

Physical Applicability and Irreversible Response in VERSF

Finite history laws, substrate relaxation and the derivation of gauge contrasts

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Scope and principal result. This paper establishes exact criteria for when the history-law construction applies and how a source-derived change enters the gauge contrasts J and T . It separates those results from conditional source models and physical coefficients that remain unevaluated. The existing approximate force relation is retained at its stated accuracy. Its leading top correction is small and nonzero; classical energy-sourced substrate relaxation produces no quadratic gauge correction about zero gauge curvature under the assumptions of Section 43. Physical numerical J and T remain unevaluated. Analytic proofs establish the general statements; the verification inventory covers specified identities, fixtures and numerical benchmarks. Measured boundary inputs are declared and do not select any source coefficient.

General reader summary

VERSF already supplies a conditional approximate relation between the strong, weak and electromagnetic force strengths. This paper asks which further effects the substrate rules require, how they would change that relation, and which proposed changes survive calculation. J and T are two combinations of the gauge-response corrections. Defining them does not by itself establish that an additional correction is physically present.

The paper first tests the history law used to describe how the substrate changes. It gives an exact test for the reversible construction, shows why the original seven-state wheel fails that test, and shows why the designated heat evolution passes under its stated assumptions. A circulating bias leaves the reversible class. Its information response can be calculated and carried through the constrained action, including the adjustment of surrounding variables.

Letting the surroundings adjust can change the relative response even when the initial cost is the same for every kind of matter, so symmetry alone cannot force J to vanish. Equally, the fact that such a change is allowed is not evidence that the physical source produces it. The paper gives the exact balance that would make J zero, shows that three specified totals of the complete response would settle both numbers, and lists precisely what still has to be evaluated or physically identified before those totals can be read off. Several of the needed definitions and intermediate reductions already exist; what is missing is their complete evaluation on one physically admitted setting.

Two correction tests give a definite stopping point. The leading effect of the heaviest quark shifts the weak-mixing prediction from 0.22733305 to 0.22736738, leaving most of the difference from the comparison value 0.23122. In the classical zero-curvature test, the substrate does adjust, but the induced contribution begins at fourth order in gauge-field amplitude and leaves the quadratic force coefficient unchanged. These results preserve the approximate relation at its declared accuracy; they neither justify a correction chosen to improve agreement nor prove that all further corrections vanish. The Higgs sector here is the substrate's mass-generation sector, not an additional independent primitive field.

Abstract

An irreducible finite Markov kernel is the Doob normalisation of an entrywise nonnegative, operator-positive, self-adjoint transfer in a fixed positive measure if and only if it satisfies detailed balance and has strictly positive spectrum. The transfer is then unique up to a positive scalar multiplier. The designated heat construction satisfies this criterion under the stated Dirichlet hypotheses. The canonical seven-state wheel is not continuously embeddable; its associated heat semigroup is a distinct admissible law. A closure-circulation tilt leaves the reversible class, with explicit affinity, spectral and entropy-production certificates. Its information return is computed and propagated through an exact constrained-action reduction.

The response analysis distinguishes current quadratures, charged readout norms and representation-resolved gauge prices. It proves that the audited symmetry and common-price assumptions do not force $J = 0$, gives the exact cancellation condition, and reduces J and T to three charge and support moments. Record composition, current conservation, completion vertices and finite matching are evaluated at their stated grades; the physical coupled response remains unevaluated. The leading top term shifts the reference weak-mixing result from 0.22733305 to 0.22736738. Classical stress-energy-sourced relaxation gives no quadratic correction about zero gauge curvature on a regular branch with the stated coupling assumptions. The result is a bounded assessment of corrections to the existing force relation: exact structural results and conditional numerical tests are retained, while physical numerical J and T remain open. Section 44 specifies the remaining source evaluation without treating established defect definitions as missing.

Results and claim-status guide

Object	What this paper establishes	Status
Applicability criterion	Detailed balance and strictly positive spectrum are necessary and sufficient in a fixed measure; the transfer is unique up to a positive scalar multiplier (Theorem 1).	Exact, self-contained
Designated heat transfer	Admissible on any finite connected Dirichlet branch; the sign condition is essential; generator fixed up to nested commutators (Proposition 2, Lemma 3, Proposition 4).	Exact given the four hypotheses
Canonical wheel and heat kernel	The original wheel is reversible but not continuously embeddable; its heat semigroup is a different law, given in closed form and admissible at every positive clock value; heat regularisation always passes (Theorems 5, 6, Lemma 7).	Exact
Circulation tilt	Leaves the reversible class for every nonzero tilt; exact affinities, spectrum, entropy production, parity, information rate on wheel and heat baseline (Propositions 8 to 13, Theorems 9, 14, 15).	Exact
Constrained response	Exact finite update on the stated constrained quotient for any returned metric change; rank-one and one-direction resolutions; direct and relaxation terms separated; frozen-direction invariance; relaxation-generated contrasts; rank bound. Tilt evenness holds under the fixed-ingredient or full parity assumptions of Sections 12, 13 and 32 (Theorems 16, 20 to 22, Corollaries 17, 23, Propositions 18, 19, 24).	Exact given the fixed ingredients
Representation and readout	Support restrictions of an invariant update; exact charged-readout norm identities with unit branch (0, 660); third-order vertex and four-fermion term (Propositions 25 to 28).	Exact on the declared branches

Object	What this paper establishes	Status
Symmetry and the zero question	Gauge symmetry, family blindness, common primitive price and fact retention do not force $J = 0$; three-channel balance $24\lambda_c + 27\lambda_w = 900\lambda_g$; protection theorem; three moments determine J and T (Theorems 29 to 31).	Exact; physical balance open
Record, phase, export, grouping	Polar evenness and commutator curvature; exact Abelian phase relation; stationary record family; export Fisher 4 sech ² s and the coboundary condition; rank-two export Gram; terminal-map invariance of the complete response (Propositions 32 to 39).	Exact restrictions; one conditional obstruction
Observables and the write	Electromagnetic contraction and its blind chirality direction; Fold universality criterion; native current signs; linear force leaves a fixed quadratic stiffness; normalised law leaves the raw weight free (Propositions 40 to 44).	Exact; conditional branches labelled
Completion and finite response	Two-loop contractions B_j , B_T ; matched candidate with negative Yukawa J contribution throughout the interval; finite attachment map and cancellation ratio; coupled momentum and gauge identities; evaluated quadratic data do not fix the mixed vertex (Propositions 45 to 52).	Algebraic identities and declared-branch transport; physical finite coefficients unevaluated
Current conservation and source kernel	Exact current and conservation reduction with its determinant; wheel-cell kernel with record mixing; known definitions separated from unevaluated physical attachments (Propositions 53, 54).	Exact on the declared cell; physical lift open
Force relation and corrections	Gauge-only 0.22733305, with leading top 0.22736738, closing 0.88% of the gap; D41 controls conditional; classical energy-sourced relaxation gives no quadratic correction (Proposition 55).	Reproduced independently; conditional continuation with declared measured boundary inputs; classical zero under stated assumptions
Physical J and T	The physical pair is not evaluated. For the normalised price, $J = 0$ exactly when $k_Q = k_u + 3k_e$. The remaining source operators, derivatives and identifications are specified in Section 44.	Open; next calculation specified

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Part A. The criterion and the designated construction

1. Position, inputs and conventions

SGW-3 [1] proves, in its Proposition 4, that two descriptions of one finite history law give identical action response under the same physical identifications whenever that law is the Doob normalisation of an entrywise nonnegative, operator-positive, self-adjoint transfer, and it defines the two central contrasts $J = \text{Tr}(O_j K)$ and $T = (1/3)\text{Tr}(A_T K)$ of the finite-loop price operator K on the 47-dimensional matter-plus-Higgs carrier. The theorem is conditional on its hypotheses, and the first question of this paper is whether the constructions the programme has actually designated satisfy them. Three inputs are inherited for that question and not re-derived: the operational heat sandwich that the history-measure source designates for physical execution [2, §56A.2]; the canonical seven-state wheel fixed by the admissibility rule HM11 [2, §§11A, 14A]; and the closure-circulation score F_c [2, §14B].

The second question is what the inherited structure, carried through the complete constrained action, determines about J and T before any numerical source return: the representation support of an invariant update, the charged completion readout, the differential order of the charged vertex, the admitted symmetries, the record and export calculus, the completion and Yukawa sector, the finite regulator response and the master action's own current and conservation residuals.

For that question the inherited inputs are the SGW-2 census and contrast operators [16], the master action and its six defect families [17], the graded source descent [18], the quadratic-release scope [19], the ordered-record preparation [20] and the primitive marked-instrument programme [22].

The argument proceeds from applicability to response and then to physical interpretation. Parts A to C establish the finite transfer criterion and its wheel and circulation benchmarks: the criterion is proved in self-contained form, applied to the designated heat transfer, and the wheel is executed exactly in its one-step and heat forms, with three independent certificates that the inherited circulation tilt leaves the reversible class and the exact information it returns on the wheel and on the heat baseline. Part D derives the constrained response to a returned metric change in closed form and resolves it into direct and relaxation modes. Parts E to H test how representation, symmetry, records, completion and propagation enter that response, each result with its exact statement and claim grade. Part I assesses specified corrections to the existing force relation and records which links of the dependency chain are established, conditional or unevaluated. No potential, clock, measure, sector weight or finite coefficient is selected to fit the comparison. The continuation in Section 43 does use its stated measured boundary inputs; that is distinct from deriving every gauge strength without experimental input.

1.1 Conventions

The state set is finite with n elements. P is a row-stochastic matrix, irreducible unless stated otherwise, with stationary law π . A positive reference measure μ is fixed. A matrix K is μ -self-adjoint when $\mu_i K_{ij} = \mu_j K_{ji}$ for all i, j ; it is operator-positive when $\langle f, Kf \rangle_\mu = \sum_i \mu_i f_i (Kf)_i > 0$ for every nonzero real f . A μ -self-adjoint matrix is diagonalisable with real spectrum, so operator positivity is the same as strictly positive eigenvalues. In Parts A to C, K denotes the history transfer, written with a double-struck $\mathbb{T}$ in the source papers. In the response sections, K denotes the attached finite-loop price where it is explicitly defined there; the two operators act on different carriers and are not identified by the shared symbol. Its generator on the clock a is $G = -a^{-1} \log K$ whenever that logarithm is defined.

If K is entrywise nonnegative and irreducible, Perron–Frobenius theory [6, Chapter 8] gives a simple eigenvalue λ equal to the spectral radius and a strictly positive eigenvector r , unique up to scale. The Doob kernel of K is

$$P_{ij} = K_{ij} r_j / (\lambda r_i),$$

which is row-stochastic because $\sum_j K_{ij} r_j = \lambda r_i$. This is the ground-state or h -transform of Doob [7], in the finite form used by Levin and Peres [5]. Detailed balance for P in π means $\pi_i P_{ij} = \pi_j P_{ji}$; a chain with this

property is called reversible. For a directed cycle γ the affinity is $A_\gamma = \ln$ of the ratio of the forward to the reversed transition product; reversibility is equivalent to $A_\gamma = 0$ for every cycle, which is Kolmogorov's criterion [8, Chapter 1].

2. The finite applicability criterion

Theorem 1 (finite applicability criterion). Let P be an irreducible row-stochastic matrix with stationary law π , and fix a positive reference measure μ . There exists an entrywise nonnegative, operator-positive, μ -self-adjoint matrix K whose Doob kernel is P if and only if (i) $\pi_i P_{ij} = \pi_j P_{ji}$ for all i, j , and (ii) $\text{spec}(P) \subset (0, 1]$. When (i) and (ii) hold, every such K has the form $K = c \cdot D_r P D_r^{-1}$ with $r_i = (\pi_i / \mu_i)^{1/2}$, $D_r = \text{diag}(r)$ and $c > 0$, and every matrix of this form is admissible. In particular K is unique up to the positive scalar multiplier c .

Proof. Necessity. Let K be admissible with Doob kernel P . Since P and K have the same support, K is irreducible, and its Perron pair (λ, r) is unique up to the scale of r . From $P_{ij} = K_{ij} r_j / (\lambda r_i)$ and $\mu_i K_{ij} = \mu_j K_{ji}$,

$$(\mu_i r_i^2) P_{ij} = \mu_i K_{ij} r_j / \lambda = \mu_j K_{ji} r_i / \lambda = (\mu_j r_j^2) P_{ji}.$$

Summing over i gives $\sum_i \mu_i r_i^2 P_{ij} = \mu_j r_j^2$, so $v_i = \mu_i r_i^2$ is an invariant measure for P . An irreducible chain has a unique invariant law, hence $\pi_i = \mu_i r_i^2 / \sum_k \mu_k r_k^2$, and the display is (i). For (ii), $P = \lambda^{-1} D_r^{-1} K D_r$ is similar to K/λ ; K is μ -self-adjoint and operator-positive, so its spectrum is real and strictly positive with largest element λ , and $\text{spec}(P) \subset (0, 1]$.

Sufficiency. Assume (i) and (ii), set $r_i = (\pi_i / \mu_i)^{1/2}$ and $K = c \cdot D_r P D_r^{-1}$. Then $\mu_i K_{ij} = c \mu_i (r_i / r_j) P_{ij} = c (\mu_i \mu_j)^{1/2} (\pi_i P_{ij}) / (\pi_j \pi_i)^{1/2}$, which is symmetric in i and j by (i); so K is μ -self-adjoint. It is entrywise nonnegative, and its spectrum is $c \cdot \text{spec}(P) \subset (0, c]$, so it is operator-positive. Finally $Kr = c \cdot D_r P 1 = c r$, so (c, r) is the Perron pair and the Doob kernel of K is $K_{ij} r_j / (c r_i) = P_{ij}$.

Uniqueness. Let K' be any admissible matrix with Doob kernel P and Perron pair (λ', r') . The necessity argument gives $\pi_i = \mu_i r_i'^2 / \sum_k \mu_k r_k'^2$, so r' is a positive multiple of r , and $K'_{ij} = \lambda' r'_i P_{ij} / r'_j = \lambda' (D_r P D_r^{-1})_{ij}$. ■

The scalar freedom is a transfer normalisation, not a change of time step. At a fixed clock a , $G(cK) = G(K) - (\log c/a)I$: the multiplier changes the additive scalar part of the generator and leaves its Doob kernel unchanged, whereas varying the clock of a nontrivial semigroup generally changes the normalised transition law.

2.1 Remarks

- The two conditions are independent. Condition (i) is the symmetry of the law in its own stationary measure; condition (ii) is the absence of zero and negative modes. A chain can satisfy either without the other, and Section 5 exhibits one that satisfies (i) and fails (ii).
- Condition (ii) implies aperiodicity, since -1 is excluded, and excludes any zero mode, so a singular P can never be the Doob kernel of an admissible transfer in any measure.
- Condition (i) does not involve μ . Changing the reference measure changes r and K but not the admissibility of P . The physical content of μ enters only through the identification of K with an action, which is the separate source-to-action step of SGW-3 and is not touched here.
- The theorem distinguishes stochastic positivity (nonnegative entries of K) from spectral positivity (positive eigenvalues of K). Both are needed, and in Section 3 they are supplied by different hypotheses.

3. The designated operational heat transfer

HMGBC §56A.2 designates the operational heat sandwich, rather than the one-step adjacency kernel, for physical execution [2, p. 95]:

$$K = M e^{-aL} M, \quad M = \text{diag}(e^{-aV/2}),$$

with L the operational generator, V the reference potential and a the clock. On a finite configuration discretisation the following four hypotheses are assumed and nothing else: (H1) L is μ -self-adjoint; (H2) $L_{ij} \leq 0$ for $i \neq j$, the Dirichlet sign condition; (H3) the off-diagonal support of L is connected; (H4) V has finite real entries. The measure, potential and clock are inherited; none is chosen here.

Proposition 2 (the designated transfer is admissible). Under (H1) to (H4), K is μ -self-adjoint, operator-positive and entrywise strictly positive. Its Doob kernel P therefore satisfies both conditions of Theorem 1 with $\pi_i = \mu_i r_i^2 / \sum_k \mu_k r_k^2$, where (λ, r) is the Perron pair of K , and SGW-3 Proposition 4 applies to P . Once the normalised law, μ and a are fixed, the generator $G = -a^{-1} \log K$ is fixed up to an additive multiple of the identity.

Proof. Self-adjointness: e^{-aL} is μ -self-adjoint by (H1), and conjugating a μ -self-adjoint matrix by the real diagonal M preserves μ -self-adjointness, since $\mu_i m_i A_{ij} m_j = m_i m_j \mu_j A_{ji}$. Operator positivity, without commuting M and L : $e^{-aL} = (e^{-aL/2})^2$ with $e^{-aL/2}$ μ -self-adjoint and invertible, so

$$\langle f, Kf \rangle_\mu = \|e^{-aL/2} M f\|_\mu^2 > 0 \quad (f \neq 0).$$

Entrywise positivity: choose $c \geq \max_i L_{ii}$. By (H2) the matrix $B = cI - L$ is entrywise nonnegative, and $e^{-aL} = e^{-ac} \sum_k a^k B^k / k!$. By (H3) every pair of states is joined by a path of positive B entries, so some power B^k has a positive (i, j) entry, and every term of the series is nonnegative; hence e^{-aL} is entrywise strictly positive, and so is K because M is positive diagonal. Perron–Frobenius then supplies the unique positive r and the simple dominant λ , the Doob kernel is irreducible, and Theorem 1 (necessity) gives detailed balance in $\pi_i \propto \mu_i r_i^2$ and spectrum in $(0, 1]$. By Theorem 1 (uniqueness), K is determined by P , μ and a up to the scale c , so $\log K$ is determined up to $(\log c) \cdot I$. ■

3.1 The sign condition is essential

Lemma 3. Hypotheses (H1), (H3), (H4) and positive definiteness of L do not imply entrywise positivity of e^{-aL} . The matrix $L = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, self-adjoint in counting measure with eigenvalues 1 and 3 and connected support, has $e^{-aL} = e^{-2a} [\cosh a, -\sinh a; -\sinh a, \cosh a]$, whose off-diagonal entries are strictly negative for every $a > 0$.

The lemma records where the two positivities come from. Positivity of the Hessian, or of L as an operator, gives spectral positivity of K and nothing else. The nonnegative entries of K come from the Dirichlet sign of the off-diagonal generator entries, hypothesis (H2), which in the source is the cone condition F37 [2, §56A.2]. The two checks are different checks on different objects, and a certificate for one is not a certificate for the other. In the counting-coordinate form used by the source, $S = \text{diag}(\pi^{1/2}) P \text{diag}(\pi^{-1/2})$ and $L = I - S$, hypothesis (H2) is the statement that S has nonnegative off-diagonal entries, which holds whenever P does.

3.2 The recovered generator

Proposition 4 (what the sandwich generator is). (a) If V is constant, $K = e^{-aV} e^{-aL}$ and the Doob kernel of K equals that of e^{-aL} . If in addition $L1 = 0$, as for $L = I - P$ with P stochastic, then the Perron pair of e^{-aL} is $(1, 1)$, the Doob transform is the identity and $P_K = e^{-aL}$ exactly. (b) In general $G = -a^{-1} \log K$ equals $L + V$ when $[L, V] = 0$, and otherwise

$$G = L + V + (a^2/12)[L, [L, V]] - (a^2/24)[V, [V, L]] + O(a^4),$$

the symmetric Baker–Campbell–Hausdorff expansion [9, Section III.4] applied to $e^{X/2} e^Y e^{X/2}$ with $X = -aV$ and $Y = -aL$.

Part (a) is why the wheel benchmark of Part B can be executed on the heat kernel directly, with no Doob step. Part (b) is a caution about attribution. The canonical generator attribution G11a identifies L ; the object that Theorem 1 fixes up to an additive constant is G , the logarithm of the designated sandwich, and the two differ by source-fixed commutator terms whenever the potential does not commute with the generator. Nothing adjustable enters: given L , V and a , the difference is determined. But a statement of the form "the generator is $L + V$ " must be read as a statement about G in the commuting case, or as a

statement about $L + V$ with the commutator terms carried separately. The verification package evaluates the expansion numerically on the wheel with a non-constant diagnostic potential and confirms the coefficient structure (Appendix B, section J).

4. What remains conditional

Proposition 2 is a finite-dimensional theorem with four hypotheses, and the conditions under which the physical construction meets them are the source's own obligations, not new inputs. They are listed here so that the scope of the result is exact.

- **Dirichlet and cone condition.** HMGBBC requires the full operational generator to pass F37, physical irreducibility and the full-carrier inheritance checks [2, §§11A.1, 55A, 56A.2]. Lemma 3 shows that positivity of a Hessian alone proves none of these. The entrywise step needs (H2) on the actual physical L .
- **Finite discretisation.** The proof requires an actual finite state set. A spatial regulator need not make continuous field values into a finite set; an operator limit needs domain, kernel, ground-state and convergence control, none of which is supplied by the finite argument.
- **Gauge and fermion phases.** A positive self-adjoint quantum transfer is not automatically a nonnegative scalar configuration kernel. HMGBBC §20.2 offers a nonnegative-determinant route and a separate reflection or transfer-positivity route; whichever is used must deliver the positive-history representation in which the Doob construction is performed. The free and static certificates in CLSN do not amount to a dynamical proof, and CLSN contains a static non-Abelian sector with a negative one-step spectral value even where a link-reflection certificate holds [3, §§17.38 to 17.50]; those are different positivity assertions about a different operator.
- **Pure-field transfer.** The Wilson transfer in Configuration Dynamics requires its integer flux-lattice coverage condition to exclude null modes and is not yet the complete source [4, Part VIII].
- **Generator attribution.** Proposition 4(b): the identification of G with $L + V$ holds in the commuting case; otherwise the commutator terms are part of the attribution.

With these named, the positive statement stands: the finite theorem has an identified source architecture that satisfies its hypotheses, and the remaining work is to show that the physical generator and measure are of the assumed form, not to find a construction.

Part B. The canonical seven-state benchmark

5. The wheel: reversibility, spectrum and non-embeddability

The HMGBBC wheel lives on the states $h, b_0, \dots, b_5$: a hub and a six-cycle rim. The hub row assigns $1/7$ to each of the seven states. Each rim row assigns $1/4$ to the hub, to itself and to each of its two rim neighbours. These entries follow from the inherited admissibility rule HM11 and are not fitted [2, §§11A, 14A]. Exact rational arithmetic gives

$$\pi = (1/31)(7, 4, 4, 4, 4, 4, 4), \quad \text{spec}(P_0) = \{1, 1/2, 1/2, -1/4, -3/28, 0, 0\},$$

with detailed balance $\pi_i P_{0ij} = \pi_j P_{0ji}$ holding exactly. The spectrum is organised by the dihedral symmetry of the wheel. The rim Fourier modes v_k , with hub component 0 and rim components $e^{i\pi k m/3}$ for $m = 0, \dots, 5$ and $k = 1, \dots, 5$, have zero rim sum, so the hub row annihilates them and the circulant rim block acts alone: the eigenvalue is $(1 + 2 \cos(\pi k/3))/4$, which is $1/2$ for $k = 1, 5$, zero for $k = 2, 4$, and $-1/4$ for $k = 3$. The remaining two-dimensional block, spanned by the hub indicator and the uniform rim vector, is $[[1/7, 6/7], [1/4, 3/4]]$ with eigenvalues 1 and $1/7 + 3/4 - 1 = -3/28$.

Theorem 5 (the one-step wheel is reversible but not embeddable). (a) P_0 satisfies condition (i) of Theorem 1 and fails condition (ii); it is not the Doob kernel of any admissible transfer in any reference measure. (b) $\det P_0 = 0$, and the negative eigenvalues $-1/4$ and $-3/28$ are simple. Hence P_0 has no real

logarithm, and a fortiori no generator Q with $P_0 = e^Q$: it is not embeddable in any continuous-time Markov semigroup. (c) No power P_0^k is embeddable, because $\det P_0^k = 0$; in particular P_0^2 has spectrum $\{1, 1/4, 1/4, 1/16, 9/784, 0, 0\}$ and retains two exact null directions. (d) The heat kernel $P_{\text{heat}}(a) = e^{-a(I - P_0)}$ is embeddable for every $a > 0$, with generator $Q = P_0 - I$, which has nonnegative off-diagonal entries and zero row sums.

Proof. (a) is the spectrum together with Theorem 1. (b) A real matrix has a real logarithm if and only if it is nonsingular and each Jordan block belonging to a negative eigenvalue occurs an even number of times (Culver [10]); P_0 fails both parts. Any embeddable stochastic matrix has $\det P = e^{\text{tr } Q} > 0$ (Elfving [11], Kingman [12]; see Davies [13] for the modern statement), which is a second, independent proof. (c) $\det P_0^k = (\det P_0)^k = 0$, and the spectrum of P_0^2 is the square of that of P_0 . (d) $Q = P_0 - I$ is a generator and e^{aQ} is its semigroup. ■

Theorem 5 replaces the observation that a logarithm of P_0 is awkward by the statement that it does not exist, and that no change of clock, choice of branch, squaring or further power can produce one. The one-step wheel and its heat kernel are therefore different kinds of object: the first is a one-step law with no continuous-time parent, the second is the continuous-time law of the canonical generator $L = I - P_0$. Restricting P_0 to the five-dimensional range of P_0 would remove the null directions, but that changes the carrier and would need its own physical justification; it is not performed here.

6. The heat kernel in closed form

For the canonical generator attribution $L = I - P_0$ on $L^2(\pi)$, with zero reference potential and an arbitrary positive clock a , Proposition 4(a) gives $P_{\text{heat}}(a) = e^{-a(I - P_0)}$ directly, with no Doob step. Because P_0 is reversible and its spectrum is rational with rational eigenvectors, the kernel can be written down exactly.

Theorem 6 (closed-form heat kernel). $P_0 = \sum_k \lambda_k \Pi_k$ with the five rational eigenprojectors of Appendix A: $\Pi_1 = 1\pi^T$ (eigenvalue 1), Π_B (eigenvalue $-3/28$), Π_H (eigenvalue $1/2$, rank 2), Π_Z (eigenvalue 0, rank 2) and Π_Q (eigenvalue $-1/4$). Hence $P_{\text{heat}}(a) = \sum_k e^{-a(1 - \lambda_k)} \Pi_k$, and its seven distinct entries are

Entry	Closed form	Value at $a = 1$
hub to hub	$7/31 + (24/31)e^{-31a/28}$	0.4816788938
hub to any rim state	$4/31 - (4/31)e^{-31a/28}$	0.0863868510
rim state to hub	$7/31 - (7/31)e^{-31a/28}$	0.1511769893
rim state to itself	$4/31 + (1/3)e^{-a/2} + (1/6)e^{-5a/4} + (7/186)e^{-31a/28} + (1/3)e^{-a}$	0.5140246682
rim state to adjacent rim state	$4/31 + (1/6)e^{-a/2} - (1/6)e^{-5a/4} + (7/186)e^{-31a/28} - (1/6)e^{-a}$	0.1334949054
rim state to rim state two apart	$4/31 - (1/6)e^{-a/2} + (1/6)e^{-5a/4} + (7/186)e^{-31a/28} - (1/6)e^{-a}$	0.0268196178
rim state to opposite rim state	$4/31 - (1/3)e^{-a/2} - (1/6)e^{-5a/4} + (7/186)e^{-31a/28} + (1/3)e^{-a}$	0.0141692961

The spectrum is $\{1, e^{-a/2}, e^{-a/2}, e^{-5a/4}, e^{-31a/28}, e^{-a}, e^{-a}\}$, strictly positive, and detailed balance in π holds exactly because $P_{\text{heat}}(a)$ is a function of the π -self-adjoint matrix P_0 . Both conditions of Theorem 1 hold, with no deleted directions. The entries are positive for every $a > 0$, as Proposition 2 guarantees, and they

are visibly so in the closed forms: each entry is a stationary value plus exponentially decaying corrections whose coefficients sum to the $a = 0$ value (1 on the diagonal, 0 off it). The limits are $P_{\text{heat}}(0) = I$ and $P_{\text{heat}}(a) \rightarrow 1\pi^T$ as $a \rightarrow \infty$.

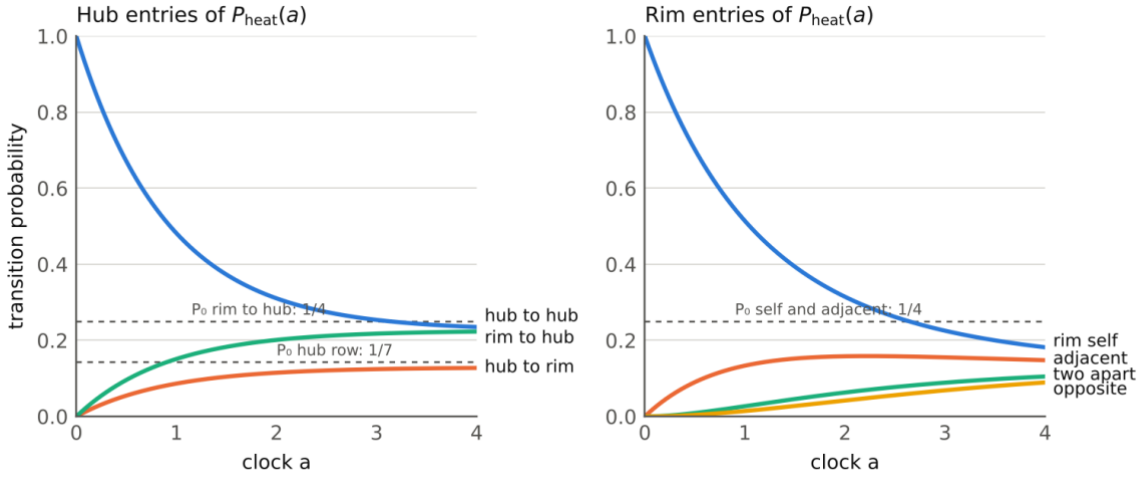


Figure 1. The seven distinct entries of the wheel heat kernel $P_{\text{heat}}(a) = \exp[-a(I - P_0)]$ as functions of the clock a , from the closed forms of Theorem 6. Dashed lines mark the corresponding entries of the one-step wheel P_0 (the two-apart and opposite rim entries of P_0 are zero). No value of a reproduces P_0 : the heat kernel is a different history law with the same stationary measure.

6.1 The diagnostic point $a = 1$

Check at $a = 1$	Return
Smallest heat eigenvalue	$e^{-5/4} = 0.2865047968601901$
Smallest transition entry (rim to opposite rim)	0.014169296126195677
Largest entry change from P_0 (hub to hub, $1/7 \rightarrow 7/31 + (24/31)e^{-31/28}$)	0.33882175097055955
Row sums and detailed balance	Exact in the closed form; floating-point residual below 10^{-15} in the series check

The third row is the one that matters. Taking the heat kernel changes the history law; it is not a normalisation of P_0 , and Figure 1 shows that no clock value reproduces P_0 . Its use is grounded in the existing §56A.2 designation, conditional on the G11a generator attribution, and the value $a = 1$ is a diagnostic unit, not a derived physical clock. In the counting-coordinate representation the same construction is $S = \text{diag}(\pi^{1/2})P_0\text{diag}(\pi^{-1/2})$, $L = I - S$; S is symmetric with Perron vector $\pi^{1/2}$, and the Doob normalisation of $e^{-a(I - P)}$ is exactly $P_{\text{heat}}(a)$. The change of representation alters neither the spectrum nor the normalised law.

