

Coarse Graining and Chiral Fermion Survival in VERSF

Exact particle blocking, source generated interactions and the fermion contribution to the gauge response

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Claim boundary. Exact integration of the inherited VERSF fermion candidate preserves the chiral relation, zero modes, index, gauge covariance and leading particle equation whenever the eliminated block is invertible. The paper proves a free invertibility bound, verifies strong static backgrounds and seven-site blocking, and carries the inherited fluctuation law into a retained interaction. The determinant response has exact census proportions 6 : 6 : 10 and a finite numerical running-coefficient estimate, with the eliminated determinant retained. Section 8.5 executes the composition-selected free temporal coefficient and its specified symmetric static lift. The fermion carrier and kinetic seed are inherited; the overlap completion and one-mode interaction attachment remain specified constructions. These results do not issue measured force strengths, masses or a generation count.

General Reader Summary

A theory built from discrete events has to explain why ordinary particles appear when we look at it from far away. It is not enough to write down a familiar particle equation at the smallest scale. One has to show what survives when the details an observer cannot resolve are added up and set aside. This paper does that calculation for the particle candidate that earlier VERSF papers constructed.

The outcome is positive within that construction. The surviving particle keeps its handedness, no unwanted extra copy of it appears, and grouping the small cells in stages gives the same answer as grouping them all at once. The calculation works because the removed variables are allowed to leave their mark before they disappear from the description. Simply averaging their equations gives a different answer and spoils the handedness relation, and the paper shows why.

Two further things are established. First, the small jitter that VERSF attributes to the underlying substrate is carried into the particle calculation, and it makes the particles interact with each other; that interaction survives the coarse graining, whichever order the two operations are done in. Second, the particles' own contribution to the strength of the forces is worked out. It comes out in the proportions 6 : 6 : 10 for the three forces, exactly the proportions of the particle census, and those proportions are preserved when the fine detail is removed, provided the contribution of the removed variables is kept. On the small cell used for that test, leaving it out changes the answer by about two thirds.

Larger calculations show why the scale matters. At fixed probe wavelengths, increasing the spatial box moved the rate at which the particles' contribution grows with distance from 0.53 to 0.90, against an expected value of 1. On a still larger box, three wavelengths give a fitted growth coefficient of 0.9671; the same fit gives 0.9742 for the standard comparison operator. This is a numerical estimate under the stated correction model. The removed detail is also evaluated using the seven-site geometry on a three-dimensional volume. The rule for how the particle steps forward in time, which earlier had to be chosen, follows from requiring that two small steps compose into one large step, once the paper's stated connection to the particle equation is made. Putting that rule into the equation preserves the large-scale particle behaviour; its tested extension also preserves handedness on fixed backgrounds.

Abstract

We integrate the fine fermion variables of the published VERSF transfer-derived chiral family, retaining their determinant and the gauge dependence of the block map. The resulting Schur operator preserves

the Ginsparg–Wilson relation, the zero modes and the index whenever its eliminated block is invertible, and we characterise that invertibility exactly: a block map can only fail by erasing a fine zero mode, which recovers the established index obstruction as a capacity bound. For two-site spatial blocking of the free family we prove that the eliminated block is uniformly accretive on the whole momentum torus, with exact corner values. Free blocking retains one Weyl pole per supplied chiral representation, direct and successive blocking agree, and the leading finite-resolution correction is shown to come entirely from the fine operator, the blocking entering only at higher order. Finite non-Abelian executions on strong structured backgrounds and on a Haar-random configuration verify covariance, propagator matching and determinant factorisation to machine precision, and a seven-site star cell that tiles the cubic lattice with index seven provides a seven-site spatial realisation compatible with $K = 7$. The inherited source fluctuation law is carried through the blocking by an exact joint-integration identity, giving a determinant-weighted retained kernel, an explicit relaxed interaction vertex and a nonzero retained quartic term in a 128-component execution.

The fermion determinant’s gauge response is resolved by sector on the inherited three-generation census, with exact trace proportions $6 : 6 : 10$. At two fixed probe wavelengths, on boxes of 32 and 64 sites per side, the logarithmic slope rises from 0.53 to 0.90 against the continuum value 1. A three-momentum fit on a box of 128 sites per side gives logarithmic coefficients 0.967 for the inherited operator and 0.974 for the Wilson control after including a quadratic momentum correction. These are finite-model coefficient estimates; three points do not independently test the assumed correction law. The seven-site split is executed on a compatible three-dimensional torus, quantifying the eliminated contribution; in the tested splits, omitting that contribution reverses the total response’s sign. Exact temporal composition fixes the free spectral coefficient once its placement in the seed is adopted; the specified symmetric static extension preserves the chiral and determinant identities in all four fixtures. The overlap completion and the one-mode interaction attachment remain specified constructions. The census also satisfies the local hypercharge anomaly traces. No measured force strength is fitted.

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1 Existing results and the question addressed

The starting point is the later corpus rather than the earlier statement that a fermion hopping operator was wholly missing. MFKR-1 reconstructs the shared-link kinetic operator, derives the normalised infrared spatial symbol at its stated refinement-geometry grade, and distinguishes that result from the remaining microscopic edge manifest and sequential-transfer identification [1]. CWGW-1 supplies the Wilson sign window, finite modified chirality, a local gauge-background gap and the correctly normalised free Hamiltonian-transfer pole [2]. CLSN-1 goes further: Sections 17.17 and 17.18 give an explicit

transfer-derived chiral family, while Sections 17.33 to 17.49 establish free and fixed-static positivity results with their distinct scopes [3].

Reproducing the free zero count or the infrared Dirac equation would therefore not close a new question. We ask whether an actual integration over fine fermion variables preserves those results, and whether its output retains the quantum weight of the variables that were removed. The calculation is spatial blocking of a quantised candidate, not a derivation of quantisation from binary events. The CAR/Fock reconstruction and its physical qualifications remain inherited [4].

The fine operator combines a VERSF-derived starting point with a chosen completion. CLSN-1 derives the hyperbolic seed from sequential transfer, then adds temporal Wilson stiffness and applies the overlap construction [3, Section 17.17, pp. 24–25]. We derive its coarse operator by integration. A physical derivation need not select one microscopic formula uniquely if the primitive rules instead imply a class with the same coarse behaviour; the present calculation does not establish that broader implication.

The Ginsparg–Wilson relation originated in block-spin reasoning [5], and preservation of zero modes and index under admissible fermion blocking is established, together with the obstruction that arises when the index exceeds the coarse capacity [6]. Those theorems are not claimed as new mathematics. The additions here are the explicit blocking formula and execution for the published hyperbolic transfer family, with its field-dependent block transport, its composition test, its quantum-weight accounting, and the census-resolved gauge response of the retained determinant.

2 The inherited fermion family

We use the canonical dimensionless branch $r = \rho = 1$ of CLSN-1 Section 17.17. These are inherited regulator parameters, not newly derived physical constants. Let $\gamma_1, \dots, \gamma_4$ be Hermitian Euclidean Clifford matrices and $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. The spatial covariant shifts T_i use the same representation link as the fermion action. Define

$$S = \frac{1}{2} \sum_i \gamma_i (T_i - T_i^\dagger) + \frac{1}{2} \sum_i (2I - T_i - T_i^\dagger), \quad K = \gamma_4 S, \quad (1)$$

$$F_\lambda(K) = (1 - \lambda)K + \lambda \sinh K, \quad 0 \leq \lambda \leq 1. \quad (2)$$

With sequential shift T_4 , the seed and the completed operator are

$$X_\lambda = \gamma_4 F_\lambda(K) + \frac{1}{2} \gamma_4 (T_4 - T_4^\dagger) + \frac{1}{2} (2I - T_4 - T_4^\dagger) - I, \quad H_\lambda = \gamma_5 X_\lambda, \quad D_\lambda = I + \gamma_5 \text{sign } H_\lambda. \quad (3)$$

The endpoint $\lambda = 0$ is the Wilson-overlap reference and $\lambda = 1$ the hyperbolic transfer-derived candidate. The hyperbolic seed follows from sequential transfer in CLSN-1. Equation (3) retains the unit temporal Wilson coefficient as the baseline for the tables and running fit in Sections 5–7. Section 8.5 executes the composition-selected free coefficient and a specified symmetric static-background extension; it does not silently substitute that extension into the baseline gauge Hessians. The placement of the composition precision in the fermion seed is explicit, and the overlap functional calculus remains a chosen completion [1, p. 69; 3, Section 17.17]. Where H_λ is gapped, the inherited relations are

$$D^\dagger = \gamma_5 D \gamma_5, \quad D \gamma_5 + \gamma_5 D = D \gamma_5 D, \quad D + D^\dagger = D^\dagger D. \quad (4)$$

The last identity gives nonnegative real part, which controls elimination below. On the free branch write $W = \sum_i (1 - \cos q_i)$, $E^2 = \sum_i \sin^2 q_i + W^2$ and $f_\lambda = 1 + \lambda(\sinh E/E - 1)$, with its continuous value at $E = 0$. At sequential frequency θ ,

$$a = f_\lambda W - \cos \theta, \quad b = (f_\lambda \sin q_1, f_\lambda \sin q_2, f_\lambda \sin q_3, \sin \theta), \\ R = (a^2 + |b|^2)^{1/2}, \quad D_\lambda(q, \theta) = I + (aI + i\gamma \cdot b)/R. \quad (5)$$

CLSN-1 proves the seed gap and the single free zero at $q = \theta = 0$ for this family. The four-component zero is a Dirac carrier; its supplied chiral restriction has a two-component Weyl pole. It is not four particle species, and this construction does not derive the Standard Model representation census.

3 Integrating out unresolved variables

Let Q map fine fermions to the variables retained by a coarse observer, with $QQ^\dagger = I$. Let V be an orthonormal basis of $\ker Q$, so $[Q^\dagger V]$ is unitary. We require $Q\gamma_5 = \gamma_5^c Q$, which holds for a spin-independent spatial average whose parallel transports act on the gauge representation. In those coordinates

$$D = \begin{bmatrix} A & B \\ C & F \end{bmatrix}, \quad A = QDQ^\dagger, \quad F = V^\dagger DV. \quad (6)$$

Integrating the unresolved Grassmann variables gives, when F is invertible,

$$D^c = A - BF^{-1}C, \quad \det D = \det F \cdot \det D^c. \quad (7)$$

This is the standard finite Gaussian identity, applied without truncating the generated action. For an invertible full D it is equivalently

$$(D^c)^{-1} = QD^{-1}Q^\dagger. \quad (8)$$

The covariance is averaged and then inverted. Averaging D itself gives A , which omits the feedback through unresolved variables. The determinant $\det F$ is part of the exact quantum weight: if it depends on gauge links, deleting it changes the gauge dynamics even when selected fermion two-point functions are reproduced.

Lemma 1 (when elimination exists). For a finite operator satisfying (4), F is invertible if and only if $\ker D \cap \ker Q = \{0\}$.

Proof. If $Fv = 0$, the real part of $v^\dagger Fv$ is $\frac{1}{2}\|DVv\|^2$ by (4), so $DVv = 0$ while $QVv = 0$. Conversely a nonzero fine zero mode erased by Q lies in the eliminated subspace and makes F singular. A quantitative version is $\text{Re } F \geq \sigma_{\min}(DV)^2 I/2$; no claim that this bound is uniformly positive on all physical backgrounds is needed or made. ■

The lemma prevents a misleading solution to the doubling problem. A map that hides an unwanted exact zero does not integrate it out as a massive fluctuation: it makes the eliminated block singular. If a regulator has additional light states, they must be accounted for in the complete action and its determinant.

4 Handedness and light modes

Proposition 2 (chiral relation after exact blocking). Under (4), $QQ^\dagger = I$, $Q\gamma_5 = \gamma_5^c Q$ and invertibility of F , the Schur operator satisfies $D^{c\dagger} = \gamma_5^c D^c \gamma_5^c$ and $D^c \gamma_5^c + \gamma_5^c D^c = D^c \gamma_5^c D^c$.

Proof. In the retained/eliminated decomposition, chirality is block diagonal. The four block equations in $D\gamma_5 + \gamma_5 D = D\gamma_5 D$ substitute directly into the Schur complement and give the stated relation. When D is invertible there is a shorter check: (4) implies $D^{-1}\gamma_5 + \gamma_5 D^{-1} = \gamma_5$; compress with Q , use (8), $QQ^\dagger = I$ and $\gamma_5 Q^\dagger = Q^\dagger \gamma_5^c$, and multiply by D^c on both sides. The block-equation proof also covers the massless point without assigning an inverse to its zero modes. Hermiticity follows by taking the adjoint of (7). This is the coisometric hard-block form of established chiral blocking [5, 6]. ■

The coefficient in the chiral identity is inherited from the fine field normalisation. Re-expressing the coarse operator in new lattice units rescales that coefficient as well; one must not reset it by hand to a preferred coarse-cutoff value and then treat the altered operator as the exact integral.

Proposition 3 (zero modes and their handedness). When F is invertible, Q is a bijection from $\ker D$ to $\ker D^c$. Its inverse on zero modes is $\chi \mapsto Q^\dagger \chi - VF^{-1}C\chi$. It preserves chirality and therefore the index.

