4 I_{49}/49$. This is incompatible, at the same physical event grain, with a finite reversible Dirichlet branch satisfying $0 < \gamma_e < 1$. The source must therefore decide the event grain and joint law rather than allowing both to be promoted simultaneously.

The hard matter-handshake lift is not yet source derived, so $\gamma_e = -1$ is retained only as a conditional discriminator branch.

39P. Revised Physical Frontier

The combined P1/P2 result changes the outstanding problem materially.

The programme no longer needs to ask for one arbitrary source embedding, nor whether finite non-common endpoint dynamics can support a nonzero current. The graded source stack already closes the native gauge/current provenance slice, and positive-conductance continuity already guarantees finite non-common current persistence.

The remaining physical release package is the gate sequence of Section 40: one unified primitive graded instrument generating every source jet together (K0A); the physical endpoint joint law and event grain fixing γ_e, the conductances and the stopping rule (K0B); the faithful (W_J, $\mathcal{K}$, K_θ) with a vanishing reciprocity residual ε_{dual} (K1); the factor/mode decision between scalar democracy and the full matrix response (K2); the continuum dimension-four attaching map (K3, K4); and canonical field normalisation with the surviving Abelian portal (K5).

Accordingly, zero-versus-nonzero current existence is no longer a frontier question. The remaining problem is **source selection of the physical magnitude, event grain and continuum normalisation**.

PART VIII: GATE-T VACUUM TOPOLOGY, CHIRAL SEVEN-TORSION AND FOLD-COMPATIBLE REFINEMENT

39Q. K7 Cycle-Lattice Reduction and the First-Cover Torsion No-Go

The microscopic K7 construction already fixes the relevant one-skeleton. Let W_7 have rim vertices v_i, $i \in \mathbb{Z}_6$, hub h, oriented rim edges $r_i : v_i \to v_{(i+1)}$, and spokes $s_i : h \to v_i$.

The six elementary wheel cycles

$$\mathbf{f_i = s_i + r_i - s_{(i+1)}}$$

form an integral basis of the complete cycle lattice. Indeed, for $x = \Sigma_i a_i r_i + \Sigma_i b_i s_i$ the condition $\partial_1 x = 0$ gives $b_i = a_i - a_{(i-1)}$, and hence $x = \Sigma_i a_i f_i$. Therefore

$$\mathbf{Z_1(W_7; \mathbb{Z}) = \ker_{\mathbb{Z}} \partial_1 \cong \mathbb{Z}^6.}$$

Let $B = (f_0 \cdots f_5)$ be the 12×6 cycle-basis matrix. Every physical two-cell attachment on this fixed one-skeleton must factor as

$$\mathbf{\partial_2^{phys} = B\, A_{phys}}$$

for an integer matrix A_{phys}. Consequently,

$$\mathbf{H_1(K_{vac}; \mathbb{Z}) \cong \mathbb{Z}^6 / \operatorname{im} A_{phys}.}$$

The topology problem is therefore not an arbitrary twelve-edge problem. It is exactly the problem of deriving an integral relation lattice inside the six-dimensional K7 cycle lattice.

The source-returned first-cover circulation supplies one natural history functional. For a completed history it counts forward minus reverse rim traversals while hub moves contribute zero. Restricted to a cycle $x(q) = \Sigma_i q_i f_i$, it becomes

$$\mathbf{K(q) = q_0 + q_1 + \cdots + q_5.}$$

Thus $K : \mathbb{Z}^6 \to \mathbb{Z}$ has Smith form (1), with $\ker K = \{ q : \Sigma_i q_i = 0 \}$ and $\mathbb{Z}^6/\ker K \cong \mathbb{Z}$. Hence **the source-returned first-cover circulation alone produces no torsion.** Taking only sign K cannot repair the problem because the sign map is not additive and therefore cannot define an integral chain quotient.

Nor can the physical attaching law simply be declared to be K mod 7. For a unit-incidence wheel cell the cycle coordinates obey $q_i \in \{0, \pm 1\}$, so $-6 \le \Sigma_i q_i \le 6$. Therefore $\Sigma_i q_i \equiv 0 \pmod 7$ implies $\Sigma_i q_i = 0$. Every such unit-incidence column lies in the ordinary rank-five kernel of K, so no collection of them can supply a full-rank six-dimensional attachment.

Theorem O (first-cover torsion no-go)

The physical order-seven topology, if present, cannot arise from the source-returned total first-cover circulation, its sign, or its mod-seven reduction while retaining ordinary unit incidence. The missing topology must depend on nontrivial relations among the six K7 cycle coordinates, rather than on their total circulation alone.

39R. Fold Relation Rank and the Oriented Attachment Algebra

The four Fold states supply six minimal unordered binary relation supports arranged into three canonical complement pairs. Choose an ordering

$R_0 = 01$, $R_1 = 02$, $R_2 = 03$, $R_3 = 23$, $R_4 = 13$, $R_5 = 12$,

so that R_(i+3) = R_i^c. Their four-atom incidence matrix is

M_Fold = [1 1 1 0 0 0 ; 1 0 0 0 1 1 ; 0 1 0 1 0 1 ; 0 0 1 1 1 0].

Exact integer reduction gives

rank M_Fold = 4, SNF(M_Fold) = diag(1, 1, 1, 2).

Any linear vacuum attachment constructed only from the unordered atom-support incidence therefore has rank at most four. It cannot supply the full-rank six-dimensional relation lattice required for finite H_1. This proves that the Fold's boundary ordering is load-bearing, not decorative.

Under the already identified connected two-neighbour cyclic locality structure, let S e_i = e_(i+1) denote oriented succession around the six Fold relations. Complementation is then J e_i = e_(i+3) = S^3 e_i. The smallest local integral attachment algebra using only the three canonically distinguished operations is

$A = aI + bS + cJ$, $a, b, c \in \{0, \pm1\}$.

There are only $3^3 - 1 = 26$ nonzero possibilities. Impose three conditions: full rank, collective order-seven index $|\det A| = 7$, and unit incidence after lifting into the physical K7 edge basis.

Exactly four coefficient triples have determinant magnitude seven,

$(-1, -1, 1)$, $(-1, 1, -1)$, $(1, -1, 1)$, $(1, 1, -1)$.

The middle pair generates physical edge coefficients of magnitude two and fails unit incidence. The surviving pair differs only by overall cell orientation.

Theorem P (minimal Fold-generated attachment uniqueness)

Within the minimal local algebra $aI + bS + cJ$ with $a, b, c \in \{0, \pm1\}$, the conditions of full rank, collective order-seven index and physical unit incidence force

$A_\chi = J - I - S$,

unique up to overall orientation. Reversal of the Fold boundary replaces S by S^{-1} and produces the conjugate attachment.

This does not yet prove that the Fold relation carrier is the physical vacuum cell carrier. It proves that if that source bridge exists and the minimal Fold operations control the local attaching law, the order-seven unit-incidence branch is no longer freely selectable.

39S. The Chiral Order-Seven Attachment and Exact Smith Form

In the K7 cycle basis the attachment law is

$c_i = f_{(i+3)} - f_i - f_{(i+1)}$,

equivalently $A_\chi = J - I - S$. Since the matrix is circulant, its Fourier polynomial is

$p(z) = z^3 - 1 - z$, $z^6 = 1$.

At the sixth roots of unity, $p(1) = -1$ and $p(-1) = -1$. For $z = e^{(\pm i\pi/3)}$, $p(z) = -2 - z$, whose conjugate product is 7. For $z = e^{(\pm 2\pi i/3)}$, $p(z) = -z$, whose conjugate product is 1. Hence

$\det A_\chi = 7$.

A 5×5 minor has determinant ± 1, and therefore

$\mathrm{SNF}(A_\chi) = \mathrm{diag}(1, 1, 1, 1, 1, 7)$.

Thus the conditional vacuum complex defined by this attachment satisfies

$H_1(K_\chi; \mathbb{Z}) \cong \mathbb{Z}/7$ and $H^1(K_\chi; \mathbb{Z}_7) \cong \mathbb{Z}_7$.

Modulo seven the attachment loses exactly one rank. In the $S e_i = e_{(i+1)}$ orientation convention an explicit nonzero holonomy cochain is

$h = (5, 4, 6, 2, 3, 1) \pmod 7$,

which obeys $A_\chi^T h = 0 \pmod 7$. The entries satisfy $h_{(i+1)} = 5 h_i \pmod 7$, and 5 generates the multiplicative group $\mathbb{F}_7^\times$. Reversing the cyclic orientation replaces 5 by its inverse $5^{-1} = 3 \pmod 7$. The two oriented branches therefore carry the conjugate primitive cycles generated by 5 and 3.

Theorem Q (conditional chiral seven-torsion theorem)

Within the minimal Fold-generated cyclic attachment class, physical unit incidence and order-seven topology force, up to overall orientation and reflection,

$A_\chi = J - I - S$,

with integral first homology $\mathbb{Z}/7$. The sevenfold character is collective: no coefficient seven occurs in an individual cell.

