

From Fold to Three Generations in VERSF

The Fold-to-K7 stitching, identity, relational, scale and coherence programmes, the species and generation census, and the bridge between them

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Claim boundary. Within an explicitly declared six-port attachment class, this paper selects the Fold-to-K7 attachment law and its order-seven structure, types the six trapped Fold faces to the six private rim continuations of the seven-site wheel with exact D_6 covariance, fixes the physical first-moment matter writer and shows that its leading matter mode is the $k = \pm 1$ wheel doublet, and reduces the remaining internal preparation problem to the spectrum of one source-derived pricing operator. It proves a timeless stable-presence theorem that calculates the largest admitted invariant support orthogonal to a source-defined absence sector, separates structural presence from commitment status and from historical record, and shows that two preserved sharp marginals force coefficient-free joint record preservation. It recovers the seven input history addresses from the recorded marks of the declared instrument, evaluates the complete relational response of the canonical wheel, and proves exact preservation of that response under retained-boundary elimination and correlated reversible refinement. It returns a conditional Planck-equivalent metric-scale closure, an admission-weight-preserving coherence attachment, an exact transport-comparison probability, a first-hit reduction of the running gate, a rigidity theorem with an explicit counterfamily, and a corrected two-scale cosmological bridge, each at its stated conditional grade. By a logically separate argument, and under the stated census premises C1 to C15 with the rank-three premise C9 inherited rather than proved, it derives the minimal anomaly-stable matter census, reproducing the classical hypercharge result, and exactly three complete generations. It then proves that the sixfold Fold carrier has a canonical antipodal three-object quotient carrying the full S_3 permutation action, that the source-selected family map $Q(z) = z^2$ factors through that quotient, and that the three terminal record keys returned by the PFDML instrument carry the same permutation type. Given the identification of family classes with terminal keys, which is stated as a separate premise B1 rather than drawn from the census premises, the admissible bridge is unique up to a phase and a relabelling; the physical test of that premise is stated and is currently failed by the EX2E implementation. No measured mass, mixing angle, gauge coupling or neutrino mass is used, and none is derived; no Fold diameter, compulsory Planck ratio or pre-fact duration is asserted.

General Reader Summary

This paper starts from the smallest structure VERSF proposes, a Fold, which is the least thing that can be told apart from its own absence, and follows it through to the matter we see. It asks how such a structure attaches to its surroundings, how its presence and identity can be tested without assuming a clock or a grid, how far apart two such structures can be said to be, what sets its size, and why the particles built on it come in the kinds and the three copies that they do.

The first answers are about shape. Once the structure attaches to its surroundings in the simplest way consistent with a few stated rules, it has six faces around a centre and a fixed sevenfold arithmetic that follows from the rules rather than being put in. Each face connects to one of six continuation routes, and the pattern that carries matter on the structure is the one that goes once around the rim. Presence is then tested in a way that needs no prior time: a structure is stable if none of the allowed ways the world can continue leaks anything out of it, and the paper gives a finite procedure that finds every such stable sector. A record that a commitment happened, a label saying where a history started, and the structure itself are shown to be three different things that must not be confused.

The next answers are about distance and size. Two structures can be related without that relation having any particular length, so the paper keeps three ideas apart: whether two things can be told apart at all,

how far apart a chosen chart draws them, and how strongly the whole network responds when they are contrasted. It shows that adding internal detail to the description need not add new places, and that the seven starting addresses of the history can be read exactly from the recorded marks. On size, it combines several of the programme's own relations and finds that they return a length within a fraction of a percent of the Planck length and a coherence length near ninety micrometres without using the measured strength of gravity, but it also shows exactly which coefficients are still assumed, why the result is sensitive to them, and why the equations as written cannot fix an absolute scale on their own.

The last answers are about counting. Starting from stated rules about how lasting records attach to the three forces, the paper recovers the list of particle kinds, shows that the known pattern of electric charges follows as a consistency condition understood since 1989, and obtains three copies from a record structure with three independent layers. It then shows that the six faces fall into three opposite pairs in a way the structure itself fixes, that the first response able to tell families apart sees exactly those three pairs, and that a separate calculation returns three record keys of the same shape. Assuming those three classes are those three keys leaves no further freedom in matching them; the paper states that this is an assumption, gives the test that would settle it, and reports that the current implementation fails the test. Nothing here predicts a mass, a mixing angle or a force strength.

Abstract

Within a declared class of six-port cyclic connector trees and unit-coefficient attachments, same-event local anchoring, genuine inter-sector incidence, full-rank closure and simple unit-incidence boundaries jointly select the complement-pair connector and the attachment law whose determinant is seven, whose Smith form has a single order-seven factor and whose supporting graph has minimum orientable embedding genus one; neither trivalence nor order seven is imposed. The six trapped Fold faces are mapped to the six private rim continuations of the seven-site wheel by an exact D_6 -equivariant intertwiner; the driven seven-site law supplies a unique stationary population whose reversal-even part is the physical first-moment matter writer, and the leading nonconstant matter mode is the $k = \pm 1$ rim doublet. The internal preparation problem factorises as a wheel doublet times a 49-dimensional internal carrier, and five exact no-go results reduce its selection to the spectrum of one source-derived pricing operator. A timeless stable-presence theorem calculates the largest common invariant subspace of the admitted continuation generators orthogonal to a source-defined absence support by finite adjoint-span closure, with structural presence, commitment status and historical record separately typed; a sharp-marginal theorem shows that an actual channel preserving two correctly typed commuting involutions preserves every joint record projector coefficient-free. The seven input history addresses are recovered exactly from the recorded marks of the declared instrument, the complete inverse-Dirichlet response of the canonical wheel is evaluated, and that response is proved invariant under retained-boundary Schur elimination and under correlated reversible refinement, with a nonreversible counterexample marking the boundary. The hierarchy, quartic-capacity and effective cosmological relations are solved jointly without measured gravitational coupling or a supplied coherence length, returning a conditional Planck-equivalent length and coherence length whose invariant content is one logarithmic matching target; a first-hit identity reduces the running gate to one conditional probability, a rigidity theorem with an explicit counterfamily shows that the displayed matching equations do not fix that target, a global rescaling shows that only the dimensionless product of the cosmological constant and the squared Planck length can be predicted by those equations, and a corrected two-scale bridge replaces a rejected literal cosmological branch.

By a logically separate argument, stated terminal-record premises yield the minimal one-generation chiral package, the classical one-Higgs hypercharge solution as a census constraint with citation, and exactly three complete generations from an inherited rank-three support-return structure through layer-local terminality and layer-exclusive attachment. The antipodal involution of the six rim states is central in the dihedral group, so the quotient action on the three antipodal classes is the full permutation action of the symmetric group on three objects; the lowest nonzero covariant family descent, which is quadratic, is constant on those classes and descends to the three cube roots of unity, placing the chiral

family vector in the complexified zero-sum sector of the three-object permutation carrier; and the three terminal keys returned by the PFDMI terminal-orbit theorem carry the same permutation type. Given the identification of family classes with terminal keys, the admissible intertwiner is unique up to a common phase and a relabelling. The identification is a premise, and its physical test is a second-order same-action condition: one primitive law must write the quadratic family event into the family response and into the terminal-key probabilities together. For the current source stack that test fails in a definite way, since the terminal-key family jet of the EX2E implementation vanishes exactly while the primitive family source tensor is nonzero. The generation count and the carrier bridge are therefore closed at representation and census level, the primitive same-action bridge and the metric-scale and Fold-scale promotions are open at their stated boundaries, and the physical normalisation of the family response lies at the same source-normalisation frontier as the open pricing operator that governs internal selection, with the map between the two normalisations not yet displayed.

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Part I. The sixfold Fold carrier: the stitching derivation

1. Purpose, scope and the explicit premise class

Part I consolidates the Gate-T topology calculation with the physical identity and attachment of the minimal Fold interface. It separates mathematical selection inside a declared class from derivation of that class by the primitive source. The organising question is whether the connector and the attaching law can be selected together, without imposing either a trivalent connector or order seven in advance.

The underlying wheel is $W_7 = C_6 \vee K_1$: six rim vertices and one hub. The notation K7 refers to the VERSF K = 7 admissibility structure, not to the complete graph on seven vertices. The cycle lattice of the wheel has rank six. Write the candidate attachment as

$$A = aI + bS + cJ, \quad J = S^3, \quad a, b, c \in \{0, \pm 1\}.$$

A_* denotes the selected mathematical candidate. Equality $A_{\text{phys}} = A_*$ requires physical source promotion. The premises are the following.

P0, declared local attachment family. The physically relevant six-sector carrier has cyclic successor $S e_i = e_{i+1}$, complement $J = S^3$, and an attachment in the displayed three-operation unit-coefficient integer family. This restriction is explicit, not a theorem about every possible Fold attachment.

P1, same-event intrinsic local anchoring. The Fold interface on which commitment is physically represented retains its own local sector rather than being represented only by a later copy or transport map. The local boundary readout satisfies $q_i \wedge (R_i) \neq 0$, hence $a \neq 0$.

P2, genuine inter-sector incidence. The attachment itself, not only an independent communication or transport law, connects distinct sectors: $(b, c) \neq (0, 0)$. The physical cyclic ordering is included in P0 and must be source-attached.

P3, full-rank closure. The six-dimensional cycle lattice is closed by six independent relations: rank $A = 6$, so its first-homology quotient is finite.

P4, simple unit-incidence boundaries. Every attached Fold boundary is one connected embedded simple cycle, with nonzero edge coefficients only ± 1 .

P5, cyclic acyclic connector. Replace the common hub by a finite connected tree whose only leaves are the six labelled ports. Suppress redundant degree-two internal vertices. Require $i \mapsto i + 1$ on the ports to extend to a tree automorphism. Internal junctions may have any degree at least three; trivalence is not assumed.

Under P0 to P5 the signs and connector are selected jointly: P0 to P3 force all three coefficients to be signs, and P4 and P5 then select the complement-pair connector and $A_* = \pm(J - I - S)$. The source must still establish these premises for the actual physical attachment. In particular, ordinary communication does not by itself prove P2, and absence of additional independent loops is a premise supporting P5 rather than a completed source derivation.

Provenance firewall. No determinant-seven condition selects the map. No scalar six-plane is identified with a 50-dimensional marked carrier by dimension counting. No Fold trap functional is renamed as the MASD closure defect. No aggregate completed-record clock is silently identified with a single primitive Fold-interface update or carrier.

2. Source returns available before the stitching argument

2.1 Primitive event grain: the K7 gate belongs to the Fold boundary

CRTF-EX2V resolves the event-grain question at source-carrier level: the seven binary K7 admissibility checks belong to the primitive Fold boundary, not to the later completed D36 history cover. The source operator contains the K7 closure word and transmissive gate but no completed-cover counter or

stopping variable [8]. The K7 admissibility structure is therefore source-typed to the primitive Fold boundary at the same primitive stage at which commitment is represented, even though the K7 electromagnetic effect and the native PFDML accepted-current mark remain separately typed observables. This typing is not an identification of the aggregate completed-cover R_4 carrier with a single primitive Fold, and it is not a marked sector map; the event-grain boundary of Section 24.2 remains in force.

2.2 The six-plane carrier bridge

In the continuum single-source construction the physical commitment bath is made from fluctuations of the same Φ -field used by the projected K7 closure coordinates. The whitened map $W = G^{1/2}$ is an exact isometric intertwiner from the projected K7 nonuniform six-plane to the corresponding nonuniform Φ -field six-plane, with all six singular values equal to one [5, corollary 6.14]. This is an exact carrier match inside that construction; its full physical promotion remains conditional on the constitutive single-source identification.

2.3 First-order scalar source descent is closed; mark selection is not

The entire zero-sum first-order scalar commitment-density carrier lies in the faithful primitive defect row space, $R_{HM4} = J_s(I - J_D^+ J_D) = 0$, so the six-dimensional scalar carrier descends through the source under the declared typing [5, corollary 6.15]. This does not select a finite commitment mark or outcome. The distinction between unmarked carrier descent and marked physical selection is essential and recurs throughout Part II.

2.4 Fold, presence and the three meanings of size

A Fold is the minimal stable physical structure distinguishable from its absence: the minimal physical structure at the interface where reversible distinguishability can become part of a persistent physical record. The Fold is not identified with the commitment event, an internal excitation, or a record that commitment occurred. Physical temporal order is constituted through facts, not supplied as a clock in which the first Fold must wait to form; auxiliary indices, operator words and solver iterations are not physical seconds or fact-creation events, and stability means preservation of structural presence under the independently admitted continuation rules.

The Fold is pre-spatial; coordinates arise through consistent relations between Fold patterns. A primitive support label identifies a structural or relational alternative without supplying a spatial address: one Fold is presence versus absence, relations between supports are here versus there. The number of supports does not determine their separation. Intrinsic interface dimension, operational resolution and metric extent must remain separate: the Fold Interface Law supplies a two-dimensional interface and the independent polarity and orientation structure within its stated framework; an operational Fold-resolution length can be assigned only after the relational description has acquired a physically normalised metric; diameter, area and thickness remain underived. The relation $\ell_* = cN_*\Delta t_{\text{tick}}$ is a report of an existing scale calibration, not a primitive formation-time law. The absence sector used to test presence must mean absence of the candidate structure, not a renamed unwritten record or commitment latch; a record can remain readable while the active carrier changes. Particle identities are stable closures of Fold structure, not single primitive Folds, and the separate collective coherence scale reported near 8×10^{-5} m is a collective record-domain quantity, not the scale of one Fold. The dimensionless readouts that appear later, the catalogue-chart gap $\sqrt{31/12}$, the minimum inverse-Dirichlet response 279/20 and the probability distinguishability value one, are not lengths.

3. The six-cycle attachment algebra

Let the six Fold relations be R_i , $i \in \mathbb{Z}_6$, ordered by the oriented successor $S e_i = e_{i+1}$, with complement $J e_i = e_{i+3} = S^3 e_i$. On the wheel let the six elementary triangular cycles be $f_i = s_i + r_i - s_{i+1}$, where r_i is the rim edge private to triangular face i and the s_i are the hub spokes. The six cycles form an integral basis of $Z_1(W_7; \mathbb{Z}) \cong \mathbb{Z}^6$. The general minimal oriented relation-to-cycle attachment is

$$\Lambda(R_i) = a f_i + b f_{i+1} + c f_{i+3}.$$

Because S is the cyclic shift, $A = aI + bS + cJ$ is a circulant; on the Fourier mode with sixth root of unity ζ it acts as the scalar $A(\zeta) = a + b\zeta + c\zeta^3$. This diagonalisation is used repeatedly.

4. Local anchoring forces a nonzero self term

The strongest formulation of the self-term requirement does not declare the private rim edge important by inspection; it quotients out everything face i shares with surrounding faces. Define the intrinsic local boundary class $L_i = C_1(\partial F_i) / C_1(\partial F_i \cap \partial F_{\neq i})$. On the wheel the two spokes of F_i are shared while r_i is unique, hence $L_i = \mathbb{Z}[r_i]$. Let q_i be the quotient evaluation onto this class. Then $q_i(f_i) = [r_i]$ and $q_i(f_j) = 0$ for $j \neq i$, and therefore $q_i \Lambda(R_i) = a[r_i]$.

Conditional local-anchor theorem. Under $P1$, $q_i \Lambda(R_i) \neq 0$. Therefore $a \neq 0$ and, within $P0$, $a = \pm 1$. ■

Within the declared wheel representation the coefficient of I measures the attachment's contribution to its own intrinsic local boundary class, and $P1$ requires that contribution to survive. The Fold's distinguishability from absence motivates the requirement but does not by itself derive the wheel representation, the quotient readout or their identification with a primitive physical interface; the argument establishes non-discardability of I under $P0$ and $P1$, and physical justification of those premises remains separate. PSAC-EX5CC shows conditionally that a topological trap bit can control a coefficient-free hard export and that, under one-direction-per-trapped-face minimality plus full wheel covariance, the private rim is uniquely selected, with same-action generation left open [5, corollaries 6.138 and 6.139]; that selection is consistent with, and does not discharge, the present requirement.

5. Genuine inter-sector attachment

Rather than requiring the attachment to retain the successor term directly, $P2$ is the weaker sufficient requirement that the same attaching map contains some nontrivial inter-sector incidence, $(b, c) \neq (0, 0)$. This distinguishes the physical claim from the purely self-sector branch $A = \pm I$, which may coexist with communication through a different operator; communication alone therefore cannot exclude it. The physical reading is that I retains the local support while S and J couple that support to other sectors; their precise coefficients follow from full rank and simple incidence within the declared family rather than from this interpretation.

6. Full rank forces both relational terms

By $P0$ and $P1$, $a = \pm 1$. Suppose $b = 0$. $P2$ then requires $c = \pm 1$ and $A = aI + cJ$. The complement J exchanges three disjoint pairs, so its $+1$ and -1 eigenspaces both have dimension three, and $\text{rank}(aI + cJ) = 3$, contradicting $P3$. Therefore $b \neq 0$. If $c = 0$, then $A = aI + bS$ with $a, b = \pm 1$ and Fourier eigenvalue $a + b\zeta$; exactly one mode, $\zeta = -a/b \in \{+1, -1\}$, vanishes, so $\text{rank}(aI + bS) = 5$, again contradicting $P3$. Thus $a, b, c \in \{\pm 1\}$.

The full-rank two-term map $S + J$ has determinant -4 and Smith form $(1, 1, 1, 1, 2, 2)$. It is excluded because $a = 0$ violates the local anchor, not because its torsion differs from seven. Calling that Smith form wrong before independent selection would reintroduce the target into the argument.

7. Three-operation non-discardability theorem

Theorem I.1 (anchored nontrivial full-rank attachment). In the declared family $P0$, assume same-event local anchoring $P1$, genuine inter-sector incidence $P2$ and full-rank closure $P3$. Then $abc \neq 0$, hence $a, b, c \in \{\pm 1\}$. The proof is the pair of rank obstructions of Section 6; it does not use a determinant-seven condition, simple-boundary enumeration or a particular connector tree. ■

The theorem does not need $b \neq 0$ as a separate assumption. It does not remove the physical premise that the attachment itself must be genuinely inter-sector. Source selection of the family and its labelled physical carrier remains load-bearing.

8. Joint selection of the connector and the signs

8.1 All reduced cyclic six-port connector trees

The enumeration of reduced trees with six labelled leaves, allowing arbitrary internal degree at least three, gives 105 trivalent six-leaf trees and 236 reduced trees when higher-valence junctions are admitted; internal labels are immaterial and the six port labels are fixed [1, S15]. Exactly three trees admit the cyclic leaf action $i \mapsto i + 1$.

Connector	Structure
Six-arm star	The six ports join one degree-six centre.
Alternating triple	Ports $\{0, 2, 4\}$ and $\{1, 3, 5\}$ join two centres connected by one connector edge.
Complement pair	Opposite pairs $\{0, 3\}$, $\{1, 4\}$, $\{2, 5\}$ join three pair vertices, which join one central vertex.

The classification has an analytic explanation. A finite tree has a centre consisting of a vertex or an edge, invariant under automorphisms. Transitivity of the cyclic leaf action forces equivalent branches. For a central vertex, after degree-two suppression, the possibilities are six one-leaf branches or three two-leaf branches. For a central edge, its two halves are equivalent rooted three-leaf branches. These are precisely the three displayed trees; the exhaustive 236-tree computation is a separate check.

8.2 A shared spoke fixes the relative self and successor sign

For any connector tree, orient x_i from port p_i to rim vertex v_i , and define the lifted wheel cycles by $f_i = x_i + r_i - x_{i+1} + P_{(i+1) \rightarrow (i)}$, where $P_{(j) \rightarrow (i)}$ is the unique connector path from p_j to p_i . An attaching boundary is $\partial_i = a f_i + b f_{i+1} + c f_{i+3}$. By Section 7 all coefficients are signs. The coefficient of x_{i+1} is $b - a$; its only possible values are 0 and ± 2 , so unit incidence forces $a = b$.

8.3 Simplicity selects the connector and the complement sign

For $a = b$ the six-arm star produces a boundary support with a four-valent junction, and the alternating-triple connector also produces a four-valent boundary junction; neither support is a simple cycle. Only the complement-pair connector survives. Orient its central edge from pair vertex $a_{i \bmod 3}$ to the centre; its coefficient in ∂_i is $-(a + c)$, and unit incidence excludes ± 2 , giving $c = -a$. Direct substitution gives one connected simple 13-edge loop for every i . Therefore the connector is the complement-pair tree and $A_* = \pm(J - I - S)$. Trivalence is a consequence of the selected connector; it is not an independent hypothesis.

8.4 The self-only-or-seven dichotomy

The full coefficient enumeration confirms that local anchoring, rank six and simple boundaries permit only $A = \pm I$, or the two signed nontrivial solutions on the complement-pair connector. P2 excludes the self-only solutions. Only after this independent selection do the invariants follow:

$$\det A_* = 7, \quad \text{SNF}(A_*) = \text{diag}(1, 1, 1, 1, 1, 7), \quad H_1 = \mathbb{Z}/7.$$

Theorem 1.2 (joint connector-and-incidence selection). Under P0 to P5 the complement-pair connector and $A_* = \pm(J - I - S)$ are jointly selected. The proof uses neither a preset trivalent tree nor order seven as a target. Its scope is the declared cyclic acyclic connector and three-operation incidence class, not every conceivable Fold network. ■

The overall sign reverses all cell orientations. A reflection of the cyclic convention instead replaces S by S^{-1} , with $J = S^3 = S^{-3}$ unchanged. These are distinct presentation changes and should not be conflated.

9. Selected refinement and supporting-graph topology

On the selected connector, six ports, three complement-pair vertices and one central vertex replace the hub. Together with the six original rim vertices the displayed graph has $V = 16$, $E = 21$ and $\beta_1 = E - V + 1 = 6$. Collapsing the connector tree recovers the original wheel and its integral cycle basis, so the lifted cell map has the same integer winding matrix and Smith invariants. All six selected boundaries are embedded simple 13-edge cycles in this refinement [1, S14, S15]; subdividing edges changes the edge count of a loop, not the attachment topology.

The graph has ten degree-three vertices and six degree-two port vertices, so there are $2^{10} = 1024$ orientable rotation systems. Exact enumeration gives the following face counts.

Faces in the rotation system	Number of rotation systems
1	320
3	664
5	40

The maximum face count is five, so $16 - 21 + 5 = 0 = 2 - 2g_{\min}$ and the minimum orientable embedding genus is one. This is the embedding genus of the selected supporting graph; it is not a derivation that the completed attached two-complex is a torus. The $\mathbb{Z}/7$ first-homology quotient arises after the two-cell attachments and is not the first homology of a bare torus.

Part II. The marked-anchor reduction and physical stitching

10. The rank-one trap and the exact continuation-faithfulness criterion

The topology source provides a sharper target than the statement that some mark must attach. In the minimal trap witness, commitment removes one orthogonal continuation direction: $\Delta \Pi_{\text{adm}}(h, e) = -|z_B(h, e)\rangle\langle z_B(h, e)|$. The forbidden support is rank one. The exact continuation-faithfulness criterion for a history-controlled physical selector Q_h is the identity

$$\sum_{\mu} K_{\mu}^{\dagger} Q_h K_{\mu} = I - \Pi_{\text{adm}}(h),$$

which is necessary and sufficient for the construction in which it is proved; complete positivity, trace preservation, gauge covariance and no-overwrite are not enough by themselves [5, corollary 6.144]. The equality must ultimately hold on an actually identified common carrier with the actual physical instrument; a fixed point of one current subinstrument, as constructed in Sections 12 and 13, is a necessary ingredient for that route but not a substitute for the full equality.

11. Boolean mark resolution fails, but the physical operator algebra has rank-one capacity

No Boolean selection from the 12-edge, 84 retained-record or 319 microscopic-mark effects can reproduce the minimal rank-one hard cut: their minimum nonzero ranks are 50, 15 and 2 respectively. Rank-one effects first appear at a finer 1092-level decomposition, but their individual identity depends on the unobserved generator frame and cannot be promoted as a source-invariant selector [5, corollaries 6.149 to 6.153]. PMS-EX28 resolves the capacity question differently: the source-invariant physical effect algebra has scalar commutant and therefore generates the full matrix algebra $M_{50}(\mathbb{C})$, so every rank-one projector is present as a non-Boolean spectral object. The problem is which history-dependent rank-one projector the source selects, not whether rank-one projectors exist [3].

12. The rank-one pullback locus

For candidate continuation edge e and supplied endpoint/current background B , let the stacked twelve nontrivial PFDML current channels be $K_{e,a;B}$. The audited execution PMS-EX31 R2 gives

$$K_e(B) = \bigcap_{a=1}^{12} \ker K_{e,a;B}, \quad \dim_{\mathbb{C}} K_e(B) = 38, \quad P(K_e) = \mathbb{C}P^{37},$$

the twelve channels having stacked rank 12 on the 50-dimensional carrier [3]. The number of maps is not by itself the rank of their stacked operator; the rank is the audited figure. For the orbit-current form, with j normalised, $\rho_j = |j\rangle\langle j|$ and Hermitian generators T_a , the channels are $K_a = i[T_a, \rho_j] = i(|t_a\rangle\langle j| - |j\rangle\langle t_a|)$ with transverse vectors $t_a = (I - \rho_j)T_a j$, so $K_a q = 0$ for every a confines q to the orthogonal complement of the current and its transverse span, and the kernel projector is $\Pi_K = I - |j\rangle\langle j| - T_{\perp}(T_{\perp}^{\dagger}T_{\perp})^{-1}T_{\perp}^{\dagger}$ on the audited transverse branch, independent of the basis chosen for that span. On K_e the nontrivial channels annihilate a ray bilaterally and the trivial positive-root channel acts as the identity, so $\Phi_{e,B}^{\dagger}(|q\rangle\langle q|) = |q\rangle\langle q|$: every normalised $\psi \in K_e(B)$ has exact rank-one pullback under the candidate instrument, while any ray with an active-sector component pulls back with higher rank. $K_e(B)$ is therefore the complete rank-one locus on that fixed background. The kernel is not a common reducing subspace of the full 84-effect terminal algebra, so preservation by this subinstrument cannot be promoted to a quotient preserving all physical effects; the terminal-mark and continuation laws must be checked separately.

13. Marked-anchor alignment and the projected-source Gram test

13.1 The alignment criterion

Let $z_B(h, e)$ be the source-returned forbidden continuation direction, with history and candidate label retained. After a physically justified carrier identification $U_{h,e}$, a fixed-point realisation on the audited current subinstrument requires $U_{h,e}z_B(h, e) \in K_e(B)$, equivalently $(I - \Pi_K)U_{h,e}z_B(h, e) = 0$. For this fixed-background fixed-point construction the criterion is exact; it is not a universal statement about every realisation of a topological trap, and the full continuation-faithfulness equality of Section 10 still has to hold on the identified carrier with the actual instrument.

13.2 A source candidate must survive projection without losing directions

Let $B_0: H_{\text{code}} \rightarrow \mathbb{C}^{50}$ be an independently source-returned linear candidate whose finite source code has dimension $d \leq 37$ and contains the history and Fold distinctions the attachment claims to preserve. Define $Y = \Pi_K B_0$ and $H = B_0^\dagger \Pi_K B_0 = Y^\dagger Y$. For every code vector v , $v^\dagger H v = \|\Pi_K B_0 v\|^2$, so $H > 0$ if and only if Y is injective on the declared code. For a single source ray this is a nonzero-overlap test; for a four-dimensional code it requires four positive eigenvalues, and four individually nonzero projected columns are not enough, since they can be linearly dependent.

13.3 Canonical polar isometry and its physical limitation

If $H > 0$, the projected map has the unique polar isometric factor $U = \Pi_K B_0 H^{-1/2}$, with $U^\dagger U = I_d$ and $\Pi_K U = U$. Its uniqueness is relative to the supplied B_0 , background and inner products; it does not establish that the primitive source selects B_0 . Polar normalisation is not automatically an admissible physical operation: the source must justify the relevant metrics, normalisation and channel, and an algebraically convenient whitening cannot be inserted to repair a failed source map or to declare preservation of the original record statistics.

13.4 Current and first-tangent seed no-go

If the image of B_0 lies entirely in the span of the current and its transverse tangents, then $\Pi_K B_0 = 0$. The current and its first transverse directions alone cannot supply this fixed-point attachment; a viable candidate must contribute an additional history- or record-sensitive component reaching K_e .

13.5 Covariant families need not be constant rays

If unitary source or background transformations give $\Pi_K(gB) = D_g \Pi_K(B) D_g^\dagger$ and $B_0(gB) = D_g B_0(B) R_g^\dagger$, then $H(gB) = R_g H(B) R_g^\dagger$ and the polar candidate satisfies $U(gB) = D_g U(B) R_g^\dagger$. A source-derived covariant attachment can therefore depend on the background. This does not prove that the archived witnesses are gauge-equivalent, nor supply their physical selection; it states what covariance would imply if returned by the source.

14. Remaining selection constraints and the background distinction

The audited report gives the same locus dimension for each edge and for all five admitted endpoint/current witnesses, but the subspace direction varies: the five-witness common intersection is zero on each edge, and the all-edge common kernel is zero for the primary background. These results exclude a universal constant ray on the tested families; they do not exclude a correctly typed, source-derived, edge- and background-dependent covariant map. The outstanding objects at this stage are: a physical History-Measure/PFDML carrier identification rather than a dimension match (D-PM41); candidate-continuation-resolved typing of that map including its relation to any proposed cyclic sector orbit (D-PM42); background provenance, meaning source selection of the endpoint/current background, a justified redundancy, or a covariant family with a source-selected state; functional and controller attachment, since the Fold trap functional, the MASD closure defect and the completed-cover controller remain differently typed; and physical normalisation and compatibility, since a positive projected Gram establishes injectivity of a linear candidate but not the source implementation of its polar normalisation, sharp record preservation, full continuation faithfulness or post-commitment causal coupling. Sections 15 to 18 close the first two.

15. One marked anchor, cyclic covariance and the atom-relabelling obstruction

If the source identifies the relevant candidate labels with the same physical six-sector orbit and returns one anchor $R_0 \mapsto e_k$, successor covariance forces $R_i \mapsto e_{k+i}$, or $R_i \mapsto e_{k-i}$ on the reflected convention; complement then follows from $J = S^3$. Choosing k fixes the starting label, and reversing the cyclic convention is distinct from the overall sign of an attached cell boundary. The cyclic action cannot be obtained merely by relabelling the four internal Fold atoms: the 24 atom permutations induce only the cycle types (1^6) , $(1^2 2^2)$, (3^2) and $(2, 4)$ on their six unordered pairs, none a six-cycle [1, S14]. A physical sixfold boundary-transport action is therefore additional source structure, not an automatic consequence of having four atoms and six pairs. One anchor propagates only after the physical orbit and intertwining law are established; the primitive atom carrier, the six binary-relation labels and the candidate-continuation labels are not identified by counting them.

