

Coarse Grained Gauge Running in VERSF

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General reader summary

VERSF describes particles and forces as behaviour that emerges when many underlying changes are considered together. This paper calculates how the force response changes when a further layer of unresolved particle fluctuations is included. It starts from the larger-scale particle equations already developed in the VERSF corpus. It does not assign a force strength to an individual fold.

The seven-state $K = 7$ construction supplies the rule for how neighbouring Higgs patterns differ. The later fermion-source result also fixes the leading spatial particle equation by requiring it to agree with the geometry produced by refinement. These are inherited VERSF results, and the paper shows where they enter the calculation.

A further result shows that the bookkeeping factors used to write the particle equations can differ between kinds and generations of matter without changing the leading rate at which the force response grows with distance. Each factor appears twice with opposite effect: once in how the particle moves and once in how it interacts with the force. The two effects cancel when the same rules are used throughout.

The paper also explains why familiar fluctuation calculations can work. On VERSF's seven-point wheel, many small random steps around its six loops build up the familiar bell-curve pattern. The usual rules for averaging these fluctuations follow, including the relationships between different loops. The same rule for carrying charge along a path is used in the field calculation: adding the contributions of closed paths gives exactly the corresponding field result. This connects the small-step description to the calculation of their combined effect.

Combining the Higgs, fermion and gauge-field contributions gives the Standard Model rates for the strong, weak and hypercharge forces on the inherited particle census. Numerical checks cover all twelve gauge directions and verify the normalisation cancellation in both retained and eliminated fluctuations. BCB supplies conservation and local state-comparison requirements that the reduction must preserve. The calculation does not determine the force strengths at a starting scale or independently establish the absence of additional light charged particles.

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Abstract

We prove that the overall Higgs kinetic residue cancels from the leading massless gauge logarithm and extend this independence to every constant positive gauge-invariant internal matter pairing on the inherited VERSF census: five Hermitian three-generation matrices and one Higgs scalar, a 46-real-parameter class. The cancellation retains the physical cutoff, measure and vertices consistently, including the full, eliminated and retained determinants. We also derive the canonical $K = 7$ wheel's joint six-cycle Gaussian limit, exact finite-history covariance and mixed Wick pairings. An exact shared-link transport identity connects its closed-history traces to the native field determinant and proper-time integral; differentiation recovers the census charge Grams. Noncommuting $SU(2)$ links and a 315-dimensional census calculation verify this statistical and operator connection.

Evaluating the resulting effective response with the inherited Higgs and fermion propagation results and gauge-ghost operators gives the parity-even one-loop coefficient $b = (-7, -19/6, 41/6)$, ordered as colour, weak and conventional hypercharge. Finite-field quadrature reproduces the twelve-generator matrix to a maximum absolute discrepancy of 3.3×10^{-9} , an implementation check rather than physical prediction accuracy. The result retains the inherited particle census, effective branch and running-spectrum window; absolute physical matching and the full chiral measure remain outside its scope. Section 6.9 provides a concrete next test: compare the flat wheel covariance with the decreasing operational covariance in their six induced interaction coefficients, then confront both with the projected inverse physical gauge kernel using the same link map, vertices and normalisations.

1 The calculation and its physical level

There are two successive reductions. The first turns substrate histories into physical fields and an effective action at a matching scale. The second integrates fluctuations of those physical fields between two scales inside the domain of that action. The present calculation performs the second reduction using the outputs of the first already supplied by the corpus.

Let $\Gamma_{\Lambda_0}[A,H,\psi]$ denote the physical effective action at a coarse-grained reference scale Λ_0 . Its gauge stiffnesses $Z_A(\Lambda_0)$, derivative corrections and threshold terms contain the modes already eliminated. Integrating the remaining fluctuations down to μ gives

$$\Gamma_\mu = \Gamma_{\Lambda_0} + \Delta\Gamma_{\mu,\Lambda_0}, \quad \mu < \Lambda_0. \quad (1)$$

A second determinant over modes already included in Γ_{Λ_0} would double count them. The scale Λ_0 is an emergent matching scale, not an energy assigned to a primitive fold.

We calculate the leading local gauge term in $\Delta\Gamma$. With $Q = T_3 + Y$ and conventional fundamental generator normalisation, define

$$\mu \, dg_A/d\mu = b_A g_A^3 / (16\pi^2) + O(g^5), \quad A \in \{C,W,Y\}. \quad (2)$$

Since $Z_A = 1/g_A^2$, the corresponding leading evolution is

$$Z_A(\mu) - Z_A(\Lambda) = [b_A / (8\pi^2)] \ln(\Lambda/\mu). \quad (3)$$

The calculation takes place above the particle-mass thresholds relevant to the stated census and below the scale where additional source carriers or uncontrolled higher-derivative effects enter. QGC calls this its running-spectrum window, P26 [1]. The window is an inherited physical premise, not a conclusion of the internal seven-state rank calculation. Absolute starting stiffnesses and the extent of that window are distinct from the coefficient b .

2 Source inputs and leading matter propagation

2.1 What is inherited and what is evaluated

The representation matrices and effective operators are inputs to this calculation. They are taken from VERSF papers rather than inferred from measured force strengths, but their use here does not rederive their source selection. The new numerical work is predominantly an application of established quantum-field methods. The Higgs normalisation cancellation is also standard mathematics; the specific connection is that the later VERSF propagation results supply the shared coefficient to which it applies.

Table 1. Provenance of the calculation inputs.

Input	Source	Use in this calculation
Gauge group and charges	VERSF particle census	Constructs the representation matrices
K = 7 Higgs gradient	HSPR and temporal attachment	Supplies the spatial rule, shared coefficient and physical distance conversion
BCB and local comparison	Conservation and gauge-structure papers	Supports covariant transport; the physical attachment and measure are stated
Physical reduced action	MASD coupled Hessian	Sets the physical starting level; a complete fold Hessian is not integrated here
Fluctuation integration	Quantum-field methods	Evaluates the effective operators; Sections 6.4–6.8 establish the native history connection
Light particle content	QGC spectrum window	Retains the stated absence of extra light charged fields
Measured force strengths	Experimental values	Not used

The review searched the 600 recovered full texts and read the controlling source sections, including the later temporal and configuration-dynamics papers. Ten controlling public PDFs were checked against their current website copies; their hashes agree. This is a corpus search and dependency review, not an independent proof audit of every paper.

Table 2. Controlling source results.

Input	Source and location	Role in the calculation
Minimal electroweak interface and particle census	ECI and SC [2,3]	One complex doublet and the inherited fermion representation
Spatial Higgs response	HSPR §§4–7 [4]	Positive spatial residue on the physical common line
Temporal attachment	ST §§6–10 [5]	The same residue multiplies time and space
Common reduced action	MASD §§23–26 [6]	Coupled auxiliary elimination and physical two-point function
Gauge and ghost Hessians	Route-M §§24.5–24.6 [7]	Background-field quadratic operators
Fermion response	CLSN, the chiral paper and MFKR [8–10,17]	Shared-link operator, source-fixed infrared spatial pole and Weyl logarithm
Physical spectrum window	QGC premise P26 [1]	No uncounted light charged carrier in the interval

2.2 The order of reduction

MASD first forms the complete physical Hessian and relaxes its admissible auxiliary block. In its notation,

$$H_{\text{red}} = H_{\varphi\varphi} - H_{\varphi\eta} H_{\eta\eta}^{-1} H_{\eta\varphi},$$

$$\Pi_H(p) = n_H^\dagger H_{\text{red}}(p) n_H. \quad (4)$$

This is a reduction of the coupled action [6,14]. Merely averaging a chosen free stencil is not a substitute for it. If a complementary physical mode is light, it cannot be classified as an eliminable auxiliary by definition.

HSPR gives the spatial response and the exact two-sided protection of the common Higgs line against the identified defect-mediated terms. With $J_{Dq_H} = 0$, every correction of the form $J_D^\dagger M J_D$ annihilates that line. The result applies to that factorisation class; it does not remove contributions acting directly on the physical tangent.

The $K = 7$ input can be exhibited before the continuum operator is written down. On HSPR's transitive seven-state carrier, S is the cyclic neighbour shift, $S^7 = I$, and the inherited endpoint-local attachment is

$$d = S - I, \quad I_H^{\text{can}} = 2^{-1/2}(I + S^{-1}), \quad A_7 = I_H^{\text{can}} d = 2^{-1/2}(S - S^{-1}). \quad (4a)$$

Consequently $A_7^\dagger A_7$ has Fourier eigenvalues $2 \sin^2(2\pi k/7)$, $k = 0, \dots, 6$. There is exactly one zero, the constant pattern; all six nonuniform directions are retained. Direct reconstruction gives the three positive pairs 0.376510, 1.222521 and 1.900969. This is a property of the specified internal carrier, not a count of physical particle species or a proof of physical mass thresholds. It is also distinct from the canonical hub-and-rim transition operator, which acts on a different carrier.