The word "conditional" is essential. The source has not yet returned the physical Fold-to-vacuum attaching map that selects this matrix.

39T. Simple-Loop Obstruction on the Bare K7 Wheel

Expand one attached cell in the physical wheel edges. Using $f_i = s_i + r_i - s_{(i+1)}$,

$$c_i = -r_i - r_{(i+1)} + r_{(i+3)} - s_i + s_{(i+2)} + s_{(i+3)} - s_{(i+4)}.$$

Every coefficient is 0 or ±1, as required. However, its support is not a simple polygon. The edges s_i, r_i, $r_{(i+1)}$, $s_{(i+2)}$ form a four-cycle through the hub, while $s_{(i+3)}$, $r_{(i+3)}$, $s_{(i+4)}$ form a triangle through the same hub. Their union is

$$\mathbf{C_4 \vee C_3},$$

two loops meeting at one point. The attachment is therefore a valid cellular attaching word but not an embedded simple Fold boundary on the bare K7 graph.

Could a different simple wheel cycle produce the same order-seven topology? Every simple cycle on W_7 is either the outer six-cycle or consists of two spokes joined by a contiguous rim path of length $k = 1, \ldots, 5$. A cyclic orbit of such cells has cycle-coordinate matrix $A_k = I + S + \cdots + S^{(k-1)}$, with Fourier polynomial $p_k(z) = 1 + z + \cdots + z^{(k-1)}$. Exact evaluation gives

$|\det A_1| = 1$, $\det A_2 = \det A_3 = \det A_4 = \det A_6 = 0$, $\mathbf{|\det A_5| = 5.}$

The $k = 5$ case is itself a seven-edge simple loop, yet it generates index five rather than seven.

Theorem R (bare-wheel simple-loop no-go)

No cyclic orbit of simple unit-incidence cycles on the unrefined K7 wheel produces order-seven first homology. In particular, the number seven of edges in a cell is not the origin of the sevenfold topology. The factor seven arises from the collective oriented inter-cell attaching algebra.

39U. Contractible Hub Desingularisation and Torsion Preservation

The preceding obstruction does not require abandoning simple Fold boundaries. It can be removed by refining the hub without adding any new homological degree of freedom.

Replace the single hub by six incidence ports $p_0, \ldots, p_5$, three complement-pair vertices a_0, a_1, a_2, and one central vertex z. Attach the ports according to the three Fold complement pairs,

$p_0, p_3 \to a_0$, $p_1, p_4 \to a_1$, $p_2, p_5 \to a_2$,

and then attach $a_0, a_1, a_2 \to z$. Call this tree T. Each p_i is connected by a refined spoke x_i to the original rim vertex v_i.

The resulting graph has

$V = 16$, $E = 21$, $\beta_1 = E - V + 1 = 6$.

The tree T is contractible. Collapsing T to one point recovers the original K7 wheel and induces a homotopy equivalence on the one-skeleton.

Let $P_{(j\to i)}$ denote the unique oriented tree path from p_j to p_i. Define the lifted wheel-cycle basis

$\tilde{f}_i = x_i + r_i - x_{(i+1)} + P_{(i+1\to i)}$.

Each satisfies $\partial \tilde{f}_i = 0$, *and under tree collapse* $\pi^*(\tilde{f}_i) = f_i$. Lift the chiral attachment by

$\tilde{c}_i = \tilde{f}_{(i+3)} - \tilde{f}_i - \tilde{f}_{(i+1)}$.

Then $\pi_*(\tilde{c}_i) = c_i$. The cycle-coordinate attachment matrix is therefore still exactly A_χ. No Smith invariant can change.

More strongly, every $\tilde{c}_i$ is now an embedded simple cycle. For $i = 0$, one oriented boundary is

$p_2 \to v_2 \to v_1 \to v_0 \to p_0 \to a_0 \to p_3 \to v_3 \to v_4 \to p_4 \to a_1 \to z \to a_2 \to p_2$.

Every vertex occurs once before return to the initial vertex. Every edge coefficient is ±1. The same cyclic construction holds for all six cells. Thus

$\tilde{c}_i \cong C_{13}$ ($i = 0, \ldots, 5$),

and since the attachment matrix is unchanged,

$\mathrm{SNF}(\tilde{A}_\chi) = \mathrm{diag}(1, 1, 1, 1, 1, 7)$, $H_1(\tilde{K}_\chi; \mathbb{Z}) \cong \mathbb{Z}/7$.

Theorem S (Fold-compatible desingularisation theorem)

The non-simple $C_4 \vee C_3$ boundary on the compressed K7 wheel can be resolved into a simple Fold-compatible boundary by a contractible hub refinement organised by the three canonical Fold complement pairs. The refinement introduces no new cycle rank and preserves the order-seven Smith factor exactly.

This is a mathematical construction theorem, not yet a physical refinement theorem. The source must still return the refinement or an equivalent physical carrier map.

39V. Genus-One Supporting One-Skeleton and the Gate-T-B Physical Boundary

Although every lifted cell boundary is now simple, the complete six-cell refined one-skeleton does not embed in a sphere.

The graph contains a subdivision of $K_{3,3}$, so by Kuratowski's theorem it is non-planar. Therefore the whole refined six-cell architecture cannot be one subcomplex of a single spherical Fold interface.

Its minimum orientable genus can nevertheless be determined exactly. The refined graph has $V = 16$ and $E = 21$. A complete finite rotation-system execution over its ten degree-three vertices finds a cellular orientable embedding with $F = 5$ faces. Hence

$\chi = V - E + F = 16 - 21 + 5 = 0$,

and for an orientable closed carrier $\chi = 2 - 2g$, so

genus($\tilde{W}$) = 1.

The $K_{3,3}$ subdivision already forces $g \geq 1$; the explicit five-face rotation system gives $g \leq 1$.

Theorem T (local-sphere / global-carrier separation)

The simple Fold boundary requirement and intrinsic K7 order-seven topology are mutually compatible, but not by identifying one minimal spherical Fold with the entire vacuum complex. Each commitment cell can be realised by a simple oriented boundary. Their common contractible refinement preserves the K7 cycle rank and the $\mathbb{Z}/7$ Smith factor. The complete six-cell one-skeleton nevertheless has minimum orientable genus one.

The resulting structural picture is therefore

local simple Fold boundaries → genus-one supporting one-skeleton → attached 2-complex with candidate $H_1 = \mathbb{Z}/7$.

One possible physical interpretation is that distinct locally spherical Fold patches glue into, or project onto, a common genus-one vacuum refinement. That interpretation is not yet a source theorem.

Gate-T-B grade after this execution

The physical vacuum problem has moved substantially, but it is not closed. The following objects are now exact mathematical returns on the candidate branch: $Z_1(W_7; \mathbb{Z}) \cong \mathbb{Z}^6$; the failure of total first-cover circulation to generate torsion; $A_\chi = J - I - S$ with $\det A_\chi = 7$ and $\mathrm{SNF}(A_\chi) = \mathrm{diag}(1, 1, 1, 1, 1, 7)$; the contractible simple-loop refinement with $\beta_1(\tilde{W}) = 6$; and genus($\tilde{W}$) = 1.

The remaining physical debt is no longer "find some possible sevenfold topology." It is the source derivation of the actual Fold-to-vacuum cell map and the physical refinement/gluing operation. If that return yields A_χ, Gate T acquires intrinsic order-seven topology by derivation. If it yields another full-rank matrix, its Smith form supersedes the candidate. If it yields no order-seven factor, the present Z_7 branch is falsified rather than repaired.

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40. Next Calculation

The next calculation should execute the faithful current-link source package, not search for another magnitude-bearing mechanism and not assume the scalar C/W/Y branch. The new Part VII results add two release gates that now precede numerical gauge normalisation.

Priority K0A: unified primitive graded provenance

Construct or identify one primitive field-dependent marked instrument, frozen before comparison, whose graded derivatives reproduce the native gauge/current response together with the record, completion and CAR/Fock source jets. The 72-dimensional native gauge/current factorisation must survive unchanged or be superseded by a stronger source-native return.

Priority K0B: physical endpoint joint-law / event-grain discriminator

Freeze the same-event endpoint law and determine the physical endpoint cross coherence γ_e. The source must decide between the hard oriented-handshake branch, a finite positive-conductance refinement law, or another derived joint law. Under a finite refinement branch, return the physical conductances and refinement/Fold stopping rule; under the hard handshake branch, prove the matter-tangent lift and event-law identification before using $\gamma_e = -1$.

Only after K0A/K0B are frozen should the numerical current/link metric be promoted.

Priority K1: physical W_prim restricted to the current-connection carrier

Return from the full history/master source, before Standard-Model output inspection,

W_J^{phys}, $\mathcal{K}^{phys}$, K_θ^{phys}

on the classified physical quotient. These objects must be derived from the same source metric and regulator branch.

