

Nonlinear History Response and Background Selection in VERSF

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General reader summary

Earlier VERSF papers describe how currents affect the chances of different events and how currents and records fluctuate together. This paper lets those hidden details change together and calculates their effect on forces. In the first model, they move the favoured background substantially, but the estimated change in the force between test currents is only about one part in a hundred million.

That tiny estimate is still consistent with no change within the calculation's uncertainty. It does not reproduce the several-per-cent changes sought in the D41 force test. A separate bound limits one positive contribution to about eight parts per million; it does not bound the full change. Two ways of weighting the hidden arrangements are compared, and all thirty internal variables are allowed to fluctuate in the force calculation.

A second model tests the shared change used as the reference for normalisation. If the field and its internal record agree across the whole closure, that reference survives specified changes to the costs of the other parts when one particular link between two parts of the source is absent. We bound the change when that link is present, and show how letting the record adjust separately lowers the response. Agreement across the whole closure remains a stated premise, rather than a result derived from the rule that a completed event leaves a persistent record.

In this second model, small fluctuations around the preferred arrangements weaken the response by different amounts in different patterns. Counting only the current fluctuations would give the opposite sign. The paper calculates this first correction and carries it into the existing test of force ratios. Connecting either model to physical force strengths still requires the source's rule for which field, current and record changes occur together, and how often. The unit cost of a primitive change remains fixed; that unit alone does not determine those probabilities.

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Abstract

In a thirty-amplitude nonlinear VERSF history model, unresolved fluctuations shift the preferred background substantially but produce a total local gauge-exchange correction of only 0.013 ± 0.009 parts per million in the colour and weak sectors and 0.012 ± 0.009 in hypercharge. The central estimate is of order one part in a hundred million and is consistent with zero at the reported precision. This declared attachment does not reproduce the collective D41 force reductions. A separate 7.6 parts-per-million upper bound applies to the isolated positive linear-pullback channel, not to the total correction. On a distinct scalar parent, retaining all 169 auxiliaries reverses the current-only fluctuation correction: the complete leading coefficient is negative in every cycle sector and changes the conditional relative-gauge balance.

The construction inherits VERSF's current-rate framework, one-bit wheel, participation law and joint score Gram. Under an explicit full-edge record-agreement premise, a constrained preservation theorem fixes the scalar reference for constant invariant weighting without a closure-completion block. Allowing that block gives sharp bounds, a criterion for common rescaling and an ordering of relaxed and constrained responses. The nonlinear auxiliary correction uses a fixed quotient chart and flat volume. An exact weight-ratio criterion specifies physical matching; the source-law audit and sufficiency control identify the joint primitive-event pushforward, physical weighting and controlled coarse-graining limit still to be evaluated. Primitive Change Normalization and the canonical weights are retained; no measured coupling selects a parameter.

1 Question and result

The preceding current-conservation paper constructs a nonzero gauge-to-cycle-current map and shows why a freely adjusting current can cancel its own residual [1]. Its finite-background results do not exclude a response produced after unresolved fluctuations are integrated. The present question is whether the reconstructed joint history model actually exhibits such a response when its quadratic approximation is removed.

The finite model exhibits a substantial change in the retained background curvature and favours a displaced background. Its force consequence is much smaller: after all thirty amplitudes are integrated, the total local exchange correction is about 0.01 parts per million in every sector, with an uncertainty of 0.009 parts per million. This is consistent with zero and far below the collective D41 reductions. A second, information-based volume tests how much the background response depends on the weighting of configurations. The two measures use the same positive history family and current core; they are finite-model choices, rather than a certified pair of physical alternatives in VERSF.

Section 6 carries this calculation into the archived gauge coordinates. It evaluates the first and second derivatives of the covariant endpoint mismatch, contracts both with the nonlinear history return, profiles the retained background, and then integrates its fluctuations. This distinguishes a change in the background from a surviving susceptibility. The attachment is explicit: history weights the absolute projected current, whereas the quadratic current defect penalises its difference from the gauge-induced value. If history instead weights that same residual alone, Proposition 1 gives exact cancellation. The source's current-rate law already motivates keeping the current entry distinct from the residual cost [11, Section 36]. The residual-only integral is a cancellation control; it is not established as an equally admissible physical source. The remaining identification concerns the projection of the complete source current, action and measure into the particular coordinates used here.

The two published predecessors calculate gauge running and native closed-history exchange [7,8]. Sections 2 to 6 evaluate the nonlinear history integral, its background feedback, 72-direction gauge contraction and local exchange kernel. Section 7 treats the distinct 226-coordinate scalar parent: shared-reference preservation, bounds on the remaining closure-completion term, the ordering of constrained responses, and the leading local auxiliary fluctuation correction. Section 8 audits the source law needed for physical matching and gives an analytic sufficiency control. Section 9 records the derivation status. Force strengths are sought after coarse graining, with the primitive normalisation carried through the same reduction.

2 The inherited history family

2.1 Source objects and participation

The source archive supplies a seven-state wheel kernel K_0 , its stationary law π_0 , six raw defect features and six bath functions [2]. The joint current construction adds two complex native cycle-current profiles, each resolved into the three participation sectors. Two real quadrature fibres implement the real bilinear convention. Each fibre has probability one half and all its features carry a factor $\sqrt{2}$.

The terminal mark counts, in the order neutral, lepton doublet, Higgs, down singlet, up singlet, quark doublet and charged-lepton singlet, are (2,6,2,9,9,18,3). Their probabilities are these counts divided by 49. The corresponding participation counts and Gram are

$$n = (36, 26, 47), \quad H = \frac{1}{49} \begin{pmatrix} 36 & 18 & 36 \\ 18 & 26 & 26 \\ 36 & 26 & 47 \end{pmatrix}.$$

The 49-address mark law is used directly. It is not replaced by a uniform law on the separate 50-coordinate computational matter carrier. The retained coordinates are the closure coordinate in each real fibre. The coordinate split is given in Table 1.

Table 1. Coordinates of the joint history model.

Coordinates	Total	Retained	Integrated
Old defect coordinates in two fibres	12	2	10
Sector current coordinates	6	0	6
Bath coordinates in two fibres	12	0	12
Total	30	2	28

2.2 The nonlinear law

Let c label the fibre, terminal mark and starting vertex. There are 98 contexts. If t is the next vertex and $F_c(t)$ is the 30-component feature vector, define

$$K_\beta(t | c) = \frac{K_0(t | c) \exp[F_c(t)^T \beta]}{Z_c(\beta)}, \quad I(\beta) = \sum_c w_c \{\log Z_c(\beta) - \mathbb{E}_0[F_c]^T \beta\}. \quad (1)$$

Here $w_c = \pi_0(x)p_r/2$. The original zero-probability transitions remain zero. This is the positive exponential tilt and baseline-to-deformed relative-entropy rate used by the source archive [2], extended to the reconstructed joint features. The context weights remain fixed in this functional; a new stationary occupation law is not recomputed at every amplitude.

The action used for the finite integration is

$$S_N(\beta) = NI(\beta) + \frac{1}{2} \eta^T H_6 \eta, \quad H_6 = I_2 \otimes H, \quad \beta = (y, z). \quad (2)$$

The six current amplitudes are η ; y contains the two retained closure coordinates and z the other 28 coordinates. The unit current core is inherited from the preceding residual-cost fixture. Its coefficient is a fixture specification. The exposure N multiplies the history cost while leaving that core unchanged. Values 7 and 49 below are concentration controls, not an executed seven-site spatial blocking law.

At the origin, $D^2 I$ agrees with the archived complete joint score Gram to 5.31×10^{-16} relative norm. Its bath-reduced current block is the inherited D41 form $I_2 \otimes (dH - \Delta \mu \mu^T)$, with $\mu = n/49$, $d = 0.0551296278772322$ and $\Delta = 0.004648154590676$. These numbers validate the source attachment; they are not inserted as target force reductions.

2.3 The integration measure

For a positive density $m(y, z)$, define

$$\Gamma_m(y) = -\log \int_{\mathbb{R}^{28}} m(y, z) e^{-S_N(y, z)} dz. \quad (3)$$

The first experiment uses $m = 1$. Knowing the transition probabilities at fixed amplitudes does not alone determine a probability law for the amplitudes themselves. The VERSF programme supplies a common operational measure and a conditional continuous proposal, together with executed joint score and bath constructions [3,4]. Section 2.4 records these inherited results. The finite choice in (3) must be

related to their coarse projection with its volume factors intact; Section 6.8 gives the exact matching condition.

Lemma 1 (normalisability of the finite family). For a finite feature family with positive context weights, positive baseline probabilities on its support and a positive-definite joint score Gram, both integration volumes tested here give finite partition integrals for every $N > 0$.

Proof. Along a unit direction v , the large-amplitude slope of $I(\tau v)$ is the weighted sum of $\max_t F_c(t)^T v - \mathbb{E}_0[F_c]^T v$. Each summand is nonnegative. A zero sum would make every supported feature constant along v , contradicting the positive score Gram. Continuity on the unit sphere gives a uniformly positive slope. Hence I grows at least linearly outside a bounded region. The current core is nonnegative, while the information-volume density in (6) is bounded above because the feature set is finite. Both integrals are therefore finite.

The archived joint Gram has minimum eigenvalue 0.00387197. The finite integrations consequently require no amplitude cutoff. Normalisability of this model does not certify an unrestricted physical field chart.

2.4 What the earlier source calculations establish

The current entry has a source precedent. The Twelve-Sector construction uses the positive law $k_e(J) = k_e(0)\exp\langle \ell_e, J \rangle$. Its Section 36 proves that the rate Hessian is the score Gram for this log-linear class; a general rate law also has a log-curvature contribution [11]. The current is therefore already an argument of the event law. Identifying that current with the six projected amplitudes in (2), and identifying its gauge dependence, remain concrete map calculations. The numerical coefficient also requires care: Section 37 derives $\lambda_J = 0.30344684305525943 g_{\text{cyc}}^2$ in its normalisation and explicitly leaves the gain to the completed source. The unit core of (2) is the preceding fixture's convention, not an absolute calibration extracted from that relation.

The fluctuation construction also predates this paper. The History-Measure theorem fixes a common quadratic operational measure and normalisation across admissible channels [3, HM11]. Parameter Closure Section 64.4.1 supplies its conditional continuous Gaussian proposal. Sections 75 and 76 go further: they execute an irreversible score-bath branch and its typed assembly [4]. Section 76 fixes that lift uniquely under its factor-blind internal-multiplicity premise, which makes each sector coefficient act equally on its operational score directions. In Section 75 the reduced score metric follows from $W_{\text{defect}} - C^T A_Q^{-1} C$; its unequal defect-sector screening is calculated from the stationary path law. Those defect sectors are distinct from the three gauge-factor coefficients sought here. The source grades this branch as conditional and distinguishes its one-bit step calibration from an action-derived commitment rate. These results are inherited at that stated scope.

