

Coarse Grained History Response and Gauge Exchange in VERSF

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General reader summary

The preceding papers connected VERSF's seven-point construction to particle propagation and to the way force strengths change with distance. This paper calculates a further part of that connection: how the complete collection of closed paths responds when the rule for carrying charge along those paths is slightly changed. The calculation includes paths of every length. It produces an explicit response for each of the six independent ways of circulating around the seven-point cell.

Short paths distinguish those six patterns strongly. As longer paths dominate, the responses become equal. The calculation also determines which contribution is lost if the six outer sites are removed without retaining their fluctuation effect. When this response is used as the rule for how the field resists change, it also fixes how strongly particles push on one another. These are consequences of the specified transport construction, with no measured force strength used to choose them.

A further calculation combines two complete sets of paths while keeping only the net effect on charge. This changes the relative colour, weak and hypercharge response because the particle families carry charge in different numbers of states. Connecting the cells through the published VERSF bridge preserves this unequal pattern: the relative proportions change by less than 0.6% in the two examples tested, although the overall response changes more. Unequal proportions also survive an arbitrarily strong connection under the model's mass rule, which pins the two joining sites rather than merging the cells. The unequal response emerges after the unresolved transport has been summed over. No force strength is assigned to an individual fold or chosen from a measurement.

The corpus already connects currents and records to the underlying action. It also proves that hidden internal structure can be averaged out without changing what an observer would measure, provided the grouping rules are obeyed; independent numerical checks reproduce that result here. The connected-cell calculation establishes that the unequal-response mechanism survives one published connection between cells. The comparison still to execute is whether the complete physical action gives this particular coarse response. That comparison belongs at the coarse-grained level. A separate exact construction shows that a chosen mixture of history lengths can reproduce the earlier six-mode candidate, although its weights have not been selected by the source.

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Abstract

We evaluate the complete quadratic gauge response of the native K7 covariant transport determinant introduced in the preceding gauge-running paper. For every positive resolvent parameter ρ , its twelve-edge Hessian has the six gauge-gradient null directions and an explicitly positive six-cycle spectrum. Four rational functions give the distinct cycle eigenvalues. They interpolate between a short-loop response proportional to the triangle Gram and a long-history response proportional to the identity. Retaining the hub and eliminating the rim gives an exact determinant decomposition: the eliminated determinant contributes only to the uniform cycle mode, with a calculated fraction of 3.6383% at the diagnostic point $\rho = 0.4$. The finite remainder after subtracting the common graph zero-mode pole is also evaluated exactly.

The inverse cycle Hessian supplies Gaussian exchange coefficients conditional on its identification with the field action. The operational covariance is excluded for the positive-resolvent family and every fixed history length, but an exact positive four-length mixture reproduces it. Common transport preserves the census Gram $(6, 13/2, 21/2)$ at the quadratic determinant stage. Nonlinear Haar composition instead gives a leading Gram proportional to $(9/4, 25/8, 73/8)$. The published two-wheel bridge yields an exact correlated extension whose leading response cannot recover the original census ratios at any nonnegative bridge conductance. For the complete native weight at fixed positive ρ , the strong-connection limit under the degree-weighted mass convention is derived exactly and retains unequal ratios while its magnitude decays as g^{-2} . Intermediate full-weight calculations corroborate the mechanism; at $g = 1/3$ the sector-ratio changes are below 0.6% at the two diagnostic parameters. The corpus's conditional-mean theorem is independently reproduced on correlated refinements, so independence is not required for response preservation. Identification with the complete physical current-record measure, physical statistics and scale matching remain distinct from these finite results.

1 Purpose and relation to the preceding papers

The programme seeks a common coarse-grained construction that determines both gauge response and the interactions that response transmits. Three preceding papers supply complementary ingredients. The charge-participation paper derives the D41 history reductions and specifies the physical gauge-attachment test [1]. The chiral-fermion paper constructs an inherited fermion candidate, proves exact coarse-graining identities and evaluates the interactions produced by a prescribed link covariance [2]. The gauge-running paper connects native covariant transport to closed-history traces and establishes the leading effective running coefficient under its stated branch and spectrum conditions [3].

The present calculation uses that connection to evaluate a complete finite response function. Its input is the native endpoint transport cost, shared compact links and canonical wheel law, rather than a chosen six-mode covariance. The computation determines the response first and then forms its inverse on the gauge quotient. This ordering gives a concrete test of the covariance candidates in [3, Section 6.9].

The source distinction is essential. CLSN derives the native transport cost while explicitly separating it from the complete action for independent links, records and other fields [4, Sections 17.65–17.67]. The determinant of that transport operator is a well-defined induced action. It is one calculable contribution to the physical problem, not a licence to discard the rest of the source. We therefore state exactly where an identification with the full field action is used.

The finite results are the rational six-cycle spectrum, its exact retained and eliminated contributions, the specified-family exclusions and an exact positive-mixture counterexample. Section 8 evaluates nonlinear composition of complete holonomy weights, first for independent cells and then for the published connected two-cell graph. Both produce unequal sector responses. The correlated calculation gives an exact bridge correction, bounds its leading sector ratios for the whole nonnegative conductance family, and derives the strong-connection limit of the complete native weight. It also identifies the distinction between the corpus's response-preserving conditional average and the retained holonomy product used here. The representation factorisation and inverse-stiffness rule are inherited structures [2–3]; their scope is preserved throughout.

2 The native cell and the response being calculated

2.1 Geometry and transport

Label the hub 0 and the rim vertices 1 to 6. The row-stochastic wheel operator T has $T_{0j} = 1/7$ for every j . A rim row has probability $1/4$ on its hub neighbour, its two rim neighbours and itself. Its stationary measure is $\pi = (7, 4, 4, 4, 4, 4)/31$ [5]. Orient the six spokes outward and the six rim edges cyclically. Let B be the 7×12 incidence matrix, with -1 at an edge source and $+1$ at its target.

The six triangle columns are $c_i = s_i + r_i - s_{i+1}$. With $C = (c_1, \dots, c_6)$, the cycle space is $\ker B = \text{Im } C$ and

$$H_\Delta = C^T C = \text{circ}(3, -1, 0, 0, 0, -1), \quad \text{spec } H_\Delta = (1, 2, 2, 4, 4, 5). \quad (1)$$

If O diagonalises H_Δ , then $V = CO \text{diag}(\ell_i^{-1/2})$ is an orthonormal edge-cycle frame. Thus $V^T V = I_6$ and $V V^T$ is the cycle projector. The six spatial modes in (1) are distinct from the twelve internal gauge generators and from the six marked-current coordinates of the RCHS construction [6].

The inherited transport difference and its normalised positive operator are

$$(D_U \varphi)_e = \varphi_{t(e)} - U_e \varphi_{s(e)},$$

$$\hat{L}_U = \Pi^{-1/2} D_U^\dagger D_U \Pi^{-1/2} / 31, \quad \Pi = \text{diag } \pi. \quad (2)$$

The factors of Π act on vertices and are tensored with the identity on internal states. At trivial links, $\hat{L}_0 = I - S$, where $S = \Pi^{1/2} T \Pi^{-1/2}$. This is the source conductance $1/31$ and its vertex measure, carried together. It is not an added multiplicity in a four-dimensional beta function.

2.2 The induced action and its parameter

For $\rho > 0$ define $H_\rho(U) = \rho I + \hat{L}_U$ and the background-dependent functional

$$\Gamma_\rho(U) = \log \det H_\rho(U) - \log \det H_\rho(I). \quad (3)$$

For a flat complex Gaussian field with this operator, (3) is its exact induced effective action. It is also the determinant functional whose closed-history representation was established in [3]. A real scalar has one half of this determinant power. Fermions and ghosts have their own operators, determinant powers and contact vertices; they are not assigned scalar statistics here.

The dimensionless ρ regulates the graph zero mode and determines the relative weighting of path lengths. We calculate the whole positive- ρ family. No value is selected from a gauge coupling, and ρ is not identified with a measured particle mass, a physical clock or a four-dimensional renormalisation scale. The value 0.4 is an inherited finite diagnostic point [3], used to exhibit numbers after deriving the functions.

With $q = (1 + \rho)^{-1}$ and $S_U = I - \hat{L}_U$, the exact history expansion is

$$\Gamma_\rho(U) = -\sum_{n \geq 1} (q^n / n) [\text{Tr } S_U^n - \text{Tr } S_0^n]. \quad (4)$$

The similarity to the covariant T_U makes these traces sums of ordered closed paths. Their total unperturbed weight is $\text{Tr } T^n$. They are not open histories sampled from π . Reversibility pairs opposite paths in the Abelian calculation. Near the identity, their second gauge variation is a positive current second moment.