The primary test is

$$\boldsymbol{\varepsilon_{dual} = \| (\mathbb{K}_J^{relaxed})^{-1} - (W_J^{phys})^{-1} - (1/2)\, \mathcal{K}^{phys} (K_\theta^{phys})^{-1} (\mathcal{K}^{phys})^\dagger \|.}$$

A same-parent execution of the current source architecture requires $\varepsilon_{dual} = 0$ within declared error. This is now the first release gate.

Priority K2: decide factor democracy rather than assume it

Compute the full matrix $\mathbb{K}_J^{relaxed}$ on the faithful factor/quadrature carrier and compare it with $K_J I_6$. Two outcomes are admissible.

K2A: factor democracy survives

$\mathbb{K}_J^{relaxed} = K_J^{phys} I_6$, $M_{JJ}^{source,phys} = (49/50) K_J^{phys} H_{CWY} \otimes I_2$.

K2B: factor democracy fails

$\mathbb{K}$_J^relaxed ≠ K_J I_6, M_JJ^source,phys = E_phys† $\mathbb{K}$_J^relaxed E_phys.

No averaging or factorwise rescaling is permitted.

The executed MASD wheel assignment provides the sharp diagnostic case. Under the common unit branch, (K_3, K_2, K_q) = (24/5, 35/6, 3/2) would imply (K_J,3, K_J,2, K_J,q) = (48/53, 35/38, 3/4). The faithful source must decide whether that mode assignment survives.

Priority K3: continuum dimension-four current/gauge attachment

Once the link/current source matrix is frozen, construct the continuum attaching map

(θ, J) → (A_μ, F_μν, j^μ)

and prove an action identity of the form

S_source^(2) → −(1/4) ∫ Z_ab F^a_μν F^(b μν) + ∫ A^a_μ j_a^μ + ⋯ .

The pass conditions are: gauge covariance/invariance; Lorentz covariance after temporal attachment; exact conservation of the continuum current; correct classification of gauge nulls; preservation or source-derived replacement of the C/W/Y geometry; one common action convention for kinetic and current terms; no measured g_s, g, g′; and regulator stability.

Priority K4: continuum operator typing

The matching must distinguish the dimension-four gauge kinetic term from higher-dimensional operators generated during subsequent Yang–Mills coarse-graining. The target boundary is −(1/4) Z_ab F^a F^b, rather than the separately running dimension-six coefficient multiplying operators such as O_DFD = Tr[(D_μ F_μν)(D_ρ F_ρν)]. The two sectors may mix under later renormalisation but cannot be identified at the microscopic matching step.

Priority K5: canonical gauge fields and Abelian portal

After Z_ab and the current vertex are returned in one convention, canonically normalise the gauge fields and only then execute the surviving Abelian portal. At that point the theory may attempt to issue g_s, g and g′ without fitting them.

Parallel temporal task

Close the independent σ-to-δ attachment,

J^(t) = s_σδ (c⋆ / a_t) J,

and construct the Lorentz four-current j^μ. This remains distinct from the interaction-strength normalisation.

Parallel Priority T0: source-derived Fold-patch to refined-K7 attachment

The Gate-T topology sequence now has one sharply defined release calculation. Starting from the actual primitive Fold/first-cover record rather than from the target Z_7 topology, derive the cellular refinement map

$\mathcal{R}$_(Fold→vac)

from local Fold boundary data into the physical K7 vacuum carrier. The calculation must return the oriented images of all six Fold relations on one common refined carrier and determine the induced cycle-coordinate attachment matrix A_phys. The physical test is then SNF(A_phys), computed only after the source map is frozen.

The candidate result to falsify or confirm is

A_phys ~ A_χ = J − I − S

up to integral basis changes, overall cell orientation and physical reflection, giving SNF(A_phys) = diag(1, 1, 1, 1, 1, 7). The refinement must simultaneously establish that each physical commitment boundary is simple on its local Fold patch, that contraction to the microscopic K7 hub reproduces the coarse wheel carrier, and that no new cycle rank is introduced by the refinement. If a common global carrier is required, its surface topology must be source-returned rather than chosen; the current minimal construction predicts genus one.

A pass would convert the present Z_7 candidate into a physical Gate-T topology theorem. A different source-returned A_phys would supersede it. Failure to produce an order-seven Smith factor would close the candidate branch negatively.

This topology calculation is parallel to, not a substitute for, K0A–K5. It does not determine the physical current/link magnitude or the continuum Yang–Mills normalisation.

Remaining provenance gates

The Fold/Gate-2-to-tangent intertwiner and the AP1b-I/II identities remain worthwhile source-provenance calculations, in the operator forms fixed by Parts II and III. They determine whether the RUDM mark law and Gate-2 cost are literally the same physical event mechanism. They no longer determine whether a current stiffness exists; Theorems F through K already answer that question positively at source-action grade.

The programme priority is therefore

W_prim → (W_J, $\mathcal{K}$, K_θ) → 𝕂_J^relaxed → M_JJ^source → Z_ab → canonical gauge fields → Abelian portal.

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41. Conclusion

RCHS-1 now moves the VERSF current-normalisation programme beyond both relative rigidity and mere existence of a positive current action.

At the primitive RUDM mark grain, the twelve sector probabilities admit no independent positive representation gains. Every same-grain positive hazard law preserving the source probabilities has the form

$\tilde{h}_A = c\, w_A$,

with one common scalar functional, and the rigidity is global: one interior ray or one ledger enforces it. Conditional on AP1b identifying the current response with that same primitive mark grain, the positive source-equivariant quadratic class then gives

$Q(J) = g\, \|J\|^2$.

The resulting C/W/Y overlap geometry is

$H_{CWY} = (1/49) \cdot [\ 36\ 18\ 36\ ;\ 18\ 26\ 26\ ;\ 36\ 26\ 47\ ]$,

positive definite and independently reached from three structurally distinct parts of the VERSF chain, with microscopic refinement unable to alter it except through the named positive-semidefinite factor-sensitive correction.

The primitive current is also prior to physical time. Its divergence-free cycle sector is nonzero and physically relevant; physical time is attached downstream as a rate coordinate rather than defining the current's existence.

The first constructive strength theorem established that complete local source relaxation leaves

$H_{JJ}^{relaxed} = (48/53)\, I_6$

on the unit diagnostic branch, while for every positive diagonal primitive weighting $K_J^{relaxed} > 0$. The primitive source therefore generates a genuine positive rank-six current action.

The new result of this revision goes further. The current stiffness is not an isolated number awaiting an independent gauge conversion. Current, link and record phase already occupy one quadratic source action. Complementary Schur reduction gives

$K_\theta = 4\, w_C + 4\, w_B\, w_R\, /\, (\, 4\, w_R + w_B\,)$,

$K_J^{relaxed} = w_J\, K_\theta\, /\, (\, K_\theta + \kappa^2 w_J/2\,)$,

hence

$1/K_J^{relaxed} = 1/w_J + \kappa^2/(2K_\theta)$.

Thus the finite-source relative normalisation between current and matched link response is no longer free. On the unit branch,

$K_\theta = 24/5$, $K_J^{relaxed} = 48/53$.

These values are complementary reductions of the same primitive parent.

The relation survives at full matrix level:

$$\left(\mathbb{K}_J^{relaxed}\right)^{-1} = W_J^{-1} + (1/2)\, \mathcal{K}\, K_\theta^{-1}\, \mathcal{K}^\dagger.$$

The eventual physical source can therefore be tested by one direct reciprocity residual rather than by comparing downstream phenomenological couplings.

The same theorem exposes a new compatibility gate. The executed MASD wheel responses (24/5, 35/6, 3/2) correspond, on the common unit branch, to dual current stiffnesses

(48/53, 35/38, 3/4).

A source with distinct physical wheel modes cannot simultaneously retain one common scalar current stiffness under a common positive separable metric. The physical W_{prim} and faithful non-Abelian carrier must decide whether factor democracy survives or whether the full matrix response replaces it.

Accordingly, the familiar scalar pullback

$$M_{JJ}^{source} = (49/50)\, K_J^{relaxed}\, H_{CWY} \otimes I_2$$

is now correctly graded as the factor-democratic branch of the more general result

$$M_{JJ}^{source} = E_{phys}^\dagger\, \mathbb{K}_J^{relaxed}\, E_{phys}.$$

This is a strengthening, not a retreat: the paper no longer hides possible factor dependence behind a common scalar. It supplies the exact test that decides whether the scalar reduction is physical.

The binary first-record calculations retain their proper role. The current F2F architecture does not change fact/no-fact occurrence,

$$D^n_{(J_{cyc})}\, \Gamma_F = 0 \quad (n \geq 1),$$

even though current changes record identity and phase. Binary occurrence is therefore not the present readout of the source current strength. This does not affect the independently derived current-link source action.

The source-provenance frontier has also moved. The supposed single source derivative $d\Phi_n$ is retyped as a graded jet, and the native representation-valued gauge/current slice now closes without an invented selector embedding: the raw current-defect and mark derivatives both have rank 144 and factorise at 2.58×10^{-15}; after Wilson quotienting the current-defect, mark and terminal responses remain rank 72 with a 1.80×10^{-15} mark reconstruction. One unified primitive instrument generating all graded sectors remains open, but the old arbitrary gauge/current embedding is no longer the live problem.