HSPR's refinement geometry assigns proper neighbour separation ℓ to the physical comparison. The endpoints of the centred numerator are 2ℓ apart, giving

$$\begin{aligned} \nabla_H &= A_7 / (2^{1/2} \ell) = (S - S^{-1}) / (2\ell), \\ |\nabla_H(p)|^2 &= \sin^2(p\ell) / \ell^2 = p^2 - \ell^2 p^4 / 3 + O(\ell^4 p^6). \end{aligned} \quad (4b)$$

This establishes the leading spatial derivative used below without choosing a force strength at the fold level. The graph normalisation is converted into proper distance; its coefficient 2 is not an extra physical coupling. At fixed physical momentum, halving ℓ reduces the leading correction by a factor tending to four, reproduced in the accompanying source check. The attachment remains exact within HSPR's declared endpoint-local class and transitive branch; writing it explicitly does not remove those upstream conditions [4, §§2.2–4].

2.3 The temporal result supplies the shared coefficient

HSPR's spatial residue is $Z_H = q_H^\dagger G_{\text{op}}^{\text{direct}} q_H > 0$. Its sequential kinetic coefficient equals this same scalar. The subsequent ST attaching theorem maps the composition parameter to local emergent time and gives

$$\Pi_H(\Omega, p) = K_H + Z_H[-\Omega^2/c_*^2 + g^{ij} p_i p_j] + \text{higher orders}. \quad (5)$$

Thus $Z_H^{\text{time}} = Z_H^{\text{space}}$. We consume this later result with its stated local-homogeneity and upstream attachment premises. We do not reopen the temporal attachment as though HSPR were the last source.

The source permits Z_H to contain an operational-flow factor and a Higgs-line ruler conversion. Neither has to equal one for the calculation below. After canonical field conversion, the physical mass is $m_H^2 = K_H/Z_H$. Changes in this ratio can affect thresholds even though the leading massless logarithm is unchanged.

2.4 From the radial response to the doublet kinetic operator

ECl derives the minimal completion representation $H \sim (1, 2)_{1/2}$ on its inherited closure branch. SC uses one such universal carrier. The source radial theorem fixes propagation in one normal direction; it does not by itself count every field in an ultraviolet regulator.

Proposition 1 (kinetic matrix on the minimal doublet). On the local, gauge-invariant, dimension-four branch with one complex Higgs doublet and a homogeneous vacuum reference, the quadratic kinetic term has the form

$$\Gamma_{H,\text{kin}} = Z_H \int d^4x (D_\mu H)^\dagger D_\mu H. \quad (6)$$

Proof. Write a constant Hermitian kinetic matrix M between the two doublet components. Gauge covariance requires $[M, t_i] = 0$ for all three weak generators. The doublet is irreducible, so M is a scalar multiple of I_2 . A term with two un-conjugated H factors has nonzero hypercharge and is forbidden. The shared temporal and spatial coefficient in (5) fixes the spacetime part. Taking $H = (0, h/\sqrt{2})$ gives the radial kinetic term $Z_H(\partial h)^2/2$ and identifies the same coefficient. ■

This argument applies at the stated operator order. Field-dependent kinetic metrics, symmetry-breaking insertions and higher-dimensional operators require their own matching. The four real doublet components enter the high-energy scalar determinant. A radial coordinate alone is not that determinant, and Goldstone directions must not be removed separately and then counted again through gauge fields.

2.5 BCB and conservation through the reduction

BCB is part of the source rationale for covariant transport. The distinguishability-conservation paper treats local state comparison as transport between internal state spaces. Under its smoothness, reversibility and metric-preserving transport premises, that comparison requires a compatible connection [15, §§3–4]. This supplies a reason within VERSF for using a covariant derivative in (6). It does not select a numerical gauge stiffness or a unique physical connection: compatible connections can have different curvature and need not be gauge-equivalent.

The matter-coupling paper separately relates conservation of its record current to gauge invariance of the current–connection interaction [16, §§5–8]. That record current must not be silently identified with each representation-valued Standard Model current. The representation and physical-current attachment used here remain those of the inherited effective branch. Probability normalisation of a seven-state transition matrix would not establish that attachment or the full BCB law.

Conservation criterion for coarse graining. Suppose the physical source action and measure are invariant under the inherited gauge transformations, and the retained/removed split transforms covariantly. Then integrating the removed variables preserves that symmetry. Indeed, in

$$\exp(-\Gamma_c[A, \varphi]) = \int d\eta \exp(-\Gamma[A, \varphi, \eta]), \quad (6a)$$

a gauge transformation is a change of integration variables with unchanged action, domain and measure. Thus $\Gamma_c[A^\theta, \varphi^\theta] = \Gamma_c[A, \varphi]$. Around the symmetric vacuum, its quadratic gauge response is transverse, $p_\mu \Pi^{\mu\nu}(p) = 0$. This is the relevant conservation test on the coarse response; it requires a symmetry-preserving regulator and measure and does not settle the global chiral-measure question.

This criterion explains why the contact term, the eliminated-mode determinant and the correct physical measure belong in the same calculation. They must collectively preserve the source symmetry. BCB constrains what a valid reduction may do; the $K = 7$ propagation construction specifies part of what is being reduced. Together they support the inputs and consistency of the running calculation, without determining its boundary stiffnesses merely by conservation.

2.6 The K7 geometry fixes the leading fermion normalisation

The later source-identity edition of MFKR supplies a stronger spatial input than a chosen nearest-neighbour stencil [17, §14.3]. On its declared $D = 3$ minimal Clifford branch, write the leading star symbol using Hermitian Clifford generators α_i and a real moment matrix M :

$$K_{IR}(p) = \sum_{ij} \alpha_i M^{ij} p_j, \quad K_{IR}(p)^2 = p^T M^T M p \, I. \quad (6b)$$

The independently derived refinement metric fixes the second-order symbol to $p^T p$ in its orthonormal frame. Requiring the graph-Dirac operator to square to that same geometry therefore gives $M^T M = I_3$. A proper rotation of M is the declared common spin-frame freedom. On the tangent-aligned positive-weight branch, M is positive semidefinite as well as orthogonal, so $M = I_3$ outright. A reflection passes the square test but requires the separate orientation identification stated in MFKR; it is not silently removed.

This inherited result fixes the leading spatial fermion coefficient without a hand-chosen directional normalisation. It also gives a stability bound: on the positive branch, $\|M - I\| \leq \|M^2 - I\| / (1 + \lambda_{\min}(M))$. The six-neighbour reference star reproduces $M = I_3$ exactly; a diagnostic $M = \text{diag}(1.2, 1, 1)$ gives moment error 0.2 and Laplacian error 0.44. These checks reproduce the theorem's algebra. They do not supply the unpublished microscopic edge manifest, select global dimensionality or establish full physical identity of the overlap representative.

MFKR's shared-link action is equally relevant here: the matter field uses the same U_e as the gauge and closure sectors, and no independent matter-link normalisation may be attached [17, §§2.3,5].

Canonicalisation of a positive matter pairing must transform the whole kinetic bilinear. Section 3.3 uses this rule to show why unequal internal pairings do not become unequal leading loop weights.

3 Matter determinants after coarse graining

In a locally flat Euclidean chart, obtained by the inherited perturbative continuation, the leading scalar Hessian is

$$K_H[A] = Z_H P_H[A], \quad P_H[A] = -D_H^2 + m_H^2. \quad (7)$$

This is the effective physical operator determined at two-derivative order. No dispersion relation at arbitrary fold momentum is specified by (7).

3.1 Normalisation cancellation

Proposition 2 (scalar residue cancellation in the leading response). For a constant positive Z_H , a consistently transformed field measure and a cutoff on physical eigenvalues, the background-subtracted scalar determinant depends on P_H , not on the overall multiplier Z_H .

Proof. At finite common regulator,

$$\begin{aligned} & \log \det[Z_H P_H(A)] - \log \det[Z_H P_H(0)] \\ &= \log \det P_H(A) - \log \det P_H(0). \end{aligned} \quad (8)$$

Equivalently, each gauge vertex carries Z_H while each scalar propagator carries Z_H^{-1} . Both the two-vertex loop and the contact term lose every factor of Z_H . The constant field-rescaling Jacobian is treated with the same regulator and measure. ■

The statement fixes the leading massless logarithm without fixing the source residue. It does not assert equality of all observables under changes of the source metric: m_H^2 , finite matching, derivatives of Z_H and other interaction terms may change. In particular, the cancellation is not a proof that a background-dependent physical metric has no response.

