

SGW-2

Physical Matter–Higgs Record Weighting and the Gauge-Coupling Closure Gate in VERSF

Representation-resolved gauge traces, conditional fact-lock response and the physical-action release test

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Claim boundary. On the inherited three-generation Standard Model census with one Higgs doublet, and in the stated K7 state-counting trace convention, SGW-2 derives the exact six-sector trace map, its rank-three image and three-dimensional kernel, and the two action contrasts that determine the relative loop response. It classifies exact preservation of committed records and evaluates one conditional protected-holonomy restriction in the complete native 18-coordinate H4 action. The stiffness increments $P_3/6$ (link and cycle) and $P_3/287$ (current) are exact in that native model, not physical gauge-coupling values. The off-equilibrium same- Φ record map, the complete MASD energetic parent, its 47-dimensional matter–Higgs embedding, the loop-action coefficient and numerical physical gauge strengths remain unreturned. The Standard Model representation content is input to this paper, not a result derived here.

General Reader Summary

This paper is about the three forces that act between the known particles, and about a question that comes before any numbers: which features of the particles could change the relative strengths of those forces, and which could not. The answer is worked out from the list of particle kinds alone, before any measured strength is used.

Different kinds of matter might be charged different prices by the underlying rule that VERSF proposes. The paper shows that only six such prices can affect the three force strengths, that three particular combinations of them are completely invisible, and that two combinations carry everything that can change the relative strengths. Measuring the three forces perfectly would therefore still not reveal the six prices. That is a limit on what measurement can do, not a gap to be filled by fitting.

The second half asks how the theory can keep changing after something has definitely happened. A recorded event does not have to freeze everything. The paper gives an exact rule for which changes leave a record intact and which would erase or mix it. On one carefully stated assumption, keeping one particular record fixed removes one way the system could relax, and in a small model this stiffens one mode by a definite amount. Whether that stiffening reaches the force strengths is left open on purpose: it could change the underlying rule and still leave the relative strengths untouched.

Nothing here predicts a force strength. The paper's contribution is to say precisely what would have to be calculated, in what order, and what would count as failure.

Abstract

On the inherited three-generation charged-matter census with one Higgs doublet, the SGW-1 finite-loop expression assumes a common matter price and a separate Higgs price. We remove that universality assumption. Gauge covariance restricts the second-curvature trace of any gauge-invariant price operator to six sector averages, one for each of the quark doublet, the up and down singlets, the lepton doublet, the charged lepton singlet and the Higgs. Applying the inherited K7 six-face normalisation, in its state-counting convention, to standard representation traces gives six exact integer trace vectors and a three-by-six integer map from sector prices to colour, weak and hypercharge responses. The map has rank three and nullity three; its common-price slice recovers SGW-1 exactly; its kernel is transverse to the

SGW-1 slice, so the non-identifiability arises entirely from removing universality; a sterile right-handed neutrino has no direct gauge trace; and family orientation is invisible at this grade. Projecting onto the inherited SGRI normal gives the exact representation-resolved transverse law. A second contrast is forced, up to scale, by requiring the down singlet, the lepton doublet and the Higgs to be invisible, and the two contrasts together determine the loop correction modulo the common gauge direction. The rooted-walk combinatorics gives the loop conversion as one sixth of a same-action coefficient that remains unreturned.

Existing source results narrow the remaining physical calculation without completing it: the primitive operational metric is internally democratic, the executed neutral local route generates no weak or Abelian split, the selected representation-valued current carrier has a non-orthogonal factor Gram, and the D41 history fingerprint fails the central sector-price pullback identities exactly. Exact Kraus intertwining classifies the operations that preserve a committed fact. A separate, conditional same- Φ holonomy identification constrains one alternating record phase, and carrying that restriction through the complete native 18-coordinate H4 action produces rank-one stiffness increments on the alternating mode in two different retained-coordinate reductions, leaving the uniform mode unchanged. The full history-dependent constrained action, including reaction-Hessian and cross-block terms, is formulated without treating a stationary phase relation as an event-time law. No numerical action contrast follows from the native calculation until the complete MASD parent and its representation-resolved physical embedding are evaluated, and the possibility that a nonzero fact-lock stiffness projects to zero in both contrasts is explicitly admitted.

Results and claim-status guide

Object	What this paper establishes	Status
Input matter census	Three charged generations and one Higgs doublet; charges and representations are inherited; the two-dimensional neutral complement of the 49-carrier is distinguished from a sterile extension.	Input, not re-derived
K7 representation response	Six vectors F_r , the 3×6 matrix R , rank 3, nullity 3, lattice basis of the kernel; SGW-1 recovered as a slice; kernel transverse to that slice.	Exact given inputs and the state-counting convention
Relative loop correction	The canonical pair of contrasts J and T determines the representation-loop response modulo the common Z direction.	Exact algebra; physical values open
Primitive metric and local tests	Internally democratic MASD metric; executed neutral local splitting vanishes; D41 fails the price pullback.	Inherited; branch limited
Fact retention	Kraus intertwining; the working state may evolve without changing a sharp recorded fact.	Exact on the declared record carrier
Protected alternating holonomy	$\delta y_3 = 0$ only on the declared same- Φ and protected-boundary branch.	Conditional physical identity
Complete native H4 response	$\Delta K_\theta = P_3/6$ and $\Delta K_v = P_3/287$ after the allowed 18-coordinate reduction.	Exact native; conditional lock
Physical carrier and source action	Full off-equilibrium record map, history rows, coupled Hessian and 47-dimensional embedding.	Open; the decisive gate

Object	What this paper establishes	Status
Normalisation and predictions	$\kappa = \alpha_{\text{loop}}/6$ combinatorially; α_{loop} , J, T, common normalisation and Z_3 , Z_2 , Z_q still open.	1/6 exact; physical values open

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Part A. The representation-resolved loop map

1. Programme position and successor target

SGW-1 reduced the reciprocal D_6 physical-work problem to four orbit coefficients and showed how their Fourier responses would determine three gauge-action coefficients. Its canonical-history calculation supplies a source-derived baseline, while the K7 closed-walk calculation isolates a representation-sensitive correction. What SGW-1 did not return was the physical representation-resolved action price or the action-to-loop coefficient needed to turn that correction into a prediction. The SGW-1 expression $\kappa[mF_m + hF_H]$, with $F_m = (36, 36, 60)$ and $F_H = (0, 3, 3)$, is exactly a two-price restriction, not a demonstrated matter-universality law. The purpose of SGW-2 is to remove that restriction, state the source information needed to calculate the weights, and test whether VERSF's retained facts can contribute a precisely identifiable correction to the same action.

Scope and original content. Sections 3 to 8 give the exact six-sector map, its two canonical contrasts and the identifiability theorem from the inherited gauge representations and the K7 rule; Sections 9 to 12 isolate the still-open physical pricing and normalisation tasks and give the direct action-trace form of the loop response. Section 13 retains the commitment and record results that Part C needs and gives an exact classification of fact-preserving operations. Sections 14 to 19 carry protected history into a constrained source action and perform the full native H4 test. The numerical stiffness calculation is a complete result on its declared native parent, not an unqualified derivation of a Standard Model coupling.

Standard of physical closure. A physical result requires (i) a single provenance chain from the primitive event through the off-equilibrium record and history map to the full MASD action; (ii) the quotient- and constraint-correct physical embedding of that action on the admitted matter–Higgs carrier; and (iii) frozen gauge-work coefficients before comparison with experiment. A record constraint that generates no non-common action contraction is a valid negative result, not an invitation to change source coefficients.

The inference map below separates the exact algebra from the source identifications still needed. Every stage marked open requires calculation rather than an interpretive identification. Figure 1 shows the same chain as a diagram: the record chain along the top, the gauge chain along the bottom, and the one open gate that joins them.

Stage	Return	Status
1. Inherited census and K7 rule	Six representation trace vectors, the map R, its kernel lattice and two canonical contrasts	Exact
2. Primitive record and holonomy branch	Fact retention and conditional alternating-phase restriction	Exact and conditional
3. Complete MASD source action	Off-equilibrium record rows, all cross-blocks, quotient and physical lift	Open
4. Physical action contrasts	J and T on the 47-component matter–Higgs carrier	Open numerically
5. Gauge work and post-freeze tests	Common normalisation, independent channels, Z_3 , Z_2 , Z_q and RG comparison	Open

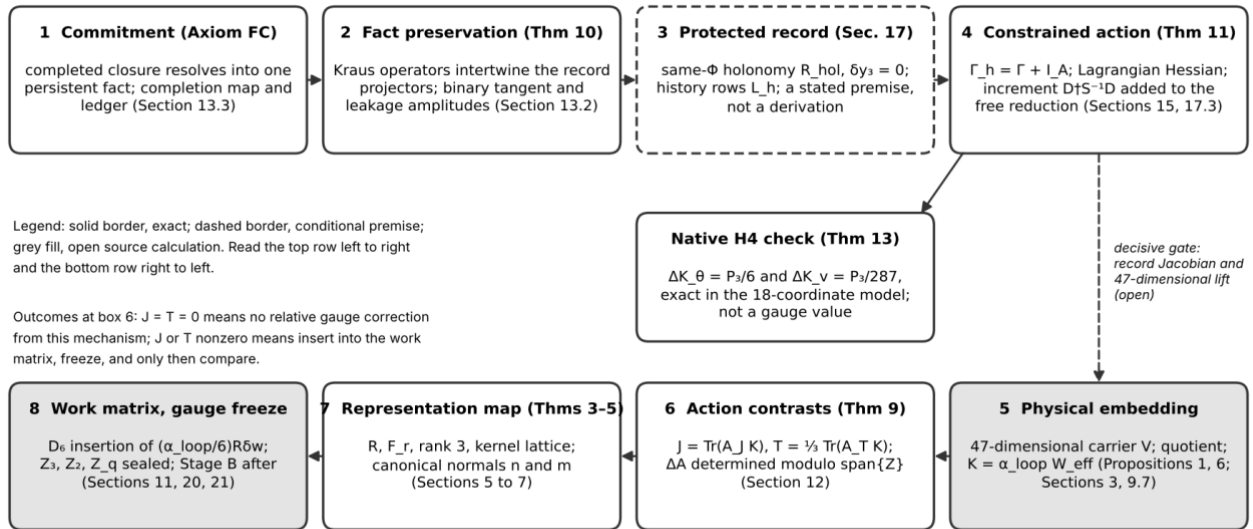


Figure 1. From commitment to gauge response. The record chain (boxes 1 to 4) and the gauge chain (boxes 5 to 8) are each exact or conditional at the grades shown; the single open gate is the physical lift from the constrained action to the 47-dimensional matter and Higgs carrier. The native H4 check is an exact model result attached to box 4, not a gauge value.

2. Frozen matter census, conventions and source firewall

2.1 Fixed minimal census

The charged-matter ledger is inherited from the generation and species census and the Fold-to-three-generations paper [4]. The calculation starts with three copies of Q_L , u_R , d_R , L_L , e_R and one complex Higgs doublet, before mass hierarchy or mixing. The hypercharges and irreducible representations in the table are Standard Model inputs to this trace computation. Their appearance in R does not independently establish a VERSF derivation of the Standard Model charge assignments or of three generations; those claims depend on the separately graded corpus.

Sector	Multiplicity	$SU(3)_c \times SU(2)_L$	Y in $Q = T_3 + Y/2$
Q_L	18 Weyl	(3, 2)	+1/3
u_R	9 Weyl	(3, 1)	+4/3
d_R	9 Weyl	(3, 1)	-2/3
L_L	6 Weyl	(1, 2)	-1
e_R	3 Weyl	(1, 1)	-2
H	2 complex components	(1, 2)	+1
N_R (hypothetical extension)	3 Weyl	(1, 1)	0

The minimal census has 45 Weyl matter components and 2 Higgs components, 47 in all; this is the carrier V on which the physical price operator acts. The completed 49-dimensional source carrier of the Fold-to-K7 continuation adds a two-dimensional neutral complement [4]. That complement is a gauge-neutral sector of the source carrier; it is not three sterile neutrinos, and the N_R row above is a different, hypothetical object. Both are gauge-neutral, both have zero direct trace and neither enters R , but the

two are not identified by notation. Where Section 13 extends operators by zero to the 49-carrier, it means the two-dimensional neutral complement.

2.2 Gauge coordinate convention

The finite-loop trace is written in conventional factor coordinates (C, W, Y) , where the Abelian generator in the covariant derivative is $Y/2$. The frozen PCN gauge vector in these coordinates is

$$Z = (6, 13/2, 21/2),$$

equivalently $(Z_3, Z_2, Z_q) = (6, 13/2, 378)$ in the primitive $q = 6Y$ convention. The SGRI normal used throughout is $n = (466, -687, 159)$ [2].