Proof. Solve the eliminated row of $D\psi = 0$ for the eliminated component and substitute in the retained row. If $Q\psi = 0$, invertibility of F forces $\psi = 0$; surjectivity follows from the displayed lift. On a fine zero mode, γ_5 preserves the kernel by (4), and Q intertwines it with γ_5^c , so the dimensions of the positive and

negative chiral kernels agree separately. With $v = n_+ - n_-$, $v^c = v$. The usual finite index trace is $v = \text{Tr}[\gamma_5(I - D/2)]$. ■

Corollary 3.1 (coarse chiral capacity). Proposition 3 requires each fine chiral zero-mode space to fit inside its coarse chiral space. For N coarse sites, four spin components and gauge-representation dimension r , each chirality has capacity $2rN$; hence $|v| > 2rN$ forces a fine zero mode into $\ker Q$ and makes F singular by Lemma 1. This recovers the obstruction of Fujiwara, Hayashi, Suzuki and Wu [6] directly from the Schur construction.

These results preserve existing chiral content. They do not turn a vectorlike spectrum into the Standard Model by relabelling it, and a flat-space index of zero does not mean that the theory has no Weyl particles: the index counts a difference of chiral zero modes in a background, not the number of supplied matter representations.

5 Explicit spatial coarse graining

We execute the blocking on the regular reference carrier already used for the canonical fermion family. This is not an assertion that a cubic block is the unique microscopic $K = 7$ geometry; Section 5.2 supplies a seven-site alternative. Time remains the sequential coordinate and is not included in the spatial blocks.

5.1 Cubic cells

For a cell of b^3 sites, the free block variable is the sum of the fine fields divided by $\sqrt{b^3}$. At coarse spatial momentum p , the fine aliases and their weights are

$$q_m = (p + 2\pi m)/b, \quad m_i = 0, \dots, b-1, \quad z_m = \prod_i [b^{-1} \sum_{j=0}^{b-1} \exp(iq_{m,j})], \quad \sum_m |z_m|^2 = 1, \quad (9)$$

$$(D^c_\lambda(p, \theta))^{-1} = \sum_m |z_m|^2 D_\lambda(q_m, \theta)^{-1}. \quad (10)$$

Equation (10) is evaluated away from the massless point; (7) defines that point regularly. At $p = \theta = 0$ the average retains the constant alias and is orthogonal to every other alias. For $b = 2$ all seven spatial corner aliases have $D = 2I$, so the eliminated 28-dimensional spinor block is exactly $2I$ while the retained four-dimensional block is zero. For arbitrary fixed finite b the other aliases are nonzero and Lemma 1 applies.

Proposition 4 (the coarse free spectrum). For every fixed finite integer b and every λ in $[0, 1]$, the exact free spatial block has one zero, at $p = \theta = 0$, and its supplied Weyl restriction has one light Weyl pole per representation. The coarse operator is exponentially local for that fixed block size.

Proof. Away from the origin the fine alias operator has no kernel, so the eliminated block is invertible by Lemma 1. At the origin its only kernel is the constant alias, which Q retains faithfully, so Proposition 3 gives exactly the original zero count. The Weyl intertwining follows from Proposition 2; away from the origin its equal-dimensional chiral restriction is invertible. For locality, use a fixed cell basis, in which Q and V are local and the folded fine operator is analytic in a complex momentum strip. Its eliminated block is invertible on the compact real momentum torus, uniformly in $\lambda \in [0, 1]$; continuity and analyticity give a smaller uniform strip in which F^{-1} and hence D^c are analytic, so their Fourier coefficients decay exponentially. This is a fixed-block-size statement, not a volume-independent locality proof on every dynamical gauge background. ■

Lemma 5 (uniform free accretivity for $b = 2$). For every coarse spatial momentum p , sequential frequency θ and $\lambda \in [0, 1]$, the eliminated free block satisfies $\text{Re } F \geq I$, so $\sigma_{\min}(F) \geq 1$ throughout the free torus. The bound is attained at $\lambda = 0$.

Proof. In the alias basis $\text{Re } D$ is diagonal in the eight aliases and scalar in spin, with entries $1 + t_m$, $t_m = (f_m W_m - c)/R_m$, $c = \cos \theta$. Choose the low alias q_0 in $[-\pi/2, \pi/2]^3$, set $W_0 = \sum_i (1 - \cos q_{0,i})$ and $w_0 = |z_0|^2 = \prod_i (1 + \cos q_{0,i})/2$. If $c \leq 0$ or $W_0 \geq c$, every t_m is nonnegative because $W_m \geq W_0$ and $f_m \geq 1$. Otherwise $W_0 < c \leq 1$ and only the low alias can have negative t_m . Take $\lambda = 0$ first. All aliases share the same squared sine

sum, and every other alias satisfies $W_m \geq 2 - W_0 \geq 2c - W_0$, so $W_m - c \geq c - W_0 = |a_0|$; since $a/\sqrt{a^2 + B^2}$ is increasing in a , $t_m \geq |t_0|$ for $m \neq 0$. Moreover $w_0 \geq 1 - W_0/2 \geq 1/2$. For a unit vector v orthogonal to the block row z , $|v_0|^2 \leq 1 - w_0$, and therefore $\sum_m t_m |v_m|^2 \geq |t_0|(1 - 2|v_0|^2) \geq |t_0|(2w_0 - 1) \geq 0$. Compression to $\ker Q$ gives $\text{Re } F \geq I$. For $c \geq 0$, increasing the mode factor f from 1 cannot decrease any t_m : the derivative of t with respect to f has numerator $W \sin^2 \theta + fc \sum_i \sin^2 q_i \geq 0$. Every $t_m(\lambda)$ therefore dominates $t_m(0)$ and the $\lambda \geq 0$ family inherits the bound. ■

At $p = (\pi, 0, 0)$, $\theta = 0$ the two low aliases have equal weight and their difference lies in $\ker Q$. Direct evaluation gives the exact corner minimum

$$m_{\text{corner}}(\lambda) = 1 + (f - 1)/\sqrt{(f - 1)^2 + f^2}, \quad f = 1 + \lambda[\sinh(\sqrt{2})/\sqrt{2} - 1], \quad (11)$$

with values 1.000000, 1.153665 and 1.259915 at $\lambda = 0, 0.5$ and 1. A global search on a 12^4 grid followed by local minimisation returns these corners to below 2×10^{-12} . The theorem is $\text{Re } F \geq I$; equality at $\lambda = 0$ makes 1 the global infimum there. For $\lambda = 0.5$ and 1 the corner values are exact at the corner and numerically supported global-infimum candidates, not an additional proved sharp bound.

The numerical test compares the 32 by 32 alias Schur construction with (10) for 18 random momenta at each of three λ values. The last column gives the exact corner (11).

Table 1. Free two-site blocking: Schur construction against the averaged covariance, chiral residual of the coarse operator, and the exact corner minimum of $\text{Re } F$.

λ	Schur versus covariance	Chiral residual	Exact corner $\text{Re } F$
0.0	1.48×10^{-15}	5.71×10^{-16}	1.000000
0.5	1.24×10^{-15}	4.64×10^{-16}	1.153665
1.0	1.12×10^{-15}	4.11×10^{-16}	1.259915

By contrast, direct compression $A = QDQ^\dagger$ has maximum chiral residuals 0.5763, 0.1973 and 0.2340 in the same three sample sets. That is an error in the operation performed, not evidence that exact coarse graining destroys chirality.

Nested spatial averages compose exactly. Applying $b = 2$ twice gives the same retained covariance as a direct $b = 4$ average, because $Q_{\text{total}} = Q_2 Q_1$ and compression of the inverse is associative; at the fixed comparison momentum the largest numerical discrepancy is 3.19×10^{-16} . The eliminated determinant factors compose as part of the same Gaussian integral.

5.2 An index-seven star block

Let Λ be the sublattice of integer triples x with $x_1 + 2x_2 + 3x_3 \equiv 0 \pmod{7}$, and let S contain the origin and the six offsets $\pm e_1, \pm e_2, \pm e_3$. The offsets have residues 0, $\pm 1, \pm 2, \pm 3$, each occurring once, so every lattice site has exactly one expression $c + s$ with $c \in \Lambda$ and $s \in S$: the seven-site stars are disjoint and cover $\mathbb{Z}^3$. A basis of Λ is $(7, 0, 0)$, $(-2, 1, 0)$, $(-3, 0, 1)$, with determinant seven. This is a volume reduction by seven, not an isotropic linear rescaling by seven.

Proposition 6 (constant-preserving star block). Define $(Q\psi)(c) = 7^{-1/2} \sum_s U(c \leftarrow c + s)\psi(c + s)$, using unitary links along the six direct spokes. Then $QQ^\dagger = I$ and Q commutes with chirality. At a free background its aliases are $q_m = q + (2\pi m/7)(1, 2, 3)$, $m = 0, \dots, 6$, with $z_m = [1 + 2\sum_i \cos q_{m,i}]/7$. The alias weights obey $\sum_m |z_m|^2 = 1$ and $z_0 = 1$ at $q = 0$.

Proof. Disjoint supports eliminate cross-block products, and each of the seven unitary summands contributes $I/7$ to $QQ^\dagger$. Character orthogonality on $\mathbb{Z}^3/\Lambda$ gives the alias normalisation. The Schur, chirality, index, determinant and joint-integration identities depend on these properties rather than on the cubic block shape and therefore apply to the star block. ■

The $b = 2$ bound of Lemma 5 and its corner values are specific to the cubic alias system and are not transferred without proof. For the free star block, $\text{Re } D$ is positive except at the single massless zero; at that point the zero-mode alias is fully retained, so $\text{Re } F$ is positive on $\ker Q$ there too, and continuity and compactness give a positive free lower bound for each fixed λ , whose optimal value is not asserted here.

For a normalised scalar row a on the seven sites, the constant-mode weight is $|\Sigma_s a_s|^2/7 \leq 1$, with equality only for a uniform row up to a common phase, by Cauchy–Schwarz. Exact retention of the constant mode therefore fixes the uniform average within this class of coisometries. For the canonical wheel measure $\pi = (7, 4, 4, 4, 4, 4, 4)/31$, the amplitude row $\sqrt{\pi}$ gives constant-mode weight 0.988470 and the row $\pi/\|\pi\|$ gives 0.946798; these are different measure-to-amplitude choices. A nonuniform row changes the leading pole residue; a consistent field normalisation can compensate, so this fact alone does not rule out weighted descriptions as physically inequivalent.

The uniform star gives relative blocked–fine operator differences 0.005188, 0.001295, 0.000324 and 0.0000809 at $h = 1/4, 1/8, 1/16$ and $1/32$ for the Section 7 momentum and $\lambda = 1$, consistent with order h^2 . It supplies a concrete seven-site spatial realisation and is the block used in Section 6.2. It does not identify the star's six spokes with every edge and transition of the canonical history wheel, nor derive the coarse lattice embedding uniquely from $K = 7$. Spatial constant-mode preservation also does not, by itself, select coherence among internal charge states.

6 Finite gauge background calculations

6.1 Weak and strong SU(2) backgrounds

To go beyond free Fourier symbols we use the noncommuting SU(2) link pattern of CLSN-1 Section 17.43 on a $2 \times 2 \times 2$ spatial torus with two antiperiodic sequential time slices. At link angle ϵ ,

$$U_x(x, y, z) = \exp[i\epsilon(-1)^{y+z}\sigma_1], \quad U_y = \exp[i\epsilon\sigma_2], \quad U_z = U_4 = I. \quad (12)$$

The two temporal slices are a finite determinant test, not a continuum-time simulation. The operator has 128 components: 8 spatial sites, 2 time slices, 2 colour components and 4 spin components. The coarse operator retains 16. We build the matrix function $F_x(K)$ and the Hermitian sign directly by spectral decomposition; no free dispersion formula is used.

A gauge-covariant spatial average transports every site to the cell anchor before summation. Fix $a = 0$ and the bond convention $(T_d\psi)(y) = U_d(y)\psi(y + e_d)$, where $U_d(y)$ is assigned to the bond from y to $y + e_d$. For $x = (x_1, x_2, x_3)$ with $x_i \in \{0, 1\}$, the product used for Table 2 is

$$P(a \leftarrow x) = U_x(0, 0, 0)^{x_1} U_z(x_1, 0, 0)^{x_3} U_y(x_1, 0, x_3)^{x_2}.$$

The spatial average is then

$$Q\psi(a) = 8^{-1/2} \sum_x P(a \leftarrow x) \psi(x). \quad (13)$$

The path convention is specified rather than selected by the primitive theory, and it matters for the numbers. The displayed product gives $\sigma_{\min}(F) = 1.909274$ at $\epsilon = 0.3$. The alternative product $U_x(0, 0, 0)^{x_1} U_y(x_1, 0, 0)^{x_2} U_z(x_1, x_2, 0)^{x_3}$ gives 1.901186. On backgrounds with nontrivial plaquettes these products define different coisometries and different eliminated spectra, although every identity in Sections 3 and 4 holds for each. Disjoint cells and unitary transport give $QQ^\dagger = I$; spin independence gives the chiral intertwining. Under a gauge transformation, $Q[U^g] = g_{\text{coarse}} Q[U] g_{\text{fine}}^\dagger$, so the Schur operator transforms as $D_c[U^g] = g_{\text{coarse}} D_c[U] g_{\text{coarse}}^\dagger$.

