

Fact Momentum, Record Overlap and the Emergence of Stable Aggregates in VERSF

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General reader summary

Every completed event in VERSF leaves a record. This paper asks whether those records, together with the field that completed events produce, can hold things together in stable clumps without any rule that favours complexity.

Two ingredients are tested. Completed events act as sources of a field that pulls like sources together over a distance, and keeping records consistent costs something when two sources sit on top of each other, which gives a push at very short range. A pair of sources then settles at a definite separation for a wide range of settings, and the paper gives the exact setting below which the pair collapses. A pair is not enough, though: a crowd of sources could still fall into a heap. The paper finds the exact strength of the push above which no crowd of any size can collapse, and shows it is always higher than the strength a pair needs, so there are settings in which pairs form but large collections do not last. Computer runs of clumps of up to twelve sources, and of larger crowds, behave as the two thresholds predict.

Locking a record once its event is complete does more than stiffen the push. A clump whose records are locked is held in the shape it had when the records were written, and two such clumps can stick together without losing their shapes, where in every case tried clumps with adjustable records merged. The same pattern of adjustable against locked appears in the programme's own equations for how records and currents respond to a change of the field, so the clumping mechanism and the force calculations share one structure.

What is not shown: that the ingredients come out of the full VERSF rules with the needed sizes, or that any of this reaches the strengths of the known forces. The paper lists the calculations that would decide each point.

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Abstract

We ask whether irreversible committed records and the field they source can produce stable aggregation without a complexity objective. A positive quadratic record sector with a relaxable record variable yields, after elimination, a pair cross-energy. We prove that this cross-energy is a pure overlap penalty only for an ultralocal record kernel: any gradient stiffness in the record sector, of the kind the record-composition defect of the master action supplies, converts part of it into a Yukawa attraction of squared mass $(\lambda + a)/\alpha$, while locking the record raises the overlap coefficient from $\lambda a/(\lambda + a)$ to λ . Combined with the finite-width scalar attraction, the pair potential has a coincident configuration that is a local maximum if and only if the coupling ratio exceeds an exactly computed threshold (0.374 at relative width 0.4), and a repulsive core above a second threshold (0.676). We then prove that the N-body energy is bounded below by a multiple of N if and only if the ratio exceeds a third threshold, $1/(2\sqrt{\pi} \mu^2 s^3)$ (3.306 at width 0.4), that the three thresholds are strictly ordered for every width, and that the gradient stiffness can deepen or weaken the pair well, depending on its range, without moving the many-body threshold. Locking the records of a formed aggregate leaves its energy and forces unchanged at the moment of commitment and adds a positive kernel that pins every constituent to its record, with an explicit stiffness; in every tested encounter two preformed six-clusters merge when their records are relaxed and associate with their internal structure preserved to five per cent when they are locked. Cluster minima for $N = 2$ to 12 and energies up to $N = 96$ behave as the thresholds predict.

The ordering of relaxed against locked response is a Loewner inequality for any positive quadratic parent, and the same inequality governs the native polar record block and the current–gauge mixed block of the source. We restate the inherited current–gauge identities with their sources, give the $K = 7$ six-cycle inverse response with a conditional 75:25 harmonic projection and the factor-separable obstruction to unequal sector corrections, write the normalised one-opportunity current–pointer–record law of the QCA-1 instrument with its Fisher Hessian, show by a two-bit example that marginals do not fix the mixed block while profiling can remove its effect, and fix the definition of the completed-history duration $\langle \tau \rangle = 21.2776829509186$ by reproducing it from the archived D36 law. No physical gauge coupling or energy closure is derived; the acceptance tests for each are stated.

1 Purpose and position

The second law permits local complexity but does not select particular structural histories. One alternative to a built-in complexity objective is that past irreversible commitments modify later interactions. We test a minimal sequence: committed sources produce a field; the field favours association; record consistency penalises excessive overlap; the combined potential may allow finite-size bound configurations. A second, harder question is whether such bound units can preserve their identities and assemble hierarchically.

The paper has one organising principle. In VERSF a record is written when an event completes and is fixed thereafter (the Fact-Lock Principle [12, Section 17.85]), while before completion the record variables are

relaxable. Every quadratic response in this paper is therefore computed twice, with the record free and with the record held, and the two are ordered. Sections 2 to 5 apply this to a candidate aggregation mechanism at the level of a pair; Section 6 proves the exact condition under which the same mechanism is stable for any number of constituents; Section 7 computes what locking the records does to a formed aggregate; Section 8 executes the cluster calculations. Section 9 shows that the same two-valued structure is already present in the source's polar record block and in its current–gauge mixed block, with the sources of each inherited identity named. Sections 10 to 12 collect the finite results on the $K = 7$ cycle response, the marked event law and completed histories that constrain how the aggregation ansatz could be attached to the source. Section 13 states the falsifiers.

The aggregation action of Sections 2 to 8 is a test ansatz. Its coefficients are model inputs, not quantities extracted from the $K = 7$ completion action, and nothing below claims otherwise. What is established is which of its qualitative features are forced by positivity and locality, which depend on parameters, and where the complete source would have to be evaluated to decide them.

2 Field-mediated attraction from committed sources

Adopt the quadratic static energy functional for a real scalar field κ coupled to a source ρ ,

$$E[\kappa, \rho] = \frac{1}{2} \langle \kappa, A\kappa \rangle - \langle \kappa, \rho \rangle, \quad A = -\nabla^2 + m_\kappa^2. \quad (1)$$

For fixed ρ and decaying boundary conditions, stationarity gives $A\kappa = \rho$. Eliminating κ gives $E_{\text{eff}}[\rho] = -\frac{1}{2} \langle \rho, A^{-1}\rho \rangle$. For two nonnegative pointlike source strengths q_a and q_b at separation r in three dimensions, the cross-energy is

$$V_\kappa(r) = -q_a q_b \exp(-m_\kappa r) / (4\pi r). \quad (2)$$

The negative sign is the statement that like-signed sources of a positive-operator scalar field attract. It is the whole of the attractive mechanism, and it is conditional: whether all VERSF commitment sources carry the same coupling sign is not established here, and Section 13 lists it as the first falsifier. The field-mediated contribution is also not the total energetic cost of forming a commitment, and the point-source idealisation is singular as r tends to zero. A previously explored VERSF parameterisation sets $m_\kappa^2 = 4/(3\xi^2)$; we use it as a model input, so that $\mu = m_\kappa \xi = 2/\sqrt{3}$, and the physical status of the scale ξ requires independent confirmation.

3 The record sector as a relaxed Schur complement

3.1 The ultralocal kernel

Let $R(x)$ be a relaxable record-response variable and $\rho(x)$ a smooth commitment-source density. The minimal positive quadratic candidate for a record-consistency and response-stiffness sector is

$$\Gamma_{\text{rec}}[R, \rho] = (\lambda/2) \int (R - \rho)^2 d^3x + (a/2) \int R^2 d^3x, \quad \lambda, a > 0. \quad (3)$$

Stationarity in R gives $R^* = \lambda\rho/(\lambda + a)$, and substitution gives $\Gamma_{\text{rec,eff}}[\rho] = (c_{\text{rec}}/2) \int \rho^2 d^3x$ with $c_{\text{rec}} = \lambda a/(\lambda + a)$. This is the Schur complement of the two-coordinate block $[[\lambda, -\lambda], [-\lambda, \lambda + a]]$ on (ρ, R) after eliminating R . For $\rho = \rho_1 + \rho_2$, subtracting the two isolated self-energies leaves the pair cross-term $c_{\text{rec}} \int \rho_1 \rho_2 d^3x$, which is positive for overlapping nonnegative sources. This is Result 1. Its derivation uses only the two quadratic terms in (3); the next subsection shows that it does not survive the addition of a spatial stiffness.

3.2 Gradient stiffness converts overlap cost into attraction

Proposition 1 (record kernel with gradient stiffness). Add to (3) the term $(\alpha/2)\int |\nabla R|^2 d^3x$ with $\alpha > 0$. Then the effective kernel after eliminating R has Fourier symbol

$$C(k) = \lambda(a + \alpha k^2)/(\lambda + a + \alpha k^2) = \lambda - \lambda^2/(\lambda + a + \alpha k^2), \quad (4)$$

and real-space form

$$C(x) = \lambda\delta(x) - (\lambda^2/\alpha) \exp(-m|x|)/(4\pi|x|), \quad m^2 = (\lambda + a)/\alpha. \quad (5)$$

For two separated smooth sources the cross-energy is $\lambda\int \rho_1\rho_2 - (\lambda^2/\alpha)\langle \rho_1, G_m\rho_2 \rangle$: a contact repulsion of strength λ and a Yukawa attraction of range $1/m$. The ultralocal result is the limit $\alpha \rightarrow 0$, in which $m \rightarrow \infty$ and $(\lambda^2/\alpha)G_m \rightarrow \lambda^2\delta/(\lambda + a)$, recovering $c_{\text{rec}} = \lambda a/(\lambda + a)$.