3 The complete closed history response

3.1 The Hessian on the gauge quotient

For a unit Abelian charge write $U_e = \exp(ia_e)$ and define $K_\rho = \partial^2 \Gamma_\rho / \partial a \partial a$ at $a = 0$. If $Q(\gamma)$ is the signed twelve-edge count of a closed path, then $BQ(\gamma) = 0$ exactly. Differentiating (4) gives

$$K_\rho = \sum_{n \geq 1} (q^n / n) \sum_{\gamma \text{ closed, length } n} w(\gamma) Q(\gamma) Q(\gamma)^T. \quad (5)$$

Proposition 1 (positive native cycle response). For every $\rho > 0$, K_ρ is real symmetric and positive semidefinite, with kernel exactly $\text{Im } B^T$. Its restriction $K_\rho^{\text{cyc}} = V^T K_\rho V$ is positive definite.

Proof. Every term in (5) is positive semidefinite and annihilates gradients. The oriented triangles have positive path weight and their six count vectors span $\ker B$. Their contribution is therefore strictly positive on that space. ■

An independent operator formula retains both the contact and two-vertex terms. Set $R = [(1 + \rho)I - S]^{-1}$, $S_e = \partial_e S_U$ and $S_{ef} = \partial_e \partial_f S_U$ at the identity. Then

$$(K_\rho)_{ef} = \text{Re } \text{Tr} [-R S_{ef} - R S_e R S_f]. \quad (6)$$

The second term need not be positive on its own. Positivity follows from the complete expression (5); the gauge-null identity provides a sensitive test of retaining the contact term.

3.2 Exact spectrum

Proposition 2 (complete resolvent spectrum). In the orthonormal cycle frame of (1), K_p^{cyc} is diagonal, with eigenvalues $(k_1, k_2, k_2, k_4, k_4, k_5)$, where

$$\begin{aligned} k_1 &= 2/[\rho(1+4\rho)(31+28\rho)] + e(\rho), \\ e(\rho) &= 3/[16(1+\rho)^2(1+2\rho)^2(1+4\rho)(5+4\rho)], \\ k_2 &= 2/[\rho(1+2\rho)(31+28\rho)], \\ k_4 &= 2/[\rho(1+\rho)(31+28\rho)], \\ k_5 &= 10/[\rho(5+4\rho)(31+28\rho)]. \quad (7) \end{aligned}$$

Proof. Dihedral covariance separates the uniform, first-harmonic, second-harmonic and alternating cycle spaces. Substituting the rational wheel matrix into (6), in each of those spaces, gives (7). Appendix A specifies the four integer vectors and the exact trace calculation, so the proof is reproducible without numerical eigenvector fitting. The alternative decomposition of k_1 follows independently from hub retention in Section 4. ■

The small and large p limits identify what the resummation does:

$$\begin{aligned} K_p^{\text{cyc}} &= [2/(31\rho)]I_6 + K_{\text{fin}} + O(\rho), \\ K_p^{\text{cyc}} &= \text{diag}(1, 2, 2, 4, 4, 5)/(56\rho^3) + O(\rho^{-4}). \quad (8) \end{aligned}$$

The first limit is dominated by long histories. The second is dominated by the shortest nontrivial loops, the triangles. Indeed their contribution is $q^3 \Sigma_i [1 - \cos(c_i^T a)]/56$. It gives precisely the triangle Gram after two derivatives.

The finite graph remainder in the first line of (8) is

$$\begin{aligned} K_{\text{fin}} &= \text{diag}(-21437/76880, -180/961, -180/961, \\ &\quad -118/961, -118/961, -528/4805). \quad (9) \end{aligned}$$

Equation (9) is a calculated subtraction constant for this finite graph. The pole being subtracted is a graph zero-mode pole, not the four-dimensional gauge logarithm. K_{fin} alone is negative and must not be inverted as a positive field stiffness. The full K_p remains positive.

3.3 Fixed history length

The closed-path covariance per step gives a second coarse-history calculation. Define

$$C_N = [N \text{Tr } T^N]^{-1} \Sigma_{\gamma \text{ closed, length } N} w(\gamma) Q(\gamma) Q(\gamma)^T. \quad (10)$$

At $N = 3$ its cycle eigenvalues are proportional to $(1, 2, 2, 4, 4, 5)$. The Perron expansion gives $V^T C_N V \rightarrow (2/31)I_6$ [3]. The exact coefficients in (A4), together with the uniform-mode formula in Appendix A, give Table 1; direct differentiation of path powers independently reproduces them. The table reports the inverse mode weights normalised to the $\ell = 1$ mode; it does not identify N with elapsed physical time or a spatial blocking factor.

Table 1. Fixed-length inverse cycle weights, normalised to the uniform mode.

History length N	$\ell = 2$ modes	$\ell = 4$ modes	$\ell = 5$ mode
3	0.500000	0.250000	0.200000
4	0.589744	0.460000	0.511111
5	0.667331	0.590621	0.574614
7	0.862809	0.838215	0.836655
14	0.997305	0.997083	0.997083
21	0.999966	0.999964	0.999964
Long history limit	1	1	1

The resolvent average and fixed-length conditioning are two specified weightings of the same path counts. They need not have the same finite response. Their agreement in the normalised long-history limit does not equate either one with a selected physical gauge measure.

4 Exact coarse graining and the eliminated contribution

Retain the hub field and eliminate the six rim fields of H_ρ . In this vertex splitting write the rim block as F and the retained Schur complement as D_c . Gaussian integration gives

$$\det H_\rho = \det F \det D_c, \quad K_\rho = K_F + K_c. \quad (11)$$

Both background-dependent determinants belong to the same reduction. The retained operator by itself does not contain the complete induced gauge action.

Proposition 3 (rank-one eliminated contribution). For this hub-retaining block, the eliminated rim determinant contributes $e(\rho)$ from (7) to the $\ell = 1$ mode and zero to the other five cycle directions. The retained contribution in that mode is exactly $2/[\rho(1 + 4\rho)(31 + 28\rho)]$.

Proof. F is the rim-rim principal block. The spoke phases occur only in the hub-rim and rim-hub off-diagonal blocks and never enter $\det F$. The rim block is a six-site covariant ring with a constant diagonal contribution from its connections to the fixed boundary. A gauge transformation on the rim removes every edge phase except their total circulation. Its determinant therefore depends only on that circulation. In the orthonormal cycle frame it lies along $\ell = 1$. Evaluation of the ring determinant, or its two derivatives, gives $e(\rho)$; subtracting it from (7) gives the retained term. ■

Table 2. Exact eliminated contribution under hub retention.

ρ	Complete $\ell = 1$ stiffness	Eliminated stiffness	Eliminated share
1	0.00689540	0.00011574	1.6785%
0.4	0.04729116	0.00172061	3.6383%
0.1	0.43688842	0.01423415	3.2581%
0.01	6.18164913	0.03370500	0.5452%

Dropping det F changes the spatial interaction pattern: it increases the inverse weight of the uniform mode while leaving the other five unchanged. The tabulated shares are properties of this particular block and action. They are not percentages of the complete Standard Model gauge stiffness and are not the eliminated-mode shares of the chiral paper’s spatial star block.

The inherited edge-subdivision identity gives another exact check [4, Section 17.65]. Split each edge into two serial edges of conductance $2/31$, preserve the ordered product of their unitary links, keep the mass term on the original boundary vertices and assign no midpoint mass. Eliminating the twelve midpoints reproduces the original unweighted operator. The midpoint determinant is field independent. We verify the complete background determinant identity on noncommuting $SU(2)$ links. This is exact static Gaussian refinement under the stated midpoint prescription; positive midpoint temporal weights would introduce the memory response already identified in [4].

5 The interaction kernel and the operational comparison

5.1 What the inverse actually supplies

Let a be the six orthonormal cycle coordinates and let $B_i = \bar{\psi} D_i \psi$ be fixed fermion currents, with any factors of i included in the vertex D_i . If the quadratic action for these link variables is $a^T K_{\rho}^{cyc} a/2$, its Gaussian integral gives

$$\int da \exp[-a^T K_{\rho}^{cyc} a/2 - a^T B] \propto \exp[B^T \Sigma_{\rho} B/2],$$

$$\Sigma_{\rho} = (K_{\rho}^{cyc})^{-1}, \quad S_{4\psi} = -B^T \Sigma_{\rho} B/2. \quad (12)$$

This evaluates the inverse-stiffness relation used in [2, Section 9.9]. It is conditional on using this induced quadratic action for the integrated field. The nonlinear fermion-link action also has a two-link contact term; its contraction must be retained alongside the quartic term. Equation (12) does not remove that term or close the chiral determinant phase.

Table 3. Cycle stiffness and conditional exchange coefficients.

Mode	Stiffness at $\rho = 0.4$	Inverse coefficient	Inverse relative to mode 1
$\ell = 1$	0.04729116	21.145603	1
$\ell = 2$ each	0.06582412	15.192000	0.718447
$\ell = 4$ each	0.08463101	11.816000	0.558792
$\ell = 5$	0.08976016	11.140800	0.526861

These coefficients refer to dimensionless, equally normalised cycle coordinates. A common multiplier of the action multiplies stiffness and divides covariance. A change of coordinates must transform the currents at the same time. The numbers are not dimensionful four-fermion coefficients or claims of equal scattering amplitudes for different particles.