Theorem 1.3 (combined conditional connector-and-stitching theorem). Assume P0 to P5 and a physically typed marked source-to-sector attachment realising those premises. Then, up to port-label convention, cyclic reflection, redundant subdivisions and overall cell orientation, the connector is the complement-pair tree and $A_{\text{phys}} = A_* = \pm(J - I - S)$; its cycle-lattice quotient has $H_1 = \mathbb{Z}/7$, its Smith form is $\text{diag}(1, 1, 1, 1, 1, 7)$, the displayed refinement has six simple 13-edge attaching cycles, and the supporting graph has minimum orientable embedding genus one. ■

The strength of the theorem is joint selection of the connector and the signs. The remaining physical premises, the restricted incidence family, cyclic carrier, acyclic six-port connector, local anchor and actual inter-sector incidence, are not reduced solely to an orientation label or a rank-one capacity argument. The classification is exact within the declared class; physical promotion remains conditional.

16. The fixed-background marked ray

Three distinctions control the physical stitching. A wheel or site selector is not automatically an internal representation selector. A source response map that is unique once a background is supplied is not the same object as a preparation law that selects the background. The orientation-odd history output and the reversal-even matter response are different projections of one driven event and must not be collapsed into one carrier.

For a history address h define the restricted history-odd operator $A_{h,e;B} = \Pi_e C_h \Pi_e$ with $C_h = i[E_i, E_j]$. The audited fixed-background executions return simple orthogonal extrema for every tested history-effect pair and edge. With $\lambda_{\max}$ the largest eigenvalue on $K_e(B)$, the operator $D_{h,e;B} = \lambda_{\max} I_{K_e} - A_{h,e;B}$ is positive semidefinite with rank 37 and one-dimensional kernel, so $\ker D_{h,e;B} = \text{span}\{\psi_{h,e;B}\}$ and $\mathbb{CP}^{37} \rightarrow \mathbb{CP}^0$: at fixed h, e and B the restricted rule selects one rank-one ray. Reversing the history changes C_h to $-C_h$ and swaps the maximal and minimal extremal rays, which are orthogonal in the audited executions, so the physical orientation is not reduced to an irrelevant sign.

Grade. The marked ray (D-PM41a) is closed at fixed-background mathematical grade, conditional on a supplied physical background and on primitive warrant for using the restricted history-odd extremal rule. The calculation derives neither input.

17. Private-rim continuation typing and exact D_6 intertwining

The wheel has six interior triangular faces Δ_i , each with one private rim edge r_i . Under full wheel covariance and the minimality condition that one trapped face exports exactly one edge direction, the private rim edge is uniquely selected: the stabilising reflection exchanges the two spokes and fixes only that rim edge [5]. Define $R|\Delta_i\rangle = |r_i\rangle$ with $R^*R = I_6$, let S_F and S_E be the successor operations on faces and private rims, and choose the reflection $M_F: \Delta_i \mapsto \Delta_{-i-1}$ with the corresponding rim reflection $M_E: r_i \mapsto r_{-i-1}$. Then

$$S_E R = R S_F, \quad J_E R = R J_F \quad (J = S^3), \quad M_E R = R M_F, \quad M S M = S^{-1}.$$

Theorem I.4 (exact D_6 typing). R is a D_6 -equivariant intertwiner between the six trapped Fold faces and the six private rim continuations, not merely a six-to-six relabelling. One anchor fixes the whole orbit up to the orientation branch. ■

The correct sixfold target is the private-rim orbit; the twelve undirected wheel edges split into six rim edges and six spokes and must not be conflated with one twelve-element C_6 orbit. Combining with Section 16 gives the structural chain $\Delta_i \rightarrow r_i \rightarrow \psi_{h,r_i;B}$. Grade: D-PM42 is closed at label and sector typing grade, conditional on the one-export-direction-per-trapped-face minimality premise and the fixed-background PFDMI construction; primitive same-action provenance of the trap-controlled export operator remains separate.

18. The physical background and the even/odd split

18.1 The five witnesses are genericity tests, not five physical vacua

The five endpoint/current witnesses all satisfy the PFDMI current residual, but that did not make them physical backgrounds. In the literal static trivial-link embedding, with $J = B^T\psi$, the MASD continuity operator is $D_T = BJ = BB^T\psi = L_W\psi$. The wheel Laplacian has spectrum $\{0, 2, 2, 4, 4, 5, 7\}$, so its static zero mode is only the vertex-constant field, and positive edge conductances do not change that kernel on a connected wheel. Every one of the five nonconstant prime-chirp witnesses therefore fails the literal static continuity zero [8, CRTF-EX2A]. This does not invalidate their role as genericity witnesses and does not exclude non-static embeddings; it removes the reading of the five as equally admissible static physical vacua.

18.2 The reversal-even writer

For the driven D36 transition law P with stationary adjoint $P^\#$, define $P_+ = (P + P^\#)/2$ and $P_- = (P - P^\#)/2$. The odd part is anti-self-adjoint in the stationary inner product and carries divergence-free circulation. The real quadratic matter-gradient response is generated by P_+ . Consuming the one-Fold, one-primitive-update premise, reversal blindness of matter magnitude and faithful identity on the even transition sector uniquely select the Reynolds conditional expectation $E_+(X) = (X + RXR^{-1})/2$. Consequently the physical matter first-moment update is

$$\rho_{n+1} = (P_+ \otimes I_{49})\rho_n.$$

The full history law remains P . The orientation-odd current is not statistically averaged away; it is retained in a differently typed history and cycle output [11].

18.3 Exact spatial mode selection

At $\alpha_* = 0.959001109719975$ a rim row of P_+ has $a = P_{\text{hub}} = P_{\text{self}} = 0.200305886207$ and $c = P_{+, i \rightarrow i \pm 1} = 0.299694113793$. On the six rim Fourier modes $\phi_k(j) = e^{2\pi i k j / 6}$, $k \neq 0$, the eigenvalue is $\lambda_k = a + 2c \cos(2\pi k / 6)$, hence

$$\lambda_{\pm 1} = 1/2, \quad \lambda_{\pm 2} = -0.099388227586, \quad \lambda_3 = -0.399082341379.$$

Theorem I.5 (leading matter mode). The leading nonconstant reversal-even matter mode is the $k = \pm 1$ doublet. The full driven history law independently selects the isolated conjugate pair $0.5 \pm 0.386111824967i$ with zero hub amplitude, equal rim amplitudes and phase step $\pm\pi/3$; its real part is the same $k = \pm 1$ spatial carrier and its imaginary part records the circulation supplied by the odd sector. The unique stationary seven-site law is $\pi = (0.189423730906, 0.135096044849, 0.135096044849, 0.135096044849, 0.135096044849, 0.135096044849, 0.135096044849)$. ■

Grade. The seven-site population background (D-PM43a) and the even-matter, odd-orientation split (D-PM43b) are closed on the stated D36/ P_+ first-moment branch. They do not select the internal representation ray.

Naming. The $k = \pm 1$ doublet is the eigenvalue-2 eigenspace $\ker(L_W - 2I)$ of the wheel Laplacian. Some K7 and PFDMI sources call it the wheel E_2 , while the flavour papers call the same primitive R_4 /flavour plane E_1 and reserve E_2 for the $k = \pm 2$ family descendant. This paper uses the flavour convention throughout: E_1 is the $k = \pm 1$ source plane and E_2 the $k = \pm 2$ carrier.

19. Exact wheel-doublet times internal-carrier factorisation

The full endpoint carrier is $\mathbb{C}^{50}$. GSJB selects one S_3 -trivial neutral void ray $|\Omega\rangle$, so the projective tangent is $T_{|\Omega\rangle}\mathbb{CP}^{49} \cong \Omega^\perp \subset \mathbb{C}^{50}$ with $\dim_{\mathbb{C}}\Omega^\perp = 49$ and central decomposition

$$49 = 2 + 6 + 2 + 9 + 9 + 18 + 3 \quad \text{over} \quad (N_R^{\text{std}}, L_L, H, d_R, u_R, Q_L, e_R).$$

On the selected wheel $k = \pm 1$ doublet the factorised endpoint innovation is $\delta\psi_v = u_v z$ with u in the doublet and $z \in \Omega^\perp \cong \mathbb{C}^{49}$. The linearised current is $j_e = \kappa q_e z$ with $q_e = (B^T u)_e$, so for every active edge the normalised PFDMI current ray is $\rho_{j,e} = |z\rangle\langle z|$: the complete spatial amplitude, sign, phase and common conductance cancel from the background current ray [8, CRTF-EX2M, EX2N]. The sixfold spatial mode fixes the spatial carrier while the internal ray remains $[z] \in \mathbb{CP}^{48}$. Representation-theoretically the wheel D_6 acts on the two-dimensional spatial factor and leaves the 49-dimensional multiplicity space untouched, $\text{Hom}_{D_6}(E_1, E_1 \otimes \mathbb{C}^{49}) \cong \mathbb{C}^{49}$: the minimal equivariant lift is unique in form with multiplicity 49, and after normalisation and global phase the residual family is $\mathbb{CP}^{48}$.

20. Exact no-go results for internal selection

20.1 The private-rim Fact cut cannot choose a representation sector. For trapped faces the export selector acts on the wheel/edge factor; on the full edge $\times$ internal carrier, $Q_{\text{Fact}} = Q_{\text{exp}} \otimes I_{49}$. For any of the seven central internal projectors P_r and $\tilde{P}_r = I_{\text{edge}} \otimes P_r$, $[Q_{\text{Fact}}, \tilde{P}_r] = 0$. The Fact operation chooses the continuation direction and leaves all internal sector populations unchanged; it cannot make the internal population vector one-hot.

20.2 A factor-blind scalar current cost cannot choose z . For normalised u in the wheel doublet the exact incidence lift gives $\sum_e |q_e|^2 = 4$, so with a factor-blind bare current norm $S_j(z) = \sum_e \|j_e\|^2 = 4\kappa^2 \|z\|^2$, which for normalised z equals $4\kappa^2$ at every point of $\mathbb{CP}^{48}$. There is no internal selector on that branch.

20.3 The rank-144 native source map is a response theorem, not a preparation theorem. On the representation-valued PFDMI source extension the current-defect derivative and the differentiated native mark law both have rank 144 on the native matrix-link slice, so the mark response factors uniquely through the reachable current-defect image; after the Wilson quotient the corresponding ranks are 72. This removes row-space embedding freedom once the source state is supplied; it does not invert the chain to choose z [7].

20.4 The neutral source is exactly dark to z at first order. At the pure neutral source $|\Omega\rangle$ the current/terminal mark law is independent of the tangent current ray z ; the nontrivial channels annihilate the neutral source. Finite occupation $|\psi_s\rangle = \sqrt{1-\beta}|\Omega\rangle + \sqrt{\beta}|z\rangle$ activates factor information at $O(\beta)$, and the native factor score can be full rank, but the score rank persists while its numerical metric changes with admissible source parameters; maximum score rank therefore does not select the endpoint ray [8, CRTF-EX2O; 10].

20.5 Common $U(49)$ covariance reduces covariance freedom but not the ray. On the reconciled projective tangent, common internal $U(49)$ covariance gives $E[z_i z_j^\dagger] = (\gamma_e/49)I_{49}$. The endpoint-coherence ambiguity reduces to a scalar γ_e rather than a free 49×49 covariance, but an isotropic covariance contains no preferred internal direction.

The wheel geometry, the private-rim Fact selector, a factor-blind current norm, the native response factorisation and isotropic endpoint covariance all fail, for different exact reasons, to select $[z]$. This identifies what the missing object is not.

21. Reduction of the internal problem to the pricing operator W_{phys}

Let W_{phys} be the source-derived Hermitian current/history pricing operator acting on the selected representation-valued internal carrier. For the factorised current $j_e = kq_e z$ the priced quadratic is

$$S_W(z) = \sum_e j_e^\dagger W_{\text{phys}} j_e = 4\kappa^2 z^\dagger W_{\text{phys}} z,$$

so the physical internal preparation problem is the Rayleigh–Ritz problem $\min_{\|z\|=1} z^\dagger W_{\text{phys}} z$. If W_{phys} respects the seven central representation sectors, $W_{\text{phys}} = \sum_{r=1}^7 w_r P_r$ with $p_r = \|P_r z\|^2$ and $\sum_r p_r = 1$, then $S_W = 4\kappa^2 \sum_r w_r p_r$. A unique lowest sector price w_s forces $p_s = 1$, and the $\mathbb{CP}^{48}$ preparation problem collapses to the projective space of the winning sector: $\mathbb{CP}^1, \mathbb{CP}^5, \mathbb{CP}^1, \mathbb{CP}^8, \mathbb{CP}^8, \mathbb{CP}^{17}$ or $\mathbb{CP}^2$ for the seven sectors in the order of Section 19. Only after that sector is selected is an intra-sector ray selector needed. The current MCFR and RCHS corpus selects the shared representation-valued current carrier and the pure-current lift at fixed current defect, and explicitly leaves the physical representation-resolved pricing operator open [11]. The narrow release gate for the internal preparation (D-PM43c) is therefore: derive the representation-resolved W_{phys} , or at minimum its seven central block prices, from the same primitive source/history action, without inserting a Hilbert–Schmidt norm or a common scalar price as a convention.

22. Ledger for Parts I and II

Object	Grade	Meaning
$A_{\text{phys}} = \pm(J - I - S)$	Exact within the declared premise class	Topological K7 attachment theorem (Theorems I.1 to I.3).
Rank-one locus $K_e(B)$	Exact at fixed B	$\dim_{\mathbb{C}} K_e = 38$; projective space $\mathbb{CP}^{37}$.
Restricted history-odd unique ray	Conditional exact	$\mathbb{CP}^{37} \rightarrow \mathbb{CP}^0$ at fixed h, e, B ; primitive rule warrant open.
Private-rim D_6 attachment	Conditional exact	Six Fold faces to six rim continuations; successor, complement and reflection intertwine (Theorem I.4).
Five prime-chirp witnesses	Resolved	Genericity witnesses; not static MASD vacua under the literal embedding.
P_+ matter writer	Exact at principle and representation grade	Unique reversal-even first-moment writer; P_- retains orientation.
Seven-site stationary population	Exact on the D36 branch	Unique π ; leading nonconstant matter mode $k = \pm 1$ (Theorem I.5).
Spatial $\times$ internal lift	Exact kinematic factorisation	$\delta\psi = u \otimes z$ with $z \in \mathbb{C}^{49}$; residual $\mathbb{CP}^{48}$.
Fact-only internal selection	No-go	$Q_{\text{exp}} \otimes I_{49}$ commutes with the central projectors.
Scalar-pricing internal selection	No-go	Current cost independent of normalised z .
W_{phys} internal selection	Open, reduced	Rayleigh–Ritz spectrum is the live internal release gate.
Fold closure to MASD curvature	Open	Functional same-action provenance still required.

Part III. Distinguishability and the identity of one Fold

23. Operational distinguishability, timeless stability and the Fold scale

23.1 Operational distinguishability and the distinction from structural stability

Mathematical inequality is not sufficient for a physical distinction. The source's observer-invariant readout requirements are retained [S10]. Let G_L be a nonempty class of physically admitted record channels at a declared operational resource or reconstructed causal scale L . States compared at different scales must have compatible embeddings in a common description; L is not an assumed pre-fact spatial coordinate. For a channel Γ define

$$d_\Gamma(\rho, \sigma) = \frac{1}{2} \|\Gamma(\rho) - \Gamma(\sigma)\|_1, \quad D^*_{\text{phys}}(L)(\rho, \sigma) = \sup_{\Gamma \in G_L} d_\Gamma(\rho, \sigma).$$

Zero distance means that none of the admitted record channels separates the states. Positive distance means that some distinguishing information is accessible within that class. A supremum of one gives the sharp operational optimum; an actually available perfect readout also requires attainment. The Fold-presence test below therefore asks for an explicit admitted effect, rather than inferring a detector from a supremum alone.

Nestedness. Under the declared inclusion $L_1 \leq L_2 \Rightarrow G_{L_1} \subseteq G_{L_2}$ one obtains $D^*_{\text{phys}}(L_1)(\rho, \sigma) \leq D^*_{\text{phys}}(L_2)(\rho, \sigma)$. This is a conditional monotonicity theorem about the admitted resource classes. It does not derive metric space or a physical duration from the inclusion itself.

Metric quotient. Each d_Γ is a pseudometric. The supremum is also a pseudometric, since symmetry is preserved and $\sup_\Gamma d_\Gamma(\rho, \tau) \leq \sup_\Gamma d_\Gamma(\rho, \sigma) + \sup_\Gamma d_\Gamma(\sigma, \tau)$. Consequently the equivalence relation $\rho \sim_L \sigma \Leftrightarrow D^*_{\text{phys}}(L)(\rho, \sigma) = 0$ defines a metric quotient of the compared states. This is an operational quotient relative to G_L , not a statement that the entire reversible substrate is physically trivial. The metric claim concerns D_{phys} ; it is not automatically transferred to the retention sup-inf functional below.

Separate requirements. Readout distinguishability concerns whether a difference can be represented. Record persistence concerns whether the representation is retained. Structural stability concerns whether the structure whose presence is being tested is preserved. A Fold requires the relevant stable physical structure and its admitted presence distinction; a durable record alone is insufficient. Spatial localisation is an additional property of a reconstructed relational geometry. It is not required of every internal physical distinction, and a conventional spatial coordinate is not a prerequisite for the pre-spatial Fold. The working distinction is therefore: Fold test, stable structural presence plus admitted distinction from absence; spatial test, a source-derived relational or metric localisation of that structure. These requirements do not declare reversible sub-Fold differences nonexistent. They separate those differences from a minimal stable Fold-level presence and from any later metric description.

23.2 Record persistence, physical minimality and the reconstructed Fold scale

Record persistence is retained as a separate test. Let $\mathcal{R}$ denote a nonempty, physically specified retention class compatible with the chosen record channels. The functional is

$$D^*_{\text{pers}}(L)(\rho, \sigma) = \sup_{\Gamma \in G_L} \inf_{R \in \mathcal{R}} \frac{1}{2} \|R\Gamma(\rho) - R\Gamma(\sigma)\|_1.$$

For two actually supplied record states the sharp persistence condition is $\frac{1}{2} \|R(\rho_R(0)) - R(\rho_R(1))\|_1 = 1$ for every $R \in \mathcal{R}$. This certifies retention of a distinction in the record carrier. It does not certify preservation of the physical structure recorded. With nested readout classes and unchanged compatible retention requirements, the functional is nondecreasing with the resource scale. No external clock is introduced by either the channel or the retention index.

Structural stability is independently required. Section 23.5 tests the candidate physical support against the actual admitted continuation rules. The Fold definition cannot be discharged solely by $D_{\text{pers}}(\emptyset, F) = 1$: a historical record can survive loss of the structure. The source must separately supply an absence sector, a physical candidate structure, a continuation law and an admitted presence readout.

Minimality is physical, not a rank convention. Let C_{src} be an independently source-admitted class of candidate physical structures, with its own notion of proper physical substructure. A candidate is eligible only if it is physically realisable, passes the stable-presence and readout conditions, and carries the required interface properties. It is minimal if no proper source-admitted physical substructure satisfies those same requirements. This is a specification of the physical test, not a derivation of C_{src} or of its realisation. Binary presence alone requires two distinguishable hypotheses; it does not derive the four-dimensional internal Fold carrier. The additional polarity and orientation requirements remain imports from [S9]. On that binary branch, one Fold equals one bit continues to describe the informational role of the binary distinction, not the Fold's identity as an event or a count of all independent physical degrees of freedom.

Only then introduce a metric scale. Once the source supplies an operational length assignment $\ell_g(C)$ to candidate structures in a reconstructed geometry, write

$$\ell_F = \inf_{C \in C_{pass}} \ell_g(C),$$

where C_{pass} contains the physically admitted candidates passing the stated structural, readout and interface tests. If the source proves that the infimum is attained, it may be written as a minimum and associated with an actual minimal realisation. A discrete solver or update index does not prove that a minimum length exists. Without a source-returned metric assignment, this formula is a target definition, not an evaluated length. The record-onset quantity

$$\ell_{dist} = \inf\{L : \exists p, \sigma, D^*_{pers}(L)(p, \sigma) > 0\}$$

remains separately meaningful in its declared scale description. For $L < \ell_{dist}$ no positive value occurs by definition. Equality $\ell_{dist} = \ell_F$ is not established by the stable-support theorem or by record persistence alone. It additionally requires the same physical scale assignment, the required sharp distinction, and a source identification of the corresponding minimal supports.

Fold-Interface Minimality (explicit physical principle). A primitive persistent record-bearing distinction requires Fold-supported physical structure. The stronger identification of its onset scale with the scale of a minimal stable, sharply readable Fold remains an additional physical identification to be established, not a consequence of the algebraic quotient or of a commitment-history latch.

Calibration grade. The relation $\ell_* = cN_*\Delta t_{tick}$ and its Planck identification remain source-reported calibration [S6]. They do not define a physical clock before facts. A structural count, if one is eventually source-derived, must be invariant under redundant descriptions and must acquire any dimensional interpretation through the reconstruction of physical measures. No such count or normalisation is returned by the timeless calculation of this part.

23.3 The unitary-attractor no-go, without a formation-time interpretation

The finite-dimensional unitary-conjugation obstruction stands. For $U_{tick}|j\rangle = e^{i\theta_j}|j\rangle$ one has

$$\mathcal{E}(|j\rangle\langle k|) = U_{tick}|j\rangle\langle k|U_{tick}^\dagger = e^{i(\theta_j - \theta_k)}|j\rangle\langle k|.$$

Every conjugation eigenvalue has unit modulus and $\|\mathcal{E}^n(O)\|_2 = \|O\|_2$. Thus strict closed unitary conjugation cannot supply the claimed norm contraction of arbitrary components into a proper Fold attractor. This rejects that mechanism; it does not reject invariant supports, internal reversible evolution or stable physical structure. The replacement stability question is whether a source-defined presence sector is preserved under the admitted physical continuations, as in Section 23.5. A reduced record or export channel may describe a different aspect of the interface without identifying the record with the structure or deriving the occurrence rule FC. No iteration index in either calculation is promoted to elapsed pre-fact time. The formation-waiting-time interpretation is not carried as a physical Fold-size derivation. The meaning of N_* , if retained, is a separately justified structural or ordering count rather than a duration or the number of rounds taken by a solver.

23.4 Planck consistency after metric reconstruction: normalisation remains open

The localisation argument is retained as an external heuristic consistency check, not a primitive VERSF derivation. In an already meaningful localisation description its order-of-magnitude steps are

$$\Delta p \gtrsim \hbar/2L, \quad E \gtrsim \hbar c/2L, \quad r_s = 2GE/c^4 = \ell_p^2/L.$$

Requiring $r_s \leq L$ gives $L \gtrsim \ell_p$. This combines quantum-localisation reasoning with a classical gravitational-radius estimate. It is not an experimentally established statement that all physical description or all structures cease below a sharply measured boundary.

Scope correction. A pre-spatial Fold cannot be excluded simply because it lacks an ordinary spatial coordinate: the source already treats such coordinates as relational outputs [S6]. The localisation estimate concerns the reconstructed measurement scale L . It is not automatically a lower bound on an unnormalised primitive structural support. A conditional comparison to ℓ_F requires the source to identify the two operational scales first. The shortcut from no conventional address to no Fold is not used.

Structural scale freedom. The stable-support equations in Section 23.5 contain null-sector and operator data but no dimensional metric normalisation. If a later effective metric g is assigned, then $g \rightarrow a^2g$ with $a > 0$ rescales lengths by a while leaving those algebraic tests unchanged. The tested structural data therefore do not fix an absolute length. This is an underdetermination result for the stated inputs, not a no-go against a more complete VERSF source fixing the normalisation. After a physically defined Fold-resolution length has been reconstructed, the comparison can be written as

$$\alpha_F = c^3 \ell_F^2 / (\hbar G) = (\ell_F / \ell_p)^2.$$

Here α_F is merely a name for this scale ratio, not the electromagnetic fine-structure constant. The present calculation does not determine it. A source-derived value of one, or an explicitly justified convention-dependent value, requires a common-source metric and physical-constant normalisation rather than an imposed Planck length. A two-sided scale argument would become meaningful only after this scale matching. Order-of-magnitude bounds remain order-of-magnitude bounds; they do not establish an exact coefficient. The Fold-Planck identification stays at calibration grade, and diameter, area and thickness remain underived.

Scale conclusion. Stable distinguishability can be tested without a clock or spatial grid that precedes facts. The length associated with that structure must be obtained from the reconstruction of spacetime measures, not from a pre-fact waiting count multiplied by assumed seconds per tick.

23.5 Timeless stable-presence theorem and finite source-selection test

23.5.1 Carrier, absence and admitted continuations

Work in a source-admitted finite-dimensional interface representation H , not in an assumed dynamical model of the Void's interior [S9; S17]. Let $N \subseteq H$ be the independently specified support representing absence of the particular candidate structure, and P_N its orthogonal projector. The absence support need not be one-dimensional. The internal Fold ground ray $|\Omega\rangle \in H_F$ of [S6, Axiom V1] is not automatically an absence-of-Fold state. Let the source supply continuation channels

$$\Phi_a(\rho) = \sum_\beta K_{a\beta} \rho K_{a\beta}^\dagger, \quad \sum_\beta K_{a\beta}^\dagger K_{a\beta} = I.$$

The indices label admitted operations and their branches, not durations. Exact stability is relative to this independently returned class. It is neither immunity to every imaginable intervention nor permission to discard a source-admitted operation because it spoils a preferred answer. The theorem and finite solver below assume one common finite carrier and closure under arbitrary composition of the listed channels. Where physical contexts or histories restrict composition, the source must retain those contexts or supply an appropriate context-labelled representation. Arbitrarily composing incompatible operations

can yield a false rejection; the unrestricted theorem must not be applied to such a source without justification.

23.5.2 Exact no-loss condition

For an orthogonal candidate projector P , structural presence is preserved when $\rho = P\rho P \Rightarrow \Phi_a(\rho) = P\Phi_a(\rho)P$ for every density operator in that support and each admitted continuation. This holds if and only if

$$(I - P)K_{a\beta}P = 0 \text{ for every } a, \beta.$$

Proof. For $|\psi\rangle \in \text{ran } P$, $\text{Tr}[(I - P)\Phi_a(|\psi\rangle\langle\psi|)] = \sum_\beta \|(I - P)K_{a\beta}\psi\|^2$. Every summand is nonnegative. Vanishing for all ψ is equivalent to the displayed identities. Conversely, those identities keep every branch and hence their sum in $\text{ran } P$. Permitted compositions preserve the same property. ■

For a trace-preserving channel this is equivalently $\Phi_a^\dagger(P) \geq P$. In the block decomposition $\text{ran } P \oplus \text{ran}(I - P)$, the no-loss identity sets the lower-left block of each Kraus operator to zero. Trace preservation makes the off-diagonal sum in $\Phi_a^\dagger(P)$ vanish and its upper-left block equal to I_P , leaving a positive complementary block. This proves the inequality; the converse follows from zero leakage probability for states in P . The equality $\Phi_a^\dagger(P) = P$ is stronger: it also prevents entry from the complementary support into P . Stability of an existing structure need not impose that extra no-entry condition. Internal states need not be fixed; it is the defining structural support that is preserved. An exact diagnostic certificate is

$$\eta_a(P) = \sum_\beta \|(I - P)K_{a\beta}P\|_{HS}^2 = 0.$$

A positive certificate rejects exact stability of this candidate for this continuation. It does not reject every candidate, approximate stability or the framework as a whole.

23.5.3 The complete stable support

Let W contain all finite products of the specified Kraus generators, including the identity. Define

$$S_{st} = \cap_{w \in W} \ker(P_N K_w), \text{ equivalently } V_N = \text{span}\{K_w^\dagger n : n \in N, w \in W\}, \quad S_{st} = V_N^\perp.$$

Theorem III.1 (timeless stable support). Under the finite common-carrier and composition hypotheses above, S_{st} is the unique largest common invariant subspace of the continuation generators contained in $N^\perp$.

Proof. The identity word gives $S_{st} \subseteq N^\perp$. If $\psi \in S_{st}$, then $P_N K_w K_{a\beta} \psi = 0$ for every word w , so $K_{a\beta} \psi \in S_{st}$. Conversely, any common invariant subspace $\mathcal{L} \subseteq N^\perp$ remains in $N^\perp$ under every word and therefore lies in every displayed kernel. Hence $\mathcal{L} \subseteq S_{st}$. ■

This calculates all compatible stable directions relative to the supplied inputs. It does not identify them with realised Folds. Where an independent physical candidate support C must also be respected, use $C^\perp$ as the forbidden support; the same construction returns the largest invariant subspace within C . Neither support is chosen to force an anticipated Fold answer.

23.5.4 Finite evaluation without a physical tick count

Starting from $V_0 = N$, calculate

$$V_{k+1} = \text{span}(V_k \cup \cup_{a,\beta} K_{a\beta}^\dagger V_k).$$

Every strict enlargement increases dimension. In $d = \dim H$, at most $d - \dim N$ strict enlargements are possible. Equality of two successive spans implies closure under every adjoint generator, so the final span is V_N . For a full-column-rank basis matrix B_N of the final span,

$$P_{st} = I - B_N(B_N^\dagger B_N)^{-1} B_N^\dagger.$$

The empty-span case uses its zero projector. This basis formula is a calculation of a subspace, not a physical normalisation operation on source states. In particular it is distinct from the proposed physical polar-normalisation issue in Section 13.3. The index k is a solver index. It is not an elapsed duration, a fact count, a formation depth or N_* . The result is unchanged by alternative Kraus representations of the

same channels and is covariant under simultaneous unitary changes of carrier, absence support and operators. Adding identity or already-generated composite channels while retaining the original generators leaves the stable support unchanged, although it may change the number of solver rounds [S17].

23.5.5 Physical readout, occurrence and minimality remain distinct

For p_F supported in S_{st} and $p_\emptyset$ supported in N , their full mathematical trace distance is one. A physical presence distinction additionally requires a source-admitted effect E such that

$$0 \leq E \leq I, \quad \text{Tr}(Ep_F) = 1, \quad \text{Tr}(Ep_\emptyset) = 0.$$

The same distinction must remain physically readable in the relevant admitted continuation contexts. The projector P_{st} is a mathematical candidate, not proof that the source permits its measurement. Physical realisability and any required interface data must also be established independently. The construction selects a largest compatible stable sector, not a minimal physical Fold. Physical minimality is the no-proper-admitted-substructure condition of Section 23.2. A rank-one occupied sector plus an absence sector already realises binary presence mathematically; therefore the four-state internal Fold structure does not follow from presence alone. The additional polarity and orientation data are still required [S9]. Hilbert-space dimension is not metric size. P_{st} must also not be identified with the rank-one locus of Section 12 merely because both are kernels. They belong to different source questions and possibly different carriers. A physical map connecting them remains a separate provenance requirement.

23.5.6 Executed diagnostics and grade

The exact checks in [S17] use constructed matrices with independently known answers:

Diagnostic	Exact return
Eight-dimensional carrier with one absence configuration, four protected labels and three unprotected labels	Stable rank four; adjoint-span ranks 1, 2, 3, 4; zero leakage.
Modified continuation rules connecting all four labels to erasure	Stable rank zero.
Equivalent Kraus representation and simultaneous rational unitary change of carrier	The same support, invariant or covariant as appropriate.
Identity and redundant composite continuations added	The same rank-four support; solver ranks change to 1, 3, 4.
Two-state continuation preserving presence while allowing entry from absence	Strict $\Phi^+(P) > P$ on the complementary direction.
Two-state presence-only control	Rank-one occupied support; binary presence does not derive $\mathbb{C}^4$.