Proof. Stationarity gives $(\lambda + a - \alpha \nabla^2)R = \lambda\rho$. Substituting into Γ_{rec} gives $\frac{1}{2}\langle \rho, [\lambda - \lambda^2(\lambda + a - \alpha \nabla^2)^{-1}]\rho \rangle$, which is (4) in Fourier space. The inverse of $\lambda + a - \alpha \nabla^2$ is α^{-1} times the Yukawa Green function of mass m , which gives (5). ■

The master action's record-composition defect compares the record transports on adjacent edges [13, Section 15], which is a lattice gradient on the record field. A record sector faithful to the master action therefore has $\alpha > 0$, and its pair response is not a pure overlap penalty: it repels at contact and attracts at separations of order $1/m$. The sign of the record cross-energy at finite separation is consequently not fixed by positivity of the record sector. This sharpens the caveat of Result 1 into a condition: the overlap penalty is the ultralocal projection of the record sector, and the composition stiffness decides how much of it survives. Section 6.3 carries the stiffness through the binding calculation.

3.3 Locking increases the retained response

Lemma 1 (relaxed and locked record). Let Γ be a positive quadratic form on (ρ, R) . The retained form on ρ obtained by eliminating R is bounded above, in the Loewner order, by the form obtained by holding R fixed. For (3) the two values are $c_{\text{rel}} = \lambda a/(\lambda + a)$ and $c_{\text{lock}} = \lambda$; for the kernel (5) they are $C_{\text{rel}} = \lambda\delta - (\lambda^2/\alpha)G_m$ and $C_{\text{lock}} = \lambda\delta$.

Proof. The Schur complement of a positive block subtracts a positive semidefinite term from the retained block. ■

Under Fact-Lock the record of a completed event is fixed. A locked record therefore resists overlap more strongly than a relaxable one, and the relaxed coefficient of Result 1 is a lower bound on the overlap cost of committed sources, not an upper bound. The aggregation mechanism is strengthened, not weakened, by the irreversibility of records. Which records are locked at the moment two sources approach each other is a question about the event sequence that the ansatz does not answer; Section 7 computes the consequences of locking once the sequence is given.

4 Finite-width sources and the pair potential

Adopt normalised Gaussian sources of physical width σ , $\rho_i(x) = q(2\pi\sigma^2)^{-3/2} \exp[-|x - x_i|^2/(2\sigma^2)]$. For separation r their overlap integral is

$$\int \rho_1\rho_2 d^3x = q^2 \exp[-r^2/(4\sigma^2)]/(8\pi^{3/2}\sigma^3), \quad (6)$$

so the ultralocal record contribution is $V_{\text{rec}}(r) = c_{\text{rec}} q^2 \exp[-r^2/(4\sigma^2)]/(8\pi^{3/2}\sigma^3)$. The attraction is the Yukawa Green function convolved with both Gaussian profiles. With $u = r/\xi$, $s = \sigma/\xi$ and $\mu = m_\kappa \xi$, in the convention where the far field is $\exp(-\mu u)/u$,

$$Y_s(u) = \exp(\mu^2 s^2)/(2u) \{ \exp(-\mu u) \text{erfc}(\mu s - u/(2s)) - \exp(\mu u) \text{erfc}(\mu s + u/(2s)) \}, \quad (7)$$

$$Y_s(0) = 1/(\sqrt{\pi} s) - \mu \exp(\mu^2 s^2) \text{erfc}(\mu s). \quad (8)$$

With normalised amplitudes $B > 0$ and $A > 0$ the dimensionless pair potential is

$$V(u) = B \exp[-u^2/(4s^2)] - A Y_s(u). \quad (9)$$

The ratio B/A is set by the record stiffness, the field coupling and the source width; it is not determined by the convolution. For unit source strength, $B = c_{\text{rec}}/(8\pi^{3/2}s^3)$ and $A = g^2/(4\pi)$, where g is the coupling of a unit source to the field, both in the energy unit of the field. With $\alpha > 0$ in Section 3.2 a second smeared Yukawa term is added to (9); Sections 5 and 6.1 treat the ultralocal case and Section 6.3 the general one.

5 Pair binding across the parameter space

The potential (9) has two exact thresholds in B/A , both computable from (8).

Proposition 2 (collapse and core thresholds). The coincident configuration $u = 0$ is a local maximum of V if and only if

$$B/A > (B/A)_c = 1/(3\sqrt{\pi} s) - (2\mu^2 s^2/3) Y_s(0), \quad (10)$$

and $V(0) > 0$ (a repulsive core) if and only if $B/A > Y_s(0)$.

Proof. The smeared kernel satisfies $(-\nabla^2 + \mu^2)Y_s = 4\pi g$, where g is the Gaussian of variance $2s^2$ with $g(0) = 1/(8\pi^{3/2}s^3)$. At the origin $\nabla^2 Y_s(0) = \mu^2 Y_s(0) - 1/(2\sqrt{\pi} s^3)$, and for an isotropic smooth function the radial second derivative is one third of the Laplacian. The Gaussian term contributes $-B/(4s^2)$ to $V''(0)/2$. The origin is a local maximum when $V''(0) < 0$, which rearranges to (10). The second statement is $V(0) = B - A Y_s(0)$. ■

Table 1 gives the thresholds for three widths and the location and depth of the interior minimum for four coupling ratios. For every ratio above the collapse threshold the interior minimum is the global minimum, the equilibrium separation grows slowly with B/A and the binding depth decreases. Table 2 gives the reference example, which lies in the middle of this range.

Table 1. Thresholds and pair minima of (9) for $\mu = 2/\sqrt{3}$. The collapse threshold is (10); the core threshold is $Y_s(0)$. Minima from bounded one-variable minimisation.

s	Collapse threshold $(B/A)_c$	Core threshold $Y_s(0)$	B/A	V(0)	u_{min}	$V(u_{\text{min}})$
0.3	0.5414	1.0680	0.8	-0.2680	0.6060	-0.3717
			1.0	-0.0680	0.7449	-0.3163
			1.5	+0.4320	0.9303	-0.2471
			2.0	+0.9320	1.0332	-0.2131
0.4	0.3740	0.6764	0.8	+0.1236	1.1262	-0.1455
			1.0	+0.3236	1.2587	-0.1240
			1.5	+0.8236	1.4545	-0.0964
			2.0	+1.3236	1.5704	-0.0825

s	Collapse threshold $(B/A)_c$	Core threshold $Y_s(0)$	B/A	$V(0)$	$u_{\min}$	$V(u_{\min})$
0.5	0.2737	0.4609	0.8	+0.3391	1.6841	-0.0606
			1.0	+0.5391	1.8217	-0.0514
			1.5	+1.0391	2.0341	-0.0395
			2.0	+1.5391	2.1640	-0.0334

Table 2. The reference example, $s = 0.4$ and $A = B = 1$.

Quantity	Calculated value
$V(0)$	+0.3236485
$V(0.5)$	+0.1289908
$V(1)$	-0.0983899
$u_{\min}$	1.2587166
$V(u_{\min})$	-0.1240260
$V(2)$	-0.0593303
$V(3)$	-0.0129140
$V(\infty)$	0

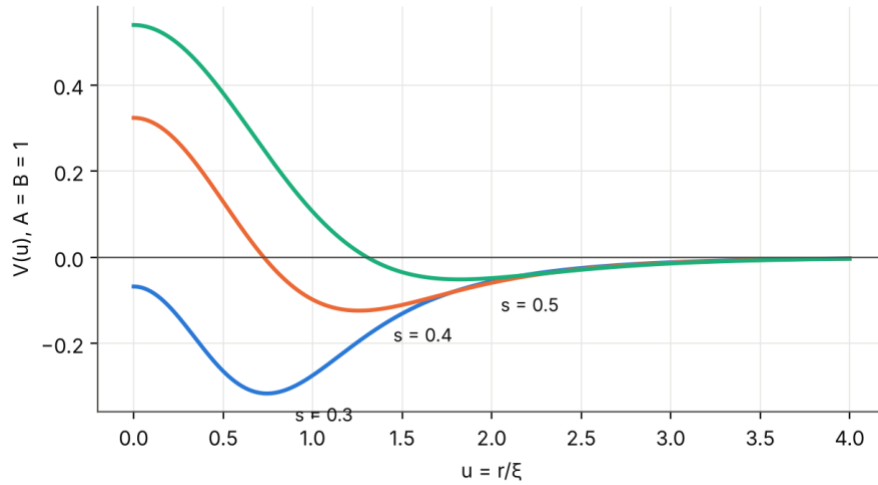


Figure 1. The pair potential (9) at $A = B = 1$ for the three widths of Table 1. Each curve has a finite-separation global minimum; the $s = 0.3$ curve has an attractive core ($V(0) < 0$) and still binds at $u > 0$, as Proposition 2 predicts.