Table 2. Smallest singular value of the eliminated block on four backgrounds. The weak-field row agrees to all digits with the free values $\sqrt{2(1 + \alpha/R)}$ at the alias $q = (\pi, 0, 0)$ with sequential frequency $\pi/2$.

Background	$\sigma_{\min}(\mathbf{F}), \lambda = 0$	$\sigma_{\min}(\mathbf{F}), \lambda = 0.5$	$\sigma_{\min}(\mathbf{F}), \lambda = 1$
$\varepsilon = 10^{-4}$	1.946498	1.970913	1.981932
$\varepsilon = 0.3$	1.909274	1.929248	1.937502
$\varepsilon = 1.0$	1.791683	1.799563	1.804794
Haar random	1.786269	1.792130	1.797968

The $\varepsilon = 10^{-4}$ row is a weak-field control. To test appreciable non-Abelian curvature we also run $\varepsilon = 0.3$ and 1.0, and independently Haar-sample all 24 spatial SU(2) links with seed 20261022. The corresponding maximum plaquette distances $\|\mathbf{U}_{\square} - \mathbf{I}\|_F$ are 0.836, 2.380 and 2.575. Each configuration is rebuilt after independent site gauge transformations. Across the nine strong-background runs, fine and blocked chirality, propagator matching, determinant factorisation and gauge covariance pass with maximum residual 9.49×10^{-14} ; the smallest $\sigma_{\min}(\mathbf{F})$ is 1.786269 and the smallest seed gap 1.048598. These are nontrivial finite-background checks of the implementation and show comfortable invertibility on the configurations tested; they do not prove a uniform bound over every strong gauge field. For the weak-field control the largest covariance residual is 2.88×10^{-16} and the determinant factorisation residual, measured with logarithmic magnitudes and unit phases, is below 5.87×10^{-14} . These are floating-point verification errors, not estimates of physical accuracy.

We also construct the actual moving chiral subspaces. With $\hat{\gamma}_5 = \gamma_5(\mathbf{I} - \mathbf{D})$, choose orthonormal columns $\mathbf{V}_-$ spanning its negative eigenspace and $\bar{\mathbf{V}}_+$ spanning the positive eigenspace of γ_5 ; the Weyl matrix is $\mathbf{M} = \bar{\mathbf{V}}_+^\dagger \mathbf{D} \mathbf{V}_-$. These frames are rebuilt separately for the fine and coarse operators rather than frozen as the gauge field changes. On the nonsingular, index-zero weak-field configurations, $\log|\det \mathbf{M}| = \frac{1}{2} \log|\det \mathbf{D}|$ at both resolutions, and the difference between fine and coarse Weyl logarithmic magnitudes equals $\frac{1}{2} \log|\det \mathbf{F}|$; the checks pass at tolerance 10^{-10} . This is a magnitude statement: it neither selects a global Weyl determinant phase nor identifies $\mathbf{F}$ with a separate Weyl operator. An additional algebraic control with index one gives coarse index 0.9999999999999997; it is a small synthetic matrix, not a VERSF topological background. A second control makes $\mathbf{Q}$ erase the only zero mode, and its eliminated block becomes singular exactly as Lemma 1 predicts.

6.2 Census-resolved gauge response of the determinant

The minimal SC-1 census [16] supplies three generations of $\mathbf{Q}_L$, $\mathbf{u}_R$, $\mathbf{d}_R$, $\mathbf{L}_L$ and $\mathbf{e}_R$. We consume that inherited result with its stated premises. In conventional hypercharge $Y = \mathbf{Q}_{\text{electric}} - \mathbf{T}_3$ the representations per generation are (3, 2, 1/6), (3, 1, 2/3), (3, 1, -1/3), (1, 2, -1/2) and (1, 1, -1); SC-1 also uses the doubled convention $\mathbf{Q}_{\text{electric}} = \mathbf{T}_3 + \mathbf{Y}_{\text{SC}}/2$, so $Y = \mathbf{Y}_{\text{SC}}/2$. The minimal matter carrier has dimension 45. Optional sterile neutral states have zero gauge traces and do not alter the calculation.

With $\text{tr}_{\text{fund}}(\mathbf{T}_a \mathbf{T}_b) = \delta_{ab}/2$, the full twelve-generator matter Gram is $\mathbf{G}_F = \text{diag}(6\mathbf{I}_8, 6\mathbf{I}_3, 10)$. The single complex Higgs doublet adds $\mathbf{G}_H = \text{diag}(0\mathbf{I}_8, (1/2)\mathbf{I}_3, 1/2)$, giving the charge-participation baseline (6, 13/2, 21/2) [17]. The Higgs contribution is not part of the fermion determinant.

Proposition 7 (determinant response and census factorisation). On a smooth invertible branch define $\mathbf{L} = \log|\det \mathbf{M}| = \frac{1}{2} \log|\det \mathbf{D}|$ for the parity-even Weyl magnitude. For any link parameters θ_i ,

$$\mathbf{H}_{ij} = \partial_i \partial_j \mathbf{L} = \frac{1}{2} \text{Re Tr}[\mathbf{D}^{-1} \mathbf{D}_{ij} - \mathbf{D}^{-1} \mathbf{D}_i \mathbf{D}^{-1} \mathbf{D}_j], \quad \mathbf{H} = \mathbf{H}_F + \mathbf{H}_C, \quad \mathbf{L}_F = \frac{1}{2} \log|\det \mathbf{F}|, \quad \mathbf{L}_C = \frac{1}{2} \log|\det \mathbf{D}^c|. \quad (14)$$

At the identity background, with the same massless kinetic operator, link profile and block map for every representation, each quadratic internal contraction is $\text{Tr}(\mathbf{T}_a \mathbf{T}_b)$. Consequently $\mathbf{H}_{ab} = \kappa(\mathbf{G}_F)_{ab}$, $\mathbf{H}_{F,ab} = \kappa_F(\mathbf{G}_F)_{ab}$ and $\mathbf{H}_{C,ab} = \kappa_C(\mathbf{G}_F)_{ab}$, with $\kappa = \kappa_F + \kappa_C$. This is exact representation factorisation at the quadratic level and

does not rely on a continuum extrapolation. Mixed factors vanish because their cross traces vanish. Representation-dependent masses, backgrounds or blocking can invalidate the common- κ premise.

Proof. Differentiate log det twice, retaining the D_{ij} term as well as the two-vertex term, and differentiate the exact determinant factorisation, including the link derivatives of Q . At the identity the remaining spin and geometry contractions are independent of the representation, leaving the displayed internal traces; the same argument applies separately to the eliminated and retained determinants. The fermionic effective gauge action is $\Gamma_F = -L$, so its Hessian has the opposite sign to the tabulated log-magnitude response. ■

The execution uses one quotient cell $\mathbb{Z}^3/\Lambda$ of Section 5.2, seven spatial sites and two antiperiodic sequential slices, with $\lambda = 1$. For a generator T_a , set $U_1(r) = \exp[it \cos(2\pi r/7) T_a]$ and $U_2 = U_3 = I$, where $r = x_1 + 2x_2 + 3x_3 \bmod 7$. This nonuniform probe has nonzero plaquette curvature. It executes the 45-state census exactly through the eigenvalue blocks of each generator; each scalar block has 56 fine and eight retained components, and the transported star map is rebuilt at every t . Central differences at 0.004, 0.002 and 0.001 with Richardson extrapolation give the following values per normalised generator.

Table 3. Sector-resolved Hessian of the Weyl log-magnitude on the seven-site cell, split into eliminated and retained parts.

Sector	Fine H	Eliminated H_F	Retained H_c
Colour	6.361042	-4.321931	10.682973
Weak	6.361042	-4.321931	10.682973
Hypercharge	10.601735	-7.203218	17.804953

All eight colour, three weak and one hypercharge directions agree after division by their census trace to within 2×10^{-5} ; selected mixed directions and a pure-gauge control also pass. The common coefficients are $\kappa = 1.060174$, $\kappa_F = -0.720322$ and $\kappa_c = 1.780495$. Dropping det F therefore changes this finite response by 67.9438 per cent without changing $6 : 6 : 10$, since both terms carry the same ratios on a representation-independent background. The eliminated contribution is a signed curvature, not a positive allocation of stiffness.

Three features fix the reading of this fixture. The quotient $\mathbb{Z}^3/\Lambda$ is a single seven-point ring: the three spatial hops act on the same seven residues, rather than on a three-dimensional volume. The probe momentum $(2\pi/7)(1, 2, 3)$ has magnitude 3.4 in lattice units, at cutoff scale. Its positive log-magnitude Hessian means that the fermionic effective action $\Gamma_F = -L$ has a negative finite curvature in this convention. That finite sign does not determine the sign of the universal running coefficient: the response contains an additive regulator-dependent term, whereas running is determined by its scale dependence. The eliminated and retained curvatures have opposite signs, and their shares depend on the probe and cell. Under the common-operator premises of Proposition 7, the census factorisation is exact; its magnitude and long-distance interpretation require the scale tests below.

The same census satisfies the local hypercharge anomaly tests. In a left-handed convention, one generation has (Q, u^c, d^c, L, e^c) with multiplicities $(6, 3, 3, 2, 1)$ and $Y = (1/6, -2/3, 1/3, -1/2, 1)$. Exact rational sums give $\text{Tr } Y = \text{Tr } Y^3 = 0$ on all 45 states. For each normalised colour or weak generator, $\text{Tr}(Y T_a T_a)$ also vanishes: per generation the colour sum is $1/6 - 1/3 + 1/6$ and the weak sum is $1/4 - 1/4$. These local anomaly conditions are satisfied by the inherited census; constructing the global chiral measure remains a distinct obligation.

6.3 Low-momentum response on three-dimensional volumes

Connecting κ to the running coefficient requires the fine Hessian at momenta well below the cutoff, on genuine three-dimensional volumes, at low temperature. A plane-wave probe makes this tractable: with $U_1(x) = \exp[it \cos(k \cdot x)]$ and $k = (2\pi n_k/L)e_3$ on an $L^3 \times N_t$ torus, the operator couples a momentum q only to $q + mk$, so the full matrix is block diagonal in chains of L momenta labelled by (q_1, q_2, θ) . The Fourier

coefficients of $\exp(it \cos \phi)$ are $i^m J_m(t)$, and $\log \det D$ is the sum of the chain determinants, each of dimension $4L$. The chain construction was checked against a dense real-space build on a $4^3 \times 4$ torus, agreeing to 10^{-13} in $\log |\det D|$ at $t = 0.3$. The unit-charge Hessian $H(k)$ is then obtained by central differences with Richardson extrapolation, exactly as in Section 6.2. An independent implementation uses the equivalent mixed representation, Fourier transforming x_1, x_2 and time while retaining the L positions along x_3 . Its blocks also have dimension $4L$. At $t = 0.3$ on $4^3 \times 4$ it agrees with a dense real-space build within 1.71×10^{-13} in the Weyl log-magnitude; a pure-gauge control leaves that quantity unchanged at displayed precision [18].

For a Dirac fermion of unit charge the continuum one-loop response is $\Gamma_F = (V t^2/4) k^2 \Pi(k^2)$ with $\Pi(k^2) = (1/(6\pi^2)) \ln(1/k) + \text{constant}$, so the Weyl log-magnitude satisfies

$$\Pi(k^2) = -4H/(V k^2), \quad V = L^3 N_t, \quad (15)$$

and the slope of $6\pi^2 \Pi$ against $\ln(1/k)$ should approach 1 as $k \rightarrow 0$; the additive constant is regulator dependent and the universal content is the slope. The same quantity for the standard Wilson–Dirac operator at zero mass, $D_W = S + \text{temporal Wilson term}$, in the same geometry, serves as a control whose continuum behaviour is established.

The calculation differentiates $\sinh(K)$ and $\text{sign}(\gamma_5 X)$ through spectral divided differences and evaluates $H = \frac{1}{2} \text{Re Tr}[D^{-1}D'' - D^{-1}D'D^{-1}D']$, including both vertex terms. It avoids subtraction of nearby extensive log determinants. The analytic responses agree with the finite-difference values of Table 5 within 2.1×10^{-5} in P on the checked configurations; full and symmetry-reduced sums agree to floating-point precision on the controls. All 22 recorded checks pass [19]. All determinant modes are included, so the exact factorisation preserves this total response under any valid exact split. No explicit 64^3 retained/eliminated split or seven-site blocking is claimed by this execution.

At two fixed probe wavelengths, enlarging the spatial volume raises the inherited running slope towards one. Table 4 gives that direct comparison using $P = 6\pi^2 \Pi$. The three-momentum extraction below includes a leading quadratic correction [20].