6.2 Passing the criterion is cheap

Lemma 7 (heat regularisation always passes). Let P be any irreducible reversible chain with stationary law π . Then for every $a > 0$ the kernel $e^{-a(I - P)}$ is reversible in π , has strictly positive entries, and has spectrum in $(0, 1]$. It therefore satisfies both conditions of Theorem 1.

Proof. Reversibility is inherited by functions of a π -self-adjoint matrix. $e^{-a(l-P)} = e^{-a\sum_k a^k P^k/k!}$, every term is nonnegative and irreducibility gives a positive term for each pair of states. The spectrum of P lies in $[-1, 1]$, so that of $e^{-a(l-P)}$ lies in $[e^{-2a}, 1] \subset (0, 1]$. ■

The lemma is the reason the heat benchmark cannot be read as evidence. Condition (ii) of Theorem 1 can always be met by passing from a reversible one-step law to its heat law, whatever the one-step spectrum; even the two-state flip, with eigenvalue -1 , has a heat kernel with positive entries and spectrum $\{1, e^{-2a}\}$. What a source must justify is therefore the law change itself, the replacement of P_0 by $e^{-a(l-P_0)}$, and not the improved spectrum. Neither squaring, lazifying nor exponentiating an arbitrary physical transfer preserves a source-derived law merely by improving its spectrum; in the wheel the replacement is justified only because §56A.2 designates the heat sandwich in the first place.

Part C. The closure-circulation obstruction and its information return

7. The tilted wheel

HMGBC §14B gives the closure-circulation score $F_C(b_i, b_{i+1}) = 1$ and $F_C(b_i, b_{i-1}) = -1$, with all other entries zero, and the row-normalised exponential deformation $P_\epsilon(i, j) = P_0(i, j)e^{\epsilon F(i, j)}/Z_\epsilon(\epsilon)$ [2, p. 31]. Apply exactly that deformation with only $\epsilon_C = \epsilon$ nonzero. The hub row is unchanged. On a rim row the four unnormalised weights, to the hub, to itself, forward and backward, are $1, 1, e^\epsilon, e^{-\epsilon}$, so

$$Z = 2 + 2 \cosh \epsilon, \quad P_\epsilon(b_i, h) = P_\epsilon(b_i, b_i) = 1/Z, \quad P_\epsilon(b_i, b_{i+1}) = e^\epsilon/Z, \quad P_\epsilon(b_i, b_{i-1}) = e^{-\epsilon}/Z.$$

Proposition 8 (the tilted wheel). For every real ϵ : (a) the stationary law is $\pi_h = 7/(7 + 6Z)$ and $\pi_b = Z/(7 + 6Z)$ on each rim state; (b) the cycle affinities are $A_{\text{triangle}} = 2\epsilon$ for every triangle (h, b_i, b_{i+1}, h) and $A_{\text{rim}} = 12\epsilon$ for the six-edge rim cycle; (c) the stationary current along each positively oriented rim edge is $j_{\text{rim}} = 2 \sinh \epsilon/(7 + 6Z)$, and the six spokes carry no current; (d) P_ϵ is reversible if and only if $\epsilon = 0$, in every positive measure.

Proof. (a) Flow into the hub is $\sum_i \pi_b/Z = 6/(7 + 6Z)$; flow out is $\pi_h \cdot (6/7) = 6/(7 + 6Z)$. The rim states are equivalent under rotation, so the uniform rim law is stationary once the hub balance holds. (b) The forward product round the triangle is $(1/7)(e^\epsilon/Z)(1/Z)$ and the reversed product is $(1/7)(e^{-\epsilon}/Z)(1/Z)$; their ratio is $e^{2\epsilon}$. Round the rim the ratio is $(e^\epsilon/e^{-\epsilon})^6 = e^{12\epsilon}$. (c) $\pi_b P_\epsilon(b_i, b_{i+1}) - \pi_b P_\epsilon(b_{i+1}, b_i) = (e^\epsilon - e^{-\epsilon})/(7 + 6Z)$. On a spoke, $\pi_h/7 = 1/(7 + 6Z) = \pi_b/Z$, so the two directions balance exactly. (d) Reversibility forces every cycle ratio to be one (Kolmogorov [8]); (b) gives $e^{2\epsilon} = 1$. Detailed balance is a property of P_ϵ alone, so the failure holds in every measure. ■

7.1 Exact rational benchmarks

When e^ϵ is rational the whole transition matrix is rational and everything can be checked in exact arithmetic. At $e^\epsilon = 2$ the rim row is $(2/9, 2/9, 4/9, 1/9)$ to (hub, self, forward, backward); at $e^\epsilon = 3$ it is $(3/16, 3/16, 9/16, 1/16)$.

Quantity	$e^\epsilon = 2$	$e^\epsilon = 3$	General ϵ
Z	$9/2$	$16/3$	$2 + 2 \cosh \epsilon$
π_h	$7/34$	$7/39$	$7/(7 + 6Z)$
π_b	$9/68$	$16/117$	$Z/(7 + 6Z)$
Triangle ratio $e^{2\epsilon}$	4	9	$e^{2\epsilon}$
Rim ratio $e^{12\epsilon}$	4096	531441	$e^{12\epsilon}$

Quantity	$e^\epsilon = 2$	$e^\epsilon = 3$	General ϵ
Rim current j_{rim}	3/68	8/117	$2 \sinh \epsilon / (7 + 6Z)$
Entropy production σ (Section 9)	$(9/17) \ln 2 = 0.366960$	$(32/39) \ln 3 = 0.901425$	$24\epsilon \sinh \epsilon / (7 + 6Z)$

These are exact benchmark obstructions, not inferred physical current magnitudes. For small ϵ the current is linear in the tilt with coefficient fixed by the wheel: $j_{\text{rim}} = (2/31)\epsilon + O(\epsilon^3)$, since $Z \rightarrow 4$ and $7 + 6Z \rightarrow 31$. The value $e^\epsilon = 2$ is a convenient rational point and carries no physical meaning.

8. The spectral certificate

A reversible chain is similar to a self-adjoint matrix and therefore has real spectrum. A complex eigenvalue is a basis-free certificate that no reference measure makes the chain self-adjoint, and it can be checked without tracing cycles. For the tilted wheel the certificate is explicit, because the tilt respects the rotation symmetry of the rim even though it breaks the reflection symmetry.

Theorem 9 (spectrum of the tilted wheel). For every real ϵ the eigenvalues of P_ϵ are 1, $1/7 - 1/Z$, and

$$\lambda_k = [1 + 2 \cosh \epsilon \cos(\pi k/3) + 2i \sinh \epsilon \sin(\pi k/3)] / Z, \quad k = 1, \dots, 5,$$

with eigenvectors the rim Fourier modes of Section 5. The imaginary parts are $\pm(3^{1/2}) \sinh \epsilon / Z$ for $k = 1, 2, 4, 5$ and zero for $k = 3$. Consequently P_ϵ has four non-real eigenvalues for every $\epsilon \neq 0$, is not similar to any self-adjoint matrix, and is not the Doob kernel of any μ -self-adjoint transfer for any μ . At $e^\epsilon = 2$ the spectrum is exactly

$$\{1, -5/63, -1/3, 1/2 \pm i \cdot 3^{1/2}/6, -1/18 \pm i \cdot 3^{1/2}/6\},$$

with characteristic polynomial $(x - 1)(x + 5/63)(x + 1/3)(x^2 - x + 1/3)(x^2 + x/9 + 7/81)$ and complex moduli $3^{-1/2}$ and $7^{1/2}/9$.

Proof. The rim block of P_ϵ is circulant with entries $1/Z$ (self), e^ϵ/Z (forward) and $e^{-\epsilon}/Z$ (backward), so the Fourier mode v_k is an eigenvector of the rim block with eigenvalue $[1 + e^\epsilon e^{i\pi k/3} + e^{-\epsilon} e^{-i\pi k/3}]/Z = [1 + 2 \cosh(\epsilon + i\pi k/3)]/Z$, which expands to the display. For $k \neq 0$ the mode has zero rim sum, so the hub row annihilates it, and its hub component is zero, so the hub column does not act; hence it is an eigenvector of P_ϵ . The two remaining eigenvalues belong to the block on the hub indicator and the uniform rim vector, $[[1/7, 6/7], [1/Z, 1 - 1/Z]]$, which has trace $1/7 + 1 - 1/Z$ and eigenvalue 1, so the other eigenvalue is $1/7 - 1/Z$. At $\epsilon = 0$ the display reduces to the spectrum of P_0 . The $e^\epsilon = 2$ values follow by substitution: $Z = 9/2$, $\cosh \epsilon = 5/4$, $\sinh \epsilon = 3/4$. ■

The theorem also locates the irreversibility. The $k = 0$ block and the $k = 3$ alternating mode stay real for every ϵ ; the tilt rotates only the $k = \pm 1$ and $k = \pm 2$ pairs into the complex plane, by an angle whose tangent is $2 \sinh \epsilon \sin(\pi k/3) / (1 + 2 \cosh \epsilon \cos(\pi k/3))$. This is a one-line test a referee can run on any proposed physical kernel: compute the spectrum, and if any eigenvalue is non-real the kernel is outside the class of SGW-3 Proposition 4 regardless of the measure.

9. Entropy production and parity

Cycle affinities say that a chain is irreversible; the entropy production rate says by how much, and it is the quantity in which any eventual statement of output-invariance in the irreversible class would have to be made. For a stationary chain it is the Schnakenberg rate [14]

$$\sigma = \sum_{i < j} [\pi_i P_{ij} - \pi_j P_{ji}] \ln[\pi_i P_{ij} / (\pi_j P_{ji})] \geq 0,$$

with equality exactly in the reversible case.

Proposition 10 (entropy production of the tilted wheel). $\sigma(\epsilon) = 12 \epsilon j_{\text{rim}} = 24 \epsilon \sinh \epsilon / (7 + 6Z)$. All of it is produced on the six rim edges; the six spokes contribute nothing. At $e^\epsilon = 2$, $\sigma = (9/17) \ln 2 = 0.36696$ nats per step; for small ϵ , $\sigma = (24/31)\epsilon^2 + O(\epsilon^4)$.

Proof. By Proposition 8(c) the spoke terms vanish identically. On each rim edge the flux is j_{rim} and the logarithmic ratio is $\ln(e^\epsilon/e^{-\epsilon}) = 2\epsilon$, so $\sigma = 6 \cdot j_{\text{rim}} \cdot 2\epsilon$. At $e^\epsilon = 2$, $j_{\text{rim}} = 3/68$ and $2\epsilon = 2 \ln 2$, so $\sigma = 6 \cdot (3/68) \cdot 2 \ln 2 = (36/68) \ln 2 = (9/17) \ln 2$. ■

Two consequences are worth stating. First, the irreversibility of the tilted history is confined to the rim: a readout that sees only hub-to-rim and rim-to-hub transitions sees edges in exact detailed balance, although the hub's return statistics still depend on ϵ through Z . Second, σ is quadratic in the tilt, with the wheel-fixed coefficient $24/31$, so any linear-response statement about the irreversible class has a definite scale against which to be compared. Neither consequence identifies the physical tilt; both are properties of the canonical benchmark.

Proposition 11 (reversal of the tilt reverses the path law). For the wheel and for any rotation-symmetric reversible baseline, $\pi_\epsilon(i)P_\epsilon(i, j) = \pi_{-\epsilon}(j)P_{-\epsilon}(j, i)$ for every transition. Consequently reversing ϵ reverses the stationary path law, and every time-reversal-even finite-path observable has an expectation that is even in ϵ .

Proof. The stationary masses depend on ϵ only through $\cosh \epsilon$. Reversing ϵ exchanges the forward and backward rim weights e^ϵ/Z and $e^{-\epsilon}/Z$ while the hub, self and spoke weights are unchanged, so the edge flow $\pi_\epsilon(i)P_\epsilon(i, j)$ equals the reversed-tilt reversed flow entry by entry. Multiplying along a path, including the stationary endpoint factor, gives the reversal identity for path probabilities, and an even observable then has the same expectation under ϵ and $-\epsilon$. ■

This is a statement about the canonical benchmark and its baselines. It is not a parity theorem for every field-dependent physical observable, nor for the complete master action, whose record and completion coordinates need not transform under the reversal at all.

10. Reversible baselines and the cycle-sum test

Proposition 12 (the obstruction survives any reversible baseline). Let P^* be any reversible chain on a connected bidirectional support and F any score, and set $P_\epsilon(i, j) = P^*(i, j)e^{\epsilon F(i, j)}/Z_i(\epsilon)$. Then for every directed cycle γ on the support,

$$A_\gamma(\epsilon) = \epsilon \sum_{(i, j) \in \gamma} [F(i, j) - F(j, i)].$$

In particular the closure tilt gives triangle affinity 2ϵ and rim affinity 12ϵ on the heat baseline $P_{\text{heat}}(a)$ for every a , exactly as on P_0 , and repairing the spectral obstruction of Section 5 does nothing to the circulation obstruction.

Proof. In the ratio of the forward to the reversed product round γ , every row normaliser Z_i appears once in each product and cancels, and the baseline products cancel by Kolmogorov's criterion for P^* , leaving $e^{\epsilon \sum [F(i, j) - F(j, i)]}$. ■

On the heat baseline at $a = 1$ and $e^\epsilon = 2$ the numbers are: stationary law $\pi_h = 0.2147112$ and $\pi_b = 0.1308815$, triangle ratio 4 to machine precision, rim current 0.0245682 and entropy production 0.2043522 nats per step. The spectral certificate also persists in closed form. Any baseline with the wheel's rotation symmetry, that is a hub row plus a circulant rim block with entries p_0, p_1, p_2, p_3 at rim distances $0, 1, 2, 3$ and spoke entry p_h , has tilted Fourier eigenvalues

$$\lambda_k = [p_0 + 2p_1(\cosh \epsilon \cos(\pi k/3) + i \sinh \epsilon \sin(\pi k/3)) + 2p_2 \cos(2\pi k/3) + p_3 \cos(\pi k)] / Z_h, \quad Z_h = p_h + p_0 + 2p_1 \cosh \epsilon + 2p_2 + p_3,$$

with imaginary parts $\pm 2p_1 \sinh \epsilon \sin(\pi k/3)/Z_h$, nonzero whenever $p_1 > 0$ and $\epsilon \neq 0$. On the heat baseline at $a = 1$ the maximal imaginary part is 0.1625642 .

Proposition 13 (which tilts stay reversible). With P^* and F as in Proposition 12, P_ϵ is reversible for one, equivalently for every, nonzero ϵ if and only if the antisymmetric part of F is a gradient on the support: $F(i, j) - F(j, i) = \psi_j - \psi_i$ for some function ψ . In that case the stationary law is $\pi_\epsilon(i) \propto \pi^*(i) Z_i(\epsilon) e^{\epsilon \psi_i}$.

Proof. By Proposition 12 and Kolmogorov's criterion, reversibility is equivalent to zero cycle sums of the antisymmetric part, which on a connected graph is equivalent to its being a gradient. Given the gradient form, set $g_i = \pi_\epsilon(i)/(\pi^*(i)Z_i)$; detailed balance for P_ϵ reads $g_i e^{\epsilon F(i,j)} = g_j e^{\epsilon F(j,i)}$, that is $g_i/g_j = e^{\epsilon(\psi_i - \psi_j)}$, so $g_i \propto e^{\epsilon \psi_i}$. ■

This is a usable exact test for each proposed physical score direction, and it needs only the antisymmetric part of the score on an independent set of cycles. The canonical F_c fails it: its antisymmetric cycle sums are 2 on every triangle and 12 on the rim. A gradient example, with ψ twice the hub indicator and $e^\epsilon = 2$, is verified exactly in the package: the tilted wheel stays reversible with the predicted stationary law. The identification of F_c with the full physical defect deformation remains the separate HM12 and F34 obligation of the source.

11. The returned circulation information

SGW-3 consumes normalised source scores through the score covariance $W = E[ss^\dagger]$ on the complete Markov carrier, or through a separately justified complete-past construction [1, §18]. For the inherited circulation family that quantity can be evaluated exactly. Write ϵ for the declared tilt coordinate; physical defect units are not inferred from that declaration. The transition score and the returned diagonal entry are

$$s_c(i, j; \epsilon) = \partial_\epsilon \log P_\epsilon(i, j) = F(i, j) - \sum_k P_\epsilon(i, k) F(i, k), \quad w_c(\epsilon) = W_{cc}(\epsilon) = \sum_{i,j} \pi_\epsilon(i) P_\epsilon(i, j) s_c(i, j)^2.$$

The score has zero conditional mean on every row, so summed transition scores are martingale differences and distinct-time covariances vanish: on N stationary steps the Fisher information is $N w_c$, and a parameter-dependent stationary initial law adds a term that disappears from the information rate.

Theorem 14 (information rate of the wheel circulation family). $w_c(\epsilon) = 12/(7 + 6Z) = 12/(19 + 12 \cosh \epsilon)$. In particular $w_c(0) = 12/31$, $w_c(\ln 2) = 6/17$, the change from $\epsilon = 0$ to $\epsilon = \ln 2$ is $\Delta w_c = -18/527$, and $w_c(\epsilon) = 12/31 - (72/961)\epsilon^2 + O(\epsilon^4)$.

Proof. The hub row has zero score. On a rim row, $E[F] = 2 \sinh \epsilon/Z$ and $E[F^2] = 2 \cosh \epsilon/Z$, so the conditional score variance is $2 \cosh \epsilon/Z - 4 \sinh^2 \epsilon/Z^2 = 2/Z$, using $Z = 2 + 2 \cosh \epsilon$. The stationary rim mass is $6Z/(7 + 6Z)$; the product is $12/(7 + 6Z)$. ■

The information is even in ϵ , strictly decreasing in $|\epsilon|$ and tends to zero at large tilt, while the entropy production of Proposition 10 increases: in this family a more deterministic circulation carries less local information about a further change of the tilt. The value is an exact returned score coefficient, not a fitted stiffness. A physical defect coordinate ξ need not equal ϵ ; if $\epsilon = \epsilon(\xi)$ is the only dependence then $W_{\xi\xi} = (d\epsilon/d\xi)^2 w_c$, and several source coordinates require their full score matrix with its cross terms.

Theorem 15 (the heat baseline returns a different change). Let $b = P_{\text{heat}}(a)(b_i, b_{i+1})$ be the adjacent-rim entry of Theorem 6 and $z(\epsilon) = 1 + 2b(\cosh \epsilon - 1)$ the common rim normaliser under the closure tilt. Then the stationary masses are $\pi_h = 7/(7 + 24z)$, $\pi_b = 4z/(7 + 24z)$, and

$$w_c(\epsilon) = [24/(7 + 24z)] \cdot [2b \cosh \epsilon - 4b^2 \sinh^2 \epsilon/z] = 48b/31 + [24b(31 - 172b)/961] \epsilon^2 + O(\epsilon^4).$$

The quadratic coefficient is positive for $0 < b < 31/172$, zero at the threshold and negative above it. The one-step wheel has $b = 1/4$ and recovers Theorem 14; the heat baseline at $a = 1$ has $b = 0.1334949054$ and a positive change.

Proof. Baseline detailed balance on the spokes fixes the spoke mass ratio at $7/4$, and tilting changes every rim normaliser to z , so hub balance gives the stationary law. The rim conditional moments are $E[F] = 2b \sinh \epsilon/z$ and $E[F^2] = 2b \cosh \epsilon/z$; multiply the variance by the rim mass $6\pi_b$. The expansion follows from $z = 1 + b\epsilon^2 + O(\epsilon^4)$. ■

Quantity	One-step wheel	Heat baseline at a = 1
Adjacent rim entry b	1/4	0.1334949054
Information at $\varepsilon = 0$	12/31 = 0.3870967742	0.2067017890
Information at $\varepsilon = \ln 2$	6/17 = 0.3529411765	0.2180109635
Change	-18/527 = -0.0341555977	+0.0113091745
Coefficient of ε^2	-72/961 = -0.0749219563	+0.0268008082
Triangle affinity at $\varepsilon = \ln 2$	2 ln 2	2 ln 2

Identical cycle affinities therefore do not determine even the sign of the information change: the baseline law decides it, and the heat designation must be respected through the score covariance as well as through the kernel. Only W_{CC} is returned here. The full six-defect matrix may have changing off-diagonal entries, and those may appear at first order even where this diagonal channel is even; no full-action parity or cancellation is inferred from the scalar result. Part D turns the returned change into the exact change of the constrained response.

Part D. The constrained-action response

12. The exact finite constrained update

A returned change of the history metric enters the physical response only through the complete quadratic action of SGW-3 [1, §§20, 21]. Write that action on an admitted finite physical quotient as $H_0 = H_{\text{rest}} + L_D^\dagger W_0 L_D$, where L_D is the magnitude-bearing defect differential, W_0 the returned block metric with all its cross terms, and H_{rest} every remaining term of the quadratic parent. Fix the background, the hard preservation conditions, the physical attachment and the action conversion α . Let Q_0 lift a retained physical variation v to its baseline constrained minimiser and let the columns of N span every feasible auxiliary variation at fixed retained variable. Then

$$N^\dagger H_0 Q_0 = 0, \quad A = N^\dagger H_0 N > 0, \quad U = L_D N, \quad Z_0 = L_D Q_0, \quad K_0 = \alpha Q_0^\dagger H_0 Q_0.$$

Theorem 16 (exact constrained update). Let $H_1 = H_0 + L_D^\dagger \Delta W L_D$ with Hermitian ΔW , and suppose $A_1 = A + U^\dagger \Delta W U$ is positive definite. The exact change of the constrained response is

$$\Delta K = \alpha Z_0^\dagger [\Delta W - \Delta W U A_1^{-1} U^\dagger \Delta W] Z_0,$$

and the new minimising lift is $Q_1 = Q_0 - N A_1^{-1} U^\dagger \Delta W Z_0$. The formula needs neither invertibility nor positivity of ΔW , and it retains every channel cross term and the common relaxation of all admissible auxiliaries.

Proof. Every feasible variation with retained coordinate v is $q = Q_0 v + N y$. Baseline stationarity removes the H_0 cross term. Apart from the baseline retained energy, twice the quadratic energy is $y^\dagger A y + (Z_0 v + U y)^\dagger \Delta W (Z_0 v + U y)$; stationarity in y gives $y = -A_1^{-1} U^\dagger \Delta W Z_0 v$, and substitution gives the display. This is the equality-constrained Schur complement [15, Chapter 10 and Appendix A.5], specialised to the returned metric, valid for real or complex coordinates with the appropriate adjoint and unchanged under an invertible change of basis in the auxiliary space. ■

For infinitesimal ΔW the leading response is $\alpha Z_0^\dagger \Delta W Z_0$, and the subtracted relaxation term begins at second order in ΔW ; this agrees with the envelope sensitivity of the parent. If H_{rest} , L_D , the preservation

rows, the background, α or the attachment also change, their contributions must be included through the full sensitivity or a new constrained solve. The theorem is not permission to freeze quantities that the physical source changes.

13. Rank-one returns, order of the response and the native check

For one changing score coefficient let $H_1 = H_0 + t d^\dagger d$, where d is its fixed physically normalised defect row and $t = w_c(\epsilon) - w_c(0)$ is the returned change of Theorem 14 or Theorem 15. Set $z = dQ_0$ and $\beta = dNA^{-1}N^\dagger d^\dagger \geq 0$.

Corollary 17 (rank-one return). Provided $1 + \beta t > 0$,

$$\Delta K = \alpha [t/(1 + \beta t)] z^\dagger z, \quad \Delta J = \alpha [t/(1 + \beta t)] z O_J z^\dagger, \quad \Delta T = \alpha [t/(1 + \beta t)] z O_T z^\dagger.$$

Proof. The auxiliary change is the rank-one matrix $t(dN)^\dagger(dN)$, positive exactly when $1 + \beta t > 0$; its scalar resolvent in Theorem 16 gives the coefficient $t/(1 + \beta t)$, and the two prescribed traces give the contrasts. If $\beta = 0$ there is no relaxation correction. ■

Here K is the price operator on the inherited 47-dimensional carrier in sector order (Q, u, d, L, e, H) with dimensions (18, 9, 9, 6, 3, 2); it is distinct from the raw history transfer. The parent fixes [1, §1; 16, §12]

$$O_J = -P_Q + P_u + 3P_e, \quad O_T = (1/3)[-269P_Q + 445P_u + 286P_d - 396P_L + 477P_e - 396P_H].$$

Both operators are traceless, but their quadratic forms are signed: with $a_r = \|zP_r\|^2$, $zO_J z^\dagger = -a_Q + a_u + 3a_e$ and $zO_T z^\dagger = (-269a_Q + 445a_u + 286a_d - 396a_L + 477a_e - 396a_H)/3$. For nonzero α and t both contrasts vanish if and only if these two factors vanish: a nonzero circulation does not force a nonzero relative gauge correction, and positivity of the metric does not force either signed contrast to vanish. For the wheel benchmark $t = -18/527$ the multiplier is $-18\alpha/(527 - 18\beta)$, subject to $527 - 18\beta > 0$; for the heat benchmark at $a = 1$ it is $0.0113091744624\alpha/(1 + 0.0113091744624\beta)$. These are conditional source-to-action formulas; assigning z or β without their physical derivation would turn them into fitted examples.

Proposition 18 (order of the response). In the family of Corollary 17 the circulation-channel response is even in ϵ and has no linear term: $\Delta K = \alpha c_2 \epsilon^2 z^\dagger z + O(\epsilon^4)$ with $c_2 = -72/961$ for the wheel and $c_2 = 24b(31 - 172b)/961$ for the heat family.

Proof. Theorems 14 and 15 give $t = c_2 \epsilon^2 + O(\epsilon^4)$; expanding $t/(1 + \beta t)$ shows that re-relaxation first changes the coefficient at order ϵ^4 , so the quadratic coefficient is fixed by the information return and the baseline lift, and exact evenness follows from the even t . ■

The statement concerns one varying information channel with the remaining ingredients fixed. It neither proves that the total physical response is even nor excludes linear terms from other score channels, changing cross terms, preservation constraints or the rest of the action, and it does not establish $\Delta J = \Delta T = 0$ for the physical write.

13.1 Verification on the inherited alternating block

The native H4 alternating block of the parent is $H_3 = [[1, 1, 0], [1, 7, -1], [0, -1, 6]]$ on (v, θ, γ) [1, §5]. Retain v and impose the conditional record lock $\gamma = 0$. The baseline minimiser and response are $Q_0 = (1, -1/7, 0)^\top$ and $K_0 = 6/7$. Using the bond row $d = (0, 1, -1)$ purely as a structural check of the reduction formula, with $\alpha = 1$,

$$z = -1/7, \quad \beta = 1/7, \quad \Delta K = t/[7(7 + t)], \quad t = -18/527 \Rightarrow \Delta K = -18/25697.$$

Direct minimisation with the shifted θ coefficient gives $K(t) = 1 - 1/(7 + t)$, which reproduces the increment. The check does not identify HMGB's circulation coordinate with that H4 bond row; it is an algebraic control on an inherited native action, and the package also checks a two-channel update with a nonzero off-diagonal ΔW against a direct bordered constrained solve in exact rational arithmetic.

14. Sector attachment and the three-price slice

Keep the 47-dimensional carrier and the two contrast operators and put $\alpha = \alpha_{\text{loop}}$. For each sector define the matrix-valued per-state attachment on the residual carrier by $G_r = Z_o P_r Z_o^\dagger / d_r$, $d = (18, 9, 9, 6, 3, 2)$. With $\mathcal{B}$ the bracketed metric update of Theorem 16, cyclicity of the trace gives $\Delta J = \alpha \text{Tr}(\mathcal{B}C_J)$ and $\Delta T = \alpha \text{Tr}(\mathcal{B}C_T)$, where

$$C_J = -18G_Q + 9G_U + 9G_E, \quad C_T = -1614G_Q + 1335G_U + 858G_d - 792G_L + 477G_e - 264G_H.$$

Proposition 19 (ratio test). Assume $\alpha \neq 0$. The identity $C_T = rC_J$ is sufficient for $\Delta T = r\Delta J$ for every admitted finite metric change with the other ingredients fixed. It is necessary if all sufficiently small Hermitian metric directions are available; if the source changes occupy a restricted family, necessity holds only with respect to its attainable response span.

Proof. Sufficiency follows from the two trace contractions. For necessity, $\mathcal{B} = \Delta W + O(\|\Delta W\|^2)$, and the assumed identity for every small Hermitian ΔW gives $\text{Tr}[\Delta W(C_T - rC_J)] = 0$ in every Hermitian direction, hence $C_T = rC_J$. ■

A sufficient three-family reduction is $G_Q = G_L = \mathcal{L}$, $G_U = G_d = G_e = \mathcal{R}$ and $G_H = \mathcal{H}$, for which $C_J = 18(\mathcal{R} - \mathcal{L})$ and $C_T = 6[401(\mathcal{R} - \mathcal{L}) + 44(\mathcal{R} - \mathcal{H})]$; if $\mathcal{R} - \mathcal{H} = \eta(\mathcal{R} - \mathcal{L})$ the response ratio is $(401 + 44\eta)/3$ wherever $\Delta J \neq 0$, and if $\mathcal{R} = \mathcal{L}$ then ΔJ vanishes for every permitted update while ΔT can remain nonzero. Equality of scalar traces does not establish these matrix identities, and a_r is a total sector norm whose per-state weight is a_r/d_r ; omitting that division changes the physics of the comparison.

14.1 The three scalar prices and the comparison coordinate

When the effective per-state prices obey $w_Q = w_L = m_L$, $w_U = w_d = w_e = m_R$ and $w_H = h$, the inherited contraction is [16, §12]

$$J = 18\alpha(m_R - m_L), \quad T = 6\alpha(-401m_L + 445m_R - 44h), \quad T/J = (401 + 44\eta)/3, \quad \eta = (m_R - h)/(m_R - m_L).$$

Effective price pattern	J	T	T divided by J
All prices equal	0	0	undefined
$m_L = m_R = m$, Higgs h	0	$264\alpha(m - h)$	undefined
$h = m_R$, $\Delta = m_R - m_L \neq 0$	$18\alpha\Delta$	$2406\alpha\Delta$	$401/3$
$h = m_L$, $\Delta \neq 0$	$18\alpha\Delta$	$2670\alpha\Delta$	$445/3$

If h lies between m_L and m_R the nonzero-J ratio lies between $401/3$ and $445/3$. The gauge representative of a pair (J, T) modulo the common direction $Z = (6, 13/2, 21/2)$ is $v = (0, (53J - T)/528, (229J - T)/528)$, which on the matter-universal slice is $(0, -T/528, -T/528)$: a boson-fermion distinction in the complete prices contributes T while J vanishes. These equalities are an additional source statement about the effective response; uniformity inside a representation is not equality between representations.

At the one-loop comparison grade the common normalisation and the matching-scale direction are quotiented out, which constrains T and leaves J degenerate with the matching scale [35, §21]. The retained diagnostic value $T = -33.48185948$ is therefore a comparison coordinate and not an input to the action. Algebraically it would require $\alpha(h - m) = 0.1268252253$ on the matter-universal slice, or $\alpha(m_L - m_R) = 0.01391598482$ for $h = m_R$ and 0.01254002228 for $h = m_L$. These are conditional translations, not solved source weights; a physical branch must freeze its action, attachment, normalisation and complete response before the full Stage-B transport and comparison, and neither the diagnostic T nor a desired ratio may choose the susceptibilities of Section 16.