2.3 The K7 trace rule and its state-counting convention

The K7 six-face closed-walk rule assigns to each gauge factor six times the quadratic generator trace over the states of the carrier, with every state counted once. A Weyl fermion component and a complex scalar component contribute the same unit. This is not an importation of perturbative loop weights: in an ordinary one-loop count a complex scalar carries half the weight of a Weyl fermion per Dynkin index [41, 42], and under that weighting the summed trace of Section 5 would be $(36, 75/2, 123/2)$, which is not proportional to the PCN vector Z . The identity $F_{\text{total}} = 6Z$ of Section 5 therefore holds in the state-counting convention, and the relative statistics weight of the Higgs is absorbed into its price w_H .

The convention is fixed by the type of the probe, not by choice. The K7 quantity is the second curvature of a spectral functional of the link configuration, $\text{Tr } L_0^3$ on the representation carrier (Section 10), whose closed walks weight each carrier state by the trace of the link unitaries along the walk. A spectral invariant of a connection on a vector bundle has no statistics: the perturbative one-half for a complex scalar arises from the integration measure of a dynamical field, that is from which determinant the field contributes and with what spin factor, not from the connection it couples to. In VERSF that dynamical weighting is exactly the price operator W_{eff} of Section 3: every statistics-dependent, species-dependent or history-dependent weight that the primitive action assigns to a carrier state enters the loop through W_{eff} , and the geometric trace is kept pure so that Theorem 2 applies to it without exception. The same state-counting rule is the one by which the PCN vector $Z = (6, 13/2, 21/2)$ was obtained, so the two are applied consistently. What this does not supply is the source theorem that the once-counted triangle-loop functional is the PCN-normalised gauge-work term of the primitive action with coefficient α_{loop} ; that identity, Proposition 8, is the remaining justification and is carried as open. If the primitive action does assign the Higgs a different weight from matter, the difference appears as $w_H \neq m$ and is visible through the n and m contrasts of Section 12; the convention hides nothing. Any comparison with perturbative coefficients belongs to Stage B and may not be applied inside the Stage-A trace (falsifier F26).

2.4 Source firewall

Source-type firewall. A Fisher or history metric, a kinetic residue, a current-link Gram, a representation-sector loop price, a full action Hessian and a gauge-action coefficient are different objects. None becomes another because its numeric ratio is suggestive. In particular, a canonical Higgs kinetic normalisation does not set its finite-loop price to unity, and D41 history retention cannot be relabelled as a sector-action price. The common physical parent, gauge-null quotient and representation carrier must be exhibited before those types can be connected.

Ordinary perturbative Standard Model threshold and renormalisation-group effects belong to Stage B, after the source action has been frozen. Double counting the same fermion determinant or loop as both a primitive Stage-A correction and a downstream perturbative contribution is prohibited. All comparator quantities are kept out of the source construction.

3. The complete typed same-source parent

Let V be the physical representation-valued carrier, not merely a three-factor diagnostic current space, and let A contain all same-order auxiliaries permitted to relax (record, bath, completion, history and additional admitted source coordinates). After the gauge and null quotient, the physical quadratic parent has blocks

$$H_{\text{parent}} = [[W_0, B], [B^\dagger, C]], \quad C \geq 0 \text{ on the retained auxiliary quotient.}$$

The only admissible effective price operator on V is the complete Schur reduction

$$W_{\text{eff}} = W_0 - BC^\dagger B^\dagger.$$

The formula is a conditional reduction identity, not a derivation of its input blocks. Every retained, auxiliary and mixed block must come from the same primitive action on the same event grain. It is invalid to splice a matter metric, a Higgs metric and a record susceptibility from differently defined parents merely because they yield favourable force ratios. If C is singular, the source must specify its quotient and the range condition $B \cdot \ker(C) = 0$; a pseudoinverse without these conditions does not certify a physical Schur reduction. Nothing in this abstract reduction licenses identifying the native H4 record coordinates with a sector of V . That identification is a separate physical task.

Proposition 1 (same-source pricing rule). Once the physical parent, event grain and auxiliary quotient are frozen, W_{eff} is fixed. Any representation price used in the finite-loop gauge calculation must be read from W_{eff} ; it is not an independently selectable coefficient.

Proof. This is the Schur-complement identity [43, 44] applied to the declared physical quadratic parent. Selecting any other retained quadratic form after the parent is fixed would amount to replacing the source action rather than reducing it. ■

4. Gauge-covariant reduction to sector-average prices

Gauge covariance sharply reduces the information from W_{eff} that can enter a gauge-invariant second-curvature trace. For each three-generation matter multiplet $r \in \{Q, u, d, L, e\}$, its carrier has the form $\mathbb{C}_{\text{gen}}^3 \otimes V_r$, where V_r is an irreducible representation of $SU(3)_c \times SU(2)_L \times U(1)_Y$, and the five V_r are pairwise inequivalent because their hypercharges differ. A gauge-invariant Hermitian W_{eff} commutes with the gauge action. Therefore, on each r ,

$$W_{\text{eff}}|_r = A_r \otimes I_{V_r},$$

with A_r acting only on the three-copy generation multiplicity. The Higgs has one copy, so gauge covariance gives $W_{\text{eff}}|_H = w_H I_H$. Define the family-averaged matter prices

$$w_r = (1/3) \text{Tr}_{\text{gen}} A_r, \quad r = Q, u, d, L, e.$$

Theorem 2 (sector-average pricing). At the gauge-invariant finite-loop second-curvature grade, all family orientation and off-diagonal generation structure in A_r drops out of the gauge trace. The complete matter and Higgs dependence is carried by the six real sector-average prices ($w_Q, w_u, w_d, w_L, w_e, w_H$).

Proof. Each gauge generator acts as $I_{\text{gen}} \otimes T_A^{(r)}$. Hence $\text{Tr}[(A_r \otimes I)(I \otimes T_A^2)] = \text{Tr}(A_r)\text{Tr}(T_A^2)$. Mixed generation entries contribute only through $\text{Tr}(A_r)$. Inequivalent gauge sectors cannot mix in a gauge-invariant quadratic operator, by Schur's lemma [38]. ■

This is the standard Schur-lemma trace consequence on a supplied irreducible gauge census, applied to the proposed source price. Its significance is that the full physical price operator may contain generation mixing while only six averages are visible to the specified K7 gauge trace. It is not a theorem that those six physical averages have already been supplied by MASD.

5. Exact K7 representation-resolved curvature vectors

The charge traces below are standard representation theory on the input census [39, 40], summed over three generations; the VERSF content is their typed use in the same-source finite-loop correction, conditional on the stated K7 rule and a physically returned price. Appendix A gives the arithmetic.

Sector	$\text{Tr } T_c^2$	$\text{Tr } T_3^2$	$\text{Tr } (Y/2)^2$	K7 vector $F_r = 6 \times \text{trace}$
Q_L	3	9/2	1/2	(18, 27, 3)
u_R	3/2	0	4	(9, 0, 24)
d_R	3/2	0	1	(9, 0, 6)
L_L	0	3/2	3/2	(0, 9, 9)
e_R	0	0	3	(0, 0, 18)
H	0	1/2	1/2	(0, 3, 3)
N_R	0	0	0	(0, 0, 0)

Theorem 3 (exact typed K7 loop decomposition). On the inherited normalised six-face curvature profile, in the state-counting convention of Section 2.3, the representation-resolved second-curvature vectors are exactly $F_Q = (18, 27, 3)$, $F_u = (9, 0, 24)$, $F_d = (9, 0, 6)$, $F_L = (0, 9, 9)$, $F_e = (0, 0, 18)$, $F_H = (0, 3, 3)$ and $F_N = (0, 0, 0)$.

Summing the five charged matter vectors gives $F_m = F_Q + F_u + F_d + F_L + F_e = (36, 36, 60)$, and adding the Higgs gives

$$F_{\text{total}} = (36, 39, 63) = 6(6, 13/2, 21/2) = 6Z,$$

recovering the SGW-1 finite-loop balance theorem without assuming a common matter weight. The balance holds because every state is counted once; with perturbative statistics weights it would fail (Section 2.3).

6. The typed finite-loop gauge map and its two contrasts

Collect the six visible prices in the column vector $w = (w_Q, w_u, w_d, w_L, w_e, w_H)^T$. The exact representation trace table defines the linear map

$$R = [[18, 9, 9, 0, 0, 0], [27, 0, 0, 9, 0, 3], [3, 24, 6, 9, 18, 3]].$$

Let κ denote, exactly as in SGW-1, the still-unreturned common conversion from this source-native finite spectral curvature to the admitted gauge-work coordinate. Then $\Delta A_{\text{loop}} = \kappa R w$, or componentwise

$$\Delta A_C = \kappa(18w_Q + 9w_u + 9w_d), \quad \Delta A_W = \kappa(27w_Q + 9w_L + 3w_H), \quad \Delta A_Y = \kappa(3w_Q + 24w_u + 6w_d + 9w_L + 18w_e + 3w_H).$$

Theorem 4 (representation-resolved finite-loop response). Under one common loop-to-work conversion κ , the complete gauge-factor shape returned by the finite K7 matter and Higgs second-curvature is $\kappa R w$. No additional representation coefficient exists at this grade once W_{eff} has fixed the six sector-average prices.

Projecting onto the SGRI normal $n = (466, -687, 159)$ gives the exact typed transverse law

$$\Delta \mathcal{T}_{\text{loop}} = \kappa(-9684w_Q + 8010w_u + 5148w_d - 4752w_L + 2862w_e - 1584w_H) = 18\kappa(-538w_Q + 445w_u + 286w_d - 264w_L + 159w_e - 88w_H).$$

Corollary 4.1 (SGW-1 is a slice). Imposing matter universality $w_Q = w_u = w_d = w_L = w_e = m$ and $w_H = h$ gives $Rw = m(36, 36, 60) + h(0, 3, 3)$ and $\Delta \mathcal{T}_{\text{loop}} = 1584\kappa(m - h)$. SGW-1 is therefore an exact two-price slice of SGW-2, not the most general typed source response. A physical derivation may return matter universality, but it may not be assumed merely because the summed matter trace admits it.

Corollary 4.2 (the second contrast is canonical). The plane orthogonal to Z is two-dimensional and contains n . Among normals m that make the down singlet, the lepton doublet and the Higgs invisible, $m \cdot R_d = m \cdot R_L = m \cdot R_H = 0$, the solution is unique up to scale: $m_W = -m_Y$ and $m_C = -(2/3)m_Y$, that is $m \propto (-2, -3, 3)$. This m is automatically orthogonal to Z , because $R \cdot 1 = 6Z$ gives $m \cdot Z = (1/6)(m \cdot R_Q + m \cdot R_u + m \cdot R_e) = (1/6)(-108 + 54 + 54) = 0$. Hence

$$mR = (-108, 54, 0, 0, 54, 0), \quad J := m \cdot \Delta A = -2\Delta A_C - 3\Delta A_W + 3\Delta A_Y = 54\kappa(w_u + w_e - 2w_Q).$$

Proof. The three conditions read $9m_C + 6m_Y = 0$, $9m_W + 9m_Y = 0$ and $3m_W + 3m_Y = 0$; the last two coincide, leaving a one-dimensional solution space. The remaining coefficients of mR are $-2 \cdot 18 - 3 \cdot 27 + 3 \cdot 3 = -108$ for Q , $-2 \cdot 9 + 3 \cdot 24 = 54$ for u and $3 \cdot 18 = 54$ for e , which sum to zero and so give $m \cdot Z = 0$ through $R \cdot 1 = 6Z$. ■

The pair (n, m) is therefore not a convenient choice but the canonical basis of $Z^\perp$ adapted to the census: n is the inherited SGRI normal and m is the unique direction in which three of the six sectors are invisible. Section 12 converts both into trace operators on V .

7. The three-dimensional gauge-invisible source kernel

The map R contains six source prices but only three gauge-factor returns. Direct exact reduction gives $\text{rank } R = 3$ and $\dim \ker R = 3$. A basis of the integer kernel lattice $\ker R \cap \mathbb{Z}^6$, in the column order (Q, u, d, L, e, H) , is

$$u_1 = (0, 0, 0, 1, 0, -3), \quad u_2 = (0, -1, 1, 0, 1, 0), \quad u_3 = (-1, -2, 4, 3, 0, 0).$$

Each satisfies $Ru_i = 0$ exactly, and the 3×6 matrix with rows u_1, u_2, u_3 has Smith normal form $\text{diag}(1, 1, 1)$ [45], so the three vectors generate every integer kernel vector (Appendix B). The three directions have a plain reading: u_1 trades the lepton-doublet price against three times the Higgs price; u_2 trades the up singlet against the down singlet and the charged lepton; u_3 shifts the quark doublet, the two quark singlets and the lepton doublet together.