The preceding wheel result $C_w = (2/31)I_6$ is a current covariance rate. Here it supplies the leading stiffness through $K_p^{cyc} \sim C_w/\rho$, and hence $\Sigma_p \sim \rho C_w^{-1}$. The common flat shape agrees in the limit, but the magnitude and role of the two objects are different. This distinction is fixed by deriving the action before performing the Gaussian field integration.

5.2 An exact discriminator

The operational witness evaluated in [2–3] has covariance

$$\Sigma_{op} = \text{diag}(6/19, 4/17, 4/17, 3/19, 3/19, 14/103). \quad (13)$$

Proposition 4 (no operational match in the native resolvent family). In the matched cycle basis, no $\rho > 0$ and common positive multiplier make Σ_p equal to (13).

Proof. A common multiplier cancels from mode ratios. Equating the $\ell = 2$ to $\ell = 4$ ratio requires

$$(1+2\rho)/(1+\rho) = (4/17)/(3/19), \text{ hence } \rho = 25/26.$$

Equating the $\ell = 4$ to $\ell = 5$ ratio instead requires

$$5(1+\rho)/(5+4\rho) = (3/19)/(14/103), \text{ hence } \rho = 215/94. \quad (14)$$

The two values differ. ■

This is a result about a fully specified response family, not a rejection of every operational construction or every VERSF source. A different physical link map, additional coupled response or different history-length measure can change the spectrum. Such changes must be derived independently. They cannot be absorbed into a common scale to evade (14).

Corollary 4.1 (fixed-length exclusion). No fixed closed-history length has inverse cycle weights proportional to (13). At $N = 1$ or 2 the cycle Hessian is zero and has no inverse. At $N = 3$ the inverse ratio Σ_2/Σ_4 is 2. For every integer $N \geq 4$ it is at most $122/93 \approx 1.312$, strictly below the required $76/51 \approx 1.490$.

Proof. The exact coefficient formulas and the uniform bound are given in Appendix A. They apply to all lengths, rather than only the finite numerical sample. ■

For the two ratios $(\Sigma_2/\Sigma_4, \Sigma_4/\Sigma_5)$, the first three nontrivial lengths give $(2, 1.25)$, $(1.282051, 0.9)$ and $(1.129880, 1.027856)$, whereas the ratios implied by (13) are $(1.490196, 1.161654)$. Table 1 includes the $N = 4$ reversal between the last two mode weights. The approach of the second ratio to one has a small alternating correction, so it is not monotone. Proposition 4 and Corollary 4.1 exclude both specified native weightings. A freely chosen positive mixture can, however, reproduce the operational witness exactly.

Corollary 4.2 (positive mixture counterexample). Let $a_{\ell,N}$ be the q^N coefficient of k_ℓ , as in Appendix A. A positive mixture supported on $N = 3, 4, 5$ and 12 has inverse weights proportional to (13). The coefficients normalised to sum to one are approximately $(0.627252, 0.012918, 0.005689, 0.354142)$.

Proof. The exact positive integers in (A6), divided by their sum, give a mixture proportional to (19/6, 17/4, 19/3, 103/14) in the four distinct stiffness modes. Inversion gives precisely the four weights in (13), including their doublet multiplicities. ■

These are coefficients multiplying the unnormalised history contributions $a_{e,N}$, not a claim about the source's stopping probabilities. They were solved from the target witness. The example establishes attainability under positive reweighting, not source selection or an independent prediction. Proposition 5 still applies to this mixture because its spatial weighting is common to every particle component.

6 Charge participation and the twelve generator directions

6.1 The premise that collective participation changes

The inherited census has 45 left-handed Weyl components and a two-component complex Higgs representation. Its conventional-Y geometric Gram is

$$G_{tr} = \text{diag}(6I_8, (13/2)I_3, 21/2). \quad (15)$$

The two additional neutral addresses used in the 49-state D41 participation carrier have zero generators and do not alter (15). With $q = 6Y$ the last entry is 378; converting both coordinates and vertices preserves the normalised comparison.

Proposition 5 (all-length representation factorisation). Suppose every component carries the same spatial transport operator and the links act through its representation matrices T_A . At the trivial background, the quadratic determinant response is

$$\mathcal{K}_{iA,jB} = (K_p^{cyc})_{ij} \text{Re Tr}(T_A T_B). \quad (16)$$

The same statement holds at each fixed history length and for convergent common linear combinations of those history functionals. It also holds separately for the two determinant contributions in (11).

Proof. At zero background the transport operator and its inverse are identities on the representation space. A first derivative inserts one generator; a second derivative inserts the symmetrised product. Both terms of (6) therefore have the same internal trace $\text{Re Tr}(T_A T_B)$. The remaining contraction is spatial. Differentiating a closed path gives the same two insertions and proves the length-resolved statement. The hub/rim splitting acts only on spatial indices, so it preserves the factorisation. ■

This extends the inherited representation identity to the complete response evaluated here. Its 72-dimensional cycle $\times$ generator matrix has inverse $\Sigma_p \otimes G_{tr}^{-1}$. Thus, under the quadratic field-action identification, one common spatial mode has sector exchange coefficients proportional to (1/6, 2/13, 2/21). These are primitive-generator coefficients; physical amplitudes also contract the particle currents.

The 47-component determinant in this test is a geometric trace functional. It is not the Standard Model matter determinant with 45 fermions assigned bosonic statistics. The effective running calculation instead uses the separate fermion and Higgs operators, with their correct spin and statistical weights [2–3,7]. The finite census test verifies the representation contraction in (16), not an interchangeable treatment of those field measures.

For the D41 proposal, $\epsilon = 0.08431318638730784$ and the participation counts $n = (36, 26, 47)$ give $r_A = 1 - \epsilon n_A / 49$ [1]. The comparison is therefore precise:

Table 4. Common transport factorisation and the D41 target.

Quantity	Common native transport response	Conditional D41 attachment
Colour coefficient	6k	5.62833371 times a common scale
Weak coefficient	6.5k	6.20920554 times a common scale
Hypercharge coefficient	10.5k	9.65084577 times a common scale
Fractional sector change	Common to all three	6.19%, 4.47%, 8.09% reductions

Here k may be any selected spatial eigenvalue or a common matched spatial contraction of K_ρ . Changing path lengths, ρ , or the hub block changes k but leaves the ratios in the middle column unchanged. The resulting factorisation cannot generate the last column. This does not invalidate the D41 history calculation. It shows that its physical attachment requires a sector-dependent response beyond the common-geometry transport leg evaluated here.

The precise premise is stronger than a shared graph: the unperturbed propagator and the transport vertices used in Proposition 5 carry the identity on the component index, apart from the generator insertions. D41 instead supplies $R = I - \epsilon u u^T$ on the 49-state participation carrier. It is a collective, off-diagonal operator on that carrier, not a common scalar multiplier of every component. Coupling a sector through its equal-weight participation vector v_A gives $v_A^\dagger R v_A = 1 - \epsilon n_A / 49$. This leaves the internal-identity assumption of Proposition 5 and permits the D41 sector dependence. It need not change the spatial graph.

The nontrivial R alone does not select that coupling. The same R contracted with the diagonal participation density gives the common factor $1 - \epsilon / 49$, which leaves the sector ratios unchanged. Thus the question is how the physical source inserts the participation response into propagation and vertices. The charge paper's collective construction is a positive conditional realisation of that insertion, rather than an implication of $R \neq I$ by itself [1, Sections 5.8–5.10].

6.2 Two conditional weak angle benchmarks

The two named attachments have distinct consequences under the frozen matching and transport prescription of [1, Table 4]. The inputs are $\alpha_s(M_Z) = 0.1180$ and $\alpha_{em}(M_Z)^{-1} = 127.95$, with the diagnostic top input $y_t(M_Z) = 0.94$. The matching scale and common multiplier are determined from the two gauge inputs. The two-loop values below are inherited results, not fresh integrations of the finite wheel determinant.

Table 5. Inherited two-loop boundary benchmarks with the frozen comparison.

Attachment	Sector boundary up to a common factor	Two loop weak angle	Difference from 0.23122
Common transport	(6, 13/2, 21/2)	0.22736738	-1.66621%
Collective D41 clamped	(6r _C , 13r _W /2, 21r _V /2)	0.23440586	+1.37785%

The endpoints straddle the frozen comparison value. The quadratic common-transport contribution gives no transverse SGRI contrast; the collective attachment changes that contrast. Their comparison tests a particular

participation mechanism. It does not exhaust the possible coarse responses: Section 8 supplies a distinct nonlinear composition with a nonzero finite contrast. Neither the two endpoints nor that finite composition is certified here as the complete physical response.

Partial or unequal collectivity and nonseparable responses remain structurally possible until the source calculation decides them. The endpoint comparison is therefore not a bound on every allowed interpolation. In particular, the charge paper's independent-sector one-loop corner values lie outside its equal-collectivity endpoints. Any interpolation selected to land on 0.23122 would be a fit. The corresponding coupling and its transport must be returned by the source without using that comparator to choose them.