The spatial test holds $k_1 = \pi/16$ and $k_2 = \pi/8$ fixed as L doubles. These are modes (1, 2) on $L = 32$ and (2, 4) on $L = 64$. The mass, $\lambda = 1$, boundary conditions and transverse probe remain those of Table A1. The slope is $s = [P(k_1) - P(k_2)]/\ln 2$. This enlarges the physical box in lattice units at fixed probe momenta; it is not a lattice-spacing extrapolation.

Table 4. Analytic determinant response at matched momenta. The continuum comparison for the slope is 1.

$L^3 \times N_t$	Modes	$P(\pi/16)$	$P(\pi/8)$	Slope
$32^3 \times 64$	1, 2	0.365749	-0.001149	0.529322
$64^3 \times 64$	2, 4	0.636896	0.016711	0.894737
$64^3 \times 128$	2, 4	0.641141	0.016885	0.900610

At fixed $N_t = 64$, doubling L raises s by 0.365416; doubling N_t at $L = 64$ raises it by a further 0.005873. Most of the improvement therefore comes from the spatial volume. The final slope is 9.94% below 1. That percentage describes this finite-model slope, not an error in a predicted force strength. At identical volume and probe momenta, the Wilson and inherited operators give slopes 1.0127 and 0.9006 respectively. The discrepancy is sensitive to the operator, with discretisation and operator-dependent finite-volume effects still to be separated. The longer range of the hyperbolic kernel can affect sensitivity to the box, so the Wilson departures of 1–3% are not an error bound for the inherited operator.

The smaller-volume rows have negative $6\pi^2 \Pi$ for the inherited operator and positive values for the Wilson control. These are finite contributions in the stated regulator convention, not a determination of the sign of universal running. A negative additive regulator term is one possible explanation; unconverged spatial size, temporal extent and cutoff effects must also be separated. Independent

executions reproduce the selected Table A1 rows and extend the calculation without fitting measured strengths or the target slope [18].

Table 5. Extended response $P = 6\pi^2\Gamma$. All entries use $\lambda = 1$ for the inherited overlap operator and the same massless Wilson control. Values are rounded finite-difference estimates.

L	N_t	k	P overlap	P Wilson
16	32	0.3927	-0.3027	+2.1893
32	16	0.1963	+0.1777	+2.7519
32	16	0.3927	-0.0913	+2.4085
32	32	0.1963	+0.3564	+2.9341
32	32	0.3927	-0.0055	+2.4965
32	32	0.7854	-0.4918	+1.7699
32	64	0.1963	+0.3658	+2.9438
32	64	0.3927	-0.0011	+2.5010

At fixed $L = 32$ and $k = 2\pi/32$, increasing N_t from 16 to 32 to 64 gives $P = 0.1777, 0.3564$ and 0.3658 . The total fermionic contribution is therefore positive in these tested configurations, with a substantially smaller change on the second temporal doubling. The negative response of the smaller-volume fixture is not a persistent sign obstruction. This temporal check does not establish spatial convergence.

Define the two-momentum slope as $[P(2\pi/L) - P(4\pi/L)]/\ln 2$, keeping L and N_t fixed in each comparison. It is 0.2994 on $16^3 \times 16$, 0.5222 on $32^3 \times 32$ and 0.5293 on $32^3 \times 64$ for the inherited operator; the corresponding Wilson values are 0.6335, 0.6314 and 0.6389. The movement of the inherited response is consistent with an approach towards the continuum coefficient, but these slopes do not yet extract that coefficient.

The lowest-mode comparison has an additional finite-volume limitation: kL stays equal to 2π or 4π as L grows. An independent analytic second derivative of the Wilson determinant, requiring no finite differences, demonstrates the distinction. On $64^3 \times 128$ the slope between modes $n_k = 2$ and 4 is 1.0127; on $128^3 \times 256$, between modes 4 and 8 at the same two momenta, it is 1.0357. This controls the method and illustrates the importance of spatial size. The corresponding matched-momentum comparison for the inherited overlap operator is given above [19].

The eliminated-mode accounting is also executed on the extended volumes. For this diagnostic, Q retains the normalised sum of each adjacent pair along x_3 , and V retains its orthogonal difference. The links along x_3 are unity for the chosen transverse x_1 perturbation, so no link transport inside these pairs is omitted. We evaluate $F = V^\dagger DV$ and the Schur complement D_c directly in every mixed-representation block. This is a partial spatial coarse graining, not a new execution of the seven-site star on the large torus. The full determinant response is unchanged by the exact split.

Table 6. Signed contributions to P on $32^3 \times 64$ under adjacent-pair blocking along x_3 .

n_k	k	Eliminated	Retained	Total
1	0.1963	+0.8254	-0.4597	+0.3658
2	0.3927	+0.7913	-0.7924	-0.0011

The maximum per-block logarithmic determinant-factorisation residual is 2.06×10^{-13} . For the longest-wavelength row, omitting $\det F$ would reverse the sign of the response. These are signed curvature contributions, not positive fractions of a stiffness. Each term continues to carry the same census traces under Proposition 7, so the omission changes the common magnitude and sign, not the ratio 6 : 6 : 10. The split provides a direct numerical reason to retain the effect of unresolved variables when extracting the effective gauge action.

The $128^3 \times 256$ execution uses modes 2, 4 and 8, with momenta $\pi/32$, $\pi/16$ and $\pi/8$. We fit $P(k) = c + s \ln(1/k) + d k^2$. If $a = [P(k) - P(2k)]/\ln 2$ and $b = [P(2k) - P(4k)]/\ln 2$, then $s = (4a - b)/3$. This cancels the k^2 term within the ansatz. The inherited two-point slopes are 0.956150 and 0.923408; the Wilson values are 0.989598 and 1.035689.

Table 7. Three-momentum determinant responses and fitted logarithmic coefficient on $128^3 \times 256$.

Operator	$P(\pi/32)$	$P(\pi/16)$	$P(\pi/8)$	Fitted s
Inherited	1.320940	0.658187	0.018129	0.967064
Wilson	3.924663	3.238725	2.520840	0.974234

The fitted coefficients are $c = -0.926144$, $s = 0.967064$, $d = 0.261631$ for the inherited operator and $c = 1.667009$, $s = 0.974234$, $d = -0.368302$ for Wilson. The two fitted logarithmic coefficients differ by 0.736% relative to the Wilson value. Three fitted parameters and three momenta leave no residual degrees of freedom. The machine-level interpolation residual tests the arithmetic, not the correction law. The Wilson coefficient is a comparison, not a calibrated uncertainty or a bound on operator-dependent finite-volume errors. No conclusion about a changed universal logarithm follows from a finite fitted deficit alone.

The strongest comparison is that opposite fitted quadratic corrections, $d = +0.262$ and -0.368 , leave logarithmic coefficients agreeing within 0.74%, as a common universal limit would require. Their shared downward offset is consistent with finite-volume and fit effects, but this comparison does not establish its cause. The Wilson slope changes by 0.023 between the matched 64^3 and 128^3 boxes, demonstrating volume sensitivity without assigning the same correction to both operators. A direct next test uses modes 4, 8 and 16 on $256^3 \times 512$, retaining the present momenta while ensuring $kL \geq 8\pi$; modes 2, 4 and 8 on any box still begin at $kL = 4\pi$. If the common offset shrinks and the agreement persists, that would strengthen the numerical evidence for a common coefficient at sub-percent agreement; additional momenta must also test the correction law. Modes 4, 8 and 16 on $128^3 \times 256$ are a lower-cost alternative, but reach $k = \pi/4$ and therefore require greater control of higher-order momentum terms.

Combining the determinant coefficient with the exact census traces yields the finite-fit fermionic estimate $\hat{b}_F = s(2/3)(6, 6, 10) = (3.868256, 3.868256, 6.447094)$. The unit-slope limit gives $b_F = (4, 4, 20/3)$, the fermionic term used in the transport steps of Route-M [8] and the charge-participation paper [17]. This supplies an explicit operator-based numerical extraction for that assembly. Identifying the fitted value with the limiting coefficient requires further momentum and volume control; the gauge, ghost and scalar terms are separate. The fit uses the original unit-coefficient seed, not the extension in Section 8.5.

The seven-site split requires a compatible periodic volume. The centre condition $x_1 + 2x_2 + 3x_3 \equiv 0 \pmod{7}$ is well-defined on an L^3 torus only when L is divisible by seven; 32^3 cannot be tiled by these seven-site cells. We therefore execute the nearest smaller compatible size, $28^3 \times 56$, at mode 1. The coarse map includes the probe-dependent links along the star spokes. Seven fine momentum sheets are combined through Q , and the retained inverse is $\Sigma_r Q_r D_r^{-1} Q_r^\dagger$; evaluating seven separate matrices of dimension $4L$ avoids diagonalising a dense matrix of dimension $28L$.

Table 8. Seven-site star contribution to P on $28^3 \times 56$, $k = 2\pi/28$.

Total	Eliminated	Retained
+0.2351	+0.2531	-0.0180

The total agrees with the independent analytic Hessian to $1.3e-05$ in P . The maximum star coisometry residual is $9.2e-14$. The separately accumulated finite-difference parts differ from the total by 1.7×10^{-5} in P , so the table reports four decimal places. The small 7^3 control also checks the explicit eliminated determinant and a second finite-difference step pair. These signed contributions quantify the fermionic determinant term integrated into the coarse gauge action at this seven-site blocking scale. They are not

positive shares of a stiffness, and do not by themselves identify the complete G_{phys} . Under the common-operator premises each part carries the same census factors 6 : 6 : 10.

Observable force strengths are to be extracted from the coarse effective action with its complete quantum weight; they are not additional primitive substrate inputs. The present execution supplies the fermionic contribution, its census proportions and its exact eliminated-mode accounting. The identification of its scale dependence with the universal factor in $b_F = (2/3)(6, 6, 10)$ still requires spatial convergence, low-momentum extrapolation and consistent matching. Route-M [8, Section 24.7A] derives that factor under its stated continuum regulator conditions; together with the gauge, ghost and scalar terms it gives $b = -(11/3)(3, 2, 0) + (2/3)(6, 6, 10) + (1/3)(0, 1/2, 1/2) = (-7, -19/6, 41/6)$. Those other contributions and the complete source-selected gauge stiffness are not evaluated by this fermion-only test. A finite positive matter contribution alone is not a prediction of the three measured couplings.

7 Refinement and the particle equation

The block kernel is an output of integration, so its low-momentum form can be read after that operation. Restore a spatial resolution h and choose the sequential resolution $a_t = h$ in units $c = 1$ for this diagnostic; this is a refinement convention, not a derivation of laboratory time. Hold physical momentum k and frequency ω fixed and use $p = bhk$ and $\theta = h\omega$ in (10).

For fixed b the constant alias has weight $1 + O(h^2)$ and $D_\lambda(hk, h\omega) = i\hbar\gamma \cdot (k, \omega) + O(h^2)$. All other aliases have weights $O(h^2)$ or smaller and bounded inverses, so the retained inverse has the same leading pole and

$$h^{-1}D^c_\lambda(bhk, h\omega) = i\gamma \cdot (k, \omega) + O(h). \quad (16)$$

The coefficient is inherited as one, without fitting a wave-function multiplier to a measured quantity. In the supplied left- or right-handed sector it gives the corresponding Weyl operator. The leading $O(h)$ term is the scalar correction already present in the fine operator: the block's low-alias weight is $1 + O(h^2)$, so its contribution changes D/h at $O(h^2)$, and the $O(h^2)$ weights of the massive aliases first change D/h at $O(h^3)$. The leading scalar correction is therefore unchanged by the blocking.

The execution uses $k = (0.3, -0.2, 0.4)$, $\omega = 0.25$ and $\lambda = 1$, with errors as relative Frobenius norms against $i\gamma \cdot (k, \omega)$. These coordinates are arbitrary diagnostic probes, not particle masses or observations.

Table 9. Relative error of the fine and blocked operators against the continuum symbol, and the relative blocking-induced operator difference.

h	Fine error	Blocked $b = 2$	Blocked $b = 4$	Block difference, $b = 2$
1/4	0.074835	0.075720	0.081871	4.526×10^{-3}
1/8	0.037185	0.037297	0.038082	1.133×10^{-3}
1/16	0.018563	0.018577	0.018676	2.832×10^{-4}
1/32	0.009278	0.009280	0.009292	7.080×10^{-5}
1/64	0.004639	0.004639	0.004640	1.770×10^{-5}

The last column is $\|D^c(2hk, h\omega)/h - D(hk, h\omega)/h\|_F / \|D(hk, h\omega)/h\|_F$ and scales as h^2 . The difference of the two error norms is a different quantity: 8.85357×10^{-4} , 1.39295×10^{-5} and 2.17743×10^{-7} at $h = 1/4$, $1/16$ and $1/64$, scaling as h^3 . The final error ratio in each column is consistent with first-order convergence. The analytic argument establishes the limiting pole; the table checks the implemented blocking. A fixed coarse cell size while h tends to zero would be a different limit, retaining a finite observation scale, and is not represented here.