Theorem 5 (gauge non-identifiability). Even an exact measurement of all three finite-loop gauge curvatures cannot reconstruct the six underlying representation prices. The source-price problem has a three-dimensional gauge-invisible kernel at this grade.

A three-dimensional kernel is an identifiability obstruction, not a freedom to fit parameters. A successful physical theory must return the admissible weight operator from its independent action; using measured couplings to choose one of the invisible directions would defeat the source-first objective. The first kernel vector is especially instructive: a coordinated shift of the lepton-doublet and Higgs prices leaves the complete three-factor curvature unchanged, so the Higgs price cannot be inferred from gauge data without a source theorem, even when the matter census is fixed.

Corollary 5.1 (the kernel is transverse to the SGW-1 slice). On the two-price slice $w = m \cdot 1_m + h \cdot e_H$, where 1_m is the indicator of the five matter sectors, $Rw = mF_m + hF_H$ with $F_m = (36, 36, 60)$ and $F_H = (0, 3, 3)$ linearly independent. Hence R is injective on the slice and $\ker R$ meets it only at zero. The non-identifiability of Theorem 5 is entirely a consequence of removing matter universality; within SGW-1's two-price model the inversion from gauge data is unique.

8. Sterile neutrality, the neutral complement and family-orientation blindness

8.1 Sterile-neutral extension invariance

A right-handed sterile neutrino carries no $SU(3)$, $SU(2)$ or $U(1)$ charge. Therefore $F_N = (0, 0, 0)$ and its central price does not enter R . The minimal 45-Weyl census and a 48-row neutral-extended matter carrier give the same finite-loop gauge second-curvature at this grade. The same statement holds for the two-dimensional neutral complement of the 49-dimensional source carrier (Section 2.1). Neutral physics can still affect the result indirectly through a same-source Schur reduction of the charged-sector prices, but not through a direct gauge trace.

8.2 Family orientation is invisible to the gauge trace

Theorem 2 also implies that CKM or PMNS orientation, generation relabelling and any off-diagonal family price matrix with the same trace do not change the second-curvature vector. That is appropriate: masses and mixings are downstream of the census and are not imported into the Stage-A gauge-price derivation. A later threshold calculation may depend on the mass spectrum; that belongs to Stage B rather than to the primitive representation trace R .

Part B. Source constraints on the physical prices

9. Existing source constraints on the physical prices

9.1 The selected representation-valued current carrier does not yet select its price

MCFR-1 and the Fold-to-K7 continuation select the shared representation-valued current carrier and the pure-current lift at fixed current defect, but explicitly leave the physical representation-resolved H_θ/H_{CMR} pricing operator open [4, 5]. This selection matters for SGW-2 because the physical current is not three orthogonal C/W/Y ledgers. On the minimal 45-Weyl matter carrier let P_C , P_W and P_Y be the projectors onto the coloured, weak-doublet and hypercharged states, of ranks 36, 24 and 45. The normalised support operators $S_C = P_C/6$, $S_W = P_W/\sqrt{24}$ and $S_Y = I_{45}/\sqrt{45}$ give the exact common-current injection $u(d_C, d_W, d_Y) = d_C S_C + d_W S_W + d_Y S_Y$ and the Hilbert–Schmidt pullback Gram

$$G_{45} = [[1, \sqrt{6}/4, 2/\sqrt{5}], [\sqrt{6}/4, 1, 4/\sqrt{30}], [2/\sqrt{5}, 4/\sqrt{30}, 1]], \quad \det G_{45} = 11/120,$$

whose off-diagonal entries are $\text{Tr}(P_C P_W)/(6\sqrt{24}) = 18/(6\sqrt{24})$, $\text{Tr}(P_C)/\sqrt{6\sqrt{45}}$ and $\text{Tr}(P_W)/\sqrt{24\sqrt{45}}$. The scalar-factor ledger has Gram I_3 . No label-preserving factorwise rescaling can turn I_3 into G_{45} , because diagonal rescaling cannot create the exact off-diagonal overlaps; the squared Frobenius mismatch is $\|G_{45} - I_3\|_F^2 = 41/12$. Therefore the 187/334 scalar-current operational chart is not the physical SGW-2 current-price carrier on the selected MCFR branch.

The Fold-to-K7 continuation writes the pricing operator on the shared carrier as

$$W_{\text{phys}} = \sum_r w_r P_r$$

when central-sector pricing is sufficient, with the full Hermitian W_{phys} retained if source-returned generation structure exists inside a fixed gauge representation [4]. SGW-2 uses precisely the same kind of object after all same-source auxiliaries have been reduced. The key point is that the selected current injection and its carrier geometry are fixed; the physical price operator on that carrier is not.

Proposition 6 (conditional common-parent identification). If the gauge-work loop and the matter and current preparation response are restrictions of one primitive event action on the same representation-valued physical carrier, then the gauge-visible W_{eff} must be the appropriate central restriction of its physical $W_{\text{phys}}/H_\theta/H_{\text{CMR}}$ operator. The existence of the common parent is a hypothesis to be checked through the actual embeddings and all cross-blocks, not an identification established by notation.

In particular, neither the selected non-orthogonal G_{45} current Gram nor the D41 history-screening matrix may be used as an inverse map to produce six sector prices. A successful common-parent calculation must return them as its own sector projections. MCFR-1 also closes the population-level matter state update under PCN, reversal-even matter typing and the stated linear projection premises, with $P_+ = (P + P^\#)/2$. Its canonical CP realisation on site populations does not select a unique coherent extension or the energetic and history price. Section 13.4 carries that distinction forward [5].

9.2 Higgs defect-silence removes one large class of Schur corrections

HSPR-1 proves that the common Higgs line q_H satisfies $J_D q_H = 0$ [8]. Consequently $q_H^\dagger J_D^\dagger M J_D q_H = 0$ for every admissible operator M on the primitive defect carrier. If the SGW-2 auxiliary coupling reaches the Higgs sector only through that same defect differential, its Schur correction on the common Higgs line vanishes exactly: $\delta w_H^{(\text{defect})} = 0$ on the same-metric, same-defect channel. This does not set $w_H = 1$. HSPR's canonical spatial Higgs kinetic residue and SGW-2's gauge-loop price are differently typed. The theorem says only that a broad class of defect-mediated history and bath corrections cannot be the origin of a nontrivial Higgs loop price.

9.3 Primitive first-order record and completion reshaping is absent

The sequential-source audit returns exact zero first-order C/W/Y–terminal-record cross-blocks on the primitive same-mark law and exact zero first-order C/W/Y–completion cross-blocks on the factorised

same-mark law. Therefore the representation split required by SGW-2 cannot be manufactured by promoting those vanishing first-order blocks. A nonzero source price difference must arise from the representation-resolved energetic and history operator itself, a higher-order or multi-cell term, a nonfactorising parent, or another independently derived source channel.

9.4 D41 is a history fingerprint and fails the sector-price pullback

QGSC-1 closes the D41-only gauge-screening shortcut: the retention belongs to a current and history Schur response, and complementary reduction of the same parent does not multiply the gauge or link stiffness [10]. On one real E2 quadrature the exact D41 history metrics are $M_0 = g_{cyc}^2 dH_{CWY}$ and $M_1 = g_{cyc}^2 [dH_{CWY} - \Delta \mu \mu^T]$, with $d = 0.0551296278772322$, $\Delta = 0.004648154590676$, $\mu = (36, 26, 47)^T/49$ and $H_{CWY} = (1/49)[[36, 18, 36], [18, 26, 26], [36, 26, 47]]$. The generalised spectrum has two protected eigenvalues 1 and one shortened eigenvalue $\lambda_{D41} = 1 - (\Delta/d)(47/49) = 0.9191281682$. This is an exact E2 history and current statement, not a gauge-action coefficient [16].

There is also an exact sector-price pullback no-go. For an operator W on the 47-dimensional carrier define its factor pullback

$$K_{XY}(W) = \text{Tr}(P_X W P_Y), \quad X, Y \in \{C, W, Y\}.$$

Let a gauge-covariant central price correction on the occupied Q, u, d, L, e, H carrier be $W_{cent} = \sum_r \delta w_r P_r$; it commutes with every P_X . Because every coloured sector is hypercharged and every weak sector is hypercharged, $P_C P_Y = P_C$ and $P_W P_Y = P_W$. Therefore the pullback of any central operator obeys

$$K_{CC} = K_{CY}, \quad K_{WW} = K_{WY}.$$

The D41 rank-one term $\mu \mu^T$ violates both identities: $\mu_C^2 = 1296/2401$ while $\mu_C \mu_Y = 1692/2401$, and $\mu_W^2 = 676/2401$ while $\mu_W \mu_Y = 1222/2401$. Hence neither the D41 correction nor the D41-reduced factor metric is the pullback of any diagonal Q, u, d, L, e, H price operator. The conclusion is independent of d, Δ, g_{cyc} and all gauge data. D41 may constrain a future common parent as a history fingerprint, but it cannot be inverted into SGW-2 sector prices.

9.5 MASD fixes the primitive metric to be internally democratic

MASD-1 does not leave the primitive residual metric as six species-selectable coefficients [13, 27]. On the complete primitive residual carrier its operational metric is induced by the stationary substrate measure as

$$W_{op} = \text{diag}(\pi) \otimes I_{\text{orientation}} \otimes I_{\text{internal}}.$$

The induced defect blocks are $W_{XY} = \iota_X^\dagger W_{prim} \iota_Y$, so all same-order cross terms come from typed overlap in one common metric rather than from separately chosen couplings. The occupied terminal-support weights are likewise fixed uniformly by the operational measure and may not be chosen separately by particle species.

Theorem 7 (primitive internal democracy). Before typed defect embedding and auxiliary reduction, the direct primitive price is proportional to the identity on the internal carrier. Therefore a $Q/u/d/L/e$ representation split cannot be inserted at W_{prim} ; it must be produced by the typed embeddings, a bath or history Schur term, a nonlocal loop response or another independently derived higher-order source effect.

9.6 Executed local quadratic no-go for the weak and Abelian representation split

The CLSN source audit executes the two most direct local routes [3]. At the neutral common background the faithful gauge generators annihilate the void ray, so the linearised constitutive current defect has no linear gauge-link term and its gauge-current and gauge-density mixed Hessians vanish. Independently, for the minimal faithful-link completion parent the gauge-completion mixed derivative remains bilinear in the left and right matter coordinates and vanishes at the symmetric background:

$$b_{A,C} = \text{Re}[x_L^\dagger i E_{SM}(T_A) T_C x_R] \big|_{x_L = x_R = 0} = 0.$$

The corresponding Gaussian determinant is gauge independent on that single open-link channel, so the source susceptibility multiplying the exact representation ratios is $\chi_c = 0$. Thus the weak factor 1 and primitive-q factor 18 are representation ratios for a future nonzero mechanism, not physical susceptibilities on the executed local branch. The separately proposed terminal +8 is also not source-derived by the executed record Gram.

Corollary 7.1 (local quadratic no-go). On the executed neutral local current and completion branch, the representation-dependent weak and Abelian Schur correction vanishes. A physical non-common pricing term must come from a nonzero matter background, a closed holonomy or nonlocal loop, a controlled higher-order or multi-cell reduction, or the full bath-resolved history operator.

9.7 The remaining price operator is one typed bath-Schur object

HMGBBC reduces the unresolved energetic pricing task to the complete bath-resolved master generator [7]. At its visible quadratic boundary the candidate operator is

$$W_{\text{bath}} = W_{\text{core}}(0) - C^+(\Omega_Q^2)C.$$

For SGW-2 the physically relevant object is the gauge-covariant representation projection of this operator onto the admitted Q, u, d, L, e, H carrier after the Defect-Lift Obligation, quotient and terminal-record projection are satisfied: $W_{\text{phys}} = P_{\text{rep}} W_{\text{bath}} P_{\text{rep}}$, with the six gauge-visible prices read as sector averages of its gauge-covariant blocks. The order matters: the bath-Schur object must be projected onto the irreducible representation blocks first; a C/W/Y factor-space Fisher or current metric may not be inverted afterwards to manufacture sector prices. HMGBBC states that the full typed lift and complete bath-resolved physical certificate have not been executed. The open problem is therefore neither six arbitrary weights nor a current-factor inversion: it is one matrix-valued bath-Schur operator, built after the full Defect-Lift, physical quotient, terminal-record projection and any source-selected history constraints, followed by its central Q, u, d, L, e, H projection or, for the non-common loop part, the two contractions of Section 12. None of these operations has been evaluated on the full physical committed background.