7 The coupled physical action and force strength test

7.1 Assemble the physical response before inversion

The full physical comparison requires the Hessian of the complete coarse-grained action, reduced on its actual physical support. If a denotes retained gauge coordinates and η denotes unresolved source variables, that response is

$$K_{\text{phys}} = \Gamma_{aa} - \Gamma_{a\eta} \Gamma_{\eta\eta}^{-1} \Gamma_{\eta a},$$

$$\Sigma_{\text{phys}} = P_6 K_{\text{phys}}^{-1} P_6^T. \quad (17)$$

Here P_6 is the prescribed linear map from the full physical gauge quotient to the six matched cycle coordinates. Exact gauge nulls are removed before inversion. At nonzero backgrounds, changing supports and embeddings must be differentiated as required by the source. The evaluated K_p is the contribution of (3); substituting it for the entire K_{phys} in (17) is an additional induced-action hypothesis. Conversely, one cannot add its inverse to the inverse of another stiffness to obtain the full interaction. Actions are assembled before the physical inverse is taken.

The corpus already gives the coupled master action and current-link reduction [6,8]. It identifies the HX1 record-phase coordinate as a tangent of the independently varied record morphism $\text{Pol}(\Phi)$, and derives its link-record Schur response [15, Section 46R]. The unit-branch block $[[3,-1],[-1,2]]$ returns the relaxed link stiffness $5/2$; its general weights are inherited metric pullbacks, not independently fitted sector parameters. The corpus also rules out arbitrary additional gauge potentials once all complete-source inputs are fixed [4, Section 17.67]. The task here is to evaluate that complete source under the specified coarse map. The endpoint transport determinant alone does not contain all of those contributions.

The gauge-relation paper supplies the useful contrast $t = (466, -687, 159)$, orthogonal to both $Z = (6, 13/2, 21/2)$ and the inherited one-loop running vector [9]. For any contribution proportional to Z ,

$$t \cdot \Delta A = 0, \quad A = (g_C^{-2}, g_W^{-2}, g_Y^{-2}). \quad (18)$$

Equation (16) therefore gives no transverse correction to that relation when the same geometric census functional is used in each sector. This is a zero result for the specified contribution, not a calculation of all physical J and T . Correct fermion, Higgs and gauge statistics, unequal propagation, mixed source terms and the physical matching prescription can supply further terms.

The numerical finite electric and magnetic comparison between two chiral operators is already present in CLSN [4, Sections 17.19–17.20]. Physical Applicability states why that relative comparison and the universal logarithm do not determine the complete finite coupled action [10, Sections 38–41]. The graph subtraction (9)

cannot replace that calculation. In particular, no identification of ρ with a physical energy scale, no Standard Model threshold computation and no new weak-mixing-angle transport is performed here. The measured discrepancy is not used to select any coefficient.

The finite native response and its conditional exchange calculation are closed within the specified determinant model. Section 8 additionally composes its full statistical weight; this operation is not included in the quadratic Schur reduction (17). The complete physical construction must identify both the action and the admissible integration over unresolved fields.

7.2 A joint test on cycles and generators

The six spatial cycles and twelve internal generators define 72 retained coordinates a_{iA} . On a fixed physical quotient, the hypothesis that the two responses separate is

$$K_{\text{phys}} = K_{\text{spatial}} \otimes G_{\text{phys}}. \quad (19)$$

This hypothesis is additional to positivity and ordinary rank 72. Proposition 5 proves it for the common-transport determinant. A collective participation Gram can also satisfy (19), so a pass does not decide between the two rows of Table 5. If (19) fails, the participation response depends on the spatial mode and neither a six-mode test nor a twelve-generator test is complete by itself.

Criterion 6 (separability of the physical kernel). Realign the entries of the 72×72 matrix into a 36×144 matrix,

$$\mathcal{R}(K)_{(ij),(AB)} = K_{iA,jB}. \quad (20)$$

A nonzero K is a single Kronecker product if and only if $\text{rank } \mathcal{R}(K) = 1$. For the physical interpretation in (19), verify that its factors are real symmetric and positive on their respective quotients. Fix their reciprocal scale ambiguity by a stated normalisation, for example $\text{Tr } K_{\text{spatial}} = 6$, together with the source's frozen baseline when comparing reductions.

Proof. For (19), $\mathcal{R}(K) = \text{vec}(K_{\text{spatial}})\text{vec}(G_{\text{phys}})^T$, an outer product. Conversely, reshaping the two vectors of a rank-one realignment gives the two factors. Their physical symmetry and positivity are separate checks. ■

For a numerical calculation, whiten the generator coordinates with the frozen conventional-Y baseline G_0 from (15), using $W = I_6 \otimes G_0^{-1/2}$. If s_j are the singular values of $\mathcal{R}(WKW^T)$, report

$$E_{\text{sep}} = [\sum_{j \geq 2} s_j^2 / \sum_{j \geq 1} s_j^2]^{1/2}. \quad (21)$$

Whitening does not change the exact rank condition. It makes the numerical convention explicit and prevents a hypercharge coordinate conversion from silently changing the audit. Also report the residual of the best product in each C , W and Y diagonal block, divided by that block's norm, and each cross-block residual divided by the geometric mean of the two diagonal-block norms. A small global residual must not conceal a sector-specific failure.

The accompanying controls distinguish this test from ordinary matrix rank. They use the computed spatial matrix at $\rho = 0.4$, the baseline or collective Gram, and a positive perturbation confined to the uniform spatial mode and hypercharge. These are algebraic test inputs, not three claimed physical source returns.

Table 6. Realignment controls on cycles and generators.

Algebraic control	Ordinary rank	Realignment rank	E_{sep}
Common transport product	72	1	6.2×10^{-16}
Collective D41 product	72	1	5.9×10^{-16}
Positive mode-dependent hypercharge control	72	2	0.005682

The last control has a hypercharge block residual of 0.01922 despite its smaller global residual. On a genuine source return, (21) is therefore the first structural test. If it passes, verify the required scalar sector blocks of G_{phys} and define $\delta_A = 1 - g_A/Z_A$, where g_A is the sector coefficient under the frozen matched baseline. The charge paper's pattern and magnitude tests then require $\delta_c/36 = \delta_w/26 = \delta_v/47 = \epsilon/49$ for the maximal D41 target. The spatial factor is separately tested against the claimed transport model. If separability fails, the full 72-dimensional kernel and its inverse are retained in the comparison. No physical response is reconstructed by imposing the product form after a failure.

7.3 The volume calculation and its source conditions

A native scalar volume test must retain both space and time: a three-dimensional static determinant does not produce the four-dimensional massless scalar logarithm. The target $b_{H,A} = (1/3) \text{tr}_H(T_A^2) = (0, 1/6, 1/6)$ is already derived from the inherited effective scalar operator in [3, Sections 2–4]. A native execution would strengthen that result's operator provenance. It must include both the contact and two-vertex terms.

CLSN supplies the endpoint covariant incidence and conductance [4, Section 17.65]. HMGBC gives a two-wheel bridge with conductance g , including $g = 1/3$, as a graph-lift demonstration [13, Appendix I]. HSPR and ST supply the Higgs spatial principal symbol and shared temporal coefficient on their stated branch [11–12]. These distinct constructions do not by themselves specify the full periodic scalar operator, its physical quotient and finite-time measure. Replacing them with an unrestricted nearest-neighbour stencil would introduce an additional model.

For star offsets $\{0, \pm e_1, \pm e_2, \pm e_3\}$, the centre lattice $x_1 + 2x_2 + 3x_3 = 0$ modulo 7 partitions the infinite cubic lattice. An ordinary periodic cubic torus requires a side divisible by seven: 63^3 or 70^3 permits this partition; 64^3 does not. The partition fixes the block map, not the inter-cell conductances.

The executable source input is therefore the multi-cell operator, temporal transfer, physical support, retained star map and proper-length matching. Once supplied, a complete determinant Hessian on $L^3 \times N_t$, with the eliminated determinant retained, can test the Ward nulls, volume stability and scalar logarithm at matched transverse momenta. This volume calculation is separate from the finite composition executed below.

8 Nonlinear composition of the native holonomy weight

8.1 The action and the specified coarse variable

Proposition 5 concerns the Hessian of a transport determinant before integrating over unresolved gauge links. It does not assert that taking that Hessian commutes with nonlinear composition of the complete statistical weight. This section calculates the latter operation for the same native determinant. The finite model has three explicit restrictions: only one rim link is variable in each wheel, the other eleven links are fixed to the identity, and two identical holonomy weights are composed independently with normalised Haar measure. The retained variable is their product $U = U_2 U_1$. These restrictions define the calculation; they are not derived here as the physical spatial blocking map.