There are consequently two different measures to connect. The transition family in (1) weights the next event at fixed β . The integral in (3) also weights different values of β . The common operational measure constrains that second operation, but its expression in these coordinates must include the projection, any eliminated modes and their volume factors. It need not be the determinant volume of the history Fisher matrix in Section 5. The History-Measure theorem's HM12 separately records the obligation to identify the physically normalised embedding into the exponential family. The later Changing-Link Source distinguishes fixed-input uniqueness from selection of the continuous increment law in Sections 17.67 to 17.79 [12].

These source results narrow the task to identifying a weighted coarse restriction of an existing construction. The reconstruction of the joint Gram in Section 2.2 establishes a local connection to the archived branch. Equality of that Gram alone does not establish equality of the nonlinear probability

weights away from the origin. The finite integrations quantify precisely the regime in which that distinction can matter.

3 Exact reduction identities

Proposition 1 (current translation with a fluctuating background). Suppose a probe a enters a current contribution only through $r = j - A(a, x)a$, where x denotes other background variables. If the current ranges over a full linear space with translation-invariant volume and the integrand has no additional dependence on a , then

$$\int dj e^{-S(y, j - A(a, x)a, x)} = \int dr e^{-S(y, r, x)}. \quad (4)$$

Proof. At fixed a, x , translate j by $A(a, x)a$. The Jacobian is one and the domain is unchanged. No quadratic expansion or constant response coefficient is used.

The resulting integral can still depend on y and x . It can therefore reweight a background that couples to the probe elsewhere in the complete action. Field-dependent measures, constrained current domains and other current couplings must also be transformed. Proposition 1 removes an isolated shift; it does not remove this indirect feedback or exclude the full physical response.

This preserves the fixed-block Gaussian result in the history-measure construction [3, Section 15]. A hidden determinant that is independent of the retained probe contributes only a constant. When its precision depends on the retained configuration, its determinant must be carried into the response, together with the measure.

Proposition 2 (response of the integrated measure). Put $V = S_N - \log m$. Under the regularity conditions allowing differentiation under the integral,

$$\partial_i \Gamma_m = \langle V_i \rangle, \quad \partial_i \partial_j \Gamma_m = \langle V_{ij} \rangle - \text{Cov}(V_i, V_j). \quad (5)$$

Proof. Differentiate the normalised expectation defining the first derivative. Differentiating its normalising integral subtracts the product of first moments, giving the covariance term.

Equation (5) is the calculation performed here. Averaging local Hessians alone would omit a substantial part of the response. It is also the ordinary-variable counterpart of the log-average and cumulant construction used for the fermion interaction in [5, Section 9.2].

4 Nonlinear integration and the moving background

4.1 Flat coordinate volume

The Gaussian comparator is the exact Schur complement of $Q_N = NW_{\text{joint}} + \text{diag}(0_{12}, H_6, 0_{12})$, eliminating the same 28 coordinates as the nonlinear integral. Table 2 reports the two retained diagonal entries at $y = 0$.

Table 2. Flat-volume curvature at the original reference point. Uncertainties are one-standard-error block estimates.

N	Quadratic	Nonlinear coordinate 1	Nonlinear coordinate 2
1	0.211616	0.027839 ± 0.000329	0.027564 ± 0.000265
7	1.305235	0.829335 ± 0.007556	0.822980 ± 0.006663
49	6.190795	5.695448 ± 0.129019	5.722292 ± 0.136132

At $N = 1$, the mean local Hessian is about 0.0403 and 0.0402 and the score covariance subtracts about 0.013, leaving a response near 0.0277. The nonlinear-to-Gaussian ratios rise from approximately 0.131 to 0.633 and 0.922 across the three exposures. This trend is consistent with a narrowing distribution approaching its local approximation. It does not establish a spatial continuum limit.

The runs use four independent chains. Their largest split-chain diagnostic is below 1.002. Free-energy reweighting at two small retained displacements agrees with the score-covariance calculation, allowing for the finite-sample covariance correction. The reported Monte Carlo errors do not include uncertainty in the amplitude measure or physical source identification.

4.2 The reference point is not stationary

At unit exposure, the first derivatives are approximately $(-0.237, -0.233)$. Thus the nonlinear return cannot consistently be summarised by its Hessian alone. A marginal-mode search, followed by independent sampling and a well-supported local reweighting, locates a mode near $(14.04, 13.89)$ in the specified retained coordinates. The two local curvatures there are approximately 0.00758 and 0.00787.

The initial long-distance reweighting had poor overlap and was not accepted as a stationary result. Independent sampling at the candidate point was used to refine it; the final reweighting has effective importance sample size 30,918 of 32,768. The quoted location is a numerical estimate of the mode of this coordinate density. It is not a physical vacuum, and its last printed digits do not represent its statistical accuracy.

The unrestricted amplitudes remain valid parameters of the positive finite history family. Their large displacement is not evidence that a local physical field chart remains valid over the same range. This distinction matters when using a strongly fluctuating calculation to motivate physical coarse graining.

5 Dependence on the fluctuation measure

5.1 A specified information volume

To test the weighting rather than merely vary an arbitrary sector coefficient, consider the metric on the hidden amplitudes

$$M_{zz}(\beta) = N \sum_c w_c \text{Cov}_{K_\beta}(F_{c,z}, F_{c,z}) + C_{zz}, \quad m_l(\beta) = \sqrt{\det M_{zz}(\beta)}. \quad (6)$$

Here C is the constant current-core metric from (2). In the natural exponential-family coordinates, $M_{zz} = S_{N,zz}$. Under a hidden-coordinate change it is pulled back as a metric tensor. Its determinant volume then carries the required Jacobian. Recomputing an ordinary coordinate Hessian after a nonlinear change would not be the same prescription.

The density m_l is a natural sensitivity control built from the specified model. It is not identified with the physical operational volume, and covariance under coordinate changes does not uniquely select it: multiplication by any positive scalar function gives another covariant volume. Both the flat volume and (6) can be transported consistently between charts; they are different measures in the original chart.

5.2 Determinant cancellation at the saddle

Proposition 3 (leading determinant cancellation). Suppose the hidden action has a nondegenerate conditional minimum $z_*(y)$ and a controlled Laplace expansion. Up to a retained-independent constant, the leading approximation is

$$\Gamma_m(y) = S_N(y, z_*) + \frac{1}{2} \log \det S_{N,zz}(y, z_*) - \log m(y, z_*) + \text{higher terms.} \quad (7)$$

For (6), the two logarithmic terms cancel at the saddle.

Proof. The Gaussian integral around z_* contributes the inverse square root of the hidden Hessian determinant. The prefactor contributes $m(y, z_*)$. Taking the negative logarithm gives (7); substitution of (6) gives the cancellation.

This cancels the leading volume contribution to background displacement, not all nonlinear corrections. At the origin the remaining leading curvature is the Gaussian Schur complement. Direct saddle calculations recover that complement at two finite-difference step sizes. At unit exposure the flat-volume saddle expansion is quantitatively poor: its predicted first derivative is about -1.92 , against the fully integrated value near -0.237 . It is used to identify the mechanism, not as a substitute for the nonlinear integral.

5.3 Full nonlinear measure comparison

The second integration samples $\exp[-S_N + 1/2 \log \det M_{zz}]$. Both derivatives of the log volume are included in (5). The metric determinant and its derivative are computed using a QR factorisation of weighted centred features; this avoids loss of accuracy from forming a nearly ill-conditioned Gram matrix.

Table 3. Unit-exposure response under two measures on the same history family, evaluated at $y = 0$.

Quantity	Flat coordinate volume	Information volume
Mean score 1	-0.23728 ± 0.00074	-0.31639 ± 0.01827
Mean score 2	-0.23331 ± 0.00085	-0.31711 ± 0.01753
Curvature 1	0.027839 ± 0.000329	0.096142 ± 0.040254
Curvature 2	0.027564 ± 0.000265	0.095654 ± 0.044812

Both information-volume mean scores are decisively nonzero. The shift away from the reference therefore survives cancellation of the leading determinant contribution. This is the resolved nonlinear result of the second measure test. Its largest split-chain diagnostic is 1.0060. The curvature entries subtract local terms near 1.95 and score covariances near 1.86, leaving appreciably larger Monte Carlo uncertainty than in the flat-volume run. Their central values differ from the flat-volume values, but this run does not decisively resolve that curvature difference in each direction. Neither measure is selected by proximity to a force target.

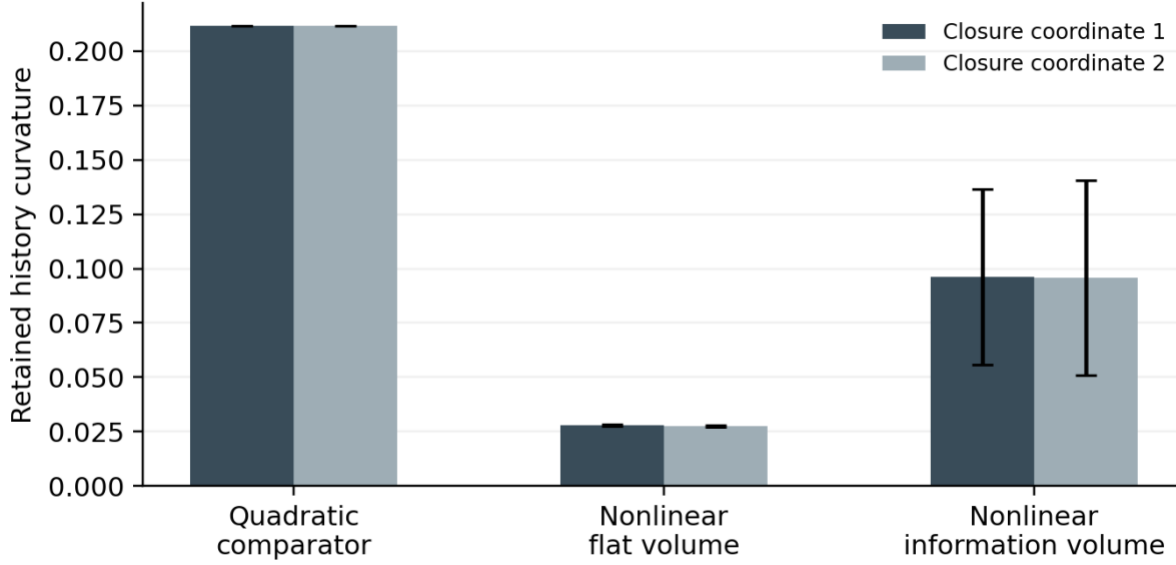


Figure 1. Retained history curvatures at the reference point. The Gaussian comparator uses the same action and hidden-coordinate split. Error bars show Monte Carlo uncertainty only; these entries are not gauge-sector stiffnesses.

6 Connection to the gauge response

6.1 The background and the probe map

Proposition 4 (a nonlinear retained map). If the physical probe enters the reduced model through $y = f(a)$, differentiation gives

$$\partial_a \partial_b [\Gamma_m(f(a))] = J_{ia} (\Gamma_m)_{ij} J_{jb} + \sum_i (\Gamma_m)_i \partial_a \partial_b f_i, \quad (8)$$

Here $J_{ia} = \partial_a f_i$.