10. Exact rooted-loop conversion and the remaining action coefficient

Primitive Change Normalisation fixes one completed primitive physical change as one unit of dimensionless physical cost and removes the arbitrary common ruler on the primitive six-cycle carrier. The source-native K7 spectral probe nevertheless contains an additional combinatorial multiplicity: $\text{Tr } L_U^3$ sums all six rootings and orientations of each triangular closed walk, the standard closed-walk count for a graph operator [51]. Define the once-counted positive triangle-loop functional Φ_{loop} by summing one copy of $\dim(V) - \text{Re Tr } U_p$ for each of the six physical triangle faces. Then the closed-walk identity can be rewritten, up to a field-independent constant, as

$$\text{Tr } L_U^3 = C'_{\text{graph}} + 6\Phi_{\text{loop}}, \quad \text{hence } \kappa = \alpha_{\text{loop}}/6.$$

Proposition 8 (rooting conversion versus action coefficient). The once-counted triangle-loop functional is sixfold smaller than the rooted and oriented closed-walk expression: $\kappa = \alpha_{\text{loop}}/6$. This is an exact combinatorial identity in the inherited K7 normalisation. It is not a proof of $\alpha_{\text{loop}} = 1$, which additionally requires identifying the once-counted loop as the PCN-normalised physical gauge-work term of the very same source action.

If the once-counted triangle-loop functional is independently proved to be the PCN-normalised primitive gauge-work term, then $\alpha_{\text{loop}} = 1$ and $\kappa = 1/6$. Until that same-action identity is proved, the physical coefficient remains open even though the rooting conversion is closed. This distinction prevents a diagnostic spectral invariant from being silently promoted to the physical action.

There is also an exact no-double-count check. For the common six-price vector $1 = (1, 1, 1, 1, 1, 1)^T$, $R1 = 6Z$, so $(\alpha_{\text{loop}}/6)R1 = \alpha_{\text{loop}}Z$. If $\alpha_{\text{loop}} = 1$ on the same PCN physical loop, this reconstructs the already-

normalised bare gauge vector rather than an additional correction. The physically relevant finite-loop reshaping is therefore the non-common part. Writing $w = \tilde{w}1 + \delta w$, the relative contribution is

$$\Delta A_{\text{rep}} = (\alpha_{\text{loop}}/6)R\delta w,$$

while the common component is parallel to Z and is exactly SGRI-null. This is the form to be inserted into the later work-matrix correction unless the full source returns an additional independently typed term. For this non-common loop observable, α_{loop} and W_{eff} need not be recovered separately if the same action supplies their product K directly. Section 12 makes this statement precise without fixing the product by definition or removing physical normalisation requirements.

11. Handoff to the D_6 physical work matrix

SGW-1 fixes the extraction from a physical reciprocal D_6 work matrix [1]. After the source analysis above, the finite-loop representation piece entering that handoff is not an arbitrary $\kappa R w$ but the non-common response $(\alpha_{\text{loop}}/6)R\delta w$, plus any separately source-derived correction from channels not represented in the K7 spectral loop. If the source-returned correction preserves D_6 symmetry, write $D_{\text{phys}} = D_N + \Delta D_{\text{typed}}$, where D_N is the frozen factor-blind neutral baseline and ΔD_{typed} is reconstructed from the same-source non-common representation response and any independently returned residual channel. Its four Fourier stiffnesses are $\omega_0, \omega_1, \omega_2, \omega_3$, and the gauge boundary is

$$Z_3 = 6\omega_2, \quad Z_2 = (13/2)\omega_3, \quad Z_q = 378\omega_0, \quad Z_Y = Z_q/36.$$

The $\lambda = 2$ (ω_1) mode remains a source-control observable rather than a fourth Standard Model gauge coupling. The SGRI work coordinate remains

$$\mathcal{T}_{\text{work}} = 2796\omega_2 - (8931/2)\omega_3 + (3339/2)\omega_0 = 9474d_1 - 8388d_2 + 8931d_3.$$

If the physical typed source breaks D_6 , the four-orbit compression is invalid and the full work matrix must be returned. No symmetry may be imposed merely to preserve the simple extraction formulas. This extraction is only the endpoint of a fully typed physical reduction. In particular, a protected-record increment on the native H4 carrier may be positive and nonzero while giving $J = T = 0$ after representation projection, either because it is gauge-common or because it is invisible to the two contrasts. Section 19 makes this a concrete success-or-failure test rather than treating stiffness as synonymous with gauge splitting.

12. Direct action contrasts on the physical parent

The representation map is fixed. What remains of this loop mechanism after quotienting the common gauge direction is fully specified by two linear functionals of a physically returned, consistently normalised action-weighted operator

$$K = \alpha_{\text{loop}} W_{\text{eff}}$$

on the 47-dimensional charged matter-plus-Higgs carrier. These are action traces, not a chosen fit or an inversion of the three-factor current Gram. Only the product K enters this loop response; separate determination of α_{loop} and W_{eff} is unnecessary if the physical action returns K directly. This reparameterisation does not determine K and does not remove the need for a physical normalisation on the same event grain.

12.1 The two central trace operators

Take the canonical pair of normals of Corollary 4.2, $n = (466, -687, 159)$ and $m = (-2, -3, 3)$, with $nR = (-9684, 8010, 5148, -4752, 2862, -1584)$ and $mR = (-108, 54, 0, 0, 54, 0)$. Convert each sector average to a trace using its dimension: for each sector divide its row-contraction coefficient by six times the sector dimension (18, 9, 9, 6, 3, 2). This defines two fixed central operators on V ,

$$A_J = -P_Q + P_u + 3P_e, \quad A_T = -269P_Q + 445P_u + 286P_d - 396P_L + 477P_e - 396P_H,$$

and the two contrasts are

$$J = m \cdot \Delta A = \text{Tr}(A_J K) = 9\alpha_{\text{loop}}(w_u + w_e - 2w_Q), \quad T = n \cdot \Delta A = \Delta \mathcal{T}_{\text{loop}} = (1/3)\text{Tr}(A_T K).$$

Both operators have trace zero ($-18 + 9 + 9 = 0$ and $-4842 + 4005 + 2574 - 2376 + 1431 - 792 = 0$), so adding any scalar multiple of the identity to K leaves J and T unchanged.

12.2 Reconstruction modulo the common direction

The two normals span the plane orthogonal to Z . Fix the representative of the class $\Delta A + \text{span}\{Z\}$ with vanishing colour component, $v = \Delta A - (\Delta A_C/6)Z = (0, v_W, v_Y)$. Then $J = 3(v_Y - v_W)$ and $T = -687v_W + 159v_Y = -528v_W + 53J$, so

$$v = (0, (53J - T)/528, (229J - T)/528).$$

Thus J and T determine exactly the two-dimensional equivalence class of ΔA modulo $\text{span}\{Z\}$. This class describes the non-common part of this loop contribution. It does not determine the total common gauge normalisation, contributions from other channels, or the final physical coupling ratios.

12.3 Direct contraction of the physical parent

Suppose the complete source reduction returns one normalised positive action parent with retained matter and Higgs carrier V and all admitted auxiliaries, including the history and record channels. Write its quadratic blocks and reduced operator as

$$H_{\text{act}} = [[K_0, L], [L^\dagger, C]], \quad K_{\text{eff}} = K_0 - LG_C L^\dagger,$$

where G_C denotes the Moore–Penrose pseudoinverse [52] of the supported auxiliary block C after the physical quotient, C is positive semidefinite and L annihilates $\ker C$, so that G_C is applied only on the admissible quotient. All same-order cross-blocks, physical embeddings, constraint reactions and field-dependent terms must already have been included, and the reduced K_{eff} has to be identified with the physical K of this section, not merely with a numerically similar Fisher or current matrix. Then

$$J = \text{Tr}(A_J K_0) - \text{Tr}(G_C L^\dagger A_J L), \quad T = (1/3)[\text{Tr}(A_T K_0) - \text{Tr}(G_C L^\dagger A_T L)].$$

These identities follow by substituting the Schur complement and cycling the finite-dimensional trace. They allow the two outputs to be evaluated without constructing every entry of K_{eff} , retain the same physical parent and do not license a factor-space metric inversion. They do not assign values to J or T . If the available parent prices W_{eff} instead, the same formulas apply to its blocks with α_{loop} outside the entire expression, because for $\alpha > 0$ pseudoinversion on the same quotient obeys $\text{pinv}(\alpha C) = \text{pinv}(C)/\alpha$; this algebra carries a common action coefficient consistently but cannot determine it or justify setting it to one. The zero-coefficient case gives $J = T = 0$. Because A_J and A_T commute with the gauge action, their traces are unchanged by gauge averaging of the already reduced K , which permits direct contraction after the proper reduction; gauge averaging and Schur reduction are not assumed to commute.

12.4 Conditional democratic retained core

The simplification $K_0 = k_0 I$ is an additional physical hypothesis about the retained, embedded core. The primitive MASD metric may be internally democratic (Theorem 7) without this stronger statement being true after physical embedding. Only on that explicitly stated branch does the direct core disappear from both contrasts. Define the sector relaxation traces $z_r = \text{Tr}(G_C L^\dagger P_r L) \geq 0$, each the squared Frobenius norm of $(C^+)^{1/2} L^\dagger P_r$. Then

$$J = z_Q - z_u - 3z_e, \quad T = (269z_Q - 445z_u - 286z_d + 396z_L - 477z_e + 396z_H)/3.$$

On this branch the sign of J is fixed by whether the quark-doublet relaxation exceeds the up-quark plus three times the charged-lepton relaxation. Positivity alone does not choose that ordering.

Theorem 9 (two-contrast sufficiency for the specified loop). Under the inherited K7 map and one normalised physical K , the pair (J, T) uniquely reconstructs the representation-loop response modulo the

common Z direction. The proof is the exact sector trace reduction of Section 12.1 and the invertible two-normal map of Section 12.2. The theorem neither evaluates K nor fixes the total common normalisation, additional action channels or the physical gauge couplings.

A matter-universal operator gives $J = 0$ and $T = 264\alpha_{\text{loop}}(m - h)$, but $J = 0$ alone does not imply matter universality. A scalar K on all 47 matter-plus-Higgs components gives $J = T = 0$. More generally, a nonzero correction δK is invisible to relative gauge strengths whenever both $\text{Tr}(A_J \delta K) = 0$ and $\text{Tr}(A_T \delta K) = 0$. In particular a positive native H4 constraint need not survive the physical projection with either contrast nonzero. Appendix G verifies the algebra; the physical input remains unevaluated.

Part C. Records and the constrained action

13. Established commitment and record response

This section keeps the committed-fact, completion, population and writer results that the physical action question needs, and gives the exact classification of operations that preserve a written fact. It distinguishes four carriers that must not be silently equated: a completed fact ledger, a classically resolved terminal mark, a retained ordered history, and the independent MASD record morphism Φ . The mathematical conservation of any one carrier is not automatically a physical constraint on every degree of freedom in the others. Sections 17 and 18 use the separate conditional same- Φ holonomy identification at precisely that grade. The inherited finite results on the bond phase, write support, orientation sensitivity, ordered-history sign, preparation sign, polar bond response and the current-instrument derivative are collected in Appendix F; the ledger of Section 23 grades them.

13.1 Sign and carrier conventions

Here $\sigma \in \{-1, +1\}$ denotes the committed orientation under investigation. It is distinct from the refinement coordinate in MASD's ∂_σ notation and from a density matrix, written ϱ below. The preparation-bias sign, the realised path circulation σ_{path} , a completed edge or type label, the occupied-address set Ω , and the continuous MASD record morphism Φ are not identified unless the source supplies the required map. The canonical wheel W_7 has one hub and six rim vertices; it is not the complete seven-vertex graph.

13.2 Exact operations that preserve the same committed fact

Let $\{P_r\}$ be the complete orthogonal partition of the physical record carrier, and let $\mathcal{E}(\rho) = \sum_\alpha K_\alpha \rho K_\alpha^\dagger$ be trace preserving. The label must include the full fact or history prefix, rather than only a coarse chirality bit. Erasure outputs belong in the output complement and cannot be discarded by postselection. HCMR's adjoint record-invariance condition [20, §25] gives the exact classification [28]; it is the finite-dimensional case of the preserved-subspace characterisation for completely positive maps [46 to 49].