A useful regulated form is the physical proper-time interval

$$\Delta\Gamma_H[A] = -\int_{\Lambda^{-2}\mu^{-2}} dt/t \operatorname{Tr}[\exp[-tP_H(A)] - \exp[-tP_H(0)]]. \quad (9)$$

If the unnormalised operator $Z_H P_H$ is used, its proper-time bounds must be Λ^{-2}/Z_H and μ^{-2}/Z_H . The substitution $t = Z_H s$ returns (9). Holding the raw bounds fixed while varying Z_H would change the physical scales being compared.

3.2 The Higgs logarithm

For a commuting constant curvature $F_{12} = BT$ in the inherited representation, the scalar orbital heat trace per unit four-volume is

$$\operatorname{Tr} \exp(-tP_H)/V = \exp(-m_H^2 t) (4\pi t)^{-2} \operatorname{tr}_H[(tBT)/\sinh(tBT)]. \quad (10)$$

Expanding $x/\sinh x = 1 - x^2/6 + O(x^4)$ and using (9) gives

$$\Delta\Gamma_H/V = [B^2 \operatorname{tr}_H(T^2)/(96\pi^2)] \int_{\Lambda^{-2}\mu^{-2}} dt \exp(-m_H^2 t)/t + O(B^4). \quad (11)$$

For $m_H/\mu \rightarrow 0$, the integral is $2 \ln(\Lambda/\mu)$. Matching $\Delta\Gamma/V = \Delta Z B^2/2$ yields

$$b_{H,A} = (1/3) \operatorname{tr}_H(T_A^2) = (0, 1/6, 1/6). \quad (12)$$

The trace includes both complex components of the one doublet. It is not a sum over the seven internal states of the source carrier.

This coefficient also follows from the local heat-kernel coefficient [11]. The calculation here uses an effective proper-time regulator on physical fields. It does not identify that regulator with an uncomputed primitive history measure. Its relevance is that the coefficient of the marginal gauge operator is fixed within the inherited effective theory, independently of the scalar normalisation.

3.3 Unequal matter pairings and generation mixing

On the inherited massless census, each fermion multiplet $r \in \{Q, u, d, L, e\}$ has carrier $\mathbb{C}_{\text{gen}}^3 \otimes V_r$. The five V_r are inequivalent gauge irreducibles. A homogeneous gauge-invariant positive kinetic pairing therefore has the form $Z_r \otimes I_{V_r}$, with Z_r a positive Hermitian 3×3 matrix. The Higgs contributes one positive scalar. This is the same representation-theoretic decomposition used by SGW-2, applied here to kinetic pairings rather than its separately defined loop-price operator [18, §§2.4,4]. The Hermitian parameter count is $5 \times 9 + 1 = 46$.

Proposition 3 (independence of constant internal kinetic pairings). Fix the leading massless covariant operator, the light representation census and a physical regulator with a consistently transformed measure. Every positive constant pairing in this class gives the same leading one-loop matter gauge coefficient. No equality of the five Z_r , or of their eigenvalues, is required.

Proof. At this order, the gauge-dependent operator in sector r is $K_r[A] = Z_r \otimes D_r[A]$. At a common finite regulator with N_r non-generation coordinates,

$$\log|\det K_r[A]| = N_r \log \det Z_r + 3 \log|\det D_r[A]|. \quad (12a)$$

Background subtraction removes the first term. Equivalently, for gauge derivatives a, b , the same shared-link rule gives $K_a = Z \otimes D_a$ and $K_{ab} = Z \otimes D_{ab}$, so

$$\begin{aligned} & \operatorname{Tr}(K^{-1}K_{ab} - K^{-1}K_a K^{-1}K_b) \\ &= 3 \operatorname{Tr}(D^{-1}D_{ab} - D^{-1}D_a D^{-1}D_b). \end{aligned} \quad (12b)$$

The Weyl determinant weight and scalar determinant weight are then applied as in Section 4. For the Higgs, Proposition 2 supplies the scalar version. Thus the fermion trace (6,6,10) and Higgs trace (0,1/2,1/2) retain their usual statistics weights throughout this pairing class. ■

The cancellation survives a flavour-independent retained/removed split. If D has eliminated block F and Schur complement D^c , then K has eliminated block $Z \otimes F$ and Schur complement $Z \otimes D^c$. Each background-subtracted determinant separately loses the constant pairing factor. Discarding the eliminated determinant remains unjustified: its response is generally nonzero even though its pairing dependence cancels.

Scope is essential. The result concerns constant internal kinetic pairings at fixed leading operator and physical spectrum. A change of geometric principal symbol, a spacetime-dependent pairing, a changed canonically normalised mass or an additional light species is a different physical change. Scale-dependent wave-function factors require the corresponding renormalisation treatment. Regulator Jacobians and finite local matching are treated consistently; no claim of universal starting gauge stiffness follows. In particular, the D41 response and SGW-2 finite-loop prices are not identified with Z_r , and their proposed physical screening is not rejected by this cancellation.

4 Complete one loop gauge response

The gauge and ghost operators follow from the same admitted Yang–Mills action in background Feynman gauge [7]. The fermion term uses the inherited Weyl magnitude and the hyperbolic overlap universality result reviewed in Appendix A. The scalar term is (12).

For $x = Bt$ times a representation eigenvalue, the combined orbital and spin traces, including determinant powers, are

$$\begin{aligned} k_{g+gh}(x) &= [x/\sinh x] \cosh(2x), \\ k_{\text{Weyl}}(x) &= -[x/\sinh x] \cosh x, \\ k_H(x) &= x/\sinh x. \end{aligned} \quad (13)$$

The gauge vector has spin eigenvalues $(-2,0,0,2)$, the squared Dirac operator has $(-1,-1,1,1)$, and the scalar has zero spin. The determinant weights are respectively 1/2 for a real gauge field, -1 for its complex ghost, $-1/4$ for the squared Dirac operator representing a Weyl magnitude, and $+1$ for a complex scalar. These weights give the three functions in (13).

Their x^2 coefficients are $11/6$, $-1/3$ and $-1/6$. With the sign convention in (2), b is minus twice that coefficient times the representation trace. Thus

$$b_A = -(11/3)C_2(G_A) + (2/3)Z_A^F + (1/3)Z_A^H. \quad (14)$$

The charge traces on the 45 left-handed fermion states are (6,6,10); those on the one Higgs doublet are (0,1/2,1/2). The adjoint Casimirs are (3,2,0). Their separate contributions are shown below.

Table 3. Complete effective one loop assembly.

Contribution	Colour	Weak	Hypercharge
Gauge and ghost	-11	-22/3	0
Three fermion generations	4	4	20/3
One complex Higgs doublet	0	1/6	1/6
Complete leading coefficient	-7	-19/6	41/6

Proposition 4 (complete leading running on the inherited effective branch). In the spectrum window stated in Section 1, with the gauge, fermion and scalar operators supplied by the cited branch, the leading parity-even gauge response has $b = (-7, -19/6, 41/6)$. The overall constant Higgs residue does not enter it.

Proof. The background-field determinant is the sum of the three weighted traces in (13). Expanding them and evaluating the inherited generator traces gives (14) and the table. Proposition 2 removes the unknown scalar multiplier. Appendix A supplies the fermion logarithm on the explicit family. ■

The coefficient triple was already present in QGC and Route-M. The source connection supplied here is the use of the later shared temporal/spatial Higgs response and the cancellation that permits its contribution to be evaluated before its overall operational residue is known. Neither a new matter census nor a new primitive-to-physical selection theorem is claimed.

The unweighted charge Gram $(6, 13/2, 21/2)$ used in the participation paper is a different object. Spin and statistics turn it into the weighted response above. For primitive $q = 6Y$, both the Abelian charge trace and b acquire a factor 36, with $g_q = g_V/6$. No physical prediction changes under that conversion.

5 Executed calculations

5.1 The full gauge coefficient matrix

The calculation constructs the 45-state fermion generators, the doublet generators, and the adjoint matrices obtained from commutators. It evaluates the finite-field functions (13) and integrates (9) over $\Lambda/\mu = 4$. The probe has $F_{12} = B\Sigma_a V_a T_a$. This is a constant-curvature test of the quadratic coefficient; it is not a test of arbitrary spacetime-dependent non-Abelian fields.