Proof. Apply the chain rule twice.

The second term is essential when the chosen reference has a nonzero mean score. For example, with one retained quantity $f(a) = y_0 + 1/2 a^T B a$, the first derivative of the map vanishes at $a = 0$, yet the response is $\Gamma'_m(y_0)B$. A vanishing linear map therefore does not establish a vanishing quadratic response. This is an exact statement about composite observables, not a choice of B for fitting the forces.

Conversely, when the retained coordinates are independent physical coordinates and their integrated action is stationary, the second term vanishes. The mode of a marginal coordinate density and a stationary point under a constrained or composite physical probe need not coincide. The mode found in Section 4 must not be substituted for the physical gauge background without that identification.

6.2 A specified gauge attachment

Let $a \in \mathbb{R}^{72}$ contain six cycle amplitudes for each of twelve generators. The archived parent [1, Section 13.3, equations (49)–(50)] contains the vector-current defect, the link term and its coupled record phase. Eliminating the quadratic record phase and the current components orthogonal to the participation span leaves the baseline K_* and the following finite extension:

$$\Psi(y, h) = -\log \int d\xi d\eta \exp\{-NI(y, \xi, \eta) - 1/2 (\eta - h)^T H_6 (\eta - h)\}, \quad (9)$$

The resulting gauge action is $\mathcal{G}(a, y) = 1/2 a^T K_* a + \Psi(y, h(a))$. Here ξ contains the other 22 hidden history coordinates. At $h = 0$, (9) is exactly the flat-volume model already sampled in Sections 4 and 5. All its old-current and bath cross terms remain inside I . The constant quadratic record and orthogonal-current determinants are retained in the normalisation; the nonlinear hidden integral is not replaced by a determinant.

The attachment in (9) implements the inherited distinction between current-dependent history rates and a covariant current residual: history depends on η , while the current defect depends on $\eta - h$. Its exact identification with the source requires the current projection and joint weight described in Sections 2.4 and 6.8. The fixture fixes the endpoint preparation, cycle chart and unit current core used to evaluate that map. No independently reduced D41 penalty is added. The residual-only alternative, with the entire integrand a function of $\eta - h$, is exactly the translation control of Proposition 1.

Let Q be the archived orthonormal spatial cycle frame, T the native participation tangent, $T^T T = H_6$, and $L = H_6^{-1} T^T$. The common endpoint vector v and all generator matrices are taken from the same archive. The finite map is

$$U_e(a) = \exp\left(i \sum_{\ell, b} Q_{e\ell} a_{\ell b} T_b\right), \quad J_e(a) = \frac{v - U_e(a)v}{31}, \quad h(a) = L \text{realify}(Q^T J(a)). \quad (10)$$

The cycle chart is held fixed in this finite test. Moving-chart terms, endpoint variations and the gauge dependence of the physical volume would be required in a different physical attachment. The first derivative $B = Dh(0) = LA_c$ reproduces the archived map. The second derivative $C_{i,\alpha\beta} = \partial_\alpha \partial_\beta h_i(0)$ follows from $\partial_\alpha \partial_\beta J_e = Q_{e\ell} Q_{em} \{T_a, T_b\} v / 62$. Independent differences of the exponentiated links check both derivatives.

6.3 The complete response of this extension

Proposition 5 (nonlinear current susceptibility and gauge pullback). At $h = 0$, define $u = \partial_h \Psi$ and $R = \partial_h^2 \Psi$. For the flat-volume attachment (9),

$$u = -H_6 \langle \eta \rangle, \quad R = H_6 - H_6 \text{Cov}(\eta, \eta) H_6, \quad \Delta K = B^T R B + \sum_i u_i C_i. \quad (11)$$

Proof. Differentiating the current core gives the first two identities by Proposition 2. The chain rule gives the third. Equivalently, translate $\eta = r + h$ inside the integral. Then $u = \langle NI_\eta \rangle$ and $R = \langle NI_{\eta\eta} \rangle - \text{Cov}(NI_\eta, NI_\eta)$. These equivalent expressions include the full nonlinear history and its cross terms.

The translated estimator has much lower variance here than subtracting two order-one covariance terms. Direct finite differences of the log integral along two independent mixed gauge directions reproduce (11). The reconstructed quadratic limit independently agrees with the Schur complement of the joint source Gram. Thus the calculation tests the gauge response itself, rather than inferring it from a scalar history curvature.

For sector A , let $\mathcal{J}_A$ contain its generator directions in all six cycles. Table 4 reports $\text{Tr}_{\mathcal{J}_A} \Delta K / \text{Tr}_{\mathcal{J}_A} K_*$. These dimensionless trace diagnostics do not assume that the fixed prepared background is isotropic. An exact average over the product group's adjoint action removes cross-sector blocks and makes each non-Abelian generator block isotropic; it preserves all three reported traces. This is an invariant projection control, not a selected physical background ensemble. Hypercharge uses $q = 6Y$ consistently in numerator and denominator, so the factor 36 cancels.

Table 4. Total gauge-Hessian correction in parts per million of the corresponding baseline sector trace. Errors are conditional Monte Carlo standard errors.

Background	Colour	Weak	Hypercharge
Original reference	2.299 ± 0.010	2.297 ± 0.010	2.304 ± 0.010
Relaxed closure mode	0.005 ± 0.009	0.003 ± 0.010	0.005 ± 0.010

At the reference point, the mean-score term supplies nearly all of the approximately 2.30 parts per million. At the relaxed background, the total is consistent with zero at the quoted resolution. The uncertainties are conditional on the reported numerical background and omit physical-source uncertainty. They do not establish an exactly vanishing correction. The physical target reductions of several per cent are not obtained by this fixture.

6.4 Allowing the background to follow the probe

Holding the relaxed y fixed is distinct from allowing it to adjust as the gauge probe changes. At a stable conditional mode, let $L_y = \Psi_{yy}$ and $X = \Psi_{yh}$. The local profiled susceptibility is

$$R_{\text{prof}} = R - X^T L_y^{-1} X, \quad \Delta K_{\text{prof}} = B^T R_{\text{prof}} B + \sum_i u_i C_i. \quad (12)$$

This follows by differentiating $\Psi_y(y_*(h), h) = 0$ and substituting $dy_*/dh = -L_y^{-1} X$. We evaluate the full eight-coordinate Hessian for (y, h) from the same integral, including all score covariances. Its two-coordinate background block is positive. The six eigenvalues of R_{prof} are positive in the evaluated return, ranging from 0.000311 to 0.002409.

Table 5. Linear-pullback part of the sector trace correction, in units of 10^{-10} . The unresolved mean-score term in Table 4 is separate.

Closure treatment at the mode	Colour	Weak	Hypercharge
Held at its mode	1.880 ± 0.025	0.680 ± 0.005	14.298 ± 0.090
Allowed to follow the probe	1.514 ± 0.010	0.666 ± 0.004	14.282 ± 0.089

The positive linear-pullback contribution is statistically resolved and remains unequal across sectors after profiling. The full total correction is less precisely determined because it also contains the mean-score term. It would be incorrect to turn Table 5 into a claimed precision measurement of the complete response, or into a prediction of the three force strengths.

6.5 Bounds and the part that preserves the sector ratios

Proposition 6 (uniform nonlinear susceptibility bound). For a convex history cost in (9), flat amplitude volume and $H_6 > 0$, the susceptibility to an independently displaced current mean obeys

$$0 \leq R \leq H_6, \quad 0 \leq B^T R B \leq B^T H_6 B. \quad (13)$$

Proof. The action inside (9) is jointly convex in the hidden coordinates and h . Marginalisation preserves log concavity [9], so Ψ is convex in h and $R \geq 0$. The covariance identity (11) gives $R \leq H_6$. Congruence by B gives the second inequality. The same argument applies after convex minimisation over y , wherever its local profile is smooth.

For the archived B, H_6, K_* , the largest generalised eigenvalue of $(B^T H_6 B, K_*)$ is 7.6448×10^{-6} . Thus this linear-pullback channel cannot acquire an arbitrarily large correction by increasing nonlinear fluctuations or exposure. The bound does not include $\sum_i u_i C_i$, a nonconstant amplitude measure, a changing physical current map or other gauge-dependent source blocks. Equation (11) and Table 4 explicitly retain the term outside the bound.

Corollary 1 (shared mean response preserves sector ratios). On the archived equal-address endpoint preparation, if each three-sector mean vector has the form $u_\mu = c_\mu m$, where $m = (36, 26, 47)^T / 49$, then the invariant projection of $\sum_i u_i C_i$ factorises as a spatial matrix times the representation Gram. It changes a common scale, not the relative sector coefficients.

Proof. Since $H^{-1}m = e_Y$, contraction through $L^T u$ selects the charged participation vector in each current profile. The link second derivative then contains the symmetrised generator product. The gauge-orbit average of the prepared endpoint expectation is $\text{ReTr}(T_a T_b) / 50$, because the preparation has equal norm per internal address. The remaining profile and cycle factors are purely spatial. This gives the stated factorisation. The two independent profile controls reproduce it to residuals below 1.3×10^{-17} .

The nonzero means at the original background are close to this shared direction. This explains the nearly common large entry in Table 4. Unequal force corrections would have to reside in the mean component outside this direction, in the current susceptibility, or in other physical gauge couplings. The calculation separates these possibilities without choosing one to match a measured force.

6.6 Integrating the background fluctuations

If only the gauge coordinates are retained at the chosen coarse scale, the two closure coordinates must also be integrated. Their most probable value is not a substitute for their distribution. Define

$$\bar{\Psi}(h) = -\log \int_{\mathbb{R}^2} e^{-\Psi(y, h)} dy = -\log \int_{\mathbb{R}^{30}} e^{-NI(\beta) - \frac{1}{2}(\eta - h)^T H_6(\eta - h)} d\beta. \quad (14)$$

This uses the same action and flat volume as (9), with no added width or chosen background value. Normalisability follows from Lemma 1. It changes which variables remain resolved; it does not identify the resulting volume with the physical source measure.

Proposition 7 (background fluctuation correction). Let $u(y) = \Psi_h(y, 0)$ and $R(y) = \Psi_{hh}(y, 0)$. With the expectation over the normalised marginal law proportional to $e^{-\Psi(y, 0)}$, the full response is

$$\bar{u} = \langle u(y) \rangle_y, \quad \bar{R} = \langle R(y) \rangle_y - \text{Cov}_y(u(y), u(y)), \quad \Delta \bar{K} = B^T \bar{R} B + \sum_i \bar{u}_i C_i. \quad (15)$$

Proof. Differentiate the first integral in (14) twice using Proposition 2, then apply the link-map chain rule. Equivalently, apply Proposition 5 directly to the thirty-dimensional integral. Fubini's theorem identifies the two orders of marginalisation. The covariance term is negative semidefinite. This does not imply an ordering relative to R evaluated at a single mode, because its average also changes.