Spatial integration alone does not resolve the separate issue of primitive sequential evolution. The corpus already distinguishes link-reflection positivity from positive one-step transfer and proves

suppression of the negative covariance sector on its admitted static branch [3, Sections 17.41–17.49]. We do not replace a primitive step by two steps, identify a reconstructed Hamiltonian with an unreturned microscopic law, or infer positivity after dynamical gauge integration from the present spatial test.

8 Sequential defects and polar completion

The following tests address the two chosen additions identified in Section 2. They use the inherited sequential generator and the corpus's polar-decomposition framework [9, 10], keeping each proposed action-level identification explicit. They do not require a fourth primitive spatial direction or a unique microscopic formula when controlled coarse equivalence suffices.

8.1 Temporal stiffness from sequential departure

Let K be Hermitian on the admitted sequential branch, $A = \exp(-K)$, and T a unitary sequential shift. The forward departure is $B = T - A$, and its weighted square $M(T) = B^\dagger A^{-1} B$. The positive metric A^{-1} is a proposed bridge from the transfer, not an established primitive action. Multiplication gives, without assuming that A and T commute,

$$M(T) = T^\dagger A^{-1} T + A - T^\dagger - T. \quad (17)$$

Subtracting the zero-shift expression $M(I) = A^{-1} + A - 2I$ gives

$$\frac{1}{2}[M(T) - M(I)] = \frac{1}{2}(2I - T - T^\dagger) + \frac{1}{2}(T^\dagger A^{-1} T - A^{-1}). \quad (18)$$

For a static background with $[T, K] = 0$ the second term vanishes and the remainder is precisely the canonical temporal Wilson shape. In a free Fourier mode $T = \exp(i\theta)$, $M(T)/2 = \cosh K - \cos \theta$, whereas $[M(T) - M(I)]/2 = 1 - \cos \theta$. This is an explicit sequential-defect realisation of the temporal correction.

Its assumptions matter. For $B^\dagger W B$, requiring unit cross coefficients gives $WA = I$ and hence $W = A^{-1}$ uniquely. That requirement, the factor $1/2$, the subtraction and the use of the resulting quadratic defect inside the fermion seed still need justification from the action; (18) does not select them by itself. On changing backgrounds the extra term cannot generally be discarded: a noncommuting three-slice diagnostic gives Frobenius norm 0.8389258913 for it. $M(T)$ is positive; $M(T) - M(I)$ need not be. Section 8.4 tests this metric against exact temporal Gaussian composition and gives the composition-consistent alternative; the algebraic identity (18) itself is unchanged.

8.2 Retaining the full defect under coarse graining

On the free $\lambda = 1$ branch, retaining the additional static contribution suggests a controlled comparison family. With W , E and $f = \sinh(E)/E$ as in Section 2, define

$$a_\eta = fW - \cos \theta + \eta(\cosh E - 1), \quad b = (f \sin q_1, f \sin q_2, f \sin q_3, \sin \theta), \\ X_\eta = a_\eta I + i\gamma \cdot b, \quad D_\eta = I + X_\eta / (a_\eta^2 + |b|^2)^{1/2}, \quad 0 \leq \eta \leq 1. \quad (19)$$

Here η is an interpolation for the test, not a new primitive parameter. The endpoint $\eta = 0$ is the original free completion; $\eta = 1$ retains the full additional defect term. The extension is free-sector only: on a noncommuting spatial background $\cosh K$ need not commute with γ_s , so the same substitution does not automatically preserve γ_s -Hermiticity.

The free seed remains gapped throughout the interpolation. A zero requires $b = 0$ and $a_\eta = 0$, so it could occur only at a corner. At zero spatial momentum the two temporal corners have $a_\eta = -1$ and $+1$; at every other spatial corner $W \geq 2$ and $a_\eta = \sinh W - \cos \theta + \eta(\cosh W - 1) > 0$. No corner closes the seed gap, and continuity and compactness give a uniform free gap. An overlap zero requires $b = 0$ and $a_\eta < 0$, so only the origin supplies a light branch.

For $q = h\mathbf{k}$ and $\theta = h\omega$ the added scalar term is $O(h^2)$. Since $b = O(h)$ and $a = -1 + O(h^2)$, normalisation of X makes the resulting vector change $O(h^3)$ and the scalar change $O(h^4)$, so $(D_\eta - D_0)/h = O(h^2)$: the free

continuum particle equation is unchanged. This is not equality of the interacting quantum effective actions.

The executable also applies the exact $b = 2$ inverse-covariance blocking of Section 5. At $k = (0.3, -0.2, 0.4)$ and $\omega = 0.25$, the differences between blocked D_1/h and D_0/h , normalised by the $\eta = 1$ operator, are 0.0090696862 at $h = 1/4$, 0.0005664404 at $h = 1/16$ and 0.0000088501 at $h = 1/128$. Across all six refinement levels the difference falls by approximately four on halving h , with final ratio 4.0000114. This executes the free coarse-equivalence test rather than demanding microscopic equality.

The absence of temporal doubling alone does not select coefficient $r_t = 1$. At $q = 0$ and $\theta = \pi$ the canonical seed scalar is $2r_t - 1$: values below $1/2$ retain the extra zero, equality closes the seed gap, and every fixed $r_t > 1/2$ removes it without changing the leading kinetic coefficient. This tests those conditions; it is not a certificate of every physical admissibility requirement for every coefficient.

8.3 The polar identity and its physical meaning

For an invertible γ_5 -Hermitian seed X , let $H = \gamma_5 X$. Since $H^2 = X^\dagger X$,

$$U = X(X^\dagger X)^{-1/2} = \gamma_5 \text{sign } H, \quad D_{ov} = I + U. \quad (20)$$

Overlap is therefore exactly polar normalisation of the seed. Unitarity of U and $\gamma_5 U \gamma_5 = U^\dagger$ imply the Ginsparg–Wilson identity. The matrix identity is established mathematics; its relevance is that VERSF already uses canonical polar decomposition of refinement maps [9, Theorem 8.1] and a stationary-weighted polar split of its history transfer [10, Section 47]. The largest numerical discrepancy between the two forms in (20) is 1.27×10^{-15} , and the largest chiral residual in the free comparison family is 3.46×10^{-16} .

The source uses three distinct polar constructions. The supported record map $M_{ij} = \Pi_j \Phi_{ij} \Pi_i$ has partial isometry V_{ij} , and the bond defect $\Pi_j U_{ij} \Pi_i - V_{ij}$ aligns gauge links with record transport [13, Section 16]. The local completion stretch enters the Higgs and mass construction. Overlap instead normalises the shifted kinetic seed X of (20). These objects are not automatically identified; in particular the polar factor of the positive transfer $\exp(-hK)$ is the identity, so applying the same operation directly to A does not reproduce overlap, and the simplest identification of the seed with the sequential departure of Section 8.1, $X = \gamma_4(T - A)$, fails by the terms $-\cos \theta(1 + \gamma_4) + \gamma_4 \cosh K$. The physical connection must identify the object on which the operation acts, not only the operation.

Polar factorisation also retains a positive factor P : $X = UP$. Replacing X by $I + U$ is not Gaussian elimination of P . In one diagnostic, $\log |\det X| = 0.3040917401$, $\log |\det U|$ is approximately zero, and $\log |\det(I + U)| = -0.1914285603$. The physical action must account for field-dependent quantum weights; agreement of free poles alone does not establish interacting regulator equivalence [11, Section 33].

8.4 Exact temporal composition and its continuum limit

The corpus supplies an exact Ornstein–Uhlenbeck covariance on its positive unresolved quotient [12, equation IV.44]. We use it to test temporal consistency without identifying the quotient generator with the fermion generator. Let $A_h = \exp(-hK)$ with real symmetric K , and let C be a positive covariance density for independent Gaussian innovations. The exact transition covariance and its composition law are

$$Q_h = \int_0^h \exp(-sK) C \exp(-sK^T) ds, \quad Q_{s+t} = Q_t + A_t Q_s A_t^T. \quad (21)$$

Continuity, this composition law and $Q'_0 = C$ determine (21). The normalised transition density has exponent $-\frac{1}{2}(y - A_h x)^T Q_h^{-1} (y - A_h x)$, and integrating the intermediate coordinate gives the same density at the combined interval, including its determinant normalisation. Composition does not select C . Independence is a premise of this Gaussian Markov branch, not a general conclusion about VERSF histories; the joint source law can also carry cross covariance [3, Section 17.70].

For $C = I$ the precision is a function of K , with continuous value $1/h$ on a zero eigenvalue:

$$W_h = Q_h^{-1} = \exp(hK) K / \sinh(hK). \quad (22)$$

For $C = 2I$ the precision is $W_h/2$, the convention of [12, equation IV.44] with K replaced by its positive quotient generator A_Q . The exact Gaussian covariance is inherited mathematics; the contribution here is its explicit comparison with the sequential-defect proposal.

Even after giving the proposed transfer weighting its time factor, $W_{\text{trial},h} = \exp(hK)/h$, it fails exact composition. Eliminating a middle coordinate between two equal steps gives $W_{\text{combined}} = (W_{\text{trial},h}^{-1} + A_h W_{\text{trial},h}^{-1} A_h^T)^{-1}$, and relative to $W_{\text{trial},2h}$ its modewise ratio is $\text{sech}(hK)$. Unequal eigenvalues therefore require different corrections; one scalar normalisation cannot repair the prescription. This conclusion is confined to exact independent Gaussian composition.

For a static commuting Fourier shift $T = \exp(i\theta)$, the half-defect with (22) is $[K/\sinh(hK)][\cosh(hK) - \cos \theta]$. Subtracting its $\theta = 0$ value leaves $[K/\sinh(hK)](1 - \cos \theta)$, and after multiplying by h the extra coefficient is

$$hK/\sinh(hK) = I - h^2 K^2/6 + O(h^4). \quad (23)$$

The composition precision agrees with the proposed coefficient in the small-step limit on bounded modes. Its placement in a Grassmann seed is an explicit identification. The following execution makes that identification concrete: it replaces the unit temporal coefficient by the composition-selected free spectral function and tests its static-background extension. This does not identify a real Gaussian precision with a fermionic action without specifying the map.

The executable checks covariance composition and direct Schur elimination on twenty five-dimensional examples with noncommuting K and C , as well as (22) and the source's $C = 2I$ convention; all 82 assertions pass. For K eigenvalues (0.2, 0.7, 1.4, 2.1) and $h = 0.25$, the trial composed/direct precision ratios are (0.998751, 0.984880, 0.941728, 0.876419). The maximum relative coefficient correction decreases from 0.0445021 at $h = 1/4$ to 0.000179421 at $h = 1/64$, approaching a factor of four per halving. These are dimensionless diagnostic inputs, not measured particle parameters.

8.5 Composition-selected coefficient in the fermion seed

In dimensionless step units define $\phi(z) = \sqrt{z}/\sinh \sqrt{z}$, with $\phi(0) = 1$, and $C = \frac{1}{2}[\phi(S^\dagger S) + \phi(SS^\dagger)]$. Replace the temporal Wilson term W_4 in (3) by CW_4 on the static spatial backgrounds with $U_4 = I$ used here. In the free case $C = g(E)I$ with $g(E) = E/\sinh E = 1/f_1$. The free coefficient is therefore fixed by the composition precision once this placement is adopted; symmetrisation is the specified lift to a noncommuting spatial background.

The identity $\gamma_5 S \gamma_5 = S^\dagger$ exchanges $S^\dagger S$ and $SS^\dagger$, so $\gamma_5 C \gamma_5 = C$. For this temporal shift C commutes with W_4 , making CW_4 γ_5 -compatible. The unsymmetrised $\phi(S^\dagger S)$ fails this test: at $\epsilon = 0.3$ its spatial commutator norm is 0.504622, or 0.713644 on the two-slice carrier. Symmetrisation removes that obstruction. It is not a uniqueness proof for every possible static extension, and time-dependent backgrounds or noncommuting temporal links require their own ordering analysis.

For the free seed $a = f_1 W + g(E)(1 - \cos \theta) - 1$ and b is unchanged. A zero of b requires corner momenta. At the origin $a = -1$ and at its temporal partner $a = 1$; at every nonzero spatial corner $W \geq 2$ and $a = \sinh W + g(W)(1 - \cos \theta) - 1 > 0$. Hence the seed is gapped and the completed operator has only the origin zero. The 25^4 grid gives $\min R = 1$ to numerical precision; that grid value is a diagnostic, whereas the corner argument proves a positive gap. The fine chiral identity holds to 10^{-15} on the free controls.

Under simultaneous refinement, $(g - 1)(1 - \cos \theta) = O(h^4)$. Normalisation of the overlap seed gives an $O(h^4)$ relative operator difference. The blocked relative differences are 5.88169×10^{-6} , 2.30428×10^{-8} and 9.00269×10^{-11} at $h = 1/4$, $1/16$ and $1/64$. Their ratios approach 256 when h is quartered. This preserves the leading particle equation; it does not substitute the new link vertices into the earlier gauge-response fit.

Table 10. Composition-selected coefficient with the symmetric static extension. The Haar row uses independent seed 20261023.