For a unit-charge phase α on the edge between rim sites 1 and 2, the exact scalar determinant ratio is

$$\begin{aligned} \det H_\rho(e^{i\alpha}) / \det H_\rho(1) &= 1 + h_\rho(1 - \cos \alpha), \\ h_\rho &= (256\rho^4 + 896\rho^3 + 1152\rho^2 + 655\rho + 144) \\ &\quad / [32\rho(1 + \rho)^2(1 + 2\rho)^2(5 + 4\rho)(31 + 28\rho)]. \quad (22) \end{aligned}$$

Only the two entries on that edge carry a phase. Expansion of the determinant therefore gives a constant plus a cosine; evaluating it at $\alpha = 0$ and π fixes (22). With $z = (1 + \rho)^{-1}$, its small- z behaviour is $h_\rho = z^3/56 + O(z^4)$, the native triangle coefficient, but (22) retains every history length.

On the full 47-state census representation, diagonalise the group element U in that representation, with eigenphases α_j . Using this induced action as a statistical weight gives

$$\begin{aligned} S_h(U) &= \sum_{j=1}^{47} \log[1 + h(1 - \cos \alpha_j)], \\ w_h(U) &= \exp[-S_h(U)] / \int \exp[-S_h(V)] dV. \quad (23) \end{aligned}$$

The integration group is $SU(3) \times SU(2) \times U(1)$, using integer charge $q = 6Y$. The census characters descend through the usual central Z_6 quotient, so using that quotient gives the same normalised integrals. This is the geometric census determinant already used in Section 6, with equal component weighting. It is not a replacement for the complete chiral-fermion, Higgs and gauge functional measure with their physical statistics.

The specified integration over the unresolved factor is

$$\begin{aligned} w_h^{(2)}(U) &= \int w_h(V) w_h(V^{-1}U) dV, \\ S_h^{(2)}(U) &= -\log[w_h^{(2)}(U) / w_h^{(2)}(1)]. \quad (24) \end{aligned}$$

This positive convolution retains all nonlinear dependence of (23). It is a static two-factor composition, distinct from a temporal noise law or a seven-star volume refinement. The source supplies the joint multiplication $(U_2, j_2)(U_1, j_1) = (U_2 U_1, j_2 + U_2 j_1)$ [4, Section 17.70]. Projection to U is a homomorphism, so independent joint increments give convolution of their gauge marginals even when U and j are correlated within each increment. Such internal correlation does not invalidate (24). Section 17.71 excludes the stronger prescription of one state-independent joint covariance across every compatible factorisation, not all gauge-marginal convolutions. The restrictions still to identify physically are the marginal weight (23), the retained map and the complete source measure. Sections 8.5–8.6 test response preservation and an explicit connected-cell extension.

8.2 An exact positive coarse response

Proposition 7 (coarse curvature of the complete weight). Let w be a positive, smooth, central weight with $w(U^{-1}) = w(U)$. For a unit generator direction A , the curvature of (24) at the identity is

$$G_A^{(2)} = \int |L_A w(V)|^2 dV / \int w(V)^2 dV \\ = \langle (L_A S)^2 \rangle_{w^2} \geq 0. \quad (25)$$

Here L_A is the left-invariant derivative in the same compact charge coordinate as the fine action. Gauge invariance makes the response scalar within each simple adjoint block and removes cross-sector quadratic terms.

Proof. The first derivative of $w * w$ at the identity vanishes. Haar integration by parts gives its second derivative as $-\int |L_A w|^2$. Dividing by $(w * w)(1)$ in the second derivative of $-\log(w * w)$ gives (25). Equivalently, Cauchy–Schwarz makes the identity a maximum of $w * w$. Adjoint invariance supplies the block form. ■

The response is strictly positive in the census directions because the weight is nonconstant along each factor. Equation (25) is an identity for this effective action, not an identification of an unrelated record Fisher matrix with a physical action Hessian. As a control, replacing the full compact action by a Gaussian on its Lie algebra gives $G^{(2)} = G^{(1)}/2$ and preserves every sector ratio. The distinction therefore comes from the nonlinear compact weight and the integration in (24).

8.3 The census determines the unequal leading response

At small h , the nonconstant part of w_h is $h f(U) + O(h^2)$, where $f = \sum_R m_R \text{Re } \chi_R$. The entries below are inequivalent conjugate pairs of representations of the full product group. Lepton and Higgs doublets belong to the same pair: $SU(2)$ is pseudoreal and their hypercharges have opposite signs. The Higgs doublet is counted with the same unit weight as a Weyl doublet in this geometric census; its physical statistics enter only in the complete functional measure. Their coefficients must therefore be added before the convolution squares them. Taking a real character accounts for the two orientations of a loop; it does not add mirror particles to the census.

Table 7. Census representations and their leading convolution coefficients.

Census contribution	Representation with $q = 6Y$	m_R	d_R	$m_R^2/(2d_R)$
Quark doublets	(3, 2, 1)	3	6	3/4
Conjugate up singlets	(3, 1, -4)	3	3	3/2
Conjugate down singlets	(3, 1, 2)	3	3	3/2
Lepton and Higgs doublets	(1, 2, ± 3), one conjugate pair	4	2	4
Conjugate charged-lepton singlets	(1, 1, 6)	3	1	9/2

Proposition 8 (representation reweighting under two-factor composition). In the model (23)–(24), the fine and coarse sector coefficients in conventional Y normalisation satisfy

$$\begin{aligned} G_A^{(1)} &= h Z_A, \\ G_A^{(2)} &= h^2 \Sigma_R [m_R^2 / (2d_R)] \text{Tr}_R(T_A^2) + O(h^3). \\ Z &= (6, 13/2, 21/2), \quad Z^{(2)} = (9/4, 25/8, 73/8). \end{aligned} \quad (26)$$

Proof. Schur orthogonality gives $\chi_R * \chi_S = \delta_{RS} \chi_R / d_R$. Every listed pair is non-self-conjugate on the full group because its U(1) charge is nonzero, so $\text{Re } \chi_R * \text{Re } \chi_R = \text{Re } \chi_R / (2d_R)$. Cross terms between inequivalent pairs vanish. The order- h^2 nonconstant term of $w * w$ is therefore $\Sigma m_R^2 \text{Re } \chi_R / (2d_R)$; the order- h^2 term of w itself contributes only a constant when convolved with 1. Taking the Hessian of its negative logarithm and assembling the inherited traces gives (26). The fine identity follows directly from (23) at $\alpha = 0$. ■

The factor d_R is the dimension of the full colour $\times$ weak $\times$ hypercharge representation, not a dimension chosen separately in each gauge sector. The calculation therefore uses the shared particle census. No independent colour, weak or hypercharge parameter has been supplied. Normalised to the weak coefficient, the leading coarse pattern is

$$G_C^{(2)} : G_W^{(2)} : G_Y^{(2)} \rightarrow 18/25 : 1 : 73/25 = 0.72 : 1 : 2.92. \quad (27)$$

This differs from the fine pattern $12/13 : 1 : 21/13$. A common change in flux or volume normalisation cannot remove the difference. For the contrast of Section 7.1, its leading value is $(466, -687, 159) \cdot G^{(2)} = (705/2)h^2 + O(h^3)$, whereas its fine value vanishes. This is a finite response contrast, not a calculated physical SGRI correction.

8.4 Full determinant evaluation and physical scope

Weyl integration evaluates (25) on a four-angle torus: two SU(3) angles, one SU(2) angle and the primitive U(1) angle. The Vandermonde and sine-squared factors supply the non-Abelian Haar densities. A second evaluation uses the radial group Laplacian and integration by parts. These independent derivative expressions agree after quadrature refinement; the complete logarithm in (23) is retained.

Table 8. Fine and coarse sector ratios from the full native determinant.

Response	G_C/G_W	G_V/G_W	Quadrature check
Fine determinant, any p	0.923077	1.615385	Exact
Coarse limit $h \rightarrow 0$	0.720000	2.920000	Exact
Coarse full determinant, $p = 1$	0.725804	2.888625	48^4 and 64^4
Coarse full determinant, $p = 0.4$	0.754205	2.731322	64^4 and 80^4

Equation (22) gives $h_p \approx 0.00507265$ at $p = 1$ and 0.02889713 at $p = 0.4$. Relative to the leading-limit ratios, the full-weight values raise G_C/G_W by 0.81% and 4.75%, and lower G_V/G_W by 1.07% and 6.46%, respectively. The small- h theorem is therefore quantitatively relevant to these native diagnostic weights; the comparisons measure the higher-order effects at the two points.

The $h \rightarrow 0$ row is the exact theorem, not an extrapolation of the two finite rows. The finite rows test the nonlinear operation on the all-history determinant. The small- h regime is a broad compact-group weight, not

a claim about weak physical gauge-field fluctuations or a continuum limit. The parameter ρ is still dimensionless and is not an energy or measured coupling.