The profile correction $X^T L_Y^{-1} X$ in (12) and the covariance in (15) agree for a joint Gaussian. They need not agree for a nonlinear history. Direct sampling of all thirty amplitudes evaluates (15) without approximating the marginal law by a Gaussian. The two closure means are 21.01 and 20.83, with standard deviations 14.92 and 14.48. Their broad distribution is not represented by the conditional mode near (14.04, 13.89). These are amplitude coordinates, not physical length or energy scales.

Table 6. Isolated linear-pullback correction in units of 10^{-10} . The two rows are independent Monte Carlo evaluations; the total also contains the mean-score term.

Background treatment	Colour	Weak	Hypercharge
Local mode profile	1.514 ± 0.010	0.666 ± 0.004	14.282 ± 0.089
Full background integral	1.421 ± 0.009	0.629 ± 0.004	13.497 ± 0.083

Full marginalisation reduces the three linear-pullback entries by 6.2, 5.5 and 5.5 per cent relative to the local mode profile. The six current-susceptibility eigenvalues remain positive, ranging from 0.000294 to 0.002208. This is a resolved change in the isolated susceptibility. The complete gauge response still includes the mean-score term; its exchange consequences and uncertainty are reported in Table 7.

The integral can be differentiated after any constant translation $\beta = q + Dh$. Write E for the injection of the six current coordinates, so $E^T \beta = \eta$, and $S_0 = NI + \eta^T H_6 \eta / 2$. The equivalent score and local Hessian are

$$s_D = D^T \nabla S_0 - H_6 \eta, \quad M_D = ND^T I'' D + (E^T D - I)^T H_6 (E^T D - I). \quad (16)$$

Then $\bar{u} = \langle s_D \rangle$ and $\bar{R} = \langle M_D \rangle - \text{Cov}(s_D)$. Integration by parts gives $\text{Cov}(\nabla S_0) = \langle S_0'' \rangle$ and $\text{Cov}(\nabla S_0, -H_6 \eta) = -EH_6$. Completing the covariance square shows that $D = \langle S_0'' \rangle^{-1} EH_6$ minimises the score covariance among constant lifts. An independent pilot estimates this matrix and fixes it before production evaluation. This changes the estimator, not the action or response. The six mean-score errors fall by factors of 28.2 to 50.3 compared with the direct estimator $-H_6 \eta$ on the same production samples. Finite differences of the log integral agree with the lifted first and second derivatives. Thus the improvement does not come from dropping the second link-map derivative or fitting a force target.

6.7 From stiffness to interaction strength

Let $\bar{K} = K_* + \Delta \bar{K}$ and $t = B^T \bar{u}$. The local quadratic gauge action is $t^T a + a^T \bar{K} a / 2$. Coupling a test current j to a and performing this Gaussian integral gives

$$\log \frac{Z(j)}{Z(0)} = -j^T \bar{K}^{-1} t + \frac{1}{2} j^T \bar{K}^{-1} j, \quad \varepsilon = \bar{K}^{-1}. \quad (17)$$

Thus the connected test-current response is the inverse stiffness, even when the expansion point has a small nonzero linear term. The corresponding quadratic stationary point is $a_* = -\bar{K}^{-1} t$. Equation (17) is exact for this local quadratic action. It is not an exact integration of the full nonlinear gauge field. The hidden-variable integrations used to obtain $\bar{K}$ have already been performed; averaging inverses of conditional Hessians would define a different operation.

Proposition 8 (exchange bound and scalar extraction). For the isolated positive linear contribution $P = B^T \bar{R} B$, if $0 \leq P \leq \alpha K_*$, then

$$\frac{1}{1 + \alpha} K_*^{-1} \leq (K_* + P)^{-1} \leq K_*^{-1}. \quad (18)$$

After invariant projection, write the three generator blocks as $K_A = z_A S_A$, with $z = (6, 13/2, 378)$ in the primitive hypercharge convention. The ratios of the sector exchange coefficients are independent of the same nonzero spatial test current j if and only if the positive matrices S_A are mutually proportional. Equivalently, the matrix with columns $\text{vec}(S_A)$ has rank one.

Proof. Positive matrix inversion reverses the Loewner order, giving (18). For the second statement, exchange in sector A is $j^T S_A^{-1} j / z_A$. A constant ratio for every j implies proportional quadratic forms by polarisation, hence proportional matrices; the converse is immediate. The rank test expresses the same proportionality in vectorised form.

Proposition 6 supplies $\alpha = 7.6448 \times 10^{-6}$, so this isolated channel reduces any quadratic test-current exchange by at most 7.645 parts per million. The bound is independent of the nonlinear fluctuation amplitude within the stated convex family. It excludes neither the mean-score term nor additional physical source couplings. It makes no statement about a nonconstant physical volume.

The archived baseline has $S_A = S$. We evaluate the invariantly projected result as a control and define $X_A = S_A^{-1}$, $X_0 = S^{-1}$. For an arbitrary spatial current, the fractional change in its normalised sector exchange is

$$\epsilon_A(j) = \frac{j^T (X_A - X_0) j}{j^T X_0 j}. \quad (19)$$

Its extrema are the generalised eigenvalues of $(X_A - X_0, X_0)$. Table 7 reports the trace average $\text{Tr}[S(X_A - X_0)]/6$; the supplement retains the full matrices, extrema and ratio tests. This definition differs from the unweighted stiffness traces in Tables 4 to 6 and includes the required matrix inversion.

Table 7. Mean fractional exchange change in parts per million after the stated invariant projection. The total includes both response terms. Errors are conditional Monte Carlo standard errors.

Sector	Mean exchange change (parts per million)
Colour	0.013 ± 0.009
Weak	0.013 ± 0.009
Hypercharge	0.012 ± 0.009

Using the same trace averages, the fractional departures of the colour-to-weak and hypercharge-to-weak exchange ratios from their baseline values are $(0.40 \pm 1.57) \times 10^{-9}$ and $(-1.06 \pm 2.01) \times 10^{-9}$. The shared uncertainty partly cancels, but neither departure is statistically resolved. These are conditional finite-model ratio tests, not measured force ratios. The total exchange changes are small compared with the proposed per-cent force adjustments. The supplement reports central spatial extrema with their jackknife errors, rather than treating the extrema as confidence limits. The matrix calculation passes inverse, dual-source coordinate-change and hypercharge-conversion checks below 10^{-12} . The positive linear term also admits an exact six-by-six Woodbury reduction of the 72-direction inverse, verified against direct inversion; the full mean-score term is retained separately.

The rank and eigenvalue tests are provided for the finite six-cycle carrier. Their central residuals are not statistical detections of nonfactorisation, and a finite-cell shape dependence would not by itself exclude scalar couplings in a long-wavelength limit. Extracting physical couplings requires the source-selected quotient, spatial normalisation and matching scale. The VERSF quantum-completion framework likewise distinguishes coupling boundary data from their subsequent loop transport [10].

6.8 Physical matching

For the complete physical quotient, write $\mathcal{V}(a, z) = \mathcal{S}(a, z) - \log m_{\text{phys}}(a, z)$. The required return remains

$$(G_{\text{phys}})_{ab} = \langle \mathcal{V}_{ab} \rangle - \text{Cov}(\mathcal{V}_a, \mathcal{V}_b). \quad (20)$$

The gauge-induced current, record and closure derivatives, second derivatives of composite currents, moving charts and physical measure all belong in (20). Section 6.3 executes this identity for (9) and (10); it does not identify that declared extension with the complete source. In particular, the gauge dependence of the retained old defect coordinates is represented by their specified integration here, not by the full physical defect map.

The source-to-fixture comparison can be stated without demanding a new microscopic mechanism. Let $d\nu_{\text{src},a}$ be the unnormalised weight of the complete source on its unresolved physical quotient at fixed retained gauge coordinate a . Let $\mathcal{P}_a$ map that quotient to the thirty amplitudes β , including the background and cycle projection. Its pushforward integrates the remaining unresolved variables. The tested full gauge weight is

$$d\nu_{\text{test},a}(\beta) = e^{-\frac{1}{2}a^T K_* a - NI(\beta) - \frac{1}{2}(\eta - h(a))^T H_6(\eta - h(a))} d\beta. \quad (21)$$

Criterion 1 (identification of the weighted reduction). If $(\mathcal{P}_a)_* \nu_{\text{src},a} = c \nu_{\text{test},a}$ in a neighbourhood of the reference gauge field, with $c > 0$ independent of a , then the source and tested free energies have identical gauge derivatives there. This is a sufficient identification of the finite gauge response. It includes the retained gauge baseline, all eliminated determinants and the absolute partition weight; equality of separately normalised probability densities would not suffice.

Proof. A pushforward preserves the total integral. The two partition integrals therefore differ only by c , so their negative logarithms differ by a gauge-independent constant.

More generally, suppose the projected source weight is absolutely continuous relative to (21), and write its density ratio as $w(a, \beta)$. Let $\langle \cdot \rangle_{\text{test},a}$ denote expectation in the normalised tested law. Then

$$\begin{aligned} \Gamma_{\text{src}}(a) &= \Gamma_{\text{test}}(a) - \log \langle w(a, \beta) \rangle_{\text{test},a}, \\ G_{\text{src}} - \bar{K} &= -\nabla_a^2 \log \langle w \rangle_{\text{test},a} \Big|_{a=0}. \end{aligned} \quad (22)$$

This follows by taking the ratio of the two partition integrals. It gives a necessary and sufficient test for equality of their Hessians under the stated regularity and common gauge-coordinate assumptions: the last Hessian in (22) must vanish. Equality of their first derivatives requires the corresponding gradient to vanish as well. A nontrivial source weight can therefore preserve the force Hessian, but this must be checked through the integrated weight. The equation does not license choosing w to fit measured forces.

A first local check uses the inherited Gaussian source. On a positive quotient, if its precision is Q_{src} and the linear coarse map is P , the projected covariance and precision are

$$C_\beta = P Q_{\text{src}}^{-1} P^T, \quad Q_\beta = C_\beta^{-1}. \quad (23)$$

The second inverse requires a full-row-rank map on the retained image. In coordinates splitting retained and eliminated variables, it is the ordinary Schur precision, including cross blocks. Comparing it with Q_N tests the local amplitude law; comparing $D^2 I$ with the archived Gram tests the history part alone. Beyond this check, the projected source must reproduce the current map and its two gauge derivatives, the context weights and the nonlinear volume in the same chart. A moving projection belongs inside $\mathcal{P}_a$ and w , rather than being omitted from the derivatives.

Equations (21) to (23) turn source matching into an explicit calculation. If the source returns a computable w with adequate overlap, the existing samples can evaluate its correction by reweighting with propagated uncertainty; poor overlap requires direct sampling of the source weight. If the projected source assigns weight outside the tested support, this reweighting is unavailable and its integral must be constructed directly. No numerical source weight is supplied by the present calculation. The established result is the finite response and exchange kernel, together with the exact test that transfers it to the inherited source construction.