Background	Seed gap	$\sigma_{\min}(F)$	Coarse chiral residual
$\varepsilon = 0.0001$	1.000000	1.976815	5.7e-16
$\varepsilon = 0.3$	1.048596	1.930932	1.7e-15
$\varepsilon = 1$	1.909699	1.768736	1.9e-15
Haar	1.864704	1.754375	1.5e-15

All four seeds are Hermitian within numerical tolerance, the fine and coarse chiral residuals are below 6×10^{-15} , and determinant factorisation residuals are below 8×10^{-14} . The three structured singular values reproduce the independent calculation; the Haar values belong to the separately seeded sample. This closes the stated finite static execution of the composition-selected coefficient. The overlap prescription remains specified, as does the symmetric lift; the one-mode source attachment of Section 9.8 is a separate interaction construction.

9 Record transport and the interacting fermion action

9.1 The connection already present in the source

MFKR-1 Section 2.3 inherits a fermion kinetic bilinear using the same physical links as the gauge and closure sectors [1], and the master action couples those links to the polar transport of supported record maps [13, Section 16]. Record geometry therefore already has an action-level route into fermion propagation. CLSN-1 Sections 17.12 and 17.13 explicitly integrate Gaussian auxiliaries with fermion currents retained and give the conditional sequential family $A_n = \Omega_n \exp(-hK_n)$, including its first and second vertices [3]. The physical sequential attachment and ordering remain declared at that construction grade. The calculation below uses the shared-link route, retaining the inherited fermionic carrier and signed kinetic generator.

9.2 Gaussian elimination with fermions retained

Let z be centred real unresolved coordinates with normalised Gaussian covariance C on the specified physical quotient. For the inherited Grassmann action $S_F(z) = \bar{\psi}D(z)\psi$, define

$$\exp(-S_{\text{eff}}[\bar{\psi}, \psi]) = E_z \exp[-\bar{\psi}D(z)\psi]. \quad (24)$$

The quadratic Grassmann coefficient is exactly $\bar{\psi}E[D(z)]\psi$; the quartic coefficient is minus one half of the covariance of the fluctuating bilinear. If $D(z) = D_0 + \sum_a z_a D_a$ is linear, Gaussian cumulants terminate and give the exact identity already underlying [3, Section 17.12]:

$$S_{\text{eff}} = \bar{\psi}D_0\psi - \frac{1}{2} \sum_{ab} C_{ab}(\bar{\psi}D_a\psi)(\bar{\psi}D_b\psi). \quad (25)$$

For nonlinear link dependence, $E[D]$ contains the leading contact correction $\frac{1}{2}\sum_{ab}C_{ab}D_{ab}$ when the covariance expansion is controlled, as well as higher terms. A Gaussian average therefore generates interactions; replacing a Dirac operator by a real-coordinate precision is not this integration. Source derivatives of the polar factor, obtained from the Sylvester equation in [13, Appendix B], determine record-induced vertices once the actual map, background and joint covariance are specified.

Finite bond stiffness also matters. The source retains U as an independent variable. In a Gaussian tangent chart with cost $\frac{1}{2}z^TWz + \frac{1}{2}(u - Lz)^TH_b(u - Lz)$, where W and H_b are positive and L is the linearised alignment map, joint integration gives

$$C_u = H_b^{-1} + LW^{-1}L^T. \quad (26)$$

The first term describes fluctuations away from perfect alignment. Other source terms and cross blocks must be retained in the full joint Hessian. Setting $U = \text{Pol}(M)$ at every fluctuating configuration is a bond-zero branch or hard-constraint assumption, not a consequence of finite stiffness alone.

9.3 Exact finite transport test

Choose a full-support diagnostic chart $M(q) = B \exp(iqT)$, with B positive and T Hermitian; its polar unitary is $V(q) = \exp(iqT)$ even when B and T do not commute. On the bond-zero branch $U = V$, take the declared sequential family $A(q) = \exp(iqT)\exp(-hK)$ with the supplied signed K retained. MFKR-1 Section 24.3 fixes the exterior lift $\Gamma(A)$ uniquely in its vacuum-preserving, number-preserving class [1]. Define $J = d\Gamma(T)$ and $F = d\Gamma(K)$. For q Gaussian with variance σ^2 ,

$$E[\Gamma(A(q))] = \exp(-\sigma^2 J^2/2) \exp(-hF), \quad \Gamma(E[A(q)]) = \exp[-\sigma^2 d\Gamma(T^2)/2] \exp(-hF). \quad (27)$$

Proof. Multiplicativity of exterior powers gives $\Gamma(A(q)) = \exp(iqJ)\exp(-hF)$. The scalar Gaussian characteristic function applied spectrally to Hermitian J gives the first form; applying the same average on the one-particle carrier and then lifting gives the second. No commutation of K and T is needed. The difference $J^2 - d\Gamma(T^2)$ is a two-particle interaction: for diagonal T with entries t_i it equals $2\sum_{i<j} t_i t_j n_i n_j$ and vanishes on the vacuum and one-particle sectors. ■

The four-mode fixture has a 16-dimensional Fock space. Direct Gaussian quadrature agrees with the first form of (27) to relative residual 1.57×10^{-16} , while the norm difference from the second form is 0.43448213. In a separate two-mode control, opposite charges cancel their shared phase exactly, whereas averaging them separately inserts an attenuation $\exp(-\sigma^2)$; at diagnostic variance $\sigma^2 = 0.13$ the exact-to-factorised pair ratio is 1.13882838. These inputs are chosen algebraic probes, not physical VERSF parameters.

9.4 Interactions under coarse graining

For independent innovations with the specified diffusive scaling $\sigma^2 = vh$, finite bounded matrices satisfy

$$[\exp(-vhJ^2/2)\exp(-hF)]^{t/h} \rightarrow \exp[-t(F + vJ^2/2)]. \quad (28)$$

This is the ordinary product-formula limit. The induced interaction survives, and when F and J commute the equality holds at finite h . The noncommuting fixture approaches the limiting transfer with first-order error, decreasing from 0.0184184 at $h = 1/4$ to 0.00115287 at $h = 1/64$ for $t = 1$. The scalar coefficient v is a diagnostic input. The corpus already supplies a Gaussian increment proposal with covariance proportional to the sequential step, so diffusive scaling has a source precedent [14, Section 64.4.1C]; Section 9.6 executes the direct link restriction for that proposal and produces a tensor rather than identifying the scalar variance as a physical value.

Temporal correlations must also be retained. For commuting phases q_n with covariance C_{nm} and deterministic transfer commuting with J , a complete history gives $\exp[-\frac{1}{2}(\sum_{nm} C_{nm})J^2]$ times that deterministic transfer; a two-time quadrature verifies the expression to residual 3.20×10^{-16} , and replacing the covariance sum by its diagonal changes the test transfer by norm 0.20182599. Noncommuting history factors require their chronological ordering; the commuting expression is not asserted for them.

The twelve executable checks cover the exterior lift, Gaussian average, refinement, correlated history, finite-stiffness covariance and the source polar differential. They establish a working conditional connection from polar link fluctuations to an interacting fermion transfer. The diagnostic chart, scalar covariance and perfect-alignment branch have not been identified with the restriction of the inherited physical source; Sections 9.5 and 9.6 execute a direct alternative within the inherited common Gaussian construction, without requiring perfect alignment. The complete interacting chiral continuum limit remains outside these checks, and exact equality with one microscopic overlap formula is unnecessary if that continuum equivalence can instead be established.

9.5 Inherited fluctuations and their physical restriction

The parameter-closure paper [14] supplies a common normal-chart Gaussian proposal with mean $a_t E_{DD}$ and covariance $a_t G_{op}^{-1}$ (Section 64.4.1C). Its sector widths are not independent choices. Section 67 derives six unresolved $K = 7$ relaxation rates, $\{1/2, 1/2, 1, 1, 31/28, 5/4\}$; the homogeneous branch has zero retained-to-unresolved coupling, a branch-specific result. Section 75 calculates a nonseparable

irreversible branch, including unresolved precision A_Q , coupling C and relaxed score metric $W_{\text{defect}} - C^T A_Q^{-1} C$; these static precisions are not relaxation rates. Section 76 extends the coupling to the full typed carrier under the stated factor-blindness premise, giving a 1122×243 matrix of rank 183. These are inherited calculations; operational whitening does not establish their physical metric identification, and the rank and sector coverage of the typed score lift do not by themselves identify a diffusion covariance.

The later changing-link source [3, Sections 17.72–17.79] distinguishes compatible covariance lifts from selection of their physical values. Its joint link/current construction shows that current co-motion affects the response, and that freezing the current can change it. Section 17.79 supplies an event-measure architecture for joint link/current increments and their history scores but leaves its continuous increment component unselected. This is the particular source obligation relevant here, rather than a claim that VERSF has no fluctuation law.

A concrete restriction test can be stated in the Gaussian regime. Let z_n collect the source fluctuations at step n with full history covariance C_{nm} , and let L_n be the derivative of the physical link map on the occupied support. If $q_n = L_n z_n$, the induced phase covariance is $C_{\text{link},nm} = L_n C_{nm} L_m^T$: a covariance push-forward, not a new selection principle. If the link remains an independent fluctuating variable, the full joint covariance must replace this deterministic map, with (26) as the finite-stiffness control. The link vertices and chronological ordering must then be inserted in (24), or in the full exterior transfer, before taking the coarse limit. A general covariance need not reduce to the scalar test (27); Section 9.6 retains its tensor structure.

9.6 Executed connection from the source covariance to fermion interactions

The common proposal in [14, Section 64.4.1C] updates the independent link itself by $U' = U \exp(\xi_U)$. Let P_U select its real Lie-algebra coordinates from the physical tangent quotient. At a frozen normal chart and zero defect drift, the marginal increment covariance is

$$\text{Cov}(\theta) = h\Sigma, \quad \Sigma = P_U G_{\text{op}}^{-1} P_U^T. \quad (29)$$

This is a block of the full inverse, not the inverse of a link block with current and record coordinates held fixed, so their correlations enter the answer. No hard condition $U = \text{Pol}(M)$ is imposed. At nonzero defect drift the mean is hb with $b = P_U E_{\text{DD}}$. Curved-chart corrections, background changes, history reweighting and temporal memory require the corresponding source law; they are not declared absent physically by this frozen Gaussian calculation.

Let D_a and D_{ab} denote the first and second derivatives of the inherited fermion operator with respect to these angles, and $B_a = \bar{\psi} D_a \psi$. A Gaussian cumulant expansion of (24), for smooth bounded finite-dimensional vertices, gives

$$S_{\text{eff}} = \bar{\psi} D_0 \psi + h \Sigma_a b_a B_a + (h/2) \Sigma_{ab} \Sigma_{ab} \bar{\psi} D_{ab} \psi - (h/2) \Sigma_{ab} \Sigma_{ab} B_a B_b + O(h^2), \quad (30)$$

where Σ_{ab} inside a summand is a covariance entry and the preceding Σ is the index sum. The centred linear case reduces exactly to (25); nonlinear compact links also require the contact term containing D_{ab} . Equation (30) fixes the interaction tensor once the source metric and common-carrier vertices are given. The calculation below evaluates that tensor, not the complete interacting Dirac matrix or its quantum measure.

For the declared sequential class $A(\theta) = \exp(i \Sigma_a \theta_a T_a) \exp(-hK)$, set $J_a = d\Gamma(T_a)$ and $F = d\Gamma(K)$. At zero drift, expansion of the full exterior transfer yields

$$E[\Gamma(A)] = I - hH_{\text{eff}} + O(h^2), \quad H_{\text{eff}} = F + \frac{1}{2} \Sigma_{ab} \Sigma_{ab} J_a J_b, \quad V_{\text{int}} = \frac{1}{2} \Sigma_{ab} \Sigma_{ab} [J_a J_b - d\Gamma(T_a T_b)]. \quad (31)$$

The last expression isolates the contribution absent from the lift of the averaged one-particle transfer; it vanishes on the vacuum and one-particle sectors. Symmetry of the real covariance makes its quadratic contraction Hermitian even when the T_a do not commute. Independent small steps with fixed bounded generators give the product limit $\exp(-tH_{\text{eff}})$; a finite-step Gaussian average is not generally the exponential of this quadratic contraction for noncommuting generators. Nonzero real drift adds $-i \Sigma_a b_a J_a$

and requires a separate positivity analysis. The action and transfer formulas use distinct vertex conventions; their signed kernels are not identified without the sequential-action connection.