This closes a finite question: coarse graining complete native holonomy weights can generate unequal sector coefficients from a common fine spatial rule. Proposition 5 survives unchanged, but it cannot be extended from the quadratic determinant to all subsequent nonlinear gauge-field integrations. The source already supplies current-record composition and conditional response-preservation results. The following sections state their scope and evaluate a correlated graph extension. The calculation retains one holonomy with twelve generator directions, not the full six-holonomy, 72-direction physical kernel; it does not assign a physical energy to ρ or return D41 fractions or a weak-angle value.

8.5 What the source already establishes about correlated reduction

The Fold-to-K7 stitching theorem [14, Sections 23F–23G] treats a specified map from fine states to retained addresses. Let L lift retained observables to the fine catalogue, let μ be the stationary fine measure, and let π be its retained marginal. The conditional-mean map is $C = D_\pi^{-1} L^T D_\mu$, with $CL = I$. For a reversible fine law S_f satisfying $S_f L = L T_o$, put $Q_f = D_\mu(I - S_f)$ and $Q_o = D_\pi(I - T_o)$. The inherited theorem gives

$$f = Lg + \eta, \quad C\eta = 0, \quad f^T Q_f f = g^T Q_o g + \eta^T Q_\eta \eta,$$

$$\min_{Cf=g} f^T Q_f f = g^T Q_o g. \quad (28)$$

Reversibility and the intertwining identity make the cross-energy term vanish; positivity gives the minimum at $\eta = 0$. Neither equal internal weights nor independent internal states are required. An independent reconstruction uses unequal conditional weights and correlated reversible fluxes on 14, 21, 28 and 63 states. Direct inverses reproduce all 21 retained pair responses in each case to at most 3.38×10^{-14} , with minimum response 279/20. These constructed refinements test the theorem; the 63-state example is not identified with the driven FCHI carrier.

The nonreversible control in [14, Section 23G] preserves the forward coarse law but not its stationary-adjoint counterpart. Its minimum-cost pair response changes from 4 to 800/191, a 4.712% increase, also reproduced independently. Exact forward grouping alone is therefore insufficient. Equation (28) fixes conditional means; Section 4 fixes boundary values; equation (24) retains a group product. Their physical identification cannot be inferred from a shared matrix size or graph.

The current-record composition in Section 8.1 and the record tangent in Section 7.1 are established source ingredients. The physical comparison concerns their complete retained law. Product Haar measure is an invariant reference measure; the source pushforward supplies its additional density [16, Section 25]. For any smooth positive joint action S_g , the retained-product weight and its curvature are

$$W_g(U) = \int \exp[-S_g(V, V^{-1}U)] dV,$$

$$G_{A,g} = \langle S_{g,AA} \rangle - \langle S_{g,A} \rangle^2 + \langle S_{g,A} \rangle^2. \quad (29)$$

Derivatives act on $U = \exp(itT_A)$ at $t = 0$ and expectations use the joint weight at $U = I$. This follows by differentiating $-\log W_g$. For a correlated action, the score-square identity (25) cannot simply replace (29). Correlation can preserve or change the retained response; the response itself is the test.

8.6 Exact composition on the published two cell bridge

HMGBC gives two canonical wheels connected between rim vertices 1 and 8 by conductance $g = 1/3$, with unit conductances on every wheel edge and self-loop [13, Appendix I]. Keep U_1 on edge $1 \rightarrow 2$ and U_2 on edge $8 \rightarrow 9$,

with all other links fixed to identity. For conductance matrix C_g and degree matrix D_g , the unnormalised native operator is $A_p = (1 + p)D_g - C_g(U)$. Its determinant ratio equals that of the normalised operator because the degree factors are field independent. The source explicitly identifies this as a graph-lift demonstration, not the complete physical lift.

For representation R define $D_R(U) = I + h[I - (R(U) + R(U)^+)/2]$, with $h = h_p$ from (22). The single-wheel bridge-vertex Green coefficient at trivial holonomy is

$$m_p = (1/6)\{(7p + 6)/[p(28p + 31)] + 2/(4p + 2) + 2/(4p + 4) + 1/(4p + 5)\}. \quad (30)$$

Set $a_g = g(1 + p)$, $\xi = gm_p$, $\kappa = 1 + a_g m_p$ and $p = \kappa^2 - \xi^2$. Here p is a positive scalar, distinct from any history-length weighting.

Proposition 9 (exact connected determinant). Up to a field-independent factor, the determinant on the connected graph in representation R is $\det P_R$, where

$$P_R = [D_R(U_1) + a_g m_p I] \cdot [D_R(U_2) + a_g m_p I] - \xi^2 I. \quad (31)$$

Proof. Removing the bridge vertex of a wheel removes its only phased edge. The cofactor is consequently field independent, so its bridge-vertex Green block is $m_p D_R(U)^{-1}$. Eliminating every other vertex leaves the two diagonal blocks $D_R(U_i) + a_g m_p I$ and off-diagonal blocks $-\xi I$. Their block determinant is (31), including for noncommuting U_1 and U_2 . ■

Use $S_g = \sum_R m_R \log[\det P_R(U_1, U_2)/\det P_R(I, I)]$ in (29), with the same census multiplicities as Table 7. At $g = 0$ this returns the independent model. With the bridge diagonal loading retained but its off-diagonal connection removed, it gives an independent control with $h_{\text{eff}} = h/(1 + a_g m_p)$. This separates loading from the actual correlation. In a scalar phase block, $\partial_{\alpha_1} \partial_{\alpha_2} \log P = -h^2 \xi^2 \sin \alpha_1 \sin \alpha_2 / P^2$, which is generically nonzero for the connected graph.

The mass term in A_p is pD_g , so each bridge vertex carries mass loading $p(4 + g)$. The leading two-vertex bridge block has eigenvalues pg and $(p + 2)g$. Both diverge at fixed $p > 0$, freezing the two bridge-vertex fields to zero, a Dirichlet limit. This degree-weighted mass convention is therefore essential to the decaying strong-connection response in Section 8.7. Keeping the mass independent of g and identifying vertices 1 and 8 would instead define a merged-cell model, which is not evaluated here. The diagonal-loading-only control above isolates the loading contribution from the off-diagonal connection.

Proposition 10 (leading bridge correction). Treating h as the expansion variable in (31), with the displayed bridge parameters fixed, the leading coarse sector vector is

$$G_g^{(2)} = h^2[\kappa^2 Z^{(2)} + (\xi^2/2)Z]/p^2 + O(h^3), \quad (32)$$

where Z and $Z^{(2)}$ are defined in (26).

Proof. Write $D_R(U_i) = I + hA_i$. Expansion of $\log \det P_R$ gives a first-order term $(h\kappa/p)\text{Tr}(A_1 + A_2)$, the two individual second-order terms $-h^2 \kappa^2 \text{Tr}(A_1^2 + A_2^2)/(2p^2)$, and the mixed term $-h^2 \xi^2 \text{Tr}(A_1 A_2)/p^2$. The first-order characters convolve as in Proposition 8. For each nonzero-charge census representation, Schur orthogonality

gives $\int \text{Tr}[\text{Re } R(V) \text{ Re } R(V^{-1}U)]dV = \text{Re } \chi_R(U)/2$, where $\text{Re } R = (R + R^+)/2$. The mixed term therefore contributes $\xi^2 Z/(2p^2)$; the individual second-order terms integrate to constants at this order. This proves (32). ■

Propositions 8 and 10 describe the small native weights quantified beside Table 8. In (32), h is the expansion variable with the bridge parameters held fixed; it is not an independently adjustable physical parameter at fixed ρ . The finite results in Table 9 use h_ρ and the complete determinant. All coefficients use conventional Y normalisation.

Table 9. Connected cell response and diagonal loading control.

ρ	Joint weight	G_c/G_w	G_v/G_w
1	Independent wheels	0.725804	2.888625
1	Diagonal loading only	0.725264	2.891556
1	Connected bridge $g = 1/3$	0.725691	2.888812
0.4	Independent wheels	0.754205	2.731322
0.4	Diagonal loading only	0.750350	2.753041
0.4	Connected bridge $g = 1/3$	0.751364	2.746564

For the connected graph, the unnormalised sector vectors are (0.00004994985, 0.00006883072, 0.00019883899) at $\rho = 1$ and (0.00191360977, 0.00254684788, 0.00699508195) at $\rho = 0.4$. Relative to independent wheels, the coefficients decrease by 17.34–17.36% across the three sectors at $\rho = 1$ and by 20.90–21.64% at $\rho = 0.4$. The colour-to-weak and hypercharge-to-weak ratios change by at most 0.5581% across these two examples. These are changes within the finite model, not uncertainties on physical force strengths.

The connected construction therefore preserves the unequal-response mechanism while modifying its exact coefficients. Equation (32) explains the modification through a calculated mixture of two inherited trace patterns. No measured force strength or sector-specific adjustment selects that mixture. The corpus’s current-record action and response-preservation theorem remain the basis for the physical comparison; this graph test does not identify its full nonlinear pushforward with either finite weight. The remaining comparison is between coarse responses, without demanding a force scale at the level of an individual fold.