7 The scalar parent: shared reference, preservation and fluctuations

7.1 Shared normalisation in the complete quadratic fixture

A complementary test concerns the reference against which a reduced response is normalised. It uses the archived 226-coordinate master model [13,14], containing closure, current realisation, transport conservation, record composition, bond compatibility and scalar completion, together with the potential Hessian. Its physical quotient has dimension 187. We retain eighteen cycle coordinates, six in each of three scalar channels, and eliminate the other 169. This fixture is distinct from the thirty-amplitude nonlinear integral and the twelve-generator gauge attachment in Sections 2 to 6.8.

CLSN-1 Sections 17.100 and 17.101 give the common-action family and its canonical-history weights [12]. Writing x and y for link and record phase variations on the cycle quotient, and $L = B_C^\top B_C$, its active quadratic cost is

$$S_{CRB}(x, y) = \frac{a}{2} x^\top L x + \frac{b}{2} y^\top L y + \frac{c}{2} \|x - y\|^2, \quad a = b = \frac{12}{31}, \quad c = \frac{21}{124}. \quad (24)$$

The full calculation uses $\mathcal{H} = G_V + J^\top W J$, with J the archived six-defect Jacobian and G_V its potential Hessian. The diagonal weights are $w_C = w_R = 12/31$, $w_B = w_T = 21/124$ and $w_J = w_H = 75/434$. These define the canonical orthogonal-defect restriction. They do not set the physical mixed blocks $l_x^\top w_{\text{prim}l_Y}$ to zero by derivation.

Proposition 9 (preservation of the reference and relaxed response). On this archived quadratic branch, the shared reference $y = x$ and the relaxed response are

$$K_{\text{ref}} = (a + b)L, \quad y_* = c(bL + cI)^{-1}x, \quad K_{\text{rel}} = aL + cI - c^2(bL + cI)^{-1}. \quad (25)$$

They are unchanged by any positive metric confined to the current, conservation and completion residuals, including cross terms among those residuals. At fixed $m = (x + y)/2$, the full quadratic minimum is the shared configuration $x = y = m$.

Proof. On a cycle variation, the endpoint lift $\delta\rho = 0$ and current lift $\delta j = -\kappa x$ cancel the linear current-realisation and conservation residuals. The completion residual and potential Hessian also vanish on this lift. For the reference, the bond residual vanishes because $y = x$. For the relaxed configuration, minimising (24) over y gives (25), while the same lift cancels the additional residuals. Their positive cost therefore cannot lower or change the minimum. At fixed mean, write $x = m + d$ and $y = m - d$. Since $a = b$, (24) becomes $am^\top L m + ad^\top L d + 2c \|d\|^2$, uniquely minimised at $d = 0$. The cancelled-residual lift attains this bound in the full parent.

Both assertions are checked by independent full-matrix elimination; the fixed-mean check uses the constrained covariance of all 187 quotient coordinates. Gaussian integration retains the auxiliary determinant, with $\log\det\mathcal{H}_{\text{quotient}} = \log\det\mathcal{H}_{\text{aux}} + \log\det K_{\text{rel}}$. On this constant quadratic parent the auxiliary determinant is independent of x . The fixed-mean assertion depends on the equal diagonal

weights and the stated restriction on mixed blocks. Section 7.3 instead defines the committed reference by imposing full edge agreement before elimination.

The cycle spectrum is (1,2,2,4,4,5). The ratio of relaxed to reference stiffness on an eigenmode is

$$r(\lambda) = \frac{k_{\text{rel}}(\lambda)}{(a+b)\lambda} = \frac{8\lambda + 7}{16\lambda + 7}. \quad (26)$$

Table 8. Remaining quadratic response relative to the shared reference, using the inherited sector-to-mode assignment of CLSN-1 Section 17.101.

Sector assignment	Cycle eigenvalue	Remaining response
Colour	4	39/71 = 54.93%
Weak	5	47/87 = 54.02%
Hypercharge	1	15/23 = 65.22%

These are stiffness fractions within the stated scalar-channel fixture. They are not the small additive corrections in Tables 4 to 7, the D41 participation reductions, or percentages of observed force strengths. No multiplication of these different returns is justified without a common physical attachment. The sector assignment is inherited, rather than selected anew by the 226-coordinate test.

7.2 Finite changes and mixed source terms

The quadratic preservation theorem has two calculable boundaries. First, construct a finite shared curve $U_e = \exp(i\epsilon v_e)$ and $\Phi_e = U_e$, where v is a normalised cycle vector. Hold the endpoint amplitudes at unity and the scalar completion at its archived vacuum. Bond alignment and the norm potential are exact on this curve. If $D_U \rho$ is the covariant endpoint difference, its adjoint is the divergence, and

$$\text{Re}\langle \rho, D_U^\dagger j \rangle = \text{Re}\langle D_U \rho, j \rangle. \quad (27)$$

Thus exact current realisation $j = \kappa D_U \rho$ and exact conservation $D_U^\dagger j = 0$ would imply $\kappa \|D_U \rho\|^2 = 0$. A generic finite holonomy acting nontrivially on the chosen matter state cannot keep both residuals zero, despite their simultaneous first-order cancellation.

Minimising the combined current and conservation cost over j at each point gives a positive fourth-order term. With q_s the sum of v_e^2 over edges incident at vertex s , and L_0 the wheel vertex Laplacian,

$$S_{JT}(\epsilon) = d_{JT} \epsilon^4 + O(\epsilon^6), \quad d_{JT} = \frac{w_J w_T}{8} q^\top (w_J I + w_T L_0)^{-1} q. \quad (28)$$

For eigenvalues 1,2,4,5, the coefficients are respectively 0.01235328, 0.01224106, 0.01313267 and 0.01292785. Halving ϵ from 0.1 to 0.05 reduces these costs by factors between 15.9790 and 15.9806, approaching the fourth-order factor 16. This is a restricted finite-curve calculation with fixed endpoint and record backgrounds, not a full nonlinear minimisation. As Proposition 2 makes explicit, higher-order terms can contribute to the coarse quadratic response after fluctuations are integrated.

Second, a mixed term between a cancelled residual and a nonzero bond, closure or composition residual lies outside Proposition 9. The canonical eight-score history matrix has transport-bond correlation singular value 1/7 [3, Section 14F.4]. To test preservation independently of the unconstructed physical embedding, we construct positive metrics pairing a vertex-transport doublet with an edge-cycle bond doublet. The pairings respect the wheel's rotations and reflections and retain the linear gauge nulls. Their shared-reference cost is unchanged, but auxiliary elimination gives

$$c_{\text{eff}} = c \left(1 - \frac{1}{49}\right) = \frac{36}{217} \quad (29)$$

on the paired doublet. Pairing the first spatial doublet changes its $\lambda = 2$ fraction from 23/39 to 10/17. In a separate test, pairing the second doublet changes its $\lambda = 4$ fraction from 39/71 to 17/31; the other modes remain unchanged in each test.

These are mathematical controls. Their field-level pairings are constructed, not supplied by the physical history law. They establish that positivity, wheel symmetry and preservation of the shared reference do not alone protect the relaxed response. The complete physical overlap blocks must be evaluated on the same carrier before either correction can be interpreted physically. In the scalar representation, the transport and bond residuals carry different gauge charges, so this bare pairing is not a gauge-invariant candidate for the physical metric. The invariant composition–bond control in Section 7.3 changes relaxation and the fixed-mean minimum while preserving the constrained reference.

7.3 Primitive event and the normalisation reference

Primitive Change Normalization fixes one unit of cost for the native E2 event and its specified gauge image [12, Section 17.106]. The bond defect compares the physical link with the polar record map [13, Section 16]. Fact-Lock fixes the identified event-local record, with renewal of the active event carrier [12, Sections 17.82 to 17.85]. These statements do not themselves identify that record with the polar record on every edge.

Premise (agreement of the committed polar record). On the closure under consideration, the polar record agrees with the link on every edge of a committed state. In the linearised scalar model this means equality of their complete edge-phase variations, including the gradient components. This is an attachment premise, not a consequence established here from Fact-Lock. It imposes the bond condition, not the vanishing of every master-action defect.

Proposition 10 (constrained reference and endpoint difference). Impose full edge-phase agreement before eliminating auxiliary variables. On a fixed positive quotient, let K_{com} be the Hessian of the resulting constrained quadratic action on the retained cycle coordinate z . Then

$$\begin{aligned} S_{\text{com}}(z) &= 1/2 z^{\top} K_{\text{com}} z, \\ S_{\text{com}}(z + e) - S_{\text{com}}(z) &= z^{\top} K_{\text{com}} e + 1/2 e^{\top} K_{\text{com}} e. \end{aligned}$$

On the canonical branch, $K_{\text{com}} = (a + b)L = K_{\text{ref}}$. Equality of a and b is unnecessary for this constrained identity. The increment-only expression is the cost about the zero reference, or the centred quadratic response. An arbitrary endpoint action difference is not, by itself, the dynamical work or probability cost of a primitive transition.

Proof. Substitute equality of the two edge-phase fields into the parent action, then minimise over its remaining auxiliaries. The Schur complement of this constrained parent defines K_{com} . Expanding the quadratic form gives the endpoint identity. On the canonical branch the bond residual vanishes, the current and conservation residuals have a cancelling lift, and the completion and potential terms attain zero. The remaining closure and composition cost is $1/2 (a + b)z^{\top} Lz$.

The order of these operations matters. Eliminating the link and record gradients independently, and only then setting their cycle coordinates equal, allows an uncanceled gradient bond residual. If the unconstrained cycle Hessian is

$$\begin{aligned}
K_{\text{cyc}} &= \begin{pmatrix} A & X \\ X^\top & B \end{pmatrix}, \\
K_{\text{rel}} &= A - XB^{-1}X^\top, \\
K_{\text{shared,cyc}} &= A + B + X + X^\top,
\end{aligned}$$

the last matrix need not equal K_{com} . With $x = m + d$ and $y = m - d$, the gradient in the difference coordinate at $d = 0$ is

$$\nabla_d S_{\text{cyc}}(m + d, m - d)|_{d=0} = (A - B + X - X^\top)m.$$

Thus the fixed-mean minimum need not be shared when a mixed block is present, or when $a \neq b$. Holding the cycle record fixed gives $K_{\text{lock}} = A$ in this reduced chart. On the canonical branch,

$$\frac{k_{\text{rel}}(\lambda)}{k_{\text{com}}(\lambda)} = \frac{8\lambda + 7}{16\lambda + 7}, \quad \frac{k_{\text{lock}}(\lambda)}{k_{\text{com}}(\lambda)} = \frac{1}{2} + \frac{7}{32\lambda}.$$

These distinguish relaxed, fixed-record and constrained shared responses. Assigning them to pre-commitment, locked-record and Fact-to-Fact physical processes additionally requires the source event map.

Proposition 11 (preservation under constant invariant weighting). Consider the linearised scalar-channel wheel model at the common-zero background, with the canonical potential, fixed closure and composition blocks aI and bI , and a positive constant complex-linear metric on the residuals. Require invariance under every vertex gauge transformation. If the closure-completion block W_{CH} vanishes, the constrained reference remains $K_{\text{com}} = (a + b)L$ under all remaining admitted mixed blocks.