15. Bond and composition calculus; changes confined to a frozen direction

The master action keeps the gauge transport U and the record transport Φ independent [17]. On a fixed occupied support with equal source and target rank, write the left polar decomposition $M = BV$ with $B > 0$ and V unitary between the occupied spaces. The inherited polar differential is the Sylvester equation [17, Appendix B; 21]

$$E = \delta M V^\dagger, \quad \Omega = \delta V V^\dagger, \quad B\Omega + \Omega B = E - E^\dagger.$$

For a pure stretch $\delta M = \delta B \cdot V$ with Hermitian δB , $\Omega = 0$; for a pure angular variation $\delta M = B(iX)V$ with Hermitian X , $\Omega = iX$ even when B and X do not commute. With U and the supports fixed, the bond defect $D_B = U - V$ has $\delta D_B = -iXV$, and under a common Hilbert-Schmidt norm on a full-support block $X = xP_r$ costs $d_r x^2$: the same direct per-state phase cost in every sector. This is a conditional direct-bond calculation, not equality of the fully relaxed physical prices. The record composition and its independent linear-coordinate derivatives are [19, §9L]

$$D_R = \Phi_{12}\Phi_{01} - \Phi_{02}, \quad D_R'[p] = p_{12}\Phi_{01} + \Phi_{12}p_{01} - p_{02}, \quad D_R''[p, q] = p_{12}q_{01} + q_{12}p_{01}.$$

On a compatible triangle with central sector amplitudes $\Phi_{e,r} = b_{e,r}V_{e,r}$, $b_{02} = b_{12}b_{01}$ and $V_{02} = V_{12}V_{01}$, scalar angular variations give $\delta D_{R,r} = ib_{02,r}V_{02,r}(y_{12} + y_{01} - y_{02})$: the direct per-state squared norm is b_{02}^2 times the squared angular mismatch, so stretch-independence of the polar angular derivative does not remove record-amplitude dependence from the composition response. RCHS-24 supplies the composition covectors $\Phi_{12}^\dagger \tau_R$, $\tau_R \Phi_{01}^\dagger$ and $-\tau_R$, with $\tau_X = \Sigma_Y W_{XY} D_Y$, and their common cross-metric [19]. At a common defect zero the action contribution is the Jacobian pullback; away from it the terms in D_R'' , all other defect jets, field-dependent metrics, support motion and constraint reactions also enter, and bond and record fields must be relaxed together in one parent.

Theorem 20 (changes confined to a frozen direction). Let $Lq = 0$ be the fixed linearised fact constraint on the physical quotient. If $H(c) = H_0 + L^\dagger X(c) + X(c)^\dagger L$, then every feasible quadratic form is unchanged, and whenever the constrained minimisation is well posed the reduced operator K and its fixed-attachment J and T are independent of c .

Proof. For every feasible q , $q^\dagger L^\dagger Xq + (Xq)^\dagger Lq = 0$, so the objective is pointwise identical on the same feasible set and its minimum at each retained probe is identical. This covers mixed terms touching a frozen row, not only its squared stiffness. ■

On the inherited six-mode parent on (v, θ, y) with $G = 3I - S_6 - S_6^\dagger$, normalised alternating vector f_3 and $P_3 = f_3 f_3^\dagger$, vary only the alternating record-composition coefficient and impose the conditional same- Φ lock $f_3^\dagger y = 0$. The alternating block is $[[1, 1, 0], [1, 7, -1], [0, -1, \kappa]]$ with $\kappa = 1 + 5c$, and eliminating the complete allowed auxiliary set gives:

Retained coordinate	Free response	Locking increment	Total with the record held
Alternating link θ_3	$6 - 1/\kappa$	$1/\kappa$	6
Alternating current v_3	$(6\kappa - 1)/(7\kappa - 1)$	$1/[7(7\kappa - 1)]$	6/7

At $c = 1$ these are $35/6 + 1/6 = 6$ and $35/41 + 1/287 = 6/7$; the full 18-coordinate bordered solves confirm invariance of the entire retained six-by-six matrices, and the other modes keep their previous responses. This is an exact native cancellation whose coefficients are not physical prices. The theorem does not freeze every record direction because one fact is retained: a physical write can also change allowed composition directions, cross blocks not annihilated by L , the background, the constraint map or the attachment, and in nonlinear constrained coordinates H_0 is the Lagrangian Hessian with its reaction terms.

16. The response resolved by relaxation mode

Return to Theorem 16 with the fact-preserving baseline lift Q_0 and the allowed auxiliary columns N , and let the complete information change follow one fixed Hermitian direction $\mathcal{V}$, including any off-diagonal blocks: $\Delta W = t\mathcal{V}$, with every other ingredient fixed and no sign restriction on $\mathcal{V}$. Define $S = A^{-1/2}U^\dagger\mathcal{V}UA^{-1/2}$ and $B = A^{-1/2}U^\dagger\mathcal{V}Z_0$; stability means $I + tS > 0$ on a connected interval containing $t = 0$.

Theorem 21 (one-direction response). $\Delta K = \alpha[tZ_0^\dagger\mathcal{V}Z_0 - t^2B^\dagger(I + tS)^{-1}B]$. With Π_λ the spectral projectors of S for its distinct eigenvalues and, for $i = J, T$, $d_i = \text{Tr}(O_i Z_0^\dagger\mathcal{V}Z_0)$ and $b_{i,\lambda} = \text{Tr}(O_i B^\dagger \Pi_\lambda B)$,

$$\Delta J/\alpha = t d_J - t^2 \sum_\lambda b_{J,\lambda}/(1 + t\lambda), \quad \Delta T/\alpha = t d_T - t^2 \sum_\lambda b_{T,\lambda}/(1 + t\lambda).$$

Proof. Factor $A + tU^\dagger\mathcal{V}U = A^{1/2}(I + tS)A^{1/2}$ in Theorem 16 and substitute; no inverse of $\mathcal{V}$ is needed. ■

The first term is the direct response of the baseline lift; the second is the change caused by every allowed relaxation mode, whose signed contrast can be nonzero when the direct contrast vanishes. The eigenvalues λ are not the sector weights and their number is not six. A nonlinear deformation $\Delta W(\epsilon)$ that changes direction as well as magnitude needs Theorem 16 itself.

Theorem 22 (exact zero and fixed-ratio conditions). Under the hypotheses of Theorem 21 with $\alpha \neq 0$, ΔJ vanishes identically on an open stable interval if and only if $d_J = 0$ and $b_{J,\lambda} = 0$ for every distinct λ , and $\Delta T = r\Delta J$ throughout that interval if and only if $d_T = rd_J$ and $b_{T,\lambda} = rb_{J,\lambda}$ for every distinct λ .

Proof. Dividing the response identity by t and taking the limit isolates the direct coefficient. The remaining identity is a linear combination of $(1 + t\lambda)^{-1}$; multiplying by the denominator product and evaluating at each pole isolates every coefficient, a possible $\lambda = 0$ contributing a constant that then also vanishes. These are rational identities, so the poles need not lie in the stable interval. The converse is substitution; the ratio claim follows by applying the argument to $\Delta T - r\Delta J$. ■

A ratio is defined only where $\Delta J \neq 0$, a cancellation at one t can be accidental, and the total pair additionally requires $T_0 = rJ_0$ for a fixed total ratio.

Corollary 23 (positive deformations). If the added form is $tR^\dagger R$ with $t > 0$, put $Z_R = RQ_0$, $U_R = RN$ and $S_R = U_R A^{-1} U_R^\dagger$. Then $\Delta K = \alpha t Z_R^\dagger (I + tS_R)^{-1} Z_R$; for $\alpha > 0$ it is positive semidefinite and vanishes exactly when $RQ_0 = 0$. Nonzero RN alone does not establish a driven response.

Theorem 20 is recovered when R factors through the frozen constraint, since $LQ_0 = LN = 0$; conversely RQ_0 can vanish with $RN \neq 0$, a case the package checks in a dense constrained parent. The three objects are therefore distinct: the full fact-preserving tangent, the auxiliary directions N available at fixed probe, and the physically driven lift Q_0 . The rank of a composition differential on an allowed tangent, without its attachment and reduction, does not determine J or T .

16.1 Universal direct coupling and unequal relaxation

A complete algebraic construction shows why direct per-state universality is insufficient. Retain $x \in \mathbb{C}^{47}$ and relax $a \in \mathbb{C}^{47}$ with $H_0 = I$ and record differential $R = [I, D_\beta]$, $D_\beta = \text{diag}(\beta_r^{1/2})$ with $\beta_r \geq 0$ constant on each sector; take β_L on Q and L , β_R on u , d and e , and β_H on H . The direct added block is tl on all 47 states, yet eliminating the a coordinates gives the per-state increment $w_r(t) = t/(1 + \beta_r t)$.

Proposition 24. $\Delta J = 18\alpha t^2(\beta_L - \beta_R)/[(1 + \beta_L t)(1 + \beta_R t)]$ and $\Delta T = 6\alpha t[-401/(1 + \beta_L t) + 445/(1 + \beta_R t) - 44/(1 + \beta_H t)]$. For $\beta_L \neq \beta_R$ the ratio is $(401 + 44\eta(t))/3$ with $\eta(t) = [(\beta_H - \beta_R)/(\beta_L - \beta_R)] \cdot (1 + \beta_L t)/(1 + \beta_H t)$; for $\beta_L = \beta_R = \beta_M$, $\Delta J = 0$ and $\Delta T = 264\alpha t^2(\beta_H - \beta_M)/[(1 + \beta_M t)(1 + \beta_H t)]$.

Proof. The Schur complement subtracts $t^2 D_\beta (I + t D_\beta^2)^{-1} D_\beta$ from $(1 + t)I$; subtracting the baseline leaves $w_r = t/(1 + \beta_r t)$, and the sector contractions of Section 14.1 give the display. ■

Both direct contrasts vanish at order t ; unequal allowed relaxation first generates them at order t^2 , and if $t = O(\epsilon^2)$ the generated contrasts begin at $O(\epsilon^4)$ with every other ingredient fixed. The package evaluates $(\beta_L, \beta_R, \beta_H) = (1, 2, 4)$ at $t = 1/10, 1, 2$ and returns $\Delta J/\alpha = -3/22, -3, -24/5$ and $\Delta T/\alpha = -2323/154, -1829/5, -1784/3$, with ratios $2323/21, 1829/15$ and $1115/9$: deliberately unselected numbers showing

that direct universality does not enforce a fixed relaxed ratio. The construction neither forces nor excludes the matter-Higgs or left-right susceptibility difference of the physical source.

17. Rank, representation restriction and averaging

On the positive-update branch with frozen attachment and preservation tangent, Corollary 23 writes $\Delta K = \alpha t Z^\dagger (I + tS)^{-1} Z$, the Gram operator of $(\alpha t)^{1/2} (I + tS)^{-1/2} Z$. Hence $\text{rank}(\Delta K) = \text{rank}(Z) \leq \text{rank}(R)$: relaxation can change eigenvalues and contrasts but cannot raise the rank above that of the activated attachment, and the rank-one expression of Corollary 17 stays rank one after its scalar screening.

Proposition 25 (support of an invariant update). On the gauge-invariant branch an operator commuting with the gauge group is $\bigoplus_r (B_r \otimes I_{n_{\{r\}}})$ with gauge dimensions $n = (6, 3, 3, 2, 1, 2)$ and family multiplicities $m = (3, 3, 3, 3, 3, 1)$, so $\text{rank}(\Delta K) = \sum_r n_r \text{rank}(B_r)$. If the operator is also family blind, $\Delta K = \sum_r c_r P_r$ and the rank is the sum of the dimensions of the sectors with $c_r \neq 0$.

Sector	Gauge rank minimum	Family-blind rank	$\Delta J/c$	$\Delta T/c$
Q	6	18	-18	-1614
u	3	9	9	1335
d	3	9	0	858
L	2	6	0	-792
e	1	3	9	477
H	2	2	0	-264

For gauge invariance alone the only possible rank-one block is the charged-lepton singlet e , whose family matrix can have rank one; since $O_J = 3I$ and $O_T = 159I$ on that block, such a change obeys $\Delta T = 53\Delta J$. With family blindness rank one is impossible and the unique rank-two support is the Higgs doublet, giving $\Delta K = cP_H$, $\Delta J = 0$ and $\Delta T = -264c$, negative on the positive-update branch, with colour-subtracted gauge correction $(0, c/2, c/2)$. Rank three selects e , rank four is impossible, and rank five is $e \oplus H$. These restrictions apply to ΔK itself, not to a representative $\Delta K + c_0 I$ after discarding a common part, and covariance alone does not give commutation at a fixed oriented background: the classification needs an invariant branch or a justified invariant reduction, and equal family averages do not make an operator family blind.

Averaging an already reduced rank-one seed $\Delta K = cz^\dagger z$ over the gauge and family groups gives $c\sum_r (a_r/d_r)P_r$, which preserves the J and T contractions because both contrast operators are central, but can raise the rank from one to 47; the six averaged weights are therefore not evidence that the original channel was a complete invariant response. The order of operations also matters: for a positive parent with responding auxiliaries, $\text{Schur}(\langle H \rangle) \geq \langle \text{Schur}(H) \rangle$, because the left side minimises the averaged form over one auxiliary while the right side averages separate minima. An exact example couples a gauge-singlet auxiliary of stiffness $c > b^2$ only to a normalised Higgs vector v through b : gauge averaging gives $\text{Schur}(\langle H \rangle) = I_{47}$ and $\langle \text{Schur}(H) \rangle = I_{47} - (b^2/2c)P_H$, a positive gap of $b^2 P_H/(2c)$ that is invisible to J and gives $\Delta T_{\text{gap}} = -132b^2/c$. The constants are diagnostic, not master-action parameters.

Two consequences for the source follow. The next evaluated source object must carry its gauge representation as well as its numerical score: on the invariant branch the image of the activated attachment must support every irreducible representation appearing in the complete response, and a

genuinely gauge-singlet linear score channel cannot intertwine directly into the charged 47-state carrier without multiple tensor components, a transforming background or a nonlinear contribution. And the six defect labels of the master action are six kinds of residual, not six scalar directions; neither label counting nor a binary terminal record supplies the rank premise.

Part E. The charged completion, differential order and the symmetry question

18. The charged completion readout and its exact norm identities

The master action supplies an actual map from the record completion to the charged fermion bilinear [17, §§31 to 35]. Its ingredients are the local completion strain X_H , the polar orientation V_H and the raw terminal-address maps $A^{(0)}_{fX}$; they define $O_H = V_H^\dagger X_H$ and $M_f = \Lambda_\star A^{(0)\dagger}_{fL} O_H A^{(0)}_{fR}$. Along a smooth source family on a fixed occupied-address carrier, with the raw maps and the conversion $\Lambda_\star$ fixed, the response insertion is $\dot{M}_f = \Lambda_\star A^{(0)\dagger}_{fL} [\dot{V}_H^\dagger X_H + V_H^\dagger \dot{X}_H] A^{(0)}_{fR}$, which carries both orientation and stretch; a threshold crossing needs a finite reconstruction of the occupied carrier, not differentiation through a rank jump, and the dot is a response derivative on a declared family, not a fractional event. The executed unit charged branch $Y_u = Y_d = Y_e = I_3$ [17, §47G] is degenerate and is not the physical mass hierarchy, but its readout fixes the charged representation support.

Write the charged readout maps as $A_u: u \rightarrow Q$, $A_d: d \rightarrow Q$ and $A_e: e \rightarrow L$, of shapes 18×9 , 18×9 and 6×3 , and embed them with their adjoints into a Hermitian 47-state operator F with zero Higgs block: F has the blocks A_u , A_d in the Q row, A_e in the L row, and their adjoints below the diagonal. F is a one-particle representation of the charged readout, not a positive classical Hessian on Grassmann fields and not yet the loop price K . Let $G = F^2$ and let x_u , x_d , x_e be the squared Frobenius norms of the three blocks.

Proposition 26 (exact sector norm identities). The sector traces of G are $\text{Tr}(P_Q G) = x_u + x_d$, $\text{Tr}(P_u G) = x_u$, $\text{Tr}(P_d G) = x_d$, $\text{Tr}(P_L G) = \text{Tr}(P_e G) = x_e$ and $\text{Tr}(P_H G) = 0$, and therefore

$$J_G = \text{Tr}(O_J G) = 3x_e - x_d, \quad T_G = \text{Tr}(O_T G) = (176x_u + 17x_d + 81x_e)/3.$$

Proof. The quark-doublet trace is $x_u + x_d$ and the up-singlet trace is x_u , so the up contribution cancels from J_G ; in T_G each charged map occurs on both chiral sides with coefficients $(445 - 269)/3$, $(286 - 269)/3$ and $(477 - 396)/3$. No family-blindness assumption or gauge average is needed. ■

For the scalar-compatible construction with a normalised Higgs orientation h and its conjugate $\tilde{h}$, $A_u = C_u \otimes I_3 \otimes \tilde{h}$, $A_d = C_d \otimes I_3 \otimes h$ and $A_e = C_e \otimes h$, and with $u = \text{Tr}(C_u^\dagger C_u)/3$, $d = \text{Tr}(C_d^\dagger C_d)/3$ and $e = \text{Tr}(C_e^\dagger C_e)/3$ the colour multiplicity gives $(x_u, x_d, x_e) = (9u, 9d, 3e)$, hence $J_G = 9(e - d)$ and $T_G = 528u + 51d + 81e$. On the executed unit branch the combined right-to-left charged map has rank 21, F and G have rank 42, G has unit eigenvalues there and vanishes on the three unpaired left-lepton directions and the two Higgs states, and

$$(J_G, T_G) = (0, 660): \quad \text{left charged spaces } (-18, -2010), \quad \text{right charged spaces } (18, 2670).$$

The unit branch cancels chirality in J_G but does not give the matter-universal value 264; the difference of 396 comes from the three left-lepton directions that this charged completion does not read, and the separate neutral symmetric completion of the master action is retained, so nothing here says that neutrinos are massless. A radial write of amplitude s multiplies these Gram results by $|s|^2$ only if its insertion is s times the unit readout, and the physical s is not selected.

Proposition 27 (sign restriction of the readout Gram). $T_G \geq \max(9J_G, -17J_G/3)$, because $T_G - 9J_G = (176x_u + 44x_d)/3 \geq 0$ and $T_G + 17J_G/3 = (176x_u + 132x_e)/3 \geq 0$; an e -only map attains the first boundary and a d -only map the second, and every nonzero charged readout has $T_G > 0$.

If a separate source theorem established a positive contribution γG to K , it would imply $\Delta T > 0$ for that contribution; no such theorem exists, and a Schur subtraction or a different source metric cannot be

assigned this sign by analogy. The same source also warns against mistaking readout size for action cost: on its fixed-support minimal scalar-stretch family $X_H = (\phi_H/\Lambda_*)P_H^R$ the sixth defect $D_H = X_H - (\phi_H/\Lambda_*)P_H^R$ vanishes identically while the scalar readout is nonzero, so W_{HH} alone cannot price that tangent, and a pure positive stretch at fixed support rotates no polar transport and changes no spatial bond or composition [25; 17]. The remaining bosonic response needs the nonfactorising blocks of the master action [17, §47J]: the bond and completion differentials, every cross-metric block and the full auxiliary inverse, with reaction and residual-curvature terms at a displaced background.

19. The charged vertex enters at third order

A distinction already established in the graded audits [26; 18] decides how the readout enters: the CAR/Fock vertex and the bosonic Hessian occupy different derivative orders. Let x be the bosonic source fields in a smooth fact-preserving chart and $\psi, \bar{\psi}$ independent Grassmann fields, and retain the charged bilinear $\Gamma_F(x, \bar{\psi}, \psi) = \bar{\psi}D_F(x)\psi$, with D_F containing the completion map and its charged address projections.

Proposition 28 (differential order and the first induced term). (a) At $\psi = \bar{\psi} = 0$, $\Gamma_{F,ij} = \Gamma_{F,i\psi} = \Gamma_{F,i\bar{\psi}} = 0$ while $\partial^3\Gamma_F/\partial x_i\partial\bar{\psi}\partial\psi = \partial_i D_F$; the readout F and its Gram $G = F^2$ therefore do not enter a classical bosonic Schur complement as an auxiliary coupling and its square, and retaining further bosonic auxiliaries cannot turn that zero block into G . (b) Let η be all relaxing bosonic coordinates on the same leaf with complete auxiliary Hessian A and $b_a = \bar{\psi}(\partial_a D_F)\psi$. Then the auxiliary equation $A\delta\eta + b + O(\psi^4) = 0$ gives

$$\Gamma_{\text{red}} = \Gamma_{B,\text{red}} + \bar{\psi}D_F(z, \eta_0)\psi - \frac{1}{2}b^T A^{-1}b + O(\psi^6),$$

a four-fermion exchange term that leaves the degree-two fermion coefficient unchanged; an external bosonic change of z still changes that coefficient through $dD_F/dz_i = \partial_i D_F - (\partial_a D_F)(A^{-1}H_{\eta})^a$.

Proof. (a) is differentiation of a bilinear at zero fermion fields; it holds for nonlinear dependence of D_F on x . (b) The quadratic bosonic displacement contributes $+\frac{1}{2}b^T A^{-1}b$ and the expanded bilinear $-b^T A^{-1}b$; higher bosonic derivatives and the second auxiliary derivative of D_F begin at degree six, and the Grassmann-even bilinears make the ordering unambiguous. ■

Consequently the unit result $J_G = 0$, $T_G = 660$ is correct in its stated type and supplies neither a tree-level bosonic price nor a theorem that the full physical J cancels. If a bare nonsingular complex Grassmann Gaussian is the chosen parent and its quantum integration is required, its contribution is $-\text{Tr} \log D_F$, with bosonic Hessian $\partial_i\partial_j[-\text{Tr} \log D_F] = \text{Tr}(SD_{F,j}SD_{F,i}) - \text{Tr}(SD_{F,ij})$ for $S = D_F^{-1}$: a propagator-weighted expression that is not F^2 , that needs the regulated kernel, measure and phase prescription, and that must not be added again to an effective action already containing it. The reported CY3' residues of the reduced candidate, 0.373223, 0.381823 and 0.493124 for u, d, e [24], concern that reduced candidate and not the full 47-dimensional differentiation. Classical auxiliary elimination establishes the zero direct quadratic contribution and the first four-fermion term; it does not return numerical J or T or the survival of the readout cancellation in the physical J .

20. Symmetry does not force J to vanish

With $K = \alpha_{\text{loop}}W_{\text{eff}}$ and $w_r = \text{Tr}(P_r W_{\text{eff}})/d_r$ the exact contraction is $J = 9\alpha_{\text{loop}}(w_u + w_e - 2w_Q)$, so for nonzero α_{loop} a physical zero requires $w_u + w_e = 2w_Q$. Gauge symmetry permits different prices on inequivalent representations; family blindness equates copies inside a sector and says nothing between sectors; O_j itself commutes with gauge and family transformations, so averaging leaves it unchanged. The question is whether a common retained core, a common auxiliary metric and complete relaxation together force the relation. They do not.

Theorem 29 (gauge symmetry, family blindness, a common primitive price and fact retention are jointly insufficient). (a) Let T_a , $a = 1, \dots, 8$, be the colour generators on the 47-state carrier with fundamental normalisation and stack them as $\mathcal{C} = (T_1, \dots, T_8)^T$, so that $\mathcal{C}^+\mathcal{C} = C_3 = (4/3)(P_Q + P_u + P_d)$. For $s \geq 0$ and $m > 4s/3$ the parent $H_s = [[mI_{47}, s^{1/2}\mathcal{C}^+], [s^{1/2}\mathcal{C}, I_{376}]]$ is positive, gauge covariant and family blind,

and its exact Schur complement $W_{\text{eff}} = mI - sC_3$ gives $J = 12\alpha_{\text{loop}}s$ and $T = -772\alpha_{\text{loop}}s$. Tensoring with the identity on a committed ledger preserves every ledger projector. (b) With the colour and weak quadratic Casimirs and the primitive charge $q = (1, 4, -2, -3, -6, 3)$, whose eigenvalues on the sectors are $C_3 = (4/3, 4/3, 4/3, 0, 0, 0)$, $C_2 = (3/4, 0, 0, 3/4, 0, 3/4)$ and $q^2 = (1, 16, 4, 9, 36, 9)$, the contractions are $\text{Tr}(O_J C_3, C_2, q^2) = (-12, -27/2, 450)$ and $\text{Tr}(O_T C_3, C_2, q^2) = (772, -4005/2, 30846)$. The symmetry-allowed subtraction family $W_{\text{eff}} = mI - \lambda_C C_3 - \lambda_W C_2 - \lambda_q q^2$ with nonnegative λ returns

$$J = \alpha_{\text{loop}}[12\lambda_C + (27/2)\lambda_W - 450\lambda_q], \quad T = \alpha_{\text{loop}}[-772\lambda_C + (4005/2)\lambda_W - 30846\lambda_q],$$

so colour-only or weak-only relaxation gives positive J , Abelian-only relaxation gives negative J , an exact zero in this family requires the balance $24\lambda_C + 27\lambda_W = 900\lambda_q$, and on that balanced subfamily $T = (\alpha_{\text{loop}}/25)(26928\lambda_W - 39864\lambda_C)$ remains independent.

Proof. (a) $\mathcal{C}$ is an intertwiner, so a common coupling is covariant; positivity follows from the positive auxiliary identity and the reduced eigenvalues $m - 4s/3$ and m ; the contractions are $\text{Tr}(O_J C_3) = -12$ and $\text{Tr}(O_T C_3) = 772$ with the subtraction sign. (b) is the same Schur reduction extended to colour-adjoint, weak-adjoint and Abelian auxiliary copies, with the traces evaluated sector by sector. ■

A common price before relaxation need not remain common afterwards, even with the inherited representation, gauge covariance and identical family treatment. The construction is a mathematical sufficiency test and not the physical parent; it does not prove that the physical J is nonzero, and identifying its coupling with a term of the master action would need source provenance that it does not supply. No measured coupling or target entered the balance condition, which is a condition to derive from a physical parent, not a fitting prescription.

Theorem 30 (protection and the exact balance). If the complete admitted matter response is proportional to the identity on all 45 matter states while the Higgs has its own price, $W_{\text{eff}} = mP_{\text{matter}} + hP_H$, then $J = 0$ and $T = 264\alpha_{\text{loop}}(m - h)$ exactly, and this survives auxiliary relaxation when the full coupled matter parent and the allowed constraint space factor identically over those 45 states. For a general complete price parent with retained core W_0 , auxiliary block A and coupling B on the admitted quotient, $W_{\text{eff}} = W_0 - BA^{-1}B^\dagger$, and with the nonnegative relaxation totals $s_r = \text{Tr}(P_r BA^{-1}B^\dagger)$,

$$J/\alpha_{\text{loop}} = \text{Tr}(O_J W_0) + s_Q - s_u - 3s_e.$$

With a common retained core the required equality is $s_Q = s_u + 3s_e$. Matter universality is sufficient but not necessary, since the balanced subfamily of Theorem 29 has unequal prices with $J = 0$. Positivity makes the individual totals nonnegative; it does not impose their weighted equality, and the colour counterparent violates it. A theorem that the physical J must vanish therefore does not follow from the audited gauge symmetry, family blindness, common primitive pricing and fact retention; the physical value may still be zero through a stronger source identity or a calculated cancellation, and the exact source balance that would have to overcome the obstruction is displayed.

21. Three moments determine J and T ; the source-balance protocol

Write $k_r = \text{Tr}(P_r K)$ for the sector totals of the price with the loop coefficient included, and define the support indicators $P_C = P_Q + P_u + P_d$, $P_W = P_Q + P_L + P_H$ and $P_Y = I_{47}$, together with the data $C = \text{Tr}(P_C K)$, $W = \text{Tr}(P_W K)$, $N = \text{Tr} K$ and $I = \text{Tr}(P_Q K)$. The last indicator is a support, not the charge operator. A returned indicator form $G_{AB} = \text{Tr}(P_A K P_B)$ has the exact shape $[[C, I, C], [I, W, W], [C, W, N]]$, with $k_Q = I$, $k_u + k_d = C - I$, $k_L + k_H = W - I$ and $k_e = N - C - W + I$.

Theorem 31 (the minimal charge moments). (a) The map from the six sector totals to the indicator data has rank four, with kernel directions $v_{\text{ud}} = (0, 1, -1, 0, 0, 0)$ and $v_{\text{LH}} = (0, 0, 0, 1, 0, -1)$. The second is invisible to both contrasts; a displacement a along the first gives $\delta J = a$ and $\delta T = 53a$, and the indicator data determine only $T - 53J = (286C - 396W)/3$. For a positive semidefinite price with $E = N - C - W + I$ the exact feasible interval is $-I + 3E \leq J \leq C - 2I + 3E$ with $T = 53J + (286C - 396W)/3$. (b) With the charge-squared moment $B_q = \text{Tr}(q^2 K)$,

$$12O_J = q^2 - 4P_C - 9P_W, \quad 12O_T = 53q^2 + 932P_C - 2061P_W,$$

$$J = (B_q - 4C - 9W)/12, \quad T = (53B_q + 932C - 2061W)/12, \quad J = 0 \Leftrightarrow B_q = 4C + 9W.$$

The augmented moment map has rank five, its remaining kernel is the invisible L-H direction, and $k_u = (B_q - 24I + 32C + 27W - 36N)/12$.

Proof. Direct evaluation on the six sectors: on Q the right side of the first identity is $1 - 4 - 9 = -12$, on u it is $16 - 4 = 12$, on d $4 - 4 = 0$, on L $9 - 9 = 0$, on e 36, on H $9 - 9 = 0$, which are $12O_J$; the second identity is checked the same way, and the ranks follow from the kernel vectors. ■

A positive information-loss example sets every per-state price to m except $p_u = m + \delta$ and $p_d = m - \delta$: the indicator data stay (36m, 26m, 47m, 18m) while $J = 9\delta$ and $T = 477\delta$, and $B_q = 378m + 108\delta$, so assuming the universal value 378m would assume the cancellation.

On a 45-state matter return with $W_{45} = \text{Tr}((P_Q + P_L)K)$ and $B_{45} = \text{Tr}(q^2 P_{45} K)$ the formulas are $J = (B_{45} - 4C - 9W_{45})/12$ and $T = (53B_{45} + 932C - 2061W_{45})/12 - 132k_H$, with $W = W_{45} + k_H$ and $B_q = B_{45} + 9k_H$ on a proved 47-state completion.

Through a coupled parent every moment is $\text{Tr}(PK) = \alpha_{\text{loop}}[\text{Tr}(PW_o) - \text{Tr}(PBX)]$ with $AX = B^+$, so that $12J/\alpha_{\text{loop}} = \text{Tr}(L_J W_o) - \text{Tr}(A^+ B^+ L_J B)$ for $L_J = q^2 - 4P_C - 9P_W$. The reduction to three moments is useful only after their same-price identification; it does not promote a current screening into a gauge stiffness.

21.1 The source-balance protocol

The calculation requires the complete attached response on a source-selected branch; neither a readout Gram nor symmetry alone supplies it.