Theorem 10 (exact fact-preserving operations). For a sharp record carrier and a trace-preserving channel,

$$\mathcal{E}^+(P_r) = P_r \text{ for every } r \Leftrightarrow P_s K_\alpha P_r = 0 \text{ for all } s \neq r \Leftrightarrow K_\alpha = \sum_r P_r K_\alpha P_r.$$

Proof. For ψ in the range of P_r and $s \neq r$, the invariant expectation is $\sum_\alpha \|P_s K_\alpha \psi\|^2 = 0$, and every nonnegative term vanishes. Conversely the block form and trace preservation give the adjoint identity. ■

Preservation of just one occupied class requires only no outflow from that class; the displayed equivalence preserves every possible written class. When renewal enlarges the carrier, the corresponding condition is $P_{r,\text{out}} K_\alpha = K_\alpha P_{r,\text{in}}$. For a unitary flow $\exp(-itA)$, exact retention is equivalent to $[A, P_r] = 0$. If the sector dimensions are d_r , the allowed real Hermitian space has dimension $\sum_r d_r^2$. On a ledger-times-working carrier this permits $K_\alpha = \sum_r |r\rangle\langle r| \otimes K_{\alpha r}$, with each fixed- r family normalised. Working-state evolution, dissipation, renewal and history-dependent backreaction can occur inside the same fact sector. These are compatible operations, not a claim that arbitrary independent MASD field variations solve the dynamics. The statement is exact for a given sharp record carrier and channel; it is not an assertion that the complete physical MASD process realises all allowed transformations.

Binary solution and the physical tangent pullback

On the inherited binary chiral carrier, $P_\pm = (I \pm \sigma_z)/2$ and $A = (a_0 I + a_x \sigma_x + a_y \sigma_y + a_z \sigma_z)/2$. With $b = (a_x + ia_y)/2$ and $E_{\text{off}} = |-\rangle\langle +|$, direct multiplication gives $P_- A P_+ = b E_{\text{off}}$, so the real constraint matrix on (a_0, a_x, a_y, a_z) is

$$L_{\text{fact}} = \frac{1}{2}[[0, 1, 0, 0], [0, 0, 1, 0]], \quad \text{rank } L_{\text{fact}} = 2.$$

Thus $a_x = a_y = 0$, while I and σ_z remain. The source pure-phase flow $U_\gamma = \exp[-iy(P_+ - P_-)/2]$ preserves $P_\pm$ exactly for every γ [20, §46S.5]. This does not identify the independent MASD/HX1 polar phase with that

binary rephasing. First-order probability invariance alone would miss the forbidden directions: writing $a^2 = a_x^2 + a_y^2 + a_z^2$ (the two-level transition formula [47]),

$$p_{\text{flip}}(t) = ((a_x^2 + a_y^2)/a^2) \sin^2(ta/2) = \frac{1}{4}(a_x^2 + a_y^2)t^2 + O(t^4).$$

The $a = 0$ case is continuous. Every direction has $p'_{\text{flip}}(0) = 0$, so the constraint must be imposed on leakage amplitudes. For a physical post-commitment continuation $K_\alpha(q)$, stack the real and imaginary parts of all forbidden blocks $\mathcal{L}_{\alpha sr}(q) = P_s^{\text{out}} K_\alpha(q) P_r^{\text{in}}$, $s \neq r$. At a preserving configuration define $J_{\text{rec}} = D\mathcal{L}$. Differentiate the projectors too if the physical readout moves; a simultaneous gauge-frame change is an equivalent description. Then

$$T_{\text{compatible}} = T_{\text{admissible}} \cap \ker(J_{\text{rec}}), \quad R_{\text{rec}} = J_{\text{rec}} E_\eta, \quad F_{\text{rec}} = -J_{\text{rec}} E_{\text{ret}},$$

where $\delta q = E_{\text{ret}} X + E_\eta \eta$, so that $R_{\text{rec}} \eta = F_{\text{rec}} X$ is precisely the condition used in Section 15. On a regular constant-rank patch, after the source constraints and gauge quotient, the kernel is the tangent of the exact preserving family. At a singular point it is only a necessary linear test. The binary block-diagonal flows integrate exactly. Independent real constraint rows, the complete action and all same-order cross blocks must be retained.

13.3 Completion and renewal under the Fact Commitment axiom

PMS-EX58 declares Axiom FC: completed admissible closure resolves into one persistent fact. The axiom specifies occurrence and persistence, not a probability, phase, coupling or Standard Model label. In the source's $K = 7$ implementation the completion boundary is $T_{\text{rec}} = T_{\text{cover}}$; the selected E2 carrier has a unique C_6 -covariant unordered chiral projector pair P_+ , P_- up to output relabelling [22, §28]. The following completion CP map is distinct from the MASD record morphism Φ . For either chiral outcome $r = +$ or $r = -$, writing $c_r = \exp(ix_r)$ for the otherwise irrelevant branch phase, the minimal completion and branch-preserving renewal give

$$C_r(\rho) = \text{Tr}(P_r \rho) |C_r\rangle \langle C_r|, \quad \forall |C_r\rangle = c_r |\psi_r\rangle \otimes |r\rangle_L, \quad W_r = c_r (|\psi_r\rangle_{\text{work}} \otimes |r\rangle_L) \langle \psi_r|, \quad \sum_r W_r^\dagger W_r = I_{E2}.$$

The composite Kraus operator writes the selected branch into an orthogonal ledger and reopens the selected working ray. The completeness identity is on the declared E2 input carrier. Ledger continuation preserves the written branch. This Fold/R4 fact-production subchain is closed relative to FC; the source retires the request for an additional pre-fact gate-firing coefficient. Its closure does not determine coherent branch phases, the complete post-commitment joint history, physical sector prices or the loop coefficient, and it does not identify the chiral branch with a path-circulation label without a source map. HCMR §46P supplies the general record-preserving renewal isometry $V_{\text{renew}} |\psi; e, r\rangle = |e, r\rangle_L \otimes |\psi_{\text{open}}\rangle$, under which every ledger projector is conserved by $U_{\text{next}} = I_L \otimes U_{\text{work}}$ [20]; it does not by itself identify the sign label, and full reopening of the working carrier does not prove full restoration of the HX1/MASD record-phase tangent [20, §46P.1].

13.4 Primitive matter population update and internal observables

MCFR-1 identifies one D36 Fold with the primitive physical update grain under PCN. Reversal-even matter typing, $\text{Hom}_{D_6}(A_2, E_2) = 0$ and the stated linear projection that retains the even component select the population writer [5]

$$P_{\text{even}} = \frac{1}{2}(P + P^\sharp), \quad K_{yx} = V(p_{yx}) T_{yx}, \quad T_{yx} = |y\rangle \langle x| \otimes I_{49},$$

where P_{even} is the operator denoted P_+ in MCFR, $p_{yx} = P_{\text{even}}(y|x)$, and T_{yx} is the site-to-site internal-identity operator. The transition Kraus operators give a canonical trace-preserving CP realisation on site populations. The I_{49} factor belongs to that source's declared carrier; it is not silently identified with SGW-2's 47-dimensional matter-plus-Higgs carrier. This closes the population update at declared-premise plus representation grade, without selecting a unique coherent off-diagonal extension or the full joint history–matter instrument. The inherited one-Fold E2 innovation covariance $0.206275412545 I_2$ (precision 4.847887528915), portal $x = 0.08798754585$ and current-score candidate $g_{\text{cyc}} = 27.3244965042$ retain their stated branch conditions and are not promoted to SGW sector prices or α_{loop} .

Let $\mathcal{E}$ be the complete canonical channel with these Kraus operators, and let O be any internal operator. Row stochasticity of P_{even} gives the exact adjoint identity

$$\mathcal{E}^\dagger(I_7 \otimes O) = \sum_{x,y} p_{yx}(|x\rangle\langle x| \otimes O) = I_7 \otimes O,$$

since at each fixed x the sum over y gives one. The identity holds for arbitrary input states, including site-internal correlations, and implies preservation of the internal marginal, $\text{Tr}_{\text{site}}\mathcal{E}(\rho) = \text{Tr}_{\text{site}}\rho$. Every composition of complete channels with scalar internal Kraus factors has the same property. Extend A_j and A_T from the 47-dimensional charged matter-plus-Higgs carrier by zero on the two-dimensional neutral complement of the completed 49-dimensional source carrier (Section 2.1). Both extended observables are fixed, and any supplied internal price operator is also fixed under this adjoint pullback. Thus this operation alone generates no new price difference. It does not establish $J = T = 0$: an already non-common K is equally preserved. The statement is restricted to the canonical factorised, unconditioned channel; a selected outcome may change a conditional internal state when the input is correlated, transition labels are not identified with the entire physical R4 record, and an observable-preservation identity is not an identity between a channel and an action Hessian [5, Appendix F.5].

13.5 The writer carried through the old fact

For the declared separate-ledger append construction, let h be the complete old history prefix. For any normalised working instrument, including a history-dependent family $L_{r\alpha}(h)$, define

$$\mathcal{K}_{r\alpha}: |h\rangle \otimes |\psi\rangle \mapsto |h\rangle \otimes |r\rangle \otimes L_{r\alpha}(h)|\psi\rangle, \quad P_{h,\text{out}}\mathcal{K}_{r\alpha} = \mathcal{K}_{r\alpha}P_{h,\text{in}}.$$

The output projector reads the old prefix and is the identity on the new mark and work factors. The off-diagonal old-history blocks vanish identically for every source-field value, so their directional derivatives also vanish, including current, endpoint, normalisation and rate responses: $J_{\text{old}} = 0$, $R_{\text{old}} = 0$, $F_{\text{old}} = 0$. These are zero rows on the working parameters of this declared continuation, not the full physical MASD constraint matrix. Old-ledger retention alone adds no lost-relaxation penalty and decides neither $s_{\text{old}} = 0$ nor $s_{\text{old}} = 1$ for an event-specific master phase; HCMR's $s_{\text{fresh}} = 0$ remains intact [20, §46R]. The writer can change working variables without erasing an earlier ledger fact, but that retention does not determine whether the independently varying MASD/HX1 record phase remains available or what energetic penalty its preservation carries. The protected-holonomy restriction of Section 17 is an independent premise, not a derivative of this spectator ledger. The full current-instrument derivative that supports these zero rows, including the polar rotation and the archived edge-9 witness, is in Appendix F.6.

14. Native action with a prescribed record relation

The FRC-1 source extraction inherits the native H4 slice on (v, θ, y) , where v is a current combination, θ a normalised link-mode amplitude and y the native record phase. At face-incidence eigenvalue $\ell > 0$, its unit-convention action is [25, §2]

$$\Gamma = \frac{1}{2}[(v + \theta)^2 + \ell\theta^2 + (\theta - y)^2 + \ell y^2], \quad H_{\text{native}} = [[1, 1, 0], [1, \ell + 2, -1], [0, -1, \ell + 1]].$$

For a declared local relation $y = s\theta$, stationarity in v gives $v = -\theta$. Here s denotes only the slope of this record relation, not the universal gauge-conversion factor used elsewhere. Substitution and completion of the square yield $\Gamma(\theta; s) = \frac{1}{2}k_\ell(s)\theta^2$ with

$$k_\ell(s) = (\ell + 1)(1 + s^2) - 2s = \ell(\ell + 2)/(\ell + 1) + (\ell + 1)(s - 1/(\ell + 1))^2.$$

The first term in the completed-square expression is the freely relaxed native stiffness; the second is the extra cost of prescribing a record relation rather than allowing its stationary response. This is a tangent-space calculation within one inherited H4 quadratic parent, not the stochastic energy of a commitment event or the full six-defect MASD Hessian. A measured force coupling cannot be read from its numerical coefficient.

14.1 Alternating mode and inherited limiting cases

Let $G = 3I - S - S^\dagger$ be the inherited six-cycle operator; its eigenvalues are $\ell = 3 - 2\cos(2\pi k/6)$, that is (1, 2, 2, 4, 4, 5) for $k = 0, \pm 1, \pm 2, 3$. The alternating mode has $\ell = 5$ and

$$k_{m=3}(s) = 35/6 + 6(s - 1/6)^2.$$

Its inherited native ratio $s = 1/6$ gives $35/6$. Fixing the phase during a subsequent tangent variation means $\delta y = 0$ and gives stiffness 6, an increase of $1/6$. These are different boundary conditions, not two physical predictions. On $\ell = (1, 2, 2, 4, 4, 5)$ the freely reduced spectrum is $(3/2, 8/3, 8/3, 24/5, 24/5, 35/6)$, while the fixed-phase tangent stiffnesses are $(2, 3, 3, 5, 5, 6)$. These native spectra are inherited checks [5, 25]. The $1/6$ is used only as the native normalised θ -to- y ratio; a map from cycle coefficients to normalised edge amplitudes must retain its own normalisation. No numerical physical J , T , PCN conversion or loop coefficient follows by identifying these distinct carriers.