Proof. In this scalar representation, closure and completion are neutral. A composition residual on the triangle $(h, i, i + 1)$ has charge $g_{i+1} - g_h$, a bond residual on an edge has charge $g_t - g_s$, and current and conservation residuals carry their endpoint vertex charges. Constant complex-linear cross blocks therefore pair closure with completion, composition with the matching chord bond, and current with conservation. On the full agreement constraint the bond residual is identically zero, so the composition-bond term drops out. When $W_{CH} = 0$, all other auxiliary residual costs are positive and have a simultaneous zero-cost lift. Their minimisation leaves exactly the fixed closure and composition quadratic form.

For complex residual coordinates, an intertwiner with coefficient $r + is$ is represented over the reals by $\begin{pmatrix} r & -s \\ s & r \end{pmatrix}$. Equal-charge selection and this complex structure must both be respected. The verification condition is $[W, J_v] = 0$ for every vertex generator J_v ; commutation with the common complex structure alone is insufficient. Equivalently, the real metric obeys $D(g)^\top W D(g) = W$ for every allowed gauge rotation.

Within this specified class, the closure-completion block is the remaining exception. Eliminating completion changes the closure block by

$$W_{CC} \mapsto W_{CC} - W_{CH}(W_{HH} + V_H)^{-1}W_{HC}.$$

On a separated matched mode this reads $a \mapsto a - w_{CH}^2/(w_H + v_H)$. It is not a general scalar replacement for coupled modes. The proposition does not classify background-dependent covariant metrics or the full non-Abelian typed carrier. Coefficients that transform with background fields can permit additional invariant terms; neither their absence nor their physical weighting is derived by this constant-metric control.

Table 9 compares the canonical model with a composition-bond block of strength $0.03I_2$ on each matching complex pair [21]. Full constrained reduction preserves $K_{\text{com}} = (a + b)L$ in both cases. The last column shows what changes if only the cycle coordinates are equated after elimination.

Table 9. Relaxed response and order-of-constraint control. Entries are ratios of quadratic forms on the indicated unit wheel mode.

Cycle eigenvalue	Canonical relaxed / committed	Mixed relaxed / committed	Mixed cycle-only shared / committed
1	0.652174	0.651532	0.993136
2	0.589744	0.586255	0.996568
4	0.549296	0.546335	0.998284
5	0.540230	0.537627	0.998627

The eigenvalue-two and eigenvalue-four entries each occur twice. One hundred positive invariant random metrics, with the fixed closure and composition blocks and $W_{CH} = 0$, preserve the constrained reference to below 3×10^{-16} relative Frobenius error [21]. The composition–bond example has a nonzero fixed-mean gradient and changes the relaxed fractions, despite preserving the constrained reference. These checks concern the stated scalar model, not a returned physical metric.

Corollary 2 (bounded change of the constrained reference). Retain the constant metric and full-edge constraint of Proposition 11, holding the closure and composition blocks $W_{CC} = aI$ and $W_{RR} = bI$ fixed, but allow $W_{CH} \neq 0$. Assume the completion potential has Hessian $V_H \succcurlyeq 0$ and no additional mixed potential term. Let $F = C_f Q$ map the six cycle coordinates to triangle circulations, so $L = F^\top F$. Define

$$\Delta_C = W_{CH}(W_{HH} + V_H)^{-1}W_{HC}, \quad \chi^2 = \|\Delta_C\|_2/a, \quad \eta = \frac{a}{a+b}\chi^2.$$

Then $0 \leq \chi^2 < 1$, and the exact constrained reduction obeys

$$K_{\text{com}} = K_{\text{ref}} - F^\top \Delta_C F, \quad (1 - \eta)K_{\text{ref}} \preccurlyeq K_{\text{com}} \preccurlyeq K_{\text{ref}},$$

$$K_{\text{ref}}^{-1} \preccurlyeq K_{\text{com}}^{-1} \preccurlyeq (1 - \eta)^{-1}K_{\text{ref}}^{-1}.$$

Proof. The positive closure–completion principal block gives $W_{CH}W_{HH}^{-1}W_{HC} < aI$. Adding a nonnegative completion potential can only decrease this Schur subtraction. The full agreement constraint removes the composition–bond block, while the other residuals admit their cancelling lift as in Proposition 11. Eliminating completion therefore gives the displayed formula. Congruence by F gives the bounds, and inversion reverses the positive-matrix order.

For the canonical $a = b$, every committed quadratic response remains strictly above half its reference value, and its inverse remains strictly below twice the reference inverse. The bound is sharp in this metric class: with $V_H = 0$, $W_{HH} = w_H I$ and $W_{CH} = t\sqrt{aw_H} [I_6 \ 0]$, where $0 \leq t < 1$, one obtains $K_{\text{com}} = (1 - t^2/2)K_{\text{ref}}$. The values $t = 0.2, 0.5, 0.8$ give reference fractions 0.98, 0.875, 0.68, respectively. These are controls of the bound, not selected physical couplings. More generally, a returned value of χ directly bounds the uncertainty: $\chi \leq \sqrt{0.02}$ guarantees at most a one per cent decrease of this reference when $a = b$.

Because F is invertible on the six-cycle quotient, the change is a common multiplier if and only if Δ_C is proportional to I_6 . Its rank is exactly the rank of W_{CH} . Thus a returned closure–completion block determines both the size of the change and which cycle combinations it affects. Scalar normalisation alone cannot describe a non-scalar subtraction. These conclusions concern the constrained scalar response; its identification with a physical gauge stiffness remains the attachment question of Criterion 1.

Lemma 2 (ordering of relaxation and agreement). For the same positive quadratic parent, at a fixed retained link-cycle coordinate,

$$0 < K_{\text{rel}} \leq K_{\text{shared,cyc}} \leq K_{\text{com}}.$$

Proof. The admissible configurations are nested: arbitrary record variation, equality of link and record cycles, and equality on every edge. Minimising the same positive action over these successive restrictions gives the ordering. Explicitly, use the full edge mismatch $d = \phi - \theta$ and eliminate the other auxiliaries first. Write the remaining Hessian on (x, d) as

$$\hat{\mathcal{H}} = \begin{pmatrix} C & D \\ D^\top & E \end{pmatrix}, \quad K_{\text{com}} = C, \quad K_{\text{com}} - K_{\text{rel}} = DE^{-1}D^\top \geq 0.$$

Here $E > 0$ on the stated quotient. Equality of relaxed and committed responses holds exactly when $D = 0$. Equating only cycles permits the gradient part of d to relax and gives the intermediate matrix. The identity provides a direct diagnostic of the effect of enforcing agreement before elimination.

The full 74-field matrices verify both results [22]. Five matched-mode controls and a separate ensemble of 100 positive invariant metrics with nonzero closure-completion blocks reproduce the committed formula, full-mismatch Schur identity and intermediate reduction to below 4×10^{-16} relative Frobenius error. Each metric commutes with all seven vertex generators; the smallest ordering margin is -1.1×10^{-15} , consistent with round-off. The controls include noncommuting mixed blocks and nonnegative completion potentials. These are quadratic bounds; field-dependent volume terms and the nonlinear determinant correction in Section 7.4 require their own reduction.

The remaining identification is precise: derive the full-edge agreement premise for the actual event-local record and identify the PCN specified gauge image with this constrained response. Section 17.107C describes a momentum impulse with continuous connection and bath coordinates; it neither proves the premise nor supplies an endpoint displacement or a transition cost from the quadratic state action alone. The closure-only sign forcing on the separate Abelian stationary slice [15] likewise does not decide this attachment.

The preservation theorem supplies a conditional local input to Criterion 1. Field-dependent determinants, the physical mixed blocks and nonlinear measure still enter the complete reduction through (22), as Sections 7.4 to 8.2 make explicit. PCN's unit remains fixed throughout; no additional conversion constant is introduced.

7.4 Fluctuations of the complete nonlinear auxiliary block

Proposition 9 concerns the Hessian of the relaxed action. It does not imply that the number and widths of the available auxiliary arrangements are independent of the retained field. The nonlinear residuals in the archived master action [13] allow that distinction to be evaluated. We keep the same six defect families, potential, canonical diagonal weights and flat background as Section 7.1. The calculation uses their full exponential link and record dependence, rather than replacing every residual by its first derivative.

The finite model is specified on the fixed linear quotient chart used by the archive, with flat auxiliary coordinate volume. The three scalar channels separate exactly, and the four scalar-completion coordinates contribute a field-independent factor. Retaining six cycle coordinates in each channel leaves 55 auxiliaries per channel and four completion auxiliaries, hence 169 in total. This is a local calculation on that chart. It does not identify the chart volume with the physical operational measure, reinstate the removed directions as nonlinear physical modes, or establish convergence of an unrestricted integral over the entire configuration space.

Let x denote the eighteen retained coordinates and let $\eta_*(x)$ be the nearby auxiliary minimum. Define $S_*(x) = S(x, \eta_*(x))$ and $\mathcal{H}_\eta(x) = S_{\eta\eta}(x, \eta_*(x))$. A smooth local cutoff equal to one around these minima defines an unambiguous local integral. Introduce a formal concentration Ω to order its expansion. It is distinct from the history exposure N in (2), and is not a selected physical exposure, the number of wheel vertices, or an executed spatial blocking factor.

Proposition 12 (response from eliminated fluctuations). For the stated local integral, with positive auxiliary Hessian, the action per concentration has the expansion, up to terms independent of x ,

$$\frac{\Gamma_\Omega(x)}{\Omega} = S_*(x) + \frac{1}{2\Omega} \log \det \mathcal{H}_\eta(x) + O(\Omega^{-2}), \quad K_\Omega = K_{\text{rel}} + \frac{K_\Pi}{\Omega} + O(\Omega^{-2}). \quad (30)$$

Here $K_\Pi = D_x^2 \log \det \mathcal{H}_\eta / 2$, with every derivative taken along the relaxed background. For a retained direction v ,

$$v^\top K_\Pi v = \frac{1}{2} \text{Tr}(\mathcal{H}_\eta^{-1} \dot{\mathcal{H}}_\eta - \mathcal{H}_\eta^{-1} \dot{\mathcal{H}}_\eta \mathcal{H}_\eta^{-1} \dot{\mathcal{H}}_\eta). \quad (31)$$

Proof. Gaussian integration about the nondegenerate local minimum gives the determinant in (30); higher local cumulants first enter the action per concentration at order Ω^{-2} . Differentiating the log determinant twice gives (31). The dependence of η_* on x is included in both derivatives. Thus a constant Hessian at the reference point does not imply a constant auxiliary determinant on neighbouring backgrounds.