All twelve coordinate directions and all sixty-six pair sums are evaluated. The pair sums recover the off-diagonal entries. Values at $B/\mu^2 = 0.03$ and 0.015 are combined to remove the leading B^2 error in the extracted coefficient. The expected matrix is

$$B_{ab} = \text{diag}(-7 I_8, -(19/6)I_3, 41/6). \quad (15)$$

The maximum absolute entrywise difference is 3.28×10^{-9} , including the mixed entries. The coefficient is calculated from the heat functions and matrices; the values in (15) are used only for the subsequent check. A second evaluation uses 60-decimal arithmetic and an independent composite Simpson quadrature.

Table 4. Finite field determinant evaluation.

Sector	Gauge and ghost	Fermion	Higgs	Total
Colour	-11.000000	4.000000	0.000000	-7.000000
Weak	-7.333333	4.000000	0.166667	-3.166667
Hypercharge	0.000000	6.666667	0.166667	6.833333

The quoted numerical agreement is implementation accuracy within this effective calculation. It is not experimental precision or an uncertainty estimate for the physical branch premises. The preceding paper's finite-volume fermion slopes 0.967 and 0.974 are numerical counterparts of the universality argument in Appendix A; no new large-volume fermion fit is reported here.

5.2 Higgs normalisation and an independent logarithm

The raw Higgs multiplier is varied through 1/16, 1/4, 1, 4 and 16. With the physical cutoffs held fixed, every case gives $b_{H,W} = b_{H,Y} = 0.166666666660$, to the displayed precision, and zero colour response. The range is a test of the cancellation, not a source-derived range of allowed physical metrics.

An independent momentum-space calculation uses the scalar vacuum-polarisation difference

$$\begin{aligned} & \Pi_H(k_1) - \Pi_H(k_2) \\ &= [T(R)/(16\pi^2)] \int_0^1 dx (1-2x)^2 \ln[(m^2+x(1-x)k_2^2)/(m^2+x(1-x)k_1^2)]. \quad (16) \end{aligned}$$

At $m = 0$, the x integral supplies 1/3 and the same coefficient $T(R)/3$ follows. Numerical quadrature at three external momenta verifies this logarithm independently of the heat functions. Finite masses turn it into a threshold-dependent response, as expected. The sign in (16) is the positive matter contribution to the stiffness difference when the external scale is lowered.

5.3 Thresholds are a separate physical question

For a common scalar mass, the proper-time scheme gives the finite-shell fraction

$$F_H(m;\mu,\Lambda) = [2\ln(\Lambda/\mu)]^{-1} \int_{\Lambda^{-2}\mu^{-2}}^{\infty} dt e^{-m^2 t}/t. \quad (17)$$

The scalar contribution is $F_H(0,1/6,1/6)$. For $\Lambda = 4\mu$ the values are as follows.

Table 5. Scalar threshold response for a fixed physical scale interval.

m/μ	Fraction F_H	Weak or hypercharge scalar coefficient
0	1.000000	0.166667
0.5	0.920782	0.153464
1	0.734882	0.122480
2	0.375282	0.062547
4	0.079126	0.013188
8	0.001363	0.000227

These are dimensionless threshold controls, not predicted VERSF masses. Their finite values are scheme dependent; the leading massless coefficient is not. A charged completion mode with a physical threshold

in the interval must be included. A finite internal rank calculation cannot establish that every such threshold is absent.

5.4 Executed tests of unequal matter pairings

The one-generation gauge commutant is reconstructed from the twelve generator matrices and has dimension five. Including arbitrary Hermitian generation mixing gives the 45-dimensional fermion pairing class, plus the Higgs scalar. Eight independently generated positive pairings on all 45 fermion states, with condition numbers up to 941, preserve every entry of the twelve-generator charge-response matrix to a maximum error of 4.7×10^{-14} .

A separate check uses the inherited hyperbolic-overlap operator on a 2^4 regulator with noncommuting SU(2) links and a finite link perturbation. Four three-generation pairings include unequal eigenvalues and complex off-diagonal mixing, with condition number up to 1,024. The background changes in $\log|\det K|$ are 0.424829518 in total, 0.397433412 in the eliminated block and 0.027396106 in the retained Schur complement. Across the pairings the maximum cancellation error is 7.6×10^{-13} . The contact-plus-bubble Hessian agrees to below 10^{-14} . The selected-site split is a finite regulator control of Proposition 3, not an asserted native $K = 7$ coarse map.

An inconsistent control changes propagation normalisations but leaves the interaction vertices unchanged. Its colour, weak and hypercharge responses become 0.578125, 0.753906 and 0.159766 times their correct values. This demonstrates why a kinetic metric cannot be inserted as a sector loop weight while omitting its required appearance in the vertices. It does not test a physical source-price operator of the different type defined in SGW-2.

6 Scale composition and source fluctuation calculus

6.1 Integrating a range in one step or two

The proper-time expression is additive on adjacent intervals:

$$\Delta\Gamma_{\mu,\Lambda} = \Delta\Gamma_{\mu,M} + \Delta\Gamma_{M,\Lambda}, \quad \mu < M < \Lambda. \quad (18)$$

At finite probe field, before taking the quadratic limit, the complete gauge, fermion and Higgs calculation gives a residual of 7.9×10^{-16} for the split $1 \rightarrow 2 \rightarrow 8$. This verifies the composition of the effective scale integration. It does not identify physical proper time with the source's discrete opportunity parameter.

6.2 The determinant retained by coupled reduction

At a finite common regulator, split a quadratic operator into retained and eliminated variables:

$$K = \begin{bmatrix} A & B \\ B^\dagger & C \end{bmatrix}, \quad K^c = A - BC^{-1}B^\dagger,$$

$$\log \det K = \log \det C + \log \det K^c. \quad (19)$$

Consequently its complete second gauge variation obeys the same sum. Derivatives of a background-dependent block map or measure belong to the calculation. The Schur complement describes the relaxed retained action; the eliminated determinant describes the fluctuation contribution of the removed variables. These are complementary parts of Gaussian integration.

A separate finite check uses a seven-site loop of the emergent doublet field with noncommuting SU(2) link perturbations, retaining its uniform block coordinate and relaxing the complement. This loop is an

effective-field regulator control, not an asserted physical $K = 7$ fold action. Its dimensionless determinant Hessian separates into

$$\text{total} = 0.02973233, \text{ eliminated} = -1.17064856, \text{ retained} = 1.20038089. \quad (20)$$

The sum agrees to below 10^{-8} across the tested Higgs normalisations. The negative eliminated contribution is allowed: the two terms are not probabilities or positive fractions of a force strength. The control illustrates why retaining only the relaxed quadratic kernel can miss part of the fluctuation response.

6.3 What may be absorbed into matching

Modes with a physical gap above the interval generate local matching terms and suppressed low-energy corrections. Modes that remain light generate additional running and must stay in the census. Both statements concern the spectrum after the physical reduction, including its constraints and measure. Neither follows from counting all algebraic patterns in an arbitrarily extended fold-level stencil.

Higher-derivative operators in Γ_{Λ_0} can change finite matching and produce scale-suppressed corrections. In a controlled interval $\mu < \Lambda \ll \Lambda_0$, their insertions carry additional powers of physical momentum over Λ_0 and do not change the leading massless logarithmic coefficient of the dimension-four theory. This statement requires bounded effective coefficients and the inherited spectrum window; it cannot be used to erase an actual extra light pole.

$K = 7$ enters explicitly through the source gradient (4a), its proper-distance limit (4b), and the upstream closure and representation constructions. BCB enters through the local comparison and conservation conditions of Section 2.5. The seven-state count is not a multiplicity of seven independent Higgs doublets or an extra factor multiplying the scalar loop. The present running coefficient concerns what has survived the source's physical coarse graining.

6.4 Gaussian fluctuations from discrete K7 histories

A standard calculation can acquire a VERSF explanation when its statistical rule follows from the source dynamics after coarse graining. The canonical wheel provides an explicit example. We use its published transition law [19, §§6–8]: the hub moves to each of the seven states with probability $1/7$; a rim state moves to itself, the hub or either rim neighbour with probability $1/4$. Its stationary distribution is $\pi = (7, 4, 4, 4, 4, 4, 4)/31$. These are inherited canonical-wheel inputs, not fitted transition probabilities.

Let $g(i, j)$ be $+1$ for a clockwise rim transition, -1 for an anticlockwise rim transition and zero otherwise. Start the chain in π and define $J_N = \sum_{n=1}^N g(X_{n-1}, X_n)$. This is a dimensionless circulation observable on the existing cycle. Every state's conditional mean increment is zero, so the increments are martingale differences, although their squared sizes are correlated. Their stationary variance is $12/31$ and hence $\text{Var } J_N = 12N/31$ exactly. No Gaussian law or independence of successive states is assumed. The count N refers to wheel updates, without a new identification with physical time or completed Facts.