8.7 Bridge strength bounds and the complete strong connection limit

The source bridge construction is defined for nonnegative conductance g , with $g = 1/3$ its published diagnostic value [13, Appendix I]. The dependence on g can therefore be examined without changing its graph law. Let B_g be the coefficient of h^2 in (32). It has the form $B_g = (\kappa^2/p^2)[Z^{(2)} + \lambda Z]$, where

$$\lambda = \xi^2/(2\kappa^2) = (1/2)\{gm_\rho/[1 + g(1 + \rho)m_\rho]\}^2,$$

$$0 \leq \lambda < 1/[2(1 + \rho)^2] < 1/2. \quad (33)$$

Corollary 10.1 (no restoration of the census ratios at leading order). For every finite $g \geq 0$ and $\rho > 0$, the h^2 coefficient satisfies

$$\begin{aligned} 18/25 &\leq B_{C,g}/B_{W,g} < 14/17, \\ 115/51 &< B_{Y,g}/B_{W,g} \leq 73/25, \\ (466, -687, 159) \cdot B_g &= (705/2)\kappa^2/p^2 > 0. \end{aligned} \quad (34)$$

In particular, these ratios cannot equal the census values $12/13$ and $21/13$.

Proof. The common positive factor cancels in the ratios. Differentiating $(9/4 + 6\lambda)/(25/8 + 13\lambda/2)$ gives a positive numerator $33/8$; differentiating $(73/8 + 21\lambda/2)/(25/8 + 13\lambda/2)$ gives the negative numerator $-53/2$. Substituting $\lambda = 0$ and the strict upper bound $1/2$ gives (34). The contrast follows from $t \cdot Z = 0$ and $t \cdot Z^{(2)} = 705/2$. The endpoint inequalities $14/17 < 12/13$ and $115/51 > 21/13$ exclude restoration of the baseline. ■

Corollary 10.1 bounds the leading coefficient B_g of h^2 over all $g \geq 0$; it does not bound the complete finite- h response. Proposition 11 instead takes $g \rightarrow \infty$ at fixed $\rho > 0$ and fixed native $h = h_\rho$, under the degree-weighted mass convention of Section 8.6.

Proposition 11 (complete native strong connection limit). For fixed $\rho > 0$, write $c = \rho(\rho + 2)$. The full coarse curvature defined by (29) and (31) obeys

$$\begin{aligned} G_g^{(2)} &= h_\rho^2 \{ (1 + \rho)^2 Z^{(2)} + Z/2 \} \\ &/[g^2 m_\rho^2 c^2] + O(g^{-3}). \end{aligned} \quad (35)$$

Thus the magnitude tends to zero as g^{-2} , while the ratios tend to those of $Z^{(2)} + Z/[2(1 + \rho)^2]$, which remain separated from the original census ratios.

Proof. With $D_i = D_R(U_i)$, equation (31) is exactly $g^2 m_\rho^2 c l + g m_\rho (1 + \rho)(D_1 + D_2) + D_1 D_2$. In $\log \det P_R$, the first-order term in $1/g$ is $(1 + \rho)\text{Tr}(D_1 + D_2)/(g m_\rho c)$. At second order the mixed term is $-\text{Tr}(D_1 D_2)/(g^2 m_\rho^2 c^2)$. The remaining terms depend on only one factor and integrate to constants at this order. Character convolution of the first-order terms supplies $(1 + \rho)^2 Z^{(2)}$; the mixed term supplies $Z/2$, as in Proposition 10. Since the group is compact and ρ is fixed and positive, the expansion and its two generator derivatives are uniform, so Haar integration preserves the stated remainder. This proves (35) without a small- h assumption. ■

Table 10 evaluates the complete determinant at intermediate conductances and gives the exact limiting ratios. The $g = 3$ results are included in the archive. The finite calculations use 32^4 and 48^4 grids, agreeing in each displayed ratio to below 10^{-8} . Evaluations at $g = 1000, 2000$ and 4000 , with the first two inverse- g corrections removed, reproduce the coefficient of g^{-2} in (35) to a relative error below 4.1×10^{-7} .

Table 10. Complete native response as the bridge is strengthened.

ρ	Bridge conductance g	G_c/G_w	G_v/G_w
1	1	0.727169	2.878504
1	10	0.747211	2.746741
1	100	0.759925	2.663722
1	∞ limit	$16/21 \approx 0.761905$	$167/63 \approx 2.650794$
0.4	1	0.750965	2.744682
0.4	10	0.774939	2.575536
0.4	100	0.788476	2.481390
0.4	∞ limit	$494/625 = 0.790400$	$4627/1875 \approx 2.467733$

Although Proposition 11 depends on ρ , its colour-to-weak limits at $\rho = 1$ and 0.4 are $16/21 \approx 0.761905$ and $494/625 = 0.790400$, both strictly inside $(18/25, 14/17)$. This supplies an explicit consistency check between the fixed- h strong-connection result and Corollary 10.1. The corresponding hypercharge-to-weak limits also lie inside the band in (34).

The finite scan is numerical evidence between the proved limits, not a theorem covering every intermediate full-weight response. The less-than-0.6% change in Section 8.6 refers specifically to $g = 1/3$; it is not asserted throughout this family. Taken together, (34) and (35) establish that the unequal response is not confined to a single bridge value: it persists throughout the leading conductance family and in the complete strong-connection limit. These are finite graph statements, without assigning g or ρ a measured force strength or a physical energy scale.

9 Numerical verification

The principal numerical script passes 96 checks. An independent symbolic script verifies the four rational spectra, their common pole and the three vanishing eliminated-mode contributions, using exact arithmetic. The checks use no measured gauge coupling, mass or mixing angle.

The separate structural script passes 80 checks: exact fixed-length coefficients against an independent recurrence through $N = 64$, the all-length exclusion bound, collective and diagonal carrier contractions, the three realignment controls, endpoint percentage arithmetic and the periodic star constraint. Its weak-angle values are inherited benchmarks, explicitly distinguished from newly executed transport. Those percentages use the frozen comparison value; it does not enter the native response or the factorisation controls.

The nonlinear script passes 36 checks, including the exact four-length mixture, the full one-rim determinant identity, the census convolution coefficients and the Gaussian control. At $\rho = 1$, the 48^4 and 64^4 quadratures agree in $G^{(2)}/h^2$ to 2.3×10^{-12} ; at $\rho = 0.4$, the 64^4 and 80^4 values agree to 6×10^{-15} . The squared-gradient and radial-Hessian expressions provide independent derivative checks. A 24^4 triangle-only control exhibits hypercharge aliasing and is not used for the reported full-determinant values.

The correlated-source scripts pass 77 and 4 checks. They reconstruct the inherited conditional-mean theorem and its nonreversible control, compare (31) with full $14d \times 14d$ determinants for $d = 1, 2, 3$ and 6, and check all fundamental SU(2) and SU(3) generator derivatives by finite differences. At $p = 0.4$, the 64^4 and 80^4 connected and loading-only quadratures agree in G/h^2 to below 7.7×10^{-14} . An independent small- h extrapolation reproduces (32) to below 5×10^{-8} . These 81 checks are distinct from the preceding scripts. The bridge-family script adds 40 checks covering exact rational bounds, noncommuting full-matrix determinants at stronger conductances, intermediate complete-weight quadratures and the full native strong-connection limit. It records separately the exact leading-order statements and the numerical finite-parameter results. The six counted numerical suites report 333 checks in total; the independent symbolic verification is additional.

Table 11. Independent finite-model verification.

Verification	Evaluated result
Path derivative sum against action Hessian	Agreement of independent constructions
Positive response and gauge quotient	Six positive cycle modes and six gradient nulls
Noncommuting SU(2) background	Maximum link commutator norm 1.37149
Complete non-Abelian action difference	0.05518184775
Eliminated plus retained action differences	$0.00016374481 + 0.05501810295$
Full census determinant	329×329 matrix agrees with independent spectral sum
Source edge subdivision	Reduced operator and background determinant preserved
Gaussian current integration	Six inverse coefficients reproduce the quartic term
Correlated conditional averaging	All 84 comparisons, comprising 21 pairs in each of the four refinements, agree within 3.38×10^{-14}
Connected two-wheel composition	Full matrices and reduced determinant agree
Leading bridge correction	Independent quadrature confirms both trace contributions
Bridge conductance family	Exact leading bounds and complete strong-connection limit

The source response is checked through six values of p from 0.001 to 10. Scalar finite differences provide a separate check of the contact-plus-bubble expression. A recurrence differentiates every path power through length 200; the weighted sum agrees with the determinant Hessian at $p = 0.4$. Fixed-length moments through $N = 49$ independently resolve the long-history limit.

The non-Abelian fixture uses independent SU(2) matrices on all twelve edges. Its determinant, hub Schur complement and serial refinement are evaluated without assuming commuting links. The 329-dimensional census fixture is a mixed-generator directional probe; it is distinguished from the noncommuting edge fixture. The representation matrices are constructed explicitly from the three-generation census and Higgs doublet.