The current part can be checked independently using the corpus's exact current-conservation reduction [16, Proposition 53]. With D_U the covariant incidence and $\kappa = 1$ on this fixture,

$$C_J(U) = w_J I + w_T D_U D_U^\dagger, \quad \frac{w_T}{w_J} = \frac{49}{50}, \quad \Gamma_J^{\text{det}}(U) = \log \det C_J(U). \quad (32)$$

The coefficient is one because the current is complex: the equivalent real integral contributes one half of the log determinant of its doubled real precision. Sylvester's determinant identity reduces the field-dependent calculation to the seven-vertex matrix $I + (49/50) D_U^\dagger D_U$. A separate analytic link differentiation verifies its Hessian and gauge-null directions. This exact elimination formula is inherited; its evaluation at these weights and comparison with the complete auxiliary determinant are the additional calculation.

Table 10. Curvature of the eliminated determinants on a unit link-cycle direction. The last column is the complete fluctuation coefficient divided by the unchanged reference stiffness $2a\lambda$. These are coefficients of the formal expansion, not physical force corrections.

Cycle eigenvalue	Current only	All auxiliaries	Relative coefficient
1	0.125245	-0.250406	-0.323441
2	0.161817	-0.186443	-0.120411
4	0.194707	-0.110390	-0.035647
5	0.202957	-0.089915	-0.023228

The eigenvalue-two and eigenvalue-four entries each occur twice. All six modes are evaluated, including their off-diagonal curvature. The full fluctuation matrix is diagonal in the wheel mode basis to the numerical resolution, and the doublet degeneracies hold. The current contribution is positive, while the

remaining variables more than cancel it, making the complete coefficient negative in every cycle sector. Retaining only the current determinant would therefore give the wrong sign on this parent.

The auxiliary minima are computed from analytic action gradients and Hessians, including the residual-times-second-derivative terms away from the common zero. The smallest auxiliary Hessian eigenvalue in the test is above 0.17020 and the largest stationarity residual is below 2×10^{-13} . Four perturbation sizes from 0.02 to 0.0025 and Richardson extrapolation determine the displayed curvatures. The independent trace formula (31), direct log-determinant differences, exact current split and recovered tree response agree within the archived tolerances [17]. This closes the leading local fluctuation calculation for the entire specified auxiliary block. It is not an evaluation of the expansion remainder at $\Omega = 1$.

7.5 The part that changes the relative force response

The existing sector-to-mode assignment maps colour, weak and hypercharge to cycle eigenvalues 4, 5 and 1 [12]. On this conditional attachment, the response fractions in Section 7.1 acquire the explicit leading terms

$$\begin{pmatrix} r_c(\Omega) \\ r_w(\Omega) \\ r_Y(\Omega) \end{pmatrix} = \begin{pmatrix} 39/71 \\ 47/87 \\ 15/23 \end{pmatrix} - \frac{1}{\Omega} \begin{pmatrix} 0.0356468 \\ 0.0232280 \\ 0.3234409 \end{pmatrix} + O(\Omega^{-2}). \quad (33)$$

The denominators remain the pre-existing shared reference. No extra conversion is introduced. The three corrections are not proportional to the three starting fractions, so this effect cannot be absorbed into a common rescaling of that candidate response. Equation (33) is an asymptotic statement on the scalar fixture; it does not assign three physical Standard Model coefficients or license substitution of $\Omega = 7$ merely because the wheel has seven vertices.

The corpus already supplies a precise way to test the relative part [18]. In conventional hypercharge coordinates, set $Z = (6, 13/2, 21/2)$ and $n = (466, -687, 159)$. Then $n \cdot Z = 0$ and $n \cdot (-7, -19/6, 41/6) = 0$. The source-side functional on the candidate is

$$\mathcal{T}_\Omega = \sum_A n_A Z_A r_A(\Omega) = 212.238782 - \frac{535.928327}{\Omega} + O(\Omega^{-2}). \quad (34)$$

This carries the evaluated correction into the same algebraic contrast used by the relative-gauge test. In particular, its nonzero coefficient proves that the fluctuation term has a component outside the plane spanned by the unscreened charge Gram and one-loop scale running. A common multiplier of the already screened candidate would also change its nonzero $\mathcal{T}$; the stronger statement that (33) is not such a multiplier follows separately from the unequal ratios of its correction to its starting fractions. Neither Ω nor an overall factor is selected by comparing (34) with measurements.

The remaining measure dependence can also be stated explicitly. For a smooth positive density $\mu(x, \eta)$ independent of Ω on the same local chart, the leading fluctuation term in (30) becomes

$$K_{\text{fl}}^\mu = D_x^2 \left[\frac{1}{2} \log \det \mathcal{H}_\eta(x) - \log \mu(x, \eta_*(x)) \right]. \quad (35)$$

Consequently the physical measure must be carried with the source, as required by Criterion 1. A density that changes the leading exponential action requires that action to be reduced again. The calculation establishes a non-common response from the complete nonlinear auxiliary block in the declared local model. Physical promotion still requires the primitive-event attachment, the actual typed metric and operational measure, the faithful generator carrier, and a controlled coarse-grained limit. Its

new content is the evaluated fluctuation term and its relative-gauge contraction, rather than another adjustable normalisation or a fit to the weak angle.

8 The source law required for matching

8.1 From primitive events to the complete history law

The finite integrals above do not yet evaluate the complete physical history weighting. Their local response is established, but the source law that must enter Criterion 1 is a law for the jointly changing fields and retained records. The primitive construction in PJCK-1 and CLSN-1 specifies how a positive event measure produces that law [12, Section 17.79; 19]. It also locates the part that is not returned by the present calculation.

Let z be the complete configuration, including the carried record and any memory needed to define the next transition. Write α for a discrete primitive mark and h for its joint configuration increment. A finite positive event measure can be disintegrated into its mark marginal m_z and a conditional probability v_z :

$$\Lambda_z(d\alpha, dh) = m_z(d\alpha) v_z(dh | \alpha), \quad F_{U,J}((U, J), (V, \eta)) = (VU, \eta + VJ). \quad (36)$$

The second formula is the inherited link/current part of the update. The record, matter, support and completion updates must belong to the same full map. Pushing the event measure through that map gives the off-diagonal raw transfer; its source survival terms and reference measure must also be retained. Perron/Doob normalisation then supplies the stochastic history kernel. Disintegration introduces no independently adjustable measure: it identifies the conditional allocation of changes that the single primitive source must determine.

The supplied discrete wheel probabilities and composition law do not by themselves evaluate v_z on the full configuration carrier. An equivalent return would be an explicit map from the primitive event carrier into that configuration, carrying the already fixed primitive measure with it. PCN fixes the common primitive action unit, and the canonical scalar coefficients in Section 7.1 remain inherited. Neither result is replaced by a new overall multiplier. The unfinished task is the joint change law and its physical embedding, including the background on which it is evaluated.

HMGBG gives the corresponding operational heat transfer [3, Sections 56A.1 to 56A.5]:

$$\mathbb{T}_\tau = M_{e^{-\tau V_{\text{ref}}/2}} e^{-\tau L_{\text{op}}} M_{e^{-\tau V_{\text{ref}}/2}}. \quad (37)$$

Here M_f denotes multiplication by f . The operational generator, reference potential and full transported defect increments must come from the same source. The reference potential is specified upstream of an independently chosen defect metric. Consequently, replacing the operational geometry by the flat quotient chart and the reference potential by the already weighted residual-square action defines an additional model. A Gaussian heat calculation with those replacements cannot establish physical source matching.

The inherited continuous proposal supplies a minimum-norm displacement for a given defect increment [4, Section 64.4.1]. It does not become autonomous evolution simply by substituting the instantaneous defect for that increment. The tested substitution produces a projector drift with only one nonzero relaxation rate, inconsistent with the inherited endpoint rates [20, Section IV.16]. This excludes that shortcut; it does not exclude the complete source dynamics.

The physical history length M is distinct both from the exposure N in (2) and from the formal concentration Ω in (30). HMGBG defines the defect metric from the score of an actual path law. For a history of M steps, its per-step rate is

$$W_{\text{prim}} = \lim_{M \rightarrow \infty} \frac{1}{M} \mathbb{E}_0[S_M S_M^\top], \quad S_M = \partial_\zeta \log d\mu_\zeta^{(M)} \Big|_{\zeta=0}. \quad (38)$$

The source parameter ζ in (38) denotes a typed defect tilt. The full source fixes its normalisation and its field dependence. Hidden trajectories can carry memory and must be eliminated with the complete path weight. Increasing Ω in one fixed-dimensional integral does not perform this operation, and setting Ω equal to seven is not a source derivation. The resulting history information must then be connected to the physical action and gauge response on the same carrier; a score covariance alone is not that connection [3, Section 16; 20, Sections IV.14 to IV.15].

8.2 The limit of selection by unit cost and symmetry

We can test exactly whether unit cost and gauge covariance suffice to determine the omitted conditional law. The following control is a mathematical sufficiency test in twelve generator coordinates. It is not an alternative physical VERSF source, nor an identification of those coordinates with the complete primitive event carrier.

Proposition 13 (unit cost and gauge covariance). A unit-cost constraint and invariance under the colour, weak and Abelian gauge actions do not alone determine the directional distribution or covariance of increments.

Proof. In an orthonormal real generator chart, let P_C, P_W, P_Y be the orthogonal sector projectors of ranks 8,3,1. On the unit sphere use normalised uniform surface measure $d\sigma$ and define

$$A = \frac{P_C}{8} - \frac{P_W}{3}, \quad p_\epsilon(\xi) = 1 + \epsilon \xi^\top A \xi, \quad \|\xi\|^2 = 1, \quad |\epsilon| \leq 1. \quad (39)$$

Since $\text{Tr} A = 0$, the density integrates to one. Its minimum is positive, it is even in ξ , and its sector norm-squares are gauge invariant. Every increment has unit cost, including away from $\epsilon = 0$. The standard unit-sphere fourth moment, or direct rotational averaging, gives

$$\mathbb{E}_\epsilon[\xi \xi^\top] = \frac{I_{12}}{12} + \frac{\epsilon A}{84}. \quad (40)$$

The trace remains one, while the colour and weak covariances change unequally. This proves the insufficiency of the two stated conditions. No value of ϵ is selected or installed in the physical calculation.

The control does not test all VERSF premises. In particular, the actual primitive event map, allowed move set, current/record correlations and source measure could select one law or make all admitted laws equivalent for the retained observables. The attempted inference from unit cost and gauge covariance does not establish either outcome. PCN's fixed unit is preserved throughout; the unresolved allocation is not a fractional primitive cost or an adjustable common normalisation.

The physical force-strength calculation therefore remains unfinished at the evaluation of the complete source weight in Criterion 1. The established finite-model results and their uncertainty remain as reported. Closing the physical calculation requires the joint transition law or an equivalent source-derived response, followed by the full current-and-record reduction and the controlled coarse-graining limit. Neither the source audit nor Proposition 13 supplies new physical force coefficients.

9 Derivation status

Table 11. Results and their remaining physical identification.