Proposition 5 (Gaussian circulation limit on the canonical wheel). Put $Y_N = J_N/\sqrt{12N/31}$. Then Y_N converges to a standard Gaussian. Every fixed even moment converges to $(2m-1)!!$ and every odd moment vanishes. In particular, the limiting fourth and sixth moments are 3 and 15.

Proof. Weight each transition by $\exp[\text{tg}(i, j)]$. Rotational symmetry reduces its moment-generating function exactly to the hub and common-rim subspace:

$$\begin{aligned} M_N(t) &= (7/31, 24/31) R(t)^N (1, 1)^T, \\ R(t) &= [1/7 \quad 6/7; \quad 1/4 \quad 1/4 + (1/2)\cosh t]. \quad (21) \end{aligned}$$

At $t = 0$ the eigenvalues are 1 and $-3/28$. The eigenvalue $\lambda(t)$ continuing 1 is therefore analytic near zero. Writing $d(t) = 1/4 + (1/2)\cosh t$ gives

$$\lambda(t) = [1/7 + d(t) + \sqrt{((1/7 - d(t))^2 + 6/7)}]/2,$$

$$\log \lambda(t) = (6/31)t^2 + (433/59582)t^4 + O(t^6). \quad (22)$$

The corresponding analytic prefactor in M_N equals one at zero; the other eigenvalue contributes an exponentially decaying term uniformly nearby. Substitution of $t = s/\sqrt{12N/31}$ gives

$$\lim_{N \rightarrow \infty} E \exp(sY_N) = \exp(s^2/2). \quad (23)$$

The convergence holds uniformly in a complex neighbourhood of $s = 0$, so derivatives give the moment limits. Reversing the rim orientation gives the exact odd-moment cancellation. ■

Four derivatives of the same two-state expression give an exact finite-history correction:

$$E[Y_N^4] = 3 + 433/(372N) - [1372/(961N^2)][1 - (-3/28)^N]. \quad (24)$$

Thus the leading departure is proportional to $1/N$. Direct seven-state differentiation through order six gives the following values; rational two-state moments and independent weighted path enumeration check the calculation.

Table 6. Emergence of Gaussian circulation moments from discrete K7 updates.

Updates N	$E[Y_N^4]$	$E[Y_N^6]$
1	2.583333	6.673611
7	3.137146	16.664169
49	3.023160	15.338923
343	3.003381	15.050546
2,401	3.000485	15.007265
Gaussian limit	3	15

The powers of seven are reporting points for accumulated histories, not a claim that this sum is the physical seven-site spatial block map. At $N = 2,401$ the relative fourth- and sixth-moment discrepancies are 0.0162% and 0.0484%. These are deviations of this current's moments from their Gaussian limits, not errors on predicted force strengths. The full and reduced characteristic functions agree within 10^{-12} at three probes and four history lengths.

Equation (23) generates the scalar Gaussian pairing rule used in Wick calculations: differentiating $\exp(s^2/2)$ produces the pairings of every even moment. Here that rule follows from the inherited discrete dynamics for this observable. Gaussian statistics also occur in classical systems, so the result by itself supplies neither quantum phases nor fermionic statistics. Nor does it identify $12/31$ with an inverse gauge stiffness. The programme's quantum-state structure and physical field attachment must accompany the statistical limit in a quantum-field application.

The corpus already supplies a conditional Gaussian amplitude proposal and a quadratic marked-innovation theorem [14, §§IV.13–IV.15]. The wheel calculation gives a direct non-Gaussian-step example with a controlled large-history limit. The following extension treats every cycle jointly and derives the associated determinant rule. It does not require every nonlinear source history to be Gaussian.

6.5 The complete conserved current and its Gaussian limit

Orient the twelve nonself edges as six outward spokes and six clockwise rim edges. Let B be the 7×12 incidence matrix, with -1 at the source and $+1$ at the target, and $e(i)$ the i th vertex unit vector. Write Q_N for the twelve signed edge counts along a stationary N -step history. Conservation is a pathwise identity:

$$BQ_N = e(X_N) - e(X_0). \quad (25)$$

Let D have as its six rows the positively oriented elementary triangles. The cycle basis is inherited from [19, §4.3; 20, §39Q]. Define the vertex Laplacian $L = BB^T$, its Moore–Penrose inverse L^+ , and the cycle projector

$$H_\Delta = DD^T = \text{circ}(3, -1, 0, 0, 0, -1),$$

$$P_{\text{cyc}} = D^T H_\Delta^{-1} D = I_{12} - B^T L^+ B. \quad (26)$$

H_Δ has eigenvalues $1, 2, 2, 4, 4, 5$, so P_{cyc} has rank six. Equation (25) gives the exact decomposition $Q_N = P_{\text{cyc}} Q_N + B^T L^+ (e(X_N) - e(X_0))$. The second term is bounded independently of N . This is conservation of these path counts, rather than an independent derivation of the complete BCB axiom.

Proposition 6 (exact current covariance and endpoint separation). Let T be the canonical wheel transition matrix and $\Pi = \text{diag } \pi$. At stationary start the mean signed current is zero, and for every positive integer N ,

$$\text{Cov}(Q_N) = (2N/31)P_{\text{cyc}} + 2B^T L^+ \Pi (I_7 - T^N) L^+ B. \quad (27)$$

The two projected components are uncorrelated. In particular, the cycle covariance equals $(2N/31)P_{\text{cyc}}$ exactly, whereas the endpoint covariance remains bounded.

Proof. Every directed nonself edge has stationary flow $\pi_i T_{ij} = 1/31$. For any $a \in \ker B$, the conditional mean of the signed increment a_{ij} is zero at every vertex: all outgoing nonself probabilities at that vertex are equal and the oriented divergence vanishes. Thus the cycle increments are martingale differences, with stationary one-step covariance $(2/31)P_{\text{cyc}}$. Summation gives the first term of (27). Detailed balance gives the same zero mean conditional on the arrival vertex. The Markov property then makes a cycle increment uncorrelated with any vertex function at any time, including both endpoints. Finally, the endpoint difference has covariance $2\Pi(I_7 - T^N)$, which gives the second term under the fixed map $B^T L^+$. ■

Proposition 7 (joint Gaussian law on the cycle quotient). As N tends to infinity, $Q_N/\sqrt{N}$ converges to a centred Gaussian supported on $\ker B$, with covariance $\Sigma = (2/31)P_{\text{cyc}}$. Every fixed mixed moment converges to the corresponding Gaussian pairing sum.

Proof. For an edge probe a , form $T(a)_{ij} = T_{ij} \exp(a_{ij})$, with antisymmetric edge marks and zero self marks. Its Perron eigenvalue is simple and analytic near $a = 0$. The first derivative vanishes by detailed balance; (27) gives the Hessian of $\log \lambda$ as $(2/31)P_{\text{cyc}}$. Reversal of stationary paths makes its expansion even. The analytic spectral projector has prefactor one at zero, while the other spectral contributions decay geometrically. Therefore, uniformly for complex a in a fixed neighbourhood after scaling,

$$E \exp(a^T Q_N / N^{1/2}) \rightarrow \exp(a^T P_{\text{cyc}} a / 31). \quad (28)$$

Analytic differentiation gives every fixed mixed-moment limit, with $O(1/N)$ corrections whose coefficients depend on the probes. ■

An exact invariance identifies the null directions. Adding a vertex potential difference to the edge probe gives

$$T(a + B^T \varphi) = \text{diag}(e^{-\varphi}) T(a) \text{diag}(e^\varphi). \quad (29)$$

Its Perron eigenvalue is unchanged for every probe for which it is defined. At finite history length the change is the endpoint term $\phi(X_N) - \phi(X_0)$; it disappears after division by $\sqrt{N}$. This is a graph-current

counterpart of invariance under a gradient shift. It does not identify the wheel with the full non-Abelian gauge theory.

Care is needed when writing six coordinates. The triangle observables DQ_N/VN have covariance $(2/31)H_\Delta$, while the coefficients $H_\Delta^{-1}DQ_N/VN$ in the triangle basis have covariance $(2/31)H_\Delta^{-1}$. In an orthonormal cycle basis V , $z_N = V^T Q_N/VN$ has covariance $(2/31)I_6$. These are different coordinate descriptions of the same law, not competing physical normalisations. The outer circulation of Section 6.4 is the sum of the six triangle observables and recovers variance $12/31$ per update.

The factor $2/31$ and divergence-free marked-current compensation already occur in the representation-resolved current construction [20, §§36–37]. They are not claimed as discoveries here. Equation (27), the joint limit and its consequences concern the unperturbed canonical wheel's signed counts. They do not select the marked source's perturbation strength g_{cyc} or turn its current-score metric into an energetic gauge stiffness.