Choose one source-selected physical branch and action level. In an admitted fact-preserving chart let z be the typed 47-state response coordinate after the pricing attachment is established and η every relaxing auxiliary; neither replaces a Grassmann field. Assemble the complete positive price parent $H_{\text{price}} = [[W_o, B], [B^+, A]]$ on its physical quotient, with A the reaction Hessian in a constrained formulation and the compatible quotient inverse where null directions are present, solve $AX = B^+$ once with the full coupled A, and read $c_r = \text{Tr}(P_r W_o)$, $s_r = \text{Tr}(P_r BX)$, and form the raw totals $c_r - s_r$ by subtracting each relaxation total from its retained-core total. With the normalisation of this section, $k_r = \alpha_{\text{loop}}(c_r - s_r) = \text{Tr}(P_r K)$, the physical contractions are $J = -k_Q + k_u + 3k_e$ and $T = [-269k_Q + 445k_u + 286k_d - 396k_L + 477k_e - 396k_H]/3$, so $J = 0$ exactly when $k_Q = k_u + 3k_e$. If the assembled parent already returns the normalised K, the k_r are read from it directly and α_{loop} is not applied a second time; a source identity is required for any conversion between an unnormalised current cost and this gauge price.

- Fix the preparation, marked instrument, occupied support and physical write on one declared branch; carry the exact fact-preserving constraints and gauge quotient; establish whether the fixed-background response is invariant and family blind, since covariance of a family alone establishes neither.
- Derive the attachment and the retained and auxiliary differentials of all six residual families C, J, T, R, B, H, retaining the full six-channel metric, the substrate potential and the admitted history and bath terms, with the designated heat baseline wherever the history construction is invoked; at a displaced background include residual curvature, metric derivatives, moving polar and support maps and constraint reactions.
- Assemble W_o , B and A from the same-source derivatives and compute the six pairs (c_r, s_r) ; where the source has not fixed an attachment or block, name that operator and derive it before assigning numbers. A Casimir ansatz is permitted algebra, not evidence that the parent has that form.
- Test the exact J balance first and then contract T. For a nonzero result fix α_{loop} through the same-action identity of SGW-2 on the same event grain unless the parent already returns the normalised K; never set the coefficient to one silently.
- For a finite physical write include the baseline and every induced change of background, constraints and attachment; use the fixed-ingredient update of Section 12 only where its premises

hold; return $J = J_0 + \Delta J$ and $T = T_0 + \Delta T$; apply the full Stage-B transport before any comparison with the diagnostic coordinate, which is never an input from which to solve a source parameter.

Classical relaxation and quantum integration are distinct: on a bare branch requiring a fermion determinant, derive the regulated propagator-weighted effective Hessian of Section 19; on an already effective branch do not add it or a reduced bath contribution a second time. A completed calculation returns the provenance and the six sector totals, establishes or refutes the exact J cancellation on that branch, and supplies T with its normalisation status; it may establish $J = 0$ without fixing the overall scale.

Part F. Record composition, export and admission

Parts D and E fix how a returned metric change and a returned price reach J and T . This Part asks what the spatial record maps, their composition, their export and their grouping can and cannot contribute, using the definitions of the master action [17], the record-stretch classification of HM4 [29], the ordered-record preparation [20] and the primitive marked-instrument programme [22]. Every result is an exact restriction on a declared branch or an explicit compatibility test; none assigns a physical sector weight.

22. Composed record stretches and polar curvature

Work on one rank-stable occupied carrier with a fixed operational inner product and Hermitian matrices S_i at the vertices, and take the stretch family $A_i(\epsilon) = e^{\epsilon S_i}$, $M_{ij} = A_i A_j^{-1}$, $V_{ij} = \text{Pol}(M_{ij})$. Then $M_{jk} M_{ij} = M_{ik}$ exactly, so the spatial record-composition defect vanishes, while the polar factors need not compose; choosing $U_{ij} = V_{ij}$ also makes the bond defect zero on this family without making the closure defect zero.

Proposition 32 (polar evenness, commutator curvature and the restricted channels). (a) $M_{ij}(-\epsilon) = M_{ij}^{-\dagger}(\epsilon)$ and $\text{Pol}(M^{-\dagger}) = \text{Pol}(M)$, hence $V_{ij}(-\epsilon) = V_{ij}(\epsilon)$ and the polar triangle holonomy $H_{ijk} = V_{ki} V_{jk} V_{ij}$ is even in ϵ at finite amplitude. (b) $V_{ij}(\epsilon) = I + (\epsilon^2/2)[S_i, S_j] + O(\epsilon^4)$ and $\log H_{ijk} = (\epsilon^2/2)[S_j - S_i, S_k - S_i] + O(\epsilon^4)$, so the vacuum-relative closure residual is $D_c = \epsilon^2 C_{ijk} + O(\epsilon^4)$ with $C_{ijk} = (-i/2 |p|)[S_j - S_i, S_k - S_i]$. (c) If all S_i commute, in particular on the gauge-invariant family-blind sector-scalar branch $S_i = \sum_r S_{ir} P_r$, every polar link is the identity. (d) $\det V_{ij} = 1$, so near the identity $\text{Tr}(q[-i \log H_{ijk}]) = 0$ on every sector-preserving branch: positive sector-preserving stretches supply no local hypercharge polar phase. (e) Two-state control: $S_0 = 0$, $S_1 = a\sigma_x/2$, $S_2 = b\sigma_z/2$ give exactly $H_{012} = \exp(-i\theta\sigma_y)$ with $\theta = \arctan[\tanh(a\epsilon/2) \tanh(b\epsilon/2)]$, so an isolated unit-area closure norm $w\theta^2$ equals $wa^2b^2\epsilon^4/16 + O(\epsilon^6)$.

Proof. (a) $A_i(-\epsilon) = A_i(\epsilon)^{-1}$ for positive Hermitian A_i ; if $M = BV$ then $M^{-\dagger} = B^{-1}V$ has the same unitary factor. (b) The first-order part $S_j - S_i$ is Hermitian and changes only the positive factor; the second-order anti-Hermitian part is half the commutator; analyticity and evenness give the display, and the three polar factors multiply in traversal order. (c) Commuting positive factors make every M_{ij} positive Hermitian. (d) $\det M_{ij} > 0$ for positive A_i , so $\det V_{ij} = \det M_{ij} / |\det M_{ij}| = 1$, sector by sector when the S_i preserve the sectors. (e) Two links have positive Hermitian record maps and trivial polar factor; the third gives the stated rotation by direct polar decomposition. ■

Unequal scalar stretches between Q , u , d , L , e and H therefore do not activate this polar-curvature route; this is a statement about stretch fields, not about unequal sector weights of an already derived action, and it is not a statement about the hypercharge kinetic coefficient or about J . For a gauge-compatible special case in which the S_i are real combinations of the faithful colour, weak and primitive-charge generators, the q part commutes with the others and drops out of the polar factor exactly, while colour or weak noncommutators can remain. Simultaneous reversal of all stretches gives identical polar links for $\sigma = \pm 1$, so a binary orientation cannot be encoded this way; other record observables can still distinguish the two.

A nonzero second derivative of a residual is not a Hessian contribution at a common defect zero: the curvature above begins at ϵ^2 and its isolated norm at ϵ^4 , while other terms of the master action can contribute at ϵ^2 . The correct nonlinear relaxation is the ordinary stationary higher-derivative reduction. With s the stretch direction, η every relaxing coordinate, $A = \Gamma_{\eta\eta}$, $g = \Gamma_{\eta ss}$ and $c = \Gamma_{ssss}$ at a stationary origin

where s is a null direction of the complete constrained quadratic form, the stationary auxiliary response $\eta = -s^2 A^{-1}g/2 + O(s^3)$ gives

$$\Gamma_{\text{red}}(s) = \Gamma_0 + (s^4/24)[c - 3g^T A^{-1}g] + O(s^5).$$

If the complete action is a squared residual with frozen positive W , first residual coefficient $D = s^2 a/2 + L\eta + \dots$, then $c = 3a^T W a$, $g = L^T W a$, $A = L^T W L$ and the relaxed coefficient is $3a^T [W - W L (L^T W L)^{-1} L^T W] a$, which removes precisely the residual component that an order- s^2 auxiliary response can cancel. The rational control $A = [[3, 1], [1, 2]]$, $g = (1, 2)$, $c = 15$ gives $g^T A^{-1}g = 2$ and relaxed fourth derivative 9. If the physical stretch direction is not a null of the complete quadratic form, the quadratic and cubic terms must be retained instead. $C_{ijk}^{\dagger} C_{ijk}$ is a curvature norm operator and not the price K ; contracting it with O_j and O_T would repeat the identification excluded in Section 19.

23. Supported composition and the exact phase relation

For a general invertible record family with $M_{ij} = A_j A_i^{-1}$ and right polar decompositions $A_i = G_i P_i$, unitary equivariance gives $V_{ij} = G_j \text{Pol}(P_j P_i^{-1}) G_i^{\dagger}$, so the polar triangle holonomy is $H_V = G_i H_P G_i^{\dagger}$ with $\det H_V = 1$: vertex unitary factors add no loop eigenvalues and cannot restore a local hypercharge phase on that branch. The master action uses the supported maps $M_{ij} = \Pi_j \Phi_{ij} \Pi_i$, for which

$$R_s = M_{jk} M_{ij} - M_{ik} = \Pi_k D_R \Pi_i - \Pi_k \Phi_{jk} (I - \Pi_j) \Phi_{ij} \Pi_i;$$

ambient composition does not establish supported composition unless the leakage through the excluded intermediate support vanishes, and fixed support rank does not by itself preserve the committed fact.

Proposition 33 (exact Abelian phase relation). Assume each supported M invertible and U transporting the occupied support unitarily. With $X_R = M_{ik}^{-1} R_s$, $E_e = V_e^{\dagger} U_e = I + V_e^{\dagger} D_{B,e}$ and $H_U = U_{ik}^{\dagger} U_{jk} U_{ij}$,

$$\det H_U = [\det(I + X_R) / |\det(I + X_R)|] \cdot \det E_{ij} \det E_{jk} / \det E_{ik},$$

and on compatible continuous logarithm branches, with $\chi = \text{Im Tr log } H_U$ and $b_e = \text{Im Tr log } E_e$, $\chi = \text{Im Tr log}(I + X_R) + b_{ij} + b_{jk} - b_{ik}$ modulo 2π . When the supported maps and links preserve the six sectors and the Abelian link phases are generated by q , the sector phases satisfy $\chi_r = d_r q_r f_q$ with $f_q = \sum_r q_r \chi_r / 378$, so supported composition and bond alignment together force zero local Abelian flux. At a nonzero background,

$$\dot{\chi} = \text{Im Tr}[(I + X_R)^{-1} \dot{X}_R] + \sum_e s_e \text{Im Tr}[E_e^{-1} \dot{E}_e], \quad \dot{X}_R = M_{ik}^{-1} \dot{R}_s - M_{ik}^{-1} \dot{M}_{ik} X_R, \quad s = (+1, +1, -1),$$

which at a supported composition zero with aligned bonds reduces to $\dot{\chi} = \text{Im Tr}(M_{ik}^{-1} \dot{R}_s) + \sum_e s_e \text{Im Tr}(V_e^{\dagger} \dot{D}_{B,e})$.

Proof. $\det \text{Pol}(M) = \det M / |\det M|$ and multiplicativity of the determinant give the determinant identity exactly, without commuting the factors; the additive form is its logarithm on a common branch; the hypercharge projection uses $\sum_r d_r q_r^2 = 378$ and the vanishing of the colour and weak determinant contributions; the derivative is matrix differentiation of $\log \det$ with the Sylvester polar derivative inside $\dot{E}$. ■

The relation is a compatibility condition on the magnitude-bearing six-defect attachment: the Abelian part of the closure variation must equal the displayed combination of supported record and bond variations. It constrains the field variations and does not determine the derivatives from their labels. If the complete positive Hessian H_Q on the admitted quotient is known and L maps its coordinates to independent observable phase variations $f = Lz$, minimising the same action at fixed phase gives $K_{\text{phase}} = (L H_Q^{-1} L^T)^{-1}$, $z_{\text{min}} = H_Q^{-1} L^T K_{\text{phase}} f$ and $\Gamma_{\text{red}} = \frac{1}{2} f^T K_{\text{phase}} f$, which prices observable phase variations but is not automatically the 47-state price. The master action keeps link, current and spatial record maps independent and favours bond alignment energetically rather than imposing it [17]; setting both R_s and D_B to zero while asking the same restricted construction for nonzero Abelian curvature would be inconsistent.

24. Stationary record amplitudes and the balance test

Start at a common six-defect zero q_0 with the substrate potential at its minimum, on a neighbourhood with fixed occupied projectors and stable polar ranks. Keep ρ , U , J and the local completion Φ_H fixed and change only the realised spatial record maps by $\Phi_{ij}(s) = c_{ij}(s)\Phi_{ij}(0)$ with $c_{ij}(s) = \exp[s(h_j - h_i)] > 0$ for real vertex values h_i .

Proposition 34 (an exact stationary family and its complete response). (a) The endpoint factors obey $c_{jk}c_{ij} = c_{ik}$, positive scalar multiplication leaves polar factors unchanged, and therefore $D_{R,t}(q_s) = c_{ik}(s)D_{R,t}(q_0)$, $D_{B,e}(q_s) = D_{B,e}(q_0)$ and $\text{Pol}(M_e(q_s)) = \text{Pol}(M_e(q_0))$: every member is a common zero and a stationary minimum of the same displayed bosonic action, even for a smooth field-dependent metric, with $n - 1$ scalar directions on a connected graph before boundary restrictions. (b) With S_s acting as $c_{ik}(s)$ on each composition residual and as the identity on the other families, $D(F_s q) = S_s D(q)$, $\tilde{W}_s = S_s^\dagger W(q_s) S_s$, and the pulled-back Hessian is $H_s = H_V + D_0^\dagger \tilde{W}_s D_0$. (c) With the transported leaf and attachment, the exact update of Theorem 16 applies with $\Delta W = \tilde{W}_s - \tilde{W}_0$. (d) For a fixed metric, $R_h = dS_s/ds$ at 0 multiplies block $t = (i, j, k)$ by $h_k - h_i$; with $p_{a,t} = 2\alpha \text{Re Tr}[O_a Z_t^\dagger (WZ)_t]$ and ∂_R the incidence matrix with +1 at k and -1 at i ,

$$\dot{F}_a = \sum_t (h_k - h_i) p_{a,t} = h^T \partial_R p_a, \quad a \in \{J, T\},$$

so invariance under every allowed rescaling is equivalent to the balance vectors $\partial_R p_J$ and $\partial_R p_T$ annihilating the allowed h -space. For a field-dependent metric add $\alpha \text{Tr}[O_a Z^\dagger \tilde{W}_h Z]$; metric covariance $S_s^\dagger W(q_s) S_s = W(q_0)$ is a sufficient, stronger-than-necessary cancellation.

The displayed common-zero equations therefore do not select the scalar amplitudes of spatial record transport on the fixed-support branch, now shown by an explicit field family of one action. Physical admission is a separate test: an actual retained fact is preserved only if its full source-defined constraint permits the amplitude change, fixed record amplitudes at boundaries may remove the family, and neither positivity nor primitive internal democracy establishes the covariance identity. Constancy requires these conditions along the whole connected family, and constancy would not prove $J = 0$.

25. Protected records and the export response

For the same rescaling $\Phi_{ij}(\lambda) = \exp[\lambda(h_j - h_i)]\Phi_{ij}(0)$, scalar endpoint factors telescope along any path even when the matrices do not commute: $\Phi_V(\lambda) = \exp[\lambda(h_{\text{last}} - h_{\text{first}})]\Phi_V(0)$, so every closed path product, with its full matrix, determinant and spectrum, is exactly preserved, and so is every edge polar factor. The protected cycle observables of the same- Φ route therefore remain fixed and the inherited history rows satisfy $B_{\text{hist}} V_h = 0$ for $V_h = \partial_\lambda F_\lambda(q_0)$. RCHS-3 classifies the 18 record-stretch nulls as three copies of the incidence-coboundary space [18, §3.3]; the scalar family integrates that type of direction finitely. A nonzero open record pinned from i to j forces $h_j = h_i$, and with B_{pin} the incidence of pinned paths, $n - \text{rank}(B_{\text{pin}}) - 1$ nontrivial directions survive; protected closed cycles add no pin.

Preserving a recorded label does not fix its weight. HCMR-P1 gives the ordered-record likelihood $L_r(\eta) = \|M_r \eta\|^2$ and conditional preparation $\mu_r = L_r \mu_{FS} / Z_r$ [20]; multiplying one resolved record operator by a scalar c_r independent of η multiplies L_r and Z_r by $|c_r|^2$ and leaves μ_r unchanged, so the normalised post-record ray is unchanged while the record evidence and occurrence weight change.

Proposition 35 (the native export pair and a conditional compatibility obstruction). (a) The complete record/export dilation of PFDML, $c_{\text{rec}}(s) = \text{sech } s$, $c_{\text{exp}}(s) = \tanh s$ with $p_{\text{rec}} + p_{\text{exp}} = 1$ [22, Theorem 23], has conditional Fisher information per opportunity $I_{ss} = 4 \text{sech}^2 s$ at every regular point with both outcomes supported; the export score has zero mean conditional on the other labels. (b) For opportunity probabilities v_e and a tangent with $g_e = ds_e/d\lambda$, the export tag contributes $I_{\lambda\lambda} = 4 \sum_e v_e \text{sech}^2(s_e) g_e^2$; under the attachment hypothesis $g_e = h_{\text{target}} - h_{\text{source}}$ this is $h^T L h$ with $L = 4B \text{diag}(v_e \text{sech}^2(s_e)) B^T$, whose kernel on a connected positive-weight graph is $\text{span}\{1\}$: every nontrivial rescaling is visible. (c) On the common-zero branch $D_0 V_h = 0$ and $V_h^\dagger H_{\text{bos}} V_h = 0$. If the native instrument were the same normalised history law whose Fisher form supplies this complete bosonic quadratic on the same variables with a nonzero common conversion, the nonnegative export contribution would require $D_s(q_0) V_h = 0$ on every sampled edge, so

the injective incidence attachment of (b) cannot simultaneously be that six-defect realisation on this family; no choice of the cross metric W repairs the conflict, because $D_o V_h$ vanishes before W acts.

Statement (c) is a conditional compatibility obstruction, not a contradiction inside the theory: if the actual source has a nonzero export response along the family, at least one proposed identification must change, the family being excluded by further constraints, additional terms lifting it, the export coordinate not being that spatial amplitude, or the export Fisher form not being the claimed action curvature. RCHS-24 already distinguishes raw transfer curvature, normalised-history Fisher information and influence-action curvature [19, §9H.4]. Conversely $g(h) = 0$ proves blindness only of this readout at this order. A useful exact sufficient test for invariance of J and T on the family is positive diagonal similarity of the raw transfer: with $T_\lambda = G_\lambda^{-1} T_o G_\lambda$ and $r_\lambda = G_\lambda^{-1} r_o$ the Doob kernel is unchanged, $P_\lambda = P_o$; but this identity must be related to the spatial rescaling by the source, and equal J , T need the equivalence on a neighbourhood of the retained probes with constraints, auxiliaries, measure, conversion and attachment transported.

26. The export quotient, cycle export amplitudes and matter sectors

Proposition 36 (the coboundary condition on the export map). PFDMI §19.3 requires the classified flat real-stretch coboundaries, $\ker B_C = \text{im } B^T$ on each twelve-edge factor carrier (HM4 Theorem L [29]), to be physical redundancies [22]. On a resolved export channel with $p_R = \text{sech}^2 s$, $p_E = \tanh^2 s$ at a regular depth $s \neq 0$, invariance gives $0 = \partial_\lambda p_R = -2 \text{sech}^2 s \tanh s \cdot g$, hence $g = D s_e(q_o) V_h = 0$ on every sampled edge. At $s_o = 0$ both first derivatives vanish for any g , but $p_E(\lambda) = \lambda^2 g^2 + O(\lambda^3)$ and $D_{KL}(p(0) \| p(\lambda)) = 2 \log \cosh s(\lambda) = \lambda^2 g^2 + O(\lambda^3)$, so exact invariance of a dark branch still requires $g = 0$. In logarithmic amplitude coordinates $\ell \in \mathbb{R}^{12}$ with $\ell \mapsto \ell + B^T h$, the derivative G of the export-depth vector must satisfy $GB^T = 0$, equivalently $G = LC$ with $L = GC^T(CC^T)^{-1}$ for the cycle rows $C = B_C$; the direct attachment $s = \ell$ fails.

Once identified with the classified redundancy, the endpoint rescaling cannot generate a resolved export response, and the incidence attachment is excluded on that branch by a requirement already in the corpus. For the three factors the thirty-six stretch coordinates have an eighteen-dimensional quotient, so an export derivative with twelve independent rows can descend; the formula $K(A, s)$ of the instrument does not by itself identify the variation of s under a change of Φ , and holding s fixed as an independent argument gives a zero partial derivative without executing that identification. A redundancy must preserve the complete response with probes and auxiliaries transported, $D_o^+ [S_h^+ W_h S_h - W_o] D_o = 0$ on the admitted probes, and $K_{\text{phys}}(F_h q) \simeq K_{\text{phys}}(q)$ with $\Delta_h J = \Delta_h T = 0$ after the physical identification; equal values along an orbit do not make either value zero, and coboundary blindness does not equate the left and right sector weights.

Proposition 37 (cycle export amplitudes at zero export). On the unit-residual scalar branch the three RCHS10 rim-stretch forms [30], $\ell_{0,3} = -(9j_c^2 + 2\sqrt{3} j_c j_s + 7j_s^2)/33708$, $\ell_{1,4} = -(6j_c^2 + 10j_s^2)/33708$ and $\ell_{2,5} = -(9j_c^2 - 2\sqrt{3} j_c j_s + 7j_s^2)/33708$, each repeated twice, sum round the oriented rim to the invariant $\chi(j) = -(4/2809) \|j\|^2 + O(\|j\|^4)$, giving the composed capacity $P_{E,\text{cycle}} = 1 - \exp(2\chi) = \kappa \|j\|^2 + O(\|j\|^4)$ with $\kappa = 8/2809$. No continuously differentiable real depth $\sigma(j)$ with $\sigma(0) = 0$ can implement this capacity, because $\tanh^2 \sigma = (v^T j)^2 + o(\|j\|^2)$ has quadratic coefficient of rank at most one while κI_2 has determinant $\kappa^2 > 0$; the same holds on every rim edge, whose quadratic coefficients have eigenvalues $6/16854$ and $10/16854$. An export amplitude vector $b(j) = B j + o(\|j\|)$ instead needs $\text{Re}(B^+ B) = \kappa I_2$, realised for instance by $b_E(j) = (2\sqrt{2}/53)(j_c + i j_s)$ with $a_R(j) = 1 - (4/2809) \|j\|^2 + O(\|j\|^4)$, and for any positive Q with $R^T R = Q$ by $a(j) = \exp(-u/2)$, $b(j) = f(u) R j$, $f(u) = [(1 - e^{-u})/u]^{1/2}$, $u = j^T Q j$, which are smooth with $a^2 + \|b\|^2 = 1$ exactly.

The source capacity fixes a nonzero real Gram for the export jet; it fixes neither the export phase, carrier nor matter attachment, and two real quadratures do not by themselves require two distinguishable destinations. The coefficient $8/2809$ belongs to the scalar branch and does not supply the 47-state sector map.

Proposition 38 (matter-sector test of the export amplitudes). Let $L_\mu(x)$ include all current and terminal outputs with $\Sigma_\mu L_\mu^+ L_\mu = I$, differentiable at the cycle origin, and let $E_\mu(x) = b(x) L_\mu(x)$ with $b(0) = 0$. Then the changing current drops out of the first export derivative, $G_{ab} = \frac{1}{2} \Sigma_\mu (B_{\mu a}^+ B_{\mu b} + B_{\mu b}^+ B_{\mu a}) = \text{Re}(b_a^* b_b) I$, and for the displayed representative $G_{11} = G_{22} = \kappa I$, $G_{12} = 0$; restricted to the 47-state carrier this gives $k_r^G = \kappa d_r$, J_G

$= \kappa(-18 + 9 + 9) = 0$ and $T_G = 0$, with the Higgs included. The literal identification of the cycle origin with zero native current fails regularity: on the Higgs weak doublet the saturated native law gives $S = P_H/2$, $\eta = 2$, $C_0 = I - P_H$ and $C_1(x) = [(x_1^2 - x_2^2)/(\sqrt{2}(x_1^2 + x_2^2))]\sigma_2$ exactly for $x \neq 0$, so the composed export derivative $D(v) = (\kappa/2)^{1/2}(v_1 + iv_2)(v_1^2 - v_2^2)/(v_1^2 + v_2^2)\sigma_2$ has $D(1, 1) = 0$ but $D(1, 0) + D(0, 1) \neq 0$ with norm $\kappa^{1/2} = 0.05336655$; the unsplit channel itself has different limits along the two rays, differing by operator norm $1/2$.

The regular branch has no export norm contrast, the same-zero branch has no Fréchet derivative, and neither identifies the complete price with the Gram: background current response can remain sector dependent even when an added scalar export norm is universal. The next source return along this route is the native current along the committed write, $j(x) = j_0 + Rx + o(\|x\|)$, from $j = \psi_t - U\psi_s$ and the commitment constraints; a nonzero j_0 permits the regular derivative, a zero j_0 needs a law regular there, and the two-dimensional cycle label alone supplies neither.

27. Record lock, terminal maps and the complete response

A stored record phase and the connection coupled to it are different coordinates. On the phase-faithful protected branch $R_{hol}(\gamma) = \exp(i\gamma)$ with $\gamma = 6\sqrt{6}$, preserving the record fixes $\delta\gamma = 0$ and imposes nothing on an independent connection coordinate θ , and the free stationary relation $\gamma = \theta/6$ must not be differentiated as a constitutive identity after γ is locked. On the inherited four-coordinate parent of RCHS-1 [38, §§39A to 39F] with $c = \lambda w_C$, $d = w_B$, $e = \lambda w_R$, $g = w_J$: eliminating the current u first, a freely relaxing record has $\gamma = d\theta/(d + e)$ and a fixed record has $\delta\gamma = 0$, so

$$k_f = c + de/(d + e), \quad k_h = c + d, \quad a_0 = \kappa^2 g/2, \quad K_{p,free} = gk_f/(k_f + a_0), \quad K_{p,record} = gk_h/(k_h + a_0), \quad 0 < K_{p,free} < K_{p,record} < g.$$

In the alternating-mode unit control $\lambda = 5$ with all weights and κ equal to one, $k_f = 35/6$, $k_h = 6$, $K_{p,free} = 35/38$, $K_{p,record} = 12/13$, the increment is $1/494$, its fraction of the remaining gap $1/39$, and $\theta^*(p) = \sqrt{2} p/13$. For the generator-dependent branch of Section 30, fixing the record rescales the contrasts by $(k_f + gb^2S)/(k_h + gb^2S)$ at fixed state: attenuated, not erased, with the pointwise ratio unchanged. A record lock is therefore not a full connection lock, and the general reaction-Hessian formula $K_{fact} = A - BC^{-1}B^T + D^TM^{-1}D$ with $D = F - LC^{-1}B^T$, $M = LC^{-1}L^T$ remains the regular-branch calculation.

Proposition 39 (terminal maps and the invariance of the complete response). (a) Take states $(f, a, b) \in \{0, 1\}^3$ with uniform measure and the fixed reversible kernel $K = I_f \otimes A \otimes B$, $A = [[3/4, 1/4], [1/4, 3/4]]$, $B = [[2/3, 1/3], [1/3, 2/3]]$, so the fact f never changes. The three lumpable terminal maps keep a , keep b or keep the parity $a \oplus b$; with $\Delta_r = (I - K_r)/4$ and $R_r = (I + \Delta_r)^{-1}$ their nonconstant responses are $8/9$, $6/7$ and $24/29$, so consistency conditions alone do not force a unique terminal completion.

(b) For the fixed microscopic probe $\psi = (\chi_a + \chi_b)/\sqrt{2}$ and the single response operator $R = (I + (I - K)/4)^{-1}$, $\langle \psi, R\psi \rangle_\pi = 55/63 = 4/9 + 3/7$, and under each map the retained and omitted parts are $(4/9, 3/7)$, $(3/7, 4/9)$ and $(0, 55/63)$: the complete response is the same.

(c) For a differentiable common-support history law and a fixed terminal map, the score decomposes as $S = E[S|R] + S_{cond}$ with $E[S_{cond}|R] = 0$, so $W_{full} = W_{record} + W_{cond}$ with $W_{cond} \geq 0$, including every cross entry; a terminal law carries all the information exactly when $S = E[S|R]$ almost surely.

(d) For a coupled quadratic action with Hessian $[[A, B], [B^T, C]]$ and source injection $G = (G_x, G_y)$, exact elimination gives $G^TH^{-1}G = G_{eff}^TS^{-1}G_{eff} + G_y^TC^{-1}G_y$ with $S = A - BC^{-1}B^T$ and $G_{eff} = G_x - BC^{-1}G_y$, and $\det H = \det C \cdot \det S$ with $\partial_u \log \det H = \text{Tr}(H^{-1}H_{uu} - H^{-1}H_u H^{-1}H_u)$ independent of the split.

(e) Two calculations that use the same complete history law, committed-fact surface, normalised defect variations, probes, action and measure, and implement their record groupings as exact reductions retaining the conditional, source and same-grade determinant terms, return the same K and hence the same J and T .

Proof. (a) and (b) are exact rational calculations; the resolvent split $R = CR_E + (I - Q)R(I - Q)$ with $Q = CE$ follows from reversibility and strong lumpability. (c) is conditional orthogonality of the score,

equivalently the KL chain rule differentiated twice. (d) is completion of the square and the determinant factorisation. (e) follows from (c) and (d) applied with the same physical derivatives, normalisation and attachment; the non-Gaussian case needs the exact conditional action rather than the displayed quadratic truncation, and a regulator or history-rate limit needs its convergence certificate. ■

Uniqueness of terminal coordinates is therefore not a prerequisite once the complete physical problem and its exact reduction are fixed, and different partial responses of different retained probes are not evidence of different couplings. Equality between representations is not $J = T = 0$ within one representation, and the result neither proves matter universality nor fixes the sector weights, the loop scale or the comparison boundary.

Part G. Observables, normalisation and the write

28. The fine structure constant and the same price

With the conventional hypercharge generator $Q = T_3 + Y$ and the SGRI inverse squared couplings $A = (g_3^{-2}, g_2^{-2}, g_Y^{-2})$, the electromagnetic coupling obeys $1/(4\pi\alpha_{em}) = e^{-2} = A_2 + A_Y$ at one scale in one scheme [34; 35]; the primitive convention $q = 6Y$ requires $Z_Y = Z_q/36$ before the sum is formed, and the loop conversion α_{loop} is a separate quantity. On the 47-dimensional carrier let $p_r = \text{Tr}(P_r K)/d_r$. SGW-2 gives $\Delta A_{loop} = R p/6$ with its frozen representation matrix R [16, §§5, 6, 11, 18]; adding the weak and hypercharge rows gives the exact electromagnetic contraction of the same price.

Proposition 40 (the electromagnetic contraction and its blind direction). (a) $E_{loop} = \Delta A_2 + \Delta A_Y = 5p_Q + 4p_U + p_d + 3p_L + 3p_e + p_H = \text{Tr}(A_{em}K)$ with $A_{em} = (5/18)P_Q + (4/9)P_U + (1/9)P_d + (1/2)P_L + P_e + (1/2)P_H$, a positive central operator of trace 17. (b) With the common coordinate $c = \Delta A_3/6 = p_Q/2 + (p_U + p_d)/4$ and $Z = (6, 13/2, 21/2)$, $\Delta A = cZ + (0, (53J - T)/528, (229J - T)/528)$ and $E_{loop} = 17c + (141J - T)/264$. (c) On the three-price branch with $p_L = m - \delta$ on Q, L , $p_R = m + \delta$ on u, d, e and $p_H = h$, all six prices strictly positive for $m > |\delta|$, $h > 0$: $E_{loop} = 16m + h$, $c = m$, $J = 36\delta$, $T = 264(m - h) + 5076\delta$ and $\Delta A(\delta) - \Delta A(0) = (0, -6\delta, 6\delta)$, so the electromagnetic and colour responses are fixed while the contrasts move with $\Delta T = 141\Delta J$. (d) If $p_L = p_R = m$ is proved, the further variation $m \mapsto m + \epsilon$, $h \mapsto h - 16\epsilon$ preserves E_{loop} and moves T by 4488ϵ .