The endpoint-current frontier has narrowed in the same way. The 49-dimensional GSJB innovation is exactly the projective tangent of the same PFDMI/MASD endpoint carrier.

Common internal covariance collapses P2 to one scalar γ_e, and the weighted-Dirichlet theorem proves that every connected positive-conductance finite non-common innovation has strictly positive current capacity at every finite refinement depth. The exact common mode is the only finite current-invisible spatial sector. Thus physical uncertainty now controls the magnitude and event law, not the existence of a finite non-common current.

The physical frontier is now sharply defined. The missing object is not another current mechanism and not an arbitrary current-to-gauge conversion. It is the faithful source package

$(W\_J, \mathcal{K}, K\_\theta)$

returned by the physical W_prim, together with the factor/mode assignment and the continuum map to

$-(1/4)\ Z\_ab\ F\text{^}a\_\mu\nu\ F\text{^}(b\ \mu\nu) + A\text{^}a\_\mu\ j\_a\text{^}\mu.$

The dimension-four kinetic normalisation must remain distinct from the higher-dimensional Yang–Mills operators produced under later coarse-graining. Only after that continuum attachment and canonical field normalisation may the surviving Abelian portal be executed and physical gauge couplings be issued.

The independent Gate-T topology branch has also moved from existence arguments to a concrete attaching-map problem. On the physical K7 wheel every two-cell attachment factors through the six-dimensional integral cycle lattice. The source-returned total first-cover circulation cannot generate torsion, the unordered Fold relation carrier has insufficient rank, and cyclic simple wheel plaquettes cannot generate order seven. Combining the source-returned complement pairing with the conditional oriented boundary successor S, so that $J = S^3$ in that ordering, instead gives the unique minimal unit-incidence order-seven candidate $A\_\chi = J - I - S$, with $SNF(A\_\chi) = diag(1, 1, 1, 1, 1, 7)$. The apparent conflict between this collective torsion and the Fold's simple commitment boundary is not fundamental: a contractible hub refinement organised by the three Fold complement pairs turns every attached cell into a simple 13-edge loop without changing β_1 or the Smith form, and the common refined one-skeleton has minimum orientable genus one. Thus local simple Fold boundaries and global intrinsic $\mathbb{Z}_7$ topology are mathematically compatible, but the source must still derive their physical refinement/gluing map.

The current-normalisation and topology results remain deliberately separated. The former has reached a source-generated positive current action and current-link reciprocity; the latter has reached an essentially unique candidate order-seven attaching algebra with a minimal simple-loop refinement. Neither result may be used to fill the remaining provenance gap of the other.

The paper's strongest present statement is consequently:

> **VERSF fixes the primitive relative C/W/Y current geometry, generates a positive rank-six current action, and fixes its finite-source current-link relative normalisation through an exact Schur reciprocity theorem. Independently, the Gate-T vacuum problem has been reduced to a sharply**

constrained Fold-to-K7 attaching map whose minimal oriented unit-incidence branch carries intrinsic $\mathbb{Z}/7$ torsion and admits a simple-loop refinement whose one-skeleton has minimum embedding genus one. What remains is physical source selection: the faithful current/link metric and continuum gauge normalisation on one branch, and the source-derived Fold-to-vacuum refinement and attaching map on the other.

Appendix A: Exact Component Ledger

Component A	d_A	(C, W, Y)
e_R^triv	1	(0, 0, 1)
e_R^std	2	(0, 0, 1)
N_R^std	2	(0, 0, 0)
L_L^triv	2	(0, 1, 1)
L_L^std	4	(0, 1, 1)
H	2	(0, 1, 1)
d_R^triv	3	(1, 0, 1)
d_R^std	6	(1, 0, 1)
u_R^triv	3	(1, 0, 1)
u_R^std	6	(1, 0, 1)
Q_L^triv	6	(1, 1, 1)
Q_L^std	12	(1, 1, 1)

Summing the uncentred signature products gives

$\Sigma_A\, d_A\, \chi(A)\, \chi(A)^T = [\ 36\ 18\ 36\ ;\ 18\ 26\ 26\ ;\ 36\ 26\ 47\ ]$.

Entry accounting: (C, C) = 3 + 6 + 3 + 6 + 6 + 12 = 36; (W, W) = 2 + 4 + 2 + 6 + 12 = 26; (Y, Y) = 49 − 2 = 47 (all dimensions except N_R^std are hypercharged); (C, W) = 6 + 12 = 18 (the Q_L components only); (C, Y) = 36 and (W, Y) = 26, since every coloured and every weak component carries hypercharge.

Appendix B: Selector Jacobian

For

$p_A = d_A / 49$

and log gains $g_A = e^{\gamma_A}$,

$p_A(\gamma) = e^{\gamma_A}\, p_A / \Sigma_B\, e^{\gamma_B}\, p_B$.

Thus

$\partial p\_A / \partial \gamma\_B = p\_A (\delta\_AB - p\_B)$,

or

$J\_g = \mathrm{diag}(p) - p\, p^T$.

Because every p_A > 0, Lemma 3 gives

rank J_g = 11, ker J_g = span{(1, …, 1)},

for any strictly interior twelve-category law, so the rank statement is a structural property of interiority and not an accident of the particular dimension vector (1, 2, 2, 2, 4, 2, 3, 6, 3, 6, 6, 12).

The common-gain null is exactly the statement that conditional mark probabilities cannot determine an absolute event clock.

Appendix C: Machine Certificates and Independent Verification

C.1 Selector and factor-Gram certificate

The associated frozen calculation package records:

- twelve-sector first-fact selector rank: **11**;
- selector nullity: **1**;
- common-direction null alignment: **1.000000000000000**;
- nonuniform-gain first-fact total variation: **0.048501275510**;
- 20,000-ray mean conditional total variation: **0.047797636485**;
- exact integer factor Gram: 49 H_CWY = [36 18 36 ; 18 26 26 ; 36 26 47];
- magnitude-bearing event-Gram residual: $\mathbf{1.132 \times 10^{-15}}$.

C.2 Attachment-audit certificate

The 49-dimensional direct-sum implementation with the twelve source dimensions records:

- common scaled block-unitary maximum effect residual: $\mathbf{1.148 \times 10^{-15}}$;
- summed effect residual: $\mathbf{2.197 \times 10^{-15}}$;
- RUDM conditional-probability residual: $\mathbf{1.110 \times 10^{-16}}$;
- scalar Euclidean-cost conditional-probability residual: $\mathbf{5.551 \times 10^{-17}}$;
- measured common rate ratio: **0.750511728837068**, equal to the predicted e^(−a_t Q) to all printed digits;
- nonuniform twelve-sector attenuation total variation: **0.072429283693**, so sector anisotropy is visible at the effect level;

- quadratic-current factorisation residual: max_A | λ_A − λ_0 $\|J\|^2$ w_A | = **8.882×10^{-16}**;
- normalised quadratic current marks reproduce the current-ray projector weights to **5.551×10^{-17}**.

C.3 Independent verification layer

This paper carries an independent verification layer, executed in exact and floating arithmetic separately from both frozen packages:

- component ledger sum: Σ d_A = 49, confirmed;
- integer Gram reconstructed from the ledger by exact arithmetic: identical to the certificate matrix, entry by entry;
- Jacobian rank 11, nullity 1, null alignment with (1, …, 1) equal to 1 to machine precision, confirmed;
- eigenvalues of M_JJ^event recomputed from the exact rational matrix: 0.314682300680, 1.010864036230, 7.572412846763, agreeing with the certificate to all stated digits;
- equal-factor marked-rate precision f^T M_JJ^event f = 1076/147 exactly, decimal 7.319727891156, confirmed;
- leading principal minors of 49 H_CWY: 36, 612, 4428, all positive, so H_CWY is positive definite; trace 109, determinant 4428;
- reproducible nonuniform-gain diagnostic with declared gain g_A = d_A: exact first-fact total variation 1140/4949 = 0.230349565569; seeded 20,000-ray mean conditional total variation 0.232893;
- handoff matrix M_0 = K_cyc^(2) H_CWY recomputed entrywise from K_cyc^(2) = 0.212610974570295 and the exact Gram: maximum deviation from the stated entries below 5×10^{-13}; axle precision $f^T M_0 f$ = 0.389063620132 confirmed, with exact form K_cyc^(2) · 269/147;
- Theorem 6 refinement identity verified numerically on a randomised factor-sensitive splitting of all twelve components: M^micro reproduces M^coarse + Σ_A λ_A I^ref(A) to finite-difference precision, with the correction positive semidefinite, and the factor-blind limit returning M^micro = M^coarse exactly;
- centred conditional-mark check: with $\mu = (36, 26, 47)^T/49$, the exact identity 2401 (H_CWY − $\mu \mu^T$) = [468 −54 72 ; −54 598 52 ; 72 52 94] is confirmed by rational arithmetic, and the equal-factor conditional-mark precision f^T 4(H_CWY − $\mu\mu^T$) f = 5200/7203 = 0.721921421630, so the uncentred/centred typing note of Section 9 is exact;
- attachment-audit operator algebra independently rebuilt with declared reproducible parameters: a random per-sector block-diagonal unitary with common norm h_0 returns E_A = h_0 P_A and Σ_A E_A = h_0 I to ≤ 1.0×10^{-15} and the RUDM conditional law to 5.6×10^{-17}; a declared scalar cost with a_t Q = ln(4/3) leaves the conditional law unchanged to 1.2×10^{-16} with rate ratio exactly 0.75; a declared per-sector attenuation d_A produces exact first-fact total variation 1140/4949, coinciding with the Section 7 diagnostic because both are the g = d gain; the current-

ray factorisation $\lambda\_A = \lambda_0 r^2 \|P\_A z\_J\|^2$ holds to 3.6×10^{-15} and the normalised marks reproduce the projector weights to 2.8×10^{-17}.