The protected model permits changes of internal state without loss of structural presence. Separate exact checks distinguish structural presence from a historical record flag and verify prefix-order properties of appended facts. None is a numerical primitive VERSF execution. The rank-four result was deliberately built into a known-answer model; it is a validation of the test, not a derivation of four Fold states.

Grade. Exact finite-dimensional structural theorem and executed algebra diagnostics. The actual Fold-absence support, structural continuation class, realisation, separating Fold-presence effect and physical metric normalisation are not supplied by this stability calculation. The intrinsic interface and minimality arguments remain inherited at their stated source grades rather than rederived by invariant-subspace

algebra. Separately, Section 31 recovers the history-address projectors of the declared marked instrument. Those address effects are not the missing Fold-presence effect and do not settle its structural or spatial interpretation.

24. Commitment-status quotients, event grain and the native cover controller

24.1 Commitment status is not structural presence

The source's persistent commitment latch is retained:

$$\kappa F_{n+1} = \kappa F_n \vee \Theta(C_{n+1} - C_*), \quad \kappa F_0 = 0.$$

It records whether the declared commitment threshold has been crossed [S12]. The index labels the stated history or controller description; it is not an assumed physical time preceding facts. Let Q_0^κ, Q_1^κ be its orthogonal history-status projectors. Use explicitly typed output labels in

$$T_\kappa(\rho) = \text{Tr}(Q_0^\kappa \rho) |0_\kappa\rangle\langle 0_\kappa| + \text{Tr}(Q_1^\kappa \rho) |1_\kappa\rangle\langle 1_\kappa|.$$

This is a CPTP measure-and-record channel. Histories of the same definite status have the same status output; different statuses have orthogonal outputs. If the actual retention maps preserve those status sectors, that historical distinction persists. These labels are not renamed Fold absent and Fold present. The statement that a commitment occurred and the statement that this Fold structure is present require different physical observables unless a source bridge identifies them. In a diagnostic structure-record tensor product,

$$P_{\text{presence}} = P_S \otimes I_R, \quad P_{\text{history}} = I_S \otimes Q_R.$$

These are generally different operators. The logical possibilities of structure present with record unwritten, and structure absent with historical record retained, demonstrate that the identification is not automatic. The diagnostic examples do not assert that both preparations are admitted by VERSF [S17]. Nor does separating the observables require the structure to be temporally earlier than commitment; they may be jointly realised. For the structural test of Section 23.5, use the source-returned absence support N and candidate physical support. When the absence representation is one ray and the Fold carrier has been independently established as $\mathbb{C}^4$, one may write the bookkeeping space

$$H_{\text{book}} = \mathbb{C}|\emptyset_F\rangle \oplus H_F.$$

Its direct-sum orthogonality represents hypotheses; it does not derive the microscopic absence sector, make the associated projection measurable or prove stability. The null direction is not a fifth internal Fold state. The Fold's internal vacuum or ground ray $|\Omega\rangle \in H_F$ is a further, differently typed object [S6, Axiom V1; S17]. The primitive Fold-level assertion is stable presence versus absence, established only when the structural stability and physical-readout conditions pass. Here rather than nowhere is shorthand for that assertion, not a coordinate assigned before spatial relations exist. Here rather than there needs distinguishable relational supports, and two supports still do not imply a metric separation $2\ell_p$. Fold existence is therefore not deduced directly from the commitment-history channel. The channel proves its own status distinction; the Fold-presence identification remains a physical source requirement.

Related but separate address result. Section 31 recovers $P_x \otimes I_{\text{int}}$ by grouping the actual source-labelled outcomes of the declared history instrument. It measures the input history address, not whether a Fold exists or whether that address remains occupied. It therefore supplements, rather than reverses, the status-presence distinction made here. Neither the input address nor a persistent departure record is identified with a Fold-location observable by notation.

24.2 Primitive Fold and completed-record clocks are not interchangeable

On the declared D36/PFDMI branch, the completed record occurs at

$$T_{\text{rec}} = T_{\text{cover}} = \inf\{n \geq 7 : X_n = X_0, \{X_0, \dots, X_n\} = V(W_7)\}.$$

The source reports a minimum of seven Fold updates and a mean of approximately 21.277682950918 updates [S4, §119A.3; S13, §2]. Its order is pre-completion evolution, first complete cover, completion operation, then persistent record. The completed R4 binary carrier therefore does not automatically identify a single primitive Fold-interface update or carrier. Use κ^{rec} for this aggregate record status. No equality $\kappa F = \kappa^{\text{rec}}$, equality of their closure complexes, or same-event identification of their marks is assumed.

Counting convention. T_{cover} is a stopping index in the declared aggregate history process. Its minimum and mean are reported in that source's update-count convention, not in seconds supplied before facts. This mathematical controller is not the timeless stable-support solver, and neither count supplies the primitive Fold's metric scale.

24.3 Exact current-only insufficiency witness

Let h be the hub and 0, ..., 5 the rim sites. Consider

$$h_A = (0, h, 0, 1, 2, 3, 4, 5, 0), \quad h_B = (0, 1, 2, 3, 4, 5, 0, 0, 0).$$

Both histories have eight steps, identical start and endpoint, and identical aggregate net current on every edge. The out-and-back hub excursion in h_A cancels in net current, while the self-transitions in h_B contribute no spatial current. But h_A first completes the closed cover at step eight, whereas h_B never visits the hub. With the archived probabilities $u = P(\text{rim} \rightarrow h) = P(\text{rim} \rightarrow \text{self}) = 0.200305886206958$, $p = P(\text{rim} \rightarrow \text{forward}) = 0.522615879874872$, and uniform hub row,

$$\Pr(h_A \mid X_0 = 0) = up^6/7 = 0.0005830305105664, \quad \Pr(h_B \mid X_0 = 0) = u^2p^6 = 0.0008174911017329.$$

Both histories have positive source probability. Thus no function of step count, endpoint and aggregate net edge current reproduces native cover status for every admitted history [S13; S15]. If self-transitions are omitted from the spatial trace graph, both traces also have first Betti number one; neither trace graph is automatically the primitive Fold closure complex. This does not prove that the unknown MASD D_C factors through net current. It excludes that shortcut, not every possible history-sensitive MASD realisation.

24.4 The native visited-set controller

Set $M_0 = \{X_0\}$ and retain the initial vertex. Then

$$M_{n+1} = M_n \cup \{X_{n+1}\}, \quad \kappa^{\text{rec}}_{n+1} = \kappa^{\text{rec}}_n \vee 1[X_{n+1} = X_0 \text{ and } M_{n+1} = V(W_7)].$$

This realises the existing completed-cover condition without inventing a scalar threshold. The visited-set register stores the history needed to recognise completion; the latch stores that completion has occurred. One persistent bit after completion is not, by itself, sufficient memory to calculate readiness throughout an arbitrary pre-cover history. This is a sufficient controller construction, not a proof of minimal visited-set memory. The source must still attach this history and controller information to the physical MASD variables and continuation law. The mathematical controller does not establish that physical attachment or turn a completed-cover record into one primitive Fold.

24.5 Ordered facts supply an order rank, not a prior clock

On the declared append-only history branch of [S4, PMS-EX25], let $h = (f_1, \dots, f_n)$ and $h' = h|f$ be an actual retained fact history and its extension. Define $h \leq h' \Leftrightarrow h$ is a prefix of h' . The relation is reflexive, antisymmetric and transitive. The rank $q(h) = |h|$ satisfies $q(h|f) - q(h) = 1$. Thus ordering and an extension count are constructed from facts in the declared history architecture. No background variable t , seconds per extension or pre-fact equation dN/dt is needed for this statement. This is consistent with the timeless fact-order distinction of PMS-EX18 [S4; S17]. A prefix order is not yet a complete relativistic spacetime, a universal order of all spacelike events or a physical elapsed-time metric. Clock comparison and causal or spatial reconstruction remain source tasks. The solver index k in Section 23.5 is not $q(h)$:

the former counts steps of a mathematical proof procedure; the latter counts actual facts on the declared history branch.

25. From the trap trigger to a four-state Fold-interface record

The Fold Interface Law derives the minimal Fold as a two-dimensional commitment interface with two independent binary structures: commitment polarity $b \in \{0, 1\}$ and local orientation $\omega \in \{-1, +1\}$. Their independence gives four sharp Fold atoms and $H_{\text{Fold}} = \mathbb{C}^4$ [S9]:

$$P_{b\omega} = |b, \omega\rangle\langle b, \omega|, \quad \sum_{b,\omega} P_{b\omega} = I_4.$$

The repeatable rank-one commitment instrument is $I_{b\omega}(\rho) = P_{b\omega}\rho P_{b\omega}$. To separate the persistent status latch from the one-time commitment event, define the first-passage firing variable

$$j_n = \kappa F_n - \kappa F_{n-1},$$

which equals 1 only at the unique $0 \rightarrow 1$ transition. At that event, provided the four-outcome commitment instrument on the Fold carrier is physically admitted at the correct event grain, it can resolve the unresolved interface state ρ_{pre} . The branch weights and post-event states are

$$p(b, \omega) = \text{Tr}(P_{b\omega}\rho_{\text{pre}}), \quad \rho_{\text{post}}(b, \omega) = P_{b\omega}\rho_{\text{pre}}P_{b\omega}/p(b, \omega) = |b, \omega\rangle\langle b, \omega|.$$

The topological threshold supplies the trigger; it does not supply the identity of the outcome. This distinction is forced by symmetry: κF is invariant under the polarity and orientation $\mathbb{Z}_2$ actions, while the four Fold atoms are permuted by them. No symmetry-respecting deterministic function of κF alone can select one of the four atoms: κF records that a commitment happened; (b, ω) records which fact happened. Exact microscopic retention of the four resolved atoms requires four orthogonal record pointers in a minimal pure Stinespring realisation. One convenient isometry is

$$V_F = \sum_{b,\omega} P_{b\omega} \otimes |R_{b\omega}\rangle, \quad V_F^\dagger V_F = I.$$

The isometry correlates the four sectors with orthogonal pointers without destroying total information. It does not by itself select one realised outcome or derive FC. The Fold carrier, the commitment event acting on it, and any later completed-cover readout must retain their distinct physical typing.

26. Polarity is the selected alternative, and a real fact forces a polarity-odd source component

The commitment and cost analysis supplies a type correction. A commitment θ maps a live alternative set C to one realised singleton $\{c_k\}$, and factorises into selection and exclusion: selection identifies which alternative became the fact; exclusion removes the others from the live set [S10]. For a binary commitment supported on a minimal Fold interface, take $C_F = \{D_0, D_1\}$. The Fold polarity is the interface-domain index selected by that commitment: $\theta_F: \{D_0, D_1\} \rightarrow \{D_b\}$, $b = k$. The Encoding Principle requires the realised outcome itself to be observer-invariantly representable. The fact-producing source therefore cannot be only an unsigned event count. If J_0 and J_1 are the two source states corresponding to the two physical commitment outcomes, fact separation requires $J_0 \neq J_1$. Their unique even and odd decomposition is

$$J_+ = \frac{1}{2}(J_0 + J_1), \quad J_- = \frac{1}{2}(J_0 - J_1), \quad J_b = J_+ + (-1)^b J_-,$$

and because $J_0 \neq J_1$ the polarity-odd component is necessarily nonzero: $J_- \neq 0$. This does not require a second primitive commitment source. It requires one primitive fact source with a richer mark alphabet, while the scalar density remains only its outcome-blind marginal. This agrees with the fact-separating Single-Source audit [S4].

26.1 Exact four-character projector decomposition

Define the commuting involutions on the four Fold atoms $S_b|b, \omega\rangle = (-1)^b|b, \omega\rangle$ and $O_\omega|b, \omega\rangle = \omega|b, \omega\rangle$, and their product $J_F = S_b O_\omega$. The sharp Fold projector has the exact Walsh or character decomposition

$$P_{b\omega} = \frac{1}{4}[I + (-1)^b S_b + \omega O_\omega + (-1)^b \omega J_F].$$

The four components have a clean interpretation: occurrence I , commitment polarity S_b , orientation O_ω , and the combined R4 character J_F . The coefficients are fixed by the joint binary projector algebra; they are not fitted. A first-passage marked-source write can therefore be represented structurally as

$$J_n^{(b,\omega)} = j_n q_n P_{b\omega},$$

with q_n the source-normalisation or current factor appropriate to the physical layer. This formula is a structural marked-source form; it is not by itself proof that the frozen pre-fact master action physically contains every required component. The audits of Section 27 show that the specifically tested outcome-blind source and embedding routes cannot supply it, and Section 28 shows why occurrence itself does not need to be supplied.

27. Source-completion audits: what cannot generate the missing mark

The source audits make the negative results sharp on their tested routes: the missing Fold identity cannot be recovered by algebraic manipulation of the outcome-blind orientation source, by tuning a transverse amplitude, or by changing an ordinary scalar κ background. These no-gos concern the specifically audited routes; they are not a theorem excluding every future source completion.

27.1 Algebraic no-go for the outcome-blind orientation source

Let O_ω be the orientation involution with $O_\omega^2 = I$. If the pre-fact source algebra contains only occurrence and orientation, then

$$A_0 = \text{Alg}^*\{I, O_\omega\} = \{\alpha I + \beta O_\omega\}.$$

Every element of A_0 is even under Fold-polarity reversal. The pure polarity operator S_b is odd. Therefore $S_b \notin A_0$ and $J_F = S_b O_\omega \notin A_0$. This is stronger than a failed first derivative: no polynomial, inverse, spectral function, or other algebraic manipulation of polarity-even source objects can manufacture the missing polarity-odd character. The obstruction is closure of the specified polarity-even algebra under its operations, not a matter of expansion order; universal physical superselection is not claimed.

27.2 A nonzero transverse gain is required for a continuum embedding, but its magnitude is not selected

The most economical two-direction marked embedding has the form

$$U_{(a,c)}(r, s) = a e_R + c s e_O, \quad J_{(a,c)} = \text{diag}(a, c).$$

It separates all four marked states iff $a \neq 0$ and $c \neq 0$. The MA1 normal form returns $a = 0$, $c = 0.578064557803$, rank 1. Every nonzero a makes the classical map injective, but the scalar count marginal contains no equation fixing its value, and symmetry fixes the transformation law rather than the physical gain. An arbitrarily chosen a would be fitting, not derivation [S4, PMS-EX41].

27.3 Ordinary κ -background rescue is impossible

The fundamental commitment field is governed by the linear equation $(\square + m^2)s = \rho_{\text{committed}}$. For $s = s_0 + \epsilon h$ about any background, including anisotropic or inhomogeneous configurations, the fluctuation operator remains $L_{s_0} = \square + m^2$ and $\partial L_{s_0} / \partial s_0 = 0$. A nonuniform scalar κ background cannot turn the inadmissible polarity mode into a zero mode. A type-correct anisotropic vector could cancel the Fold-Hessian divergence algebraically, but the source would have to return both that vector and the relevant cross-sector vertex independently; choosing them from the desired cancellation equation would be target fitting [S4, PMS-EX42].

27.4 Mark capacity exists, but capacity cannot name the mark

The corrected EX43-R2 audit establishes that the working microscopic instrument is not unmarked: 319 physically resolved marked operators exist, each has a unique conditioned state up to phase, the classical record-copy isometry is exact, and the marks coarse-grain to 84 persistent edge-and-terminal-type records. But the terminal-type index is explicitly firewalled from the later PSAC R4 sign, the exact-lift family is highly non-unique, and the unmarked Doob kernel is blind to the marked decomposition. Capacity and label counting cannot identify a PFDMI label as Fold polarity; that typing has to be returned by the source amplitudes or projectors or not claimed [S4].

Source note. The complete EX43-R2 report is available in full [S16]. Its executed negative concerns the specified branch-blind sequential R4-to-PFDMI route: equal detailed-label laws for the two R4 signs remain equal under later deterministic or stochastic regrouping. This is not a claim that no physical marks or generic conditioned-state history memory exist.

28. Category correction: fact production is primitive, not a missing pre-fact coupling

The cumulative PMS paper resolves a category error in the framing of the frontier. Its minimal Fact-Commitment axiom FC states: when an admissible primitive closure reaches its completed-record condition, exactly one allowed completion alternative resolves into a persistent fact. FC supplies only occurrence and persistence. It does not choose the alternative, probability law, R4 basis, phase, coupling strength, Standard Model label, or measured number [S4, PMS-EX58]. Around FC, the source stack derives the completed-record clock $T_{\text{rec}} = T_{\text{cover}}$, the unit-cost W_7 two-plane, the C_6 -covariant binary R4 projector pair up to output exchange, the minimal pure-record completion CP maps up to Kraus phase, and branch-preserving renewal up to branch phases. The demand for the frozen pre-fact master action to generate a separate nonzero gate-firing coefficient is therefore not made: that demand was attempting to derive the primitive fact-production rule itself.

Category correction. The scalar pre-fact action does not need to manufacture the existence of a committed fact. Fact production at completed admissible closure is the explicit primitive FC axiom. What remains derivational is the carrier, basis, timing, outcome map, persistence, and the post-commitment couplings that let a retained fact affect later dynamics.

This axiomatic boundary does not identify the completed-record event with one primitive Fold-interface update or carrier, install an independent same-event orientation record, or derive post-commitment coupling.

29. Sharp marginals, four-sector support and coefficient-free Fold-record preservation

29.1 Two commuting labels do not by themselves give four states

On the nondegenerate two-dimensional R4 carrier, put $Z_{R4} = \text{diag}(1, -1)$. Every Hermitian involution commuting with it is $\pm I$ or $\pm Z_{R4}$. Each gives only two nonzero joint sectors. An independent four-state Fold record cannot be obtained by attaching a second commuting label to that same two-dimensional carrier. A reflection that swaps the chiral R4 rays instead satisfies $FZ_{R4}F^\dagger = -Z_{R4}$. The operation reversing orientation is not thereby an independent commuting orientation-record observable. On a minimal four-dimensional joint carrier, the construction

$$R = Z \otimes I, \quad O = I \otimes Z, \quad B = RO = Z \otimes Z$$

gives the four rank-one projectors $Q_{(r,\omega)} = \frac{1}{4}(I + rR)(I + \omega O)$, $r, \omega \in \{\pm 1\}$. The Fold labels are recovered by $b = (1 - r\omega)/2$. This is an exact construction, not a derivation of its same-event physical installation [S14].

29.2 Four-sector support is an independent acceptance condition

For commuting Hermitian involutions R, O , require all four joint sectors to be nonzero. On a four-dimensional carrier their ranks are

$$\text{rank } Q_{(r,\omega)} = \frac{1}{4}[4 + r\text{Tr } R + \omega\text{Tr } O + r\omega\text{Tr}(RO)].$$

Thus all four ranks are one exactly when $\text{Tr } R = \text{Tr } O = \text{Tr}(RO) = 0$. Alternatively, source-realised independent polarity and orientation reversals act as $(r, \omega) \mapsto (-r, \omega)$ and $(r, \omega) \mapsto (r, -\omega)$. Their transitive action forces equal joint-sector ranks. On a four-dimensional carrier, completeness then fixes each rank at one. These must be physical symmetries on the common carrier, not merely names assigned to outcomes. Availability of all four sectors across admitted preparations is not a requirement that every individual preparation assigns positive probability to all four.

29.3 Sharp-marginal preservation theorem

Let $B = S_b$ and $O = O_\omega$ be the commuting intrinsic Fold observables, and set $R = BO$. Let $\mathcal{E}$ be an actual CPTP source-to-record channel. Its output must have correctly typed commuting Hermitian involutions $R_{\text{out}}, O_{\text{out}}$.

Theorem III.2 (sharp-marginal preservation). Suppose $\mathcal{E}^+(R_{\text{out}}) = R$ and $\mathcal{E}^+(O_{\text{out}}) = O$. Then $\mathcal{E}^+(R_{\text{out}}O_{\text{out}}) = RO = B$, and every joint record projector is preserved: $\mathcal{E}^+[\frac{1}{4}(I + rR_{\text{out}})(I + \omega O_{\text{out}})] = \frac{1}{4}(I + rR)(I + \omega O)$.

Proof. Write $\mathcal{E}^+(X) = V^+(X \otimes I)V$ with $V^+V = I$. For an involution pair satisfying $\mathcal{E}^+(A_{\text{out}}) = A_{\text{in}}$, let $\Delta = (A_{\text{out}} \otimes I)V - VA_{\text{in}}$. Expansion gives $\Delta^+\Delta = 2I - \mathcal{E}^+(A_{\text{out}})A_{\text{in}} - A_{\text{in}}\mathcal{E}^+(A_{\text{out}}) = 0$. Hence $(A_{\text{out}} \otimes I)V = VA_{\text{in}}$. Applying this to R and O yields the product identity, and expanding the joint projectors proves the remaining formula. This is the specialised sharp-observable preservation argument in [S15]. ■

No additional mixed-character coefficient is needed once an actual channel preserves both sharp marginals. The theorem fixes joint classical record statistics; it does not prove full quantum-coherence transport, select a realised outcome, construct the source channel, or identify the primitive and completed-record event grains. FC remains the explicit occurrence rule.

29.4 Discrete readout is distinct from continuum gain

A source-selected Hermitian polarity-odd operator X_b with no zero eigenvalues defines $S_b = \text{sgn}(X_b) = X_b|X_b|^{-1}$. Nonzero real rescaling changes at most the overall sign, that is, the names of the two outcomes. Its magnitude remains relevant to a physical continuum coupling. This functional-calculus observation supplies neither a missing X_b nor its physical attachment. Combined with the sharp-marginal theorem, it separates discrete record identity from continuum response magnitude without hiding either source requirement.

Identity gate. Establish a common physical event grain, a source-native channel, the correct independent binary transformation laws, four nonzero joint sectors, and exact preservation of both sharp marginals. Product-character and joint classical-record preservation then follow coefficient-free. An orientation involution commuting with a binary R_4 matrix, by itself, is not a sufficient PASS criterion.

Part IV. Relational readout, native history addresses and refinement

30. The relational question and the quantities that must remain separate

30.1 Presence is not an interval

The anchor definition of Section 2.4 is unchanged: a Fold is the minimal stable physical structure distinguishable from its absence. A relation between two supports is a different object from the presence of either support. The count of its endpoints does not fix its metric length. For a two-element set $\{a, b\}$, the assignment $d(a, a) = d(b, b) = 0$, $d(a, b) = d(b, a) = \delta$ is a metric for every $\delta > 0$. Thus two endpoints establish a possible nontrivial interval but do not imply 2δ , $2\ell_p$, or any other numerical separation. If an unweighted graph represents their relation by one elementary edge, the graph distance is one edge; identifying that graph distance with a physical length is a further source requirement [S22]. The absence state is not a spatial point through which one measures a path between Folds. Nor is a hub assigned coordinate zero thereby an absent Fold. The distinction between absence, an occupied reference, internal ground state and historical record in Sections 23 to 24 remains in force.

30.2 Conditional hexagonal realisation

The calculation in [S22] tests the following additional realisation: a locally Euclidean two-dimensional support geometry, a positive minimum separation δ , and simultaneous realisation of the maximum number of neighbours at separation δ . These are metric and realisation assumptions, not consequences of interface dimension alone. For two neighbour displacements v_i, v_j of norm δ ,

$$\|v_i - v_j\|^2 = 2\delta^2(1 - \cos \theta) \geq \delta^2.$$

Hence consecutive neighbour directions are separated by at least $\pi/3$. At most six neighbours fit around the reference support. If six are realised, all angular gaps are $\pi/3$. The hexagon describes the relations, not a literal hexagonal Fold tile. For successive primitive displacements u, v , the Gram matrix is

$$G_{\text{hex}} = \delta^2 \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}.$$

If the pattern also extends by the declared primitive integer translations, then $d(m, n)^2 = \delta^2(m^2 + mn + n^2)$ for $(m, n) \in \mathbb{Z}^2$. The identity $4(m^2 + mn + n^2) = (2m + n)^2 + 3n^2$ shows that the smallest nonzero squared value is one in native units, attained in six directions. This is a conditional distance rule, not a derivation of δ . The primitive translation-cell area $\sqrt{3}\delta^2/2$ is not an independently derived area of one Fold. Global lattice continuation, a support interpretation and physical normalisation remain additional requirements [S22].

30.3 Distinguishability and distance dictionary

The calculations of this part use several quantities that cannot be substituted for one another:

Quantity	Meaning	What is not established by it
D_{phys}	Distinguishability through the admitted readout class of Section 23.1.	A spatial length or a Fold diameter.
d_{read}	Euclidean distance of a declared two-coordinate catalogue chart.	Identification with the physical operational metric or spatial metric.
$R, d_{\text{resp}} = VR$	Inverse-Dirichlet contrast response and its associated diagnostic metric.	Identification with D_{phys} , the chart distance or a physical ruler.
δ_{rel}	Minimum distance between source-admitted physical supports after a positional metric is supplied.	A numerical value before that metric and its normalisation are derived.

Quantity	Meaning	What is not established by it
ℓ_F	Operational Fold-resolution scale in the sense of Section 23.2.	Equality to δ_{rel} , a metric diameter, or ℓ_p without the corresponding physical bridges.

A perfect probability distinction of one, a dimensionless chart gap and a graph-response gap answer different questions. The role of the following sections is to calculate their source-relative content while keeping those meanings separate.

31. Source-native reconstruction of the history-address readout

31.1 Marked outcomes supply the input-address projectors

FCHI-1 specifies the recorded history instrument

$$H_{y \leftarrow x} = \sqrt{(T_{xy})} |y\rangle\langle x|, \quad \sum_y T_{xy} = 1.$$

Its outcomes retain the ordered label $y \leftarrow x$. The effect determining the probability of a mark is $E_{yx} = H_{y \leftarrow x}^\dagger H_{y \leftarrow x} = T_{xy} P_x$, with $P_x = |x\rangle\langle x|$. Coarse-grain the actual recorded outcomes by retaining the origin and forgetting the destination. Completeness then gives

$$\sum_y E_{yx} = P_x.$$

This is a sum of effects in the declared instrument, not a detector inferred from a preferred coordinate chart. An input supported in address x yields the grouped outcome x with probability one. Different address inputs have disjoint full-mark distributions and total-variation distance one [S19; S25]. The resulting commutative address algebra is $A_{hist} = \text{span}\{P_x\} \cong \mathbb{C}^7$. Its seven minimal nonzero projections identify the retained history-address alternatives up to relabelling. They do not, merely by existing, identify seven spatial positions or seven individual Folds.

Scope of the readout. The result concerns the input address in the stated measurement context. A recorded departure is not proof of continued occupation, and a sharp address measurement is not the structural-presence test relative to the absence sector of Section 23.5. The source-native outcome labels are load-bearing: an arbitrary unobserved Kraus representation of a channel cannot be promoted to physical recorded outcomes.

31.2 Internal completeness preserves the address algebra

For a complete internal instrument $\{N_\alpha\}$,

$$K_{yx\alpha} = H_{y \leftarrow x} \otimes N_\alpha, \quad \sum_\alpha N_\alpha^\dagger N_\alpha = I_{int}.$$

Summing over both destination and internal outcome gives $\sum_{y,\alpha} K_{yx\alpha}^\dagger K_{yx\alpha} = P_x \otimes I_{int}$. For FCHI-1's nine-dimensional neutral factor this yields seven rank-nine address projectors on a 63-dimensional carrier. Neither the 63 carrier dimensions nor the 217 product outcomes are counts of spatial locations. The identity holds throughout the admitted neutral-parameter range; the output polar unitaries cancel from the completeness calculation. No special value of a survival parameter is interpreted as a clock [S19; S25]. The same conclusion holds for an edge-dependent complete internal instrument, $K_{yx\alpha} = \sqrt{(T_{xy})} |y\rangle\langle x| \otimes A_{yx\alpha}$ with $\sum_\alpha A_{yx\alpha}^\dagger A_{yx\alpha} = I_{int}$. It also survives refinement of a recorded outcome whenever the refined effects sum to the original effect. Additional internal observables may remain available; they are not additional elements of this particular retained address partition merely because they are distinguishable.

31.3 Mark preservation is stronger than unmarked coarse dynamics

The canonical wheel admits five rotation- and reflection-compatible exact groupings of its unmarked transition law, with 1, 2, 3, 4 and 7 retained labels. Their existence is established by the exhaustive 877-partition calculation in [S24]. The coarser groupings discard some marked origin information. Requiring

the complete recorded-mark likelihood to factor through the grouping leaves exactly one of the 877 partitions: all seven input labels remain separate [S25]. This agrees with the projector identity above and does not contradict the five unmarked groupings. On the canonical kernel, keeping only the destination gives effects $E_y = \sum_x T_{xy} P_x$. The one-use total-variation separations of distinct input addresses then belong to $\{1/4, 3/7, 1/2, 3/4\}$ rather than one. Erasing the classical origin register changes the accessible instrument while leaving the quantum channel obtained by summing over all marks unchanged. Complete origin-labelled records, not a channel alone, support the sharp history-address conclusion.

32. Canonical source, spectral chart and full relational response

32.1 Keep the canonical and driven sources separate

HMGB-1 supplies the reversible canonical catalogue $K = \{h, b_0, b_1, b_2, b_3, b_4, b_5\}$, with

$$T_0(h, y) = 1/7 \text{ for every } y, \quad T_0(b_i, h) = T_0(b_i, b_i) = T_0(b_i, b_{i-1}) = T_0(b_i, b_{i+1}) = 1/4,$$

and all other entries zero. Its stationary law and spectrum are

$$\pi = (1/31)(7, 4, 4, 4, 4, 4, 4), \quad \text{spec}(T_0) = \{1, 1/2, 1/2, -1/4, -3/28, 0, 0\}.$$

This is the explicit numerical source of [S23 to S25]. HMGB grades its construction at the canonical-catalogue level, not as the complete physical MASD/CAR/Fock history law [S18]. FCHI-1's numerical history kernel is the driven, nonreversible kernel. Applying FCHI's general marked-operator prescription to T_0 is an explicitly declared calculation; it is not a rerun of the driven FCHI or frozen EX2E arrays [S19; S25].

32.2 Two-coordinate cyclic readout

Within the declared class of linear two-dimensional readouts intertwining the wheel rotation and reflection with the usual planar action and separating all labels, the candidate is unique up to scale and simultaneous coordinate orientation:

$$X(h) = (0, 0), \quad X(b_i) = c(\cos(\pi i/3), \sin(\pi i/3)).$$

The unscaled coordinate functions have stationary inner-product Gram matrix $(12/31)I_2$. Requiring unit norm in that inner product fixes the chart convention

$$c = \sqrt{31/12}, \quad \Delta_{\text{read}} = \sqrt{31/12} = 1.6072751268.$$

The first harmonic is a $1/2$ -eigenmode of T_0 . Its geometric normalisation is a chart choice; repeated kernel application contracts the mode and is not the isometric refinement used below. The second harmonic identifies opposite rim labels and therefore fails the stated injectivity requirement [S23]. Among the 21 unordered pairs, squared chart distances are $31/12$ for twelve pairs, $31/4$ for six pairs and $31/3$ for three pairs. Two neighbouring radial vectors have Gram matrix $H_{\text{read}} = (31/12)[[1, 1/2], [1/2, 1]]$. No factor here is a Planck multiple, metre, diameter or independently derived physical metric normalisation. The coordinate zero at the hub is an occupied catalogue reference, not Fold absence. The two coordinate functions jointly separate the catalogue. Their polynomial-function spaces have dimensions 1, 3, 6 and 7 at total degrees zero through three. Thus two coordinates can generate all catalogue indicator functions; a two-dimensional coordinate chart does not imply only two independent catalogue observables. This is an algebraic observation, not proof that every such function is a spatial observable [S24].