Proof. (a) is the row sum of R divided by six and its per-state form; (b) follows from the inherited reconstruction modulo Z , since $(53J - T + 229J - T)/528 = (141J - T)/264$; (c) and (d) are direct contraction. ■

An isolated electromagnetic loop response therefore fixes $141J - T$ only if c is independently known, and otherwise one combination of three quantities. At the one-loop fixed-census grade the complete ledger is $A(\mu) = SZ + \tau b + v(J, T) + \Delta_{other}$ with $b = (-7, -19/6, 41/6)$, $\tau = \ln(\mu^*/\mu)/(8\pi^2)$ and $v(J, 0) = (7J/264)Z + (J/44)b$, so A depends on $S + 7J/264$, $\tau + J/44$ and T , and $1/(4\pi\alpha_{em}(\mu)) = 17S + (11/3)\tau + (141J - T)/264 + E_{other}$: the known degeneracy of J with the normalisation and matching scale in electromagnetic coordinates, not a new freedom. The chirality cancellation is exact for the boundary trace map and under identical one-loop affine transport; it is not an all-orders statement. The two-loop gauge matrix of SGRI has electromagnetic row sums $B_{em} = (80/3, 31/3, 113/9)$ [35, §10], and at one point a variation with $dA/d\delta = (0, -6, 6)$ changes the gauge part of $dE/d \ln \mu$ by $-[62/A_2^2 - 226/(3A_Y^2)]/(128\pi^4)$, generally nonzero, so a measured low-energy α_{em} acquires indirect sensitivity to the boundary chirality split through higher-order transport. The constant is connected to the same source price through one explicit contraction; it cannot replace the unevaluated representation-resolved price, and no deduction that $\alpha_{loop} = \alpha_{em}$ or that physical J vanishes follows.

29. Universal Fold response

MCFR-1 gives the same-event matter writer $P_+ \otimes I_{49}$ at its premise-plus-representation grade and keeps the physical action price separate [36]; the master action uses one primitive metric for all six defect families and prohibits species coefficients, while including joint derivatives, potential, cross terms and

auxiliary relaxation [17]. Three objects must be separated: the write operation, a fixed cost per committed Fold, and the fully relaxed quadratic response that enters J and T.

Proposition 41 (count terms and the universality criterion). (a) On a fixed-support branch with fixed integer Fold count N and γ independent of the continuous coordinates x , $\Gamma_{\text{count}} = \gamma N$ has $D_x \Gamma_{\text{count}} = D_x^2 \Gamma_{\text{count}} = 0$; a single new Fold changes it by the finite increment γ with zero field derivatives on either regular branch.

(b) Let $Q(x, \eta) = \frac{1}{2}x^\dagger A x + \text{Re}(x^\dagger B \eta) + \frac{1}{2}\eta^\dagger C \eta$ be the full quadratic parent on one regular quotient with relaxed response $R = A - BC^\dagger B^\dagger$, M the independently supplied positive physical normalisation form, E the injection of the selected particle-pattern coordinates, $G = E^\dagger M E$ and $V = EG^{-1/2}$. Then $K = V^\dagger R V = G^{-1/2} E^\dagger R E G^{-1/2}$, and $E^\dagger R E = c E^\dagger M E$ with one common c if and only if $K = cI$.

(c) If a repeated pattern has N identical local contributions with response r_0 and norm μ_0 , its normalised coefficient $p_N = N r_0 / (N \mu_0) = r_0 / \mu_0$ is independent of N , also after repeating an identical complete local auxiliary parent.

(d) If the universal response holds on all 47 components, $p_r = c$ for every sector and $J = 9c(1 + 1 - 2) = 0$, $T = c(-1614 + 1335 + 858 - 792 + 477 - 264) = 0$; if it holds on the 45 matter components only, $J = 0$ and $T = 264(m - h)$.

The exact condition is proportionality of the complete response to the physical norm on the participating patterns; M and E must have independent provenance, since defining $M = R/c$ would make the condition vacuous, and equality of unnormalised costs is insufficient when the norms differ. The source-level test is $E^\dagger[A - BC^\dagger B^\dagger - cM]E = 0$ on the chosen image; the sources checked return neither this equality for the Fold-to-47-state attachment nor an admitted counterexample. The universality proposal is therefore viable but unproved, it would imply $J = 0$ for universal matter pricing and $T = 0$ only when the Higgs shares the price, and it is stronger than necessary, since the balance of Theorem 30 also permits $J = 0$ with unequal prices. A mechanical analogy suggesting arrangement-dependent response establishes a possibility, and Fold locality alone establishes locality, not the scalar equality.

30. The native current response

The substrate current defect and its general variation are [17, §§4, 13] $D_{J,e} = J_e - \kappa_e(\rho_t - U_e \rho_s)$ and $\delta D_{J,e} = \delta J_e - \delta \kappa_e(\rho_t - U_e \rho_s) - \kappa_e(\delta \rho_t - \delta U_e \rho_s - U_e \delta \rho_s)$. Holding current, amplitude and coefficient fixed isolates one connection column, $\partial_\theta D_{J,e} = i \kappa_e T_e U_{e,*} \rho_s$ for $U_e(\theta) = \exp(i\theta T_e) U_{e,*}$: the generator is present before any species-dependent price is introduced. Here θ is one surviving real connection or cycle coordinate after the gauge quotient; obtaining a column $v = ibq\psi$ requires a typed pullback intertwining the generator with q , with a common coefficient b that includes profile, constitutive coefficient and amplitude.

Proposition 42 (the single-mode branch). Assume a normalised 49-complex-dimensional current chart with metric gI , $g > 0$, a surviving real mode with column $v = ibq\psi$, $b > 0$, $\|\psi\| = 1$, and the reduced parent $Q(x, \theta) = (g/2)\|x + v\theta\|^2 + (k/2)\theta^2$ with $k > 0$ [38, §§8, 39E]. In real coordinates the 99-dimensional Hessian reduces to $R_{\mathbb{R}} = gI_{98} - [g^2/(k + g\|v\|^2)]v_{\mathbb{R}}v_{\mathbb{R}}^T$, a real phase removing one real rank, and the per-complex-state sector price $p_r = \text{Tr}(P_r^{\mathbb{R}} R_{\mathbb{R}})/(2d_r)$ gives, on the seven sectors (Q, u, d, L, e, H, N) of dimensions $(18, 9, 9, 6, 3, 2, 2)$ with $q = (1, 4, -2, -3, -6, 3, 0)$ and A_j, O_T extended by zero,

$$J_{\text{cur}}(\psi) = -(g\eta/2) S_J/(1 + \eta S), \quad T_{\text{cur}}(\psi) = -(g\eta/2) S_T/(1 + \eta S), \quad S = \psi^\dagger q^2 \psi, \quad S_J = \psi^\dagger A_J q^2 \psi, \quad S_T = \psi^\dagger O_T q^2 \psi, \quad \eta = gb^2/k,$$

with the exact moments $\text{Tr } q^2 = 378$, $\text{Tr}(A_J q^2) = 450$, $\text{Tr}(O_T q^2) = 30846$, $\text{Tr}(A_J q^4) = 13950$ and $\text{Tr}(O_T q^4) = 886530$. For ψ Haar-uniform on the unit sphere of $\mathbb{C}^{49}$ the means of J_{cur} and T_{cur} are strictly negative for every finite positive g, k, b , with $E[J_{\text{cur}}]/g = -(225/49)\eta + (3681/98)\eta^2 + O(\eta^3)$, $E[T_{\text{cur}}]/g = -(15423/49)\eta + (6273159/2450)\eta^2 + O(\eta^3)$ and $E[T_{\text{cur}}]/E[J_{\text{cur}}] = 5141/75 + (1936/625)\eta + O(\eta^2)$; the flat-modulus control $|\psi_i|^2 = 1/49$ gives $J_{\text{cur}} = -225g^2b^2/(49k + 378gb^2)$ and $T_{\text{cur}}/J_{\text{cur}} = 5141/75$ exactly. A newly imposed lock $\theta = 0$ with all else fixed reverses the branch contribution exactly.

Proof of the sign statement. The coordinates $|\psi_i|^2$ are exchangeable Dirichlet(1, ..., 1). With $s = 1/\eta$, $a_i = q_i^2$ and $F(a_i) = a_i E[x_i/(s + \sum a_i x_i)]$, pairing each sample with its swap on two coordinates gives the exact identity $[ax - by]/D + [ay - bx]/D' = (a - b)[(x + y)C + 2(a + b)xy]/(DD') > 0$ for $a > b$, so F is strictly increasing in the charge square; then $E[J_{cur}] = -(9g/2)[F(16) + F(36) - 2F(1)] < 0$, and the T numerator is a positive sum of adjacent differences. The expansions use $E[(\psi^\dagger A \psi)(\psi^\dagger B \psi)] = (\text{Tr } A \text{Tr } B + \text{Tr}(AB))/(49 \cdot 50)$. ■

Unresolved isotropic averaging therefore does not erase this conditional mechanism, and averaging after reduction is a different operation from averaging the parent. Physical admission is still required: whether the fact-preserving quotient admits the mode, whether the write locks it, whether the typed attachment returns $v = ibq\psi$ with a common b and metric gl , and a record-dependent preparation need not have the exchangeability used in the proof. The contrasts are contractions of the declared current quadratures and no identity here makes them the 47-state finite-loop prices; Section 42 shows that the isolated shifted-current block is matter blind under relaxation and Gaussian elimination, and that the conservation residual and full cross metric supply the larger action in which the question must be asked. The conditional signs and ratios are not promoted to gauge predictions.

31. Source variations on the shared link and the six defect derivatives

The master action varies the connection, current, endpoint and record variables independently and uses the same represented link in the fermion kinetic and bosonic bond actions [17]; no second particle-to-link coupling exists. For Q , u and e with family-inclusive dimensions 18, 9, 3 and primitive charges 1, 4, -6, a left-trivialised variation $\delta U_r = iX_r U_r$ has $X_Q = I_{fam} \otimes (a_C \otimes I_2 + I_3 \otimes a_W + a_q I_6)$, $X_u = I_{fam} \otimes (a_C + 4a_q I_3)$ and $X_e = -6a_q I_{fam}$, with traceless Hermitian a_C , a_W . The CAR/Fock lift uses the same generator through second quantisation, $\delta U \Gamma_{kin} = -\sum_{e,r} \bar{\Psi}_{t,r} \gamma^e i X_{e,r} U_{e,r} \Psi_{s,r} + h.c.$, and the representation trace gives the diagnostic vertex norm balance $N_J = -\text{Tr } X_Q^2 + \text{Tr } X_u^2 + 3\text{Tr } X_e^2 = -3\text{Tr } a_C^2 - 9\text{Tr } a_W^2 + 450a_q^2$, that is $(-3/2, -9/2, 450)$ with fundamental norm one half: vertex norms, not J values.

The current responds with the link. Holding current, endpoints and coefficient fixed gives the link-only column of Section 30; the residual-shell lift instead compensates the link at fixed endpoints, $\delta j_r = -ikX_r U_r \psi_{s,r}$, giving $\delta D_{J,r} = 0$, an available tangent of that construction and not a kinematic constraint, distinct from the MCFR minimum-action lift at nonzero current defect. A current-defect zero does not imply a terminal-score zero. The other five first variations are, with the logarithmic holonomy defect of the master action and $Q_p = H_p \cdot^{-1} H_p(U)$:

$$D_C(p) = (-i/|p|) \text{Log}_0(Q_p), \quad \delta D_C(p) = (-i/|p|) D \text{Log}_{Q_{\{p\}}}[H_p \cdot^{-1} \delta H_p], \quad \delta H_p = \sum_e H_{after}(\delta U_e) H_{before},$$

$$\delta D_T = \partial_\sigma(\delta \rho) + \partial_U(\delta J) + (\delta \partial_U)J, \quad \delta D_R = (\delta \Phi_{jk})\Phi_{ij} + \Phi_{jk}(\delta \Phi_{ij}) - \delta \Phi_{ik},$$

$$\delta D_B = (\delta \Pi_t)U\Pi_s + \Pi_t(\delta U)\Pi_s + \Pi_t U(\delta \Pi_s) - D \text{Pol}_M[\delta M], \quad \delta D_H = DX_H[\delta \Phi_H] - (\delta \phi_H/\Lambda^*)P_H - (\phi_H/\Lambda^*)\delta P_H,$$

with $M = \Pi_t \Phi_{tr} \Pi_s$ and δM its product rule. Reference, face area and branch are fixed in the closure display; inverse links are differentiated as inverses in order; polar and logarithmic differentials need their certified constant-rank support. On the compensated path with fixed endpoints, supports, spatial record maps and completion morphism, the closure, transport and bond derivatives generally survive, and the bond derivative contains the projected connection column.

At a common zero of all defects, with the one-half quadratic convention, $H_{bos} = L_D^\dagger W_{prim} L_D + H_V$ with $L_D = D_X D$; nonzero defects add residual-curvature, metric-derivative and constraint-reaction terms, with fermionic or effective contributions included once at their order.

On a regular quasistatic constrained branch the response system is the bordered operator $[[H_L, C^\dagger], [C, 0]]$ acting on $(\dot{X}, \dot{\lambda})$ with forcing from the explicit action and constraint variations; this static system is not a dynamics postulate, and the retained response may equally be supplied by an independently work-normalised susceptibility, whose same-action, same-grade value and normalisation are the unresolved input [4; 19].

32. The physical write and the normalised write law

The PSAC orientation source on its stated branch gives, on the combined closure/record line, $h_o = \sigma\alpha^*(1, 0)$ with preparation affinity approximately 0.959001109719975, which is not the finite-loop coefficient. Write the complete action with the source term as $\Gamma_a(z) = \Gamma_o(z) - a\ell(z)$, a labelling a source family for differentiation and neither time nor a fraction of a committed fact; on a fixed logarithm branch the primitive Abelian projection is $f_q(p) = \text{Tr}(qD_C(p))/378$, locally affine in the ambient link angles.

Proposition 43 (what a write can and cannot change at quadratic order). (a) For $\Gamma_a = \frac{1}{2}x^T Mx + x^T B\eta + \frac{1}{2}\eta^T A\eta - a(f^T x + g^T \eta)$ with $A > 0$ on a fixed affine fact space, $\eta_* = A^{-1}(ag - B^T x)$ and $\Gamma_{\text{red}} = \frac{1}{2}x^T R x - a(f - BA^{-1}g)^T x - (a^2/2)g^T A^{-1}g$ with $R = M - BA^{-1}B^T$ independent of a : a linear force does not change a fixed quadratic stiffness, so if $K = \alpha_{\text{loop}} R$ with fixed attachment the increments of C , W , B_q , J and T vanish while their baselines need not.

(b) A newly imposed fact restriction $Fx + L\eta = r$ is a different operation and adds $R_{\text{fact}} = R + D^T S^{-1} D$ with $D = F - LA^{-1}B^T$, $S = LA^{-1}L^T$.

(c) On a smooth fixed-rank fact leaf with the retained probe held at $x = 0$, η_a solving $\Gamma_{a,\eta} = 0$, $v = d\eta_a/da = A^{-1}\ell_\eta$, $w = d^2\eta_a/da^2 = A^{-1}(2\ell_{\eta\eta}v - \Gamma_{a,\eta\eta\eta}[v, v])$, $E = (I, -A^{-1}B^T)^T$, $N = (0, I)^T$, $H_1 = \Gamma_{a,zz\eta}[v] - \ell_{zz}$ and $H_2 = \Gamma_{a,zz\eta\eta}[v, v] + \Gamma_{a,zz\eta}[w] - 2\ell_{zz\eta}[v]$, the reduced derivatives are $R' = E^T H_1 E$, $G = N^T H_1 E$ and $R'' = E^T H_2 E - 2G^T A^{-1}G$.

(d) If the complete action, constraints, preparation and probe transform covariantly under $a \mapsto -a$ and the contrasts are invariant, $J'(0) = T'(0) = 0$ and $J(a) = J(0) + (a^2/2)J''(0) + O(a^4)$, with $J'' = (\alpha_{\text{loop}}/12)\text{Tr}(L_J R'')$ and $T'' = (\alpha_{\text{loop}}/12)\text{Tr}(L_T R'')$, $L_T = 53q^2 + 932P_C - 2061P_W$.

Proof. (a) and (b) are completion of the square. (c) The auxiliary rows of HE vanish; differentiating that condition gives $E' = -NA^{-1}N^T H_1 E$, so the first derivative has no stationary-lift contribution and the second has the two equal negative relaxation terms displayed. (d) is evenness. ■

The last relaxation term is negative semidefinite but the complete second derivative need not have a fixed sign, and freezing the auxiliary inverse or lift omits part of the response. A reflection can exchange fixed-sign fact sectors rather than act within one, so parity alone does not prove the premises of (d), and the preparation affinity near 0.959 is not within a controlled small- a range without an error bound. The inspected source does not return the attached H , forcing and mixed third and fourth derivatives for this write.

The history route requires instead the derivative of the complete raw transfer along the physically normalised bond/record deformation, $\mathbb{T}_h = d\mathbb{T}_{\text{raw}}(z(\epsilon))/d\epsilon$ at zero, which HMGB keeps as the HM12 and F34 obligation [2]. On a finite enlarged Markov carrier with simple Perron root λ , right vector r and edge law $\rho(i, j) = \pi(i)P(i, j)$, the complete normalised score is $S_a(i, j) = F_a(i, j) + u_a(j) - u_a(i) - \partial_a L$ with $F_a = \partial_a \log \mathbb{T}$, $u_a = \partial_a \log r$, $L = \log \lambda$, and $W_{ab} = \sum_p S_a S_b$. Twice differentiating row normalisation and averaging with π gives the exact identity $W_{ab} = \partial_a \partial_b \log \lambda - \sum_p B_{ab}$ with $B_{ab} = \partial_a \partial_b \log \mathbb{T}$, while the raw free energy $f_{\text{raw}} = -\log \lambda/a_t$ has $\partial_a \partial_b f_{\text{raw}} = -(W_{ab} + E_p B_{ab})/a_t$: raw curvature and Fisher curvature differ. Multiplying the transfer by a state-independent scalar $e^{-f(\epsilon)}$ with $f = \epsilon^T M \epsilon/2$ leaves r , P , every normalised finite-history probability and W unchanged, while shifting $\partial_{ab} \log \lambda$ and B_{ab} by $-M_{ab}$ and the raw curvature by M_{ab}/a_t . Normalised statistics therefore do not reconstruct the raw scalar weight; a fixed source action and measure forbid changing that factor, and the example changes no physical J or T at fixed complete data.

33. Primitive Change Normalisation and the work-to-action conversion

CLSN-1 adopts Primitive Change Normalisation as a declared physical premise: one completed primitive change has unit cost and its exact E_2 -to-gauge-cycle image carries the same cost, which on the one-opportunity event carrier fixes the common conversion $s_{\text{PCN}} = 1/t = 46.376498312836$ with $t = 0.0215626456584632$ [3, §§17.106 to 17.108]. The identification is stronger than a unit convention and is not derived from entropy; an independently normalised pre-screening work return that disagreed with it would falsify it on that carrier, while genuine downstream same-source relaxation remains a physical

output. Identifying a particular finite-loop term with this event is still required before its separate loop coefficient is set to one.

Proposition 44 (electric work and the inverse expansion). (a) The closed-loop work reconstruction of CLSN-1 [3, §§17.80, 17.81], $D_{ab} = [W_b(p + c_a) - W_b(p - c_a)]/(2q) = c_a^T Q c_b$, returns $D = C^T Q C$ for the electric Hamiltonian $H_E(p) = \frac{1}{2} p^T Q p$; it probes a retained electric inverse kinetic form, whereas the direct extraction of SGW-1 assumes a face-curvature action Hessian in the PCN unit [41, §5]. With $L_E = \frac{1}{2} E^T M E$, $p = M E$ and $Q = M^{-1}$, and in orthonormal cycle coordinates $\hat{Q} = G^{-1/2} D G^{-1/2}$, $\hat{M} = G^{1/2} D^{-1} G^{1/2}$, $G = C^T C$. (b) For a fixed-coordinate expansion $Q(\eta) = Q_0 + \eta Q_1 + \eta^2 Q_2 + O(\eta^3)$ with $M_0 = Q_0^{-1}$, $M_1 = -M_0 Q_1 M_0$ and $M_2 = M_0 Q_1 M_0 Q_1 M_0 - M_0 Q_2 M_0$. (c) Under simple-factor isotropy in conventional Y coordinates, with $Z_A(\eta) = Z_A^{(0)} + \eta a_A$ and $Q_A(\eta) = 1/Z_A^{(0)} + \eta q_A^{(1)}$, $a_A = -(Z_A^{(0)})^2 q_A^{(1)}$, so

$$J = 2(Z_3^{(0)})^2 q_3^{(1)} + 3(Z_2^{(0)})^2 q_2^{(1)} - 3(Z_Y^{(0)})^2 q_Y^{(1)}, \quad T = -466(Z_3^{(0)})^2 q_3^{(1)} + 687(Z_2^{(0)})^2 q_2^{(1)} - 159(Z_Y^{(0)})^2 q_Y^{(1)},$$

and on the conditional unscreened primitive baseline (6, 13/2, 21/2), $J = 72 q_3^{(1)} + (507/4) q_2^{(1)} - (1323/4) q_Y^{(1)}$ and $T = -16776 q_3^{(1)} + (116103/4) q_2^{(1)} - (70119/4) q_Y^{(1)}$.

These are conditional extraction identities: inverse-work and action corrections differ in sign and normalisation, higher orders mix lower-order responses, a screened baseline must use its own leading values, and for $q = 6Y$ the inverse coefficient converts in the opposite direction to $Z_q = 36Z_Y$. They must not be applied to a current Fisher tensor, an untyped link mass, a total kinetic coefficient or a running correction counted again in Stage B. The controlling direct-action target remains the bond and completion differentials, their cross metric and the full connected auxiliary inverse on the physical background [17, §47J]; the history route additionally needs the HM12 transfer derivative.

Part H. Completion, Yukawa transport and the finite response

This Part asks what the completion sector contributes to J and T through the channels the source already supplies: the mass bridge, the momentum-dependent gauge response of the sequential chiral completion, the canonical Yukawa vertices and their two-loop running, the finite regulator matching, the coupled record and completion kernel and its gauge derivatives, and the master action's own current and conservation residuals. Running contributions are evaluated exactly on the declared branches; finite coefficients are reduced to explicit maps from unevaluated integrals to the two contrasts; and the evaluated quadratic action is shown not to fix the mixed completion response. None of these results is a numerical physical J or T .

34. Same-side completion blocks and canonical factors

Proposition 45 (returns of the mass bridge and canonical factors). (a) With $R = K_{\text{int}}^{-1}$ on the admitted intermediate support, $a = G_{\text{out}}^+ l$ and $b = G_{\text{in}} r$ for normalised external vectors, the static Schur term is the negative of $S = [[L, C], [C^*, R_s]]$ with $L = a^+ R a$, $C = a^+ R b$, $R_s = b^+ R b$; if $R \geq 0$ then $L, R_s \geq 0$ and $|C|^2 \leq L R_s$, so a nonzero bridge forces nonzero same-side returns on a positive branch, while an invertible indefinite response can have zero diagonal returns and a nonzero bridge.

(b) The selected-line control $R = \text{diag}(r, a, b)$, $G_{\text{out}} = \alpha(1, x, 0)$, $G_{\text{in}} = \alpha(1, 0, y)^T$ keeps both gate coefficients equal to α and the omitted cross routes zero, yet $C = \alpha^2 r$, $L = \alpha^2(r + ax^2)$, $R_s = \alpha^2(r + by^2)$: the same protected mass permits unequal returns, and $x = y = 0$ is not established by the selected-line theorem.

(c) For energy-independent blocks, $Q(E) = EI - H_{\text{eff}}(E) = BRB^+ + E(I + BR^2 B^+) + O(E^2)$: the static mass uses R , the energy normalisation its second moment.

(d) If the intermediate readout is a fermionic block of the same regulated action, $D(\epsilon, U) = [[D_\epsilon, \epsilon V], [\epsilon W, D_i]]$, then with $S = V D_i^{-1} W$ and $P = D_\epsilon^{-1}$, $\delta \Gamma_+ = \epsilon^2 \text{Re Tr}(PS) + (\epsilon^4/2) \text{Re Tr}(PSPS) + O(\epsilon^6)$ relative to zero insertion, and $\partial_a \partial_b \delta \Gamma_+ = \epsilon^2 \text{Re Tr}(P_{ab} S + P_a S_b + P_b S_a + P S_{ab}) + O(\epsilon^4)$; a purely cross-chiral insertion has zero first trace on a block-diagonal massless baseline, so its first determinant effect is fourth order in the gate parameter.

(e) Constant invertible canonical factors $D_Z(U) = LD_O(U)R$ with $\partial_a L = \partial_a R = 0$ change Γ_+ by $-\text{Re} \log \det L - \text{Re} \log \det R$ and contribute nothing to $\partial_a \partial_b \Gamma_+$: $\Delta J = \Delta T = 0$ on a fixed consistent regulator and measure.

Statement (e) does not cover momentum-dependent factors, source-dependent transformations, changed canonical masses and thresholds or new interactions, and it does not set the baseline or total contrasts to zero; rescaling a propagator while leaving its vertices unchanged would create a spurious response. The mass bridge itself is neither replaced nor assigned a gauge price by analogy [44].

35. Momentum-dependent response and the common-kernel cancellation

The sequential chiral completion of CLSN-1 supplies gauged transfer, frequency dependence, the moving-projector Weyl determinant and finite electric and magnetic matching [3, §§17.13 to 17.20; 47]. For a same-side product $\Sigma = BRC$ with $R = K^{-1}$, $R_a = -RK_aR$ and $R_{ab} = RK_aRK_bR + RK_bRK_aR - RK_{ab}R$, the complete derivatives $\Sigma_a = B_aRC + BR_aC + BRC_a$ and Σ_{ab} (nine terms) include gate derivatives, intermediate propagation and contact terms, and on a regular covariant support pure-gauge consistency gives $\delta_\Lambda \Sigma = i[\Lambda, \Sigma]$, $\delta_\Lambda^2 \Sigma = -[\Lambda, [\Lambda, \Sigma]]$, which does not determine the transverse finite response. The constructed vertices are $A_n = \Omega_n \exp(-a_t K_n)$, $E_a = -a_t \int_0^1 \exp(-a_t(1-s)K) K_a \exp(-a_t s K) ds$, $A_a = \Omega_a E + \Omega E_a$, $A_{ab} = \Omega_{ab} E + \Omega_a E_b + \Omega_b E_a + \Omega E_{ab}$; the chiral completion also differentiates its sign function and moving Weyl projectors, and freezing those projectors fails the recorded Ward check. The projected Weyl Hessian is $\Gamma^{W,+}_{ab} = \text{Re} \text{Tr}(M^{-1} M_b M^{-1} M_a - M^{-1} M_{ab})$ with $M_\lambda = \bar{V}^* D_\lambda V_\lambda$, replayed with differences 1.44×10^{-15} and 2.06×10^{-14} from the archive.

Proposition 46 (common-kernel cancellation and the relative matching coefficients). Assume a flat symmetric background, one common massless kinetic and regulator family for every inherited Weyl matter sector, the shared-link coupling and no sector-dependent completion insertion. Then the parity-even quadratic gauge response factorises as $\Pi_{a,\mu\nu}(q) = I_a \Pi^*_{a,\mu\nu}(q)$, with the three-generation indices per generator (colour, weak, conventional hypercharge) equal to (3, 9/2, 1/2) for Q, (3/2, 0, 4) for u, (3/2, 0, 1) for d, (0, 3/2, 3/2) for L and (0, 0, 3) for e, total (6, 6, 10). Since $j \cdot (6, 6, 10) = 0$ and $t \cdot (6, 6, 10) = 264$, $J_{\mu\nu}(q) = 0$ and $T_{\mu\nu}(q) = 264 \Pi^*_{a,\mu\nu}(q)$ at every external momentum, including contact terms, for either compared kernel and for their difference. The inherited candidate-minus-reference coefficients, in the convention $\Delta\Gamma = (C_E/2) \int \Sigma F_{ti}^2 + (C_M/2) \int \Sigma_{i<j} F_{ij}^2 + \dots$, are $C_E = +0.0003893315522607$ and $C_M = -0.0010120513222149$, giving $\Delta A_X = C_X(6, 6, 10)$, $\Delta J_X = 0$ and $\Delta T_X = 264 C_X$: $+0.1027835298$ electric and -0.2671815491 magnetic, with working uncertainty scales of about 0.0023 and 0.0099 that are quadrature and extrapolation estimates, not bounds.

These are relative regulator matching coefficients, not physical J or T; electric and magnetic values cannot be averaged into one Lorentz-invariant value without the physical matching, no interface or loop multiplier is inserted, and neither is comparable with the diagnostic -33.48 . When the finite response admits a sector decomposition $c_f = c^* + \delta c_f$ for matter with a separate Higgs coefficient normalised to its index (0, 1/2, 1/2), $J = 9(\delta c_u + \delta c_e - 2\delta c_Q)$ and $T = 264(c^* - c_H) - 1614\delta c_Q + 1335\delta c_u + 858\delta c_d - 792\delta c_L + 477\delta c_e$; the common momentum response drops out of J and the remaining test is the finite completion and record remainder on the physical branch; if the response does not decompose, the three gauge coefficients are contracted directly.

36. Completion contractions at two-loop order

The master action supplies the mixed completion route $Y_f = R_{fL} E_{fL}^\dagger Y^* E_{fR} R_{fR}$ with $Y^* = \Lambda^* D_{\Phi_{\{H\}}} (V_H^\dagger X_H)$, $f = u, d, e$ [17, §§32 to 36]; a shared completion operator does not make its sector compressions equal. For the quantum calculation the returned vertices must be the canonically normalised Standard-Model Yukawa matrices, in the renormalisable effective action with no additional active sterile-neutrino Yukawa or record particles; that identification and its finite matching are not derived here.