Two stiffnesses of the uniform mode. The uniform mode $\ell = 1$ appears with two different numbers in this paper. The HX1 phase census of Section 16.1 uses the (θ, y) block $[[3, -1], [-1, 2]]$ with the current coordinate absent, which relaxes to $y = \theta/2$ and stiffness $5/2$. The H4 parent above includes v , and reducing it as well gives $\ell(\ell + 2)/(\ell + 1) = 3/2$ for the same mode. The two values are different reductions of different parents, not a discrepancy, and neither is a physical coupling.

15. Constrained reduction of the complete quadratic parent

The native calculation extends to a frozen quadratic parent after the physical quotient. Let x denote retained coordinates and η the auxiliaries, with C positive definite on the auxiliary quotient. Let $R_{\text{rec}}\eta = F_{\text{rec}}x$ be the source-derived linearised condition for the written record, with redundant constraint rows removed. These matrices are distinct from the SGW representation map R and vectors F_r . Retain every same-order cross block:

$$H = [[A, B], [B^\dagger, C]], \quad G_{\text{free}} = A - BC^{-1}B^\dagger, \quad S_{\text{rec}} = R_{\text{rec}}C^{-1}R_{\text{rec}}^\dagger, \quad D_{\text{rec}} = F_{\text{rec}} + R_{\text{rec}}C^{-1}B^\dagger, \\ G_{\text{write}} = G_{\text{free}} + D_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}.$$

Theorem 11 (write-constrained quadratic response). Under the stated positive-quotient and constraint hypotheses, the physical constraint adds exactly $D_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}$ to the freely reduced parent, so that

$$\Delta J = \text{Tr}(A_J D_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}), \quad \Delta T = (1/3)\text{Tr}(A_T D_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}),$$

where the two trace formulas are meaningful only once the retained coordinates x have been identified with the physical matter and Higgs carrier V , in the same normalisation as $K = \alpha_{\text{loop}} W_{\text{eff}}$, so that A_J and A_T act on them. On any other retained coordinate set, including the native H4 parent of Section 18, the increment $D_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}$ is exact but ΔJ and ΔT are undefined; the identification of coordinates with V is the open physical lift of Sections 3 and 9.7, not part of this theorem.

Proof. The free stationary lift is $\eta_0 = -C^{-1}B^\dagger x$. Completing the square leaves the cost $\frac{1}{2}(\eta - \eta_0)^\dagger C(\eta - \eta_0)$. The constraint becomes $R_{\text{rec}}(\eta - \eta_0) = D_{\text{rec}}x$, whose least-cost solution is $\eta - \eta_0 = C^{-1}R_{\text{rec}}^\dagger S_{\text{rec}}^{-1} D_{\text{rec}}x$, the equality-constrained quadratic minimum in its bordered-system form [53]. Substitution gives the displayed positive semidefinite correction. ■

The formula assumes this quadratic expansion and linear constraint; a nonlinear constrained background with nonzero multipliers requires its constrained-Hessian terms as well (Section 17.3). A record constraint that agrees with the free stationary lift creates no additional quadratic stiffness. A nonzero constrained-Schur increment is positive semidefinite on the specified retained coordinates, but the signed gauge contrasts of its physical projection may vanish or have either sign. This distinction prevents the loss of a relaxation direction from being mistaken for an automatically successful electroweak splitting. The theorem does not return $A, B, C, R_{\text{rec}}, F_{\text{rec}}$ from the physical source, nor establish their SGW carrier identification. It is an evaluation formula for those source outputs, with no additional fitted stiffness. Appendix G records the native and independent constrained-solve checks.

16. Phase-constraint census, sharp pullback and the input split

16.1 The explicit phase-constraint census

PSAC-1-R19 executed this census in EX5BT to EX5BU, §§28BF to 28BG [21]; it is inherited. The returned explicit constraints do not remove the selected uniform-rim HX1 phase. On the scalar branch $\Phi_e(t) = \exp(itf_e)\Phi_e$, real f_e , both Gram products $\Phi_e\Phi_e^\dagger$ and $\Phi_e^\dagger\Phi_e$ are unchanged exactly, so with the other independent fields and occupied terminal addresses fixed the strengthened MASD Gram operators, rank-stable occupied projectors, positive polar factors and completion stretches remain fixed [27, §§8A, 16]; the scalar phase still changes the polar isometry and generally the record-composition defect, $\delta D_B = -if_e V_e$ with the link fixed. These are finite-cost off-vacuum fluctuations; the vacuum condition that all defects vanish is not a hard constraint on post-commitment fluctuations.

Let A_{exp} contain the returned hard-constraint differentials on the declared HX1 phase carrier, with its inherited inner product and gauge quotient. For the normalised selected mode the source census gives $A_{\text{exp}}\phi_{\text{old}} = 0$. Let G_A be the Moore–Penrose inverse of $A_{\text{exp}}A_{\text{exp}}^\dagger$ on its supported range; G_A is not an additional constraint. Then

$$P_{\text{exp}} = I - A_{\text{exp}}^\dagger G_A A_{\text{exp}}, \quad P_{\text{exp}}\phi_{\text{old}} = \phi_{\text{old}}, \quad s_{\text{exp}} = 1.$$

The inherited unit uniform-mode Hessian is $[[3, -1], [-1, 2]]$, with free relaxation $y = q/2$ and stiffness $5/2$. Its published explicit constraints leave this relaxation available, so their additional lost-relaxation cost is $\Delta k_{\text{exp}} = \frac{1}{2}(1 - s_{\text{exp}}^2) = 0$. The zero extra cost is not zero bond stiffness or zero SGW response. The tested uniform HX1 5/2 block is distinct from the alternating H4 35/6 block and from the 47-dimensional SGW carrier (Section 14.1). HCMR's weighted locked-minus-relaxed value $w_B^2/(w_B + w_R)$ remains physically unevaluated [20, §46R]. The protected-holonomy identification of Section 17 constrains $m = 3$ conditionally; the native full-carrier calculation of Section 18 leaves the uniform mode unchanged. The two results coexist without setting physical old-phase survival for every mode.

16.2 Sharp-pullback obstruction for the fixed resolved writer

The next question is whether a more general readout can install the missing exact continuation cut after the existing writer. PSAC requires the exported output to pull back to the forbidden continuation projector [21, §28BP]. Let $\mathcal{E}$ be a finite-dimensional trace-preserving channel with Kraus operators K_μ , let $0 \leq Q \leq I$ be any output effect, and let P be an input orthogonal projector.

Theorem 12 (sharp output pullback). If $\mathcal{E}^+(Q) = \sum_\mu K_\mu^\dagger Q K_\mu = P$, then $QK_\mu = K_\mu P$ for every μ , and hence $[P, K_\mu^\dagger K_\nu] = 0$ for all μ, ν ; in particular P commutes with every grouped physical outcome effect $E_r = \sum_\alpha K_{r\alpha}^\dagger K_{r\alpha}$. Consequently, if the common commutant of the physical outcome effects contains only scalar multiples of I , an output effect can pull back to a sharp input projection only for $P = 0$ or $P = I$.

Proof. For u in $\text{ran } P$, completeness gives $\sum_\mu \langle K_\mu u, (I - Q)K_\mu u \rangle = \langle u, (I - P)u \rangle = 0$, so positivity forces $QK_\mu u = K_\mu u$. For v in $\ker P$, $\sum_\mu \langle K_\mu v, QK_\mu v \rangle = 0$, hence $QK_\mu v = 0$. The two subspaces give $QK_\mu = K_\mu P$. Its adjoint implies $PK_\mu^\dagger K_\nu = K_\mu^\dagger QK_\nu = K_\mu^\dagger K_\nu P$. ■

A later trace-preserving quantum operation followed by an export test is included, because its adjoint returns an effect Q . The theorem covers all projection ranks and does not assume a Boolean selection of recorded labels [30]; the positivity step is the standard one for effects and Kraus decompositions [46, 47]. PSAC §28BQ reports a scalar common commutant for the native 84 physical effects on the same 50-dimensional input carrier, with terminal ranks (15, 15, 19, 15, 22, 22, 31); finer resolved marks do not supply the missing sharp projection. Conditional on that certificate and retention of the full effect family, the required identity $\sum_\mu K_\mu^\dagger Q_h K_\mu = I - \Pi_{\text{adm}}(h)$ cannot hold for a nontrivial forbidden projector solely through output processing of the fixed resolved writer. Strictly positive scalar edge weights preserve the commutant. This application uses the reported certificate; the 84 native matrices have not been independently reconstructed here. The executed 50-dimensional carrier is not replaced by the 49-dimensional completed source or the 47-dimensional charged SGW carrier.

The resolution assumption is essential. The channel sums unresolved Kraus alternatives and retains outcome labels as classical blocks; it does not retain coherent access to every environment alternative. A coherent pre-resolution lift is a different channel and can evade the obstruction. The theorem is not

automatically a statement about a single-edge conditional subinstrument or a history-dependent modification. It concerns exact exclusion, not approximate suppression. It narrows the PSAC fork: a non-Boolean selector applied only after this unchanged resolved writer is excluded, while a source-selected input operator or a full history and configuration carrier remains possible.

16.3 A consistent input split and the remaining source identification

Suppose the physical source returns an orthogonal forbidden-continuation projector P_h on the writer's actual input carrier, distinct from the old-history spectator-ledger projectors of Section 13.5. Let f_{ret} and f_{exp} be orthogonal unit flags in a two-dimensional output space. An admissible input-controlled split would have Kraus operators

$$A_{h\mu} = f_{\text{ret}} \otimes K_\mu(I - P_h), \quad B_{h\mu} = f_{\text{exp}} \otimes K_\mu P_h, \quad \Sigma(A_{h\mu}^\dagger A_{h\mu} + B_{h\mu}^\dagger B_{h\mu}) = I, \quad \Sigma B_{h\mu}^\dagger B_{h\mu} = P_h.$$

Completeness follows by inserting $\Sigma_\mu K_\mu^\dagger K_\mu = I$ on each side of the two complementary projections. No commutation of P_h with the working Kraus operators is required. Controlling the construction by orthogonal old-history projectors preserves each old fact exactly. The split changes the input channel and its cross-sector coherence; it is therefore outside Theorem 12's fixed-output-processing class. PSAC supplies a rank-one topological admissibility change, a history-controlled retain-and-export isometry and sufficient export-fibre dimension in its witnesses [21, §§28BK to 28BP]; the Fold interface supplies the commitment and boundary-preservation setting [31]. The unreturned identification is the restriction or embedding of that complete-configuration continuation support on the actual writer input, together with its source-derived activation; alternatively the full configuration carrier must be retained explicitly. A projector chosen just to remove ϕ_{old} would assume the result. A source-derived input selector remains necessary for the stronger marked-carrier implementation of this split. The protected-holonomy route developed next instead keeps the physical record observable on its declared same- Φ carrier, so conditional local Fact-Lock need not wait for a selected 50-dimensional marked ray [32 to 35]. Either route still requires a physically typed constraint differential, the complete action and the SGW embedding before numerical J and T can be issued.

17. Protected fact holonomy and the history rows in the complete action

17.1 The conditional phase restriction

The $m = 3$ comparison, commitment-write, carrier-bridge and protected-holonomy calculations supply a conditional route beyond the classically resolved-mark limitation [32 to 35]. Under the separately specified Gate-3 phase-faithfulness and isolation-trap premises, a completed event retains a boundary cycle with nonzero $\mathbb{Z}_7$ charge [36]. The inherited Fold-to-K7 attachment transmits the alternating $m = 3$ direction, but identifying that direction with the independently variable MASD record phase remains the same- Φ constitutive premise [34]. The equations below state the consequence of assuming that identification, not an unconditional derivation of it:

$$R_{\text{hol}}(y_3) = \exp(i\beta_{\text{hol}}y_3), \quad \beta_{\text{hol}} = 6V6; \quad R_{\text{hol}} = \exp(2\pi i q/7), \quad y_3(q) = \pi q/(21V6) \bmod \pi/(3V6).$$

Here β_{hol} is a record-readout gain, distinct from the loop conversion $\kappa = \alpha_{\text{loop}}/6$. Preserving the already written value gives $\delta R_{\text{hol}} = i\beta_{\text{hol}}R_{\text{hol}}\delta y_3 = 0$, equivalently $\delta y_3 = 0$. Locally, preserving the holonomy readout fixes the old alternating record coordinate. This follows exactly from the nonzero readout derivative on the conditional same- Φ branch; the finite holonomy period represents global equivalence, not a free infinitesimal direction. The restriction does not fix the renewed phase of a later event or all record modes, and the marked-ray selector can remain open at this observation grade without supplying the missing physical energetic embedding.