32.3 Full canonical Dirichlet response

Retain all nonconstant source modes, rather than only the planar chart. Put $D_\pi = \text{diag}(\pi)$ and $Q_0 = D_\pi(I - T_0)$. Detailed balance makes Q_0 symmetric, positive semidefinite and rank six, with kernel spanned by the common constant. Its scalar quadratic form is

$$\mathcal{E}_0(f) = f^T Q_0 f = \frac{1}{2} \sum_{x,y} \pi_x T_{0xy} (f_x - f_y)^2.$$

For a pair contrast $b_{xz} = e_x - e_z$, the associated inverse-cost response is

$$R_{xz} = \sup_{\mathcal{E}_0(f) > 0} (f_x - f_z)^2 / \mathcal{E}_0(f) = b_{xz}^T Q_0^+ b_{xz}.$$

Equivalently, the minimum quadratic cost for $f_x - f_z = 1$ is $1/R_{xz}$. The inverse is taken only on the nonconstant subspace. The square root of R admits a Euclidean response representation, but is not automatically a spatial length, an operational trace distance or the physical Fisher metric [S24].

Pair type	Squared chart distance	Full response R
Hub to rim	31/12	279/20
Adjacent rim labels	31/12	341/20
Rim labels two steps apart	31/4	93/4
Opposite rim labels	31/3	124/5

The normalised squared-distance ratios are respectively (1, 1, 3, 4) and (1, 11/9, 5/3, 16/9). The two constructions are not conflicting measurements of a physical length; neither has been physically identified with that length. Their different ratios rule out identifying them merely by one scalar rescaling. The full response Gram has rank six; that is not a claim of six spatial dimensions [S24].

32.4 The same marks return the response operator

For each directed mark define $b_{xy} = e_y - e_x$. Then

$$Q_0 = \frac{1}{2} \sum_{x,y} \pi_x T_{oxy} b_{xy} b_{xy}^T.$$

Thus the declared recorded addresses and their stationary mark weights return the same complete quadratic response. Self-marks have zero signed incidence, but have nonzero outcome effects and participate in address recovery. The 14-by-31 arrival and departure matrix has rank thirteen; its signed 7-by-31 difference has rank six. These are incidence ranks, not spatial dimensions or metric factors. The combinatorial relation to PFDML's source and target attachment remains at that source's declared-class grade [S20; S25].

33. Internal description can refine without adding retained addresses

Let a fine description of an existing address x be (x, z) , with normalised conditional weights $w(z|x)$. Set

$$\mu(x, z) = \pi_x w(z|x), \quad (Lf)(x, z) = f(x), \quad (CF)(x) = \sum_z w(z|x) f(x, z).$$

Then $CL = I$, and $\|Lf\|_{\mu}^2 = \sum_{x,z} \pi_x w(z|x) |f(x)|^2 = \|f\|_{\pi}^2$. The lift is therefore exactly isometric in the declared stationary inner products. The retained chart obeys $X_{\text{fine}}(x, z) = X(x)$, and its minimum nonzero gap remains $v(31/12)$. The exact checks in [S23] use unequal conditional weights and 1, 2, 4, 8 and 16 internal descriptions per address, giving 7, 14, 28, 56 and 112 total descriptions while retaining seven chart images. Zero distance within a fibre means zero distance in this retained readout, not equality under every physical observable. An internal state, historical record or additional variable is not automatically another position. Conversely, this construction does not establish that every fine variable in the actual source is merely internal. The retained address map is one of the hypotheses, not an inference from the desired gap. For a positive physical fine metric G_n , after removing its verified null directions, the minimum-cost quotient formula of [S21] reads

$$K_n = (DQ_n G_n^{-1} DQ_n^T)^{-1}, \quad \min_{DQ_n v = d} v^T G_n v = d^T K_n d.$$

Pseudoinverses require the fixed-rank and null-compatibility conditions of the source. With a varying metric this initially specifies a local tangent cost; a finite distance needs control along admitted paths. It does not identify a history address with a physical position or a Dirichlet response with a Fisher metric.

34. Complete retained-boundary response under internal elimination

34.1 Exact Schur-reduction condition

For a connected positive graph response, split the variables into retained boundary labels B and internal variables I . Assume every internal connected component meets the retained set, so the internal block is invertible: $Q_f = [[Q_{BB}, Q_{BI}], [Q_{IB}, Q_{II}]]$. Minimisation at fixed retained boundary values gives

$$f_I^{\text{opt}} = -Q_{II}^{-1}Q_{IB}f_B, \quad Q_{\text{eff}} = Q_{BB} - Q_{BI}Q_{II}^{-1}Q_{IB}.$$

Theorem IV.1 (retained-boundary reduction). For a comparison with an original connected response Q_c on the same labelled retained set and in the same units, all retained pair responses agree if and only if $Q_{\text{eff}} = Q_c$.

The forward direction uses the fact that the full pair-response matrix determines the centred inverse Q_c^+ , and hence Q_c , on its nonconstant subspace. This is standard Schur or Kron reduction applied to the canonical source. The exact verification in [S24] checks all 119 proper retained subsets of the seven-label catalogue containing at least two labels.

34.2 Retained response does not constrain every newly exposed variable

A constructed diagnostic replaces each edge of coefficient $1/31$ by a series chain of r edges, each of coefficient $r/31$. Eliminating its intermediate variables reproduces the original response exactly. If the intermediate variables are instead exposed as independent terminals, their minimum response separation decreases.

Total variables	Minimum R over original seven labels	Minimum R with inserted variables exposed
7	279/20	13.95
19	279/20	11.2375
31	279/20	8.438889
43	279/20	6.684375
91	279/20	3.608594

For one segment of a subdivided spoke the exact expression is $R_{\text{segment}} = 31/r - 341/(20r^2) \rightarrow 0$. These are artificial refinement controls, not admitted sub-Fold objects. Their force is logical: retained-response preservation does not prove that the introduced variables are additional locations, but it also does not exclude that interpretation. The physical support readout must settle the distinction [S24].

35. Correlated internal refinement preserves the full retained response

35.1 Hypotheses and notation

Let a finite fine catalogue Y map to retained labels X through an independently specified map $p: Y \rightarrow X$. Let S_f be its row-stochastic transition matrix and $\mu > 0$ a stationary distribution. Define

$$\pi_x = \sum_{a:p(a)=x} \mu_a, \quad (Lg)_a = g_{p(a)}, \quad C = D_\pi^{-1}L^T D_\mu.$$

Then C is conditional averaging within fibres, $CL = I$, and L is an isometry between the stationary inner products. Assume the fine law is reversible and $S_f L = L T_o$. In entries, the total probability of reaching retained address y is T_{oxy} , independently of which internal description of x was used. This exact grouping condition must be tested on the source. It is not guaranteed by symmetry or by the dimensions of the matrices. Set $Q_f = D_\mu(I - S_f)$ and $Q_o = D_\pi(I - T_o)$.

35.2 Whole-response preservation theorem

Every fine observable with retained conditional means g has the unique decomposition $f = Lg + h$ with $Ch = 0$. Reversibility and the intertwining identity imply $\langle Lg, (I - S_f)h \rangle_\mu = \langle L(I - T_o)g, h \rangle_\mu = 0$ and $L^T Q_f L = Q_o$. Consequently

$$f^T Q_f f = g^T Q_o g + h^T Q_f h.$$

Theorem IV.2 (whole-response preservation). Since Q_f is positive semidefinite and $h = 0$ is admissible, $\min_{Cf=g} f^T Q_f f = g^T Q_o g$. The complete retained contrast response therefore agrees with the original: $\sup_{f: f \wedge T Q_{-f} f > 0} ((Cf)_x - (Cf)_z)^2 / (f^T Q_f f) = (e_x - e_z)^T Q_o^+ (e_x - e_z)$.

This is not limited to independent duplication or equal internal weights. Internal correlations may be nonzero while the retained response remains exactly unchanged [S25].

Different coarse constraints. Section 34 fixes the values at retained boundary vertices. This section fixes conditional means over fibres. Both are valid minimum-cost calculations, but their constraints are different and must not be interchanged without a source identification.

35.3 Executed correlated refinements and the retained gap

The construction in [S25] uses unequal conditional weights, correlated inter-fibre transitions and internal self-step memory. It verifies the stationary marginal, exact grouping, lift isometry, full coarse-operator recovery and vanishing cross-energy block in rational arithmetic.

Internal descriptions per retained address	Fine dimension	Retained addresses	Minimum retained R
2	14	7	279/20
3	21	7	279/20
4	28	7	279/20
9	63	7	279/20

An independent numerical inversion of each complete fine operator checks all 21 retained pairs; the archived maximum error is below 4×10^{-14} . The 63-dimensional constructed reversible refinement is not FCHI's driven 63-dimensional history-neutral carrier. Equal dimension is not a physical identification. The theorem is independent of the number of internal descriptions. For any family satisfying the hypotheses in one common normalisation, the retained response gap is preserved uniformly. The table verifies examples; the theorem carries the statement beyond those examples. Neither proves a positive lower bound on new physical supports that are invisible to the retained map.

36. The nonreversible boundary must be tested separately

The marked-address projector identities of Section 31 do not require reversibility. The response-preservation theorem of Section 35 does. For a nonreversible fine law, define its stationary adjoint $S_f^{(\mu)} = D_\mu^{-1} S_f^T D_\mu$. The scalar quadratic response uses the symmetric operator

$$Q_f^{\text{sym}} = D_\mu [I - (S_f + S_f^{(\mu)})/2].$$

A sufficient extension of Section 35 is preservation of the retained subspace by both the forward and stationary-adjoint kernels, with the corresponding coarse laws: $S_f L = L T_c$ and $S_f^{(\mu)} L = L T_c^{(\mu)}$. The stationary adjoint is a mathematical construction. It does not reverse a physically committed fact or supply a pre-existing clock. Forward preservation alone is insufficient. The exact four-state control in [S25] is

$$S_f = [[2/5, 1/10, 2/5, 1/10], [1/10, 2/5, 2/5, 1/10], [1/4, 1/4, 1/10, 2/5], [1/4, 1/4, 1/10, 2/5]].$$

Group the first two and last two labels. The matrix is positive and doubly stochastic and has the exact forward coarse law $T_c = \frac{1}{2}[[1, 1], [1, 1]]$. Reverse-kernel preservation fails. The coarse and fine-minimised symmetric quadratic operators are

$$Q_c = \frac{1}{4}[[1, -1], [-1, 1]], \quad Q_{\text{eff}} = (191/800)[[1, -1], [-1, 1]].$$

Their pair responses are 4 and 800/191, respectively. Thus a forward coarse transition law can be exactly preserved while the minimum-cost relational response changes. The reversible canonical proof cannot be transferred without further checking to the driven FCHI/PFDML source [S25].

37. Physical promotion and the meaning of a minimum

37.1 What is supplied

The declared marked instrument supplies the retained input history-address effects. The canonical catalogue supplies exact transition weights and a complete quadratic response. The spectral chart supplies a distinct, explicitly normalised two-coordinate representation. The refinement calculations establish precise conditions under which retained addresses and their responses survive additional internal description. The unresolved readout is specifically the physical localisation of Fold structure, not the availability of history-address observables.

37.2 What remains to be identified

A physical source map must determine which history-address, internal and record observables describe the same physical support and which describe different supports. It need not be a one-to-one map from the seven catalogue labels to seven locations. A closure-state label may represent a history role, relation or configuration rather than an individual Fold position. The same source must determine the operational spatial metric of those supports, the permitted refinement maps and their common normalisation. A chart rescaling, an inverse-Dirichlet normalisation or the number of endpoints cannot substitute for that physical assignment. The 31/12 and 279/20 numbers belong to different calculated diagnostics and cannot each be called the minimum physical Fold separation. Once an actual positional set P_n and metric d_n are source-returned, a fundamental minimum requires a statement of the form

$$\inf_n \inf_{x \neq y \in P_n} d_n(x, y) > 0,$$

with the same physical normalisation and a correct treatment of genuinely new supports. Finite cardinality or positivity of a gap at every separate level is insufficient. Exact preservation of old addresses controls only those retained addresses. Refinement that changes the physical support set requires its own test.

37.3 No conflation with the other identity gates

The projectors $P_x \otimes I_{\text{int}}$ do not identify the structural absence projector, the four intrinsic Fold atoms, a rank-one trap selector, the rank-one current-kernel locus, or a completed R4 polarity. They act on differently typed carriers and answer different questions. In particular, source-address recovery does not close the polarity and continuation-faithfulness gates of Sections 10 to 29. No physical time is inferred from a kernel index, internal multiplicity, graph-subdivision count or solver round. Ordered facts retain the role specified in Section 24.5. No spatial diameter, interface area, absolute ruler or Fold-Planck ratio is returned by [S22 to S25].

Relational-refinement conclusion. A more detailed mathematical description need not contain smaller physical things or shorter physical intervals. The declared recorded instrument already supplies exact retained history addresses, and suitably compatible refinements can preserve their full relational response. A fundamental spatial minimum additionally requires the source to identify actual supports, the metric governing their relations and the status of any newly resolved supports. These results establish those distinctions as executable tests, not as a numerical size prediction.

Part V. Conditional metric-scale closure and physical coherence attachment

38. What the Planck-scale calculation is, and is not, predicting

38.1 Keep the structural and metric objects separate

The anchor definition is unchanged: a Fold is the minimal stable physical structure distinguishable from its absence. A commitment event, a retained record, a history address, an admissible transport configuration and a metrically certified process are not interchangeable descriptions of that structure. The scale programme contains a conditional numerical closure as well as the One-Fold calibration. Its output concerns a proposed metric identity-collapse or certification scale and the conventional Planck combination. It does not determine the diameter of a primitive Fold, identify seven history labels with seven spatial locations, or turn the collective coherence length into a single-Fold size. The following dictionary controls the calculation:

Symbol	Meaning in this part	Identification not supplied by notation
ℓ_F	Operational Fold-resolution length after physical support and metric reconstruction	Not an assumed diameter or catalogue distance
δ_{rel}	Minimum nonzero separation between physically identified supports, when a uniform minimum is established	Not automatically ℓ_F or twice ℓ_F
ℓ_*	Identity-collapse or fact-supporting metric scale in the scale-closure papers	Not automatically the primitive Fold's metric extent
ℓ_e	Emergence or reference length entering Route M	Its ratio to ℓ_* needs physical justification
ξ	Coherence observable entering the hierarchy and capacity relations	The two uses must describe the same physical observable before the equations are combined
L_Λ	Declared de Sitter reference length $\sqrt{3/\Lambda_{\text{cos}}}$	An external cosmological input, not an inferred Fold count
$\ell_{\text{P,cand}}$	Candidate value of $\sqrt{\hbar G/c^3}$ returned by the joint conditional system	Not a completed first-principles Fold-size prediction

The companion Fold, Fact, Planck, and Closure [S30] separates structural distinction, commitment, metric certification and collective closure. Its terminology is not imported wholesale: that paper sometimes calls the Fold provisional or not yet realised, whereas this paper retains the stable-structure definition above. The useful contribution consumed here is the separation of properties and scale questions, not a silent change of the Fold definition. These distinctions also do not impose a universal sequence of formation stages timed by a prior clock.

38.2 The effective-regime certification bound

The Planck-time paper [S29] supplies a conditional certification argument within an already available effective causal description:

$$\Delta t \geq \hbar/E_{\text{dyn}}, \quad E_{\text{dyn}} \leq E_{\text{grav}}, \quad E_{\text{grav}} \leq \eta(c^5/G)\Delta t.$$

For positive energies and duration the algebra gives $\Delta t^2 \geq (\kappa/\eta)(\hbar G/c^5)$. The quantum distinguishability bound, energy-budget bridge and no-trapping safety condition retain the applicability assumptions of [S29]. In particular, the safety condition restricts the admitted certification protocols; it is not proved to characterise every conceivable deep-substrate process. The result is a lower bound on the effective duration in that class, not a proof of attainment, temporal discreteness or a pre-fact waiting period.

Spatial-scope amendment. A lower bound on Δt , together with $R \leq c\Delta t$, does not alone imply a universal lower bound on R . A spatial conclusion additionally needs a justified light-crossing identification or a separate localisation-energy condition. The statement $R_{\min} \sim \ell_P$ is therefore retained only with that extra physical scope. It is not used here as an independent proof that $\ell_F = \ell_P$. The certification calculation and the numerical closure below answer different questions. The former gives conditional Planck scaling in an effective regime. The latter tests whether a proposed hierarchy and common scale matching return the magnitude of that metric threshold without supplying measured G or a measured ξ .

39. Joint Route-M, capacity and cosmological closure

39.1 Retain the numerical coefficients instead of setting them by notation

Write the Route-M hierarchy as $\Theta := \ln(\xi/\ell_e)$. On the constant-coefficient branch of [S27 to S28],

$$\Theta = (1/(2b))(1/g_o^2 - 1/p_c), \quad g_o^2 = 1/128, \quad b = 7/8.$$

Here b is the probability-running coefficient being tested. It is unrelated to the incidence coefficient called b in Part I. Θ is used instead of the hierarchy symbol Λ , so that it cannot be confused with the cosmological constant $\Lambda_{\cos}$. Retain the capacity coefficient explicitly:

$$\rho_\Lambda \xi^4 = B\hbar c, \quad B > 0.$$

The coherence paper [S8] distinguishes its defined threshold $\chi(\xi) = 1$ from its unreturned physical mode-spectrum prefactor. Setting $B = 1$ defines a particular quartic-unit length. Establishing that the physical coherence observable used by Route M has that same normalisation is an additional matching requirement. Similarly, retain the constants in the proposed common-tension relations [S27]:

$$\ell_*^2 = C_{UV}\hbar c/T_{\max}, \quad G = C_{IR}c^4/T_{\max}.$$

Here $T_{\max}$ is a force or tension, not the pressure variable of a separate constitutive model. Eliminating it gives $\alpha_* := C_{UV}/C_{IR}$ and $\ell_*^2 = \alpha_*\hbar G/c^3$. Define $q := \ell_e/\ell_*$ and $\ell_P^2 := \hbar G/c^3$. Thus

$$\xi = qV(\alpha_*) \ell_P e^\Theta.$$

The identifications $q = 2$ and $\alpha_* = 1$ are explicit benchmark premises. The tension cancellation does not establish equality of the UV and IR coefficients, and the two-endpoint argument of Section 30 does not establish $q = 2$.

39.2 Solve the positive system without a supplied coherence length

Use the declared cosmological reference $L_\Lambda = \sqrt{3/\Lambda_{\cos}}$. The effective cosmological vacuum relation, explicitly imported into this cross-paper calculation, is

$$\rho_\Lambda = \Lambda_{\cos}c^4/(8\pi G) = 3c^4/(8\pi GL_\Lambda^2).$$

Here ρ_Λ is energy density. This follows the energy-density convention recorded in the closure audit [S37], with c^4 ; it is an explicit dimensional-convention amendment to the coherence paper's printed expression, not a VERSF derivation of the cosmological vacuum relation. Combining the capacity and cosmological relations gives

$$\xi^4 = (8\pi B/3)\ell_P^2 L_\Lambda^2, \quad \xi^2 = \sqrt{(8\pi B/3)} \ell_P L_\Lambda.$$

The hierarchy gives a second expression, $\xi^2 = q^2\alpha_*\ell_P^2 e^{2\Theta}$. Solving on the positive branch yields

$$\ell_{P,\text{cand}} = (\sqrt{(8\pi B/3)/(q^2\alpha_*)}) L_\Lambda e^{-2\Theta}.$$

The corresponding outputs are

$$\xi_{\text{cand}} = (v(8\pi B/3)/(qV(\alpha_*))) L_{\Lambda} e^{-\Theta}, \quad G_{\text{cand}} = (8\pi B c^3/(\hbar \Lambda_{\text{cos}} q^4 \alpha_*^2)) e^{-4\Theta}.$$

Measured G , measured ℓ_P , and a measured or chosen coherence length are not inputs to this solution. The externally supplied cosmological scale, effective c and $\hbar$, the common-observable identification and all displayed dimensionless coefficients remain inputs. Absence of an explicit measured G is not a proof that every source assumption or cosmological inference is independently derived. The archive [S37] checks symbolic elimination and an independent numerical solution. The three equations are nondegenerate in positive logarithmic variables. Their algebraic determination is conditional on the physical independence and selection of their inputs; it does not turn proposed matching assumptions into source theorems.

39.3 Numerical benchmark and its actual scope

Use the rounded source reference $L_{\Lambda} = 1.7 \times 10^{26}$ m, the middle digest value $p_c = 0.1865$, and the declared branch $g_o^2 = 1/128$, $b = 7/8$, $B = 1$, $q = 2$, $\alpha_* = 1$. Then $\Theta = 70.0788969743393...$ and

Quantity	Conditional output
Planck-equivalent length	1.66032×10^{-35} m
Coherence length	90.3858 μm
Gravitational coupling	7.04319×10^{-11} m ³ kg ⁻¹ s ⁻²
Dimensionless combination $\Lambda_{\text{cos}} \ell_{P,\text{cand}}^2$	$2.86158639 \times 10^{-122}$

The 88 μm coherence example is not supplied to this calculation. The 90.4 μm value is an output, not evidence that an intrinsic coherence boundary has been measured. Against the fixed comparison values recorded in [S37], the residuals are about 2.73% in length and 5.53% in G ; these are benchmark residuals, not agreement within experimental uncertainty or an independently estimated theoretical error. Holding the other benchmark assumptions fixed and using only the narrower threshold interval in the digest of [S27] gives:

p_c	ξ_{cand}	$\ell_{P,\text{cand}}$
1/6	130.15 μm	3.44262×10^{-35} m
0.1865	90.39 μm	1.66032×10^{-35} m
1/5	73.50 μm	1.09787×10^{-35} m

This is a conditional envelope, not a confidence interval. The microphysical companion [S28] also discusses a broader working range, approximately [0.17, 0.30]; the narrower digest interval must not be represented as a universal source-selected bound. The threshold and coherence inputs cannot be varied independently after imposing the joint closure. The archived sensitivity audit gives

$$\partial \ln \ell_{P,\text{cand}} / \partial \ln b = 2\Theta = 140.16.$$

A diagnostic increase of 1% in b changes this candidate length by a factor of approximately 4.01; a decrease of 1% gives a factor of approximately 0.243. These changes test sensitivity to the dynamical coefficient, not uncertainty in counting fourteen channels. They show why an exact count cannot be substituted for an independently evaluated probability-running coefficient.

Interpretation of the numerical agreement. The benchmark's proximity to the comparison Planck length is a conditional numerical result. Its evidential strength depends on independent justification of the inputs and the final invariant coefficient combination. Given the sensitivity factors above, the reported

2.73% length residual cannot be interpreted as a demonstrated theoretical accuracy. A physical prediction requires the relevant running and normalisation to be determined independently of the desired comparison value.

39.4 The normalisation-invariant target

The joint result can be written

$$\Lambda_{\text{cos}} \ell_{\text{P, cand}}^2 = (8\pi B / (q^4 \alpha_*^2)) e^{-4\Theta}.$$

Define the combined quantity

$$\Theta_{\text{phys}} = \Theta + \ln(q/2) + \frac{1}{2} \ln \alpha_* - \frac{1}{4} \ln B. \quad \text{Then} \quad \Lambda_{\text{cos}} \ell_{\text{P, cand}}^2 = (\pi/2) e^{-4\Theta_{\text{phys}}}.$$

The subscript names the physical matching target; it does not certify that its numerical value has been derived. Section 47 shows that K itself is not invariant under all consistent length-convention changes (an ℓ_e rescaling by v and a ξ rescaling by u give $K' = (v/u)K$), so statements about the numerical value of K hold in the fixed convention stated here; the object invariant across conventions is $\Omega := \Theta + \ln K$, identical to Θ_{phys} , through $J_\Lambda := \Lambda_{\text{cos}} \ell_P^2 = (\pi/2) e^{-4\Omega}$. For example, the consistent bookkeeping change $\xi' = u\xi$ requires $B' = u^4 B$ and $\Theta' = \Theta + \ln u$, leaving the displayed combination unchanged. Setting an intermediate coefficient to one by a scale convention moves its content elsewhere; it does not remove the physical matching obligation [S38]. Compulsory prediction would therefore be established either by fixing the individual physical quantities or by showing that all admitted implementations yield the same combined value. The latter can suffice even where intermediate conventions differ. No such full-source rigidity result is returned.

39.5 Inverse formulation and central-value matching targets

The forward calculation is stiff in b ; the same stiffness makes the inverse evaluation informative. The final invariant relation can be read backwards: given a cosmological constant and the measured Planck length,

$$\Theta_{\text{phys}}^{(\text{req})} = -\frac{1}{4} \ln(\Lambda_{\text{cos}} \ell_P^2 / (\pi/2)).$$

In terms of the benchmark parameters,

$$\Theta_{\text{phys}}^{(\text{req})} = (128 - 1/p_c) / (2b) + \ln K, \quad K = (q/2) v(\alpha_*) B^{-1/4},$$

so solving for any one parameter requires fixing the others; the calculation does not separately determine b , p_c , q , α_* and B . The cosmological reference is constructed as $\Lambda_{\text{cos}} = 3\Omega_\Lambda H_0^2 / c^2$, with $\Omega_\Lambda = 0.6847$ and $H_0 = 67.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from the Planck 2018 base- Λ CDM analysis (TT, TE, EE + lowE + lensing), using $1 \text{ Mpc} = 3.0856775814913673 \times 10^{22} \text{ m}$ and CODATA 2018 constants [S46]. These cosmological quantities are model-inferred within Λ CDM, not direct measurements independent of cosmological assumptions. The second column below is a constructed sensitivity scenario: the SH0ES value $H_0 = 73.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with the Planck $\Omega_\Lambda = 0.6847$ held fixed. The central-value spread between the two columns is not a confidence interval. The inverse calculations solve for the value each selected parameter would require to reproduce the observational central values, holding the remaining parameters fixed. These values are matching targets, not independently derived predictions, and the table reports central-value calculations only; its displayed decimal places are not demonstrated precision.

Quantity	Planck-based branch	SH0ES- H_0 sensitivity scenario
Λ_{cos}	$1.0891 \times 10^{-52} \text{ m}^{-2}$	$1.2806 \times 10^{-52} \text{ m}^{-2}$
$L_\Lambda = v(3/\Lambda_{\text{cos}})$	$1.65966 \times 10^{26} \text{ m}$	$1.53060 \times 10^{26} \text{ m}$
$\Theta_{\text{phys}}^{(\text{req})}$	70.080338	70.039860

Quantity	Planck-based branch	SH0ES-H ₀ sensitivity scenario
Benchmark Θ_{phys}	70.078897	70.078897
Inverse-required b ($K = 1$, p_c fixed)	0.874982	0.875488
Required K ($b = 7/8$, p_c fixed)	1.001442	0.961716
Inverse-required p_c ($b = 7/8$, $K = 1$)	0.186588	0.184154

Forward re-evaluation with the more precise Planck-based cosmological reference and unchanged benchmark coefficients gives $\ell_{\text{P,cand}} = 1.62092 \times 10^{-35}$ m, a length residual of approximately +0.29% and a gravitational-coupling residual of approximately +0.58%. These results are distinct from the rounded-reference residuals of +2.73% and +5.53%: most of the rounded-reference length residual was numerical rounding of L_Λ , not a property of the coefficients. The inverse-required running coefficient under the fixed benchmark assumptions is 0.874982 on the Planck-based branch and 0.875488 in the SH0ES-H₀ scenario, a spread of 0.058% of the coefficient between the two constructed scenarios. The benchmark value 7/8 lies between these two inverse-required central values. This location is a conditional numerical observation at central values, holding $K = 1$ and $p_c = 0.1865$ fixed; observations constrain the combined hierarchy and normalisation rather than the running coefficient alone. At the adopted central values and fixed hierarchy parameters, exact matching requires $K = 1.001442$, approximately 0.144% above unity, in the fixed length convention stated above (Section 47). This displacement is not an observational tolerance or a theoretical error bound.

Coherence normalisation. The inherited value $\xi_M = 84.071785948 \mu\text{m}$ is the collective coherence length of [S8], obtained from the vacuum-energy throughput condition within that source's own construction, with its order-unity mode-spectrum prefactor retained as an explicit matching obligation; it has not been established to share the cosmology, observable definition or normalisation of the present capacity calculation. The capacity relation at $B = 1$ with the Planck-based inputs returns $\xi = 88.11 \mu\text{m}$. Matching ξ_M alone would require $B_{\text{req}} = (84.07/88.11)^4 = 0.829$, which changes K by the factor $B_{\text{req}}^{-1/4} = 1.048$. This is a conditional matching calculation: the resulting change in K is a normalisation difference requiring reconciliation, and no stronger conclusion is drawn until the common-observable identification and the relevant uncertainties are established.

Uncertainty structure. Observational uncertainty propagates through $d\Theta_{\text{phys}}^{(\text{req})} = -\frac{1}{4} d\ln \Lambda_{\text{cos}} - \frac{1}{2} d\ln \ell_p$. A first-pass propagation of the published Planck 2018 marginal uncertainties ($\Omega_\Lambda = 0.6847 \pm 0.0073$, $H_0 = 67.36 \pm 0.54$), ignoring their mutual correlation, gives $\sigma(\Theta_{\text{phys}}^{(\text{req})}) = 0.0048$ within the adopted model, with the CODATA Planck-length contribution negligible. On that first pass the Planck-branch central displacement of 0.00144 lies well inside the observational spread; a treatment retaining the published parameter correlations has not been performed. Four contributions are kept distinct: numerical rounding of reference inputs; observational uncertainty within the adopted model; differences between constructed scenarios; and unresolved theoretical choices among b , p_c , q , α_* and B . The cosmological reference introduces a quantified dependence in the benchmark; its contribution cannot yet be ranked against the unresolved theoretical dependence on the running, threshold and normalisation coefficients. The principal numerical comparison is the length residual: approximately +0.29% at Planck-based central values with unit coefficients. The corresponding displacement in Θ_{phys} (0.002%) may be quoted as a diagnostic, but a small relative difference in a logarithm does not establish equivalent accuracy in the physical output.

40. Which coefficient choices are fixed, and which remain conditional

40.1 The first-differentiation prior is not a fitting parameter

The Post-Commitment source [S35, Section 46AE] states Pre-Fact Indifference explicitly: on the declared first-differentiation closure carrier, there is no primitive datum that assigns different prior weights to unresolved raw alternatives. Given the $K = 7$ alphabet, normalisation gives $p_o(w) = 1/128$ for $w \in F_2^7$. For $P_{full} = |1111111\rangle\langle 1111111|$ only the corresponding diagonal entry is needed: $\text{Tr}(p_{pre}P_{full}) = 1/128$. A fully maximally mixed density matrix, seven independent fair physical coins and an absence of off-diagonal coherence are not required by this calculation. The statement is exact under the source's foundational premise and alphabet identification. It is not an independent derivation of that premise, and it does not require every later conditioned state to have the same diagonal law. The joint bit-flip-neutrality argument in [S28] is a sufficient symmetry realisation of uniformity. It should not be promoted into a mandatory symmetry of every multi-cell state. In particular, the common-flip symmetry used in Appendix B is an additional joint-law hypothesis, not a consequence of the one-cell prior alone.