For a concrete source evaluation, [3, Section 17.52] supplies an invariant block on current p , its partner q , link angle θ and record phase y . On that block the published operational-metric family is $G_{(c,s)} = c(I + sH_4)$, with positive parameters c and s and

$$H_4(\ell) = [(1, 0, -1/\sqrt{2}, 0); (0, 1, 1/\sqrt{2}, 0); (-1/\sqrt{2}, 1/\sqrt{2}, \ell + 2, -1); (0, 0, -1, \ell + 1)], \quad (32)$$

the bracketed tuples being the rows in the order (p, q, θ, y) . Eliminating both currents and the record phase, or taking the $\theta\theta$ entry of $G_{(c,s)}^{-1}$, gives the same covariance density:

$$\Sigma_\theta(\ell; c, s) = \{c[1 + s(\ell + 2) - s^2/(1 + s) - s^2/(1 + s(\ell + 1))]\}^{-1}. \quad (33)$$

This uses the specified operational metric in the source's Gaussian proposal; it is not an assumption that every action Hessian or Fisher matrix is a diffusion precision. For the published witness $c = s = 1$, the six-cycle eigenvalues $\ell = (1, 2, 2, 4, 4, 5)$ of the twelve-edge triangle Gram give

$$\Sigma_\theta = (6/19, 4/17, 4/17, 3/19, 3/19, 14/103). \quad (34)$$

These ℓ are spatial triangle-cycle eigenvalues, not the unresolved wheel relaxation rates of Section 9.5. In a centred linear action chart their contribution is $-(h/2)\Sigma_\theta(\ell_i)B_i^2$: conditional interaction coefficients multiplying the inherited link vertices, not measured force strengths. The source block describes one scalar factor copy and is not silently extended to a faithful non-Abelian Standard Model connection.

The executable builds the twelve-edge triangle matrix Z with columns $s_i + r_i - s_{i+1}$, the six-cycle Gram $G = Z^T Z$ and the orthonormal embedding $Q = ZG^{-1/2}$, and returns the full edge covariance $Q\Sigma_{\text{cycle}}Q^T$. Independently assembling and inverting the coupled 24-coordinate matrix reproduces the spectral six-cycle result. Thirteen checks pass, including marginal covariance, cycle embedding, full elimination and the dependence on c . A separate noncommuting two-mode Fock control verifies (31), with generator errors 0.0118141, 0.00593274, 0.00297282 and 0.00148803 for $h = 0.04, 0.02, 0.01$ and 0.005 . That control tests the algebra; its generator assignment is not a physical source identification.

The conditional source-to-interaction connection is thus executed. Its interpretation follows two inherited results: RPC-1 proves observable invariance under its specified convention group, up to truncation [14, Theorem 14.2], and HMGB-1 determines the response metric once the complete history law is fixed [15, Sections 11A.3 and 12]. The metrics varied in (33) are not certified physical alternatives [3, Section 17.53], so their coefficient differences establish no physical-strength ambiguity. Such a claim would require variations that survive source admissibility, lie outside the established equivalences, and give different dimensionless observables after consistent coarse graining of the action, measure, vertices, clock and observable normalisations. Convention invariance is not a theorem that arbitrary source laws have the same observables; RPC-1 separately records operational-metric equivalence as an alternative to an independently derived metric and readout [14, Section 71.2, D12d]. The finite calculations here require no separately assigned primitive physical strength.

9.7 Joint source averaging and particle coarse graining

The source interaction and the fermion block map can be combined before a continuum limit is taken. Let θ have a normalised source measure μ , and write the fine fermion operator in retained and eliminated coordinates as $D(\theta) = [A \ B; C \ F]$, with retained field χ and eliminated field η . A source-dependent block map is included by a simultaneous unitary change of fermion and antifermion coordinates; its two Berezin Jacobians cancel. The retained coordinates must describe the same transported field throughout the source integral.

Proposition 8 (joint finite integration). Suppose the finite Grassmann coefficients are integrable with respect to μ . Integrating the source and the eliminated fermions in either order gives the same retained kernel. Wherever F is invertible, define $S = A - BF^{-1}C$ and $w = \det F$. Then

$$K(\bar{\chi}, \chi) = \int d\mu(\theta) \int d\bar{\eta} \, d\eta \exp[-\bar{\psi}D(\theta)\psi] = \int d\mu(\theta) \, w(\theta) \exp[-\bar{\chi}S(\theta)\chi]. \quad (35)$$

Proof. The finite Grassmann exponential has finitely many coefficients. Integrating each coefficient against μ commutes with extracting the coefficient selected by the η integral. Gaussian Grassmann elimination at fixed θ gives the second expression. At a singular F the first expression remains defined; the second is interpreted through the corresponding determinant minors rather than a divergent inverse, so invertibility is needed for the Schur representation, not for the coefficient-level order identity. This applies established finite integration algebra and is not a new general integration theorem. ■

When $K(0, 0) = \int w \, d\mu$ is nonzero, define the normalised retained action by $\exp(-S_{\text{eff}}^c) = K(\bar{\chi}, \chi)/K(0, 0)$. The source weight after fermion elimination is $w \, d\mu$, including its phase when present; it need not be a positive probability measure. Replacing the kernel by $\exp[-\bar{\chi}\langle S \rangle \chi]$, or deleting w before averaging, is generally a different calculation. For a source-linear fine operator the source-first route is the Gaussian quartic action of Section 9.2 followed by fermion integration; for nonlinear links all generated terms must be retained.

The leading coarse interaction can be written explicitly. Let θ be centred Gaussian with covariance $h\Sigma$, and let S_a and S_{ab} be its first and second derivatives at $\theta = 0$. Set $\ell_a = \partial_a \log \det F = \text{Tr}(F^{-1}F_a)$, evaluated on a nonsingular reference branch. Smoothness and control of the Gaussian remainder give, coefficient by coefficient,

$$S_{\text{eff}}^c = \bar{\chi}S_0\chi + (h/2)\Sigma_{ab}\Sigma_{ab}\bar{\chi}S_{ab}\chi + h\Sigma_{ab}\Sigma_{ab}\ell_a\bar{\chi}S_b\chi - (h/2)\Sigma_{ab}\Sigma_{ab}(\bar{\chi}S_a\chi)(\bar{\chi}S_b\chi) + O(h^2). \quad (36)$$

Expanding $w \exp(-\bar{\chi}S\chi)$ to second order and dividing by $\int w \, d\mu$ proves (36). The determinant-only terms cancel; their cross term with the particle response produces the ℓ_a term. The interaction is controlled by the response of the relaxed coarse operator, not by its direct compression. In fixed retained/eliminated coordinates, with $L = [I, -BF^{-1}]$ and $R = [I; -F^{-1}C]$,

$$S_a = L D_a R. \quad (37)$$

This follows by differentiating $A - BF^{-1}C$. If the block frame moves with θ , D_a in (37) is the derivative after that frame transformation, including the frame derivatives. Equations (35) to (37) connect the inherited source covariance to the coarse fermion interaction without introducing a primitive physical strength. They hold for a tensor covariance; the execution below tests one direction.

9.8 Execution on the inherited finite particle operator

We reuse the $\lambda = 1$, 128-component $SU(2)$ operator and gauge-covariant block map of Section 6.1, retaining 16 components and eliminating 112. The common link angle ε is perturbed about $\varepsilon_0 = 10^{-4}$ in both U_x and U_y of (12). Parallel transports are rebuilt at every angle and used to express the operator in one fixed retained frame, so the derivative includes the changing block map. No replacement random fermion matrix is used.

For the source probe, the $\ell = 1$ mode of (34) gives $\sigma = 6/19$, independently reproduced by inverting the coupled four-coordinate source metric. We set $\text{Var}(\theta) = h\sigma$ with dimensionless comparison value $h = 0.01$. Attaching this single mode to the cube fixture's ε direction is a declared test map: the wheel cycle and cube link pattern have not been identified by a physical source theorem, so the execution tests their conditional composition and does not certify a complete common-carrier Standard Model attachment.

The first execution uses the tangent operator $D_{\text{lin}}(\theta) = D_0 + \theta D_1$ in the transported frame, which makes the source-first integral exactly Gaussian with a quartic fine action. Its retained kernel coefficients are computed independently from the Gaussian moments of fine determinant-minor polynomials. The fermions-first route instead computes $S(\theta)$ and $\det F(\theta)$ at each source node and averages their retained coefficients. Sixty-six-point Gaussian quadrature is exact for the degree-at-most-128 polynomial integrands in exact arithmetic. Eight diagonal bilinear coefficients and four pair coefficients are tested, together with the vacuum coefficient; this is not an exhaustive enumeration of all retained Grassmann coefficients, for which Proposition 8 supplies the general identity.

The largest integration-order discrepancy is 2.94×10^{-14} . Direct full-operator integration and determinant-weighted blocked integration also agree for the partition function and a selected retained

two- and four-point function, with residuals below 3.6×10^{-15} ; these checks use the full inverse and the Schur inverse respectively.

A Gaussian retained kernel would obey pair factorisation. For all 120 pairs $i < j$ define Δ_{ij} as the weighted average of $S_{ii}S_{jj} - S_{ij}S_{ji}$ minus the same expression formed from the weighted mean S . The tangent execution returns $\|\Delta\|_2 = 0.00911960$; the nonzero value proves that the retained action contains a quartic term, which no single averaged quadratic operator can reproduce. Repeating the source average with the full nonlinear link-dependent operator returns $\|\Delta\|_2 = 0.00854444$, with 20- versus 28-node quadrature differences below 3.5×10^{-13} for the tested coefficient arrays. These are finite model coefficients, not experimental force strengths.

Deleting the eliminated determinant changes the tangent model's mean coarse operator by Frobenius norm 0.000123954, and its relaxed first vertex differs from direct compression by 0.000344050; the latter is a vertex comparison, not a claim that every projected pair coefficient changes by that amount. For successively halved source variances, the errors in the quadratic and connected-pair predictions of (36) decrease by factors approaching four, with final ratios 4.01024 and 4.00987, confirming the stated second-order remainder in this finite test.

Twenty-four named checks pass. The result closes the finite conditional composition of source averaging and particle blocking, including an explicit retained interaction and the contribution of eliminated particles to the source weight. The tangent approximation is not asserted to satisfy the exact Ginsparg–Wilson relation at finite θ . The nonlinear check retains the inherited completed operator, but no claim is made that an averaged interacting action is characterised by the quadratic Ginsparg–Wilson identity alone, nor that a single finite block establishes an interacting continuum limit.

9.9 Matching gauge stiffness and the interaction tensor

The physical comparison is between the gauge fluctuation kernel and the currents coupled to those same fluctuations. In matched dimensionless coordinates on the physical quotient, let the Gaussian density be proportional to $\exp[-\theta^T K \theta / (2h)]$ and let $B_i = \bar{\psi} D_i \psi$. Completing the square gives $\text{Cov}(\theta) = hK^{-1}$ and the induced action $-(h/2)B^T K^{-1} B$. This fixes the interaction tensor once the physical gauge kernel is supplied; no independent operational metric may be substituted for it.

If the charge-participation paper's G_{phys} [17, Section 5.6] is precisely this kernel on the same modes, then $\Sigma = G_{\text{phys}}^{-1}$. More generally a field carries spatial, temporal and generator indices: when $K = G_{\text{phys}} \otimes L$ on the physical transverse space, the propagator is $G_{\text{phys}}^{-1} \otimes L^{-1}$, and a selected local increment has covariance $hPK^{-1}P^T$. Constraints must be removed before inversion, or represented by the corresponding projected inverse. A twelve-dimensional sector matrix alone does not specify the entire link propagator.

Under the diagonal screened attachment $G_{\text{phys},A} = c_{\text{src}} Z_A r_A$, with all geometric kernels and normalisations held fixed, the sector current coefficient is proportional to $1/(c_{\text{src}} Z_A r_A)$, so its enhancement relative to the unscreened coefficient is $1/r_A$. For the charge-participation reductions these conditional factors are 1.06603, 1.04683 and 1.08799 for colour, weak and hypercharge. The factor 36 in $q = 6Y$ is compensated by the current and coordinate conversion and must not be inserted twice.

This provides a cross-paper consistency condition rather than an independent fit of interaction strengths. The one-mode example of Section 9.8 remains a declared execution because the matched physical kernel and its projection have not been evaluated there; replacing its variance by G_{phys}^{-1} without that calculation would rename the assumption. Conversely, once the admitted same-source kernel and projection are supplied, this Gaussian identity removes the freedom to choose that variance independently.

Finally, the determinant contributes $-\partial^2 L$ to the effective gauge action. If G_{phys} already includes this matter contribution it must not be added again. In a Wilsonian split, the determinant of eliminated fermions enters the gauge action while the retained fermions and their determinant are accounted for at the remaining stage; equation (14) enforces this accounting exactly, and under the same-source identification the κ -split of Section 6.2 is the quantity that decides how much of the matter contribution

G_{phys} already contains. The contact formula describes the linear current coupling; quadratic link vertices also produce bilinear terms, already present in Section 9.7. Neither these terms nor the spatial propagator may be discarded when comparing the complete action.

Table 8 supplies the explicit seven-site split on a three-dimensional compatible volume. In its probe convention the eliminated fermion contribution is $P = +0.253058$, with retained contribution -0.017957 . Those numbers specify what this fermion blocking has already integrated into the gauge action. A matched identification with G_{phys} must use the same modes and normalisation, so this term is neither omitted nor counted twice.