Two qualifications. First, for $s = 0.3$ and B/A at or below $Y_s(0)$ the core is attractive, yet the interior minimum is still global; a positive core is not required for a finite-separation pair, only the curvature condition (10). Second, these are statements about a two-body radial potential. Whether many constituents interacting through (9) are stable is a separate question with a separate threshold, which the next section answers exactly.

6 Many-body stability

For N constituents interacting pairwise through (9) the energy is $E_N(x_1, \dots, x_N) = \sum_{i < j} V(|x_i - x_j|)$. The classical notion of stability for such a system is that $E_N \geq -bN$ for some constant b , for every N and every configuration [17, 18]. Without it, suitably dense configurations have energies that fall like $-N^2$, so that no collection of many constituents has a finite energy per constituent and no large aggregate, however it is assembled, can be stable. With it, the energy per constituent is bounded below for every N ; that is a thermodynamic statement, and it does not by itself say that an aggregate of a given N has a stable equilibrium geometry, which is a separate question that Section 8 addresses numerically for N up to 12. A pair minimum says nothing about either property, and the reference example of Table 2 turns out to lack the first.

6.1 The exact threshold

Proposition 3 (many-body stability threshold). The three-dimensional Fourier transform of the pair potential (9) is

$$\tilde{V}(k) = 4\pi \exp(-s^2 k^2) [2\sqrt{\pi} B s^3 - A/(k^2 + \mu^2)]. \quad (11)$$

The potential is stable in the sense above if and only if

$$B/A \geq 1/(2\sqrt{\pi} \mu^2 s^3), \quad (12)$$

and when (12) holds, for every N and every configuration,

$$E_N \geq -N V(0)/2 = -(N/2)[B - A Y_s(0)]. \quad (13)$$

Proof. The Gaussian term of (9) has transform $8\pi^{3/2} B s^3 \exp(-s^2 k^2)$. The smeared Yukawa is the Yukawa Green function, with transform $4\pi/(k^2 + \mu^2)$, convolved with two unit Gaussians of variance s^2 , each with transform $\exp(-s^2 k^2/2)$; this gives (11). The bracket in (11) is increasing in k^2 and the prefactor is positive, so $\tilde{V}(k) \geq 0$ for every k if and only if $\tilde{V}(0) \geq 0$, which is (12). If $\tilde{V} \geq 0$ then $\sum_{i,j} V(x_i - x_j) = \int \tilde{V}(k) |\sum_i \exp(ik \cdot x_i)|^2 d^3 k / (2\pi)^3 \geq 0$, and subtracting the N diagonal terms gives (13) [17]. Conversely, if $\tilde{V}(0) = \int V d^3 x < 0$, place N constituents independently and uniformly in a ball Ω whose volume $|\Omega|$ is large compared with the range of V . The expected energy is $N(N-1)|\Omega|^{-2} \int_{\Omega} \int_{\Omega} V(x-y) d^3 x d^3 y / 2 = N(N-1)[\int V d^3 x + o(1)] / (2|\Omega|)$, which is below $-cN(N-1)$ for a $c > 0$ independent of N ; some configuration does at least as well, so E_N/N is unbounded below [18, Section 3.2]. ■

Condition (12) says that the integrated repulsion $8\pi^{3/2} B s^3$ must be at least the integrated attraction $4\pi A/\mu^2$. By Lemma 2 below the many-body threshold exceeds the core threshold, so every stable member of the family (9) has a repulsive core and the bound (13) is strictly negative, proportional to the height of that core. Table 3 gives the three thresholds for the widths of Table 1 together with the pair minimum at the many-body threshold itself: pair binding persists there, at a larger separation and a smaller depth than in Table 1. The reference example $B/A = 1$ at $s = 0.4$ binds a pair but is not stable for large N . The example $B/A = 4$ at the same width lies above the threshold: its pair minimum is $u_{\min} = 1.8018$ with $V(u_{\min}) = -0.0599 A$, and (13) gives $E_N \geq -1.6618 A N$ for every N . Figure 2 shows the three thresholds across the width range.

Table 3. The three thresholds of (9) for $\mu = 2/\sqrt{3}$, and the pair minimum at B/A equal to the many-body threshold (12).

s	Collapse (10)	Repulsive core $Y_s(0)$	Many-body stability (12)	$V(0)$ at (12)	$u_{\min}$ at (12)	$V(u_{\min})$ at (12)
0.3	0.5414	1.0680	7.8360	+6.7680	1.3790	-0.1257
0.4	0.3740	0.6764	3.3058	+2.6294	1.7434	-0.0650

s	Collapse (10)	Repulsive core $Y_s(0)$	Many-body stability (12)	$V(0)$ at (12)	$u_{\min}$ at (12)	$V(u_{\min})$ at (12)
0.5	0.2737	0.4609	1.6926	+1.2317	2.0904	-0.0368

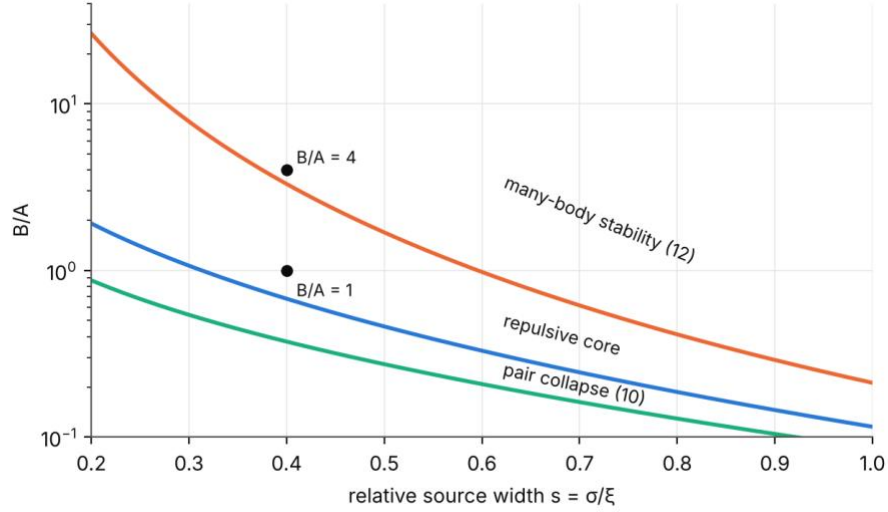


Figure 2. The three thresholds of (9) against the relative source width. Below the lowest curve the pair collapses; between the lowest and the highest a pair binds but a large enough collection does not; above the highest curve every N is stable. The two marked points are the examples of Tables 2 and 5.

6.2 Ordering of the thresholds

Lemma 2 (three strictly ordered thresholds). For every width $s > 0$,

$$(B/A)_c < Y_s(0) < 1/(2\sqrt{\pi} \mu^2 s^3). \quad (14)$$

Proof. For the second inequality, $Y_s(0) = \int g(x) \exp(-\mu|x|)/|x| d^3x$ with g the Gaussian of Proposition 2, and $0 \leq g(x) \leq g(0)$ with strict inequality away from the origin, so $Y_s(0) < g(0) \int \exp(-\mu|x|)/|x| d^3x = 4\pi g(0)/\mu^2 = 1/(2\sqrt{\pi} \mu^2 s^3)$. For the first, write $x = \mu s$ and $e(x) = \exp(x^2) \operatorname{erfc}(x)$; with (8) and (10) the inequality is equivalent to $\sqrt{\pi} e(x) < 2(1 + x^2)/[x(3 + 2x^2)]$. The right side is the third convergent of Laplace's continued fraction for $\sqrt{\pi} e(x)$ [19, 7.1.14], whose convergents bound the function alternately from above and below for $x > 0$, the third from above. Appendix A confirms the inequality numerically on x from 10^{-3} to 10^2 . ■

Three regimes therefore exist for every width. Below $(B/A)_c$ the pair collapses. Between $(B/A)_c$ and the many-body threshold a pair binds, with an attractive core below $Y_s(0)$ and a repulsive one above it, while a sufficiently large collection collapses. Above the many-body threshold every N has a bounded energy per constituent. The width of the intermediate window depends strongly on the source width: the ratio of the core threshold to the many-body threshold is 0.022 at $s = 0.1$, 0.205 at $s = 0.4$, 0.545 at $s = 1$ and 0.958 at $s = 5$. For wide sources the three thresholds coalesce, because the smeared attraction becomes a Gaussian of the same width as the overlap term and the potential is a single bump whose sign, curvature at the origin and integral change together; for narrow sources pair binding is cheap and many-body stability is expensive. The physical source width is therefore not a detail of the ansatz but the quantity that decides whether pair binding and many-body stability can be had at the same coupling.