C.4 AP1b-I audit verification

The Part III spectra and kernel claims are independently reverified in exact and floating arithmetic:

- Q_comp reconstructed from A_comp: equal to the stated integer circulant; factorisation $Q\_comp = (I + P)^T(I + P)$ exact; spectrum {0, 1, 1, 3, 3, 4} matching the closed form $4 \cos^2(\pi k/6)$ to machine precision; Q_comp a = 0 and (I + P) a = 0 exactly for the alternating mode;
- $Q\_Gate2,on = Q\_comp + \mathbb{1}\mathbb{1}^T$: spectrum {0, 1, 1, 3, 3, 10}; Q_Gate2,on a = 0 to 5.6×10^{-17}; inertia (5 positive, 0 negative, 1 zero) confirmed;
- W_wheel: exact rational eigenvalues {3/31, 39/248, 45/248, 54/217, 12/31, 12/31} confirmed by symbolic computation via the two singleton and two 2 × 2 symmetry blocks; all strictly positive, so the inertia mismatch (6, 0, 0) versus (5, 0, 1) underlying Section 31 is exact.

C.5 Part IV verification

The Part IV exact claims that are checkable without the frozen packages are independently reverified:

- Hodge additivity of the CRTF split: 1 + 2/3 = 5/3 exactly, and the cycle share 1/(5/3) = 3/5, so the 60% statement is exact rational arithmetic on the certified Hodge norms;
- the MASD half of the generator match: for the K = 7 wheel (hub plus six rim vertices, twelve edges) the incidence identity $B B^T = L\_(W_7)$ holds exactly in integer arithmetic, so with $C = (1/31) I_{12}$ the identity $B C B^T = (1/31) L\_(W_7)$ is automatic; the independent certificate content is the HMGBC equality $Q_7 = \Pi(I - T_0) = (1/31) L\_(W_7)$, consumed at its source grade;
- the Theorem E conversion algebra: the chain σ = s_σδ δ, t_phys = (a_t/c⋆) δ gives ∂_σ = (a_t/(s_σδ c⋆)) ∂_t, and the round-trip factor (s_σδ c⋆/a_t)(a_t/(s_σδ c⋆)) simplifies to 1 symbolically, so the continuity equation retains its exact form with J^(t) = s_σδ (c⋆/a_t) J and no residual factor.

C.6 Binary first-record curvature certificate

The BCTR/RCCSJ calculation supplies the following exact or machine-certified identities:

- $D\_(J\_cyc)\ \Gamma\_F |_0 = 0$ by divergence-free outgoing-score compensation;
- local m = 2/4 spatial Grams $G\_h = (3/112) I_2$ and $G\_(b\_j) = (5/384) I_2$;
- stationary coefficient (7/31)(3/112) + 6 (4/31)(5/384) = 1/62 exactly;
- uniform, nonuniform and full-cycle coefficients 3/62, 1/62 and 2/31;
- equality of the binary-rate curvature and RCCSJ marked Fisher matrices;
- full representation-resolved rank six for g_cyc ≠ 0;

- numerical scale conversion using the inherited K_cyc^(2): (2/31) / 0.212610974570295 = 0.30344684305525943;
- conditional unit-normalised birth target λ_birth = 1/3 and implied g_cyc = 1.048088753531807, explicitly dependent on the same-event and same-normalisation premises of Section 38.

The numerical identities verify the algebra of the positive witness. They do not select the physical gain or replace the completed-source extraction.

C.7 Part V independent verification

The Part V arithmetic checkable without the frozen packages is independently reverified in exact and symbolic arithmetic:

- stationary reduction over the common denominator: (7/31)(3/112) = 3/496 and 6 (4/31)(5/384) = 5/496, summing to 8/496 = 1/62 exactly; full-cycle additivity 3/62 + 1/62 = 2/31 exactly;
- scale conversion (2/31)/K_cyc^(2) = 0.303446843055259448, agreeing with the certificate to all printed digits, with the exact form λ_J = (2/(31 K_cyc^(2))) g_cyc^2;
- the conditional gain in closed form: g_cyc^2 = (1/3)/λ_J(1) = 31 K_cyc^(2)/6 = 1.098490035280, so g_cyc = √(31 K_cyc^(2)/6) = 1.048088753531807, agreeing with the certificate to all printed digits;
- the occurrence/type Fisher decomposition verified symbolically on a two-parameter, three-type family: the full categorical Fisher matrix equals ($\nabla h\, \nabla h^{T}$)/(h(1−h)) + h F_q identically, with the cross terms vanishing through Σ_A ∇q_A = 0, and ∇Γ_F = ∇h_F/(1 − h_F) confirmed;
- rank six of the curvature blocks: H_CWY is positive definite of rank three (minors 36, 612, 4428), so H_CWY $\otimes I_2$ has rank six for every nonzero gain;
- Lemma V.1 verified numerically on a randomised eight-transition log-linear rate family with first-order compensation imposed: the finite-difference Hessian of the total rate matches the rate-weighted score Gram to finite-difference precision and the first derivative vanishes at the base point.

C.8 BCTR-3 through BCTR-5 source verification

The published-source binary and feedback calculations supply:

- all-orders serial occurrence response D_J^n NR = D_J^n h_F = D_J^n Γ_F = 0;
- conditional first-record Jacobian rank 6;
- edge/history marginal rank 0, with maximum residual 1.373319860609179 × 10^{-13};
- terminal/type marginal rank 6;
- exact eligibility-contrast identity (1/2) D_J^2 Γ_(F,x) = h_x (G_(0,x) − G_(1,x));
- minimum exact positive eligibility carrier: three E_2-equivalent score copies;
- one-copy and two-copy minimum target residuals 0.138464949041 and 0.041156112028;

- exact scale extraction $\lambda_J = 7.624971107276631\ \alpha$ conditional on a source-returned eligibility contrast;
- self and fact effect factorisation residuals 2.776×10^{-17} and 2.220×10^{-16};
- trace-preserving renewal feedback Hessian 0_{6x6};
- conditional 84-record Fisher rank 6, but target-shape residual 0.604817373322.

C.9 BCTR-6 mixed-source census and phase precursor

The deeper source execution supplies:

- physical current Gram residual to $(49/50)\ H_{CWY} \otimes I_2$: 5.299×10^{-16};
- exact EPMD current-to-record-phase block $H_{(J_{cyc},y)}^{eff} = -(\sqrt{2}/11)\ I_6$;
- current-to-record-phase rank 6;
- current-to-record-capacity rank 0, with Schur cross norm 3.964×10^{-31};
- phase-mediated uniform stretch response $s_{rim}^\star = -(2/8427)\ \|J\|^2$;
- CMHRE history–record cross norm 0 at both regulator levels;
- XI-3 and XI-4 maximum intergroup source-Hessian cross norms 0;
- projected defect Jacobian rank 183 on 226 coordinates, leaving nullity 43;
- current-to-history-eligibility reachability in the frozen nonzero source graph: false.

These certificates establish an action-level rank-six precursor and the frozen-source history-eligibility cut. They do not select a physical λ_J.