Item	Result established here	Physical identification
Inherited source structure	Current-rate law, common operational measure and executed conditional bath branch identified	Carry their stated scope into the finite projection
Joint nonlinear history law	Positive family with the archived joint Hessian	Nonlinear weighted projection in Criterion 1
Isolated current shift	Exact cancellation under the stated translation conditions	Check the full measure, domain and other couplings
Coupled fluctuations	Nonlinear response and background displacement computed	Project the inherited continuous law and all cross blocks
Gauge entry	Both derivative terms evaluated in 72 directions	Identify the source current and gauge map in the finite chart
Background treatment	Mode profile and full thirty-amplitude marginal compared	Carry the physical quotient and its volume through the projection
Test-current exchange	Total correction 0.013 ± 0.009 ppm (colour, weak), 0.012 ± 0.009 ppm (hypercharge); consistent with zero	Physical spatial normalisation and matching
Response estimation	Independent pilot fixes an equivalent low-variance lift	No additional physical assumption
Shared feedback	Mean-response factorisation proved; linear channel bounded	Identify sector-dependent source couplings
Measure dependence	Two volumes evaluated; leading determinant cancellation proved	Match the projected operational measure rather than choose a new volume
Source matching	Exact weight-ratio criterion; required joint increment law identified	Evaluate the full primitive pushforward and its physical reduction
Shared normalisation	Canonical reference preserved; constrained preservation theorem under the stated agreement premise and metric restriction	Derive full-edge record agreement and identify the PCN specified gauge image
Additional source terms	Six defect families and potential; complete leading local auxiliary correction evaluated	Physical metric and measure; control of the nonlinear remainder

Item	Result established here	Physical identification
Mixed source blocks	Invariant composition–bond control changes relaxation; closure–completion change has a sharp bound and a common-rescaling criterion	Return the physical typed metric and identify the constrained scalar response with the gauge response
Auxiliary fluctuations	Complete leading correction negative on all cycle modes; current-only contribution positive	Physical quotient measure and controlled coarse limit
Relative-gauge contribution	Conditional transverse coefficient -535.928327 at order $1/\Omega$	Physical event attachment and faithful generator extension
Unit cost and symmetry	Exact sufficiency control leaves unequal directional covariances	Complete primitive rules must select the joint change law
History limit	Local concentration and physical history length distinguished	Evaluate actual path weighting and controlled coarse graining
Force-strength triple	No physical triple issued; complete history calculation unfinished	Use the source law with the same PCN unit and faithful gauge map

Criterion 1 is the physical matching condition. Proposition 9 proves preservation on the canonical quadratic branch; Propositions 10 and 11 give the constrained endpoint identity and its preservation under the specified constant invariant metrics. Corollary 2 bounds the remaining constant-metric correction and Lemma 2 orders the three responses. Proposition 12 evaluates the leading local nonlinear auxiliary correction. Section 8.1 identifies the joint source law needed for physical matching; Proposition 13 tests the insufficiency of unit cost and gauge covariance alone. The full-edge agreement premise and its attachment to PCN remain to be derived from the source event map. The canonical scalar weights and PCN's fixed unit are retained, with no additional conversion constant.

10 Conclusion

The nonlinear history calculation yields a total local exchange correction of 0.013 ± 0.009 parts per million for colour and weak and 0.012 ± 0.009 for hypercharge, despite substantial background displacement. At this precision the correction is consistent with zero, and the declared attachment does not produce the collective D41 force reductions. The isolated positive linear-pullback contribution has a separate 7.6 parts-per-million bound. The shared mean contribution preserves sector ratios after invariant averaging. The marginal mode shifts by about 14 units in each retained amplitude coordinate, while the fully integrated fractional exchange change is only of order 10^{-8} .

The scalar parent supplies a complementary result. Under the stated record-agreement premise, the committed reference is preserved when the constant invariant metric has no closure–completion block. Corollary 2 quantifies the remaining correction when that block is present, identifies exactly when it is a common rescaling, and bounds the corresponding inverse response. Lemma 2 measures the reduction obtained by allowing record mismatch. On the canonical diagonal branch, complete auxiliary fluctuations add a non-common leading correction, reversing the current-only sign and giving an explicit contraction with the inherited relative-gauge functional.

The physical continuation is the joint configuration-change law with its event map and measure, evaluated through Criterion 1 and a controlled coarse-graining limit. PCN retains its fixed unit throughout. The paper establishes the finite nonlinear responses, their conditional gauge contraction and quantitative normalisation bounds; those calculated objects provide the inputs and checks for that source evaluation.

Appendix A Numerical methods and reproduction

The raw source arrays are preserved in `source_reference.npz`. Reconstruction verifies the bath, defect and current scores, the participation Gram and the complete joint Hessian. The source has seven vertices, seven marks and two fibres; transitions remain on the original support. Centring the raw features at the reference changes neither the exponential family nor its relative-entropy rate.

Hamiltonian Monte Carlo uses four chains and a fixed metric obtained by whitening the hidden Hessian at the origin. Step-size adaptation is confined to warmup. The flat-volume runs retain 8,192 samples per chain at unit exposure and 4,096 at exposures 7 and 49. The information-volume run retains 2,048 samples per chain after 1,024 warmup steps. Analytic hidden gradients are checked by finite differences at two nonzero amplitudes. The volume derivatives in the two retained directions are checked at two step sizes.

Uncertainties for the curvature use a delete-a-block jackknife that retains the covariance subtraction. Split-chain diagnostics are reported for sampled coordinates and response observables; they diagnose the computations rather than certify all tails. Curvature from small-displacement free-energy reweighting is compared with the derivative estimator. The marginal-mode search reports overlap diagnostics and retains the unsuccessful initial estimate as a diagnostic, not an accepted result.

A constant translation of the hidden integration coordinates provides an equivalent estimator. For a fixed matrix B , the derivatives $D_i = \partial_{y_i} + B_{ki} \partial_{z_k}$ can replace the derivatives in (5), because $z = q + By$ has unit hidden Jacobian. A lift fixed by the Gaussian conditional minimum is evaluated as a control. Its results agree within Monte Carlo uncertainty; it does not improve the curvature precision of this information-volume run. Retained finite-difference steps 0.001 and 0.002 agree to below 10^{-5} on the checked sample. This estimator equivalence is separate from the physical current translation in Proposition 1.

The computational supplement contains the source arrays, executable scripts, numerical results and a SHA-256 manifest. The principal scripts are `test_nonlinear.py`, `information_volume.py`, `laplace_controls.py`, `response_checks.py`, `gauge_feedback.py`, `background_profile.py`, `full_background.py`, `prepare_response_lift.py` and `gauge_exchange.py`. The controls change the exposure or measure explicitly. No observed coupling or screening target is used to choose a parameter.

The gauge extension uses the archived native tangent, dual map, endpoint preparation, generators, cycle frame and coupled parent. Gauge derivatives are checked directly against exponentiated links; mixed-direction finite differences of the full log integral check the response. The local mode uses importance reweighting with effective sample size 30,918. Block jackknife estimates recompute the covariance and profile Schur complement in every deletion. Table 5 reports errors of its isolated linear-pullback term; Table 4 reports the full correction conditional on the numerical mode. Neither uncertainty includes physical attachment or measure uncertainty. The split-chain diagnostic for the added current scores and Hessians is below 1.0003 in both ensembles. Exact group averaging, Gaussian regression identities and shared-mean factorisation are separate controls. The supplemental inputs and output arrays retain each contribution independently.

The full-background run retains 8,192 samples in each of four independent chains after 2,048 warmup steps. Its separate pilot retains 1,024 samples per chain after 1,536 warmup steps. The pilot seed differs from the production seed; its constant lift is frozen throughout evaluation. The production HMC target is unchanged by this estimator choice. No divergences occur in the full-background run. Its maximum split-chain diagnostic is 1.0010; for the response scores and Hessians it is 1.0002. Block jackknife estimates propagate the score covariance and matrix inversion. All quoted errors are conditional on the model, gauge map and fixed pilot, not estimates of physical-source uncertainty. Independent differences of the reweighted log integral check the lifted score and Hessian. Matrix-inverse, dual-source coordinate changes and Y to $q = 6Y$ conversions are checked separately. Inversion and invariant projection retain their declared order.

The preservation archive [14] contains `run_completion.py`, the master-vacuum module, four source arrays, numerical results and a SHA-256 manifest. All 83 checks pass, including Jacobian regeneration, both full-quotient reductions, the auxiliary determinant, positive-metric controls and finite-curve coefficients. Matrix-identity residuals are below 2×10^{-15} . Its transport-bond pairings are algebraic controls and are not admitted as constant gauge-invariant scalar blocks.

The committed-constraint archive [21] contains `committed_change.py`, the independent `check_claims.py` wrapper, numerical results and source excerpts. Section 7.3 uses its complex-linear equal-charge metric construction, corrected transposed Schur forms and full-edge constrained elimination. The verification reproduces Table 9, the endpoint background term and the 100 positive invariant-metric preservation controls with the closure-completion block absent, all below 3×10^{-16} relative Frobenius error. It uses 74 real fields and 87 real residual coordinates on the scalar wheel; it is a separate check from the 226-coordinate archive. The fixed-mean derivative follows the upper-right-block convention stated in Section 7.3. Neither archive identifies the physical primitive event or the complete physical weighting.

The response-bound archive [22] contains `calculate_bounds.py`, the unchanged scalar model source, `bounds_results.json` and a SHA-256 manifest. It checks Corollary 2 and Lemma 2 by full 74-field elimination. Five matched-mode cases verify sharpness; a separate ensemble of 100 positive invariant metrics varies the closure-completion block, complex composition-bond and current-conservation blocks, and a nonnegative completion potential. The committed formula, full-mismatch Schur identity and intermediate reduction agree to below 4×10^{-16} relative Frobenius error across that 100-metric ensemble. The random seed is 202610267. Every vertex-generator commutator is checked separately. The potential-free matched-mode case is the bound-saturating limit within the declared metric class, not a selection of the physical metric.

The nonlinear auxiliary archive [17] contains `calculate.py`, analytic action derivatives, the source master-vacuum module and arrays, the full mode table, a source manifest and the manuscript extension. All 47 checks pass. They include the finite nonlinear action and Hessian against the archive, finite-difference derivative checks, recovery of the tree response, complete current/remaining-auxiliary determinant factorisation, the independent log-determinant trace identity, two step-extrapolation sequences, cycle off-diagonals and doublet degeneracies. The independent current Hessian has the required gauge nulls, and finite current determinants are gauge invariant. Physical measure selection and the asymptotic remainder are outside these verification checks.

Sections 8.1 and 8.2 contain a source construction audit and an analytic sufficiency control, not an additional numerical history simulation. For a uniform unit vector in d dimensions, rotational invariance gives $\mathbb{E}[\xi_i \xi_j \xi_k \xi_l] = c(\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$. Contracting twice and using $\|\xi\|^4 = 1$ fixes $c = 1/[d(d+2)]$. Contracting with the trace-zero matrix A at $d = 12$ gives (40), without a fitted parameter or Monte Carlo estimate. The earlier numerical check counts certify only their declared models; they do not certify the complete physical weighting discussed in Section 8.1.

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