6.3 The gradient-stiffened potential and the source-level condition

Proposition 4 (binding and stability with gradient stiffness). With the record kernel (5), the field attraction (2) and Gaussian sources of relative width s , the pair potential is

$$V(u) = B_\lambda \exp[-u^2/(4s^2)] - A Y_{\mu,s}(u) - D Y_{v,s}(u), \quad v^2 = (\lambda + a)\xi^2/\alpha, \quad (15)$$

with $Y_{m,s}$ the kernel (7) of mass m , $B_\lambda = B(\lambda + a)/a$ and $D = 2v\pi s^3 v^2 (\lambda/a) B$, where $B = c_{\text{rec}} q^2 / (8\pi^{3/2} s^3)$ is the ultralocal amplitude of (9). Its Fourier transform is

$$\tilde{V}(k) = 4\pi \exp(-s^2 k^2) [2\sqrt{\pi} B_\lambda s^3 - D/(k^2 + v^2) - A/(k^2 + \mu^2)], \quad (16)$$

in which both subtracted terms decrease with k^2 , and $2v\pi B_\lambda s^3 - D/v^2 = 2v\pi B s^3$. The potential (15) is therefore stable for every N if and only if the ultralocal amplitude satisfies (12), whatever the value of α : the gradient stiffness does not move the many-body threshold. In the variables of the record sector the condition reads

$$\lambda a / (\lambda + a) \geq g^2 / m_\kappa^2 = 3g^2 \xi^2 / 4, \quad (17)$$

for unit source strength: the relaxed overlap coefficient must be at least the integrated field attraction.

Proof. The record cross-energy of two Gaussian sources under (5) is λ times the overlap (6) minus (λ^2/α) times the Yukawa Green function of mass m convolved with both profiles, which is (15) with $D = \lambda^2 q^2 / (4\pi \tilde{\alpha})$ and $\tilde{\alpha} = \alpha/\xi^2$; the stated forms of B_λ and D follow from $c_{\text{rec}} = \lambda a / (\lambda + a)$ and $v^2 = (\lambda + a)/\tilde{\alpha}$. Each term of (16) is the transform of the corresponding term of (15). The symbol (4) satisfies $C(k) \geq C(0) = c_{\text{rec}}$, and $-A/(k^2 + \mu^2)$ is increasing in k^2 , so the bracket of (16) is increasing and its sign is decided at $k = 0$, where it equals $2v\pi B s^3 - A/\mu^2$. The same statement in the kernel variables is $\exp(-s^2 k^2)[C(k) - g^2/(k^2 + m_\kappa^2)] \geq 0$ for all k if and only if $C(0) \geq g^2/m_\kappa^2$, which is (17). ■

Table 4 carries the stiffness through the binding calculation at the reference width, with $a/\lambda = 1/2$ and the ultralocal amplitude held fixed, so that the many-body margin $2v\pi B s^3 - A/\mu^2$ is the same in every row (-0.5231 for $B/A = 1$, $+0.1575$ for $B/A = 4$). The finite-separation minimum survives for every v and returns to the ultralocal values as $v \rightarrow \infty$, but its depth is not monotone in the stiffness. At $B/A = 4$ every finite v binds more deeply than the ultralocal case (from -0.060 A to -0.229 A at $v = 2$). At $B/A = 1$ the well deepens for $v \geq 1$ (to -0.156 A at $v = 2$) and is shallower than the ultralocal -0.124 A at $v = 0.5$, where it is -0.098 A: there the contact height has tripled to $B_\lambda = 3B$ while the record attraction, of range 2ξ , is spread too thinly to compensate at the pair separation. The limit $v \rightarrow 0$ at fixed B sends D to zero and the range to infinity, leaving the contact repulsion and the field attraction alone, so a weakly stiffened record sector always binds less than the ultralocal one. The curvature condition at the origin is satisfied in every row. The question left by Section 3.2 is therefore answered within the ansatz: the composition stiffness can strengthen or weaken pair binding, according to whether the range of the record attraction is short or long compared with the pair separation, and it leaves the condition for many-body stability untouched either way. The last column anticipates Section 7.

Table 4. The potential (15) at $s = 0.4$, $a/\lambda = 1/2$ and $A = 1$, with the ultralocal amplitude B fixed; $v = \infty$ is the ultralocal case of Table 1. k_{pin} is the pinning stiffness (20).

B/A	v	B_λ	D	V(0)	u_{min}	V(u_{min})	k_{pin}
1	0.5	3	0.1134	+2.2095	1.6563	-0.0979	0.157
	1	3	0.4537	+1.9880	1.5688	-0.1302	0.555
	2	3	1.8150	+1.5391	1.4428	-0.1562	1.621

B/A	v	B _λ	D	V(0)	u _{min}	V(u _{min})	k _{pin}
	4	3	7.2600	+0.9685	1.3298	-0.1529	3.439
	8	3	29.040	+0.5624	1.2736	-0.1366	5.093
	∞	1	0	+0.3236	1.2587	-0.1240	0
4	0.5	12	0.4537	+10.8672	1.9392	-0.1262	0.629
	1	12	1.8150	+9.9811	1.7943	-0.2034	2.219
	2	12	7.2600	+8.1854	1.6520	-0.2294	6.482
	4	12	29.040	+5.9030	1.5976	-0.1526	13.757
	8	12	116.16	+4.2787	1.6812	-0.0830	20.374
	∞	4	0	+3.3236	1.8018	-0.0599	0

7 Fact-Lock as kinetic protection

Lemma 1 compared the relaxed and locked record coefficients for a given configuration. The physical sequence is different: a configuration forms while its records are relaxable, and the records are then locked at their relaxed values when the events complete. Let $x^L = (x_1^L, \dots, x_N^L)$ be the configuration at commitment, ρ_L its source density and $R_L = R^*(\rho_L)$ the record it leaves. For the record sector of Proposition 1 with the Gaussian sources of Section 4, write $E_{\text{rel}}(x)$ for the energy with the record relaxed and $E_{\text{lock}}(x)$ for the energy with the record held at R_L ; the field κ remains relaxed in both, since the lock applies to records and not to fields.

Proposition 5 (Fact-Lock pinning). (a) At $x = x^L$ the two energies and their gradients coincide: locking changes neither the energy nor the forces at the moment of commitment. (b) For every later configuration,

$$E_{\text{lock}}(x) - E_{\text{rel}}(x) = \frac{1}{2} \sum_{i,j} D [Y_{v,s}(|x_i - x_j|) - 2Y_{v,s}(|x_i - x_j^L|) + Y_{v,s}(|x_i^L - x_j^L|)] \geq 0, \quad (18)$$

which is $\frac{1}{2}(\rho - \rho_L, K_v(\rho - \rho_L))$ for the positive-type record Yukawa kernel K_v of (5). (c) The Hessian at the lock configuration increases by a positive semidefinite matrix,

$$H_{\text{lock}} = H_{\text{rel}} + D \mathcal{Y}, \quad \mathcal{Y}_{ij} = -\nabla \nabla Y_{v,s}(x_i^L - x_j^L), \quad \mathcal{Y}_{ii} = (k_{\text{pin}}/D) I, \quad (19)$$

$$k_{\text{pin}} = (D/3)[1/(2\sqrt{\pi} s^3) - v^2 Y_{v,s}(0)] > 0. \quad (20)$$

In particular the six rigid modes of the relaxed aggregate acquire positive stiffness: a locked aggregate is pinned to the position and orientation at which its records were written, and each constituent is pinned individually to its own record with stiffness k_{pin} . (d) Displacing the whole aggregate rigidly by d costs $\sum_{i,j} D [Y_{v,s}(|x_i^L - x_j^L|) - Y_{v,s}(|x_i^L - x_j^L + d|)]$, which increases to the total record energy $\sum_{i,j} D Y_{v,s}(|x_i^L - x_j^L|)$ as $|d| \rightarrow \infty$. (e) Since $E_{\text{lock}} \geq E_{\text{rel}}$ everywhere, the many-body condition (12) for the relaxed system remains sufficient after locking.