40.2 The direct probability gate replaces channel-count insertion

RMOP [S33] distinguishes the counting prescription 14/16 from an evaluated physical beta coefficient. For the actual adjacent-regulator probabilities and scales, with $h_n = \ln(k_n/k_{n+1}) > 0$, the required calculation is

$$b_{n \rightarrow n+1}^{(phys)} = (p_n^{-1} - p_{n+1}^{-1})/(2h_n).$$

For Route M's declared OR block event,

$$p_B = 1 - \Pr(C_1 = 0, \dots, C_m = 0) = 1 - e^{-\Delta_m} \prod_{j=1}^m (1 - p_j), \quad \Delta_m := \ln[\Pr(C_1 = 0, \dots, C_m = 0) / \prod_{j=1}^m (1 - p_j)].$$

No independence approximation is made. On the diagnostic choices $p_n = 1/128$, $m = 16$, $k_{n+1}/k_n = 1/2$, returning 7/8 would require

$$p_B = 1/(128 - (7/4)\ln 2) = 0.007887244431012...$$

This is an inverse test certificate, not a measured or source-derived probability. Identical copies give $p_B = 1/128$ and $b = 0$; independent redraws give $p_B = 1 - (127/128)^{16}$ and $b \approx 86.2161$. These constructed alternatives demonstrate what local marginals fail to fix, not competing physical VERSF predictions [S38 to S41]. One successful block step would not prove a constant coefficient over the full hierarchy. With a physically defined scale coordinate and a running $b(p)$, the continuous form is

$$\Theta = \int_{p_o}^{p_o - c} dp / (2b(p)p^2).$$

Using this integral requires the corresponding smooth-running regime. The constant-coefficient expression is its constant- b special case. Block multiplicity, scale ratio, coherence event, joint law and endpoint threshold must be selected before comparison with the desired hierarchy.

40.3 Capacity and tension matching do not force all unit coefficients

The quartic arguments [S8; S43 to S44] retain their scaling-level status. Restoring the factors suppressed in those arguments, write $E_{total} = v\mu L^4$, $E_{dyn} = fE_{total}$, $\tau = tL/c$ and $R = rL$. Within the stated quantum-transition and entropy-capacity applications, the separate necessary conditions are

$$x(L) := \rho L^4 / (\hbar c) \geq 1/(2fvt), \quad x(L) \geq \ln 2 / (2\pi vr).$$

At unit factors the right-hand sides are $1/2$ and $\ln 2 / (2\pi)$, not the same exact number. The calculations share a quartic dependence; they do not prove that the stable physical record process exists at either bound or that its operational B equals one. These durations describe an effective causal process, not the formation time of the primitive Fold. The common-tension argument likewise removes an unknown absolute tension but leaves C_{UV}/C_{IR} . The two saturation profiles displayed in [S27], $\tanh(s/s_c)$ and $s/(s + s_c)$, have the same initial normalised slope and ceiling but different approaches to it. Neither reaches that ceiling at finite argument. A finite identity-collapse criterion and its source normalisation are therefore not selected by the qualitative saturation statement alone [S37]. An additional compatibility relation is available. Retain the amplitudes in the leading UV/IR balance [S31]:

$$F(L) = a_{UV} \ell_*/L + a_{IR} L/L_\Lambda, \quad D := a_{UV}/a_{IR}.$$

Its minimum gives $\xi^2 = D\mathcal{L}_*L_\Lambda$. If this is the same physical ξ as the capacity observable, consistency requires

$$D^2\alpha_* = 8\pi B/3.$$

Thus $D = B = \alpha_* = 1$ cannot all be imposed as exact values in that common convention. This is an additional matching constraint, not a contradiction in the source's scaling relation and not a complete coefficient selection.

41. Probability selection and admission-weight-preserving attachment

41.1 Use the recorded source rather than introduce another statistical tilt

The retained-record theorem [S34, Section 10A] applies a completely positive operation to each retained event, $J_a(\sigma) = \sum_\alpha K_{a\alpha}\sigma K_{a\alpha}^\dagger$. For an actual ordered retained record h , compose these operations to obtain J_h . Given the incoming preparation σ_o ,

$$\sigma_h = J_h(\sigma_o)/\text{Tr } J_h(\sigma_o).$$

The probability of the retained record must be positive for this conditioning to be defined. Under the source's stated Fubini–Study preparation ensemble, its conditioned average is $\bar{\sigma}_h = J_h(\bar{\sigma}_o)/\text{Tr } J_h(\bar{\sigma}_o)$. This mean is sufficient for a subsequent probability that is linear in the density state. It is not a claim that every nonlinear directing functional can be recovered from the mean, or that the retained record by itself selects a pure amplitude. The ordered-likelihood and no-additional-tilt results in [S34] fix the conditioning rule at their stated predictive-consistency and measure-separation grades. They remove an independently adjustable exponent applied to the record likelihood. They do not fix all coefficients in the microscopic instrument or identify the Route-M observable. The generic Gibbs diagnostics in Appendix B therefore cannot be used to reintroduce a free record-conditioning temperature.

41.2 An admitted-link carrier has already passed some original tests

Appendix D.1 of Two-Planck [S45] defines three edge-existence and invertibility tests, one triangle-holonomy test, two embedding-match tests and one orientation test. On a carrier consisting only of admitted invertible links, the first three tests have already passed. In a uniform Boolean control, conditioning the seven-bit alphabet on those three tests leaves sixteen words:

$$\text{Pr}(A) = 1/8, \quad \text{Pr}(\text{full} \mid A) = 1/16, \quad \text{Pr}(\text{full}) = (1/8)(1/16) = 1/128.$$

The physical admitted-link law is not thereby proved uniform. The control demonstrates a conditioning distinction: counting already-passed tests as unresolved, or forgetting the admission weight, changes the event being evaluated [S41]. The RMOP integrability theorem [S33] supplies a separate restriction. On its physical link branch, tetrahedral compatibility follows from the common underlying transport and carries no extra obstruction probability. This does not make individual triangle curvature vanish. Nor does it provide an independently populated pre-integrability source. The admission operation introduced below must not be interpreted as reopening that obstruction sector; it is a general attachment formula for a genuinely source-specified admission stage, if one exists at the relevant physical grain.

41.3 Pull the coherence test back through the actual admission operation

Let A_{adm} be the actual trace-nonincreasing completely positive operation admitting a candidate to the carrier on which the remaining coherence test acts. For incoming state ρ_o , set $r_{\text{adm}} = \text{Tr } A_{\text{adm}}(\rho_o)$ and $\rho_{\text{adm}} = A_{\text{adm}}(\rho_o)/r_{\text{adm}}$ when $r_{\text{adm}} > 0$. Let $0 \leq C \leq I$ be the source-defined remaining coherence effect. Then

$$\rho_{\text{coh}} = r_{\text{adm}} \text{Tr}(C\rho_{\text{adm}}) = \text{Tr}[CA_{\text{adm}}(\rho_o)], \quad \text{equivalently} \quad E_{\text{coh}} = A_{\text{adm}}^\dagger(C), \quad \rho_{\text{coh}} = \text{Tr}(E_{\text{coh}}\rho_o).$$

Complete failure has effect $I - A_{\text{adm}}^\dagger(C)$, including both rejection and admitted-but-incoherent outcomes. No normalisation factor is discarded. A physical failure completion is part of a realised instrument, not merely a formal complement chosen after fitting the answer. The original coherent holonomy class

remains to be physically specified. A convenient tolerance, the bright port of a newly chosen comparison, a current-pointer event or the native cover controller cannot be substituted for it without a source-level event-identification theorem.

42. The joint block effect and its exact evaluation

42.1 Logical predicate versus physical joint readout

The K7 full-word selector is $P = P_{\text{full}}$, $Q = I - P$. On m correctly interpreted output word registers, the all-failure predicate is

$$R_{\text{fail}} = Q^{\otimes m}, \quad R_{\text{any pass}} = I - Q^{\otimes m}.$$

These operators define the questions asked of the output state; they do not require the state to factorise. The physical existence of these registers and the correspondence between their words and the Route-M tests remain part of the attachment. Let Φ_B be a trace-preserving joint source-to-block readout, with rejected branches explicitly represented as failure outcomes rather than discarded. Its input-side failure effect is $E_{\text{fail},B} = \Phi_B^\dagger(Q^{\otimes m})$. With the retained-record state,

$$p_B(h) = 1 - \text{Tr}[E_{\text{fail},B} J_h(\sigma_0)] / \text{Tr} J_h(\sigma_0).$$

If the joint map is subnormalised instead, missing admission branches must be included before using this complement. Independent normalisation at each cell would lose the correlations and weights the calculation is intended to determine. For this observable, the adjoint image of one specified effect is sufficient; full reconstruction of every action of Φ_B is not mandatory. This is a computational reduction, not a licence to select the effect by its desired expectation.

42.2 Backward effect recursion

Where the source supplies an ordered implementation, let F_j be its actual failure operation at stage j , including the relevant state update and memory. Define

$$H_m = I, \quad H_{j-1} = F_j^\dagger(H_j). \quad \text{Then} \quad E_{\text{fail},B} = H_0, \quad \text{Pr}(\text{all fail}) = \text{Tr}(H_0 \sigma_h).$$

The recursion follows from trace duality, and $0 \leq H_j \leq I$ follows from positivity and trace-nonincreasing failure operations. If each stage is completed by its corresponding success operation, the first-success effects plus the all-failure effect sum to the identity. This provides an exact normalisation check without enumerating every microscopic failure history [S41]. The archived noncommuting five-stage controls agree with forward propagation and explicit history sums. They verify the method, not the physical spatial-block law. In particular, m spatial cells are not automatically m chronological observations. A serial implementation must be derived or shown equivalent to the relevant joint physical event; its memory cannot be discarded merely to reduce the calculation.

42.3 Fixed single-cell effects do not fix joint probabilities

The constructed instrument family

$$J_w^{(\lambda)}(\rho) = \text{Tr}(P_w \rho) [\lambda P_w + (1 - \lambda)I/128], \quad 0 \leq \lambda \leq 1,$$

has the same word effects and the same uniform stationary state for every λ . Its serial sixteen-observation all-failure probability is nevertheless $(127/128)[1 - (1 - \lambda)/128]^{15}$. The exact-retention and independent-redraw limits reproduce the different beta diagnostics in Section 40.2. Matching 7/8 within this deliberately constructed family would require $\lambda = 0.999357136056941\dots$, obtained by solving backwards from the target. This value is not adopted as a VERSF parameter. The example establishes that preparation and probability effects alone leave a joint-update obligation [S41].

43. Transport comparison and its explicit probability attachment

43.1 Consume the balanced comparison at its actual source grade

RCHS-29-R1 [S36, Sections 12 to 14] supplies a normalised comparison of two unitary transports U_A, U_B : $H_s \rightarrow H_t$ with the same endpoints:

$$M_+ = (U_A + U_B)/2, \quad M_- = (U_A - U_B)/2.$$

The subscripts distinguish these transports from the integer attachment matrix A_* of Part I. RCHS-29 explicitly assumes balanced coherent route preparation and recombination before which-route information becomes an irreversible record. It uses a fresh probe for each trial. The physical source selection of that operation, preparation and trial schedule remains open in that paper. Direct expansion gives $M_+^\dagger M_+ + M_-^\dagger M_- = I$. Thus

$$E_- = \frac{1}{4}(U_A - U_B)^\dagger (U_A - U_B), \quad p_- = \text{Tr}(\rho E_-), \quad E_+ = I - E_-.$$

The factor 1/4 is fixed by this declared balanced comparison's amplitude normalisation. It is not fitted to a desired mismatch cost or beta coefficient. Choosing the balanced comparison as a physical operation remains a premise. Under simultaneous endpoint transformations $U_A \mapsto G_t U_A G_s^\dagger$, $U_B \mapsto G_t U_B G_s^\dagger$, the effect transforms as $E_- \mapsto G_s E_- G_s^\dagger$. With the matching transformation of ρ , the probability is invariant. This covariance does not select the probe or remove an independent physical state variable.

43.2 Apply the comparator to the written substrate paths

Appendix G of [S32] specifies $U_A(\epsilon) = e^{i\epsilon H_2} e^{i\epsilon H_1}$ and $U_B(\epsilon) = e^{i\epsilon H_1} e^{i\epsilon H_2}$, with Hermitian traceless generators. The transport parameter ϵ is not a pre-fact elapsed duration. The source expansion gives $U_A - U_B = \epsilon^2[H_1, H_2] + O(\epsilon^3)$. The combined calculation [S42] therefore returns

$$E_- = (\epsilon^4/4)[H_1, H_2]^\dagger [H_1, H_2] + O(\epsilon^5), \quad p_- = (\epsilon^4/4)\text{Tr}(\rho[H_1, H_2]^\dagger [H_1, H_2]) + O(\epsilon^5).$$

This is an explicit defect-to-probability relation for the stated comparison experiment. It is not a general identity between a Fold trap, the complete MASD action and a probability. For the Appendix G diamond contribution, with Hilbert–Schmidt norm and the declared test state $\rho = I_8/8$,

$$\Delta_C^{(\text{diamond})} = p_c^2 \|U_A - U_B\|_{\text{HS}}^2, \quad p_- = \Delta_C^{(\text{diamond})}/(32p_c^2),$$

where p_c here is the amplitude of Appendix G of [S32], not the Route-M threshold. This 1/32 includes the probe dimension and chosen norm. An anisotropic source preparation requires the trace pairing above. RCHS-29's original occupied 49-dimensional probe in a 50-dimensional representation is not silently substituted for the different eight-component test.

43.3 Exact finite specialisation and verification grade

The verification in [S42] chooses $H_1 = L_{12} = i(|1\rangle\langle 2| - |2\rangle\langle 1|)$ and $H_2 = L_{23} = i(|2\rangle\langle 3| - |3\rangle\langle 2|)$ in $\mathbb{C}^8$, with $\rho = I_8/8$. These are allowed test generators and a declared isotropic test state, not a source-selected physical Hessian or vacuum. The five other component directions are spectators for this example; neither eight components nor the active three-plane counts physical Folds or spatial dimensions. Writing $c_\epsilon = \cos \epsilon$, exact matrix multiplication gives

$$p_-(\epsilon) = (3 - c_\epsilon)(1 - c_\epsilon)^2(1 + c_\epsilon)/16, \quad p_-(\epsilon) = \epsilon^4/16 - \epsilon^6/96 - \epsilon^8/320 + O(\epsilon^{10}).$$

At the exact test point $\epsilon = \pi/3$, $p_- = 15/256$. This is a comparison probability, not the K7 full-word probability and not a replacement bare coupling. The packaged execution checks completeness, controlled-route implementation, endpoint covariance, positivity and the exact formula. Completeness residuals are below 1.4×10^{-15} . The matrix and symbolic control scripts were rerun for this integration, reproducing the archived result. This rerun verifies the declared finite experiment; it is not a rerun of physical PFDMI, HCMR or full RCHS source arrays.

44. A smaller loop and a subdivided original loop are different observables

44.1 Local phase halving

Replacing ϵ by $\epsilon/2$ in the two-factor path pair defines a smaller local comparison. At small phase, $p_-(\epsilon/2) \approx (1/16)p_-(\epsilon)$. This reproduces the algebra of the local phase-halving expression in Appendix G of [S32]. Its factor sixteen is a fourth-power suppression factor; it is not a count of independent physical cells.

44.2 Transport-preserving subdivision

Instead split each original edge into m equal segments while preserving its complete end-to-end transport:

$$(e^{i\epsilon H_a/m})^m = e^{i\epsilon H_a}, \quad a = 1, 2. \quad \text{Therefore} \quad U_A^{(m)} = U_A, \quad U_B^{(m)} = U_B, \quad E_-^{(m)} = E_-.$$

The two different generators need not commute for this identity: subdivision composes each original edge with its own generator. The result assumes no new selection, measurement, noise or state-update operation is inserted between the segments. Such an insertion would be additional dynamics, not merely another description of the same transports. The exact comparison audit at $\epsilon_0 = 0.4$ gives:

Subdivision level	Smaller-loop probability	Same original-loop probability
0	1.5554148×10^{-3}	1.5554148×10^{-3}
1	9.9325462×10^{-5}	1.5554148×10^{-3}
2	6.2395522×10^{-6}	1.5554148×10^{-3}
3	3.9046212×10^{-7}	1.5554148×10^{-3}
4	2.4411519×10^{-8}	1.5554148×10^{-3}

The full-path operator residual is below 7.5×10^{-15} over the seven tested levels [S42]. This is a finite transport-preserving refinement test, not a proof of the complete physical continuum limit.

Interpretive amendment. The local suppression statement of [S32] cannot be transferred to the unchanged end-to-end loop while omitting the concatenation of its segments. Those calculations use different observables. This sharpens the interpretation of the source's formula without denying its local expansion.

44.3 Consequence for physical running

RMOP [S33] proves that a projectively consistent history measure assigns the same probability to the same event pulled back through refinement. A nonzero physical running must therefore reflect a changed physical blocked observable, not a relabelled description of an unchanged event. The executed path example realises that distinction explicitly: smaller local comparison, same-loop subdivision and Route-M OR block are three different observables. Neither the comparator's $1/4$, the eight-component probe's $1/32$, nor the phase-halving factor $1/16$ establishes $b = 14/16$. A physical coefficient requires the scale-dependent event, its selected joint state and the regulator mapping of Section 40.

45. What the partial attachment supplies, and the remaining physical selection

45.1 Comparison success is not an exact structural certificate

For scalar relative phase $e^{i\phi}$, the balanced comparison gives $p_- = \sin^2(\phi/2)$ and $p_+ = \cos^2(\phi/2)$. At $\phi = 0.2$, $p_+ \approx 0.99003$, and sixteen conditionally independent fresh bright outputs still have probability approximately 0.85192. Thus a bright outcome can occur with nonidentity transport. It is a valid probabilistic mismatch readout, not a proof that every original coherence condition has passed. If a source theorem identifies this particular port outcome with the relevant coherence event, its probability can be used. Without that theorem, the port, the full-word selector, stable structural presence and

metric certification remain separate. On an admitted-link stage, its input-side bright effect is $A_{\text{adm}}^+(E_+)$, retaining the admission weight exactly as in Section 41.

45.2 Probe and field correlations retain their physical meaning

For fresh independent probes conditional on a fixed common field configuration X , the all-dark probability is a product of the conditional probabilities. With a fluctuating common field law,

$$\Pr(\text{all dark}) = \int \prod_j p_{-}^{(j)}(X) d\mu(X).$$

It is generally not the product of the separately averaged probabilities. These assumptions describe the declared RCHS comparison experiment; they are not an assertion that the physical Route-M block factorises in that manner. A recycled probe or a shared quantum memory uses a different joint operation and must be evaluated with that memory retained.

45.3 An admissibility selector is not automatically an admission instrument

The field-configuration selector of [S32], under its stated hypotheses, maps into M_{adm} and obeys $\Delta[\Pi_{\text{adm}}(\text{GF})\Psi] = 0$. Testing the same enforced property after selection passes by construction. It does not return an unconditioned success probability. Nor does the symbol Π establish that this configuration map is a linear orthogonal projector or a trace-decreasing quantum operation. A compatible physical realisation could exist, but it must return its preparation, outcomes and weights. No such realisation is inferred merely from idempotence or convergence of a mathematical flow.

45.4 Present return and next decisive calculation

The calculation of this part supplies a normalised probability effect for the declared transport comparison. The next physical test is to specify the coherence event, admission operation, preparation, joint block operation and adjacent-regulator map from the source, independently of the desired running coefficient. With those inputs fixed, evaluate the single-scale and block probabilities and calculate the resulting running. The result must be allowed to differ from 7/8 and to vary across the hierarchy. Agreement at one step would establish that step only; the metric-scale calculation requires the accumulated hierarchy and its common physical normalisation. The original probability architecture need not be reinvented. The retained-record source fixes the conditioned density state once its instrument and preparation are specified. The K7 alphabet supplies the full-word selector at its stated grain. The OR rule fixes the logical block predicate. The balanced comparator supplies one explicit transport-sensitive component with a checked refinement behaviour. The unreturned physical object is the same-source, admission-weighted joint coherence effect on the actual adjacent-regulator carrier, with the physical criterion, preparation, memory and refinement map established before evaluating its probability. This task must not be replaced by choosing a Gibbs penalty, fitting the comparator phase, adjusting a recovery basin, or borrowing an unrelated accepted-current event. A returned block law must be tested over the relevant hierarchy, not only at one step. The physically selected endpoint p_c , common coherence observable and invariant normalisation combination must then be supplied. Even that metric-scale closure must be connected independently to the operational Fold scale ℓ_F before it becomes a Fold-resolution prediction.

Combined scale-and-coherence result. The scale programme gives a conditional Planck-equivalent metric-scale estimate and a normalised transport-to-probability construction. It does not make the beta coefficient, capacity prefactor or Fold-scale identification compulsory. Its advance is that the remaining selection is an explicitly typed source, effect and refinement calculation, while several substitutes (channel counting, independently normalised admission, fixed-event subdivision and a bright comparison outcome treated as exact closure) are excluded for the proposed use.

Part VI. Direct running, normalisation rigidity and record-geometry cosmology

This part executes, at calculation level, the first three scale tasks of the priority programme of Section 66: a direct first-hit reduction of the RMOP running gate, a rigidity analysis for the normalisation combination, and an audit of the internal record-geometry cosmological route. The requested three-way closure is not established. What is established: an exact reduction of the required probability return, a conditional rigidity theorem with an explicit counterfamily showing the displayed matching equations are insufficient, two corrections to the finite-record cosmology source together with a rejection of its literal branch, a corrected two-scale bridge between the microscopic coherence length and the macroscopic record-curvature radius, and a scale non-identifiability result showing the remaining scale freedom in the displayed equations. No joint microscopic VERSF source dataset was executed; constructed examples are not proposed physical laws, and inverse requirements are expressly distinguished from independent derivations throughout [S47].

46. Direct derivation of the RMOP gate as a first-hit observable

Let the physical scale step have $h_n = \ln(k_n/k_{n+1}) > 0$. Suppose the source supplies a jointly readable block of coherence indicators $C_1, \dots, C_m$, with equal marginals $p_n = p$, $0 < p < 1$, and the coarse event is their OR. Choose one member as the reference cell. Define the conditional rescue probability

$$r_n = \Pr(C_2 \vee \dots \vee C_m = 1 \mid C_1 = 0).$$

Here rescue names a conditional event in a joint readout. It does not assume a chronological ordering of spatial cells or a new physical mechanism. The two disjoint ways for the block to succeed give $p_{n+1} = p + (1 - p)r_n$. Substitution into the RMOP gate yields the exact identity

Theorem VI.1 (first-hit identity). With $p_{n+1} = p + (1 - p)r_n$ and $h_n > 0$,

$$b_n = (1/p - 1/p_{n+1})/(2h_n) = (1 - p)r_n/(2h_n p[p + (1 - p)r_n]).$$

Thus the minimal remaining probability return, once the reference marginal, block semantics and scale ratio are established, is the probability that the remaining block succeeds conditional on that reference cell failing. This is a reduction of the required observable, not a source selection of its probability.

46.1 Physical effect and admission weights

Let the actual trace-preserving joint readout be Φ_B , with the source's preparation and memory retained. On its recorded output, let P_j be the coherence projector in register j and $Q_j = I - P_j$. Define

$$E_1 = \Phi_B^\dagger(P_1 \otimes I), \quad E_{\text{new}} = \Phi_B^\dagger[Q_1 \otimes (I - \bigotimes_{j=2}^m Q_j)].$$

The OR effect is exactly $E_B = E_1 + E_{\text{new}}$, and for the source state σ_h , $p = \text{Tr}(E_1 \sigma_h)$, $d_n = \text{Tr}(E_{\text{new}} \sigma_h)$, $r_n = d_n/(1 - p)$. This identity requires the reference marginal of the joint readout to match the original physical single-cell event. When admission is trace decreasing, rejection must be included in a completed readout or retained through the adjoint pullback of Section 41. Independently normalising admitted states would alter p and d . Noncommuting microscopic tests require the actual joint instrument; multiplying unimplemented microscopic effects does not define it.

46.2 What 7/8 would require

For the diagnostic choices $p = 1/128$, $m = 16$, $h = \ln 2$, obtaining $b = 7/8$ requires

$$p_{n+1} = 1/(128 - (7/4)\ln 2) = 0.007887244431012115\dots, \quad d_n = 0.000074744431012115\dots, \quad r_n = 0.000075332969838982\dots$$

The last value is about 0.00753%. It is an inverse test target. There is no source result here assigning that probability to the actual physical block. Setting an adjustable correlation to produce it would not derive b.

46.3 Sharp non-identifiability under the supplied marginal information

From $p \leq p_B \leq \min(1, mp)$, the displayed diagnostic branch admits

$$0 \leq b \leq (128 - 8)/(2 \ln 2) = 60/\ln 2 = 86.5617024533...$$

Both bounds, and every intermediate value, are attained by legitimate joint word laws with uniform one-cell seven-bit marginals:

- Choose one of 128 words uniformly and repeat it in all sixteen cells. Then $p_B = 1/128$.
- Choose an ordered sixteen-tuple of distinct words uniformly. Then $p_B = 16/128$.
- Mix these two laws with weight t on the distinct-word law. Then $p_B = (1 + 15t)/128$.

Each law is invariant under permutation of cells and under applying the same seven-bit XOR shift to every cell. Thus even that joint symmetry and the uniform local prior do not select a beta value. For 7/8 the mixture weight would have to be 0.000637819144637, again inverse-selected. These laws prove insufficiency of the listed marginal and symmetry conditions. They are not asserted to satisfy the complete microscopic VERSF action. A physically selected joint channel can exclude them; that channel is precisely the remaining input.

46.4 Scope of a possible constant-beta theorem

A further restriction follows from linearity. On one fixed carrier, assume fixed effects E and F give $p = \text{Tr}(E\sigma)$ and $p_B = \text{Tr}(F\sigma)$. Requiring the same positive b and h for every preparation in a convex family would require

$$p_B = f(p) = p/(1 - 2bhp), \quad f''(p) = 4bh/(1 - 2bhp)^3 > 0$$

on the physical domain. But both probabilities are affine in mixtures of preparations. If two allowed preparations have different p and their mixtures are allowed, strict convexity gives a contradiction. Consequently a positive constant- b law cannot hold on that entire convex family with fixed effects. A physical constant- b theorem must concern a selected state trajectory, a restricted family, or appropriately changing scale observables. This is not an objection to state-dependent running. Finally, the exact discrete hierarchy identity is

$$\sum_{n=0}^{N-1} b_n h_n = \frac{1}{2}(p_0^{-1} - p_N^{-1}), \quad \bar{b} := \sum_n b_n h_n / \sum_n h_n.$$

Whenever $\bar{b} > 0$, this fixes $\sum_n h_n = (p_0^{-1} - p_N^{-1})/(2\bar{b})$; constant running is a sufficient special case, not a necessary condition. The relation remains an identity until the source independently determines the running and the scale steps. Identifying the sum with $\ln(\xi/\ell_e)$ still requires the physical endpoint and regulator-to-length correspondence.

47. Rigidity of K and of the physical invariant

The coefficient audit permits consistent changes of characteristic-length convention:

$$\xi' = u\xi, \quad \ell_e' = v\ell_e, \quad \ell_*' = r\ell_*, \quad B' = u^4 B, \quad q' = (v/r)q, \quad \alpha_*' = r^2 \alpha_*, \quad \Theta' = \Theta + \ln u - \ln v.$$

For the requested combination, $K = (q/2)v(\alpha_*)B^{-1/4}$ and $K' = (v/u)K$. Therefore K alone is not invariant under all these conventions. Once the relevant physical definitions are fixed, its value can meaningfully be selected. Across changing conventions, the quantity whose rigidity matters is

$$\Omega := \Theta + \ln K, \quad J_\Lambda := \Lambda_{\cos} \ell_p^2 = (\pi/2)e^{-4\Omega}.$$

The intermediate identity-collapse length cancels out. A convention-cancellation identity is not a theorem that all physically different source realisations yield the same Ω .

47.1 Conditional global rigidity theorem

Theorem VI.2 (conditional global rigidity). Let independently specified source equations $F(c) = 0$ define a smooth, path-connected admitted parameter manifold M in a fixed physical convention. Let f be either $\ln K$ or Ω . Suppose $df_c(v) = 0$ for every $c \in M$ and $v \in T_c M$. Then f is constant on M . Conversely, a constant f has zero derivative on every tangent.

Proof. Integrate $df(c(t))/dt = 0$ along any piecewise smooth path in M . At regular points $T_c M = \ker DF_c$, so the condition is equivalently $\nabla f(c) \in \text{rowspan } DF_c$. ■

Disconnected branches require an additional equality check between their constants, and singular strata require their own admissible-path analysis. One source-derived reference value is also needed: constancy alone does not prove $K = 1$. This is the explicit connected-manifold statement of the local criterion in the coefficient audit; it does not claim new invariant-subspace mathematics.

47.2 The displayed matching equations fail to establish K-rigidity

The inherited amplitude-restored UV/IR condition $D^2\alpha_* = 8\pi B/3$ implies

$$K = (q/2)(8\pi/3)^{1/4}\alpha_*^{1/4}/\sqrt{D},$$

which still contains unselected physical amplitudes. Even if $q = 2$ and $\alpha_* = 1$ are held fixed, every positive t gives a solution $B = t$, $D = \sqrt{(8\pi t/3)}$, $K = t^{-1/4}$:

B	D	K
0.5	2.046653416	1.189207115
1	2.894405018	1
2	4.093306832	0.840896415

If Θ is held fixed, this is a family of different conditional predictions satisfying the displayed coefficient constraint. It proves that constraint is insufficient. If B changes only because ξ is relabelled and Θ changes consistently, Ω remains fixed instead. Physical variations and convention changes must be tested separately. No claim is made that the full source admits the displayed physical family.

48. Attempted internal cosmological closure

48.1 Two lengths with different definitions

Use ξ_M for the microscopic Route-M coherence length and R_{rec} for the macroscopic record-curvature radius. The cosmology paper uses ξ for the latter, while the scale synthesis uses it for the former. Restoring distinct symbols, its proposed cosmology is

$$\Lambda_\infty = 3/R_{\text{rec}}^2, \quad R_{\text{rec}} = \kappa r_s, \quad \Lambda_{\text{eff}}(a) = \Lambda_\infty F(\chi(a)), \quad F(\chi) = 1/(1 + \chi^n).$$

Its Sections 3.1 to 3.2 explicitly describe the sound-horizon identification as a hypothesis and admit other bounded monotone functions. Dimensional analysis fixes an inverse-square form, not a positive numerical curvature amplitude for a physically specified ruler. A factor 3 may define a curvature-radius convention; the relation of that radius to the sound horizon still requires physical derivation.