C.10 Current-stiffness and current-link reciprocity verification

The constructive Part VI results carry the following exact and numerical checks, all independently reverified in this paper's symbolic layer:

1. the displayed general $H_4(w)$ is exactly the Hessian of the stated sum-of-squares source action $\Gamma_4^{(2)}$, verified symbolically, so the block is a priced common action and not an assembled matrix;
2. Schur elimination of q, θ from the unit four-variable Hessian gives $H_{pair}^{(J,y)} = (1/11)[\ 10\ -\sqrt{2}\ ;\ -\sqrt{2}\ 53\ ]$ exactly;
3. its determinant is $\det H_{pair}^{(J,y)} = 528/121 = 48/11$;
4. Schur elimination of y gives $10/11 - (\sqrt{2}/11)^2/(53/11) = 48/53$; the phase-screening decrement and fraction are 2/583 and 1/265, with $583 = 11 \cdot 53$ and $(264/265)(10/11) = 48/53$ exactly;
5. the general link stiffness is $K_\theta = 4\ w_C + 4\ w_B\ w_R/(4\ w_R + w_B)$, and the retained current-link determinant is $\det H_{J\theta}^{red} = w_J\ K_\theta$, both verified symbolically;
6. complementary Schur elimination gives $K_J^{relaxed} = w_J\ K_\theta/(K_\theta + \kappa^2 w_J/2)$, and the reciprocal form $1/K_J^{relaxed} - 1/w_J - \kappa^2/(2K_\theta) = 0$ is verified symbolically;
7. expanding the reciprocity form reproduces exactly the Theorem G closed formula $8\ w_J\ (\ w_B\ w_C + w_B\ w_R + 4\ w_C\ w_R\ )/\Delta_\kappa$, verified symbolically, so the two derivations of $K_J^{relaxed}$ are one identity;

8. on the unit branch $K_\theta = 24/5$ and $K_J^{relaxed} = 48/53$;
9. the formula is homogeneous of degree one in the weights, and $\partial K_J^{relaxed}/\partial\kappa = -16\,\kappa\, w_J^2\,(w_B + 4\, w_R)(w_B\, w_C + w_B\, w_R + 4\, w_C\, w_R)/\Delta_\kappa^2 < 0$, confirming Remark G.3;
10. the full-matrix Woodbury identity $(\mathbb{K}_J^{relaxed})^{-1} = W_J^{-1} + (1/2)\,\mathcal{K}\, K_\theta^{-1}\, \mathcal{K}^\dagger$ verifies numerically on a randomised complex six-dimensional package (positive-definite W_J, K_θ; complex $\mathcal{K}$) to residual 3.0×10^{-15} in this paper's independent layer; the frozen certificate records matrix reciprocity residual 2.64×10^{-16} and complementary link-block recovery residual 1.18×10^{-16};
11. on unit weights the mode family gives $K_\theta(4) = 24/5$, $K_\theta(5) = 35/6$, $K_\theta(1) = 3/2$ and $K_J(4) = 48/53$, $K_J(5) = 35/38$, $K_J(1) = 3/4$, verified exactly;
12. for every positive weighting $dK_\theta/d\lambda = w_C + w_B^2\, w_R/(\lambda\, w_R + w_B)^2 > 0$ and $dK_J/d\lambda > 0$, proving the factor-democracy/distinct-wheel-mode incompatibility under the common separable branch;
13. pullback through the certified physical current Gram gives $M_{JJ}^{source} = (49/50)\, K_J^{relaxed}\, H_{CWY} \otimes I_2$; on the unit branch $(49/50)(48/53) = 1176/1325 = 0.8875471698113208$; and since H_{CWY} is positive definite of rank three, the pulled-back Hessian has rank six for every $K_J^{relaxed} > 0$.

These checks certify the current-link reciprocity theorem and its compatibility gate. They do not select the physical W_prim, the faithful factor branch or the continuum gauge coupling.

C.11 Graded P1 native-provenance verification

The Part VII P1 execution records: correctly typed graded-source descent rather than one ordinary $d\Phi$ matrix; gauge quadratic descent residual 2.5266×10^{-15}; full-action completion descent residual 1.8867×10^{-15}; record-phase row-space residual 3.1352×10^{-15}; shared-link CAR/Fock residual 1.7935×10^{-15}; CAR completion common-direction residual 1.1487×10^{-15}; native matrix-link coordinate count 144; restricted current-defect rank 144; native marked-law rank 144; raw mark factorisation residual 2.5840×10^{-15}; raw terminal reconstruction residual 2.2823×10^{-15}; Wilson-quotient current-defect / mark / terminal ranks 72 / 72 / 72; physical mark reconstruction residual 1.8038×10^{-15}; physical terminal reconstruction residual 1.8755×10^{-15}.

These checks close P1a on the source-extension branch; they do not promote the deterministic endpoint witness or close P1b.

C.12 Weighted-Dirichlet P2 verification

The general P2 theorem is analytic. Its canonical and random-conductance checks record: arbitrary positive $K = \mathrm{diag}(\kappa_e)$ on the connected wheel gives $\mathrm{rank}(L_K) = 6$ and one spatial common-mode kernel; canonical spectrum recovered as $\{0, 1/2, 1/2, 1, 1, 31/28, 5/4\}$; $\gamma_{spoke}(1) = 0.486616791079$ and $\gamma_{rim}(1) = 0.502983670412$; $\gamma_{spoke}(2) = 0.815845031782$ and $\gamma_{rim}(2) = 0.813224908401$; 200 random positive-conductance draws × four finite depths = 800 finite-time tests; minimum tested edge correlation 0.003013491826; maximum tested edge correlation 0.995035145673; minimum global

Dirichlet capacity 0.291956503239; maximum spectral-formula residual 5.33×10^{-15}; spectral-bound violations 0.

The canonical branch and persistence bounds are additionally reverified in this paper's independent symbolic/numeric layer: the stationary-mass wheel with 1/31 conductances reproduces the spectrum $\{0, 1/2, 1/2, 1, 1, 31/28, 5/4\}$ and all four benchmark correlations $\gamma_{\mathrm{spoke}}(1)$, $\gamma_{\mathrm{rim}}(1)$, $\gamma_{\mathrm{spoke}}(2)$, $\gamma_{\mathrm{rim}}(2)$ to every printed digit; L_K annihilates $M^{(1/2)}1$ to machine precision; and an independent 200-draw random-conductance, random-mass sweep at three finite depths confirms $\mathcal{D}_K(s) > 0$ and the two-sided spectral bounds with zero violations. The random sweeps check implementation only. The persistence statement follows from the positive spectral representation of $\mathcal{D}_K(s)$.

C.13 Gate-T seven-torsion and refinement verification

The Part VIII calculations are independently checked in exact integer arithmetic and finite graph enumeration, and every item below is additionally reverified in this paper's own symbolic/enumerative layer:

- the K7 wheel cycle basis $f_i = s_i + r_i - s_{(i+1)}$ has rank six and spans $\ker_{\mathbb{Z}} \partial_1$ exactly;
- the first-cover circulation row $K = (1, 1, 1, 1, 1, 1)$ has Smith form (1); its ordinary kernel has rank five and torsion-free quotient;
- the Fold atom-support matrix has rank four and Smith form $\mathrm{diag}(1, 1, 1, 2)$;
- for the 26 nonzero matrices $aI + bS + cJ$ with $a, b, c \in \{0, \pm1\}$, exact determinant enumeration finds exactly four triples with determinant magnitude seven, $(-1, -1, 1)$, $(-1, 1, -1)$, $(1, -1, 1)$ and $(1, 1, -1)$; the middle pair produces a physical spoke coefficient of magnitude two (the $s_{(i+1)}$ coefficient is $b - a$) and fails unit incidence, leaving only $A_\chi = J - I - S$ and its overall negative;
- exact arithmetic gives $\det A_\chi = 7$ and $\mathrm{SNF}(A_\chi) = \mathrm{diag}(1, 1, 1, 1, 1, 7)$;
- the edge expansion of one cell has exactly seven nonzero unit coefficients, $-r_i - r_{(i+1)} + r_{(i+3)} - s_i + s_{(i+2)} + s_{(i+3)} - s_{(i+4)}$, confirming unit incidence and the $C_4 \vee C_3$ support;
- modulo seven, $A_\chi^T (5, 4, 6, 2, 3, 1)^T = 0$ with one-dimensional nullspace, and the holonomy ratio $h_{(i+1)} = 5\, h_i \pmod 7$ is confirmed entrywise;
- for cyclic orbits of simple wheel cycles the determinant census is $|\det A_1| = 1$, $\det A_2 = \det A_3 = \det A_4 = \det A_6 = 0$, $|\det A_5| = 5$, so no bare-wheel simple-cycle orbit gives order seven;
- the complement-pair hub refinement has $V = 16$, $E = 21$, $\beta_1 = 6$; all six lifted attaching cycles are explicitly reconstructed and each contains exactly thirteen nonzero unit-incidence edges, with connected support and every support vertex of degree two, hence each is a simple C_{13};
- tree contraction reproduces the original K7 cycle basis and leaves the cycle-coordinate attachment matrix equal to A_χ, so the Smith form is unchanged;
- the refined common graph is non-planar, with the following explicit $K_{3,3}$ subdivision witness, independently reverified: take the bipartition $\{v_2, v_4, z\}$ and $\{v_3, a_1, a_2\}$, with

the nine subdivision paths v_2–v_3; v_2–v_1–p_1–a_1; v_2–p_2–a_2; v_4–v_3; v_4–p_4–a_1; v_4–v_5–p_5–a_2; z–a_0–p_3–v_3; z–a_1; z–a_2. The paths are pairwise edge disjoint, their interiors $\{v_1, p_1\}$, $\{p_2\}$, $\{p_4\}$, $\{v_5, p_5\}$, $\{a_0, p_3\}$ are mutually disjoint and avoid the six branch vertices, and each of the nine branch pairs is joined exactly once, giving the genus lower bound $g \geq 1$;

- exhaustive orientable rotation-system evaluation over all $2^{10} = 1024$ rotation assignments of the ten degree-three vertices gives maximum face count F = 5, hence $16 - 21 + 5 = 0 = 2 - 2g$ and g = 1 exactly. One explicit five-face witness, in cyclic edge order at each degree-three vertex with rim edges r_i : v_i → v_(i+1), refined spokes x_i, port edges e_i and tree edges g_j, is: v_0 (r_0, r_5, x_0); v_1 (r_0, r_1, x_1); v_2 (r_1, r_2, x_2); v_3 (r_2, r_3, x_3); v_4 (r_3, r_4, x_4); v_5 (r_4, x_5, r_5); a_0 (e_0, e_3, g_0); a_1 (e_1, g_1, e_4); a_2 (e_2, e_5, g_2); z (g_0, g_1, g_2); the six degree-two ports carry their unique rotation. Face tracing of this system closes with exactly five faces, giving the matching upper bound $g \leq 1$.