Proof. With $R_L = \lambda(\lambda + a - \alpha \nabla^2)^{-1} \rho_L$, the locked energy of the record sector is $\frac{1}{2} \lambda \int \rho^2 - \lambda \int \rho R_L + \frac{1}{2} \int [(\lambda + a) R_L^2 + \alpha |\nabla R_L|^2]$, and the relaxed energy is the same expression with R_L replaced by $R^*(\rho)$. Their difference is $\frac{1}{2} \lambda^2 (\rho - \rho_L, (\lambda + a - \alpha \nabla^2)^{-1} (\rho - \rho_L))$, which is nonnegative and, for Gaussian sources, equals (18) with the D of (15). It vanishes together with its gradient at $\rho = \rho_L$, which gives (a). Differentiating (18) twice at $x = x^L$ gives (19); the $i = j$ blocks are $-D \nabla \nabla Y_{v,s}(0) = -D Y_{v,s}''(0) I$, and the Laplacian identity in the proof of Proposition 2, with μ

replaced by v , gives (20); positivity of k_{pin} is the second inequality of Lemma 2 with μ replaced by v . (d) is (18) at $x_i = x_i^L + d$, and (e) follows from (b) and (13). ■

For the six-constituent minimum of (15) at $s = 0.4$, $B/A = 4$ and $v = 2$ ($D = 7.2600$, $k_{\text{pin}} = 6.482$), the relaxed cluster has energy $E_6 = -3.0284$ A, six zero modes and a softest internal mode of stiffness 1.735; after locking, every one of its eighteen modes has stiffness at least 4.346, the former translations at 4.346 and the former rotations at 5.311, and the eigenvalues of D^2U lie between 4.346 and 10.990. The locked Hessian (19) agrees with central differences of the locked energy to 7×10^{-5} .

Three consequences follow. First, binding and commitment are distinct events: the configuration must first be bound by the relaxed potential, since locking does not move it, and locking then freezes what the relaxed dynamics produced. Second, the record attraction that helped form the aggregate is converted, not lost: between mobile constituents only the contact repulsion $\lambda\delta$ remains, which is Lemma 1, while the attraction reappears as an external restoring potential toward the recorded configuration, which is the kinetic persistence that a record of a completed event should confer. Third, the result is stated for the ansatz's spatial record density, and its physical reading depends on the carrier question of Section 9.1: if the locked record density is anchored in space, constituents are pinned to the place of the event; if it co-moves with the aggregate, the pinning acts on the internal configuration only. Both readings protect the internal structure, and Section 8.2 uses the second for the cluster test. Equation (18) is the complete energy difference within the ansatz, including the stored record energy $\frac{1}{2}\sum_{i,j} D Y_{v,s}(|x_i^L - x_j^L|)$ and the work of (d); what it omits is whatever the complete source adds to the record sector beyond (3) and the gradient term.

8 Clusters: from a pair to a hierarchy

A stable pair at finite separation is necessary but insufficient. To demonstrate a hierarchy of interacting aggregates, a many-body calculation must establish: (i) normal-mode stability of finite clusters after removing translational and rotational zero modes; (ii) identifiable internal records that survive a cluster–cluster encounter; (iii) a secondary local minimum or kinetic protection separating associated clusters from complete rearrangement; and (iv) a conservative or driven energy and entropy balance for the actual commitment process. Coarse-grained effective source charges alone do not guarantee any of the four. This section executes (i) to (iii) for the ansatz; (iv) remains open.

8.1 Finite clusters

Table 5 gives the lowest energy found for $N = 2$ to 12 constituents under (9) at $s = 0.4$ for the two examples of Section 6, with the energy per constituent, the energy to remove one constituent, the softest nontrivial normal mode and the mean nearest-neighbour separation. Each minimum is the best of 60 ($N \leq 6$) or 150 ($N \geq 7$) random starts together with starts seeded from the $(N - 1)$ -body minimum, relaxed by a quasi-Newton method with the analytic gradient; they are the lowest minima found, not certified global minima. Every minimum is normal-mode stable, with the rigid modes at zero to 10^{-7} and the softest nontrivial mode positive. The removal energy grows with N at both ratios, so every cluster in the table is bound against losing a constituent.

The two ratios differ in the way the thresholds predict. At $B/A = 1$, below the many-body threshold, the energy per constituent falls steadily and the nearest-neighbour separation shrinks from the pair value 1.259 to 1.009 by $N = 12$: the clusters compress as they grow. At $B/A = 4$, above the threshold, the energy per constituent falls more slowly and the separation shrinks only from 1.802 to between 1.64 and 1.69 at $N = 11$ and 12. Table

6 and Figure 3 continue both sequences to $N = 96$ with sixteen random starts at each size, which gives upper bounds on E_N rather than minima. At $B/A = 1$ the bounds follow $E_N/N \approx -0.153 - 0.0335 N$, that is $E_N \approx -0.0335 N^2$, and the nearest-neighbour separation falls to 0.44: this is the N^2 collapse of Proposition 3 in operation. At $B/A = 4$ the bounds follow $E_N/N \approx -0.733 + 1.25 N^{-1/3}$, a bulk energy with a surface correction, every value lying above the bound (13), $E_N/N \geq -1.662$.

Table 5. Cluster minima of (9) at $s = 0.4$. Energies in units of A ; stiffnesses in units of A/ξ^2 ; the removal energy is $E_{N-1} - E_N$.

N	E_N (B/A = 1)	E_N/N	Removal	Softest mode	Mean nn	E_N (B/A = 4)	E_N/N	Removal	Softest mode	Mean nn
2	-0.1240	-0.0620	0.1240	1.212	1.259	-0.0599	-0.0299	0.0599	0.948	1.802
3	-0.3721	-0.1240	0.2481	0.909	1.259	-0.1797	-0.0599	0.1198	0.711	1.802
4	-0.7442	-0.1860	0.3721	0.606	1.259	-0.3593	-0.0898	0.1797	0.474	1.802
5	-1.1798	-0.2360	0.4356	0.267	1.192	-0.5537	-0.1107	0.1943	0.281	1.783
6	-1.7451	-0.2908	0.5653	0.557	1.200	-0.7974	-0.1329	0.2437	0.417	1.774
7	-2.3213	-0.3316	0.5762	0.194	1.121	-1.0331	-0.1476	0.2357	0.274	1.767
8	-2.9934	-0.3742	0.6721	0.021	1.052	-1.2826	-0.1603	0.2495	0.075	1.748
9	-3.7348	-0.4150	0.7415	0.134	1.057	-1.5504	-0.1723	0.2678	0.177	1.736
10	-4.5421	-0.4542	0.8072	0.108	1.006	-1.8509	-0.1851	0.3005	0.209	1.720
11	-5.4555	-0.4960	0.9135	0.089	1.015	-2.1637	-0.1967	0.3128	0.011	1.641
12	-6.4007	-0.5334	0.9452	0.068	1.009	-2.5392	-0.2116	0.3755	0.159	1.692

Table 6. Upper bounds on E_N/N for larger N (sixteen random starts), with the mean nearest-neighbour separation of the configuration found.