6.6 Mixed pairings and the determinant rule

Equation (28) gives the full Wick rule. If X is the limiting vector and Σ its covariance, then

$$E[X_a X_b X_c X_d] = \Sigma_{ab}\Sigma_{cd} + \Sigma_{ac}\Sigma_{bd} + \Sigma_{ad}\Sigma_{bc}. \quad (30)$$

This statement includes correlations between distinct loops. For the unit-variance triangle observables $Y_{i,N} = (DQ_N)_i/V(6N/31)$, adjacent triangles have correlation $-1/3$ and nonadjacent triangles have zero correlation. Their limiting mixed fourth moments are therefore $11/9$ and 1 respectively. Zero covariance does not imply independence at finite N , as the table shows.

Corollary 1 (Gaussian determinant from the limiting history measure). In any orthonormal cycle basis the limiting z has density $(31/(4\pi))^3 \exp(-31z^T z/4)$. For a fixed positive semidefinite six-dimensional quadratic probe A ,

$$\lim_{N \rightarrow \infty} E \exp(-z_N^T A z_N/2) = \det(I_6 + 2A/31)^{-1/2}. \quad (31)$$

Proof. The probe is bounded and continuous, so Proposition 7 permits taking its expectation in the limiting measure. Diagonalising A and performing the six normalised one-dimensional Gaussian integrals gives (31). ■

For any fixed linear image $y = Fz$, its covariance is $\Sigma_y = (2/31)FF^T$ and the same result is $\det(I + \Sigma_y A)^{-1/2}$. Null directions remain on their support. Thus a fixed linear coarse map transports the derived Gaussian rule and its covariance together. It does not require the microscopic steps to be Gaussian. The inverse-square-root determinant is the bosonic Gaussian factor used in a one-loop calculation; fermionic and ghost determinant powers still come from their own field statistics and measures.

For a numerical test, take two adjacent unit-variance triangle observables, covariance $\Sigma_y = \begin{bmatrix} 1 & -1/3 \\ -1/3 & 1 \end{bmatrix}$, and the positive test matrix $A = \begin{bmatrix} 0.70 & 0.15 \\ 0.15 & 0.40 \end{bmatrix}$. Its determinant limit is 0.669816421 . These probe coefficients are numerical controls, with no identification as force couplings.

Table 7. Mixed cycle moments and the quadratic determinant limit.

Updates N	Adjacent mixed fourth	Opposite mixed fourth	Quadratic expectation
7	1.579573	0.589731	0.710596
49	1.285533	0.926036	0.676176
343	1.231519	0.989117	0.670715
2,401	1.223555	0.998439	0.669945
Gaussian limit	1.222222	1	0.669816

The mixed moments come from differentiated seven-state transfers. The quadratic expectation is independently integrated from their characteristic function, with quadrature convergence below 3.3×10^{-13} and a three-step path-enumeration check. At $N = 2,401$ its relative deviation from (31) is 0.0192%. The exact full covariance is checked by forward path-moment recursion and direct three-step enumeration; its largest reported relative residual is 1.5×10^{-14} . All 19 joint-current checks pass.

This closes the joint statistical calculation for the specified source process. Conservation selects the accumulating cycle space; its limiting measure supplies mixed pairings and Gaussian integration. The six cycles are graph coordinates. Their connection to the representation-valued gauge links is given by the native transport operator in the following sections, with the field and history variables kept distinct.

6.7 Native links connect circulation to field transport

The connection uses a source operator already present in the corpus. CLSN's native transport primitive is $F_{tr} = \sum_e k_e \|\psi_t - U_e \psi_s\|^2$, with $k_e = 1/31$ on the canonical wheel [8, §17.65]. MFKR uses the same shared-link incidence in the matter kernel [17, §2.3]. For a unitary representation R , define

$$(D_U \psi)_e = \psi_{t(e)} - R(U_e) \psi_{s(e)},$$

$$\hat{L}_U = (\Pi^{-1/2} \otimes I) D_U^\dagger D_U (\Pi^{-1/2} \otimes I) / 31. \quad (32)$$

The hats denote the orthonormal vertex coordinates obtained from the inherited measure Π . The background-independent change of measure cancels from determinant ratios. This is the canonical wheel transport sector, distinct from HSPR's transitive internal carrier in (4a).

Proposition 8 (native transport and the circulation probe). Let T_U have diagonal blocks T_{ii} . On an oriented edge $e: i \rightarrow j$, set $(T_U)_{ij} = T_{ij} R(U_e)^\dagger$ and $(T_U)_{ji} = T_{ji} R(U_e)$. Then

$$S_U = (\Pi^{1/2} \otimes I) T_U (\Pi^{-1/2} \otimes I), \quad \hat{L}_U = I - S_U. \quad (33)$$

For scalar links $U_e = \exp(ia_e)$, T_U is exactly the characteristic transfer $T(-ia)$ of Proposition 7. Its powers sum the same paths with their accumulated phase $\exp(-ia^T Q_N)$. For matrix links, the scalar phase is replaced by the ordered product of link matrices.

Proof. Expand $D_U^\dagger D_U$ edge by edge. The diagonal is the nonself degree, and the off-diagonal blocks are $-R(U_e)^\dagger$ and $-R(U_e)$. Using $\pi_i T_{ij} = 1/31$ gives (33), including the hub and rim self probabilities. Multiplication of T_U then gives the ordered path sum directly. ■

Taking the trace closes the path. In the scalar case this gives the exact identity

$$\text{Tr } T(-ia)^n = \sum_{\gamma \text{ closed}, |\gamma|=n} w(\gamma) \exp[-ia^T Q(\gamma)],$$

$$w(\gamma) = \prod_{(i,j) \in \gamma} T_{ij}, \quad i_n = i_0. \quad (34)$$

Every closed path has $BQ(\gamma) = 0$. These are unnormalised closed-history weights, with total $\text{Tr } T^n$, rather than the stationary open-history measure used in (27). After normalisation, their long-history covariance rate is again $(2/31)P_{\text{cyc}}$: in $\text{Tr } T(a)^n$ the dominant spectral projector has trace one, so the same Perron expansion proves the limit. The executed rate residual falls from 0.00246 at $n = 7$ to 3.7×10^{-15} at $n = 49$.

The source's known geometric map is consistent with these coordinates. If V spans the orthonormal E_2 face doublet, its transverse link basis $R_{\text{cyc}} = D^T V/2$ satisfies $BR_{\text{cyc}} = 0$, $R_{\text{cyc}}^T R_{\text{cyc}} = I_2$ and $DR_{\text{cyc}} = 2V$ [14, §VII.29]. These inherited identities are reproduced exactly to numerical precision. They relate link and closure coordinates; no freely selected linear lift is needed for that geometric conversion.

6.8 Closed histories give the determinant and its charge factors

Let $d = \dim R$ and introduce a positive dimensionless regulator ρ . Set $H_\rho(U) = \rho I + \hat{L}_U$ and $q_\rho = 1/(1+\rho)$. The regulator makes the finite determinant well defined, including the unlinked constant mode.

Proposition 9 (closed-history representation of the native determinant). For any unitary links,

$$\Delta \log \det H_\rho = -\sum_{n \geq 1} (q_\rho^n/n) [\text{Tr } T_U^n - d \text{Tr } T^n]. \quad (35)$$

Equivalently, the same quantity is the background-subtracted proper-time integral of the native heat trace. Its discrete-history expansion is

$$\exp(-t\hat{L}_U) = \exp(-t) \sum_{n \geq 0} (t^n/n!) S_U^n. \quad (36)$$

Proof. Unitary transport is a contraction in the Π norm, so S_U is Hermitian with spectrum in $[-1,1]$.

Expanding $\log(I - q_\rho S_U)$ gives the absolutely convergent series (35); similarity preserves traces. Expanding $\exp(tS_U)$ proves (36). Multiplying its background difference by $-\exp(-\rho t)/t$ and integrating gives q_ρ^n/n term by term. ■

The t in (36) is a heat-kernel parameter; it does not assign physical time to a wheel update.

For a complex scalar field, Gaussian integration gives $\Delta \Gamma = \Delta \log \det H_\rho$; for a real scalar it gives one half of that value. Equation (35) therefore connects a field determinant to circulation-weighted closed histories of the same covariant operator. The gauge links are probes of the path currents, not an assertion that path counts equal field amplitudes. The cycle Gaussian limit and the field Gaussian integral are different constructions connected by (33)–(36).