Proposition 47 (completion contractions). (a) In the unbroken canonical limit the one-loop self-energy family matrices multiplying the common momentum kernel are $Y_u Y_u^\dagger + Y_d Y_d^\dagger$ on Q, $2Y_u^\dagger Y_u$ on u, $2Y_d^\dagger Y_d$ on d, $Y_e Y_e^\dagger$ on L and $2Y_e^\dagger Y_e$ on e. (b) At zero Higgs background the Yukawa vertex first enters gauge running

at two loops. With $h_f = \text{Tr}_{\text{family}}(Y_f^\dagger Y_f)$ and $A = (g_3^{-2}, g_2^{-2}, g_1^{-2})$, the complete published coefficient [46] gives $dA/d \ln \mu|_{\text{Yuk}} = 2Dh/(16\pi^2)^2$ with

$$D = [[2, 2, 0], [3/2, 3/2, 1/2], [17/6, 5/6, 5/2]], \quad jD = (0, -6, 6), \quad tD = (352, 34, 54),$$

rows colour, weak, conventional hypercharge, columns u, d, e; contracting the five matrices of (a) with the per-generation indices reproduces D. Defining B_j and B_T as the Yukawa contributions to the derivatives of $j \cdot A$ and $t \cdot A$,

$$B_j = 12(h_e - h_d)/(16\pi^2)^2, \quad B_T = 2(352h_u + 34h_d + 54h_e)/(16\pi^2)^2.$$

(c) If the normalised readout matrices of Section 18 are these canonical vertices, $x_u = 3h_u$, $x_d = 3h_d$, $x_e = h_e$, so $J_G = 3(h_e - h_d)$, $T_G = 176h_u + 17h_d + 27h_e$ and $(B_j, B_T) = [4/(16\pi^2)^2](J_G, T_G)$: a legitimate quantum-order use of the Gram contractions that does not identify K with G.

B_j vanishes exactly when $h_e = h_d$ at the stated scale; the direct up-sector coefficient cancels, whereas $B_T > 0$ if any charged Yukawa is nonzero. On the executed unit candidate $Y_u = Y_d = Y_e = I_3$, $B_j = 0$ and $B_T = 2640/(16\pi^2)^2$. The conditional shared-kernel relation $y_d = 9y_e$ of QCMP at one matching point [44, §14] gives a first-line contribution $B_j^{(e,d)} = -960y_e^2/(16\pi^2)^2$, which with $y_e = \sqrt{2} \alpha_{\text{int}}^2/(8\pi)$ is $-15\alpha_{\text{int}}^4/(128\pi^6)$; the factor nine is the proposed attachment coefficient, other generations can cancel it in the family trace, and the lepton norm of CLAS, $h_e = y_e^2(1 + \exp(32/3) + \exp(64/3)/144)$ [45, §15], retains its discrete, kernel, scale and matching premises. Along a regular write coordinate s , $dh_f/ds = 2 \text{Re Tr}(Y_f^\dagger dY_f/ds)$ and $\partial_s B_j = 24 \text{Re Tr}(Y_e^\dagger \partial_s Y_e - Y_d^\dagger \partial_s Y_d)/(16\pi^2)^2$; a pure unitary record-basis motion $\partial_s Y = A_L Y - Y A_R$ preserves h exactly, a change of completion stretch or normalised attachment need not, and the universal completion resolvent $R_\Omega = (I + L_\Omega)^{-1}$ contributes $\partial_s R_\Omega = -R_\Omega(\partial_s L_\Omega)R_\Omega$ as one term only.

The archived static completion block of Configuration Dynamics [4, §XV.12], in $x_H = (\phi_H, X_{H,0}, X_{H,1}, X_{H,2})^T$ with defects $d_i = X_{H,i} - \phi_H$ and isolated potential Hessian $\text{diag}(2, 0, 0, 0)$, has $J_H = [[-1, 1, 0, 0], [-1, 0, 1, 0], [-1, 0, 0, 1]]$, $H_H = J_H^T J_H + H_V = [[5, -1, -1, -1], [-1, 1, 0, 0], [-1, 0, 1, 0], [-1, 0, 0, 1]]$, $q_H = \frac{1}{2}(1, 1, 1, 1)^T$ with $J_H q_H = 0$; an added defect force $s^T d$ gives the exact relaxed response $\delta x_H = -H_H^{-1} J_H^T s = (0, -s_0, -s_1, -s_2)^T$ and $q_H^T \delta x_H = -(s_0 + s_1 + s_2)/2$. The archived bath map has zero completion-defect columns and this block has zero cross-block to the other coordinates, so its forcing gives zero displacement and zero induced bilinear through the single completion leg; the zero is static, assumes frozen record and fermion coefficients, and does not remove the background Yukawa interactions.

At a common defect zero the full record and completion response is $H = \sum_{X,Y} L_X^\dagger W_{XY} L_Y + H_V + H_{\text{quantum}}$ with $H_{\text{red}} = H_{xx} - H_{x\eta} H_{\eta\eta}^{-1} H_{\eta x}$, each quantum contribution once; for the completion-mediated running part of J the source must return $h_e(\mu^*)$, $h_d(\mu^*)$ and their write derivatives, T needs h_u as well, and the finite boundary needs the local remainder after matching and thresholds. The diagnostic -33.48 cannot be compared with a running derivative or with the static zero.

37. The full-family candidate and the matched balance

The full-family candidate assembled in MCHY-1C, CLAS, QCMP and the quark grid [49; 45; 44; 50] closes the cancellation question for that candidate without choosing a flavour frame. With $R = \exp(16/3)$ and $c = \sqrt{2} \alpha_{\text{int}}^2/(8\pi)$, the candidate matrices are $Y_e^C = c \text{diag}(1, R, R^2/12)$ and $Y_d^C = 9c \text{diag}(1, 20, 5R)$; the quark entries are pre-RG coefficients, and a shared algebraic kernel is not a common matching convention.

Proposition 48 (exact candidate balance, matching threshold and matched transport). (a) $h_e^C = c^2 E$ and $h_d^C = c^2 D$ with $E = 1 + R^2 + R^4/144 = 12824538.9850$, $D = 81(401 + 25R^2) = 86908417.8962$, $F = E - D = -74083878.9111$ (paired generations contribute -80, +10501.6972 and -74094300.6084), $D/E = 6.77672842646$, and $B_j^C = 12c^2 F/(16\pi^2)^2 = -112.879682572\alpha_{\text{int}}^4$, strictly negative for any nonzero common coupling. With norm-transfer ratios $a_f = \|Y_f^*\|_F/\|Y_f^C\|_F$ at one scale and scheme, $B_j^* = 0$ if and only if $a_d/a_e = (E/D)^{1/2} = 0.384140377703$; equal relative corrections ϵ preserve the sign if $\epsilon < 0.4449401464$.

(b) Retaining the bottom/strange rule $R/4$ with the strange/down coefficient q free, $B_j^C(q) < 0$ iff $q > 7.68278687189$; holding $q = 20$ and varying $b = y_b/y_d$ gives cancellation at $b = 397.3998513$, below the candidate $5R = 1035.6362444$.

(c) With the exact top-grid coefficient $U = 81[(6/13)^2 + (3080/13)^2 + (5R-6/13)^2(77/3)^3]$, $B_T^C = 2c^2(352U + 34D + 54E)/(16\pi^2)^2 = 27972280.8139\alpha_{\text{int}}^4$; at $\alpha_{\text{int}} = 7/960$ the candidate values are $h_e^C = 0.000114788734051$, $h_d^C = 0.000777892077083$, $h_u^C = 2.80083499009$, $B_J^C = -3.19097210278 \times 10^{-7}$ and $B_T^C = +0.0790742547236$.

(d) On the FRGM-1C matched branch [48], frozen at $\mu_R = 2$ GeV, $\mu_0 = 243.722807638$ GeV, $\mu_* = 5800$ GeV, $g_s(\mu_*) = 1$, $\alpha_{\text{int}} = 7/960$, $R_u = R_d = r_q l_3$ with $r_q = 0.580788332942$ and $(m_t, m_b, m_c) = (171.783689943, 4.213223810, 1.228341125)$ GeV, the raw singular values are $y_u^C = (1.243 \times 10^{-5}, 0.006379, 1.67356)$, $y_d^C = (2.6926 \times 10^{-5}, 0.0005385, 0.027886)$ and $y_e^C = (2.9918 \times 10^{-6}, 0.0006197, 0.010696)$, with raw down/lepton squared-norm ratio 6.7769856867 and cancellation threshold 0.3841330865. Full complex matrix evolution with $16\pi^2 Y_u' = (3/2)(H_u - H_d)Y_u + (T_Y - C_u)Y_u$, $16\pi^2 Y_d' = (3/2)(H_d - H_u)Y_d + (T_Y - C_d)Y_d$, $16\pi^2 Y_e' = (3/2)H_e Y_e + (T_Y - C_e)Y_e$, $T_Y = 3h_u + 3h_d + h_e$, $C_u = (17/12)g_Y^2 + (9/4)g^2 + 8g_s^2$, $C_d = (5/12)g_Y^2 + (9/4)g^2 + 8g_s^2$, $C_e = (15/4)g_Y^2 + (9/4)g^2$ and $16\pi^2 g_i' = b g_i^3$, $b = (41/6, -19/6, -7)$ in the order (g_Y, g, g_s) , gives:

Quantity	At μ_0	At $\mu_* = 5800$ GeV
h_u	0.9447668603	0.7192855103
h_d	0.000262404053	0.000181859135
h_e	0.000114788453	0.000119927252
h_d/h_e	2.285979521	1.516412092
B_J	$-7.10352717 \times 10^{-8}$	$-2.98027319 \times 10^{-8}$
B_T	+0.0266733522	+0.0203074891
(g', g, g_s)	(0.3419910489, 0.6504768410, 1.1793326525)	(0.3476127274, 0.6336586930, 1)

(e) With $L = \frac{1}{2}\ln(h_d/h_e)$, the exact trace equations give $16\pi^2 L' = (3/2)[\text{Tr}(H_d^2)/h_d - \text{Tr}(H_e^2)/h_e - \text{Tr}(H_u H_d)/h_d] - 8g_s^2 + (10/3)g_Y^2$, the common T_Y and weak terms cancelling and $15/4 - 5/12 = 10/3$. Positive semidefiniteness, $\text{Tr}(H_f^2) \leq h_f^2$ and $0 \leq \text{Tr}(H_u H_d) \leq h_u h_d$, with $g_s^2 \geq 1$ on the interval, bounds the norms inside the box $h_u \leq 0.945$, $h_d \leq 0.000263$, $h_e \leq 0.000130$ for every relative flavour orientation (the up and down faces have brackets at most -7.493162 and -10.327373 , and Grönwall keeps $h_e \leq 0.000128630204$), so that $L(t) \geq L(0) - 3(0.945130)t/(32\pi^2) - (4/7)\ln[g_s^2(0)/g_s^2(t)] > 0.196428$ and $h_d/h_e > 1.4812$, $B_J < 0$, for $0 \leq t \leq \ln(5800/243.722807638)$. (f) The integrated contributions are

$$\delta J_Y = \int B_J d \ln \mu = -1.525785594 \times 10^{-7}, \quad \delta T_Y = \int B_T d \ln \mu = +0.07347082431 \quad (\mu_0 \text{ to } 5800 \text{ GeV}),$$

with $J_G(\mu_*) = -0.0001857956493$ and $T_G(\mu_*) = 126.6005795$ on the Gram identity.

Statement (e) is a continuous differential-inequality proof with outward-rounded constants, not a sign inferred from sampled points; it proves non-cancellation of this Yukawa contribution on the central conditional branch and does not determine the sign of the total finite physical J . At μ_* the effective norm-transfer ratio is 0.4730317975 against the archived cancellation ratio 0.3841330865, and consistently typed matching corrections with relative Frobenius bound ϵ at every scale preserve the sign if $(1 - \epsilon)(1.4812)^{1/2} > 1 + \epsilon$, that is $\epsilon < 0.09789$, a target for a physical error bound that the unevaluated terms have not been shown to meet. Each of FRGM's five preregistered readout points gives h_d/h_e between 2.2712 and 2.4153 at μ_0 ; the proof applies to the central branch only. The quark grid identifies the adopted strange/down value 20 as an imported kaon-ratio target [50, §6.1], so the assembled grid is not certified as free of empirical flavour input; no new parameter was fitted here. The integrals are one-loop matrix and gauge transport inside the leading two-loop gauge contraction, reverse sign with

direction, and are not a finite matching constant; existing Stage-B transport must be checked for them before any addition. QCD matching is applied once at μ_0 ; the separate pure-QCD factor 0.4810030165 at 5.8 TeV is not applied again.

38. Finite matching and record relaxation

The recovered CLSN finite archive differentiates the moving-projector Weyl determinant with its contact vertices on a massless free-fermion kernel with $h = c = a_t = r = \rho = 1$; it contains no Yukawa matrix, Higgs background or record variable, and it carries the coefficients C_E , C_M of Proposition 46. At zero Higgs background the canonical Yukawa interaction is cubic and does not enter the free quadratic determinant, so the matched norms of Section 37 do not reweight C_E and C_M ; the factor r_q acts on the Yukawa matrices, not on the representation indices, and the finite Yukawa-dependent gauge response begins at two loops in this reference calculation, whose beta function fixes logarithmic dependence and not finite constants.

Proposition 49 (local relaxation, the exact finite Schur identity and the information bound). (a) For a smooth local insertion $\Delta\Gamma(a, \eta) = \frac{1}{2}\sum_{i,j} C_{ij}(\eta) \|F_{ij}(a)\|^2$ on a regular stationary branch with fixed canonical attachment and $F(0) = 0$, $\eta_0(a) = \bar{\eta} + O(a)$ and $F_{ij}(a) = L_{ij}a + O(a^2)$ give $\Delta\Gamma(a, \eta_0(a)) = \frac{1}{2}\sum C_{ij}(\bar{\eta}) \|L_{ij}a\|^2 + O(a^3)$: at the flat background $\Delta\Gamma_\eta = \Delta\Gamma_{a\eta} = \Delta\Gamma_{\eta\eta} = 0$, the direct quadratic gauge coefficient is unchanged by relaxation, and the common local fermion term retains $\Delta J = 0$.

(b) For $H_1 = H_0 + \Delta H$ on a common identified support with reference blocks $A, B, C, R = C^{-1}B^T$, $W = \Delta B^T - \Delta C \cdot R$ and $V = (I, -R)^T$, the reduced Hessian changes exactly by $\Delta S = V^T \Delta H V - W^T (C + \Delta C)^{-1} W$, which reduces to $\Delta S = \Delta A$ when $\Delta B = \Delta C = 0$.

(c) At first quantum order, $\Gamma = S_0 + \varepsilon\Gamma_1$, the reduced gauge Hessian changes by $\delta S_{ab} = V_a^T \Gamma_1 V_b + \Gamma_{1,\eta} \cdot \eta_{0,ab}$ with $\eta_{0,a} = -C^{-1}(B^T)_a$ and $\eta_{0,ab} = -C^{-1}S_{0,\eta ij} V_{ia} V_{jb}$; the second term, the finite auxiliary one-point response, vanishes for the pure F^2 insertion at the flat background but need not vanish for the remaining finite action.

(d) A common finite shift $\xi(6, 6, 10)$ added to both compared regulator actions leaves their difference unchanged and shifts each absolute T by 264ξ ; the map from the three local gauge coefficients to (J, T) has rank two, with $k = (0, (53u - v)/528, (229u - v)/528)$ giving $j \cdot k = u$ and $t \cdot k = v$ for any formal contrast shifts.

Logarithmic running, matched Yukawa norms and the relative regulator difference are therefore insufficient by themselves to determine both finite constants; the specified physical action and finite prescription must fix them, and the example is an information-sufficiency test, not a licence to fit counterterms. The remaining return is $\Gamma_{fin,xx}, \Gamma_{fin,x\eta}, \Gamma_{fin,\eta\eta}$ and $\Gamma_{fin,\eta}$ on the canonically attached matched branch, with the reference B, C and the third derivatives entering the stationary response, retaining Yukawa, Higgs, gauge, ghost and record/completion contributions at the declared order with the fixed finite prescription and physical electric and magnetic normalisation; the matched FRGM Yukawas are known conditional inputs and their QCD matching factor is not to be refitted.

39. The finite completion attachment

On the conditional matched branch, in the unbroken parity-even canonical block with no active neutrino Yukawa and with $N = (16\pi^2)^2$, the order- Y^2 exchange and contact contribution can be reduced to charge tensors. The squared-generator attachments per generation norm, in factor order $(3, 2, Y)$, are: up, left $(1, 3/2, 1/6)$, right $(1, 0, 8/3)$, scalar $(0, 3/2, 3/2)$; down, left $(1, 3/2, 1/6)$, right $(1, 0, 2/3)$, scalar $(0, 3/2, 3/2)$; charged lepton, left $(0, 1/2, 1/2)$, right $(0, 0, 2)$, scalar $(0, 1/2, 1/2)$. The right attachment includes both weak components of the Yukawa contraction and the scalar attachment the colour multiplicity. For the invariant Yukawa tensor the signed generator actions obey $q_L + q_R + q_H = 0$ and $2 \operatorname{Re}\langle q_L, q_R \rangle = \|q_H\|^2 - \|q_L\|^2 - \|q_R\|^2$, so mixed attachments reduce to squared attachments. Their contractions with j and t are: up $(-6, 6, 0)$ and $(-538, 890, -792)$; down $(-6, 0, 0)$ and $(-538, 572, -792)$; lepton $(0, 6, 0)$ and $(-264, 318, -264)$; and $L_f + R_f$ is the beta-function column of D for every sector.

Proposition 50 (the attachment map and its cancellation conditions). (a) When the subtracted parity-even kinematics factor from these tensors, $\delta A_Y = (1/N)\Sigma_f h_f(c_L^f L_f + c_R^f R_f + c_H^f S_f)$ and $N\delta J_Y = 6[(c_R^u - c_L^u)h_u - c_L^d h_d + c_R^e h_e]$.

(b) With common functions across sectors, $N\delta J_Y = 6[c_R(h_u + h_e) - c_L(h_u + h_d)]$ and $N\delta T_Y = -c_L[538(h_u + h_d) + 264h_e] + c_R[890h_u + 572h_d + 318h_e] - 264c_H(3h_u + 3h_d + h_e)$; at the 5800 GeV endpoint with $(h_u, h_d, h_e) = (0.719285510311454, 0.000181859135124261, 0.000119927252018600)$ the coefficients, including $1/N$, are -0.000173110295 (c_L) and $+0.000173095393$ (c_R) in δJ_Y , and $-0.01552349275, +0.02567723730, -0.02285182857$ for (c_L, c_R, c_H) in δT_Y .

(c) With fermion-line equality $c_L = c_R = c_F$, $\delta J_Y = -1.49013659288 \times 10^{-8} c_F$ and $\delta T_Y = 0.01015374455 c_F - 0.02285182857 c_H$; without it, cancellation requires $c_R/c_L = (h_u + h_d)/(h_u + h_e) = 1.000086087594$, an offset of about 0.00861%; the coefficient vector $(2, 2, 0)$ reproduces the logarithmic slope $B_J = -2.98027318576 \times 10^{-8}$, $B_T = +0.02030748910$.

(d) With $c_F = (c_L + c_R)/2$ and $c_A = (c_R - c_L)/2$, $N\delta J_Y = 6[(e - d)c_F + (2u + d + e)c_A]$ and $N\delta T_Y = (352u + 34d + 54e)c_F + (1428u + 1110d + 582e)c_A - 264(3u + 3d + e)c_H$, with endpoint coefficients $-1.49013659 \times 10^{-8}$, $+0.000346205688$, 0 in δJ_Y and $+0.01015374455, +0.04120073005, -0.02285182857$ in δT_Y ; the zero-J condition for this block alone is $c_A/c_F = (d - e)/(2u + d + e) = 4.30419442246 \times 10^{-5}$, and the c_A coefficient is 23233.15 times the c_F coefficient.

(e) Even with $c_A = 0$ imposed, $\det M_{F,H}^{\text{equal}} = 1584(d - e)C/N^2 = 3.40523459623 \times 10^{-10} \neq 0$ with $C = 3u + 3d + e$, so no nonzero linear combination of δJ_Y and δT_Y eliminates both unevaluated coefficients; the one invisible combination of the general two-by-three map is $(c_L, c_R, c_H) \propto (1, 1.000086087594, 0.444426362185)$, whose gauge response is parallel to the common direction $(12, 13, 21)$.

The up contribution cancels from J under fermion-line equality while the matched down/lepton imbalance survives if $c_F \neq 0$, with its sign not fixed before c_F is evaluated; the scalar attachment has zero J contraction in this block. Exchange invariance of the complete regulated and subtracted integrand, including contacts and the conjugate scalar line, is what fermion-line equality requires, and identical free dispersions do not establish it. A logarithmic slope does not determine finite constants, and the absence of a scalar attachment in that slope does not prove its finite coefficient zero. The exchange and contact terms from one fermion action are, for admitted real auxiliaries η_i , $F_{ij} = \partial_i \partial_j [-\text{Tr} \log D] = \text{Tr}(PV_i PV_j - PV_{ij})$ and $\Gamma_{\text{ex}}^{(2)}[A] = \frac{1}{2} \Sigma_{ij} G^{ij}[A] F_{ij}[A]$ with G the inverse of the complete coupled bosonic block on classified support; the functional is differentiated twice in the external field, including P, G, both vertices, supports, projectors and measure, and the local transverse term is extracted after the prescribed subtraction with the stationary pullback and tadpole of Section 38 retained. The known matched Yukawas fix particular canonical readouts, not every internal-momentum covariance.

40. Coupled momentum response and gauge derivatives

Write the complete Euclidean covariance of the record and completion fields on the classified same-fact support as $K(p) = [[R(p), X(p)], [X(p)^\dagger, H(p)]]$, with every other coupled field already reduced into the blocks. Then $S = H - X^\dagger R^{-1} X$, $G_{cc} = S^{-1}$, $G_{rc} = -R^{-1} X S^{-1}$ and $G_{rr} = R^{-1} + R^{-1} X S^{-1} X^\dagger R^{-1}$; positive Euclidean R makes the subtraction in S positive semidefinite and softens the completion kernel without fixing a finite threshold sign.

Proposition 51 (static replay, momentum expansion, gauge differentiation and the selection rule). (a) The recovered complete neutral archive [52] has the faithful 1125×334 defect differential, potential Hessian, six-channel bath map and source differential; its six input hashes replay and $C_{\kappa J_D} = J_{\kappa J}$ holds to relative residual 2.17905×10^{-15} . On $(\phi_H, X_{H,0}, X_{H,1}, X_{H,2})$ its unit-metric block H_0 and completion differential J_H are those of Section 36, with $H_0^{-1} = \frac{1}{2} 11^T + \text{diag}(0, 1, 1, 1)$, $H_0^{-1} q_H = (1, 3/2, 3/2, 3/2)^T$ and $J_H H_0^{-1} q_H = \frac{1}{2} 1_3$; the cross-block to the other 330 variables and all three completion-defect columns of C_κ vanish, so with frozen vertex $(C_{\kappa J_D, H})^\dagger R(p)^{-1} (C_{\kappa J_D, H}) = 0$ at every momentum.

(b) Along a Euclidean direction with $s = p^2$, gapped invertible R_0 , H_0 , and $R(s) = R_0 + s Z_r + O(s^2)$, $X(s) = s X_2 + O(s^2)$, $H(s) = H_0 + s Z_c + s^2 H_4 + O(s^3)$: $G_{rc}(s) = -s R_0^{-1} X_2 H_0^{-1} + O(s^2)$ and $S(s) = H_0 + s Z_c + s^2 [H_4 - X_2^\dagger R_0^{-1} X_2] +$

$O(s^3)$, so the mixed covariance begins at p^2 and changes the completion inverse kernel first at p^4 , with $G_{rc}(s)q_H = -sR_0^{-1}X_2(1, 3/2, 3/2, 3/2)^T + O(s^2)$ on the archived probe; a linear $X(p) = pX_1$ gives $-p^2X_1^+R_0^{-1}X_1$ and a gapless record mode gives $-sX_2^+Z_r^{-1}X_2$.

(c) The inherited HSPR identity $\Delta K(p) = J_D^+ F(p) J_D \Rightarrow \Delta K(p)q_H = 0$ is retained when the same defect map and protected line apply [59]; the relaxed radial tangent of the archived block is $2q_H$.

(d) With $z = (r, c)$, $D_\eta = K_{\eta\eta}$, $K_{\text{eff}} = K_{zz} - K_{z\eta}D_\eta^{-1}K_{\eta z}$, $T = (I, -D_\eta^{-1}K_{\eta z})^T$ and $U_\alpha = (K_\alpha T)_\eta$: $(K_{\text{eff}})_\alpha = T^+K_\alpha T$ and $(K_{\text{eff}})_{\alpha\beta} = T^+K_{\alpha\beta}T - U_\alpha^+D_\eta^{-1}U_\beta - U_\beta^+D_\eta^{-1}U_\alpha$. For $\Gamma = V + d^TWd/2$ the full Hessian is $K_{ij} = V_{ij} + d_i^TWd_j + d_i^TWd_{ij} + d_i^TW_jd + d_j^TW_id + \frac{1}{2}d^TW_{ij}d$, so terms vanishing at $d = 0$ can have nonzero gauge derivatives there: the common zero is imposed after differentiation, the mixed vertex needs third action derivatives and two-gauge contacts can need fourth.

(e) Under the stronger premise $X[0] = 0$ as a full operator, $(G_{rc})_\alpha = -R^{-1}X_\alpha H^{-1}$, $(G_{cc})_\alpha = -H^{-1}H_\alpha H^{-1}$ and $(G_{cc})^{\text{mix}}_{\alpha\beta} = H^{-1}(X_\alpha^+R^{-1}X_\beta + X_\beta^+R^{-1}X_\alpha)H^{-1}$, positive semidefinite in one real direction for positive R ; if $X_\alpha = C_\alpha J_H$ on the archived block then $q_H^T(G_{cc})^{\text{mix}}_{\alpha\alpha}q_H = \frac{1}{2}1_3^T C_\alpha^+ R^{-1} C_\alpha 1_3$, zero for positive R exactly when $C_\alpha 1_3 = 0$. The exchange term differentiates as $\partial_\alpha \partial_\beta \Gamma_{\text{ex}} = \frac{1}{2}\text{Tr}(G_{\alpha\beta}F + G_\alpha F_\beta + G_\beta F_\alpha + G F_{\alpha\beta})$ and the real-boson determinant contributes $(\Gamma_b)^{\text{mix}}_{\alpha\beta} = -\frac{1}{2}\text{Tr}[H^{-1}(X_\alpha^+R^{-1}X_\beta + X_\beta^+R^{-1}X_\alpha)]$.

(f) Static and infrared data do not determine the full covariance: with $s = 4 \sin^2(p/2)$, $K_0(s) = \text{diag}(1 + s, H_0 + sI_4)$ and $K_\lambda(s) = K_0(s) + \lambda s^2 v v^T$, $v = (1, -1, 1, 0, 0)^T$, the added term is positive, local, even in momentum and annihilates $Q = (0, q_H)^T$, leaves $K(0)$ and the p^2 coefficient unchanged, yet $Q^T[K_1(4)^{-1} - K_0(4)^{-1}]Q = -4/3801$.

(g) Selection rule: on an unbroken invariant background with no weak-doublet spurion and a same-fact support and auxiliary census invariant under the central weak transformation $-I$, chirality-preserving record endomorphisms and gauge adjoints have parity $+1$ and a true Higgs doublet parity -1 , so $\mathcal{P}^+K[A]\mathcal{P} = K[A]$ and $K_{+,-}[A] = 0$ for every gauge background; each block of the retained/auxiliary split intertwines the parity actions, so the Schur complement obeys the same identity and the even-record to true-doublet mixed covariance and all its gauge derivatives vanish on this branch.

The positive polar stretch X_H acts on the right carrier and is weak even; it stays in the even block and can couple to even records, and off-diagonal left-right record maps are weak odd and can couple to the true doublet, so the rule eliminates neither the completion response of Sections 36 to 39 nor J. For record endomorphisms $r_{LR}: R \rightarrow L$ of the charged carrier, transforming in $R_L \otimes R_R^*$ with hypercharge $Y_L - Y_R$, the six left-right block types are $u_R \rightarrow Q_L$ in $(1 \oplus 8, 2)$ with $Y = -1/2$, $d_R \rightarrow Q_L$ in $(1 \oplus 8, 2)$ with $+1/2$ and $e_R \rightarrow L_L$ in $(1, 2)$ with $+1/2$, whose colour-singlet parts can pair with $\tilde{H}$ or H at zero gauge background, while $e_R \rightarrow Q_L$ in $(3, 2)$ with $7/6$, $u_R \rightarrow L_L$ in $(3, 2)$ with $-7/6$ and $d_R \rightarrow L_L$ in $(3, 2)$ with $-1/6$ are excluded from any vertex with one such record, one doublet and only gauge insertions; the colour-octet parts of the first two rows are excluded only at zero gauge background, since a colour insertion can supply the adjoint tensor. The table records representation permission, not the presence of a mode or its coefficient.

The later local time attachment is supplied by the sequential-opportunity theorem [60, Theorem 6]: $\Delta t = a_t/c_*$, $Z_H^{\text{time}} = Z_H^{\text{space}} = Z_H$, with finite spatial symbol $\hat{p}_H^2 = \sum_a \sin^2(p_a \ell_a)/\ell_a^2$ and rest-pole relation $E_H = (2\hbar c_*/a_t) \text{arsinh}[(a_t/2)(K_H/Z_H)^{1/2}]$, so time attachment is not an additional unresolved normalisation here, although the common radial symbol is not the entire unbroken doublet covariance.

On a translational regulator the displacement-labelled rows give $L(p) = \sum_\delta e^{ip \cdot \delta} L_\delta$ and the common-zero block $\mathcal{K}(p) = H_V(p) + L(p)^+ W(p) L(p)$; a static matrix with displacement labels removed does not determine this function.

41. The mixed vertex and the sufficiency of the evaluated action

The physical coordinate c is the gauge-singlet positive completion stretch and ε_b denotes bond probes in fixed physical defect units. Four types are distinct: the normalised polar logarithm X_H acts on the right carrier and has weak-singlet type there, a representation-invariant scalar trace is a full gauge singlet, the Higgs-like interface has weak-doublet type [57; 58], and the local left-right completion morphism is not

spatial record transport. For a gauge-invariant scalar stretch direction C the exact local path is $X_H(c) = X_{H,0} + cC$, $B_H(c) = B_{\text{sym}}^{1/2} \exp[X_{H,0} + cC] B_{\text{sym}}^{1/2}$, $\Phi_H(c) = B_H(c) V_H$, and with occupied projectors, polar orientation and other fields fixed, $\partial_c D_B = \partial_c D_R = 0$ while $\partial_c D_H = C$; fixed projectors are stronger than an open gap, since a gapped projector may rotate.

Proposition 52 (the mixed vertex: what is fixed, what cancels and what the evaluated action cannot supply). (a) At a common defect zero the complete third variation of the displayed part is $\Gamma_{\text{arc}} = V_{\text{arc}} + d_{\text{ar}}^T W d_c + d_{\text{ac}}^T W d_r + d_{\text{rc}}^T W d_a + d_a^T W r d_c + d_a^T W c d_r + d_r^T W a d_c$.

(b) On fixed full support at an aligned background with $D_B = U - V_{\text{rec}}$, $U = \exp(iaT_A)$, $V_{\text{rec}} = \exp(irS_b)$, the completion derivative of the bond metric contributes $(\Gamma_{\text{arc}})_{\text{bond metric}} = -\text{Re}\langle T_A, (\partial_c W_{BB}) S_b \rangle$, equal to $-w_B'(0) \delta_{Ab}/2$ for $W_{BB}(c) = w_B(c)I$ and fundamental normalisation; in the commuting one-generator restriction $\Gamma_B = w_B(c)[1 - \cos(a - r)]$ and $\Gamma_{\text{arc}}|_0 = -w_B'(0)$.