N	E_N/N , B/A = 1	Mean nn	E_N/N , B/A = 4	Mean nn
16	-0.6815	0.892	-0.2445	1.679
24	-0.9582	0.787	-0.2971	1.638
32	-1.2278	0.729	-0.3350	1.598
48	-1.7624	0.622	-0.3840	1.556
64	-2.2961	0.536	-0.4196	1.538
96	-3.3632	0.441	-0.4685	1.514

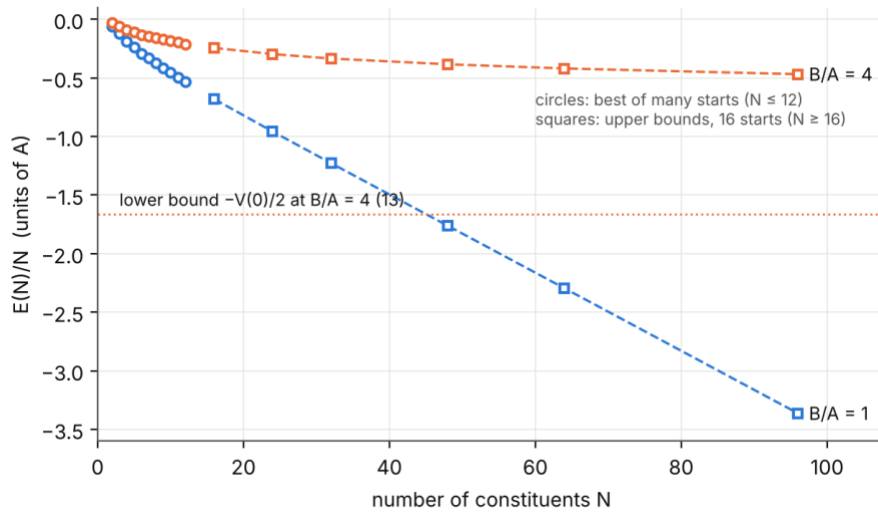


Figure 3. Energy per constituent against N for the two examples at $s = 0.4$. Below the many-body threshold the energy per constituent falls linearly in N ; above it the values approach a bulk limit and stay above the bound (13).

8.2 Cluster-cluster encounters

Requirements (ii) and (iii) were tested by placing two copies of the six-constituent minimum in contact, one randomly rotated, at a centroid separation between 0.8 and 1.2 pair lengths beyond touching, and relaxing the twelve-body energy. Identity is said to be kept when every internal distance of each cluster changes by less than ten per cent of its smallest internal distance. Table 7 gives the outcome for the two ultralocal examples, for the gradient-stiffened potential (15) at $B/A = 4$ and $v = 2$, and for the same gradient-stiffened potential with the records of both clusters locked at their formation configurations in the co-moving reading of Section 7, each cluster's record image being carried rigidly with it and the relaxation running over the twelve positions and the two rigid record frames.

Table 7. Two preformed six-clusters brought into contact at $s = 0.4$. The last column is the energy of the associated configuration relative to the two clusters at infinite separation, in units of A .

Potential and record state	Trials	Identity kept	Smallest internal distortion	Outcome	Association energy
(9), $B/A = 1$, relaxed	20	0	53%	all 20 reach the twelve-body minimum	-2.911
(9), $B/A = 4$, relaxed	20	0	20%	8 reach the twelve-body minimum, 12 other compact minima	-0.944 to -0.712
(15), $B/A = 4$, $v = 2$, relaxed	20	0	12%	9 reach the twelve-body minimum, 11 other compact minima	-3.371 to -2.507
(15), $B/A = 4$, $v = 2$, locked	12	12	1.3% to 4.8%	two clusters in contact, centroid separation 2.44 to 2.52	-2.380 to -2.278

With relaxed records no trial keeps identity under any of the three potentials: two touching six-clusters rearrange into a compact twelve-body configuration, and the gradient stiffness, which adds a second attractive range but no second repulsive one, did not change this in any tested encounter. With locked records every trial keeps identity. The two clusters settle in contact with internal distances within five per cent of their formation values, each constituent within 0.05ξ of its record, bound to each other by $2.3 A$ relative to

separation, and the associated configuration is a strict local minimum of the energy over the twelve positions and two record frames, with six zero modes for the overall rigid motion and a softest nontrivial mode of 0.525. The locked association is 1.0 A less bound than the relaxed merger, which is the price of keeping the internal records.

8.3 Status of the four requirements

(i) is established numerically for $N = 2$ to 12 at both coupling ratios; Proposition 3 decides which ratios give a bounded energy per constituent for every N , and the two statements are complementary, since neither implies the other. (ii) and (iii) are established, for the tested configurations, only with Fact-Lock: Proposition 5 supplies the kinetic protection, and the locked encounter of Table 7 realises it. With relaxed records every tested encounter ended in merger. That is evidence against a relaxed hierarchy in this ansatz, not a proof of its absence: the trials cover two six-clusters in contact at one width and three parameter points, and a metastable associated configuration elsewhere on the landscape, for other cluster sizes, widths or ratios, is not excluded by them. A potential with one repulsive core and monotonically decaying attraction offers no obvious barrier between association and merger, and a wider search would have to find one. The controls that the protocol requires, with the field coupling disabled and with the record overlap disabled, follow from the structure: without the field attraction (9) is purely repulsive and nothing binds, and without the record sector there is no contact repulsion, no lock and no protection. (iv), the energy and entropy balance of the commitment process itself, is not addressed by any calculation in this paper.

9 The same structure in the source: record and current blocks

The relaxed-versus-locked ordering of Lemma 1 is not peculiar to the ansatz. It is the structure of two blocks already present in the VERSF source, which this section restates with their origins so that the aggregation sector can be compared with them without conflating carriers.

9.1 The native polar record block

The Post-Commitment Atomic-Operation programme identifies a polar bond–record-phase block in its unit branch [14, Section 46R],

$$H_{(q,y)} = [[3, -1], [-1, 2]], \text{ relaxed } 3 - 1/2 = 5/2, \text{ locked } 3. \quad (21)$$

Allowing the record phase y to respond softens the bond mode from 3 to 5/2; holding a committed record fixed retains 3. This is Lemma 1 on a two-coordinate carrier. The record phase here is the phase tangent of $\text{Pol}(\Phi)$ and the bond is the compatibility of that record with the physical link; the aggregation sector's R is a spatial density of commitment response. The two share the Schur structure, not the carrier, and establishing a common carrier and spatial kernel is part of the derivation that would attach Section 3 to the source. The same carrier question decides between the two readings of Proposition 5.

9.2 The current–gauge mixed block

The constitutive current residual of the bosonic parent [13, Section 13; 6] is

$$D_J = J - \kappa(\rho_t - U(A)\rho_s), \quad \Gamma_J = \frac{1}{2}\langle D_J, W_J D_J \rangle. \quad (22)$$

At a common defect-zero background with fixed endpoint states, define $v_A = (\partial U / \partial A_A)\rho_s$, so that $\delta D_J = \delta J + \kappa v_A \delta A_A$. The blocks of the isolated current term are $H_{AA} = \kappa^2 v_A^\dagger W_J v_A$, $H_{AJ} = \kappa v_A^\dagger W_J$ and $H_{JJ} = W_J$. In the left-exponential link convention $v_A = i T_A U_0 \rho_s$, so the representation enters through v_A and the mixing is present at

source level. These are the gauge-to-cycle-current Jacobian and the current-and-record parent of the current-conservation paper [5, Section 13], and the following three statements are inherited from there and from Proposition 53 of [15].

If the current is free, its Schur reduction cancels the isolated gauge contribution exactly: $\kappa^2 v_A^\dagger [W_J - W_J W_J^{-1} W_J] v_A = 0$. If the rest of the same action supplies an independent positive current stiffness K_J , the retained contribution is $\kappa^2 v_A^\dagger [W_J - W_J (W_J + K_J)^{-1} W_J] v_A$; for the scalar diagnostic $W_J = 2$, $K_J = 3$, $\kappa v_A = 1$ the parent is $[[2, 2], [2, 5]]$ and the retained value is $2 - 4/5 = 1.2$, an illustration and not a coupling. At a source-driven background the Gauss–Newton term is not the Hessian; with configuration coordinates z , residual $d(z)$, metric $W(z)$, fixed residual source h and potential $V(z)$, the complete second derivative of $\Gamma = V + \frac{1}{2} d^T W d - h^T d$ is

$$H_{ij} = V_{ij} + d_i^T W d_j + (W d - h)^T d_{ij} + d_i^T W_j d + d_j^T W_i d + \frac{1}{2} d^T W_{ij} d, \quad (23)$$

with further terms when h , the support, the polar frame, the measure or the fermion determinant vary [7, Section XIII.4]. The current-only cancellation therefore does not determine the complete source-driven Hessian.

9.3 One ordering for all three blocks

Lemma 3 (ordering of retained forms). For a positive quadratic parent on (a, x) , with a retained and x auxiliary, the retained form satisfies

$$0 \leq K_{\text{free}} \leq K_{\text{constrained}} \leq K_{\text{fixed}}, \quad (24)$$

where K_{free} eliminates x without restriction, $K_{\text{constrained}}$ eliminates x against an additional positive stiffness, and K_{fixed} holds x at its background value.