The link perturbation is fixed by the inherited representation: $R(U_e) = \exp[i a_e \sum_A \theta_A t_A]$. At the trivial background the operator is a spatial matrix tensored with I_d . Applying the representation-factorisation argument of the chiral paper [10, Proposition 7, §6.2], including both contact and two-vertex terms, gives

$$\partial_A \partial_B \log \det H_\rho|_0 = \chi_\rho(a) \text{Re Tr}_R(t_A t_B), \quad (37)$$

where $\chi_\rho(a)$ is computed from the scalar spatial determinant. It also equals the q_ρ^n/n -weighted sum of the unnormalised closed-history second moments of $a^T Q$. The finite census Hessian there and the native transport Hessian here instantiate the same representation factorisation. Their spatial coefficients, fermionic sign and Weyl weight belong to their respective operators; the common assertion is the internal trace factor, not equality of the complete Hessians.

There is a second, local check of the shared normalisation. Relaxing the endpoint variation of F_{tr} about a constant vector ξ removes the gradient part of a and leaves $\|P_{\text{cyc}} a\|^2 \|t\xi\|^2/31$ at quadratic order. Let $K_{\text{tr}}^{\text{rel}}$ be the retained constant-mode kernel after this endpoint relaxation. Taking its internal trace gives

$$\partial_A \partial_B \text{Tr } K_{\text{tr}}^{\text{rel}}|_0 = (2/31)(a^T P_{\text{cyc}} a) \text{Re Tr}_R(t_A t_B). \quad (38)$$

Thus the cycle variance factor also appears in this specified native transport response. This identity does not fix an additional link-only term in the full source action, which the native transport source itself leaves separate [8, §17.65].

The execution uses a noncommuting $SU(2)$ background with maximum link-commutator norm 1.30 and $p = 0.4$. Direct determinant evaluation gives $\Delta \log \det H_p = 0.100139351201$. Independent closed-path enumeration verifies the trace identity through four steps; the full trace series and proper-time integral give the results below.

Table 8. Native transport and closed-history determinant checks.

Connection test	Numerical result
Closed-history series through length 64	Determinant discrepancy 3.2×10^{-11}
Closed-history series through length 128	Determinant discrepancy 1.1×10^{-15}
Independent proper-time integral	Determinant discrepancy 1.5×10^{-15}
Closed-current sum versus contact-plus-bubble Hessian	Maximum matrix discrepancy 4.9×10^{-17}
Census charge factors from the native determinant	Maximum Gram discrepancy 1.2×10^{-8}
Full 315-dimensional census Schur calculation	Agreement with spectral evaluation 5.9×10^{-16}

The inherited census gives $\text{Tr}_F(t_A t_B) = \text{diag}(6I_8, 6I_3, 10)$ and $\text{Tr}_H(t_A t_B) = \text{diag}(0I_8, \frac{1}{2}I_3, \frac{1}{2})$. Both are recovered from (37) and independently from the relaxed native cost (38). The internal census check identifies representation factors, not fermionic determinant statistics. Those still enter through the inherited chiral operator and its Weyl weight. All 32 operator-connection checks pass.

The handoff to Sections 3–4 is therefore explicit: the source fixes covariant incidence and shared links; path composition produces ordered closed-history traces; those traces are the heat-kernel and determinant data; differentiating the same links produces the census charge Grams. The inherited four-dimensional principal operators and spin/statistics weights then give $b_F = (2/3)(6, 6, 10)$, $b_H = (1/3)(0, \frac{1}{2}, \frac{1}{2})$ and the gauge–ghost contribution. The finite wheel is not a four-dimensional momentum regulator and does not itself generate that continuum logarithm. Its dimensionless factor $1/31$ is not multiplied into b : the physical ruler and common kinetic coefficient are carried by the existing source normalisation, with the constant-pairing cancellation proved in Section 3. This completes the operator connection while leaving absolute physical matching to the full coupled source action.

6.9 A cycle kernel discriminator for fermion interactions

Proposition 7 supplies a definite same-source candidate for the fluctuation kernel needed by the preceding interaction calculation. In an orthonormal cycle basis V , the wheel's accumulated signed-current covariance per update is $C_w = (2/31)I_6$. The chiral paper instead evaluates the link marginal of the coupled operational witness $G(c, s)$, giving [10, §9.6, equations (32)–(34)]

$$\Sigma_{\text{op}} = \text{diag}(6/19, 4/17, 4/17, 3/19, 3/19, 14/103), \quad c = s = 1. \quad (39)$$

Both are expressed on the orthonormal cycle modes with triangle-Gram eigenvalues $\ell = (1, 2, 2, 4, 4, 5)$. These six spatial modes are distinct from the twelve internal gauge-generator directions. The two matrices are not alternative coordinate expressions for the same covariance: one is flat and the other has unequal eigenvalues.

For matched link coordinates with $\text{Cov}(\theta) = h\Sigma$ and fixed currents $B_i = \bar{\psi}D_i\psi$, Gaussian elimination gives the quartic term [10, §§9.6,9.9]

$$S_{4\psi} = -(h/2) \Sigma_{ij} \Sigma_{ij} B_i B_j. \quad (40)$$

Evaluating the chiral paper's Section 9.6 formula under both kernels, with fixed currents, gives the six coefficients below. Here $S_{4\psi} = -(h/2)\Sigma_i c_i B_i^2$; the action coefficient is $-hc_i/2$. Nonlinear links also retain that paper's contact term. The wheel witness assumes equal mode transfer into link coordinates and unit common conversion.

Table 9. Six induced interaction coefficients in matched orthonormal cycle modes.

Mode and eigenvalue	Wheel interaction c_i	Operational interaction c_i
1, $\ell = 1$	$2/31 = 0.064516$	$6/19 = 0.315789$
2, $\ell = 2$	$2/31 = 0.064516$	$4/17 = 0.235294$
3, $\ell = 2$	$2/31 = 0.064516$	$4/17 = 0.235294$
4, $\ell = 4$	$2/31 = 0.064516$	$3/19 = 0.157895$
5, $\ell = 4$	$2/31 = 0.064516$	$3/19 = 0.157895$
6, $\ell = 5$	$2/31 = 0.064516$	$14/103 = 0.135922$

Normalising to the first mode removes a common scale. The wheel predicts $1 : 1 : 1 : 1 : 1 : 1$, whereas the operational witness predicts $1 : 0.745098 : 0.745098 : 0.5 : 0.5 : 0.430421$. Its largest multiplier is $309/133 = 2.323308$ times its smallest. These are coefficients of fixed, equally normalised current products, not a claim that every particle scattering amplitude is equal under a flat kernel.

The physical comparison is already specified by the chiral paper's §9.9 and the charge paper's gauge-stiffness test [21, §§5.5–5.6]. On the physical quotient, with full gauge kernel K_{phys} and projection P_6 onto these modes, it requires

$$\Sigma_{\text{phys}} = P_6 K_{\text{phys}}^{-1} P_6^T. \quad (41)$$

Remove constraints before inversion and retain the spatial, temporal and internal indices. The twelve-generator sector matrix alone does not specify this covariance. Comparing the normalised spectrum of (41) with Table 9 tests the attachment; it does not assume that $2/31$ is an inverse energetic stiffness.

The source-to-link map belongs to this test. If $\theta = Az$, then $\Sigma_\theta = (2/31)AA^T$; the positive square root $A = (31\Sigma_\theta/2)^{1/2}$ reproduces (39). Unequal mode gains can therefore change the pattern, while an orthogonal basis change and common scale cannot. Hold the physical map, current vertices, clock conversion and normalisation fixed. The operational witness is not a separately admitted physical alternative.

The marked-current construction [20, §§36–37] already carries $g_{\text{cyc}}^2(2/31)H_{\text{CWY}} \otimes I_2$. Its representation factors and unselected gain are distinct from these six spatial modes. The shared source normalisation connects the calculations without equating a marked-current response with an inverse stiffness.

The script reconstructs the carrier, wheel covariance, full coupled 24-coordinate inverse, six four-fermion coefficients and nonuniform-map control. All 18 checks pass, with maximum numerical discrepancy 1.2×10^{-15} .

7 Consequence for the VERSF derivation

The $K = 7$ gradient and refinement geometry, BCB transport requirements and coupled reduced action supply the physical setting. The inherited Higgs spatial and temporal results, fermion operator and gauge-ghost response then give the leading coefficient $(-7, -19/6, 41/6)$. The scalar multiplier and the complete constant positive gauge-invariant matter pairing class cancel from this coefficient, so its evaluation needs no numerical stiffness assigned to a primitive fold.

The wheel supplies the joint fluctuation rule and an exact connection to the field calculation. Its cycle measure gives mixed Wick pairings; its shared-link closed paths reproduce the native determinant and proper-time integral; and link differentiation produces the census charge Grams. The results and their domains are collected below.