Two scope statements govern the use of this restriction. First, the minimal completion map fixes an outcome state and its persistent branch label, while its rank-one Kraus representative is unique only up to phase [22, §28]; the FRC-1 G2 calculation shows that if the native $m = 3$ operational descent acts only as scalar mark rephasing, $M \mapsto e^{i\phi}M$ leaves $MqM^\dagger$ unchanged, so resolved probabilities and conditioned

density operators cannot detect that phase [26]. A resolved terminal mark insensitive to scalar phase does not establish a protected MASD phase, and the holonomy observable is admitted as a distinct coherent record carrier, so the resolved-mark no-go does not eliminate it. Second, the distinction between on-shell and off-shell identification is decisive: the pre-commitment relation $y_3 = \theta_3/6$ gives a stationary solution relation, not by itself a physical record observable for independent θ , y and other MASD variations. A valid full-action calculation must supply that record Jacobian on the committed field background. Substituting the stationary relation in place of the source map would assume the conclusion.

17.2 The retained history determines the constraint rows

Let h specify the retained isolation events, their surviving boundary cycles $\gamma_j(h)$, written residues q_j and transports to the current regulator. A candidate continuation $z \rightarrow z'$ must preserve the old isolation boundaries and the transported record values: $R_{j,h}(z') = R_{j,h}(z) = \exp(2\pi i q_j/7)$ for every j . On a fixed topological stratum, let $\chi(z)$ be the physically identified phase-coordinate map and $b_j(h)$ the oriented cycle row. Choose a local real lift c_j of the written phase. Then

$$f_{j,h}(z) = b_j(h)\chi(z) - c_j = 0, \quad B_h = Df_h(z_h), \quad T_h = \ker B_h \cap T_{\text{other}}.$$

If cycle representatives or transports vary with z , their derivatives are included in B_h . A boundary-destroying update is a separate discrete exclusion, not a smooth phase tangent. At a singular point the displayed kernel is only necessary and the actual tangent cone must be tested. This protected-record condition is not asserted to enumerate every admissible full MASD/CAR/Fock move; HMGBC leaves the full continuous proposal law open [7, §11A]. For the conditional scalar record chart y with event rows L_h , the restriction is $L_h \delta y = 0$, and with G_h the pseudoinverse of $L_h L_h^\dagger$ on the image of the retained history rows its orthogonal projector is

$$P_{\text{allow}} = I - L_h^\dagger G_h L_h.$$

The number of removed scalar directions is rank L_h , not the number of recorded events. Dependent constraints add no independent restriction. Distinct histories may supply different cycle and transport rows. Adding a fact narrows the local tangent when old rows remain on one common carrier; renewal must retain a separate new coordinate rather than identify it with the old one.

17.3 Insert the history restriction into the complete MASD action

At the level of exact support, continuation is restricted to the physically admitted set A_h , equivalently represented by an indicator functional that is zero on allowed configurations and infinite otherwise. A finite penalty cannot impose a sharp zero-support exclusion. The complete source action must then be differentiated at the actual committed background, allowing for nonzero old defects, field-dependent metric terms, cross-blocks and the constraint reaction. The resulting formulas are conditional evaluations of returned fields, not a new event-selection or timing law:

$$\Gamma_h = \Gamma + I_{A_h}, \quad \Gamma = \Gamma_V + \frac{1}{2}\langle D, WD \rangle + \Gamma_{\text{ferm}} + \Gamma_{\text{top}}.$$

For a real field variation ξ at the actual committed background, let D_1, D_2 denote the first and second directional derivatives of the complete defect vector and W_1, W_2 those of W . Direct differentiation gives

$$\delta^2 \Gamma_{\text{bos}}[\xi] = \delta^2 \Gamma_V[\xi] + \langle D_1, WD_1 \rangle + \langle D, WD_2 \rangle + 2\langle D_1, W_1 D \rangle + \frac{1}{2}\langle D, W_2 D \rangle,$$

with all products in the real inner product. If W is constant, its derivative terms drop out. A defect-zero vacuum has the simpler Hessian $H_V + J_D^\dagger W J_D$, but a nonzero-holonomy committed state cannot simply be replaced by that vacuum. The fermionic and topological sectors retain their separately stated grades; Grassmann variables require a legitimate effective-action reduction before a positive real Hessian solve. At a regular constrained stationary background, $\nabla \Gamma + B_h^\dagger \lambda = 0$ and $f_h = 0$, and the correct quadratic form uses the Lagrangian Hessian

$$H_L = \nabla^2 \Gamma + \sum_j \lambda_j \nabla^2 f_{j,h}.$$

The reaction term vanishes for linear constraints in a phase chart but need not vanish in general field coordinates. On the verified physical quotient, let N_h span the admissible tangent; the constrained form is $N_h^\dagger H_L N_h$. Unexplained null directions are retained for the source audit, not silently discarded. This is a static response calculation and does not itself derive the event-time law or trap-activation time. To use Theorem 11, split the complete relevant parent into retained x and all allowed auxiliaries η , write $F_h x + R_h \eta = 0$, and with $H = [[A, B], [B^\dagger, C]]$, C positive on its quotient and constraints feasible,

$$D_h = F_h - R_h C^{-1} B^\dagger, \quad M_h = R_h C^{-1} R_h^\dagger, \quad K_h = A - B C^{-1} B^\dagger + D_h^\dagger M_h^{-1} D_h.$$

Here M_h is the constraint compliance matrix. The relation to Theorem 11 is $F_{\text{rec}} = -F_h$, $R_{\text{rec}} = R_h$, $D_{\text{rec}} = -D_h$ and $S_{\text{rec}} = M_h$, so the positive correction is identical; completing the square about $\eta_0 = -C^{-1} B^\dagger x$ gives the result without dropping cross blocks. With dependent rows use $M_h^\dagger$ and require $D_h x$ in its range; singular C needs the separate null and range analysis. The constrained increment is positive semidefinite and has rank no greater than the independent constraint rank, for the same fixed local parent. It vanishes when the free stationary response already preserves old records. Several facts generally interact through the full compliance matrix rather than producing additive one-fact penalties. These facts establish the form of a source calculation, not the numerical physical sector correction.

18. Full native H4 evaluation with one protected alternating record

Use the full native H4 quadratic parent of FRC-1, with all six Fourier modes of each of the link and cycle variable θ , the current combination v and the record phase y . The three variables are physically distinct. With $G = 3I - S - S^\dagger$ the inherited six-cycle operator and $P_3 = f_3 f_3^\dagger$ the projector onto its alternating $\ell = 5$ direction, the 18-coordinate parent in the block order (v, θ, y) is

$$H_{H4} = [[I, I, 0], [I, G + 2I, -I], [0, -I, G + I]].$$

This is a common-unit native benchmark, not the already-embedded 47-dimensional MASD matter and Higgs action [25, 37]; its smallest eigenvalue is 0.4679111138, so the parent is positive. Impose only $f_3^\dagger \delta y = 0$, the conditional preservation rule for the already written alternating fact.

Theorem 13 (native H4 Fact-Lock response). Reducing the 18-coordinate parent over v and every permitted record direction at fixed θ gives a rank-one increment on the alternating link mode and no change elsewhere:

$$K_{\theta, \text{free}} = G(G + 2I)(G + I)^{-1}, \quad K_{\theta, h} = K_{\theta, \text{free}} + (1/6)P_3,$$

so the alternating θ stiffness rises from 35/6 to 6 and the extra local quadratic cost is $(\delta\theta_3)^2/12$. If instead the current combination v is retained and the allowed link and record variables relax, the Schur reduction of the same parent gives a different rank-one response:

$$K_{v, \text{free}} = I - [G + 2I - (G + I)^{-1}]^{-1}, \quad K_{v, h} = K_{v, \text{free}} + (1/287)P_3, \quad k_{v, \text{free}} = 35/41, \quad k_{v, h} = 6/7, \quad \Delta k/k = 1/245 \approx 0.408\%.$$

Proof. The parent is mode-diagonal in the Fourier basis of the six-cycle, so each eigenvalue ℓ of G reduces independently. Eliminating v and y at fixed θ gives $\ell + 2 - 1/(\ell + 1) = \ell(\ell + 2)/(\ell + 1)$; holding y fixed in the alternating mode removes the term $1/(\ell + 1) = 1/6$ there and nowhere else. Eliminating y then θ at fixed v gives $1 - 1/(\ell + 2 - 1/(\ell + 1)) = (\ell^2 + 2\ell)/(\ell^2 + 3\ell + 1)$, which is 35/41 at $\ell = 5$; holding y fixed replaces it by $1 - 1/(\ell + 2) = 6/7$, and $6/7 - 35/41 = 1/287$. ■

The other five link and cycle modes, including the uniform mode tested in PSAC, acquire no additional stiffness from this one history row, consistent with the uniform-phase census of Section 16.1: the two results restrict different modes under different premises. The added current-combination energy is $(\delta v_3)^2/574$. The increments 1/6 and 1/287 belong to two different retained-coordinate Schur reductions and must not be added, equated or identified with the physical gauge strengths. They are exact model results under the conditional record restriction; they do not select a physical J or T or the full MASD carrier. Appendix G.7 records the independent 18×18 constrained solve.

19. Scope of closure and the physical decision rule

SGW-2 has exact algebra for six-sector gauge pricing, exact mathematical fact-preservation conditions, and a complete constrained-action evaluation rule. It also has an exact native 18-coordinate response on the conditional same- Φ protected-holonomy branch. What it does not have is the full field-level action connecting that native record mode to the admitted physical matter and Higgs response.

The decisive source computation is specific. First establish the same- Φ holonomy observable and its derivative for independent field variations at the committed background, with its protected history rows and any other admissibility constraints. Next differentiate the complete MASD action there, retaining all same-order cross blocks, quotient and null conditions, potential nonzero-defect terms and reaction Hessians. Finally produce the correct representation-valued physical lift, contract the action directly with A_J and A_T , and return common normalisation and independent gauge channels separately. Neither an extra H4 example nor a diagnostic current Gram can substitute for those steps.

Two outcomes must be accepted without retuning. If the projected record response is zero in both action contrasts, the native Fact-Lock remains mathematically valid but this route does not change relative gauge strengths. If at least one contrast is nonzero, the result can be inserted into the D_6 work matrix, or the full matrix if D_6 symmetry fails, frozen and only then tested against Standard Model data. The physical α_{loop} and remaining coefficients are still to be sourced; the marked 50-dimensional writer problem remains a separate question.

Physical decision rule. The sign or magnitude of the native stiffness increment is not a substitute for the two gauge-sensitive action contractions; the programme accepts all three post-reduction outcomes below.

Source-returned physical response	Interpretation	Consequence
$J = T = 0$	The record-lock action may be nonzero but its gauge-relative projection vanishes.	No relative K7 gauge correction from this mechanism.
J or T nonzero; wrong post-freeze result	The proposed source branch changes relative strengths but fails the comparator.	Reject this frozen branch; do not retune.
J or T nonzero; consistent after full insertion	Conditional candidate for physical gauge splitting.	Requires normalisation, other channels, threshold and RG analysis and independent scrutiny.

Part D. Programme, falsifiers and ledger

20. Source-only execution contract

A genuine SGW-2 closure is finite and pre-registerable. The calculation is to be executed in the following order.

Gate	Required source return	Release condition
S1 Parent	Use the MASD common parent on the admitted event grain and derive the complete physical bath and history generator rather than selecting its blocks.	One provenance chain for W_{core} , Ω_Q^2 , C and the gauge and loop carrier.
S2 Reduction	Perform the full typed bath Schur reduction $W_{\text{bath}} = W_{\text{core}} - C^+(\Omega_Q^2)^*C$ and all required gauge and null quotients.	Defect-Lift, positivity, Ward and null and reciprocity checks pass.
S3 Pricing	Retain the full typed gauge-covariant reduction. Compute the sector averages, or contract the two central operators of Section 12 directly for the non-common loop part.	No gauge target selects the source; the quotient and physical representation typing are retained.
S4 Loop normalisation	Use $\kappa = \alpha_{\text{loop}}/6$. Return α_{loop} with W_{eff} , or return the physically normalised product K from the same action.	The exact factor $1/6$ is retained; α_{loop} is not silently set to one; common normalisation is accounted for.
S5 Typed response	Evaluate J , T and reconstruct the non-common loop correction by Section 12.2, or equivalently compute $(\alpha_{\text{loop}}/6)R\delta w$. Supply independent channels separately.	The common $R1 = 6Z$ component is not double-counted; the quotient result is not mistaken for complete coupling ratios.
S6 Work insertion	Map the returned factor response into ΔD_{typed} , or return the full D if D_6 breaks.	Work spectroscopy reproduces the same reduced action.
S7 Gauge freeze	Extract and seal Z_3 , Z_2 , Z_q .	All Stage-A hashes and inputs frozen.
S8 Stage B	Run finite scheme, thresholds and coupled RG transport.	Only then compare with experiment.