(c) For the operational realisation $W_{\text{op}} = \text{diag}(\pi) \otimes I$ with fixed typed probes, $w_B(c) = \sum_x \pi_c(x) q_B(x)$ has $\partial_c w_B = \sum_x (\partial_c \pi_c(x)) q_B(x)$, which vanishes when q_B is constant: normalised probability redistribution cannot change a state-independent bond response. The fixture $\pi_{\pm}(c) = [1 \pm \tanh(\kappa c)]/2$ with profile (1, 4) gives $w_B(0) = 5/2$, $w_B'(0) = -3\kappa/2$ and $\Gamma_{\text{arc}}|_0 = 3\kappa/2$ for every κ with the same quadratic Hessian, and zero for the profile (1, 1).

(d) Exact auxiliary relaxation need not remove the coupling: for $\Gamma = \frac{1}{2}w(c)(a - r - \eta)^2 + \frac{1}{2}m(c)\eta^2$, $\eta^* = w(a - r)/(w + m)$, $w_{\text{eff}} = wm/(w + m)$ and $\Gamma_{\text{arc}}^{\text{red}} = -(m^2 w' + w^2 m')/(w + m)^2$.

(e) The full stationary cubic reduction is $\Gamma_{\text{arc}}^{\text{red}} = \Gamma^{(3)}[t_a, t_r, t_c]$ with $t_l = (e_l, L_l)$, $L_l = -H_{\eta\eta}^{-1} H_{\eta l}$, second derivatives of η^* cancelling because $(Ht_l)_\eta = 0$; for a bilinear Gaussian bath $W^{\text{red}} = W_{\text{core}} - C^+ A^{-1} C$, $\delta W^{\text{red}} = \delta W_{\text{core}} - (\delta C)^+ A^{-1} C - C^+ A^{-1} \delta C + C^+ A^{-1} (\delta A) A^{-1} C$; the Gaussian quantum contribution $(\hbar/2) \text{STr} \log D$ has $\partial_{\text{arc}} \log D = \text{Tr}[G D_{\text{arc}} - G D_a G D_{\text{rc}} - G D_r G D_{\text{ac}} - G D_c G D_{\text{ar}} + G D_a G D_r G D_c + G D_a G D_c G D_r]$.

(f) The history law must be differentiated including its scores: with $S_B = \partial_{\epsilon_{\{B\}}} \log \mu_N$, $L_c = \partial_c \log \mu_N$ and $M_{cB} = \partial_c S_B$, $\partial_c W_{BB} = \lim N^{-1} E[L_c S_B \otimes S_B + M_{cB} \otimes S_B + S_B \otimes M_{cB}]$; for a stationary Markov execution with $h_i = \pi_{c,i}/\pi_i$, $\partial_c W_{BB} = \sum \pi_i K_{ij}[(h_i + s_c) S_B \otimes S_B + m_{cB} \otimes S_B + S_B \otimes m_{cB}]$, and for the locally normalised source $K(c, \epsilon) \propto K(c, 0) \exp[\epsilon \Xi_B(c)]$ the surviving moving-score term is $E[\Xi_{B,c} \otimes S_B + S_B \otimes \Xi_{B,c}]$. The mixed Perron/Doob derivative is $m_{cb}(i, j) = \mathbb{T}_{cb,ij}/\mathbb{T}_{ij} - \mathbb{T}_{c,ij} \mathbb{T}_{b,ij}/\mathbb{T}_{ij}^2 + \Delta_{ij}(r_{cb}/r - r_c r_b/r^2) - \lambda_{cb}/\lambda + \lambda_c \lambda_b/\lambda^2$, with r_{cb} , λ_{cb} from the bordered system $(\lambda I - \mathbb{T}) r_{cb} + r \lambda_{cb} = \mathbb{T}_{cb} r + \mathbb{T}_c r_b + \mathbb{T}_b r_c - \lambda_c r_b - \lambda_b r_c$; for $\mathbb{T} = \exp(-\tau G)$, $\mathbb{T}_{cb} = L_{\exp}[-\tau G_{cb}] + L^{(2)}_{\exp}[-\tau G_c, -\tau G_b]$ retains two ordered insertions even when $G_{cb} = 0$.

(g) A static zero does not determine the mixed score: $p(y|c, \epsilon) = \exp[y\epsilon(1 + \kappa c)]/(2 \cosh[\epsilon(1 + \kappa c)])$ has uniform law, zero completion score and $W_{BB} = 1$ at $c = \epsilon = 0$ for every κ , but $\partial_c W_{BB} = 2\kappa$. The HCMR-XI-4 instrument $K_r(\phi) = \sin \phi A_r$, $K_\emptyset = (I - \sin^2 \phi A^+ A)^{1/2}$ [53, §3] gives $W_{\text{instrument}}^{\text{BB}} = fC + f^2 b \otimes b/(1 - fQ)$ with $f = \sin^2 \phi$, hence $\partial_\phi W_{\text{instrument}}^{\text{BB}}|_0 = 0$ and $W = \phi^2 C + O(\phi^4)$: a support-boundary zero, not a regular completion-score identity, a rate per completed fact or a map from ϕ to c .

(h) The evaluated quadratic action does not fix the next derivative: the master action identifies the history log-density with the displayed action only modulo $O(\|D\|^3)$ [17, §10], the common-zero Hessian $H = V'' + J_D^+ W_o J_D$ contains W_o and not $W_{\cdot c}$, and the family $W_\kappa(c) = E_\kappa(c)^+ W_o E_\kappa(c)$ with $E_\kappa(c) = \exp[(\kappa c/2) P_B]$ agrees with W_o at $c = 0$, preserves positivity, rank and every cross block, yet has $(W_{BB})'(0) = \kappa W_{BB,0}$, $(W_{BX})'(0) = (\kappa/2) W_{BX,0}$ and changes the bare bond contribution by $-\kappa \text{Re}\langle T_A, W_{BB,0} S_b \rangle$. The two-defect model $\Gamma = \frac{1}{2} w e^{\kappa c} (a - r - y)^2 + \kappa e^{\kappa c/2} c (a - r - y) + \frac{1}{2} \hbar c^2 + \frac{1}{2} m y^2$ with $wh > k^2$ has every Hessian entry at the origin independent of κ and $\Gamma_{\text{arc}}^{\text{red}}|_0 = -\kappa w m^2/(w + m)^2$ after exact elimination of y , with Gaussian determinant $\log(w e^{\kappa c} + m)$ independent of a, r .

Three further closures sharpen the input. The primitive joint update of PJCK, $(U, j) \mapsto (VU, \eta + Vj)$ pushed through a positive event measure Λ and Perron/Doob normalised [56; 3, §17.79], has the exact mixed derivative $\mathbb{T}_{bc} f = \int f(F) d\Lambda_{bc} + \int Df(F) [F_b] d\Lambda_c + \int Df(F) [F_c] d\Lambda_b + \int (D^2 f(F) [F_b, F_c] + Df(F) [F_{bc}]) d\Lambda$ with $j'_{bc} = \eta_{bc} + V_{bc} j + V_{bj} c + V_{cj} b + V_{jj} c$; the saturated native current instrument of RCHS-29 is exactly invariant under $j \mapsto xj$, so a scalar-only completion change $j(b, c) = \chi(b, c) \tilde{j}(b)$ gives $\partial_c C_\alpha = \partial_b \partial_c C_\alpha = 0$ and $\partial_c W_{BB} = 0$ on that channel, while a mixed change with $j_c = \tilde{j}_j$ only at the reference point leaves the transverse vector $v_{bc} = P_\perp(j_{bc} - \tilde{j}_b)$

and $\rho_{bc} = (v_{bcj}^\dagger + jv_{bc}^\dagger)/t$, and the diagnostic attachment $x = b(1 + c)$ shows that changing scores enter even when the baseline has no c -response, since $W_{BB}(0, c) = (1 + c)^2 W_{xx}(0)$.

The exact Fold balance of PJCK, $x_{n+1} = Px_n + \varepsilon_n$ with $S = H^{-1}$ and $\Sigma = S - PSP^T$, differentiates to $\Sigma_c = -SH_cS - P_cSP^T + PSH_cSP^T - PSP_c^T$: an independently returned covariance response determines the action response and conversely, but with neither supplied the equation is a consistency relation; for the mean-shift experiment $K(x'|x) = N(P(c)x + bh(c), \Sigma(c))$ the complete rate is $W_{BB} = h^T Gh$ with $(W_{BB})_c = 2h_c^T Gh - h^T G \Sigma_c Gh$, a fixed-force coupling gives $h = (I - P)Sf$ and on a commuting scalar block $W^{\text{force}}_{BB} = [(1 - p)/(1 + p)]f^T H^{-1} f$ with $(W^{\text{force}}_{BB})_c = -[(1 - p)/(1 + p)]f^T SH_c Sf$, and the sector family $H_\kappa(c) = H_0 + \kappa cA$ with $A = \Pi_a/d_a - \Pi_d/d_d$ keeps H_0 , P , trace, stationarity and detailed balance while $(W_{BB})_c|_0 = \kappa \|h\|^2/[d_a(1 - p_a^2)]$.

Finally the existing EPMD/CFB1 cubic, $\Gamma^{(3)}_{jjs} = (w_R p^2/2)[s_0 F_0(j) + s_{2c} F_c(j) + s_{2s} F_s(j)]$ with $F_0 = (4/6)(j_1^2 + j_2^2)$, $F_c = (j_1^2 - j_2^2)/\sqrt{3} + 2j_1 j_2$, $F_s = -j_1^2 + j_2^2 + 2j_1 j_2/\sqrt{3}$ and $p = \sqrt{2}/53$ on the unit branch (the E_2 coefficient $2/(2809\sqrt{3})$ of PJCK), becomes in the bond coordinate $b = \theta - \gamma$, after relaxing both current helicities and the phase, $\Gamma^{(3)}_{bbs} = (\lambda_b/2)[s_0 F_0(b) + \dots]$ with $\lambda_b = w_R w_C^2/(w_C + w_R)^2$ and $H^{\text{red}}_{bb} = w_B + 4w_C w_R/(w_C + w_R)$, $\lambda_b = 1/4$ and $H_{bb} = 3$ at unit weights; the whole nonlinear action separates as $\Gamma_{\text{net}}(z) + \Gamma_H(x, h)$ with $\Gamma_H = \frac{1}{2}(x - \alpha_H h)^T W_H (x - \alpha_H h) + (\lambda_h/4)(h^2 - v^2)^2$, so the local completion leg of this cubic is exactly zero after complete auxiliary relaxation on that source, $\partial_c H^{\text{red}}_{bb} = \Gamma^{(3), \text{red}}_{bbc} = 0$, with $\chi_{0c} = 0$ for the only linearly mixing A_1 stretch.

What these results establish together is precise. The doublet selection rule does not eliminate the positive stretch; gauge covariance permits the bond-metric tensor; the normalised-history zero and the separation zero are exact within their subcases; the quadratic data of the evaluated action do not determine the cubic response, and exact stationary or Gaussian reduction need not cancel it. None establishes a nonzero complete physical mixed vertex, and none is a nonuniqueness theorem for the complete source: the κ families are sufficiency counterexamples for the evaluated data. The required return is the joint bond and completion dependence of the primitive increment law, $\mathbb{T}_c$, $\mathbb{T}_b$ and $\mathbb{T}_{cb}$ on the complete admitted carrier or their transported-increment equivalent, or an exact source symmetry proving the complete contraction insensitive to it; Section 42 shows what the master action's own residuals already supply towards that return.

42. Current conservation and the source kernel

The four-coordinate source kernel on a wheel mode, in the rotated current basis $u = (p + q)/\sqrt{2}$, $v = (q - p)/\sqrt{2}$ and with constant coefficients, is $S_4 = (w_J/2)u^2 + (w_J/2)(v + \kappa\theta)^2 + (a\ell/2)\theta^2 + (c/2)(\theta - \gamma)^2 + (b\ell/2)\gamma^2$. Eliminating both current coordinates and the record at fixed link gives $k_{\text{link}}(\ell) = a\ell + b\ell/(b\ell + c)$ and $\det H_{(u,v,\gamma)} = w_J^2(b\ell + c)$: the reduced link stiffness contains neither the current mixing nor the current weight, and the eliminated determinant is independent of the retained link. The isolated Gaussian block therefore supplies no matter-dependent gauge response through its shifted current squares; a current Schur price or its Casimir denominators cannot be identified with prices on the 47-state carrier, and the conditional signs of Section 30 are not gauge predictions. The master action also prices current conservation and includes cross-defect terms, which the following reduction evaluates.

Proposition 53 (exact reduction of the current and conservation residuals). Let D_U map vertex amplitudes to transported edge differences, $(D_U \rho)_e = \rho_{t(e)} - U_e \rho_{s(e)}$, with weighted adjoint the source divergence, and take $d_J = j - \kappa D_U \rho$ and $d_T = Z\rho + D_U^\dagger j$ with Z the refinement derivative [17, §§13, 14]. For positive diagonal current and transport metric blocks and a density stiffness H_V , $S_{JT} = \frac{1}{2}d_J^\dagger W_J d_J + \frac{1}{2}d_T^\dagger W_T d_T + \frac{1}{2}\rho^\dagger H_V \rho$. With $j' = j - \kappa D_U \rho$, $Q = Z + D_U^\dagger \kappa D_U$ and $C = W_J + D_U W_T D_U^\dagger$, exact current elimination gives the retained density kernel

$$K_\rho = H_V + Q^\dagger R Q, \quad R = (W_T^{-1} + D_U^\dagger W_J^{-1} D_U)^{-1},$$

without commuting the conductance with the graph Laplacian; for constant scalar weights, $Z = i\omega I$, $H_V = m^2 I$ and a vertex-Laplacian eigenvalue λ it specialises to $K_\rho(\lambda, \omega) = m^2 + [w_J w_T/(w_J + w_T \lambda)](\omega^2 + \kappa^2 \lambda^2)$. The current Gaussian integral retains $\frac{1}{2} \log \det C$; if $H_V = 0$ and Q is nonsingular, $\det H_{(j,\rho)} = \det W_J \det W_T |\det Q|^2$. For fixed links and Hermitian inserted generators $U_e(a, b) = U_e \exp[i(aX_e + bY_e)]$, the incidence

derivatives have source-vertex blocks $(D_a)_{e,s(e)} = -iU_e X_e$ and $(D_{ab})_{e,s(e)} = \frac{1}{2}U_e(X_e Y_e + Y_e X_e)$, and with $L = D_U^\dagger D_U$, $L_{ab} = D_{ab}^\dagger D_U + D_U^\dagger D_{ab} + D_a^\dagger D_b + D_b^\dagger D_a$; product and inverse differentiation give the derivatives of C , R and K_p . At a common defect zero with all six residuals collected as $d = E_j + L_z z$, E having I in the current row and $D_U^\dagger$ in the conservation row, exact current elimination for the full metric gives $C_{full} = E^\dagger W E$ and $H_{z,eff} = H_V + L_z^\dagger [W - W E C_{full}^{-1} E^\dagger W] L_z$.

Proof. The cross column of the shifted action is $D_U W_T Q$, and the Schur complement contains $W_T - W_T D_U^\dagger C^{-1} D_U W_T$, which equals R by the matrix inverse identity; the triangular change $j \rightarrow j'$ has unit determinant; the derivative formulas are the product rule. The package compares the direct and shifted reductions, the complete Gaussian determinant and the analytic gauge jets against exponentiated links on a noncommuting fixture, and reproduces the diagnostic specialisation on every Laplacian mode. ■

Current relaxation therefore cannot be treated as a free cancellation independently of the conservation residual, and the density kernel alone cannot price the full Gaussian response. The frequency is conjugate to the specified refinement coordinate and not automatically to duration; a scalar density mass is not the radial projector potential; and away from the common-zero locus the residual-curvature, metric-derivative and reaction terms of Sections 40 and 41 return.

The wheel carriers must also be kept apart: for the seven-vertex wheel with incidence B and the six triangle-cycle columns Z_c , $BZ_c = 0$, $\text{spec}(Z_c^\top Z_c) = (1, 2, 2, 4, 4, 5)$ belongs to cycle coordinates and $\text{spec}(BB^\top) = (0, 2, 2, 4, 4, 5, 7)$ to vertex density, whose rotation-invariant sector contains hub and rim with eigenvalues zero and seven, so the charge-cycle value one cannot be substituted for the density eigenvalue, and the sixfold angular label within one cell is not a spatial displacement between cells.

The canonical wheel ledger supplies current and completion diagonal entries $75/434$, transport and bond entries $21/124$ and a current/completion entry $-33/434$ [2, §14C], with the restored transport and bond doublets of §14F.4 having $W_{TB} = (3/248)[[1, \sqrt{3}], [-\sqrt{3}, 1]]$ and $W_{TT} = W_{BB} = (21/124)I_2$, whose defect-space Schur complement is $(36/217)I_2$, a reduction by $1/49$ (the scalar form gives $585/3472$, a reduction by $1/196$). These are benchmark data, not the authoritative physical path-law metric, which HMGB C distinguishes from both the frame-only lift and the wheel tensor lift [2, §§56A.4, 56A.5].

For the fermion gauge-exchange class the index-Casimir matrix in factor order $(3, 2, Y)$ is $M_{ij} = \sum_r T_i(r) C_j(r) = [[8, 9/4, 11/12], [6, 9/2, 1/2], [22/3, 3/2, 95/18]]$, $\det M = 99$, with $jM = (-12, -27/2, 25/2)$ and $tM = (772, -3609/2, 5537/6)$; the term has the conditional structure $\Delta A_i = \sum_j g_j^2 M_{ij} F_{ij}$ with the coefficients F_{ij} per factor pair unless equality of the external vertex operators and all contacts is established, invertibility concerning only $F_{ij} = F_j$; three responses cannot determine nine coefficients and J, T inspect two combinations; factorwise Ward identities do not prove equality across factors; record mixing may be absorbed only when it preserves the representation basis; and the common fermion index vector contributes zero to J and $264C$ to T .

The closed-history determinant is the source mechanism: for $W = A_{N-1} \cdots A_0$ and $R_+ = (I + W)^{-1}$, $\partial_{ab} \log \det(I + W) = \text{Tr}(R_+ W_{ab}) - \text{Tr}(R_+ W_a R_+ W_b)$, with every single-step contact and every ordered pair of insertions, and for the boson logarithm $-\log \det(I - W_H)$ both terms carry positive sign; the actual step operators, their first and second gauge derivatives, the closing boundary condition, measure and selected background remain required, and a holonomy susceptibility alone is not the finite local transverse coefficient.

Proposition 54 (the source kernel on the wheel cell). On the W_7 cell with incidence B , cycles $z_i = s_i + r_i - s_{i+1}$, $G = Z^\top Z = \text{circ}(3, -1, 0, 0, 0, -1)$ with $\det 320$ and spectrum $(1, 2, 2, 4, 4, 5)$, the displacement structures explicitly instantiated in the cell construction are the C_6 rotation inside the cell, whose Fourier label m has $\ell(m) = 3 - 2\cos(\pi m/3)$ with colour on $m = \pm 2$, weak on $m = 3$ and primitive hypercharge on $m = 0$, and the sequential interface index with its identification Ω_n ; the closure Jacobian has the displacement blocks $L_0^C = (1, 1)$, $L_1^C = (-1, 0)$ on (spoke, rim) with symbol $Z(m) = (1 - \omega^m, 1)$, $|Z(m)|^2 = \ell(m)$, the bond block is edge-local and the matter gradient takes the spoke to $(b_i - h)$ and the rim to $-b_i$ and $+b_{i+1}$. Per mode and generator the coupled kernel with record mixing on (u, v, θ, y) is $H(\ell) = [[w_j, 0, 0, 0], [0, w_j, \kappa w_j, 0], [0, \kappa w_j, a\ell + c + \kappa^2 w_j, -c], [0, 0, -c, b\ell + c]]$, which in the unit convention reproduces the native block $[[1, 1, 0], [1, \ell + 2, -1], [0, -1, \ell + 1]]$ of CLSN-1 and with $a = b = 12/31$, $c = 21/124$ gives

$k_{\text{link}}(4, 5, 1) = (3744/2201, 1880/899, 360/713)$, face kernel $(936/2201, 376/899, 360/713)$, record-locked stiffness $a\ell + c$, current Schur $K_v = w_j k_{\text{link}} / (k_{\text{link}} + \kappa^2 w_j)$, eliminated determinant $w_j^2(b\ell + c)$ and record-rotation mass ratio $c/b = 7/16$. The bosonic master action and the fermion transfer share only the link; at the neutral common background the void ray is invariant, $D_{j,e} = j_e - (\delta\psi_t - U_e\delta\psi_s)$ exactly, $\delta_A D_j = \delta_A D_T = 0$ and the direct record and completion derivatives vanish, so in this isolated Gaussian construction the record sector reaches the fermion loop through the per-factor link propagator; this does not set the gauge derivatives of the full conservation, closure or completion kernels to zero. On the record-following branch the bond Hessian in covariant fluctuation coordinates is $w_B \text{Tr}(T_{(A} T_{B)})$ for every background, so its first and second gauge derivatives vanish; on the record-locked branch the first derivative vanishes and the second is $\partial_C \partial_D K_{AB} = -w_B \text{Re} \text{Tr}(T_{(A} T_{B)} T_{(C} T_{D)})$ times the mode's edge profile, a Casimir-weighted contact term conditional on the Fact-Lock identification and on extending the one-channel unitary bond form to the full carrier. Under the common-kernel and flat-background assumptions of Section 35, the matter-density loop is sector democratic and has zero J contraction; this is not a cancellation theorem for the full interacting response.

The 39-check record of the source-kernel construction [66] is evidence for its declared finite models, not a differentiation of the full physical master action: its four-coordinate kernel omits the conservation coupling of Proposition 53, its scalar weight substitution does not establish the full typed lift, and supplying the master-action and history-measure papers alone does not make the physical calculation executable. What the construction fixes precisely is which objects are already defined, which are defined but unevaluated on the coupled carrier, and which identifications remain open, which Section 44 records with the status and location of each. The remaining task is to instantiate or derive the complete admitted multicell source and history law, with the typed metric, record and completion attachments, finite fermion transfer and required gauge derivatives, or to prove that a specified missing contribution cancels from the complete observables.

Part I. The force relation, the dependency chain and the conclusion

43. Correction tests on the existing force relation

The SGRI relation supplies a conditional approximate comparison of the gauge strengths [35]. J and T are two linear contrasts of the response, $\Delta J = j \cdot \Delta A$ and $\Delta T = t \cdot \Delta A$ with $j = (-2, -3, 3)$ and $t = (466, -687, 159)$ in factor order $(3, 2, Y)$, conventional hypercharge and $A_i = g_i^{-2}$; they add no independent field or adjustable coupling, so a specified correction can be propagated to the weak-mixing observable directly. The effective Higgs and Yukawa terms describe the substrate mass-generation sector and are not a second field to be added to it.

Proposition 55 (leading top contribution, D41 controls and classical substrate relaxation). (a) Freeze the SGRI boundary problem: $\alpha_s(M_Z) = 0.1180$, $\alpha^{-1}(M_Z) = 127.95$ and $A(\mu^*) = sZ$ with $Z = (6, 13/2, 21/2)$. With $x = \sin^2 \theta_W(M_Z)$, $S = 127.95/(4\pi)$, $L = \ln(\mu^*/M_Z)$ and $\tau = \ln(\mu/M_Z)$, the low boundary is $A(0) = (1/(4\pi \cdot 0.1180), Sx, S(1-x))$, the two-loop gauge flow with the leading top term is

$$dA_i/d\tau = -b_i/(8\pi^2) - [\sum_j B_{ij}/A_j - c_{t,i}h]/(128\pi^4), \quad dh/d\tau = (h/8\pi^2)[(9/2)h - 8/A_3 - 9/(4A_2) - 17/(12A_Y)],$$

with $b = (-7, -19/6, 41/6)$, $c_t = (2, 3/2, 17/6)$, $B = [[-26, 9/2, 11/6], [12, 35/6, 3/2], [44/3, 9/2, 199/18]]$ and the recorded diagnostic input $y_t(M_Z) = 0.94$ [23, §6]. Solving for x and L by equality of the three endpoint ratios $A_i(L)/Z_i$ gives $\sin^2 \theta_W(M_Z) = 0.227333051557$ with $h \equiv 0$ and 0.227367379454 with the top term, a shift of $+0.000034327897$ that closes 0.883% of the gauge-only gap to the comparison value 0.23122 and leaves 0.003852621, about 1.67% of that value; the top continuation has $L = 27.3238663942$ and $s = 0.5216608790$. (b) The D41 statistical response matrices $H = (1/49)[[36, 18, 36], [18, 26, 26], [36, 26, 47]]$, $m = (36, 26, 47)^T/49$, $W_0 = dH$ and $W = W_0 - \Delta m m^T$ with $d = 0.0551296278772322$ and $\Delta = 0.004648154590676$ [23] give clamped fractions $W_{ii}/(W_0)_{ii} = (0.9380556182, 0.9552623909, 0.9191281682)$ and relaxed fractions $(W_0^{-1})_{ii}/(W^{-1})_{ii} = (1, 1, 0.9866359992)$, both invariant under a common gain. Replacing the high-scale vector by $Z_i f_i$ under the same continuation gives:

Boundary reading	Gauge only, $\sin^2\theta_w(M_Z)$	With leading top
No extra screening	0.22733305	0.22736738
Relaxed D41 control	0.22870564	0.22874077
Clamped D41 control	0.23436810	0.23440586

(c) On a regular classical stationary branch with zero background gauge curvature, let a be the retained gauge perturbation and η the allowed geometric response with boundary conditions fixed and null directions removed. If geometry couples to the gauge stress-energy only, its leading source $Q_2[a]$ is quadratic in a , and with $S = S_g^{(2)}[a] + \frac{1}{2}\langle\eta, C\eta\rangle + \langle\eta, Q_2[a]\rangle + \dots$ stationarity gives $\eta_* = -C^{-1}Q_2[a]$ and $\Delta S_{\text{red}} = -\frac{1}{2}\langle Q_2[a], C^{-1}Q_2[a]\rangle = O(a^4)$: the mixed Hessian $S_{a\eta}$ vanishes at $a = 0$ and $\Delta Z_3 = \Delta Z_2 = \Delta Z_Y = 0$ for this channel at this order and background.

Every number in (a) and (b) was reproduced independently for this paper from the stated equations and inputs alone, with a high-order integrator and a root solve on the endpoint ratios: the gauge-only value agrees to 3×10^{-9} , the top value to 3×10^{-9} , L and s to the printed digits, and all four D41 control values to 2×10^{-8} (Appendix B, section L). The relaxed D41 control with the top term closes 35.65% of the unscreened remaining gap but stays 0.002479225 below the comparison value; the clamped control is 0.003185858 above it.

These are diagnostic sensitivities: the clamped and relaxed readings are alternative readings of one model, not additive corrections or independently admitted sources, and neither closeness nor the substrate's ability to deform identifies the D41 coordinates with the physical gauge response.

The classical relative factors in (c) are (1, 1, 1), distinct from the relaxed D41 factors (1, 1, 0.9866359992), so classical stress-energy adjustment alone does not derive that screening; the test excludes neither field-dependent quantum determinants, measure and ghost terms, quantum gravity, nonzero-curvature backgrounds, condensation nor source-derived curvature-linear gauge-record mixing, and an allowed Abelian tensor coupling still requires its actual source vertex.

The comparison value never enters the solve, the top correction is nonzero and small on the scale of the approximate agreement, and the matched FRGM transport of Section 37 must not be added a second time to this separate reference trajectory. The next physical correction must come from a specified term of the substrate action with its admitted background, derivatives and matching prescription, not from the possibility of defining a contrast or from improved agreement.

44. Dependency chain and remaining source evaluation

The ledger below runs from the primitive cell to a physical J and T and distinguishes a known definition from its physical instantiation and from an evaluated response. Established means that the cited definition or the stated calculation supplies the object at its declared grade. Conditional means that the result depends on a branch or identification that has not been physically admitted here. Unevaluated means that a formula or construction exists but has not been carried through on the required coupled carrier. Not located means that the required explicit object was not found in the material assessed for the cited construction; it is not a theorem that the complete corpus cannot supply it. An identification recorded as open in a source remains open here unless a specific later closure is supplied.

Link	Object	Status
L1	Cell geometry: W_7 , wheel, incidence, six triangle cycles, $G = Z^T Z$, stationary measure, conductance $1/31$	Established (Sections 5, 42)
L2	Canonical weights $a = b = 12/31$, $c = 21/124$; PCN $t = 0.0215626456584632$ and $s = 1/t$; generator Grams $\text{diag}(6I_8, (13/2)I_3, 378)$, $q = 6Y$, $C_{\text{norm}} = 1$	Established [3, §§17.88, 17.89, 17.101, 17.106]
L3	Applicability of the response-uniqueness theorem to the designated heat law; circulation obstruction; returned information	Established (Parts A to C)
L4	Complete constrained update of the response for any returned metric change; relaxation modes; frozen-direction invariance	Established (Part D)
L5	Cell and sequence displacements; common spatial carrier for the coupled action	Cell rotation and sequential interfaces established; the required spatial attachment is not instantiated in this calculation (M1)
L6	The six defect families C, J, T, R, B, H and their first variations	Definitions and first variations available (Section 31); their complete coupled physical realisation and higher derivatives unevaluated (M2, M4)
L7	Wheel-mode record mixing; current and conservation elimination with determinants	Derived on the stated models (Section 42); physical identification conditional
L8	Shared represented link and a compatible fermion-transfer carrier	Shared link established; an explicit operator on the chosen common carrier is still required for the cell route (M3)
L9	Gauge derivatives of the bond, conservation, closure and completion blocks	Bond derivatives vanish in the isolated record-following construction; conservation derivatives available (Section 42); other derivatives and locked contacts retain their stated conditions
L10	Identification of the evaluable cell kernel with the physical occupied-support action	Recorded open in the cited CLSN construction; not established by the cell algebra alone (M2)
L11	Closed-history assembly, completion attachment and running contributions	Assembly rules, attachment maps and declared-branch running available; coupled step data and physical finite coefficients unevaluated
L12	Normalised sector totals k_r , or the moments C, W, B_q , of the attached response; the balance $k_\alpha = k_u + 3k_e$	Physical values unevaluated; current prices, readout norms and static arrays do not substitute for this response

Link	Object	Status
L13	Physical J and T, response normalisation and subsequent transport	Physical pair open; PCN retained as a declared premise (Section 33); selected running and time attachments available (Sections 37, 40); finite-price identification and full transport of the physical return required

Item	Object	Status	Location named by the corpus
M1	Spatial displacement labels and inter-cell blocks on one carrier shared by the bosonic action and the fermion transfer	Not instantiated in the cell calculation; the C6 label is not a substitute for spatial propagation	MASD-1 (refinement coordinate); RHFU-1; RCHS-36F; HSPR-1 §§6, 8; Configuration Dynamics refinement geometry; MFKR §14
M2	Physical identification of the covariant family $F_{a,b,c}$ with the bond and record defects on the occupied support, including the complete cross metric	Recorded open in CLSN-1 §17.100; the defect definitions themselves are available	MASD-1 §47A; HMGB-1 wheel metric; RPC-1 D31; GNC-EX3A §§1, 2
M3	Explicit gauged fermion operator with its spinor and edge assignment on the selected common carrier	Sequential and reference-carrier constructions exist; their common-carrier attachment is not evaluated here	Joint Temporal Reduction and Gauged Continuation; MFKR §14; Sequential Chiral Completion
M4	Physical completion propagator, mixed vertices and the joint increment-law derivatives T_c, T_b, T_{cb} on the admitted branch	Formal vertex and derivative rules exist (Sections 18, 19, 40, 41); their branch-specific coefficients and coupled evaluation outstanding; listed as required returns in CLSN-1 Appendix B	HM4-EX4A; Multi-Regulator Full Faithful Source Assembly; HCMR-XI-4; PJCK-1
M5	Physical attachment of the current weight w_i , the per-factor current-link mixing κ_x and the native current along the committed write	Canonical wheel metric entries known (Section 42); their physical occupied-support and charged-background interpretation conditional	RCHS-1; RCHS-9; CRTF radial tangent; MCFR-1

Item	Object	Status	Location named by the corpus
M6	Admission of the refinement geometry (HMGBBC requirement HP3) and of the selected record-following or locked branch (Fact-Lock)	Physical admission not established here; the cited HP3 and Fact-Lock obligations must be resolved for the branch used (CLSN-1 §§17.58, 17.85)	HMGBBC-1; RHFU-1; RCHS-36F; CLSN-1 §17.85
M7	Finite completion coefficients c_L , c_R , c_H and the finite derivatives $\Gamma_{fin,xx}$, $\Gamma_{fin,x\eta}$, $\Gamma_{fin,\eta\eta}$, $\Gamma_{fin,\eta}$ after matching on the selected branch	Attachment maps and running available; the physical finite diagrams and coupled derivatives unevaluated (Sections 38, 39)	CLSN finite matching notes; FRGM-1C matched branch; this paper, Sections 38, 39

For the wheel-cell route, the first unexecuted construction is the common spatial and transfer attachment (M1 and M3). Even with that attachment, the physical identification in M2 remains a separate obligation. These are evaluations and identifications to complete using the existing sources; they do not require redefining the six defect families or discarding the PCN premise. A reference-carrier construction may be used instead if its common-action attachment is established. The calculation should then proceed in the following order.