Table 10. Results and scope.

Question	Result of this calculation
Does the overall constant Higgs residue enter the leading scalar logarithm?	No; cancellation proved and evaluated
Do unequal constant matter pairings change the leading running?	No; the complete 46-parameter internal class cancels
Can joint Gaussian averaging emerge from discrete $K = 7$ updates?	Yes; all six cycles, exact finite covariance and mixed Wick pairings are derived
Does the same limit give a determinant rule?	Yes; the positive quadratic probe gives $\det(I + \Sigma A)^{-1/2}$
How do the histories enter the field determinant?	Native shared-link transport gives an exact closed-path trace and proper-time identity
How can the cycle law test the interaction attachment?	With the same link map and currents, the wheel has equal weights; the operational witness has maximum-to-minimum ratio 2.323308. Compare both with the physical inverse kernel
What does one inherited Higgs doublet contribute?	$(0, 1/6, 1/6)$
What is the complete leading coefficient on the inherited spectrum window?	$(-7, -19/6, 41/6)$
Must the scalar formula be extended over a microscopic cubic momentum torus?	No; that is not an input to this effective calculation
Is the full source spectrum window derived here?	No; QGC P26 and its upstream branch premises are retained
Are the starting force strengths or all thresholds calculated?	No; they belong to physical matching

The next source calculation has a concrete target: project the physical inverse gauge kernel onto these same six cycle modes and compare the interaction coefficients in Table 9. Equal weights support the wheel attachment; the decreasing pattern supports the operational witness; a different pattern requires a different attachment. The source must determine the outcome with the link map and vertices fixed. This test directly connects the chiral paper's six-mode interaction tensor to the charge paper's physical stiffness calculation.

Appendix A The inherited fermion universality result

MFKR's $K = 7$ infrared moment closure, carried into Section 2.6, supplies the spatial principal-symbol normalisation [17]. The full finite microscopic source identity remains distinct from that inherited continuum result.

The running calculation uses the same unit temporal-coefficient family as the preceding numerical fits [8,10]. The separately tested composition-selected static temporal extension is not silently substituted into those data.

For unitary covariant shifts T_μ and Hermitian Euclidean gamma matrices, define

$$S = \sum_{i=1,3} [\gamma_i(T_i - T_i^\dagger)/2 + I - (T_i + T_i^\dagger)/2], \quad K = \gamma_4 S,$$

$$f_\lambda(K) = (1-\lambda)K + \lambda \sinh K, \quad 0 \leq \lambda \leq 1,$$

$$X_\lambda = \gamma_4 f_\lambda(K) + \gamma_4(T_4 - T_4^\dagger)/2 - (T_4 + T_4^\dagger)/2,$$

$$H_\lambda = \gamma_5 X_\lambda, \quad D_\lambda = I + \gamma_5 \text{sign}(H_\lambda). \quad (A1)$$

Gauge covariance follows from functional calculus. The relations $\gamma_4 S \gamma_4 = S^\dagger$ and $\gamma_5 K \gamma_5 = -\gamma_4 K \gamma_4$ give a Hermitian H_λ without assuming that temporal and spatial shifts commute.

On the free torus put $w_i = 1 - \cos q_i$, $W = \sum w_i$, $E^2 = \sum \sin^2 q_i + W^2$ and $F_\lambda = 1 - \lambda + \lambda \sinh(E)/E$. Then $a = F_\lambda W - \cos \theta$, $b = (F_\lambda \sin q_1, F_\lambda \sin q_2, F_\lambda \sin q_3, \sin \theta)$, and

$$R^2 = a^2 + |b|^2 \geq 1, \quad D_\lambda = I + (aI + i\gamma \cdot b)/R. \quad (A2)$$

The bound follows from $E^2 - 2W = 2\sum_{i<j} w_i w_j \geq 0$ and $F_\lambda \geq 1$. Only $q = 0$, $\theta = 0$ gives $b = 0$ and $a < 0$, so D has one free zero. Moreover $D_\lambda - D_0$ starts at cubic momentum order.

The inverse denominator $2(R+a)$ is smooth, nonnegative and has that sole zero with leading quadratic behaviour. Entire divided differences of $\sinh$ and the gapped sign calculus supply smooth gauge vertices, including the second-derivative contact term. Covariance supplies the Ward identity. On the perturbative gapped branch these conditions give the same leading covariant symbol and logarithm as the Wilson overlap reference under one-loop lattice power counting [12,13]. The Weyl magnitude contributes half the Dirac result, giving $b_F = (2/3)(6,6,10)$.

The $128^3 \times 256$ fits in the preceding paper give logarithmic coefficients 0.967 for the inherited operator and 0.974 for the Wilson control against the continuum value 1 [10, §6.3, Table 7]; these are the finite-volume numerical counterpart of the argument, consistent with a common limiting coefficient within their stated scope, without establishing convergence or a calibrated uncertainty.

The archived finite checks include fully time-dependent non-Abelian links on a 2^4 fixture, relative gauge-covariance residual 5.8×10^{-14} , and a gauge-orbit contact-plus-bubble residual below 2.2×10^{-16} . A separate finite determinant on the census gives per-generator trace factors between 6.000000025 and 6.000000078 in colour, 6.000000078 in weak, and 9.999999918 in hypercharge. These are operator and implementation controls. They do not establish the full chiral phase or an identity with every admissible microscopic source kernel.

Appendix B Scope of the scalar lattice diagnostic

The unrestricted periodic cubic operator $K_{\text{cent}}(q, \theta) = \sum \sin^2 q_i + 4\sin^2(\theta/2)$ has eight light spatial corners. A simple index-seven star average has nonzero overlap at each. Those are correct results about that explicit model.

HSPR supplies a seven-carrier construction and a physical continuum response. It does not specify that unrestricted operator as the complete Higgs measure on the physical source quotient. Its higher-order and finite-regulator claims also carry their own domain. Therefore the eight-corner result cannot be used as evidence that VERSF produces eight physical Higgs doublets or as a prerequisite to applying its already-derived effective action.

Conversely, coarse graining does not automatically remove a genuine light physical mode. If the actual source produced such a mode, equation (19) would preserve its contribution in the complete determinant and P26 would fail in the affected interval. No assertion that seven modes disappear is needed or made here.

The positive higher-derivative repair previously tested for that cubic construction remains an algebraic example. It is not used in the effective calculation and is not identified with a VERSF source term. Its script and numerical results are retained separately in the calculation archive as extension controls.

Appendix C Reproducibility

The archive contains the manuscript source, document builder, executables and outputs. Source records include public URLs and SHA-256 hashes; the manifest hashes every executable and document.

README.txt lists all eight commands, to run from the archive root with Python, NumPy and SciPy.

The `coarse_response.py` script passes 21 aggregate checks of the twelve-generator response, independent high-precision quadrature, scalar normalisation cancellation, momentum integral, thresholds, scale composition and coupled determinant split on the effective branch of Section 1.

The `k7_source_checks.py` script passes eight checks of the HSPR gradient and proper-distance limit. The `matter_pairing_checks.py` script passes 31 checks of the gauge commutant, unequal pairings and full, retained and eliminated determinants. The `k7_fermion_moment_checks.py` script passes six checks of the infrared moment identity, orientation qualification and stability bound.

The `k7_gaussian_limit.py` script passes 13 scalar-circulation checks, comparing seven-state derivatives with rational two-state moments and direct paths. It reproduces (22) and (24); its output is `k7_gaussian_results.json`.

The `k7_joint_cycles.py` script passes 19 checks of (27), the Perron Hessian, mixed moments and the quadratic determinant. Forward moments through 2,401 updates, rational projection, direct paths, an independent Poisson solve and characteristic-function integration provide complementary evaluations. Its output is `k7_joint_cycles_results.json`. Neither history script uses Gaussian step noise or Monte Carlo sampling.

The `cycle_transport_connection.py` script passes 32 checks of (32)–(38), including native incidence, non-Abelian paths, gauge covariance, the determinant series, proper-time integration and census traces. A full 315-dimensional evaluation checks the spectral method. Its output is `cycle_transport_connection_results.json`.

The `cycle_kernel_comparison.py` script passes 18 checks of Section 6.9: the common carrier, full coupled inverse, relative four-fermion coefficients and coordinate-map controls. Its output is `cycle_kernel_comparison_results.json`; the physical link attachment remains to be selected.

Check counts retain their separate scopes and are not combined into tests of the VERSF axioms. No measured force strength, weak angle or fitted mass enters. Test masses, cutoffs and probes are controls. The Appendix B scalar diagnostics are retained separately.

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