48.2 Numerical correction in the source

Using the source's 150 Mpc,

$$150 \text{ Mpc} = 4.6285163722 \times 10^{24} \text{ m}, \quad 3/(150 \text{ Mpc})^2 = 1.4003533871 \times 10^{-49} \text{ m}^{-2}.$$

The source prints approximately $1.3 \times 10^{-52} \text{ m}^{-2}$ and describes approximately 20% agreement. That arithmetic is incorrect by about three orders of magnitude. Relative to its own 1.1×10^{-52} comparison,

the corrected asymptotic value is 1273 times larger. With $F = 1$, matching the comparison requires a radius of approximately 5352 Mpc, or κ approximately 35.68. That is an inverse requirement, not an internally derived factor.

48.3 The declared occupancy law does not repair this branch

For Model 0 the source uses the present density ratio $r_0 = 0.45$ and $r_0 = \chi_0(1 + \chi_0^n)$. Solving this conditional relation gives:

n	χ_0	$F(\chi_0)$	$\Lambda_{\text{eff}} \text{ at } R_{\text{rec}} = 150 \text{ Mpc (m}^{-2}\text{)}$	Required κ
2	0.39046745	0.86770544	$1.21509425 \times 10^{-49}$	33.23599
5	0.44249344	0.98331875	$1.37699375 \times 10^{-49}$	35.38096

These rows match a density ratio, not simultaneously all absolute observed densities. Enforcing those absolute densities makes the failure sharper. On the same G convention, set the comparison dark-energy density through $\Lambda_{\text{ref}} = 1.1 \times 10^{-52}$ and let $\rho_{m0}/\rho_{DE,\text{ref}} = 0.45$. For $R_{\text{rec}} = 150 \text{ Mpc}$ and $0 \leq f_{\text{coll}} \leq 1$,

$$\chi_0 = f_{\text{coll}}\rho_{m0}/\rho_{\Lambda\infty} \leq 0.45\Lambda_{\text{ref}}/\Lambda_{\infty} = 0.0003534822\dots$$

For $n \geq 1$ this entails $F(\chi_0) \geq 1/(1 + 0.0003534822) = 0.9996466427$, whereas matching Λ_{ref} would require $F = 0.0007855160$. Thus reducing the committed fraction cannot supply the needed suppression within this occupancy definition. A different source-derived radius, normalisation or closure law is required. This is a rejection of this literal branch, not of all possible record-geometry cosmologies.

48.4 Equation-of-state correction

For the source's separately conserved effective fluid with constant R_{rec} and Model-0 occupancy $\chi \propto a^{-3}$,

$$d \ln \rho_{DE} / d \ln a = 3n\chi^n / (1 + \chi^n), \text{ so } w_{DE} = -1 - n\chi^n / (1 + \chi^n),$$

not the printed plus sign. The early-time limit is $-1 - n$, not $n - 1$. The $n = 2$ and $n = 5$ density-ratio examples give w_0 approximately -1.26459 and -1.08341 . This changes the source's equation-of-state tables and interpretations; it does not by itself perform a new observational fit. More generally,

$$w_{DE} = -1 + (2/3)d \ln R_{\text{rec}} / d \ln a + (n\chi^n / (3(1 + \chi^n)))d \ln \chi / d \ln a.$$

At constant R_{rec} with $\chi \propto f_{\text{coll}}a^{-3}$, substitute $d \ln \chi / d \ln a = d \ln f_{\text{coll}} / d \ln a - 3$. This also shows why a fixed comoving sound-horizon coordinate and a constant physical curvature radius cannot be interchanged silently. A time-varying effective-fluid density is not a strictly constant vacuum term; their identification must be specified at an epoch or asymptotic limit.

48.5 Why an acoustic calculation is not automatically input-free

The usual acoustic ruler takes the form

$$r_s(a_d) = \int_0^{a-d} c_s(a) / (a^2 H(a)) da.$$

Evaluating it requires an early-universe expansion law, sound speed, drag epoch and the corresponding matter, radiation and interaction inputs. Substituting fitted cosmological parameters into this integral replaces one external input by others. To make it an internal VERSF result, those quantities and the record-radius map must be returned by the source. No such evaluated input-free construction was located in the retrieved materials.

49. A corrected bridge between microscopic coherence and record curvature

Suppose the same physically identified effective vacuum density is used in the capacity and cosmological relations at one specified epoch, with $B > 0$, $F > 0$ and all characteristic lengths positive. Retain

$$\rho_\Lambda \xi_M^4 = B \hbar c, \quad \rho_\Lambda = \Lambda_{\text{eff}} c^4 / (8\pi G), \quad \ell_P^2 = \hbar G / c^3, \quad \Lambda_{\text{eff}} = 3F / R_{\text{rec}}^2.$$

Direct elimination gives

$$\xi_M^2 = \nu(8\pi B / (3F)) \ell_P R_{\text{rec}}.$$

Together with $\xi_M = q\nu(\alpha_*) \ell_P e^\Theta$, this gives

$$R_{\text{rec}} / \ell_P = \nu(6F / \pi) K^2 e^{2\Theta}.$$

This is the correct conditional two-scale bridge. The microscopic coherence length and macroscopic curvature radius remain different objects. It gives a concrete independent check for a future record-geometry source calculation: calculate R_{rec} / ℓ_P and F from the actual record geometry, then test this relation against independently evaluated running and normalisation. Defining R_{rec} by this equation would merely restate the scale closure of Section 39.

50. What can be predicted without an external cosmological scale

Even successful source selection of $b(p)$, its endpoint, and Ω would first determine the dimensionless observable $\Lambda_{\text{cos}} \ell_P^2 = (\pi/2) e^{-4\Omega}$. That would be substantial progress. It would not alone determine both Λ in m^{-2} and G in SI units. The displayed equations admit, for any $s > 0$,

$$(\ell_P, \ell_*, \ell_e, \xi_M, R_{\text{rec}}) \mapsto s(\ell_P, \ell_*, \ell_e, \xi_M, R_{\text{rec}}), \quad G \mapsto s^2 G, \quad \Lambda \mapsto \Lambda / s^2, \quad \rho \mapsto \rho / s^4, \quad T_{\text{max}} \mapsto T_{\text{max}} / s^2,$$

with c , $\hbar$ and dimensionless coefficients unchanged. Every hierarchy, capacity, common-tension and cosmological matching relation remains satisfied. A similarly rescaled acoustic history $H \mapsto H/s$ gives $r_s \mapsto sr_s$, so inserting a scale-free acoustic integral alone does not remove this freedom. This is a non-identifiability result for the equations examined here, not a universal prohibition on an additional source law. Internally normalised record geometry might determine Λ times a native ruler squared and the corresponding gravitational coefficient. Reporting a dimensional number in laboratory units still requires an operational relation to that ruler. The observational Planck length already contains measured G and cannot be used as an unnoticed independent anchor in a claimed simultaneous derivation of both G and Λ .

51. Concrete next source execution for the scale programme

- Return the physical pullback E_{new} (or the joint all-failure effect), preparation and admission weights on the actual adjacent-regulator carrier. Calculate d_n and b_n , with no target-selected probability.
- Repeat on the physically related scales to determine the accumulated hierarchy and source-defined endpoint. One successful step is insufficient.
- Fix operational definitions of the two characteristic lengths. Apply the rigidity criterion to Ω , or to K in a fixed convention, across the admitted source variations and branches. Supply an independently derived reference value.
- Repair the record-geometry cosmology's scale conversion and fluid sign, then calculate a macroscopic record radius from the source. Establish its physical rather than merely comoving meaning and its relationship to the microscopic ruler.
- Compare that independently obtained geometry with the two-scale bridge. Track all remaining initial-state, density, epoch and ruler inputs before claiming the external cosmological reference has been removed.

The computations documented in [S47] pass their stated mathematical checks. They do not return $b = 7/8$, $K = 1$ or a measured cosmological constant as compulsory physical outputs. The concrete results are the first-hit reduction, the scope restriction on constant-beta rigidity, the normalisation-rigidity criterion and counterfamily, the cosmology branch rejection, and the corrected two-scale matching relation.

Part VII. The species and generation census

52. Census premises

A mass matrix presupposes a list of species and generation slots, and a mixing matrix presupposes bases on those slots. Part VII therefore fixes the carrier census before any hierarchy or mixing calculation may use it. The premises are named so that each carries its own falsifier.

- C1.** Terminal species principle: a physical species is an irreducible terminal-record carrier role in the physical quotient.
- C2.** Weak binary attachment: the minimal unresolved weak attachment is a left chiral doublet.
- C3.** Charged closure partners: charged weak-doublet components require right-singlet closure partners or an equivalent declared mechanism (a charged Weyl fermion cannot self-close without violating the unbroken charge it carries).
- C4.** Colour support branching: support-coupled quark matter carries a colour-triplet branch; colourless completion matter is permitted where required.
- C5.** One-Higgs minimal closure: the minimal charged-fermion closure carrier is one weak doublet with a neutral lower component.
- C6.** Anomaly stability: the terminal census satisfies the local gauge anomalies and the global weak anomaly; the mixed gravitational–hypercharge relation is a consistency check after the gauge-sector solution.
- C7.** Census minimality: no species is admitted unless it closes a named attachment, mass, anomaly or support-return obligation; among admissible repairs the one adding the fewest Weyl fields and fewest new obligations is minimal.
- C8.** Support-return generation principle: a generation is a complete anomaly-stable species package assigned to one independent support-return completion layer.
- C9.** Rank-three completion: the physical support-return quotient has exactly three independent completion layers. Inherited, not proved here [12].
- C10.** Generation completeness: each completion layer carries a full terminal package; a partial generation is inadmissible.
- C11.** No duplicate layer without bridge: a fourth minimal generation requires a fourth completion layer, a new primitive record obligation or a new anomaly-stable sector.
- C12.** Layer-local terminality: the projected census on each completion layer itself satisfies weak attachment, colour branching, charged closure, charge assignment and anomaly stability.
- C13.** Mixing is post-census: CKM- or PMNS-type mixing acts between already-complete generation slots and cannot create a missing species.
- C14.** Ordered weak-component closure: up-type, down-type and charged-lepton closure preserve the weak upper/lower ordering.
- C15.** Layer-exclusive attachment: an irreducible terminal carrier constitutes the completion of one layer, not of several independent layers at once.

53. The minimal one-generation package

Start with one colour-triplet left-handed weak doublet. Colour gives three copies of the weak doublet, so the quark sector contributes three left-handed SU(2) doublets, and charged closure (C3) requires the two right-handed colour-triplet singlet roles for the upper and lower components:

$$Q_L = (u_L, d_L)^T, \quad u_R, \quad d_R.$$

Three left-handed SU(2) doublets are globally inconsistent by Witten's anomaly [17]: the sum of 2T over left-handed Weyl fermions must be even, and for doublets 2T = 1, so three is inadmissible. The repair must add an odd number of doublets. One colourless weak doublet adds one doublet and two Weyl fields; a second colour-triplet doublet adds three doublets, six Weyl fields and new colour obligations; the first half-integer multiplet above the doublet, $j = 3/2$, has 2T = 10 and cannot repair the parity at all, and the smallest with odd index, $j = 5/2$, has 2T = 35 and six components. Under C7 the colourless doublet is the minimal repair, and its charged lower component requires one colourless right-handed singlet:

$$L_L = (v_L, e_L)^T, \quad e_R.$$

Lemma VII.1. Under C2, C3, C4, C6 and C7 the minimal one-generation chiral matter package is $G = \{Q_L, u_R, d_R, L_L, e_R\}$. A sterile right-handed neutrino is not required by gauge anomaly cancellation, global weak parity or charged mass closure; it is absent from the minimal census and admissible in a declared neutrino-mass extension. ■

54. Hypercharge and anomaly consistency: a classical result reproduced

This section reproduces classical charge-quantisation algebra in the notation of the census. Its role is validation and overdetermination, not novelty: uniqueness of the Standard Model hypercharges from anomaly conditions together with one-Higgs Yukawa structure was established by Geng and Marshak [13], Minahan, Ramond and Warner [14], and Foot, Joshi, Lew and Volkas [15], with the Geng–Marshak reply [16]; the global weak condition is Witten's [17]; the right-handed-neutrino freedom and its relation to a Majorana mass were analysed by Babu and Mohapatra [18]. The calculation is retained because a census proposal must pass these consistency tests.

Use the convention $Q = T_3 + Y/2$. The neutral lower Higgs component has $T_3 = -1/2$ and $Q = 0$, so $Y_H = +1$. Gauge-invariant ordered closure through H and $\tilde{H} = i\sigma^2 H^*$ gives

$$Y_u = Y_Q + 1, \quad Y_d = Y_Q - 1, \quad Y_e = Y_L - 1.$$

The $SU(2)^2U(1)$ condition gives $3Y_Q + Y_L = 0$, so $Y_L = -3Y_Q$. With the closure relations inserted the $SU(3)^2U(1)$ condition $2Y_Q - Y_u - Y_d = 0$ holds identically, and the $U(1)^3$ condition $6Y_Q^3 - 3Y_u^3 - 3Y_d^3 + 2Y_L^3 - Y_e^3 = 0$ reduces to

$$-27Y_Q^3 + 27Y_Q^2 - 9Y_Q + 1 = (1 - 3Y_Q)^3 = 0,$$

a perfect cube with the single triple root $Y_Q = 1/3$, so that

$$Y(Q_L) = +1/3, \quad Y(u_R) = +4/3, \quad Y(d_R) = -2/3, \quad Y(L_L) = -1, \quad Y(e_R) = -2, \quad Y(H) = +1,$$

and $Q(u) = +2/3$, $Q(d) = -1/3$, $Q(v) = 0$, $Q(e) = -1$, with $2Q(u) + Q(d) + Q(e) = 0$. The mixed gravitational–hypercharge condition $6Y_Q - 3Y_u - 3Y_d + 2Y_L - Y_e = 0$ reduces under the same substitutions to the linear factor $1 - 3Y_Q$ and closes automatically; it is a surplus check, not a determining input, which matters in a programme that defers gravitational response. Written with all fermions left-handed, the five anomaly sums vanish: $SU(3)^3$ gives $2 - 1 - 1$; $SU(3)^2U(1)$ gives $2/3 - 4/3 + 2/3$; $SU(2)^2U(1)$ gives $1 - 1$; the gravitational sum gives $2 - 4 + 2 - 2 + 2$; and the cubic sum has integer numerator $6 - 192 + 24 - 54 + 216 = 0$ over 27. The global weak anomaly closes because three coloured Q_L doublets plus one L_L doublet give four.

Before ordered closure is imposed the anomaly system is underdetermined: with $a = Y(Q_L)$, $b = Y(u_R)$, $c = Y(d_R)$, $\ell = Y(L_L)$, $e = Y(e_R)$, imposing the gravitational condition instead gives $e = -6a$ and the cubic factorises as $18a(8a^2 + bc) = 0$, with the standard-like branch $\{b, c\} = \{4a, -2a\}$ and a degenerate branch $a = 0$. Ordered closure (C14) removes this ambiguity in the minimal one-Higgs census and fixes which singlet closes which weak component. With a sterile right-handed neutrino the closure and anomaly equations admit the continuous deformation $Y \rightarrow Y + \lambda(B - L)$, so the charged minimal census and the neutral extension have different uniqueness properties; a Majorana condition or other $B - L$ breaking can

remove the deformation [18]. The hypercharge pattern is therefore the classical consistency solution reproduced inside the census, and SC-1's premises land on it.

55. The three-generation theorem

The substrate anomaly-descent programme supplies the premise that the physical support-return quotient has three independent completion layers R_1, R_2, R_3 [12]; it is not rederived here. Anomaly cancellation alone permits any number of generations, since one generation is anomaly-stable by itself, so the count rests on two bridge premises. C12 is the local condition: project the census onto one layer; if the projection contains Q_L without its right singlets, charged closure fails; if it contains coloured weak matter without the colourless doublet, the global weak condition fails; if it contains the colourless doublet without e_R , charged-lepton closure fails. A layer that needs another layer to repair one of these defects is not terminal on its own. C15 supplies the other half: one terminal carrier cannot constitute two independent completion layers simultaneously, otherwise the layers share a constituting entry and are not independent. C12 forbids an incomplete layer; C15 forbids several layers from sharing one complete package.

Theorem VII.1 (generation count). Under C8 to C15, $N_g \geq 3$, because three independent layers each require one exclusive complete package; and $N_g \leq 3$ in the minimal census, because three complete packages populate the three layers and a fourth closes no new completion direction unless a fourth layer or a new primitive obligation is supplied (C11, C7). Hence $N_g = 3$. ■

The theorem is exactly as strong as C9. Its VERSF content is the conversion of three layers into three complete generation packages through C12 and C15. Family universality of hypercharge follows as a corollary, since the hypercharge determination of Section 54 applies to each layer separately and every layer closes through the single carrier H.

56. The full minimal census and its downstream consequences

Multiplet	$SU(3)_c \times SU(2)_L \times U(1)_Y$	Weyl components
Q_L	(3, 2, +1/3)	6
u_R	(3, 1, +4/3)	3
d_R	(3, 1, -2/3)	3
L_L	(1, 2, -1)	2
e_R	(1, 1, -2)	1

One generation contains fifteen chiral Weyl components and three generations forty-five. The minimal charged-closure carrier is one Higgs doublet with hypercharge +1; the inherited gauge algebra contains eight colour, three weak and one Abelian carrier, twelve in all, recorded but not derived. Yukawa matrices, mass eigenstates and CKM or PMNS rotations act on the three-dimensional family slot that the census supplies; they may mix completed generations but cannot reduce the number of slots. For N generations the charged-current mixing matrix carries $(N - 1)(N - 2)/2$ physical CP phases after rephasing, so $N = 3$ is the smallest count with a nonzero phase slot; this is a consistency consequence of the census, not a derivation of the observed phase.

The seven central representation sectors of Section 19, with dimensions 2, 6, 2, 9, 9, 18 and 3, are the three-generation multiplets of this table together with the Higgs doublet and a two-dimensional neutral sector: 18 = Q_L over three colours and three generations, 9 and 9 the up and down singlets, 6 the lepton doublets, 3 the charged singlets, 2 the Higgs doublet. The internal carrier on which the pricing operator of Section 21 acts is therefore the census carrier of Part VII.

Part VIII. The bridge

57. Why six Fold sectors are not six generations

Part II and Part VII count different objects. The six Fold sectors are the six local continuation labels of the D_6 geometry (Theorem I.4). A generation is one complete anomaly-stable matter package assigned to one independent support-return completion layer (C8). Nothing licenses reading the generation count off the number six, and this paper does not do so. The connection appears one level later, in the source response: the first family-producing response is quadratic in the source coordinate on the E_1 plane of Theorem I.5, and a quadratic map cannot distinguish a rim state from its antipode. Two further inputs are used: on the canonical wheel the real R_4 carrier $\ker_{\mathbb{R}}(L_W - 2I)$ is the same real D_6 source plane as the flavour axle carrier, by direct mode matching, with $\pm I$ the only norm-preserving equivariant identification, and the unique lowest-order nonzero D_6 -covariant family descent is $Q(z) = z^2$ [3, PMS-EX59]; and PMS-EX82 returns the $E_1^2 \rightarrow E_2$ cubic source term with coefficient $g_{ax} = 1/2$ in the frozen source convention, off-axle residual below 5×10^{-16} [3].

58. The canonical antipodal quotient

Index the six rim states by $j \in \mathbb{Z}_6$ with $r: j \mapsto j + 1$ and $s: j \mapsto -j$, so that $D_6 = \langle r, s \mid r^6 = s^2 = 1, srs = r^{-1} \rangle$ has order twelve. The antipodal involution is $a = r^3$.

Proposition VIII.1 (canonical three-object quotient). The subgroup $\langle r^3 \rangle$ is central in D_6 , and $D_6/\langle r^3 \rangle \cong D_3 \cong S_3$ acts on the three antipodal classes [0], [1], [2] by the full permutation action.

Proof. r^3 commutes with r , and $sr^3s = r^{-3} = r^3$ since $r^6 = 1$, so $\langle r^3 \rangle$ is central and normal. The quotient has order six, generated by $\bar{r}$ of order three and $\bar{s}$ of order two with $\bar{s}\bar{r}\bar{s} = \bar{r}^{-1}$, so it is $D_3 \cong S_3$. The antipodal classes are the cosets of $\langle 3 \rangle$ in $\mathbb{Z}_6$, that is $\mathbb{Z}_6/\langle 3 \rangle \cong \mathbb{Z}_3$, on which $\bar{r}: [j] \mapsto [j + 1]$ is the three-cycle and $\bar{s}: [j] \mapsto [-j]$ the transposition fixing [0]; together they generate S_3 . ■

The three-object structure is not inserted. The sixfold carrier comes with one involution that is central, and the quotient by it is a three-element S_3 -set with no further choice.

59. The quadratic family map factors through the quotient

Let z be the canonical complex coordinate on the E_1 source plane with rim phases $z_j = e^{i\pi j/3}$.

Proposition VIII.2 (factorisation). $Q(z) = z^2$ is constant on each antipodal class and descends to the three cube roots of unity on $\mathbb{Z}_6/\langle 3 \rangle$. The normalised family vector $q = (1, \bar{\omega}, \omega)/\sqrt{3}$, with $\omega = e^{2\pi i/3}$, lies in the complexified zero-sum sector $E_2 \otimes \mathbb{C}$ of the three-object permutation carrier $1 \oplus E_2$, and q together with $\bar{q} = (1, \omega, \bar{\omega})/\sqrt{3}$ spans that sector.

Proof. $z_{j+3} = -z_j$ and $Q(-z) = Q(z)$, so $Q(z_{j+3}) = Q(z_j)$ and Q is a function of the class $[j]$, with values ω^j . The components of q sum to $1 + \bar{\omega} + \omega = 0$, so q is orthogonal to $(1, 1, 1)/\sqrt{3}$ and lies in $E_2 \otimes \mathbb{C}$; q and $\bar{q}$ are the two C_3 characters of order three, hence independent, so they span the sector and their real and imaginary parts give a real basis of E_2 . ■

j	$z_j = e^{i\pi j/3}$	$z_j^2 = e^{2\pi i j/3}$	Antipodal class	Family value
0	1	1	[0]	1
1	$e^{i\pi/3}$	$e^{2\pi i/3}$	[1]	ω
2	$e^{2\pi i/3}$	$e^{4\pi i/3}$	[2]	$\bar{\omega}$
3	-1	1	[0]	1
4	$e^{4\pi i/3}$	$e^{2\pi i/3}$	[1]	ω

j	$z_j = e^{i\pi j/3}$	$z_j^2 = e^{2i\pi j/3}$	Antipodal class	Family value
5	$e^{5\pi i/3}$	$e^{4\pi i/3}$	[2]	$\bar{\omega}$

Which of q and $\bar{q}$ is called the family vector is the chirality choice ψ_+ or ψ_- of the source plane; the pair is the invariant object. Propositions VIII.1 and VIII.2 upgrade the observation six over two equals three into a statement about representations: the family carrier produced by the source is the quotient S_3 -set of the Fold carrier, with the S_3 action inherited from D_6 , and the family vector is the E_2 part of that carrier.

60. Terminal keys and the intertwiner

The PFDMI terminal-orbit theorem returns the closed terminal carrier at regulator levels $n = 3, 4, 5, 6$ as $P_{\text{terminal}} = K_7 \otimes I_3$ with zero terminal-block residual; after the seven-state history block is separated the terminal carrier is $R = \text{span}\{R_1, R_2, R_3\} \cong \mathbb{C}^3$ with three mutually orthogonal, exhaustive key projectors permuted freely by S_3 and no intrinsic order [4]. By Proposition VIII.1 the family side is $G = \text{span}\{[0], [1], [2]\} \cong \mathbb{C}^3$ with the quotient permutation action. Both are copies of $1 \oplus E_2$.

B1, family–terminal identification premise. The three PFDMI terminal-key rays R_1, R_2, R_3 constitute the three family rays $[0], [1], [2]$ of the antipodal quotient, that is, the three support-return class rays. B1 is the only premise introduced in Part VIII. It is distinct from C8, which assigns a generation to one completion layer and says nothing about which physical object a terminal key is; B1 is the physical identification that C8 does not supply, and it is stated separately so that it cannot be read as already contained in the census premises.

Write $P_i = |R_i\rangle\langle R_i|$ for the three key projectors on R and $P_{[j]} = |[j]\rangle\langle [j]|$ for the three class projectors on G . For this bridge, layer-exclusive attachment (C15) has one precise mathematical content: a terminal key is carried to one family class, not spread over several. Formally, a bridge $T: R \rightarrow G$ is layer-exclusive if

$$TP_i T^\dagger = P_{[\pi(i)]} \text{ for every key } i \text{ and some bijection } \pi,$$

that is, T maps each key ray onto a class ray. This is the meaning of C15 used below; it is stated as a condition on T so that what C15 contributes to the uniqueness is visible. The bijection π is the key-to-class labelling; it is fixed first, and equivariance is then imposed in the common labels it defines.

Proposition VIII.3 (uniqueness of the bridge given the identification). Assume B1. Let $T: R \rightarrow G$ be unitary and layer-exclusive with labelling π , and suppose that $\tilde{T} = P_\pi T$, which maps R to itself in the common labels, commutes with the S_3 permutation representation. Then $\tilde{T} = e^{i\phi} I$ for a single phase ϕ , and hence $T = e^{i\phi} P_\pi$.

Proof. Commuting with the full permutation representation of S_3 on three objects places $\tilde{T}$ in the commutant $\text{span}\{I, J\}$, with J the all-ones matrix, so $\tilde{T} = aI + bJ$. Equivariance and unitarity alone do not force $b = 0$: the permutation carrier is $1 \oplus E_2$, and $\tilde{T} = e^{i\alpha} P_1 + e^{i\beta} P_{E_2}$ with $P_1 = J/3$ and $P_{E_2} = I - J/3$ is unitary and equivariant for every pair of phases, with $b = (e^{i\alpha} - e^{i\beta})/3$. Layer exclusivity removes the relative phase: $\tilde{T}P_i \tilde{T}^\dagger = P_\pi P_{[\pi(i)]} P_\pi = P_i$ is a rank-one projector onto the key ray, so the column $\tilde{T}|R_i\rangle = a|R_i\rangle + b\sum_k |R_k\rangle$ has support on exactly one key, which forces $b = 0$ and then $|a| = 1$ by unitarity; equivalently $\alpha = \beta$. Hence $\tilde{T} = e^{i\phi} I$ and $T = P_\pi \tilde{T} = e^{i\phi} P_\pi$. A nontrivial permutation matrix does not itself commute with all of S_3 , which is why the relabelling is removed before equivariance is imposed rather than after. ■

What this does and does not establish. Propositions VIII.1 to VIII.3 show that both sides are canonical three-object permutation carriers, that the family side's S_3 structure comes from the Fold quotient rather than from a label, and that once the two triples are identified and C15 is read as the ray condition above, no continuous freedom remains; without C15 one relative phase between the trivial and E_2 components of the bridge survives. They do not show that the two triples are the same physical objects: any two three-element S_3 -sets carry the same permutation type, so the identification of family classes with terminal keys is exactly what premise B1 states, and nothing in C1 to C15 supplies it. The bridge is closed

at representation and census level, meaning uniqueness given the identification, and open at the level of physical identification. The next section states the test that would close the second.

61. The primitive same-action test

61.1 Statement

Because the family carrier is generated by the quadratic channel $E_1^2 \rightarrow E_2$ and not by a linear E_1 response, the same-event test is second order. Let p_{terminal} be the terminal-key probabilities of the primitive marked instrument as a function of the E_1 source amplitude, and define the terminal-key family jet

$$B_{\text{key}} = \Pi_{E_2}^{(\text{key})} D_{E_1}^2 p_{\text{terminal}},$$

the E_2 projection of the second derivative of the key probabilities with respect to the E_1 amplitude, read on R . Let B_{family} be the primitive $E_1^2 \rightarrow E_2$ source tensor returned on the family carrier. A primitive same-action bridge holds if $B_{\text{key}} \neq 0$ and B_{key} agrees with B_{family} up to the intertwiner of Proposition VIII.3. This is the closure test for the present probability-level implementation, not a necessary condition for every conceivable source realisation.

61.2 Inputs

Two exact results enter [3]. First, by PMS-EX81, every native mark and terminal probability of the EX2E installation is invariant under a common family rotation $z \mapsto e^{i\alpha}z$, because the current and Kraus operators transform by conjugation and the current residual stays on shell; this is a symmetry statement, exact to all orders, and gives zero derivatives of p_{terminal} of every order along the rotation orbit. Second, the EX2E and EX2F laws depend on the current only through the normalised projector $p_j = |j\rangle\langle j|/\langle j|j\rangle$, so they are invariant under radial rescaling $j \mapsto \lambda j$, and the E_1 amplitude enters the EX2E instrument through the current; PMS-EX81 records the radial blindness and PMS-EX108 proves it to all orders [3]. The two orbits together exhaust the punctured plane: every $z \neq 0$ in the complex E_1 coordinate is reached from a fixed z_0 by $z = cz_0$ with $c \in \mathbb{C}^\times$, and $p_{\text{terminal}}(cz_0) = p_{\text{terminal}}(z_0)$ for every such c . Hence p_{terminal} is constant on $E_1 \setminus \{0\}$, and every derivative with respect to the E_1 amplitude, of every order, vanishes at every nonzero source amplitude; in particular the full second jet, radial, rotational and mixed, is zero there. No order-by-order expansion is involved. The base point of the test is therefore any nonzero amplitude z_0 . This loses nothing on the family side: the family source enters through the exact quadratic $Q(z) = z^2$, whose second derivative is the same constant tensor at every base point, so B_{family} is base-point independent and the comparison may be made at $z_0 \neq 0$. The origin is treated separately because the normalised projector p_j is singular at zero current: a statement of the second jet at $z = 0$ additionally requires that the physical terminal-probability map extend continuously to zero current with its neutral value equal to the punctured-plane constant. The cited records establish radial and rotational blindness, not that neutral-limit identity, so the origin jet is carried as conditional on it and is not used.

61.3 Result

$$B_{\text{key}} = 0 \text{ (current EX2E family-orbit implementation), } B_{\text{family}} \neq 0 \text{ (PMS-EX82, } g_{\text{ax}} = 1/2).$$

The first equality is exact at every nonzero source amplitude and holds to all orders, not only at second order. The two outputs that one primitive event would have to write together are, in the current stack, written by different objects: the family source responds at second order and the terminal keys do not respond at all. This is a no-return result for the EX2E implementation, not a claim that a family-to-key bridge is dynamically impossible. The representation-valued source extension of RCHS-3, with its rank-72 gauge and current response after the Wilson quotient, is a different source slice; it carries the gauge and current response, not the E_1^2 family-to-key second jet, and neither supplies the missing jet nor overturns the no-return [7]. The missing object is one primitive source law whose quadratic E_1^2 event appears in both outputs.