These checks certify the finite mathematical construction. They do not supply the missing physical Fold-to-vacuum refinement map.

The numerical packages are verification certificates; none of these diagnostics substitutes for the source-provenance grades stated in the main text, and the audits' analytic verdicts on the published stack are unaffected by them.

Appendix D: Mark-Refinement Identity

For completeness, let

λ_Aα = λ_A r_Aα, Σ_α r_Aα = 1.

Then

s_F,Aα = ∂_F log λ_Aα = s_F,A + u_F,Aα, u_F,Aα = ∂_F log r_Aα.

The microscopic contribution at fixed coarse component A is

Σ_α λ_A r_Aα (s_F,A + u_F,Aα)(s_G,A + u_G,Aα).

Using

Σ_α r_Aα u_F,Aα = Σ_α ∂_F r_Aα = 0

the cross terms vanish, giving

λ_A s_F,A s_G,A + λ_A Σ_α r_Aα u_F,Aα u_G,Aα.

Summing over A,

M^micro = M^coarse + Σ_A λ_A I_A^ref,

with

$I\_A^{ref} = \Sigma\_\alpha\ r\_A\alpha\ u\_A\alpha\ u\_A\alpha^T \geq 0$,

and I_A^ref = 0 exactly when u_Aα vanishes on the support of r_A, that is, when the splitting is factor blind. This exact identity is the basis of Theorem 6 and its equivalence Corollary 6.1.

Appendix E: Attachment-Status Ledger

Object	Status after RCHS-1 and the attachment audit
Twelve source projectors P_A	exact on frozen 49D commitment tangent
RUDM relative hazards $h\_A = h_0\ w\_A$	closed at inherited source grade
Common absolute RUDM hazard h_0	open
One primitive shared-interface record before copying	strong conditional inherited pass
Twelve-projector resolution on primitive commitment carrier	source supported / exact projector algebra
Fold/K_4-to-tangent intertwiner	open
AP1a primitive carrier compatibility	partly source supported; not fully executed
AP1b decomposed into AP1b-I/II/III	operational reduction within SGPL/HMGBC, exactly derived (Part II); conjunction remains physically open
Effect-equivalence criterion $E\_A = h_0\ P\_A$	exactly derived; finite operator test
Common-parent variation mechanism (Theorem B)	exactly derived; no cost-to-probability axiom needed
Block-unitary transfer characterisation	exactly derived; transfer must lie in the sector commutant
Scalar Gate-2 cost as common multiplier $c = h_0\ e\^(−a\_t\ Q)$	exactly derived; realises Theorem 1A's surviving functional
Current-ray factorisation $h_0 = \lambda_0\ r^2$	exact identity; conditional on realised-ray equality
AP1b-I common-parent ancestry of Gate-2/current term	**not yet proved (Part III)**; wheel-metric shortcut exactly ruled out; reduced to the L_λ Jacobian return
Gate-2 A_comp provenance from the master action	open; load-bearing postulate per its source paper
$Q\_comp = (I + P)^T(I + P)$, spectrum {0, 1, 1, 3, 3, 4}, alternating kernel	exact

Object	Status after RCHS-1 and the attachment audit
Q_Gate2,on spectrum {0, 1, 1, 3, 3, 10}, same kernel	exact
W_wheel spectrum {3/31, 39/248, 45/248, 54/217, 12/31, 12/31}, positive definite	exact; full typed defect lift still open
Invertible identification Q_Gate2 ↔ W_wheel	exactly ruled out (inertia)
Faithful pullback of W_wheel, or of any positive-definite metric	exactly ruled out (Corollary 32.1)
Pre-compression Jacobian L_λ and parent pullback L_λ† H_loc L_λ	open; the named AP1b-I object (F11)
AP1b-II mark-projector and realised-ray identity	partly supported; conditional on birth-current tangent identity
Linear current-to-record amplitude D_(J_cyc) $\mathcal{K}$_A	**fails on published stack (PCRS-1R)**
Binary first-record curvature in RCCSJ/BCTR-2	exact positive existence pass; $D\Gamma\_F = 0$, $D^2\Gamma\_F \neq 0$; common gain free
AP1b-III magnitude-bearing primitive event law	event-law route fails on the present published architecture; source-action current stiffness passes separately in Part VI
Exact RCCSJ gauge-scale family	λ_J = (2/(31 K_cyc^(2))) g_cyc^2, numerically 0.30344684305525943 g_cyc^2; symbolic relation exact, gain not selected
Published serial binary occurrence response	exact zero to all orders; current changes record identity, not occurrence
Conditional first-record identity current response	nonzero / rank six
Eligibility-contrast identity	exact: $(1/2)\, D\_J^2\, \Gamma\_(F,x) = h\_x\, (\, G\_(0,x) - G\_(1,x)\,)$
Minimum full-rank eligibility carrier	three E_2-equivalent score copies / six real directions
Eligibility scale extraction	λ_J = 7.624971107276631 α if the source returns the required contrast
Record/internal signal through trace-preserving renewal	carried conditionally, but next factorised self/fact selector is exactly blind
Published feedback coefficient α_feedback	exactly zero for the tested mechanism; not a physical zero-force prediction
Conditional record Fisher shortcut	rank six but not source placed; relative target-shape residual 0.604817373322

Object	Status after RCHS-1 and the attachment audit
Physical current-cycle to record-phase precursor	exact action-level rank six; $H_{(J\_cyc,y)}^{eff} = -(\sqrt{2}/11)\, I_6$
Fully relaxed current stiffness on unit source metric	exact: $H_{JJ}^{relaxed} = (48/53)\, I_6$; rank six
Record-phase screening fraction	exact unit-branch decrement 2/583, equal to 1/265 of the pre-phase current stiffness; screening universally nonnegative (Remark G.3)
General positive-weight relaxed current stiffness	exact $K_J^{relaxed}(w) > 0$ for every positive w_J, w_C, w_R, w_B, κ in the source family
Physical C/W/Y source-action pullback	factor-democratic branch: $M_{JJ}^{source} = (49/50)\, K_J^{relaxed}\, H_{CWY} \otimes I_2$; authoritative fallback $E_{phys}^\dagger\, \mathbb{K}_J^{relaxed}\, E_{phys}$ (F7)
Current-link Schur reciprocity $1/K_J^{relaxed} = 1/w_J + \kappa^2/(2K_\theta)$	exact within the common four-variable source block
Matched link stiffness $K_\theta = 4w_C + 4w_B\, w_R/(4w_R + w_B)$	exact; unit branch 24/5, complementary reduction of the same parent as 48/53
Full-matrix reciprocity (Woodbury) and ε_{dual} release test	exact linear algebra; faithful physical execution open (F7A)
Factor-democracy / wheel-mode compatibility gate	open; distinct wheel modes ⇒ distinct K_J under a common separable metric (diagnostic triple 48/53, 35/38, 3/4)
Dimension-four vs dimension-six continuum operator typing	firewall declared; O_{DFD} coefficient separately typed
Unit diagnostic C/W/Y source-action Hessian	exact $M_{JJ}^{source,unit} = (1176/1325)\, H_{CWY} \otimes I_2$; not a physical coupling prediction (F15)
Physical primitive current metric W_{prim}	open; must fix the absolute source-action unit and hence K_J^{phys}
Continuum gauge-action attaching map	open; must derive discrete current action → $S_{gauge}[A] + S_{int}[A, j]$
Physical current-cycle to record-capacity Hessian	exact zero
Nonlinear current-squared record-stretch response	nonzero but attenuating: $s_{rim}^\star = -(2/8427)\, \|J\|^2$
CMHRE history–record and XI-3/XI-4 intergroup crosses	exact zero on the executed product/direct-sum sources
Projected source Jacobian into the missing bridge	absent / non-identifiable from the frozen defect map on a 43-dimensional kernel