Numerical residuals are implementation checks for the stated finite matrices. They are not uncertainties on a physical force prediction. All tolerances, seeds, formulas and results are included in the accompanying calculation archive.

10 Consequence for the Standard Model programme

The quadratic common-transport calculation cannot alter the sector ratios, even with every history length and the eliminated determinant retained. Nonlinear composition of the complete weight can alter them, and the unequal pattern survives the published connected two-wheel construction. This establishes a finite mechanism for unequal response during coarse graining of the inherited census and native action. The corpus already supplies current-record composition, a master-action record tangent and a response-preservation theorem under explicit grouping conditions. The physical task is to evaluate the complete retained measure and compare its response with the finite formulas, including the correct statistics and scale matching. Correlation alone is not an obstruction, and no primitive fold-level force strength is required.

Table 12. Derivation ledger and remaining physical obligations.

Item	Before this calculation	Established here	Remaining obligation
Native cycle kernel	Determinant and moment construction	Exact positive spectrum and finite remainder	Identify its share in the full physical action
Eliminated modes	Exact determinant identity	Mode-resolved retained and eliminated response	Carry the actual physical block through reduction
Operational witness	Alternative six-mode candidate	Excluded in two families; attainable by a positive four-length mixture	Derive the history weighting from the source
Quadratic common transport	Possible source of relative corrections	Ratios unchanged under Proposition 5	Include the physical nonlinear composition
Full holonomy composition	Unequal response not evaluated	Exact leading Gram and full native finite-parameter checks	Match the complete physical retained measure
Correlated source reduction	Conditional preservation theorem in the corpus	Independent reconstruction and its nonreversible boundary	Verify its hypotheses for the physical retained map
Connected cell composition	Published bridge graph	Exact determinant, leading bounds and strong-connection limit	Identify the full physical joint action and scale matching
Physical kernel	Separate cycle and generator tests	Joint 72-channel realignment criterion and tested controls	Compute the same-source kernel and apply both tests
Weak-angle comparison	Two conditional boundary calculations	Their premise and opposite-sign discrepancies are explicit	Select the coupling from the source, then transport it
Scalar volume response	Effective scalar coefficient established	Source and spacetime conditions for the native run identified	Return the physical multi-cell operator and execute the volume test

The common and D41 weak-angle benchmarks remain conditional comparisons; Section 8 supplies a further calculated response rather than a third weak-angle prediction. Its advance is an explicit unequal sector weighting produced by coarse graining of the inherited census, together with a connected-cell calculation that quantifies how the source geometry modifies it. The bridge adds a definite contribution from the original charge Gram. That contribution cannot restore the census ratios in the leading conductance family, and the

complete native model retains unequal ratios in its strong-connection limit. The particle-state content, native transport and nonlinear composition thereby constrain the response beyond a single tested connection.

Appendix A Exact spectral calculation

The symbolic derivation avoids square roots of the stationary measure. Trace invariance permits replacing the symmetric S by T . For an integer cycle profile w , let $D_{ij} = T_{ij}w_{ij}$, with signed w_{ij} , and $E_{ij} = T_{ij}w_{ij}^2$. The derivatives of the characteristic transfer are $-iD$ and $-E$. Thus, with $R_T = [(1+\rho)I - T]^{-1}$,

$$k(w) = \text{Tr}[R_T E + R_T D R_T D] / (w^T w). \quad (A1)$$

Take $w = Ct$ for the four triangle-coordinate vectors

$$\begin{aligned} t_1 &= (1,1,1,1,1), \quad t_2 = (2,1,-1,-2,-1,1), \\ t_4 &= (2,-1,-1,2,-1,-1), \quad t_5 = (1,-1,1,-1,1,-1). \end{aligned} \quad (A2)$$

They lie in the four distinct eigenspaces of H_Δ , and division by $w^T w$ gives the orthonormal-cycle coefficient. The dihedral doublets supply the repeated eigenvalues. For the eliminated determinant, replace R_T by $[(1+\rho)I_6 - T_{rr}]^{-1}$, where T_{rr} is the rim principal block, and restrict D and E to those vertices. This yields $e(\rho)$ for t_1 and zero for the other three vectors. These exact expressions prove (7), (9) and Proposition 3.

For completeness, a single rational expression for the first eigenvalue is

$$k_1 = (128\rho^4 + 512\rho^3 + 768\rho^2 + 541\rho + 160) / [16\rho(1+\rho)^2(1+2\rho)^2(5+4\rho)(31+28\rho)]. \quad (A3)$$

For the all-length exclusion, write $q = (1+\rho)^{-1}$ and let $a_{e,N}$ be the coefficient of q^N in $k_e(\rho(q))$. Equation (5) gives $a_{e,N} = \text{Tr}(T^N)(V^T C_N V)_{ee}$. Consequently inverse mode ratios are ratios of these coefficients in the opposite order. Partial fractions of (7), for $N \geq 2$, give

$$\begin{aligned} a_{2,N} &= 2/31 - (4/17)2^{-N} - (784/1581)(-3/28)^N, \\ a_{4,N} &= 2/31 - (1568/279)(-3/28)^N, \\ a_{5,N} &= 2/31 - 2(-1/4)^N + (490/93)(-3/28)^N. \end{aligned} \quad (A4)$$

At $N = 3$, $a_{4,N}/a_{2,N} = 2$. For every $N \geq 4$, positivity of the denominator and the decreasing absolute powers give the exact bound

$$\begin{aligned} a_{4,N}/a_{2,N} &\leq [2/31 + (1568/279)(3/28)^4] \\ &/ [2/31 - (4/17)/16 - (784/1581)(3/28)^4] = 122/93 < 76/51. \end{aligned} \quad (A5)$$

Thus the first required operational ratio already fails at every fixed length. The alternating terms in (A4) also explain the $N = 4$ ordering reversal without a monotonicity assumption.

For the mixture counterexample, the remaining coefficient, valid for $N \geq 3$, is

$$\begin{aligned} a_{1,N} &= 2/31 + (1/3)(-1/4)^N + [2/3 - (N+1)/4]2^{-N} - (49/279)(-3/28)^N. \\ (p_3, p_4, p_5, p_{12}) &= (786165682946537, 16190660285624, \\ &7129956964048, 443863376330752) / 1253349676526961. \end{aligned} \quad (A6)$$

All four coefficients are positive and sum to one. Direct rational substitution gives $\sum p_N a_{e,N} = [13623816175830/1253349676526961] (19/6, 17/4, 19/3, 103/14)_e$. This proves Corollary 4.2. The script

independently verifies the uniform-mode generating function, so the counterexample does not rely on rounded mode eigenvalues.

Appendix B Reproducibility

The calculation archive contains `run_response.py`, `derive_spectrum.py`, `structural_tests.py` and `nonlinear_blocking.py` in `calculations`, together with `correlated_source_test.py`, `verify_bridge_limits.py` and `bridge_family.py` in `correlated_source/calculations`. Their JSON results, the manuscript source, document builder and source manifest are included. The numerical implementation needs Python, NumPy and SciPy; the independent exact calculation additionally needs SymPy. The scripts report their own SHA-256 hashes. The archive manifest records hashes of every packaged file and of the controlling source documents used for this calculation.

Run the four scripts in `calculations` in the order listed above; they write `response_results.json`, `symbolic_results.json`, `structural_results.json` and `nonlinear_results.json`. Then run `correlated_source/calculations/correlated_source_test.py` followed by `correlated_source/calculations/verify_bridge_limits.py`. They write `correlated_source_results.json` and `bridge_limits_results.json`. Run `correlated_source/calculations/bridge_family.py` to produce `bridge_family_results.json` for Section 8.7. Their independent-wheel baseline is bundled under `correlated_source/paper/calculations`. Each script checks its assertions before returning success. The random seed is fixed at 20261024. The source definitions and all numerical inputs are included; no unarchived fitting step is required.

The finite model uses the canonical wheel law, native conductance, conventional hypercharge and the inherited representation census. Its diagnostic p values are displayed rather than inferred from observations. The native response scripts contain no measured force-strength comparator. The structural script records the inherited weak-angle benchmarks and computes their percentage differences from the frozen comparator, without fitting any response coefficient or performing a new RG integration. The nonlinear script includes the exact positive-mixture identity, the representation reweighting theorem and complete native one-rim holonomy convolution. It reports quadrature grids, independent derivative checks and a Gaussian control. Its triangle-only calculation is an additional asymptotic check, separately labelled in the JSON; the paper's finite rows use the full native determinant. The four mixture weights are constructed from the operational witness and are explicitly not source predictions. The correlated-source archive supplies the conditional-mean reconstruction, exact connected determinant, loading-only control, complete-weight Hessian, trace-mixture proof and bridge-family analysis. Its scripts and results are included without changing their verified arithmetic. The corpus inputs in [14–16] are identified by URL and file hash. Reproduction distinguishes inherited source theorems, independent controls and new finite calculations; it does not equate a graph demonstration with the complete physical pushforward.

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