62. One source-normalisation frontier for internal selection and family normalisation

Part II ended with the internal preparation problem reduced to the Rayleigh–Ritz spectrum of W_{phys} on the 49-dimensional carrier, and Section 56 identified that carrier with the census carrier. Part VIII ends with the physical normalisation of the family response open. These are candidate manifestations of one source-normalisation frontier rather than one displayed operator: W_{phys} is defined on the 49-dimensional internal carrier and controls which internal matter direction is energetically selected, while the family response is the $E_1^2 \rightarrow E_2$ object of Section 61 whose physical coefficient is the PMS normalisation $\alpha_{E_2}^{\text{phys}} = g_{\text{ax}}/(2\kappa_{E_2}^{\text{phys}})$ with $\kappa_{E_2}^{\text{phys}}$ the physical E_2 stiffness [3]. Both are set by the same primitive Hessian and pricing form of the source action, but this paper does not display the map that returns $\kappa_{E_2}^{\text{phys}}$ from W_{phys} , and the identification of the two normalisations as one coefficient is therefore a conjecture to be executed, not a result. Neither object decides whether the number of generation slots is three, which Part VII fixes, nor whether the family classes are the terminal keys, which Section 61 tests. Its shape is already constrained: the full-carrier same-action test of CLSN-1 excludes the separable H_4 branch and returns the coherent locked branch $K_\theta = \text{tG}$ at its stated grade [6]. Deriving W_{phys} , or at least its seven central block prices, from the same primitive source action, together with the physical E_2 stiffness it induces, is the calculation that would advance the Fold carrier, the census and the bridge at once.

Part IX. Unified status ledger, next calculations, falsifiers and conclusion

63. Unified status ledger

The programme keeps occurrence, discrete identity, subsequent dynamical influence and physical topology separate. The stitching and marked-anchor objects of Parts I and II are graded in Section 22; the ledger below covers Parts III to VIII. Each row names the object or gate, the return actually in hand, and its grade.

63.1 Identity, distinguishability and stable presence (Part III)

Object or gate	Return	Grade
Record-relative distinguishability	Equal readout images are indistinguishable at that record level.	Exact given the admitted channel; not an all-observable quotient theorem.
Physical distinguishability metric	$D_{\text{phys}}^*(L)$ is the supremum of single-channel pseudometrics over the admissible class G_L ; symmetric with the triangle inequality.	Operational definition; pseudometric structure exact.
Channel nestedness and monotonicity	$L_1 \leq L_2 \Rightarrow G_{L_1} \subseteq G_{L_2}$, hence $D_{\text{phys}}^*(L_1) \leq D_{\text{phys}}^*(L_2)$.	Stated axiom on the admissible class; monotonicity then exact.
Metric quotient of admitted readouts	D_{phys} gives the quotient by indistinguishability under the supplied channel class.	Exact pseudometric result; not automatically a metric result for the retention sup-inf.
Record persistence	D_{pers} tests retention after an admitted readout.	Record-level condition; does not establish continued structural presence.
Record-onset scale	ℓ_{dist} is an infimum for positive persistent record distinguishability in its declared scale description.	Operational definition; equality to a structural Fold scale is not derived.
Reconstructed Fold scale	$\ell_F = \inf_{C \in C_{\text{pass}}} \ell_g(C)$ after a source metric is supplied.	Scale target; minimum only if attained; no primitive length computed.
Readout, structural stability and localisation	Admitted readout distinguishes presence; the continuation law preserves structure; coordinates require a relational reconstruction.	Separate tests; no conventional coordinate required before a pre-spatial Fold.
Localisation consistency scope	The heuristic concerns reconstructed localisation, not an already identified primitive Fold length.	External heuristic; independent scale matching required.

Object or gate	Return	Grade
Fold-Interface Minimality	Persistent primitive record distinctions require Fold-supported structure; onset equality is a further physical identification.	Explicit principle and open bridge, not a stable-support theorem.
No pre-existing formation time	No primitive waiting duration; solver iterations and operator words are not elapsed time or facts.	Timeless source interpretation retained; waiting-time route not used.
Three meanings of size	Interface dimension, operational resolution and metric extent remain distinct.	Source interface data retained; resolution calibrated; metric extent open.
Minimal interface area	$A_F \sim \ell_P^2$ by dimensional reasoning.	Discussion expectation only; primitive interface metric underived.
Planck localisation consistency	Quantum localisation plus gravitational-radius bound give $L \gtrsim \ell_P$.	Heuristic external consistency check, not a VERSF derivation.
Planck comparison gate	Compute $\alpha_F = c^3 \ell_F^2 / (\hbar G)$ after source normalisation and scale matching.	Not evaluated; order-of-magnitude consistency does not fix an exact coefficient.
Unitary attractor mechanism	Pure unitary conjugation does not contract the full operator norm into a Fold attractor.	Exact no-go.
Structural no-loss condition	$(I - P)K_{a\beta}P = 0$ for every admitted branch.	Exact on the finite structural carrier and stated continuation class.
Subharmonic presence projector	$\Phi_a^+(P) \geq P$; equality adds a no-entry condition.	Exact for trace-preserving channels.
Largest stable non-null support	$S_{st} = \cap_w \ker(P_N K_w)$ (Theorem III.1).	Unique largest compatible invariant subspace; source physical inputs open.
Finite adjoint-span solver	$V_{k+1} = \text{span}(V_k \cup \{K_{a\beta}^\dagger\} V_k)$.	Finite algebraic termination; rounds are not physical ticks.
Representation and redundant-composition checks	Same support under Kraus mixing, representation covariance and redundant generator additions.	Exact constructed diagnostics [S17].
Structural presence versus commitment history	The physical structure projector and historical-record projector are separately typed.	No identification without a source bridge.
Physical readout and minimality	An admitted separating effect, physical realisation, interface data and no proper admitted physical substructure.	Required for promotion; not supplied by stable rank alone.

Object or gate	Return	Grade
Presence alone versus four Fold states	Binary presence can use a rank-one occupied sector; polarity and orientation provide additional interface data.	Exact presence-only control; no new derivation of the four-state Fold.
Fact-derived order rank	Prefix order and $q(h f) - q(h) = 1$.	Exact on the declared append-only branch; not seconds or universal simultaneity.
Metric normalisation freedom	$g \rightarrow a^2g$ leaves the structural test unchanged while lengths scale.	Underdetermination by tested inputs; a complete source may remove it.
Primitive Fold and aggregate record clock	Distinct grains unless source-identified; T_{cover} minimum seven updates, mean ≈ 21.2777 .	Explicit boundary; bridge open.
Net-current cover controller	Same net current, endpoint and update count can give different cover status (h_A, h_B).	Exact allowed-history counterexample; no pre-fact duration assigned.
Visited-set controller	Initial site, visited mask and persistent latch realise the declared cover trigger.	Exact construction on the D36/PFDMI record branch.
Fold-to-MASD functional or controller attachment	No automatic equality to current, traced-graph Betti number or closure defect.	Open.
FC occurrence rule	One allowed completion resolves into a persistent fact.	Primitive axiom; not derived by a Stinespring isometry.
Intrinsic four-atom Fold carrier	Independent polarity and orientation in the Fold-interface construction; exact Walsh decomposition of $P_{b\omega}$.	Existing source-level structural return.
Polarity-odd source component	$J_- \neq 0$ is forced by fact separation; $S_b \notin \text{Alg}\{I, O_\omega\}$.	Exact algebraic no-go for the outcome-blind orientation source.
Transverse gain, κ background, mark capacity	$a \neq 0$ required but unselected; κ rescue impossible; 319 marks and 84 persistent records cannot name polarity.	Exact on the audited routes [S4; S16].
Joint four-sector test	Four nonzero sectors; ranks one on the minimal carrier iff $\text{Tr } R = \text{Tr } O = \text{Tr}(RO) = 0$.	Exact algebra; same-event physical realisation open.
Sharp-marginal preservation	Two correctly typed sharp marginals force product and joint classical effects (Theorem III.2).	Exact conditional channel theorem.
Post-commitment continuum or current coupling	Physical retained-mark influence on later active dynamics.	Open; separate from record identity.

63.2 Relational readout and refinement (Part IV)

Object or gate	Return	Grade
Two supports versus interval length	Two endpoints admit any positive metric separation; an unweighted elementary link counts as one edge.	Exact metric control; no factor-two Planck derivation.
Conditional hexagonal geometry	$d^2 = \delta^2(m^2 + mn + n^2)$ under a local Euclidean, saturated six-neighbour and translation realisation.	Exact conditional geometry [S22]; δ remains an input.
Canonical catalogue source	Reversible T_0 , stationary weights (7, 4, 4, 4, 4, 4, 4)/31, spectrum {1, 1/2, 1/2, -1/4, -3/28, 0, 0}.	Existing canonical source [S18], not the full physical history law.
Two-coordinate readout	First cyclic harmonic; normalised minimum squared chart gap 31/12.	Exact within the declared chart class and normalisation [S23].
Full canonical response	$Q_0 = D_\pi(I - T_0)$; all 21 pair responses calculated; minimum $R = 279/20$.	Exact inverse-Dirichlet diagnostic [S24 to S25], not a physical spatial ruler.
Source-native input address	$\sum_y H_{y \leftarrow x} {}^\dagger H_{y \leftarrow x} = P_x$.	Exact outcome coarse-graining of the declared marked instrument [S19; S25].
Internal instrument versus address count	$\sum_{y,\alpha} K_{y\alpha} {}^\dagger K_{y\alpha} = P_x \otimes I_{\text{int}}$.	Seven rank-nine effects for the FCHI neutral factor; dimensions and marks are not positions.
Full-mark grouping audit	Exactly one of 877 input partitions preserves the full canonical recorded-mark likelihood.	Exact finite audit [S25]; unlike the five symmetry-compatible unmarked groupings.
Same-label refinement	Conditional-weight lift preserves the retained chart and stationary norm.	Exact for the stated retained-map family [S23].
Retained-boundary response	Schur reduction preserves all retained pair responses iff the effective operator equals the original (Theorem IV.1).	Exact connected positive quadratic result; 119 canonical retained subsets checked [S24].
New-terminal subdivision control	Original responses remain fixed while exposed internal-segment responses can tend to zero.	Exact constructed control [S24]; not physical sub-Fold objects.
Correlated reversible refinement	$S_f L = L T_0$ plus reversibility and correct marginal gives $\min_{\text{cf=g}} f^T Q_f f = g^T Q_0 g$ (Theorem IV.2).	Exact sufficient theorem with non-product checks at dimensions 14, 21, 28, 63 [S25].

Object or gate	Return	Grade
Nonreversible limitation	Forward grouping alone permits retained response 800/191 instead of 4.	Exact four-state counterexample; address readout remains valid without reversibility [S25].
History address to physical localisation	Identify actual Fold supports, their positional observable and metric, and refinements admitting new supports.	Open physical promotion; seven catalogue labels are not assigned seven Fold positions.
Candidate gravity-route disposition	The tested four-dimensional Gaussian phase extension gives incompatible curvature coefficients and a flat-coordinate response.	Failure of that candidate promotion [S26], not a general VERSF rejection or a Fold-size prediction.

63.3 Conditional metric-scale closure and coherence attachment (Part V)

Object or gate	Return	Grade
Conditional joint metric-scale closure	$\ell_{p,cand} = [v(8\pi B/3)/(q^2\alpha_*)]L_{\Lambda}e^{-2\Theta}$; benchmark 1.66×10^{-35} m and $\xi = 90.4$ μ m.	Exact algebra and reproduced conditional benchmark [S37]; physical coefficient selection open.
Invariant coefficient target	$\Theta_{phys} = \Theta + \ln(q/2) + \frac{1}{2}\ln \alpha_* - \frac{1}{4}\ln B$.	Exact consistent-convention cancellation [S38], not a source-selected number.
Inverse-required values	$\Theta_{phys}^{(req)} = 70.0803$ at Planck-based central values; inverse-required $b = 0.874982$ and 0.875488 across the two constructed scenarios, with $7/8$ between them; $K_{req} = 1.001442$; Planck-based forward residual $+0.29\%$.	Conditional central-value evaluations with other inputs fixed; observational uncertainty and physical coefficient selection remain separate.
First-differentiation closure prior	Uniform diagonal word weights and rank-one full-word probability $1/128$.	Exact under P14 and the K7 carrier [S35]; not a statement of multi-cell independence.
Physical beta calculation	Source joint coherence probability and actual regulator ratio enter the direct RMOP gate.	Existing exact conditional prescription [S33]; 14/16 counting is not its numerical execution.
First-step probability requirement	For $p_0 = 1/128$ and scale ratio $1/2$, $b = 7/8$ requires $p_B \approx 0.007887244431$.	Inverse diagnostic certificate [S38 to S40], not a source probability.
Block-pattern and entropy tests	Common-flip branch: $E[D_B] = 1.0095673$ and $H_{joint} \leq 7.2882832$ bits.	Conditional necessary tests [S39 to S40]; optimiser and weights are not physical predictions.

Object or gate	Return	Grade
Retained-record probability selection	$\sigma_h = J_h(\sigma_o)/\text{Tr } J_h(\sigma_o)$, with the source's conditional no-extra-tilt rule.	Existing instrument theorem at its stated grade [S34]; source and coherence event still required.
Admission-weight-preserving coherence	$E_{\text{coh}} = A_{\text{adm}}^\dagger(C)$; rejected and admitted-but-incoherent outcomes retained.	Exact conditional CP attachment [S41]; physical A_{adm} and C are not manufactured.
Joint all-failure effect	$E_{\text{fail},B} = \Phi_B^\dagger[(I - P_{\text{full}})^{\otimes m}]$ for a completed trace-preserving joint readout.	Exact predicate and pullback; no product-state assumption; physical joint readout open.
Backward block evaluation	$H_m = I$, $H_{j-1} = F_j^\dagger(H_j)$.	Exact recursion with constructed noncommuting controls [S41]; not a spatial-to-serial identity.
Balanced transport comparison	$M_+ = (U_A + U_B)/2$ and $E_- = (U_A - U_B)^\dagger(U_A - U_B)/4$.	Existing RCHS experiment [S36]; normalisation exact, physical operational selection conditional.
Substrate path-probability return	$p_- = (3 - \cos \epsilon)(1 - \cos \epsilon)^2(1 + \cos \epsilon)/16$ for the declared eight-component test.	Exact finite specialisation and reproduced verification [S42], not a vacuum or K7 probability.
Same-loop subdivision	Full transport products and the comparison effect remain unchanged.	Exact within transport-preserving subdivision without new intermediate operations [S42].
Smaller-loop comparison	Local phase halving gives leading quartic suppression.	Changed-observable calculation; neither a physical cell census nor a beta-coefficient derivation.
Physical coherence selection	Same-source admission, criterion, preparation, joint state and update and adjacent-scale observable.	Partial transport attachment supplied; complete Route-M physical return open.
Fold-scale promotion	Relate the conditional metric threshold to actual Fold support and the normalised spatial ruler.	Open physical identification; no Fold diameter or compulsory Fold-Planck ratio.

63.4 Direct running, rigidity and record-geometry cosmology (Part VI)

Object or gate	Return	Grade
First-hit RMOP reduction	$b_n = (1 - p)r_n/(2h_np[p + (1 - p)r_n])$; for 7/8 the required conditional rescue probability is $r_n = 7.5333 \times 10^{-5}$.	Exact identity and inverse target [S47]; no source result assigns that probability.
Beta non-identifiability	Uniform marginals plus permutation and XOR symmetry admit $0 \leq b \leq 60/\ln 2 = 86.56$; explicit joint laws attain the range.	Exact constructed counterexamples; the physical joint channel is the remaining input.
Constant-beta scope	A positive constant-b law with fixed effects fails on any convex preparation family by strict convexity of $p/(1 - 2bhp)$.	Exact scope restriction; state-dependent running unaffected.
Ω invariance and K convention	$\Omega = \Theta + \ln K (= \Theta_{\text{phys}})$ is convention-invariant; K alone transforms as $K' = (v/u)K$; rigidity needs a source manifold with $d\Omega = 0$ plus a reference value (Theorem VI.2).	Conditional rigidity theorem plus explicit counterfamily [S47]; displayed matching equations insufficient.
Record-geometry cosmology audit	$3/(150 \text{ Mpc})^2 = 1.4004 \times 10^{-49} \text{ m}^{-2}$, not $\approx 1.3 \times 10^{-52}$; occupancy law cannot supply the needed suppression; $w_{\text{DE}} = -1 - n\chi^n/(1 + \chi^n)$.	Verified source corrections; literal branch rejected, framework not.
Two-scale bridge	$\xi_M^2 = \sqrt{8\pi B/(3F)} \ell_P R_{\text{rec}}$; $R_{\text{rec}}/\ell_P = \sqrt{6F/\pi} K^2 e^{2\Theta}$.	Exact conditional matching relation; independent record-geometry check, not a definition.
Scale non-identifiability	A global rescaling of all lengths with $G \rightarrow s^2 G$, $\Lambda \rightarrow \Lambda/s^2$ preserves every displayed relation; $\Lambda_{\text{cos}} \ell_P^2$ is the only dimensionless combination the displayed equations could determine once Ω is source-selected.	Exact for the equations examined; an additional source law is not excluded.

63.5 Census and bridge (Parts VII and VIII)

Object or gate	Return	Grade
Minimal one-generation package	$G = \{Q_L, u_R, d_R, L_L, e_R\}$; colourless doublet is the minimal Witten repair.	Conditional (Lemma VII.1) on the declared attachment, closure and minimality premises.
Hypercharge assignments	$(1/3, 4/3, -2/3, -1, -2)$, $Y_H = +1$, from the perfect cube $(1 - 3Y_Q)^3 = 0$.	Classical result reproduced as a census constraint [13 to 18]; unique within the minimal one-Higgs content; B – L freedom reopens with v_R .

Object or gate	Return	Grade
Exactly three generations	$N_g = 3$ from C9 with C12 and C15.	Conditional theorem (Theorem VII.1); inherits C9.
Antipodal quotient $D_6/\langle r^3 \rangle \cong S_3$	Central involution; quotient acts on three classes by the full permutation action.	Exact (Proposition VIII.1); a property of the sixfold carrier.
$Q(z) = z^2$ factors; $q \in E_2 \otimes \mathbb{C}$	Family vector $(1, \bar{\omega}, \omega)/\sqrt{3}$ in the zero-sum sector of the three-object carrier.	Exact (Proposition VIII.2), given the PMS source identity; physical normalisation downstream.
Three terminal keys, $K_7 \otimes I_3$	Closed terminal carrier at regulator levels 3 to 6 with zero terminal-block residual.	Exact within the PFDML instrument [4]; same permutation type as the family quotient.
Family-class to terminal-key intertwiner	$T = e^{i\phi} P_\pi$.	Unique up to phase and relabelling (Proposition VIII.3), given the identification premise B1 and the ray reading of C15.
Primitive same-action bridge	$B_{\text{key}} = 0$ for EX2E while $B_{\text{family}} \neq 0$ ($g_{\text{ax}} = 1/2$).	Open; no-return for EX2E, exact to all orders (PMS-EX81 and EX108).
W_{phys} and the family normalisation	Rayleigh–Ritz spectrum on the 49-dimensional census carrier; shape constrained to $K_\theta = tG$ [6].	Open; candidate shared source-normalisation frontier; the $W_{\text{phys}} \rightarrow K_{E_2}^{\text{phys}}$ map remains to be derived.
Masses, CKM and PMNS values, neutrino masses	Not derived.	Downstream.

The Planck-scale one-Fold interpretation, many-Fold particle ontology and separation from the collective coherence scale retain the distinctions of Sections 2.4 and 23. The individual Fold's calibrated scale, expected interface area and external localisation heuristic are not promoted by the chart or address results. Parts V and VI add a separate conditional metric-scale prediction and a partial source-to-coherence probability attachment; they do not supply a physical Fold diameter or make the Planck coefficient compulsory. The chart normalisation, inverse-Dirichlet response, address count, transport-comparison probability and physical spatial ruler remain different objects. The probability constructions do not close polarity, stable presence, the marked trap attachment or the physical localisation of Fold supports.

64. Next identity calculation: source-native sharp-marginal and event-grain test

The immediate task is not to attach a commuting orientation matrix. First declare whether the proposed instrument acts on one primitive Fold, the completed-cover R4 record, or a later retained ledger. A channel between different grains needs an explicit source-derived semantic and dynamical map; its labels cannot identify the grains by themselves. On the correct source carrier, return the intrinsic observables $R = BO$ and O , their independent symmetry actions and their joint support. Then derive the actual source-to-record channel $\mathcal{E}$ and its physical output readouts. Test

$$\mathcal{E}^+(R_{\text{out}}) - R = 0, \quad \mathcal{E}^+(O_{\text{out}}) - O = 0.$$

Check all four joint projectors, their nonzero ranks, and their preservation under the actual retention and renewal channel. The product-character identity is then a consequence, not an extra coupling to fit.

Decision rule. PASS: source-native same-event channel, correct independent binary typing, four-sector support, and exact sharp-marginal preservation. PARTIAL: an algebraic construction or an unselected channel family passes but physical provenance or event grain is unresolved. FAIL FOR THE TESTED ROUTE: collapsed joint sectors, wrong grain, wrong transformation type, or failure of a marginal identity. None of these route-level outcomes by itself establishes or refutes all possible VERSF constructions.

65. Next attachment calculation: source-returned mixed map and projected Gram matrix

Fix the physical background or a source-derived covariant family, the candidate continuation e , the History-Measure code and its forbidden ray. Derive $B_0(h, e; B)$ from the primitive history- and record-sensitive source term before inspecting the desired kernel. Do not choose columns inside K_e to guarantee success. Compute the current and transverse vectors, verify the source branch ranks, and form

$$Y = \Pi_K B_0, \quad H = B_0^\dagger \Pi_K B_0.$$

A zero H rejects that seed for this route. Singular H identifies a lost direction of the declared linear code; a pseudoinverse cannot restore that direction. Positive-definite H returns an injective projected candidate, not a final physical PASS. This is a clean pass-or-fail experiment on VERSF itself: an actual B_0 derived from the history-sensitive source either survives projection or identifies exactly which information was lost. If polar normalisation is proposed, derive its physical metric and admissibility rather than applying it as an unearned correction. Then evaluate the complete continuation-faithfulness equality on the identified carrier, the actual terminal-record readouts, candidate covariance and the physical cyclic sector intertwining. The fixed-point property of one current subinstrument cannot replace these tests. In parallel, use the correct visited-set first-cover controller at aggregate-record grain and derive its relation, if any, to the physical Fold trap and MASD variables. Source generation of this controller and its feedback into admissible continuation are separate from storing its final bit. The identity and attachment tests are related but need not be declared strictly sequential without a source theorem: their carriers, clocks and instruments must be reconciled before either is used to promote the physical Gate-T map. The timeless structural-support projector of Section 23.5 is not automatically the current-subinstrument kernel of Section 12. Their carrier identification and physical relationship require an independent source map.

66. Priority programme: physical supports, coherent source effects, a common scale and the pricing operator

Physical support and stability. The recovered input history-address effects are available; their physical localisation remains to be established. Determine which address, internal-state and record distinctions correspond to different actual Fold supports and which describe the same support. Return the physical positional observable and common metric. The stable-presence package of Section 23.5 still needs an independently typed absence support, admitted continuation and context law and separating physical effect. Neither a history label nor a successful comparator outcome substitutes for the Fold's structural presence.

Use the actual refinement law. Keep the reversible canonical catalogue and the driven physical instrument separate. Test the appropriate complete response reduction on source-returned fine data.

Adding internal description and admitting a new support are different operations; a bound on old addresses does not constrain new supports hidden in a coarse fibre. A uniform physical minimum must include all admitted supports on the same ruler. Transport-preserving subdivision also differs from comparison of a smaller loop: the full original path products must be retained when the event is claimed unchanged.

Do not reopen the probability rule. The retained-record source supplies the conditioned density state once its instrument and preparation are fixed. Its likelihood is not multiplied by a freely chosen record temperature. The K7 source supplies the full-word selector at its stated first-differentiation grain. The balanced path comparison supplies a concrete mismatch-sensitive effect on the declared unitary branch. None of those results by itself selects the physical joint Route-M event.

Return the same-source coherence attachment. Derive the physical operation admitting the candidate, the relevant coherence criterion and its readout, and the joint operation on a physical block. Retain rejected branches and conditional admission probabilities. Do not replace this calculation by an automatically passed tetrahedral identity, a gradient-flow selector tested after enforcement, a current-pointer event, a completed cover or a port outcome whose structural meaning has not been established. On the required common carrier, return $E_{fail,B}$ and evaluate $p_B(h) = 1 - \text{Tr}[E_{fail,B}J_h(\sigma_0)]/\text{Tr} J_h(\sigma_0)$. The needed return can be the adjoint action on this one physically identified effect; full channel reconstruction is not an automatic prerequisite. An ordered implementation must preserve the actual memory and be justified for the physical block, rather than identifying spatial sites with chronological trials.

Calculate running before selecting the target. The first-hit reduction of Section 46 identifies the minimal remaining return: the conditional rescue probability of the block given reference-cell failure, on the actual carrier with admission weights retained. From the physical adjacent-regulator probabilities, block multiplicity and regulator ratio, evaluate the RMOP coefficient. Allow the result to differ from 7/8 or to vary with scale. Determine the critical endpoint and integrate or sum the returned running in the regime actually justified by the source. The pattern-diversity and entropy bounds in Appendix B are rejection tests for a proposed branch, not recipes for choosing its measure.

Close the metric normalisation. Identify the same physical coherence observable across Route M, capacity and UV/IR matching. Derive or bound the convention-invariant combination $\Omega = \Theta_{phys}$ rather than resetting B, q and α_* separately to convenient values; per Section 47, rigidity requires a source-defined admitted manifold on which $d\Omega$ vanishes, together with an independently derived reference value, and the displayed matching equations alone are insufficient. The conditional estimate of Section 39 is available as a comparison after the source return is frozen, not as a datum that selects it. The inverse evaluation of Section 39.5 makes the obligation concrete at central values: exact matching there requires $K = 1.001442$, the inverse-required running coefficient under the fixed benchmark assumptions lies within a tenth of a percent of 7/8 in both constructed scenarios, and the present coherence identification corresponds to a normalisation difference in the combination that requires reconciliation. All of these are conditional matching targets, not derived quantities. Relate the resulting metric threshold to the operational Fold-resolution length ℓ_F and to δ_{rel} before evaluating $\alpha_F = c^3 \ell_F^2 / (\hbar G) = (\ell_F / \ell_P)^2$.

Derive the pricing operator. Derive the representation-resolved current and history pricing operator W_{phys} from the common source action and evaluate its seven central-sector prices on the census carrier of Section 56. If one sector is uniquely lowest, the internal preparation problem of Section 21 collapses from $\mathbb{CP}^{48}$ to that sector in one step. Then display the map from the same primitive Hessian to the physical E_2 stiffness $\kappa_{E_2}^{phys}$, so that the family normalisation of Section 62 is either returned by the same calculation or shown to be a separate one. Alongside it, return a primitive source law whose quadratic E_1^2 family event is visible in the terminal keys, so that the same-action test of Section 61 can be passed or definitively failed.

The inherited connector premises, marked source map, projected-Gram and physical-normalisation tests, full continuation-faithfulness, event-grain reconciliation and Fold-to-MASD controller bridge of

Sections 64 to 65 remain separate. The phase-sensitive comparison does not select the integer attachment matrix or a rank-one forbidden continuation merely by sharing a dimension or label count.

Decision rule. ALGEBRAIC PASS means the declared comparison, probability pullback or scale identity is exact within its stated assumptions. SOURCE PARTIAL means that construction is available but physical preparation, event semantics or the joint source operation is unreturned. RUNNING PASS requires the actual adjacent-regulator event and joint law, not a fitted correlation. METRIC-SCALE PASS additionally requires source-selected common matching and normalisation. FOLD-SCALE PASS further requires the physical support and ruler identification and a minimum that includes all admitted supports. CENSUS PASS requires the C-premises to hold on the physical support-return quotient. BRIDGE PASS requires $B_{\text{key}} \neq 0$ equal to B_{family} up to $e^{i\phi}P_{\pi}$. Failure of one candidate does not reject all VERSF realisations; a conditional numerical match does not establish these stronger passes.

67. Falsifiers

#	Observation	Consequence
F1	A physical Fold attachment outside the family P_0 , or one failing P_1 to P_5 .	Theorems I.1 and I.2 do not apply; the order-seven structure is not selected.
F2	The trapped faces export more than one edge direction under wheel covariance.	The private-rim intertwiner is not unique (Theorem I.4).
F3	A static background other than the vertex-constant field satisfies the continuity zero on the wheel.	The witness reclassification of Section 18.1 fails.
F4	The leading nonconstant mode of P_+ is not the $k = \pm 1$ doublet.	Theorem I.5 fails; the E_1 source plane is not the matter carrier.
F5	A Fact cut, scalar current cost or isotropic covariance that selects $[z]$.	One of the five no-go results of Section 20 fails.
F6	A terminally admissible partial generation on one support-return layer, or one carrier completing two independent layers.	C12 or C15 fails; Theorem VII.1 fails.
F7	A fourth independent support-return layer, or fewer than three.	C9 fails; the count fails with it.
F8	A smaller anomaly-stable repair of the coloured weak sector than one colourless doublet.	Lemma VII.1 minimality fails.
F9	A second hypercharge solution within the minimal one-Higgs content under ordered closure.	The classical uniqueness result is misapplied.
F10	A corrected primitive implementation returns $B_{\text{key}} \neq 0$ equal to B_{family} up to $e^{i\phi}P_{\pi}$.	The same-action test passes; the identification premise B1 is promoted from premise to result.
F11	$B_{\text{key}} \neq 0$ but not equal to B_{family} up to a phase and a permutation.	Premise B1 is rejected; Propositions VIII.1 to VIII.3 stand as representation-level results.

#	Observation	Consequence
F12	A source realisation whose first nonzero family response is linear in E_1 , or whose lowest descent is not z^2 .	The C_3 family carrier derivation is void for that realisation.
F13	A terminal block not of the form $K_7 \otimes I_3$ at an audited regulator level.	The three-key side fails; the family quotient is unaffected.
F14	A continuum spatial rotation acting nontrivially as a family rotation.	The internal typing of the quotient carrier fails.
F15	A source-admitted continuation whose leakage certificate $\eta_a(P)$ is positive for every candidate support orthogonal to the returned absence sector.	No exactly stable Fold-presence sector exists for that continuation class; Theorem III.1 returns the zero support.
F16	A source-native channel whose two sharp marginals are preserved but whose joint record projectors are not.	Impossible under Theorem III.2; the channel is not CPTP or the marginals are not both sharp, so the identity gate was misapplied.
F17	A complete recorded-mark likelihood that factors through a coarser partition than the seven input labels.	The sharp history-address readout of Section 31 fails for that instrument.
F18	A reversible fine law with correct marginal and grouping whose retained response differs from Q_0 .	A hypothesis of Theorem IV.2 fails; the refinement is not a same-support refinement.
F19	A pair of coherence indicators with correlation below 0.9904 on the prescribed OR gate with $p = 1/128$.	The 7/8 running coefficient is excluded on that gate (Appendix B.2).
F20	A source-returned adjacent-regulator block law whose first-hit rescue probability differs from 7.5333×10^{-5} at $p = 1/128$, $m = 16$, $h = \ln 2$.	The running coefficient differs from 7/8; the conditional benchmark of Section 39 is re-evaluated with the returned value.
F21	An admitted source variation along which $d\Omega \neq 0$.	Ω is not rigid on that manifold; the Planck-scale closure is not compulsory (Theorem VI.2).
F22	A source-returned record radius R_{rec} and suppression F violating $R_{\text{rec}}/\ell_P = \sqrt{(6F/\pi)K^2 e^{2\Theta}}$ at the independently evaluated running and normalisation.	The two-scale bridge fails; the capacity and cosmological relations do not share one vacuum density.