Object	Status after RCHS-1 and the attachment audit
Binary first-record occurrence as gauge-strength carrier	exhausted on the current executable corpus
Saturated same-event birth target	$\lambda_J = 1/3$ at $\kappa_J = 1$, with $g_{cyc} = \sqrt{31 K_{cyc}^{(2)}/6}$; strong conditional target only
Functional relative hazard rigidity	exact conditional on AP1b
Positive quadratic current law $Q = g\|J\|^2$	exact within source-equivariant quadratic class conditional on AP1b
Coarse magnitude-bearing score support $s_{F,A} = 2\chi_F(A)$	exact for that quadratic law
H_CWY factor shape	exact
Factor-blind microscopic refinement invariance	exact, and an equivalence (Corollary 6.1)
Factor-sensitive refinement correction	exact PSD formula; physical value open
Common physical current/history scale λ_J	open; a positive source-action stiffness is now derived, but its conversion to the physical history/Fisher normalisation requires W_prim and the continuum attaching map (F9); never by temporal coordinate conversion (F-PT1)
Abelian closure portal (g_C, a_C)	open
Primitive MASD constitutive current $D_J = J - \kappa(\rho_t - U\rho_s)$	exact current definition; no physical-time coordinate required
Relative-Birth Current $\delta J = \kappa(\delta\rho_t - \delta\rho_s)$	exact at neutral common-background zero locus
Pure current-cycle $J_{cyc} \in \ker(B \otimes I_{49})$	exact nonzero divergence-free current subcarrier
Nonzero cycle current with no independent density update	exact static continuity pass at isolated MASD current-cycle grade
Physical CRTF current-cycle norm fraction	exact 3/5 of squared current-tangent magnitude
σ refinement parameter	inherited primitive update/refinement typing; not ordinary pre-existing physical time
Canonical current-divergence/refinement matching $B C B^T = Q_7 = (1/31)L_{(W_7)}$	exact on reversible K = 7 branch
Fold-opportunity-to-Lorentzian-time map $T(\delta) = \delta a_t/c\star$	source-derived on declared local homogeneous branch

Object	Status after RCHS-1 and the attachment audit
σ-to-δ source bridge	open; determine $s_{\sigma\delta}$ in $\sigma = s_{\sigma\delta}\ \delta$
Physical-time-normalised discrete current	conditional: $J^{(t)} = s_{\sigma\delta}\ (c\star/a_t)\ J$
Strict same-step current conversion	conditional on $\sigma = \delta$: $J^{(t)} = (c\star/a_t)\ J$
Continuum Lorentz four-current pushforward	open
"Flow of realised facts" interpretation	structurally supported interpretation; not promoted to theorem
Single ordinary dΦ source-provenance gate	**retyped / withdrawn**; source sectors begin at different differential orders
Graded gauge / completion / record / CAR faithful descent	pass at declared orders; machine-zero residuals in Part VII
Native P1a raw gauge/current factorisation	exact on PFDMI-EX2E source-extension branch: rank 144 current-defect and mark derivatives; residual 2.5840×10^{-15}
Native P1a Wilson physical quotient	exact on source-extension branch: ranks 72 / 72 / 72 for current-defect / mark / terminal; mark residual 1.8038×10^{-15}
P1b unified primitive graded instrument	open; must generate gauge/current, record, completion and CAR/Fock jets together without imported target arrays
GSJB 49D innovation ↔ PFDMI/MASD endpoint tangent	exact kinematic carrier reconciliation
P2 internal endpoint covariance freedom	collapsed by common U(49) covariance to scalar γ_e
Weighted positive-conductance current persistence	exact: every finite non-common innovation has $\mathcal{D}_K(s) > 0$; only spatial common mode is current invisible
Finite maximally-unresolved edge correlation	exact on finite positive-conductance Dirichlet branch: $0 < \gamma_{ij}(s) < 1$ for $s > 0$
Hard oriented matter-handshake $\gamma_e = -1$	exact conditional consequence of the oriented hard branch; physical matter/event-grain attachment still open (F18)
Physical P2 joint endpoint law / numerical γ_e	open; requires event-grain selection, conductances/stopping rule and post-record Gibbs–Haar tilt
K7 integral cycle lattice	exact: $Z_1(W_7; \mathbb{Z}) \cong \mathbb{Z}^6$
General physical two-cell factorisation	exact typing: $\partial_2^{phys} = B\ A_{phys}$; A_{phys} physically open

Object	Status after RCHS-1 and the attachment audit
First-cover total circulation as torsion source	exact fail: quotient is free $\mathbb{Z}$
K mod 7 unit-incidence attachment	exact fail for full-rank six-cell attachment
Raw unordered Fold six-relation support incidence	exact rank 4; insufficient for finite six-dimensional K7 quotient
Minimal oriented Fold attachment algebra	exact conditional classification on $\{I, S, J = S^3\}$
Chiral candidate $A\_\chi = J - I - S$	exact construction; unique up to sign/orientation in minimal local class
Candidate integral homology	exact conditional $H_1 = \mathbb{Z}/7$
Candidate flat holonomy	exact conditional $H^1(-; \mathbb{Z}_7) = \mathbb{Z}_7$
Bare-wheel candidate cell boundary	exact $C_4 \vee C_3$; not a simple Fold loop
Cyclic simple-wheel order-seven route	exact fail; determinant set gives no 7
Complement-pair contractible hub refinement	exact mathematical construction; $\beta_1 = 6$ preserved
Refined candidate cell boundaries	exact six simple C_{13} loops with unit incidence
Smith factor under refinement	exact preservation: (1, 1, 1, 1, 1, 7)
Refined common one-skeleton planarity	exact fail
Refined one-skeleton minimum orientable embedding genus	exact g = 1 (F23: not an identification of the attached complex with a torus)
Fold S^2 local-boundary / global K7 reconciliation	construction-level compatible; physical gluing/refinement source open (F22)
Physical Fold-relation → A_phys map	open; decisive Gate-T-B object (Priority T0)
Physical source selection of S versus S^{-1} branch	open (F20)
Physical K_vac order-seven torsion	not yet promoted; awaiting A_phys (F19)

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3. **The Physical C/W/Y Radial-Current Tangent Field and Existing-Instrument Score Descent in VERSF.**
4. **The Primitive Selection History-to-Curvature and Standard-Model Attaching-Map Closure Theorem in VERSF.**
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9. **Gate-2 Representation-Resolved Tensor-Lift Source Audit.**
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11. **RCCSJ-1: The Representation-Resolved Current-Cycle Source Jet and Positive-History Realisation in VERSF**, consumed both as a boundary/no-go predecessor and, in Part V, as the positive marked current-cycle witness whose Fisher metric equals the binary trigger curvature.
12. **SGPL: the event-resolved primitive marked-transfer (Kraus) architecture.**
13. **HMGBC: the history-parent generator and physical-admission architecture.**
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21. **CRTF-HMGBC Full-Typed Current-Cycle Defect-Lift and Cycle-Protection Calculation**, source of the protected current-cycle carrier, endpoint-relaxed residual and divergence-free cycle-current result.
22. **CRTF-EX2K: Linearised MASD Relative-Birth Current and Haar Universality Theorem**, source of the relative endpoint-innovation current relation.
23. **PCRS-1: The Primitive Current-Record Scale and Absolute Gauge-Normalisation Boundary Theorem in VERSF**, source of the saturated first-birth

matching equation, the action/Fisher scale firewall and the current-scale orbit boundary.

24. **BCTR-1: Binary Commitment Trigger, First-Record Rate and Gauge-Normalisation Reframing in VERSF**, source of the binary occurrence/type Fisher decomposition and the revised complete-fact-rate release object.
25. **BCTR-2: Binary First-Record Trigger Curvature of the RCCSJ Positive Current-Cycle Witness**, source of the exact hub, rim and stationary trigger curvatures and their identity with the RCCSJ Fisher metric.
26. **BCTR-3: The Published-Stack Binary First-Record Curvature No-Return Theorem in VERSF**, source of the all-orders occurrence zero and the rank-six conditional record response.
27. **BCTR-4: The Current–History Eligibility Contrast and Rank-Six First-Record Curvature Theorem in VERSF**, source of the eligibility-contrast identity, rank-six minimum-carrier theorem and the $\lambda_J = 7.624971107276631\ \alpha$ extraction.
28. **BCTR-5: The Rank-Six Record Signal, History-Selector Blindness and Eligibility-Feedback No-Return Theorem in VERSF**, source of the edge/type localisation and trace-preserving renewal blindness theorem.
29. **BCTR-6: The Current–Record-Phase Precursor and History-Eligibility Cut Theorem in VERSF**, source of the complete mixed-source census, rank-six phase precursor and frozen-source reachability result. The present paper additionally takes BCTR-6's exact current/phase pair block through the final record-phase Schur relaxation to derive the rank-six current-stiffness theorem.
30. **PMS-EX35: Record-Likelihood Doob/Instrument Realisation of History-Conditioned Transition Bias.**
31. **PMS-EX36: State-Preserving Renewal and History-Bias Survival Test.**
32. **PMS-EX37: Renewal-Uniqueness, Covariant-Memory-Family and Physical-Effect Rigidity Test.**
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