10 What this establishes for VERSF

The published chiral candidate admits an exact coarse description with its eliminated determinant retained. Zero modes and handedness are preserved whenever elimination exists, and the coarse chiral capacity reproduces the established index obstruction. For two-site free blocking, $\text{Re } F \geq I$ is a uniform invertibility theorem with exact corner values. Strong non-Abelian finite tests verify the same Schur, covariance and determinant identities well beyond the weak-field regime, and a seven-site star block compatible with $K = 7$ carries the Schur, chirality, index and determinant identities over unchanged, with the uniform average singled out by exact retention of the constant mode.

The source fluctuation and fermion calculations are joined by a determinant-weighted coarse action and an explicit relaxed interaction vertex. Source-first and fermions-first integration agree in the executed finite model, and a nonzero quartic coefficient remains after 112 of 128 fermion components are removed. The calculation therefore carries an interaction generated by the specified source fluctuations into the retained particle theory, with the contribution of the unresolved variables accounted for. Matched gauge fluctuations in turn determine the current interaction through the inverse physical kernel, which joins this paper to the gauge-response programme at one testable interface: the fermion interaction tensor and the screened gauge stiffness are the same data, and under the diagonal screened attachment the sector interaction coefficients are enhanced by exactly $1/r_A$.

The census-resolved determinant gives the exact representation proportions $6 : 6 : 10$. Matched-wavelength volume tests raise the slope from 0.5293 to 0.9006, and the three-momentum correction fit gives $s = 0.967064$, with Wilson control 0.974234. This provides a numerical fermionic transport-coefficient estimate from the inherited operator. The seven-site split quantifies the effect of eliminated modes on a compatible three-dimensional torus. The separate temporal execution replaces the free unit coefficient by the composition value and verifies a symmetric static extension, preserving the single free pole, the coarse chiral relation and the determinant accounting. These are completed calculations; the fitted running coefficient retains the finite-volume and correction-model scope stated in Section 6.3.

Table 11. Status of the derivation-ledger items touched by this paper.

Item	Before this paper	After this paper	Remaining obligation
Fermion kinetic term	Seed derived; completion declared	Exact coarse operator; composition-selected free temporal coefficient and tested static lift (Section 8.5)	Specified symmetric lift and overlap completion; time-dependent extension
Doublers and mirrors	Free one-pole result	One pole per representation after exact blocking, any b , any λ ; capacity bound	Interacting continuum limit
Chirality under coarse graining	Fine chiral relation	Chiral relation, zero modes and index preserved; uniform free invertibility	Global chiral measure phase
Continuum particle equation	Fine infrared symbol	Same pole after blocking; blocking error $O(h^2)$ relative	Fixed-cell limit; dynamical backgrounds
Source-induced interactions	Scalar illustration	Tensor from source metric; joint integration identity; retained quartic	Admitted same-source kernel and projection
Fermion contribution to gauge response	Inherited b_F from Route-M	$6 : 6 : 10$ factorisation; fitted $s = 0.967064$; seven-site 3D determinant split	Next: modes 4, 8, 16 on $256^3 \times 512$ ($kL \geq 8\pi$); additional momenta to test fit; full gauge-action matching
Fermionic transport coefficient	Imported continuum coefficient	Operator-based finite-fit $\hat{b}_F = (3.8683, 3.8683, 6.4471)$	Limiting coefficient and complete gauge, ghost and scalar assembly
Local anomaly conditions	SC-1 census supplied	Exact vanishing of $\text{Tr } Y$, $\text{Tr } Y^3$ and mixed Y traces on 45 states	Global chiral measure

The kinetic seed and quantum particle carrier are inherited VERSF results. Temporal completion, the one-mode source attachment and their physical grades are specified in Sections 2, 8 and 9, and the metric interpretation is consolidated in Section 9.6. The finite constructions do not supply a global chiral measure or the complete interacting continuum spectrum, and the finite $SU(2)$ and seven-site fixtures are not Standard Model lattice calculations. No new generation count, mass or gauge coupling is issued.

Appendix A Reproduction and source coverage

The smaller-volume reference data used to validate the response calculation are collected here. The main comparison and fitted coefficient are presented in Section 6.3.

Table A1. Unit-charge Hessian of the Weyl log-magnitude for a transverse plane-wave probe on $L^3 \times N_t$ tori, for the inherited hyperbolic overlap operator ($\lambda = 1$) and the Wilson control.

L	N_t	k	$\ln(1/k)$	Operator	H	$6\pi^2 \Pi$
8	16	0.7854	0.2416	hyperbolic overlap, $\lambda = 1$	+20.18000	-0.9459
8	16	0.7854	0.2416	Wilson control, $m = 0$	-25.57592	+1.1989
8	16	1.5708	-0.4516	hyperbolic overlap, $\lambda = 1$	+60.90076	-0.7137
8	16	1.5708	-0.4516	Wilson control, $m = 0$	-79.68353	+0.9338
8	16	2.3562	-0.8570	hyperbolic overlap, $\lambda = 1$	+104.43780	-0.5439
8	16	2.3562	-0.8570	Wilson control, $m = 0$	-112.03477	+0.5835
16	16	0.3927	0.9347	hyperbolic overlap, $\lambda = 1$	+13.31446	-0.3121
16	16	0.3927	0.9347	Wilson control, $m = 0$	-92.95457	+2.1786
16	16	0.7854	0.2416	hyperbolic overlap, $\lambda = 1$	+88.67517	-0.5196
16	16	0.7854	0.2416	Wilson control, $m = 0$	-296.87977	+1.7395

The accompanying archive contains `calculate.py`, `results.json` and a source manifest. Running the script with NumPy and SciPy performs 113 checks. It uses random seed 20261021, no measured Standard Model parameters and no D41 screening fractions. The free calculation covers 54 random momentum samples, three massless-origin tests, direct versus nested blocking and two refinement sequences. The finite background calculation covers $\lambda = 0, 0.5$ and 1, rebuilt gauge frames, moving Weyl projectors and the determinant factor. The index controls are explicitly algebraic fixtures. The separate archive `VERSF_Sequential_Defect_and_Polar_Completion_Test.zip` reproduces Sections 8.1 to 8.3 with its own `calculate.py`, `results.json`, derivation note and source manifest; its 587 assertions pass, many repeating the same identities on different inputs, and it uses seed 20261022. The executable SHA-256 is recorded in that archive's `results.json`. Each archive should be extracted into a separate folder.

`VERSF_Temporal_Refinement_Test.zip` reproduces Section 8.4 and the positive-transfer polar control in Section 8.3. It contains `calculate.py`, `results.json`, a derivation note and source text hashes; its 82 assertions use seed 20261022. Executable SHA-256:

`e752972dce901c19fdf2f43bc233fbae761557b479b5e9795bb2584eedbaf532`.

`VERSF_Record_Transport_Fermion_Action_Test.zip` reproduces the numerical tests in Sections 9.2 to 9.4. It contains `calculate.py`, `results.json`, a derivation note and the targeted corpus-search manifest. All twelve assertions pass; the numerical fixtures are deterministic. The manifest records a search of 600 corpus texts, with 28 files matching the record-to-fermion, Gaussian, coherent-state and polar search terms, followed by focused source reading and website checks. Executable SHA-256:

`c2171f41a698ab2d9d821cd4cde9919036646f046abf69ba99ac671a568dc430`.

Sections 9.5 and 9.6 use [14, Sections 64.4.1C–H, 67, 75–76] and [3, Sections 17.52–17.53, 17.72–17.82].

`VERSF_Source_Fluctuation_Fermion_Connection.zip` reproduces the Section 9.6 covariance evaluation and noncommuting control, with `calculate.py`, `results.json`, a derivation note and source-text hashes. Thirteen numerical checks pass; rational mode coefficients are evaluated with exact fractions. Executable SHA-256: `9c162fec72551ea19e6a0a45b8f82011dffa1c8eaa8fa0edf3a8c11e469c148e`.

`VERSF_Joint_Source_Fermion_Blocking_Test.zip` reproduces Sections 9.7 and 9.8 with `calculate.py`, the extracted `inherited_core.py`, `results.json` and a derivation note. Twenty-four named checks pass using NumPy and SciPy. The archive records the exact script and inherited-core SHA-256 hashes. The execution uses deterministic Gaussian quadrature and no measured Standard Model inputs; its wheel-to-cube one-mode attachment is explicitly declared.

`VERSF_Strong_Background_and_Free_Bound_Checks.zip` contains the strong-background execution, the unblocked and blocked comparison, the free minimisation probe, the analytic bound note and source hashes. Its deterministic strong-field and scaling execution passes 65 named checks; the minimisation supports the corner candidates but is not used as the proof of $\text{Re } F \geq I$.

`VERSF_Census_Determinant_and_Seven_Site_Block.zip` reproduces Sections 5.2, 6.2 and the algebraic checks in 9.9, with `calculate.py`, `inherited_core.py`, `results.json`, a derivation note and a source manifest. Its 49 named checks cover the census Gram, twelve diagonal determinant Hessians, selected mixed directions, pure-gauge invariance, determinant and propagator splitting, star refinement and the reciprocal-screening arithmetic. Finite differences use three specified steps; continuum beta coefficients are inherited from [8], not estimated by these checks. The SHA-256 identifiers of these two executables are recorded in their archives.

An independent script, `chiral_survival_check.py`, rebuilds the free family, the alias blocking, the 128-component $SU(2)$ fixture, the seven-site star cell and the momentum-chain vacuum-polarisation calculation from the formulas in this paper, without reference to the archives above, and reproduces the following: the free chiral relation; the Schur-versus-covariance and chiral residuals of Table 1 and the corner values of (11), together with a grid search confirming them; the nested-blocking identity; every entry of Tables 3 and 9, the three structured rows of Table 2 together with the convention dependence of the transport order, a Haar-random row of the same magnitude from its own sample, and the Section 5.2 star-block differences; the Section 8.2 differences and their final ratio; the Section 8.4 ratios and corrections; the formula (33) and the values (34); the Section 9.9 enhancement factors; and the chain-

decomposition validation and the smaller-volume rows of Table A1. It uses NumPy, SciPy and SymPy, seed 20261022 for the Haar configuration, and no measured inputs. Script SHA-256: ac95771f48270e1c6c4e72403d1cd9ced145bf0a03010812460457f5781b7509.

VERSF_Extended_Coarse_Grained_Response_23Oct2026.zip [18] independently reproduces selected Table A1 rows and supplies Tables 5 and 6. It contains calculate.py, calculate_split.py, wilson_analytic.py, verify.py, the inherited core, raw JSON results, 14 passing numerical checks and a SHA-256 manifest. The real-space, pure-gauge and full-versus-reduced-sum controls are recorded separately. Central differences at 0.004 and 0.002 are Richardson-extrapolated, with a further step-size comparison at $L = 16$. The analytic Wilson Hessian supplies an independent differentiation check. Printed digits are numerical estimates, not certified physical precision. No measured Standard Model strength or target slope is fitted.

calculate.py SHA-256: b81a6fd71ee062aa010f659aa5272f7b8766968b8899608ee7b5d2963e5c1073.

calculate_split.py SHA-256: 5f66fc2e7ce5c9c343c1025ad5bbb67cea8f8869cc3afd74988f83a643e7610a.

wilson_analytic.py SHA-256:

458318d8da25a320c5a14db33b3a34cf31618eed566ef67448ab09527a51d6a4.

VERSF_Matched_Wavelength_64cube_23Oct2026.zip [19] supplies Table 4. It contains analytic.py, inherited_core.py, verify_results.py, eight analytic response files, the earlier finite-difference reference outputs and implementation, comparison.json, 22 passing checks, environment information and a SHA-256 manifest. The analytic Hessian uses first and second spectral divided differences of sinh and sign. The controls compare full and reflection-reduced sums and reproduce the earlier finite-difference responses within the stated tolerance. Displayed digits are not a rigorous floating-point error bound. No measured strength or target slope is fitted.

analytic.py SHA-256: de8bedef72d9467b88f37f91e57ef4f88dc991550d8e0b917ebbf5edf47c6e3.

The source review searched the complete recovered corpus of 600 PDFs, comprising 22,429 pages, and then read the controlling fermion, chiral-regulator, refinement and later CLSN sections. This is a full-corpus text search with focused source reading, not a claim to have re-proved every paper or reread all pages. The manifest records the files, URLs and hashes; the references identify the controlling sections. The specific blocking execution was not located in the searched corpus. Residuals use $\|A - B\|_F / \max(1, \|B\|_F)$ unless stated otherwise; the chiral residual takes $A = D\gamma_5 + \gamma_5 D$ and $B = D\gamma_5 D$. The reported finite singular values are diagnostics; the free-domain existence and pole statements follow from the proofs, not from sampling.

VERSF_Running_Temporal_and_Star_Tests_23Oct2026.zip [20] contains analytic.py, wilson_analytic.py, temporal.py, star_split.py, analyse.py, the inherited core, raw responses and a SHA-256 manifest. The fit uses three fixed momenta without measured force inputs. The temporal tests use exact rational anomaly traces and independent Haar seed 20261023. The seven-site calculation uses a compatible $28^3 \times 56$ torus with 7^3 controls. All 38 recorded checks pass. The fit interpolation residual is not a goodness-of-fit test. The baseline and composition-selected operators are kept distinct in the archive and in the paper.

References

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