- 1. Fix one admitted background, occupied support, regulator, measure and closing boundary condition. Identify the common carrier and the source action whose bosonic and fermionic terms are being differentiated. Keep the generator conventions and the declared PCN event grain explicit.
- 2. Construct the complete coupled propagator and vertices from that action. Retain the six-channel metric, including the transport-bond and current-completion cross entries W_{TB} and W_{JH} , and differentiate the moving supports, record maps and completion insertions before imposing a common-defect zero.
- 3. Assemble the ordered-product response with all single-step contacts and ordered pairs of insertions. Include the required determinants, measure terms and any interacting contributions at the declared order, each once. Check the pure-gauge identities on the complete carrier, then extract the local transverse kinetic coefficients on the background's unbroken subgroup.
- 4. Establish the attachment to the normalised 47-state price before using its sector totals or the three moments of Theorem 31. If the calculation returns the three gauge coefficients directly, use their declared contractions. A projection of an arbitrary Schur complement onto the carrier is not this identification.
- 5. Test the exact J balance, evaluate T with the same normalisation, and carry the resulting physical return through matching and Stage-B transport, counting the selected Yukawa contribution once. Freeze the source result before comparing it with experiment.

On the wheel-cell route the concrete form of steps 2 and 3 is to replace the covariant family of Section 42 by the physical defects on the occupied support at the neutral common background, keeping a , b , c , t and the generator conventions; to attach the fermion operator through the shared link and evaluate the closed-history determinant with its assembly rule on the twelve-edge cell, with the record-dressed link propagator in place of the bare one; and to evaluate the trilinear completion vertex at the neutral origin and at a specified nonzero background admitted by the source, recording whether the mixed derivative $b_{A,C}$ is an open-link zero, a nonzero-background return, a closed-holonomy response or a higher-order effect.

A vanishing result excludes only the specified branch and contribution at the evaluated order; it does not exclude every Gaussian mechanism or every physical correction. A nonzero result is a prediction only after the physical attachment and normalisation of step 4 are established. A source identity may remove a particular contribution without evaluating all its coefficients; otherwise the unresolved object is the coupled response and its required derivatives, not one free scalar to be fitted.

45. Decision rules and failure conditions

Each rule names a condition that would overturn or restrict a statement of this paper and what follows if it is met.

- **F1 (sign).** If the physical operational generator on some finite branch has a positive off-diagonal entry in the inherited measure, Proposition 2 does not apply to that branch and entrywise positivity must be proved separately (Lemma 3).
- **F2 (one-step law).** If the physically identified history law on the seven-state carrier is P_0 itself rather than its heat form, the reversible theorem does not apply and no generator exists (Theorem 5).
- **F3 (cycle sums).** If the actual bond and record score has an antisymmetric part with a nonzero sum on any independent cycle, the history leaves the reversible class and SGW-3 Proposition 4 yields no $\Delta J = \Delta T = 0$ statement for it (Proposition 13); the returned information then enters through Theorem 16 with Theorems 14 and 15 as benchmarks.
- **F4 (spectrum).** If the Doob kernel of a proposed physical transfer has a non-real eigenvalue, no self-adjoint transfer in any measure produces it (Theorem 9).
- **F5 (attribution).** If the generator attribution is used as $L + V$ with non-commuting L and V , the recovered generator differs by the terms of Proposition 4(b).
- **F6 (limit and clock).** A continuum statement needs domain, kernel, ground-state and convergence control; $a = 1$ is diagnostic; the heat kernel is a different law from P_0 (Sections 4, 6, Lemma 7).
- **F7 (frozen ingredients).** If the write changes the background, constraints, lift, attachment or conversion, Theorem 16 and its corollaries do not apply unchanged; the full sensitivity or a rebuilt constrained parent is required (Sections 12, 15, 32).
- **F8 (rank premise).** The representation restrictions of Section 17 apply to ΔK itself on an invariant or justified invariant branch; label counting, a binary record or a representative $\Delta K + c_0 I$ does not supply the premise.
- **F9 (readout is not price).** Any identification of the charged readout Gram, a curvature norm operator, a current Fisher tensor or an export Gram with the price K requires a same-action theorem; without it the contractions of Sections 18, 22, 26 and 30 are not J and T (Proposition 28).
- **F10 (symmetry zero).** A claim that gauge symmetry, family blindness, a common primitive price and fact retention force $J = 0$ is refuted by Theorem 29; a physical zero requires the balance of Theorem 30 or the moment identity of Theorem 31.
- **F11 (diagnostic not input).** The comparison coordinate $T = -33.48185948$ and the comparison value 0.23122 may not select a source weight, susceptibility, finite coefficient or branch (Sections 14, 43).
- **F12 (double counting).** Running contributions, relative regulator coefficients and finite constants belong to different objects; a quantum determinant already present in an effective branch, the Yukawa transport of Section 37 and the top continuation of Section 43 are each counted once (Sections 19, 36, 38, 43).
- **F13 (sufficiency limits).** The κ families of Section 41 and the fixtures of Sections 24, 27 and 40 are counterexamples to sufficiency of evaluated data; none is an admitted alternative source, and none may be fitted.

46. Conclusion

The paper establishes a precise boundary between an applicable history construction and an evaluated physical gauge response. For an irreducible finite Markov law in a fixed positive measure, detailed balance and a strictly positive spectrum are necessary and sufficient for the transfer representation, and the transfer is unique up to a positive scalar multiplier. The designated heat construction satisfies the criterion on every finite connected Dirichlet branch. The canonical wheel fails it and has no continuous-time embedding; its heat semigroup is a distinct law that satisfies the criterion for every positive clock value. A nonzero circulation tilt leaves the reversible class, visibly in the affinities, the spectrum and a positive entropy production; its information return, with a sign threshold on the adjacent-rim entry, and the constrained response to that return are calculated exactly, with direct and relaxation modes separated and with invariance under changes confined to a frozen fact direction. Evenness of the contrasts in the tilt holds under the fixed-ingredient or full parity assumptions stated with the relevant results.

The source analysis determines how to test J and T without assigning sector prices by hand. It separates the charged readout, the current quadratures and the gauge price; the readout has exact norm identities with unit branch (0, 660) at rank 42 and enters at third derivative order, so it is not a price. It proves that the audited gauge symmetry, family blindness, common primitive price and fact retention do not enforce $J = 0$, and it identifies the exact balance and the three price moments that would settle both contrasts.

The record calculus, current conservation, completion vertices and finite matching supply explicit reductions and conditional cancellations: polar evenness and commutator curvature, the Abelian phase relation, the stationary record family with its balance test, the coboundary condition on the export map, the invariance of the complete response under exact regroupings, the two-loop contractions, a matched candidate whose Yukawa contribution to the running J coordinate is strictly negative on its interval, the finite attachment map with its cancellation ratio, and the proof that the evaluated quadratic action does not fix the mixed completion response.

Their scope matters: the isolated shifted-current block is blind at tree and Gaussian order, whereas the full conservation and interacting response has additional gauge dependence. The defect definitions, the canonical metric entries, the PCN premise, the selected running and the time attachment are available; what is not yet evaluated is their complete physically identified response on one admitted branch.

The existing approximate force relation survives the correction tests at its declared accuracy. The leading top contribution changes the reference weak-mixing value from 0.22733305 to 0.22736738, reproduced independently here, and leaves most of the comparison gap. Classical stress-energy-sourced substrate adjustment, on the regular zero-curvature branch considered here, begins at fourth order in the gauge amplitude and adds no quadratic correction. The D41 readings remain conditional sensitivity tests. These results neither justify a correction chosen to improve agreement nor prove that all further corrections vanish. Physical numerical J and T remain unevaluated; for the normalised price, $J = 0$ exactly when $k_Q = k_u + 3k_e$, and T follows from the same complete price. Section 44 states the source calculation required to determine them, or to prove that a specified contribution cancels. No fitted source parameter, universal zero, full-admission theorem or continuum closure is claimed. This closes the present paper at a substantive result: exact response criteria, evaluated correction tests and an explicit boundary on what follows from them.

Appendix A. Exact eigenprojectors of the wheel

Index the states as $(h, b_0, \dots, b_5)$ and write $d(i, j) \in \{0, 1, 2, 3\}$ for the rim distance between two rim states. The five projectors are:

- $\Pi_1 = 1\pi^T$: every row equals $\pi = (7, 4, 4, 4, 4, 4)/31$. Eigenvalue 1, rank 1.
- Π_B (eigenvalue $-3/28$, rank 1): right vector $r = (-24, 7, 7, 7, 7, 7)$, left vector $l = \pi \circ r$, normalised by $l \cdot r = 168$. Entries: hub to hub $24/31$; hub to rim $-4/31$; rim to hub $-7/31$; rim to rim $7/186$.
- Π_H (eigenvalue $1/2$, rank 2): zero on the hub row and column; rim entries $(1/3)\cos(\pi d/3)$, that is $1/3, 1/6, -1/6, -1/3$ for $d = 0, 1, 2, 3$.
- Π_Z (eigenvalue 0, rank 2): zero on the hub row and column; rim entries $(1/3)\cos(2\pi d/3)$, that is $1/3, -1/6, -1/6, 1/3$.
- Π_Q (eigenvalue $-1/4$, rank 1): zero on the hub row and column; rim entries $(1/6)(-1)^d$, that is $1/6, -1/6, 1/6, -1/6$.

Direct exact-arithmetic substitution shows that the five projectors are idempotent, mutually annihilating, sum to the identity, have traces $(1, 1, 2, 2, 1)$ and reconstruct $P_0 = \sum \lambda_k \Pi_k$. The rim projectors are the Fourier projectors of the six-cycle; they are orthogonal in counting measure and in π alike because π is uniform on the rim. The entries of Theorem 6 are read off as $\sum_k e^{-a(1-\lambda_k)} (\Pi_k)_{ij}$ with $1 - \lambda_k \in \{0, 31/28, 1/2, 1, 5/4\}$.

Appendix B. Verification record and reproducibility

The verification inventory below lists 57 checks for `sgw3_verify.py` and 104 for `sgw3_response_verify.py`, a total of 161. These are inventory counts, not a certification that the executable supplements have been run. The first script is recorded as using the Python standard library and `Fraction` for exact arithmetic, with floating-point comparisons for transcendental quantities. The second is recorded as using NumPy and SciPy for algebraic fixtures and numerical benchmarks, including the boundary-value calculations of Section 43. Analytic proofs establish the general statements. Exact-arithmetic checks test specified identities and fixtures; floating-point checks test numerical implementations at their stated tolerances.

Recorded SHA-256 identifiers: `sgw3_verify.py`,
 3dea824f39f28b8c1650c9034c4aa58eb0e26b4f80bfd847e2dea5b907fa7001; `sgw3_response_verify.py`,
 6ed09d1b4f134e0720d0587e4826d959a1a45a894608eea891a0839706c10e13. Reproduction requires the corresponding script files, dependency versions and execution output. A listed count or recorded hash alone does not establish execution or verify a general theorem.

Section	What <code>sgw3_verify.py</code> checks	Checks
A	Wheel rows, stationary law, exact detailed balance, characteristic polynomial against the displayed spectrum, rank 5 of P_0 and P_0^2 , zero determinant, simplicity of the negative eigenvalues, determinants of powers, spectrum of P_0^2	9
B	Rational eigenprojectors: completeness, idempotence, orthogonality, reconstruction of P_0 , traces, the Π_B entries	5
C	Closed-form heat kernel against a series exponential; the seven entry formulas at $a = 1$; row sums; detailed balance; smallest entry; smallest eigenvalue; spectrum; largest entry change; the $a \rightarrow \infty$ limit	15
D, E	Lemma 3 counterexample; Lemma 7 on the two-state flip	3
F, G	Tilt at $e^\epsilon = 2$ and 3: rational rows, stationary law, triangle and rim ratios, current, vanishing spoke currents, entropy production, exact characteristic polynomial, Fourier eigenvalues, maximal imaginary part, linear-response limits	16
H	Heat baseline at $a = 1$: exact triangle ratio 4, stationary law, current, entropy production, circulant Fourier eigenvalues and maximal imaginary part	4
I, J	Cycle sums of F_C ; exact reversibility of a gradient tilt with the predicted stationary law; the sandwich generator and its a^2 commutator coefficients	5

Group	What <code>sgw3_response_verify.py</code> checks	Checks
A	Contrast operators: tracelessness, $R \cdot 1 = 6Z$, $j \cdot R/6$ and $t \cdot R/6$, three-price formulas, ratios 401/3 and 445/3, universal zero, representative $v(J, T)$, diagnostic translations	12
B	Casimir and charge moments: the six contractions, quartic moments, balanced three-channel family, colour counterparent, the two moment identities, ranks 4 and 5, kernel direction, positive example, $T - 53J$	12

Group	What <code>sgw3_response_verify.py</code> checks	Checks
C	Readout Gram identities, unit branch (0, 660), chiral split, sign bounds, sector table, lepton ratio 53, Higgs rank two, averaging gap	7
D	Theorem 16 against a direct Schur difference on a rational fixture; native baseline, z , β and $-18/25697$; direct minimisation; native cancellation at three c ; Theorem 21 at two t ; Proposition 24 numbers and closed forms	12
E	A_{em} coefficients and trace 17; $E_{loop} = 17c + (141J - T)/264$; ΔA decomposition; chirality variation; $v(J, 0)$; two-loop electromagnetic row sums	6
F	Yukawa structure: jD , tD , index sums, attachment contractions, beta-function columns, $10/3$, endpoint coefficients, cancellation ratio, slope replay, c_A/c_F and the determinant, candidate E, D, F, D/E, thresholds, B_J^C , B_T^C , values at $7/960$, q and b thresholds, margins	19
G	Record calculus: two-state polar holonomy, evenness, commutator expansion on a complex fixture, quartic control, $\kappa = 8/2809$ from the six stretches, edge Gram eigenvalues, export Fisher 4 sech^2 s, KL curvature, nonadditive ray derivatives, universal Gram contractions	11
H	Terminal responses $8/9$, $6/7$, $24/29$; $2/63$ and $55/63$; the fixed probe on the full working kernel	3
I	Native current Haar expansions and ratio corrections; flat-modulus ratio; record-lock unit control; isolated current block k_{link} and determinant; $k_{link}(4, 5, 1)$	7
J	Restored transport/bond doublet (36/217) and scalar 585/3472; conservation reduction K_p on Laplacian modes and against direct elimination; index-Casimir matrix and contractions; formal contrast shifts	6
K	Inverse-expansion contractions on (6, $13/2$, $21/2$)	1
L	Two-loop gauge boundary solve with and without the leading top term; L and s ; D41 fractions; relaxed and clamped controls; quartic scaling of energy-sourced relaxation	8

The original calculation records of the continuation notes remain in the consolidated archive with their embedded reproducibility packages: representation (17), charged completion (20), action order (4), symmetry (10), composed stretches (30), record phase (16), stationary records (28), admission and export (29), export quotient (11), cycle export amplitudes (10), matter export sectors (13), fine structure constant (17), universal Fold response (17), native current (17), record lock (23), terminal maps (32), complete response (41), same-side response (16), momentum and gauge dependence (20), completion and record responses (17), full-family balance (14), matched balance (replay to 6.7×10^{-16} in r_q), finite response (17), finite attachment (22), coupled momentum (24), gauge response (24), mixed vertex (35), history derivative (24), joint source (8), primitive joint update (14), Fold balance (22), cubic attachment (30), projected cancellation (17), current conservation (12) and the gravity-response symbolic record.

The force-relation continuation of Section 43 uses the measured electromagnetic and strong-coupling boundary inputs stated there, together with its declared top input; the weak-angle comparison value is

not an input to that solve, and no source coefficient or branch is fitted to it. Deterministic fixtures test the displayed algebra; they do not select the physical background or establish the source attachment, and the floating-point residuals test only the implementation.

Appendix C. Provenance boundaries

Inherited and not re-derived: the designation of the heat sandwich for physical execution and the generator attribution G11a, the wheel entries from HM11, the closure-circulation score F_c and the row-normalised exponential deformation, the treatment of detailed balance as an optional reference condition and the positivity routes for gauge and fermion sectors [2]; the response-uniqueness theorem, the contrast operators, the native H4 action and the source-to-action identification [1]; the 47-state census, the invariant price operator, the sector trace map and the contrast normalisation [16]; the six defect families, the independent link and record variables, the occupied-support polar calculus, the charged completion readout and the complete auxiliary reduction [17]; the graded source descent [18], the quadratic-release scope and composition covectors [19], the ordered-record likelihood and conditional preparation [20], the primitive marked-instrument programme with its export pair and coboundary requirement [22], the D41 typing and the top input [23], the static certificates, finite matching and the PCN premise [3; 47], the canonical Yukawa candidates and the matched branch [44; 45; 48; 49; 50], the published two-loop coefficients [46; 51], the protected Higgs line and the time attachment [59; 60], the closure-Born renewal and the finite completion instrument [53; 54; 55] and the classical geometric response [63].

Established here: Theorems 1, 5, 6, 9, 14 to 16, 20 to 22 and 29 to 31; Propositions 2, 4, 8, 10 to 13, 18, 19, 24 to 28 and 32 to 55; Lemmas 3 and 7; Corollaries 17 and 23; the rational benchmark values; the exact reductions, contractions and identities displayed in Parts D to I; the independent replication of Section 43; and the verification inventory of Appendix B. Nothing here returns the physical numerical gauge contrasts, selects a source quantity or changes the physical identifications of the source papers.

References

- [1] K. Taylor. Source Derived Relative Gauge Correction in VERSF (SGW-3), 13 October 2026. Sections 1, 5 and 18 to 26: contrast operators, native H4 action, source scores, complete constrained reduction and conditional response uniqueness.
- [2] K. Taylor. The VERSF History-Measure, Generation-Geometry and Branch-Closure Theorem (HMGB-1). Sections 11A, 14A to 14F, 16, 17, 20 to 22, 31, 55A and 56A: canonical wheel, closure score, operational heat transfer, positivity routes, wheel metric and the authoritative path-law metric.
- [3] K. Taylor. The Physical Changing Link Source Representation Resolved History Metric and Gauge Normalisation Closure Attempt in VERSF (CLSN-1), 9 October 2026. Sections 17.13 to 17.20, 17.38 to 17.50, 17.52, 17.58, 17.79 to 17.89, 17.94 to 17.108 and Appendices A, B.
- [4] K. Taylor. Configuration Dynamics Refinement Geometry and Record Conditioned Curvature Response in VERSF. Part I, Part VIII, Parts XIII to XV including XIII.3 to XIII.8, XV.11 to XV.14 and XV.49.
- [5] D. A. Levin and Y. Peres, with E. L. Wilmer. Markov Chains and Mixing Times, second edition. American Mathematical Society, 2017.
- [6] R. A. Horn and C. R. Johnson. Matrix Analysis, second edition. Cambridge University Press, 2013. Chapter 8.
- [7] J. L. Doob. Conditional Brownian motion and the boundary limits of harmonic functions. Bulletin de la Société Mathématique de France 85 (1957), 431 to 458.
- [8] F. P. Kelly. Reversibility and Stochastic Networks. Wiley, 1979. Chapter 1.
- [9] E. Hairer, C. Lubich and G. Wanner. Geometric Numerical Integration, second edition. Springer, 2006. Section III.4.
- [10] W. J. Culver. On the existence and uniqueness of the real logarithm of a matrix. Proceedings of the American Mathematical Society 17 (1966), 1146 to 1151.
- [11] G. Elfving. Zur Theorie der Markoffschen Ketten. Acta Societatis Scientiarum Fennicae, Nova Series A, 2(8) (1937), 1 to 17.
- [12] J. F. C. Kingman. The imbedding problem for finite Markov chains. Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete 1 (1962), 14 to 24.
- [13] E. B. Davies. Embeddable Markov matrices. Electronic Journal of Probability 15 (2010), paper 47, 1474 to 1486.
- [14] J. Schnakenberg. Network theory of microscopic and macroscopic behavior of master equation systems. Reviews of Modern Physics 48 (1976), 571 to 585.
- [15] S. Boyd and L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004. Chapter 10 and Appendix A.5.
- [16] K. Taylor. Physical Matter-Higgs Record Weighting and the Gauge-Coupling Closure Gate in VERSF (SGW-2), 13 October 2026. Sections 2 to 13, 17 to 21 and 18.1 to 18.4.
- [17] K. Taylor. The VERSF Master Substrate Action and Constitutive-Descent Theorem (MASD-1). Sections 4, 7 to 10, 12 to 24, 31 to 37, 47F to 47M, 48 to 50 and Appendices A to D.
- [18] K. Taylor. RCHS-3: Graded Source-Jet Retyping and Native Current-Link Provenance, 4 September 2026. Sections 2 to 5, including the §3.3 record-stretch incidence nulls.
- [19] K. Taylor. RCHS-24: The Quadratic Gauge-Release Sufficiency and Conditional First-Fact Endpoint Reduction Theorem in VERSF, September 2026. Sections 3 to 5, 9H.4 and 9K to 9L.
- [20] K. Taylor. The Event-Level Preparation, Extension-Information and Realised Directing-State Theorem in VERSF (HCMR-P1), August 2026. Sections 5 to 15.

- [21] E. S. Gawlik and M. Leok. Computing the Fréchet derivative of the polar decomposition. arXiv:1608.04491, 2016.
- [22] K. Taylor. The Primitive Field-Dependent Marked-Instrument Programme in VERSF (PFDMI). P-PF10 to P-PF15, §19.3, Theorem 23, §§43 to 45, 52 to 53, 61, 63 and 65 to 69.
- [23] K. Taylor. CLSN-2: The Universal Gauge Conversion Factor, D41 Representation Typing and Source-Response Release in VERSF, 9 October 2026. Sections 1, 2, 6, 18, 18A and the source-response conclusion.
- [24] K. Taylor. MHPE-1 (§§3, 6, 13, 14) and FRDT-1 (§§1, 9, 10, 15): reduced six-defect metric, the CY3' comparison and the distinction from full physical differentiation.
- [25] K. Taylor. VERSF Direct Record Coupling Derivation, 3 October 2026. Fixed- and moving-support polar response and the scope of a pure positive stretch.
- [26] K. Taylor. Primitive Matter Selection in VERSF (PMS-1): graded CAR audit, differential-order distinction between a fermion vertex and a bosonic Hessian.
- [27] K. Taylor. GNC-EX3A: Bond-Polar Differential and Weak-Sector Screening Test, September 2026. Sections 2, 3, 6 and 7.
- [28] K. Taylor. RCCSJ-1: The Representation-Resolved Current-Cycle Source Jet and Positive-History Realisation in VERSF, 3 September 2026. Section 3.
- [29] K. Taylor. The Multi-Regulator Full-Faithful Source Assembly (HM4). Section XIV.1, Theorem L; Section XIV.3, Theorem M.
- [30] K. Taylor. RCHS10: Current-to-Export Quadratic-Capacity Transfer, 5 September 2026. Sections 1 to 6.
- [31] K. Taylor. EPMD-1: Edge-Resolved Primitive Marked-Action Descent, 24 July 2026. Sections 19 to 24.
- [32] K. Taylor. RCHS11: Current-Export versus Commitment-Selector Rank Mismatch. Sections 2 to 10.
- [33] K. Taylor. Completed Export Neutral Check, 24 September 2026; VERSF Symbolic Export Response and Auxiliary Reduction, 17 September 2026; VERSF Coupled Record Relaxation and Export Response, 16 September 2026.
- [34] Particle Data Group. Electroweak Model and Constraints on New Physics, 2025 review, Sections 10.1 to 10.3. <https://pdg.lbl.gov/2025/reviews/rpp2025-rev-standard-model.pdf>
- [35] K. Taylor. The Scale and Common Normalisation Free Gauge Relation in VERSF (SGRI-1). Sections 2 to 10A: conventional hypercharge, one-loop ledger, two-loop gauge matrix, frozen high-scale vector and the gauge-only continuation.
- [36] K. Taylor. MCFR-1: Matter Census and Family Current Geometry Reduction, 11 October 2026. Section 1.2, premises R4, R5 and R11.
- [37] K. Taylor. Reconciling Particles and Bits. Sections 6a, 6b and Appendix D.
- [38] K. Taylor. RCHS-1: The Twelve-Sector Current-Rigidity, Source-Generated Rank-Six Current-Stiffness and Current-Link Schur Reciprocity Theorem in VERSF, through BCTR-6, October 2026. Sections 3, 8, 39A to 39F and 39M to 39N.
- [39] K. Taylor. The Post-Commitment Atomic-Operation, Retention and Renewal Theorem, with First Bridge Execution. Sections 46P and 46R.
- [40] K. Taylor. The Renormalised Standard Model Parameter-Closure Theorem in VERSF. Sections 64.4.1B to F, 69, 72.2, 74.7, 79 and 81.
- [41] K. Taylor. Source Derived Gauge Work and Curvature Response in VERSF (SGW-1), 10 October 2026. Sections 2, 5 and 10 to 15.
- [42] K. Taylor. VERSF Two-Completion Gauge Kinetic Uniqueness Test, 6 October 2026.

- [43] K. Taylor. VERSF Full-Carrier Same-Action Shape Selection Test, 7 October 2026.
- [44] K. Taylor. The Quark Current Mass Projection Theorem in VERSF (QCMP). Section 4A.6, premises P13, P15, P16, Sections 13 and 14.
- [45] K. Taylor. The Charged Lepton Absolute Scale Theorem in VERSF (CLAS). Section 15 and the closure statement.
- [46] L. N. Mihaila, J. Salomon and M. Steinhauser. Renormalization constants and beta functions for the gauge couplings of the Standard Model to three-loop order. Physical Review D 86, 096008 (2012). arXiv:1208.3357.
- [47] K. Taylor. CLSN finite matching notes, 29 September 2026: Finite Electric and Magnetic Matching of the Sequential Chiral Completion; Frequency Dependent Gauge Response and Finite Electric Magnetic Matching; An Explicit Transfer Derived Chiral Completion and Its Finite Gauge Response.
- [48] K. Taylor. The Fermion RG Matching and Self-Consistent Quark-Threshold Theorem in VERSF (FRGM-1C), 19 July 2026. Sections 9 to 30 and Appendices A, B.
- [49] K. Taylor. The Microscopic Closure-Hessian and Full Yukawa Evaluation Theorem in VERSF (MCHY-1C). Sections 37.1 to 37.3 and 37.7.
- [50] K. Taylor. The Conditional Quark Mass-Ratio Grid. Sections 6.1 and 6.2.
- [51] G. Buchalla et al. Complete one-loop renormalization of the Higgs-electroweak chiral Lagrangian. arXiv:1710.06412, Section 6, Eq. (65).
- [52] K. Taylor. Complete Neutral Source Evaluation, 22 September 2026, Sections 2 to 4, with Complete_Neutral_Source_Reproduction.zip (SHA-256 df9795784c1018ffec6176ac3da7ff8cdaf6ec271147d88d6a3bf086f8fca820).
- [53] K. Taylor. HCMR-XI-4 Remaining Work Execution Report, 2 August 2026. Section 3.
- [54] K. Taylor. The $K = 7$ Closure-Born-TPB Renewal Construction in VERSF. Sections 1 to 3.
- [55] K. Taylor. VERSF One Link Endpoint Phase Registers and a Retained Record, 17 September 2026. Sections 1 to 3.
- [56] K. Taylor. PJCK-1: primitive joint update, exact Fold construction and innovation-score results. Sections 12 to 16.
- [57] K. Taylor. The Chiral Matter Representation Theorem in VERSF. Pages 1 and 2.
- [58] K. Taylor. The Weak-Orientation Admissibility Theorem in VERSF. Section 0.6.
- [59] K. Taylor. The Refinement-Geometry Higgs Spatial-Propagation and Kinetic-Residue Reduction Theorem in VERSF (HSPR-1), 31 August 2026. Sections 6 to 9.
- [60] K. Taylor. The Sequential Opportunity to Lorentzian Commitment Time Attaching Theorem in VERSF, September 2026. Sections 6 to 14, Theorem 6.
- [61] K. Taylor. VERSF NFDMA-2 action archive, 20 July 2026: run_nfdma2.py, action_premises.json and the accompanying paper.
- [62] K. Taylor. The Finite Master-Action Fermion Kernel Reconstruction and Source-Identity Gate in VERSF (MFKR). Sections 14 and 25 to 28.
- [63] K. Taylor. Gravity from Tensorial Closure of Record Dynamics in the VERSF Framework. Sections 8 and 9.
- [64] K. Taylor. CRTF-HMGBC HM12 Current-Score Normalisation and Doob Descent Audit, 2 September 2026.
- [65] K. Taylor. SGW-3 continuation calculation notes, 14 to 17 October 2026, with embedded reproducibility archives: representation and charged completion; composed record stretches; record

phase compatibility; stationary record amplitudes; record preservation and export; source constraints on export probabilities; cycle export amplitudes; matter sector export test; fine structure constant; universal Fold response; native current universality; record preservation and connection response; terminal-map uniqueness; complete response under record grouping; source-defined Q, u, e variations; minimal charge moments; write-driven response; write law and action cost; corpus audit and work conversion; QCMP same-side response; momentum and gauge dependence; completion and record responses; full-family completion balance; matched completion balance; finite matching and record relaxation; finite completion attachment; coupled momentum response; remaining record-completion response; mixed vertex representation and source test; bond history metric derivative; joint source action sufficiency; primitive joint update; exact Fold balance; cubic bond completion attachment; projected cancellation audit.

[66] K. Taylor. Source-kernel construction on the master-action cell, 16 October 2026, with `source_kernel_check.py` (39 checks).

[67] K. Taylor. `MASD_Current_Conservation_Reduction.py`, 17 October 2026 (three exact and nine numerical checks).

[68] K. Taylor. `SGW3_Gravity_Response_Screening_Test_17Oct2026.py`: symbolic stationary reduction, unchanged quadratic gauge Hessian and quartic scaling.