Proof. Each step restricts the set over which the positive action is minimised. ■

The three instances in this paper are: the record sector of Section 3 ($c_{\text{rel}} \leq c_{\text{lock}}$, and $C_{\text{rel}} \leq C_{\text{lock}}$ with gradient stiffness, realised configuration by configuration in Proposition 5); the polar record block ($5/2 \leq 3$); and the current–gauge block ($0 \leq 1.2 \leq 2$ in the diagnostic). The companion paper on nonlinear history response proves the same ordering for the relaxed, cycle-shared and fully constrained responses of the scalar parent [11, Lemma 2]. In every case the physical question is which variables are relaxable at the moment of the perturbation, and that is a question about the event sequence and Fact-Lock, not about the quadratic algebra.

10 The $K = 7$ cycle response and a factor-separable obstruction

The exact closed-history response of the seven-state wheel has six cycle directions with positive regulated kernel $K_{\text{cyc}} = \text{diag}(k_1, k_2, k_2, k_4, k_4, k_5)$ in the orthonormal harmonic frame, and its inverse is the Gaussian exchange kernel when the cycle field is integrated at the specified action level [4, Sections 3 and 5]. The four distinct eigenvalues are

$$\begin{aligned} k_1 &= 2/[\rho(1+4\rho)(31+28\rho)] + 3/[16(1+\rho)^2(1+2\rho)^2(1+4\rho)(5+4\rho)], \quad k_2 = 2/[\rho(1+2\rho)(31+28\rho)], \\ k_4 &= 2/[\rho(1+\rho)(31+28\rho)], \quad k_5 = 10/[\rho(5+4\rho)(31+28\rho)]. \end{aligned} \quad (25)$$

Table 8. Six-cycle kernel and inverse response at the inherited diagnostic point $\rho = 0.4$.

Mode	$k(0.4)$	Inverse response
Uniform, $m = 0$	0.0472911550	21.14560321
First harmonic, two modes	0.0658241180	15.19200000
Second harmonic, two modes	0.0846310088	11.81600000
Alternating	0.0897601609	11.14080000

The native current-cycle cochains place $3/4$ of their squared norm in the uniform harmonic and $1/4$ in the $m = 2/4$ harmonic [16, Section 24.29]. If those two harmonics are identified with the corresponding cycle modes and their score gain is held common, the conditional inverse response is

$$\chi_{\text{cyc}} = (3/4)/k_1 + (1/4)/k_4 = 18.8132024102 \quad (\rho = 0.4). \quad (26)$$

This is a mode-projection susceptibility obtained by weighting two inverse mode responses with the cochain norm fractions. The weights are a choice of observable, not an identity, and the two harmonic constructions have not been attached by the same physical current source and regulator; (26) is reported as the value that attachment would have to reproduce or replace.

With a factor-separable source attachment $B_A = b \otimes T_A$, the same χ_{cyc} multiplies every representation trace, so $Z_3 : Z_2 : Z_q = 6 : 13/2 : 378$ in the primitive convention $q = 6Y$, or (6, 6.5, 10.5) after dividing the hypercharge trace by 36. This is the exact obstruction to deriving unequal sector corrections from a universal scalar record overlap: a record sector of the form of Section 3, attached through a single spatial kernel, rescales all three sectors together. Any unequal correction must enter through a representation-dependent block, and Sections 3 and 9 contain none.

11 The marked event law and its information

The QCA-1 conditional current attachment uses a retained current coordinate j , the twelve faithful gauge generators T_a and the terminal projectors P_r [8]. On the normalised current orbit, let $p_j = jj^\dagger/(j^\dagger j)$, $K_a(j) = i[T_a, p_j]$, $S_j = \sum_a K_a^\dagger K_a$, $s_j = \text{tr}(S_j)/2$, $C_0(j) = (I - S_j/s_j)^{1/2}$ and $C_a(j) = K_a(j)/\sqrt{s_j}$ for $a = 1$ to 12, on the stratum $s_j > 0$. Then $\sum_{a=0}^{12} C_a^\dagger C_a = I$ and $\sum_r P_r = I$, and for a normalised current-carrier wavefunction $\Psi(j)$ the one-opportunity probability of current j , pointer a and terminal record r is

$$p(j, a, r) = \|P_r C_a(j) \Psi(j)\|^2, \quad \int d\mu(j) \sum_{a,r} p(j, a, r) = 1. \quad (27)$$

No Born weight is inserted beyond the specified isometry; the mixed-state form is $\text{tr}[P_r C_a p(j) C_a^\dagger P_r]$. The instrument is gauge-intertwining on its carrier. It does not select the physical current propagation or the record and field energy exchange.

With $N = j^\dagger j$ and tangent δj , the projector derivative is $\delta p_j = (\delta j j^\dagger + j \delta j^\dagger)/N - jj^\dagger(j^\dagger \delta j + \delta j^\dagger j)/N^2$, and $\delta K_a = i[T_a, \delta p_j]$. For positive $p(j, a, r; \theta)$ the score $s_\alpha = \partial_\alpha \log p$ gives the Fisher block $G_{\alpha\beta} = \int d\mu \sum_{a,r} p s_\alpha s_\beta$, unique once the prepared joint state, the instrument and the parameter path are specified. It is not an action Hessian merely because both are positive quadratic forms.

Proposition 6 (marginals do not fix the mixed block). For two binary marks $a, b = \pm 1$ with law $p(a, b \mid u, v, c) = [1 + au + bv + abc]/4$ on its positive region, the marginals $p(a \mid u) = (1 + au)/2$ and $p(b \mid v) = (1 + bv)/2$ are independent of c , while at $u = v = 0$

$$G(c) = [[1, -c], [-c, 1]]/(1 - c^2), \quad G_{uu} - G_{uv}^2/G_{vv} = 1 \text{ for every } c. \quad (28)$$

At $c = 0.6$ the diagonal entries are 1.5625 and the off-diagonal -0.9375 . Identical marginals therefore admit different mixed information responses, so the cross block of a joint law cannot be reconstructed from separately fixed marginals. In this example the profiled information in u is independent of c : the mixed block is present but has no effect on the retained u -response once v is profiled. That is a property of this law, not a general rule; in general profiling changes the retained response by G_{uv}^2/G_{vv} , and whether a mixed block in the physical joint law alters the retained gauge coefficients has to be computed from that law.

12 Completed-event histories

The native D36 closed-cover history is the first return to the starting state after all seven wheel states have been visited, with the starting state drawn from the stationary law. From the archived D36 transition law its expected duration is

$$\langle \tau \rangle = 21.2776829509186 \text{ primitive updates.} \quad (29)$$

The definition matters: with the hub as anchor the same chain gives 19.6731992 from a stationary start and 20.6188715 from the hub, and a rim anchor started on itself gives 21.4316407; only the stationary average over returns to the starting state gives (29), and Appendix A reproduces it. For the stopped-history experiment the score increments are martingale differences, so its information metric is $G_{\text{stop}} = \langle \tau \rangle G_{\text{step}}$. This is a common multiplier for the specified statistical experiment; it fixes neither the four-sector energy exchange nor a gauge-coupling conversion. The same source reports that a distinct conditional no-reinjection temporal evolution reduces the D41 bath-only percentages, which is why the temporal response has to be selected rather than assumed [9].

The completed-event energy calculation finds a field-energy gain $\Delta\langle E_{\text{field}} \rangle = 0.169012 \pm 0.011330$ in its three-direction quadratic branch, with the uncertainty a Monte Carlo standard error on that branch [10]. Its source does not determine the compensating current, record, completion, interaction and matter energies; conservation imposes a total but cannot supply the missing dynamics by subtraction. The evolving joint-event law is the interleaving of the action-selected joint propagator with the native instruments, for a history $h = (h_1, \dots, h_n)$,

$$P_{\text{phys}}(h) = \text{tr}[I(h_n) \circ U_{\text{phys}} \circ \dots \circ I(h_1) \circ U_{\text{phys}}(\rho_0)], \quad (30)$$

where U_{phys} is the complete physical update, not yet selected, and the stopping rule and its completion projector are applied when the record stopping rule is part of the event. Equation (30) specifies the remaining calculation; it is not a result for P_{phys} .

13 Falsifiers and acceptance tests

The aggregation proposal fails as a derived VERSF mechanism if any of the following holds.

- Commitment sources do not carry a common coupling sign to the completing field, so that (2) is not attractive between generic committed events.
- The projected record and completion Hessian of the source has no positive contact cross-response, or its composition stiffness α is large enough that the record Yukawa of Proposition 1 dominates at all separations where the field attraction acts.