68. Conclusion

The topological result stands. Within P0 to P5, the complement-pair connector and $\pm(J - I - S)$ are jointly selected; order-seven torsion follows afterwards. This remains a theorem within the declared attachment class, not a completed physical source installation or a literal toroidal spacetime. The marked-anchor result uses the audited rank-12 channel stack and 38-dimensional rank-one locus. Its projected-source Gram test distinguishes lost directions from an injective candidate attachment, while

physical selection, normalisation and full continuation compatibility remain separate. The history-address effects do not identify a rank-one trap ray or a Fold-polarity outcome.

The timeless stability criterion is exact on its finite common-carrier class. A support is preserved precisely when all admitted branches have zero leakage, and the largest compatible subspace orthogonal to a source-defined absence support is calculable by adjoint-span closure. The calculation does not turn a maximal invariant sector into a minimal physical Fold, a record into its structure, or a solver index into time. The relational programme contains both a conditional hexagonal distance construction and an exact canonical spectral chart, and it evaluates the complete inverse-Dirichlet contrast response; their different distance ratios make clear that a symmetry-compatible picture and a full response geometry are not automatically the same metric. The strongest identification is at the marked-instrument level: grouping actual recorded outcomes by their origin recovers P_x , and complete internal factors preserve $P_x \otimes I_{\text{int}}$. Exact Schur reduction characterises preservation of all retained boundary responses, and a reversible fine law with the correct marginal and grouping gives a whole-response conditional-mean identity even with correlated internal variables; a nonreversible counterexample shows that forward grouping alone is insufficient, and subdivision controls show that preservation of old responses does not bound newly exposed supports.

The scale audit adds a conditional quantitative result. Combining the inherited hierarchy with the quartic and effective cosmological relations returns approximately 1.66×10^{-35} m and $90.4 \mu\text{m}$ on the declared benchmark, and 1.62×10^{-35} m at Planck-based central values, without measured G or a supplied coherence length. This is stronger than an arbitrary scale calibration but weaker than compulsory physical prediction: the source has not selected the relevant hierarchy, shared coherence observable and invariant coefficient combination, the displayed matching equations admit an explicit counterfamily, and the equations examined are invariant under a global rescaling of all lengths. The retained-record law supplies the conditioned state without a Gibbs tilt; the correct coherence effect retains admission probability; the balanced same-endpoint comparison gives an exact mismatch effect whose subdivided original loop keeps its probability while a genuinely smaller comparison is quartically suppressed; and the first-hit reduction names the one conditional probability that a physical running calculation must return. The finite-record cosmology's literal branch is rejected after correcting its scale conversion and fluid sign, and a corrected two-scale bridge replaces it as a check.

Within a declared six-port class the smallest stable structure attaches to its surroundings by a law that is selected rather than imposed, with determinant seven and a supporting graph of genus one; its six trapped faces type exactly to the six rim continuations of the seven-site wheel; the physical matter writer is the reversal-even part of the driven seven-site law, whose leading mode goes once around the rim; and the internal kind of matter that sits on that mode is left to one pricing operator that the source has not yet returned. By a separate argument the minimal matter census is the Standard Model package with its classical hypercharges, repeated exactly three times because the record structure has three independent layers each needing a complete package.

The two threes are related by structure. The sixfold carrier's antipodal quotient is canonically a three-object S_3 -set; the source-selected family map factors through it and places the family vector in its zero-sum sector; the three terminal keys have the same shape; and, if the three classes are the three keys, the matching is unique up to a phase and a relabelling. That identification is a premise, its test is a second-order same-action condition, and the current implementation fails the test: the terminal keys do not see the family event that the family source produces. The paper therefore stops at a definite place. The generation count and the carrier bridge are closed at representation and census level; the primitive same-action bridge is open; and the physical normalisation of the family response and the pricing operator that governs internal selection are open at the same source-normalisation frontier, with the map between them not yet displayed. One calculation, the source derivation of W_{phys} and of a primitive law whose quadratic family event is visible in the terminal keys, would move every part of this paper at once. Nothing in it fixes a mass, a mixing angle or a coupling, and nothing in it used one.

One-line result. Fold $\rightarrow$ six-port selection $\rightarrow A_* = \pm(J - I - S)$, $\det 7 \rightarrow$ private-rim D_6 typing $\rightarrow P_+$ writer, $k = \pm 1$ carrier $\rightarrow u \otimes z$ with $[z] \in \mathbb{CP}^{48}$ reduced to W_{phys} ; stable presence by the largest admitted invariant support, addresses by origin-grouped marks, retained response 279/20, conditional scale closure $\Lambda_{\cos} \ell_P^2 = (\pi/2)e^{-4\Omega}$ with Ω open; independently, terminal-record premises $\rightarrow \{Q_L, u_R, d_R, L_L, e_R\}$ with classical hypercharges, and rank-three support-return with C12, C15 $\rightarrow N_g = 3$; bridge: $D_6/\langle r^3 \rangle \cong S_3$, $Q(z) = z^2$ factors through it, keys $K_7 \otimes I_3$ of the same type, intertwiner $e^{i\phi} P_\pi$ given the identification; same-action test currently failed ($B_{\text{key}} = 0$, $B_{\text{family}} \neq 0$).

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Appendix A. Disposition of the metric-to-gravity size attempt

The metric-attachment work is retained as a diagnostic boundary, not imported into the relational proof. The attempted four-dimensional Gaussian extension of the compact-phase source, with its declared minimal scalar coupling and fixed-coordinate cell averaging, did not yield a single Einstein–Hilbert coefficient. It was a candidate extension, not a uniquely selected physical VERSF regulator [S26]. At the largest archived box 64³, the two apparent readings were

$$\Lambda_{\text{scale}} = -0.02677964551, \quad \Lambda_{\text{shape}} = +0.00510443117.$$

The same candidate responded to an exactly flat coordinate deformation. The archive gives analytical long-wavelength sign checks and finite spectral verifications. The increasing-volume sequence is not a fixed-geometry continuum-refinement proof. The result diagnoses remaining regulator and attachment dependence in promoting that raw statistical functional into a metric-only gravitational action. It is not a statement that the complete VERSF theory has inconsistent gravity. No positive reading was selected, averaged with the negative reading, converted by absolute value or repaired by fitting a common curvature coefficient. A common Einstein–Hilbert contribution shifts both readings equally and does not remove their discrepancy. No half-Planck-length or other Fold-size prediction is accepted from that test [S26]. For an independently established physical geometric action, comparison of a native length a with the Planck scale could still take the conditional form

$$\Lambda_a = c^3 a^2 / (16\pi\hbar G), \quad \ell_F / \ell_P = (\ell_F / a) \sqrt{16\pi\Lambda_a}.$$

This is a matching identity, not an evaluated prediction. It needs both a common physical curvature coefficient and an independent Fold-to-ruler identification. Setting a numerical lattice spacing to one supplies neither. Gravity enters that particular Planck comparison through the definition of the comparison scale; it does not redefine a Fold or supply a clock before facts. Sections 30 to 37 instead develop the source's relational descriptions and their preservation. They do not use a gravitational coefficient, the measured Planck length or the failed candidate's apparent coefficients as inputs. The physical scale question remains open at the boundary stated in Section 66.

Relation to the conditional scale calculation. Section 39 is a different route: it combines the hierarchy, capacity and effective cosmological relations while retaining their coefficients. It uses neither of the failed candidate's apparent curvature readings. A conditional Planck-equivalent output from that system neither fixes the failed regulator nor proves that its own coefficient choices are physically selected. Sections 43 to 44 likewise test transport comparisons, not an Einstein–Hilbert coefficient. The three calculations retain their different objects and grades.

Appendix B. Conditional block-pattern and source-weight diagnostics

This appendix retains the exact diagnostics obtained while investigating the physical Route-M coefficient [S38 to S40]. They constrain a successful source return on their declared branch; they do not prescribe the distribution that nature must use. The full pattern readout is not needed merely to evaluate the all-failure effect of Section 42, and must not be imposed as additional physical architecture just because it simplifies a diagnostic.

B.1 Pattern-presence quotient and its extra symmetry hypothesis

Let the sixteen cells have physically aligned seven-bit words $X_j \in F_2^7$, and assume that the joint law is invariant under the same bit flip in every cell: $(X_1, \dots, X_{16})$ and $(X_1 \oplus u, \dots, X_{16} \oplus u)$ are equal in distribution for every $u \in F_2^7$. This is additional joint symmetry, not a consequence of local Pre-Fact Indifference. It permits correlations and does not assert that the physical source occupies every mathematical block configuration. The set $S_B = \{X_1, \dots, X_{16}\}$ is the minimal description retaining every question of the form: does pattern t occur in the block? Two blocks give the same answers for all targets exactly when their sets agree. The set does not retain multiplicity, order, location or every physical observable. For $D_B = |S_B|$, summing the pattern-presence indicators over all 128 targets and using the common symmetry gives

$$p_B = E[D_B]/128.$$

The beta test at 7/8 therefore requires $E[D_B] = 1 + \epsilon_D$, $\epsilon_D = 0.00956728716955\dots$. Since $1_{D_B \geq 2} \leq D_B - 1$,

$$\Pr(X_1 = \dots = X_{16}) \geq 1 - \epsilon_D = 0.99043271283045\dots$$

Agreement is not successful coherence: the shared word may fail some of its checks. Perfect copying gives $D_B = 1$ and zero running, not 7/8. A small nonzero departure from agreement must be source-selected rather than inferred from the requirement to match the coefficient.

B.2 A pairwise rejection test with weaker assumptions

Only equal coherence marginals $p = 1/128$ and the OR block rule are needed for the following bound. For every pair of binary coherence indicators, $\Pr(C_i = C_j = 1) \geq 2p - p_B$. Consequently, at the block probability required by 7/8,

$$\text{Corr}(C_i, C_j) \geq 1 - (p_B - p)/(p(1 - p)) = 0.99035737986061\dots$$

A single pair violating this lower bound excludes the 7/8 coefficient on that prescribed gate. Passing every pair does not determine the sixteen-way law. These are diagnostic thresholds, not correlations measured in or returned by VERSF.

B.3 Raw sector-weight condition

For a physically defined pattern readout on a source measure, collect the raw weights of the sectors $D_B = d$ as Z_d . On a supplied Gibbs representation they can be written

$$Z_d = \int_{D_B[\Psi]=d} e^{-\beta_{\text{stat}} F[\Psi]} D\Psi.$$

The statistical parameter β_{stat} is not the probability-running coefficient b . The identity below does not require this particular representation: nonnegative common-source sector weights suffice. The target mean diversity is equivalent to

$$\sum_{d=2}^{16} (d - 1 - \epsilon_D) Z_d = \epsilon_D Z_1.$$

Writing $Z_{\neq 1} = \sum_{d=2}^{16} Z_d$, positivity gives

$$\epsilon_D/(15 - \epsilon_D) \leq Z_{\neq 1}/Z_1 \leq \epsilon_D/(1 - \epsilon_D), \text{ that is } 0.0006382262 \leq Z_{\neq 1}/Z_1 \leq 0.0096597043.$$

The corresponding negative logarithm lies between 4.63979 and 7.35682. These are aggregate statistical-weight differences, including multiplicities and the reference measure, not individual energy

barriers, action costs or durations. For a jointly recorded pattern instrument, sector likelihood weights can instead be computed directly as $w_d(h) = \text{Tr}[E_d J_h(\sigma_0)]$. The same normalised mean formula applies. Equating these likelihood weights with a particular functional integral requires a same-source identification. An extra Gibbs factor cannot be attached after the retained-record probabilities have been fixed.

B.4 Sharp information requirement

The number of ordered sixteen-cell assignments containing exactly d distinct words is

$$g_d = (128)_d S(16, d), \quad \sum_{d=1}^{16} g_d = 128^{16} = 2^{112},$$

where $S(16, d)$ is a Stirling number of the second kind. Maximising joint Shannon entropy at the required mean diversity yields a mathematical optimiser

$$P_t(X_1, \dots, X_{16}) = e^{-t(D_B - 1)} / Z_{\text{ent}}(t), \quad Z_{\text{ent}}(t) = \sum_d g_d e^{-t(d-1)},$$

with $-\partial_t \ln Z_{\text{ent}} = \epsilon_D$. Nonnegativity of relative entropy proves that its entropy bounds every law with the same mean. The exact counting and high-precision finite optimisation in [S40] give

$$H(X_1, \dots, X_{16}) \leq 7.28828320555474... \text{ bits.}$$

Since each marginal is uniform under the common symmetry, the sum of marginal entropies is 112 bits. Their total correlation is therefore at least 104.711716794445 bits. With $U = X_1$ and $R_j = X_j \oplus X_1$, common-flip symmetry gives $P(U = u, R = r) = v(r)/128$, so $H(R_2, \dots, R_{16}) \leq 0.28828320555474... \text{ bits}$. This sharp mathematical requirement may exclude a proposed physical law. It does not establish that the maximising law is physical. Its multiplier is solved using the beta hypothesis and must not be inserted into the substrate action as a derived coefficient. The information bits here are neither committed facts nor counts of Folds. The sixteen-term optimisation is not an enumeration of 2^{112} physical configurations.

B.5 Parameter independence and the record-conditioning correction

For a fixed readout and reference measure, a specified family $F_\lambda = F_0 + \lambda A$ obeys

$$\partial E[D_B] / \partial \lambda = -\beta_{\text{stat}} \text{Cov}(D_B, A).$$

Thus a remaining parameter can be irrelevant to this observable only if the corresponding dependence vanishes over the admitted family, or cancels in the final invariant prediction. A vanishing derivative at one benchmark is not a global rigidity theorem. There is an exact finite-spectrum criterion. Let a_r be the reference weight of configurations at distinct cost E_r , and c_r their reference-weighted diversity sum. Then

$$E_\beta[D_B] = \sum_r c_r e^{-\beta E_r} / \sum_r a_r e^{-\beta E_r}$$

is identically d_* on an open β interval if and only if $c_r = d_* a_r$ at every distinct cost. The result follows from linear independence of the distinct exponentials. Extending it to continuous spectra requires the corresponding transform-uniqueness hypotheses. These diagnostics were established before the review of the retained-record source. Their correct role is limited: they may test unresolved microscopic source parameters, but they do not authorise a free exponent on the already selected record likelihood. Sections 41 to 42 use the existing instrument law directly.

Appendix verdict. The block and source-weight tests constrain a proposed physical return; none selects the required physical readout, joint measure or full scale dependence by itself. The source may satisfy them, reject the 7/8 value or require a differently typed coherence event. The numerical Planck benchmark is not promoted by satisfying a diagnostic that was solved backwards from that same benchmark's coefficient.

Appendix C. Referee objections to the census and replies

Objection: this is Geng–Marshak and related classical charge-quantisation work. The hypercharge algebra is classical and is cited explicitly [13 to 18]. The calculation is retained because a census proposal must pass those known consistency tests. The claimed VERSF contribution lies elsewhere: the terminal-record interpretation of species, the conditional three-generation argument from C9 plus C12 and C15, and the source-level R4/PMS family-carrier structure. The hypercharge result is therefore labelled a classical result reproduced as a census constraint.

Objection: the three-generation result is assumed through the rank-three premise. The number three is indeed inherited through C9, and Part VII does not claim otherwise. Its added content is the bridge from three completion layers to three complete generation packages. C12 prevents an incomplete layer from borrowing species or anomaly cancellation from another layer; C15 prevents a single constituting carrier from serving several independent layers. If C9, C12 or C15 fails, the generation theorem fails transparently.

Objection: could one generation span all three layers? Not under C15. A layer-spanning package would make the same terminal carriers constitutive of several supposedly independent completion directions. That would contradict the independence premise being used to count the layers. This is a substantive assumption and is therefore exposed to falsification rather than hidden in notation.

Objection: why mention the mixed gravitational–hypercharge anomaly when gravitational response is deferred? It is not used as the determining equation in the ordering adopted here. Ordered Higgs closure, $SU(2)^2U(1)$ and $U(1)^3$ fix the minimal root; the mixed gravitational–hypercharge relation then collapses to the same linear factor and is recorded as a surplus consistency check. This preserves the separation between the census theorem and the deferred VERSF gravitational-response programme.

Objection: are the six Fold sectors the six flavours or the three generations? Neither by count. The six sectors are continuation labels of the D_6 geometry; a generation is a complete package on one completion layer. Part VIII shows that the relation runs through the canonical antipodal quotient of the sixfold carrier, which is a three-object permutation carrier, and that the identification of its three classes with the three terminal keys is a stated premise with a stated test.

Objection: what is the one-sentence census result? Given weak binary attachment, colour support branching, ordered Higgs-mediated charged closure, anomaly stability, rank-three support-return completion, layer-local terminality and layer-exclusive attachment, the minimal VERSF Standard Model matter census contains exactly three complete anomaly-stable generations with one Higgs doublet as the universal charged-closure carrier.

Source provenance for Parts I to VI

Parts I to VI cite the Fold programme's internal sources by S-number. Each archived calculation entry below names its package and its verification script, so that a reader can request the exact artefact; the frozen EX2E package consumed by [S11] and by the executions of [3] is identified by SHA-256 3be75134298ee5226efc4e4f1f54bffe8daa6a85e4c5c8e28b5ca3d55f5308a0. Papers of the VERSF programme are held at versf-eos.com; for external circulation each entry is to carry a stable identifier (location, date and, for packages, hash) so that the provenance recorded here is reproducible rather than only transparent. Claim discipline across them: mathematical compatibility is not promoted to physical realisation or source selection; the connector theorem is exact under P0 to P5; the rank-one locus is the audited rank-12 channel stack with its 38-dimensional kernel; the timeless stability theorem assumes a finite common carrier, a correctly identified absence support and the declared composition-closed continuation class; the calculated sector is the largest compatible invariant support, not automatically a minimal Fold; record persistence does not imply continued structural presence; no pre-fact duration, prior spatial coordinate or physical interpretation of solver rounds is introduced; the conditional metric-scale closure of Section 39 is distinct from the physical identification of the primitive Fold's operational resolution with the Planck length; and physical source installation, event-grain identification, marked carrier attachment, full continuation-faithfulness, Fold-to-MASD coupling, subsequent record influence and the normalisation of spacetime measures remain open where stated.

[S1] RCHS-1: Twelve-Sector Current Rigidity, Source-Generated Current Stiffness and Current-Link Schur Reciprocity Theorem in VERSF. Gate-T attachment audit and complement-pair refinement.

[S2] The Primitive Selection History-to-Curvature and Standard-Model Attaching-Map Closure Theorem in VERSF (PSAC-1-R19). Executions EX5T, EX5U, EX5BY to EX5CE, and the fact-separating Fold-record audits.

[S3] The Physical C/W/Y Radial-Current Tangent Field and Existing-Instrument Score Descent in VERSF (CRTF-1-D6). Source-grain result CRTF-EX2V.

[S4] Primitive Matter Selection in VERSF: Cumulative Executed Theorems on Fact Commitment, Record Retention and Renewal. PMS-EX28 to EX32, PMS-EX41, PMS-EX42, the corrected EX43-R2 mark audit, PMS-EX58 Fact-Commitment and Stage-3 R4 promotion; the timeless audit imports PMS-EX18 (fact order versus clock), PMS-EX24 (record persistence), PMS-EX25 (ordered extension) and PMS-EX26 (retained record versus active carrier).

[S5] The Fold and the Record. Nearest-neighbour Fold-interface action, per-Fold $\mathbb{C}^4$ state bridge, hexagonal interface and six boundary directions plus global closure mode.

[S6] One-Fold Paper. One Fold is one stable bit; $\ell_* = c\Delta t_{\text{tick}} N_*$ is calibrated to the physical Planck length; a Fold has no coordinates by itself. The tick index is read here as an ordering parameter. Axiom V1's internal ground ray is a state inside the Fold carrier, not automatically the absence of the Fold.

[S7] Reconciling Particles and Bits. Particles are emergent stable, self-sustaining closures of many Folds rather than single Folds.

[S8] The Spacetime Coherence Length. $\xi = 8.2 \times 10^{-5}$ m is a collective commitment or coherence-domain scale fixed by the vacuum-energy throughput condition, not the characteristic scale of one Fold; the order-unity mode-spectrum prefactor and the identification with the Route-M ξ are explicit matching obligations in Part V.

[S9] The Fold Interface Law. Minimal intrinsic commitment separation requires a two-dimensional surface carrying independent commitment polarity and orientation, yielding the four-state Fold structure and $H_{\text{Fold}} = \mathbb{C}^4$.

[S10] Commitment Requires Cost. Observer-invariant distinguishability, the selection and exclusion decomposition of commitment, the Encoding Principle, substrate necessity for outcome identity.

[S11] PMS-EX31-R2: Predecessor- and Background-Audited Rank-One Kraus-Pullback Locus Test. Twelve nontrivial channels, rank 12, kernel dimension 38; not a common reducing subspace of the full terminal-effect algebra; background dependence and zero five-witness intersection; frozen EX2E package identified by SHA-256.

[S12] RCHS-13: Persistent Trap Latch, Instantaneous-Closure Sufficiency Criterion and Minimal One-Bit Memory Theorem.

[S13] RCHS-25: The Record-Clock Event-Grain Separation and Source-Stopped Endpoint Theorem in VERSF. Pre-completion and completion separation, first closed complete cover, D36 probabilities and record-clock statistics.

[S14] VERSF Gap-Closing Algebra Audit (verify_gap_audit.py, gap_audit_results.json). Exact 27-triple attachment scan, shared-edge constraints, Smith form, all 1,024 orientable rotation systems, two-dimensional joint-sector obstruction, 24-atom-permutation check.

[S15] VERSF Remaining-Gaps Follow-On Audit (tree_check.py, all_tree_check.py, verify_remaining_gaps.py). Complete 236-tree reduced connector classification and joint incidence test; sharp-marginal theorem; two D36 history probabilities; projected-source Gram and polar identities.

[S16] PMS-EX43-R2: Full-Predecessor Microscopic-Mark Sufficiency and Fold-Polarity Extraction Audit, recovered in full.

[S17] Timeless Stable-Presence Criterion and Source-Relative Selection Test in VERSF (verify_timeless_stability.py, exact_check_results.json). Finite common-carrier hypotheses, no-loss and subharmonic-projector equivalence, largest invariant support orthogonal to absence, finite adjoint-span solver, constructed diagnostics.

[S18] The VERSF History-Measure, Generation-Geometry and Branch-Closure Theorem (HMGBC-1). Canonical seven-label wheel, stationary law and spectrum; distinction between the canonical graph construction and the complete physical history operator.

[S19] FCHI-1: Full-Carrier Finite-Regulator Hamiltonian and Primitive Terminal Instrument, Sections 3.1 to 3.4. Source-and-target-labelled rank-one history marks, complete nine-dimensional neutral instrument and 217-mark product on dimension 63.

[S20] The Primitive Field-Dependent Marked Instrument Programme in VERSF, Sections 65 to 67 and 83 to 85. Endpoint, current and internal representation distinctions, complete internal instruments and source-and-target incidence construction within its declared class.

[S21] PFMJ-1: Primitive Functional Measure, Coarse-Graining Jacobian and Metric Dependence in VERSF, Section 8.

[S22] Minimal Relational Distance Calculation (Derivation.md, verify.py). Exact two-point metric control, conditional Euclidean six-neighbour argument, triangular native Gram and minimum, 624-point enumeration.

[S23] Source-Wheel Relational Readout and Same-Support Refinement (verify.py, numerical_crosscheck.py). Canonical-kernel and planar-equivariance calculations; chart normalisation 31/12; all 21 pair distances; conditional-weight isometric lifts with 7 to 112 fine descriptions.

[S24] Full-Source Relational Response and Support-Refinement Test. Full canonical Dirichlet response and all pair values; five symmetry-compatible unmarked groupings among 877 partitions; Schur reduction over 119 proper retained subsets; series-subdivision controls.

[S25] Marked History-Address Reconstruction and Relational-Response Preservation. Origin-address effects from recorded marks; canonical response from stationary mark fluxes; full response preservation under correlated reversible refinements; nonreversible forward-only counterexample; 877-partition full-mark audit.

[S26] Fold-Size Promotion Test of the Metric-Attached Phase Model (verify_fold_size.py, ir_integrals.py). Four-dimensional Gaussian extension, incompatible scale and shape curvature readings, flat coordinate test.

[S27] K. Taylor, Structural Determination of the Planck Scale and Gravitational Coupling from Information-Theoretic Closure, 58 pages. Sections 5 to 8 and Appendix D: common-tension construction; Sections 7.5 to 7.5.1: Route-M digest and numerical examples.

[S28] K. Taylor, Microphysical Foundations of Route M in the Two-Planck Framework, 26 August 2026. Sections 5 to 8: blocking prescription, UV-neutrality probability and percolation estimates.

[S29] K. Taylor, The Planck Time as a Minimum Bit-Certification Scale, 29 pages. Sections 2, 4 and 8: existing-interface and effective-causal-regime scope, energy-budget bridge and conditional duration inequality.

[S30] K. Taylor, Fold, Fact, Planck, and Closure: Structural, Realization, Metric, and Coherence Thresholds in VERSF, 21 pages.

[S31] K. Taylor, Multiple Structural Derivations of the Mesoscopic Coherence Scale ξ_{meso} , 29 pages. Sections 2 to 4: input and output structure, rounded de Sitter reference and leading UV/IR balance.

[S32] K. Taylor, Substrate Dynamics and the Higgs Ratio: A Unified VERSF Formulation, 46 pages. Sections 5.9.10 to 5.9.11 and Appendix G: ordered noncommuting transports and local phase-halving defect.

[S33] K. Taylor, The Operational Phase Bundle, Closure Effect and Direct Route-M Running in VERSF (RMOP-1) and the controlling RMOP-1R note of 29 August 2026. Operational coherence-event requirements, correlated OR-block law, unchanged-cylinder-event invariance, direct probability-running gate, current-source integrability result.

[S34] K. Taylor, Ordered Records, Realised Event States, 59 pages. Section 10A: retained-event CP operations, conditioned density state and Fubini–Study-averaged output; ordered-likelihood and no-additional-tilt results.

[S35] K. Taylor, The Post-Commitment Atomic-Operation, 121 pages, premise P14 and Section 46AE, Theorem 40A. Pre-fact word prior under Pre-Fact Indifference, exact full-word selector.

[S36] K. Taylor, RCHS-29: Current-Cycle Source Response and Phase-Sensitive Gauge Comparison in VERSF (RCHS-29-R1), September 2026, Sections 12 to 14. $M_+ = (U_A + U_B)/2$, conditional coherent route preparation and recombination, endpoint covariance, fresh-probe trial convention.

[S37] Conditional Planck Closure Audit (DERIVATION_AND_AUDIT.md, calculate.py, results.json). Simultaneous positive-system elimination, no-measured-G and no-inserted- ξ benchmark, threshold envelope and saturation-profile checks.

[S38] Coefficient Selection Audit (DERIVATION.md, calculate.py). Normalisation-invariant combinations, UV joint-neutrality calculation, inverse first-step beta target, coefficient-restored capacity conditions, UV/IR amplitude matching.

[S39] Route-M Block-Pattern Selection. Conditional common-flip pattern-set quotient, diversity identity, agreement bound, pairwise rejection test and deterministic-copying controls.

[S40] Route-M Source-Weight Rigidity. Sector-weight identity, aggregate weight bounds, exact combinatorial reduction and sharp finite entropy optimisation.

[S41] Admission-Weight-Preserving Coherence-Effect Attachment (coherence_effects.py, verify_attachment.py). Admission pullback, exact output OR predicate, retained-record contraction and backward all-failure recursion.

[S42] Source Coherence and Refinement (verify_source_comparison.py). RCHS-29 comparator applied to the written substrate path pair; L_{12} and L_{23} generators, isotropic eight-component state, two distinct refinement tests.

[S43] K. Taylor, The Quartic Capacity of Causal Regions, 11 pages. Sections 3 to 4 suppress numerical factors at scaling level.

[S44] K. Taylor, From Quantum Speed Limits to Cosmological Horizons: The Quartic Capacity Parameter of Bounded Spacetime Regions, 20 pages. Section 2 and Section 2.5.

[S45] K. Taylor, Two-Planck, 68 pages, Appendix D.1. The original seven coherence conditions: three edge existence and invertibility tests, triangle holonomy, embedding matches and orientation.

[S46] External metrological and cosmological references. CODATA 2018: $c = 299\,792\,458\text{ m s}^{-1}$ (exact), $\hbar = 1.054\,571\,817 \times 10^{-34}\text{ J s}$, $G = 6.674\,30(15) \times 10^{-11}\text{ m}^3\text{ kg}^{-1}\text{ s}^{-2}$, $\ell_P = 1.616\,255(18) \times 10^{-35}\text{ m}$. Planck 2018 results VI, base- Λ CDM chain TT, TE, EE + lowE + lensing: $H_0 = 67.36 \pm 0.54\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_\Lambda = 0.6847 \pm 0.0073$. SH0ES 2022: $H_0 = 73.04 \pm 1.04\text{ km s}^{-1}\text{ Mpc}^{-1}$. $1\text{ Mpc} = 3.0856775814913673 \times 10^{22}\text{ m}$. These values enter only Sections 39.3 and 39.5 as external reference inputs.

[S47] Direct RMOP Running, Normalization Rigidity and Record-Geometry Cosmology (calculation and source audit, 7 September 2026). First-hit reduction and probability controls, conditional rigidity theorem and K-counterfamily, finite-record cosmology corrections with literal-branch rejection, corrected two-scale bridge, global rescaling non-identifiability, executed JSON results and standard-library Python verifier. The finite-record cosmology copy audited is Cosmic-Acceleration-from-Finite-Record-Geometry, 419,239 bytes.

[U1 to U9] Marked-anchor and physical-background sources consumed in Part II Sections 16 to 21: the audited PMS-EX31 R2 and EX32 background-moduli results (rank-12 channel stack, 38-dimensional locus, reversal swap, non-gauge background-moduli audit); PFDMI (seven-state wheel, representation-valued endpoint and current carrier, D36/FCHI transition identity, exact D_6 attaching-map covariance); PSAC-1-R19 (trap-controlled export Q_{exp} , minimal covariant private-rim selector); CRTF-EX2A (static $D_T = L_W, \psi$ obstruction and positive-conductance extension); PFDMI-EX2F (D36 stationary law, isolated $k = \pm 1$ history mode); CRTF-EX2M and EX2N (49 and 50 carrier reconciliation, wheel-doublet times internal-current factorisation); CRTF-EX2O and RCHS-4 (neutral-source darkness, finite-occupation activation, U(49)-covariant endpoint-coherence reduction); RCHS-3 (rank-144 native factorisation, rank-72 Wilson-quotient response); MCFR-1 (P_+ and P_- decomposition, unique reversal-even matter writer, representation-resolved pricing left open). These are also listed as [3] to [11] in the References.