- The ratio B/A fixed by the source lies below the collapse threshold (10) at the physical source width, so the pair collapses.
- The ratio fixed by the source lies between the collapse threshold and the many-body threshold (12) at the physical width, so that pairs bind but no large aggregate is stable; by Lemma 2 this window exists at every width and is wide for narrow sources.
- The locked record density of a completed event is neither anchored in space nor carried with the aggregate in the sense of Proposition 5, so that locking supplies no pinning and the cluster encounter of Table 7 has only its relaxed outcome.
- The bound states are unstable once currents and matter are included, or the length scale ξ is inconsistent with the binding it is asked to explain.

For the attachment of Sections 9 to 12 to physical gauge coefficients, the acceptance tests are those of the series, restated for the present objects.

- Test A, same-parent mixed derivative: compute $\partial^2 \Gamma / \partial A_A \partial (J, R, C, M)$ at a source-selected background using the complete form (23), including measure and time dependence.
- Test B, instrument and action matching: derive the current–carrier–record operation of Section 11 from the evolving action, or prove the equivalence of the coarse gauge observable to the QCA-1 instrument.
- Test C, restricted Schur reduction: enforce Fact-Lock, gauge and Gauss constraints, eliminate only relaxable modes in the sense of Lemma 3, and retain determinant and contact contributions.
- Test D, exchange to gauge action: establish the low-momentum transverse kinetic term, the field and current normalisation and the refinement limit before identifying Z_3, Z_2, Z_q .
- Test E, event-energy closure: compute the current, record, completion, matter and interaction energies independently rather than as an imposed residual of the field energy.
- Test F, blind return: once A to E pass, compute the sector ratios without reference to their experimental targets.

The collective D41 branch proposes (5.63, 6.21, 9.65) in the normalised convention and other finite parents give other unequal reductions [5, 11]; nothing above selects among them. The record overlap of Section 3 is a distinct candidate that must either be recovered as the short-distance projection of the complete six-defect source or rejected as a reading of the same record dynamics.

14 Conclusions and results ledger

The record sector of a positive quadratic action produces an overlap cost when it is ultralocal and relaxable, a larger cost when locked, and a contact repulsion with a Yukawa attraction when it carries the gradient stiffness that the master action's composition defect implies. Combined with the field attraction of committed sources it binds a pair at finite separation for all coupling ratios above the curvature threshold (10), and its N -body energy is bounded below in proportion to N if and only if the ratio exceeds the many-body threshold (12), which the gradient stiffness does not move; the finite clusters computed for N up to 12 are normal-mode stable at both of the ratios examined. The two thresholds are strictly ordered at every width, so the ansatz has a regime in which pairs form and large aggregates do not, and the source width decides how wide that regime is. Locking the records of a formed aggregate leaves its energy and forces unchanged and pins every constituent to its record; this is what allowed two aggregates to associate without merging in the tested encounters, in which the relaxed potential produced merger every time. The same relaxed-versus-locked ordering governs the source's own record and current blocks, which is the structural link between the

aggregation ansatz and the gauge-response programme; the carrier link is not yet established. The finite results on the cycle response, the marked event law and completed histories constrain the attachment without selecting it.

Table 9. Results ledger.

Item	Status	Established	Remaining
Field attraction (2)	Model input	Like-signed scalar sources attract with range $1/m_k$	Sign and scale of commitment sources in the source
Overlap cost (3)	Conditional	Relaxed coefficient $\lambda a/(\lambda + a)$; locked coefficient λ	Extract λ , a from the completion Hessian
Gradient stiffness (Proposition 1)	New	Contact repulsion $\lambda\delta$ and Yukawa attraction of mass ² $(\lambda + a)/\alpha$; ultralocal limit recovers Result 1	Value of α from the composition defect
Pair thresholds (Proposition 2)	New	Exact collapse and core thresholds; global interior minimum above collapse threshold at the widths tested	Physical B/A and s
Many-body stability (Proposition 3)	New	Exact threshold (12), necessary and sufficient; bound (13)	Physical B/A and s
Ordering of thresholds (Lemma 2)	New	Collapse < core < many-body for every width; window closes as s grows	Physical s
Stiffened potential (Proposition 4)	New	Three-term potential (15); threshold unchanged by α ; source-level condition (17); pair well deepened or weakened according to the range $1/v$	Values of λ , a , α , g from the source
Fact-Lock pinning (Proposition 5)	New	Energy and force preserved at commitment; positive kernel (18); pinning stiffness (20); rigid modes stiffened	Anchored or co-moving record carrier
Finite clusters (Table 5)	New, numerical	Normal-mode stable minima $N = 2$ to 12 at both ratios, a separate fact from (12); N^2 collapse below (12) and bulk limit above it to $N = 96$	Certified global minima; physical parameters
Hierarchy (Table 7)	Conditional on Fact-Lock	Relaxed records: merger in every tested encounter; locked records: association with identity kept in every trial	Wider search for relaxed metastable associations; energy and entropy balance of commitment (iv)
Record block (21)	Inherited [14]	Relaxed 5/2, locked 3	Common carrier with the spatial record density
Current–gauge block (22), (23)	Inherited [5, 7, 15]	Mixed block at source level; free-current cancellation; constrained restoration; full Hessian	Same-parent evaluation at a physical background
Ordering (Lemma 3)	New statement	One Loewner ordering for all three blocks	Which variables are relaxable at the event
Cycle response and χ (26)	Inherited [4] and conditional	Factor-separable attachment preserves 6 : 13/2 : 378	Attach harmonics by the same source and regulator
Marked law (27), Proposition 6	Inherited [8] and new	Normalised law and Fisher block; marginals do not fix the mixed block; profiling can remove it	Compute the mixed block of the physical joint law

Item	Status	Established	Remaining
Completed histories (29)	Inherited [9], definition fixed	$\langle \tau \rangle$ reproduced; alternative definitions distinguished	Select U_{phys} in (30)
Physical couplings	Not derived	None issued	Tests A to F

Appendix A Verification

Every number in Tables 1 to 9, the data of Figures 1 to 3 and the duration (29) were produced by a NumPy and SciPy implementation of the procedures below, which runs in about five minutes. Its inputs are $\mu = 2/\sqrt{3}$, the widths and ratios of the tables, $a/\lambda = 1/2$ for Table 4, and fixed random seeds; no measured quantity enters.

Pair potential. The kernels (7) and (8) are evaluated through the scaled complementary error function; the procedure minimises (9) and (15) on the interval $[0.02, 8]$ to 10^{-10} in u , and checks the analytic threshold (10) against the numerical crossover of the global minimum (agreement to 1×10^{-4} in B/A , the residual being the difference between the curvature criterion and the equal-depth criterion). It checks the Fourier symbol (11) against numerical quadrature of $V(u) u^2 \sin(ku)/(ku)$ at $k = 0, 0.5, 1, 2$ and 4 (agreement to 10^{-11}), the ordering (14) on s from 10^{-2} to 10^2 and the continued-fraction inequality on x from 10^{-3} to 10^2 , the symbol (4) at its two limits, and the identity $2\sqrt{\pi} B_\lambda s^3 - D/v^2 = 2\sqrt{\pi} B s^3$ row by row in Table 4.

Clusters. The N-body energy uses the analytic first derivative of (7); the second derivative is a central difference of the first with step 10^{-5} . The analytic gradient agrees with central differences of the energy (step 10^{-6}) to 10^{-9} . Minima are found by L-BFGS-B with gradient tolerance 10^{-10} from the random and seeded starts described in Section 8.1; Hessians are assembled analytically from the pair blocks and their rigid modes are reported. The locked Hessian (19) is checked against central differences of the locked energy (step 10^{-4}) to 7×10^{-5} , and the positivity of (18) is checked on 200 random displacements. The encounter tests of Table 7 use the identity criterion stated in Section 8.2; the locked encounter relaxes the twelve positions and the two record frames (translation and rotation vector) with numerical gradients, and the Hessian of that 48-coordinate energy at the first converged trial is formed by central differences.

Source quantities. The procedure evaluates the diagnostic parent of Section 9.2, the record block (21), the kernel (25) and projection (26), the Fisher matrix (28), and the expected cover-and-return time of the archived D36 law under the four definitions quoted in Section 12 by solving the linear system on the 448 states (current vertex, visited set). The transition law and stationary distribution are the arrays of the signed-current archive of [5].

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