Strong success criterion. Return the complete typed physical parent and consistently normalised loop response from one source. Either evaluate W_{eff} and α_{loop} separately or directly evaluate the corresponding action product K ; for this loop's non-common part, the two contractions of Section 12 are sufficient. Then supply the common component and any independent channels required by the physical work matrix, perform the insertion, and freeze Z_3 , Z_2 , Z_q before comparison. A source theorem that K is scalar on the full visible matter-plus-Higgs carrier gives zero non-common response from this mechanism. A factor-space history matrix alone does not supply these action contractions.

21. Post-freeze Stage-B comparison protocol

SGRI-1 provides a clean downstream diagnostic because the unknown common matching normalisation and the one-loop scale direction are quotiented out [2]. On the frozen unscreened branch the one-loop

relation is not exact at M_Z ; gauge-only two-loop transport removes part of the discrepancy and leaves the recorded transverse remainder -33.48185948 in the declared comparator coordinates. That number is not a source target. Once the physical source has frozen $T = \Delta\mathcal{T}_{\text{loop}}$, either through α_{loop} and the sector prices or through the direct action contraction of Section 12, it may be compared with the Stage-B ledger. If, only for diagnostic purposes, this finite-loop term were the sole missing contribution, it would have to satisfy

$$(\alpha_{\text{loop}}/6)(-9684\delta w_Q + 8010\delta w_u + 5148\delta w_d - 4752\delta w_L + 2862\delta w_e - 1584\delta w_H) = -33.48185948.$$

On the matter-universal slice this is equivalently $264\alpha_{\text{loop}}(m - h) = -33.48185948$, or $\alpha_{\text{loop}}(m - h) = -0.1268252253$. This remains a post-freeze comparison coordinate only. It may falsify a frozen source branch; it may not be used to solve for α_{loop} or the sector prices. A precision physical comparison must use the same convention bridge, finite thresholds and complete coupled Standard Model running for all three returned coefficients. The affine one-loop SGRI equation is a diagnostic invariant, not a substitute for that final transport. Neither the sign nor the size of the remainder may be used to solve for α_{loop} , matter and Higgs sector weights or an unreturned record constraint.

22. Falsifiers and no-shortcut rules

F1, matter-universality insertion. Setting $w_Q = w_u = w_d = w_L = w_e$ without a source theorem collapses SGW-2 back to an assumed SGW-1 slice.

F2, gauge-data inversion. Using measured gauge couplings to choose among the three ker R directions violates the source firewall; R has an exact three-dimensional nullspace.

F3, Higgs kinetic and price conflation. Canonical Higgs kinetic normalisation does not imply $w_H = 1$ in the finite-loop gauge-price operator.

F4, Fisher and action reversal. A history or Fisher metric may not be renamed W_{eff} , nor may its scalar screening ratio be transferred to an action price, unless the carrier and action bridge is derived from the common parent.

F5, D41 promotion. D41 retention may not be copied into w_H , w_e , a Q/u/d/L/e/H price, or a gauge coefficient. Its exact factor-space Schur term fails the central sector-price pullback identities $K_{CC} = K_{CY}$ and $K_{WW} = K_{WY}$.

F6, rooting and action conflation. The exact $1/6$ rooted-walk conversion may not be promoted to $\kappa = 1/6$ unless the once-counted loop functional is proved to enter the PCN-normalised physical action with $\alpha_{\text{loop}} = 1$.

F7, partial Schur assembly. W_{core} , Ω_Q^2 , C, matter, Higgs, record and bath reductions may not be taken from different parents and combined afterwards.

F8, neutral direct contribution. A sterile N_R price, or a price on the two-dimensional neutral complement, may not enter the second-curvature gauge trace directly; the neutral trace is zero.

F9, D₆ forcing. If the typed source breaks D_6 after complete reduction, the full work matrix must replace the four-orbit compression.

F10, RG double counting. Ordinary perturbative determinant and threshold effects already assigned to Stage B may not be added again as Stage-A source weights.

F11, comparator feedback. The sign or size of the SGRI residual may reject a frozen source branch but may not select W_{eff} , κ or an auxiliary adjacency.

F12, nullspace overclaim. Agreement of the three gauge factors does not prove that the underlying six sector prices are uniquely correct.

F13, reviving a closed local route. The representation counts 1, 18, 4, 72 or +8 may not be promoted to physical Schur susceptibilities on the executed neutral local completion branch: $\chi_c = 0$ there and the terminal +8 is not source-derived.

F14, scalar-current carrier reuse. The orthogonal I_3 scalar-factor ledger or the 187/334 operational chart may not be identified with the selected representation-valued current carrier by a label-preserving rescaling; the exact carrier Gram is G_{45} with nonzero cross-factor overlaps.

F15, factor-to-sector inversion. A C/W/Y current or history metric may not be inverted into Q, u, d, L, e, H prices unless it lies in the image of a gauge-covariant central sector operator. The support identities $P_C P_Y = P_C$ and $P_W P_Y = P_W$ provide immediate necessary tests.

F16, write by alignment. A bond-action minimum may not be imposed as an instantaneous physical update without a source-derived event law.

F17, sign conflation. Preparation bias, committed sign and realised path circulation remain distinct until their source relation is proved.

F18, ledger and field conflation. Retaining an event label does not prove installation or survival of a MASD polar-phase direction.

F19, support proxy. An occupied-address set, occupied projector or final vertex may not replace an ordered-history sign record without a verified encoding.

F20, normalisation as writer. Positive Perron and Doob normalisation cannot be credited with eliminating wrong-sign or erasure transitions present in the raw transfer.

F21, contrast overclaim. The two contrasts determine this loop correction only modulo the common gauge direction; they do not fix total normalisation, other physical channels or final couplings.

F22, primitive and retained democracy. Internal democracy of the primitive metric does not prove $K_0 = k_0 I$ after physical embedding. The specialisation of Section 12.4 requires that additional source result.

F23, lock-to-gauge inference. A positive-semidefinite native Fact-Lock stiffness does not imply a nonzero physical J or T. The 47-dimensional embedding must be evaluated; $J = T = 0$ is a possible source result, not a failure of the native algebra.

F24, on-shell map promoted off-shell. The stationary relation $y_3 = \theta_3/6$ does not establish the same- Φ phase observable for independent field variations. It cannot supply the missing off-equilibrium record Jacobian.

F25, inherited representation content. The trace map assumes the stated Standard Model multiplets, hypercharges and K7 identity. Its algebraic closure does not itself derive those inputs from the void or solve three-generation uniqueness.

F26, statistics-weight substitution. Replacing the state-counting K7 rule by perturbative loop weights changes F_{total} away from 6Z and is a Stage-B operation; it may not be applied inside the Stage-A trace, and the Higgs statistics weight stays inside w_H .

F27, neutral-sector conflation. The two-dimensional neutral complement of the 49-carrier may not be identified with three sterile neutrinos, or either with a Fold-location observable, by notation.

23. Closure ledger

The ledger records only what the Results guide, Figure 1 and the falsifiers do not already state: the question asked, the answer in hand, and its grade. Entries graded exact in Sections 5 to 7 and 12 are summarised in one row each.

Question	Answer after SGW-2	Status
Is one matter weight fundamental?	No. Five charged-matter sector-average prices are allowed; m is the universal slice (Corollary 4.1).	Closed

Question	Answer after SGW-2	Status
Representation map, contrasts and kernel	F_r , R , rank 3, lattice basis u_1 to u_3 , canonical m , kernel transverse to the SGW-1 slice (Theorems 3 to 5, Corollaries 4.2, 5.1).	Closed; exact in the state-counting convention
Exact SGRI typed response	$(\alpha_{\text{loop}}/6)(-9684\delta w_Q + 8010\delta w_u + 5148\delta w_d - 4752\delta w_L + 2862\delta w_e - 1584\delta w_H)$.	Closed form; α_{loop} , δw open
Neutral sector and family orientation	No direct trace from N_R or the neutral complement; orientation and off-diagonal family structure invisible at this grade.	Closed
Is the physical price operator selected?	Not numerically. Primitive metric democratic (Theorem 7); current carrier G_{45} fixed; local neutral splitting zero (Corollary 7.1). Remaining gate: the central $Q/u/d/L/e/H$ projection of $W_{\text{core}} - C^+(\Omega_Q^2)^+C$.	Sharpened, open
Higgs price from HSPR?	No. Defect-mediated corrections vanish; the direct gauge-loop price is differently typed.	Reduced, open
D41 as a price or inverse map?	No. Exact history fingerprint; fails $K_{CC} = K_{CY}$ and $K_{WW} = K_{WY}$; screening $0.0551296278772322 \rightarrow 0.0504814732865564$, $\lambda = 0.9191281682$.	Closed negatively
Is κ fixed?	$\kappa = \alpha_{\text{loop}}/6$ exactly; $\alpha_{\text{loop}} = 1$ only if the once-counted loop is the PCN work term (Proposition 8).	1/6 closed; α_{loop} open
Common finite-loop component	$R1 = 6Z$; after $\kappa = \alpha_{\text{loop}}/6$ it is $\alpha_{\text{loop}}Z$, SGRI-null, not to be double-counted.	Closed algebra; conditional physical
Two direct action contrasts	$J = \text{Tr}(A_J K)$, $T = (1/3)\text{Tr}(A_T K)$; ΔA reconstructed modulo $\text{span}\{Z\}$; direct Schur contraction of the parent verified.	Exact algebra; physical values open
Fact preservation and the writer	Kraus intertwining (Theorem 10); canonical writer preserves every internal observable; spectator ledger gives $J_{\text{old}} = R_{\text{old}} = F_{\text{old}} = 0$.	Exact on the declared carriers
Commitment and population writer	FC-relative completion and renewal inherited; $P_{\text{even}} = \frac{1}{2}(P + P^*)$ closes the population update under MCFR premises; energetic price separately graded.	Inherited; closed relative to FC
Inherited finite record results	Bond alignment $y = \theta$; raw and normalised write support equivalent; 12 wheel histories, six per sign; polar bond response and writer derivative (Appendix F).	Exact on their declared branches
Native write-constrained action	$k_\ell(s) = \ell(\ell + 2)/(\ell + 1) + (\ell + 1)[s - 1/(\ell + 1)]^2$; 35/6 free, 6 fixed-phase; uniform-mode 5/2 and 3/2 reconciled.	Exact on the declared native slice
Phase census and sharp pullback	$S_{\text{exp}} = 1$, $\Delta k_{\text{exp}} = 0$ on the uniform HX1 mode; no nontrivial sharp input projection by output processing (Theorem 12), given the 84-effect scalar commutant.	Inherited exact; certificate inherited

Question	Answer after SGW-2	Status
History-dependent input split	Completeness, exact export and old-fact retention once P_h is supplied.	Construction verified; source identification open
Protected $m = 3$ holonomy	Preserving $R_{hol} = \exp(i6V6 y_3)$ imposes $\delta y_3 = 0$ on the same- Φ branch.	Exact, conditional
Constrained action and native H4 response	Theorem 11 increment $D_{rec}^\dagger S_{rec}^{-1} D_{rec}$; Theorem 13: $\Delta K_\theta = P_3/6$, $\Delta K_v = P_3/287$, other modes unchanged.	Exact formula and exact native result; physical inputs open
Physical Z_3, Z_2, Z_q	Not issued; require physical J, T, common normalisation, independent channels and work insertion.	Open

24. Conclusion

SGW-2 derives the exact representation-resolved K7 loop response for the inherited minimal Standard Model census, in the stated state-counting convention. Its six matter and Higgs traces form a rank-three map with a three-dimensional invisible price kernel whose lattice basis is displayed; the kernel is transverse to the SGW-1 two-price slice, so the non-identifiability is entirely the price of dropping universality. The SGW-1 matter-universal relation is one special slice. Two central action traces, the inherited SGRI normal and the forced second normal that blinds the down singlet, the lepton doublet and the Higgs, determine the representation-loop correction modulo the common gauge direction, but neither returns a numerical physical coupling without the same-source action operator.

The source audits narrow the remaining calculation. The primitive operational metric is internally democratic, the tested neutral local route does not generate the needed split, the selected current carrier is not an orthogonal three-factor ledger, and D41 does not satisfy the central sector-price pullback identities. The $1/6$ rooted-loop normalisation is exact; the common action coefficient α_{loop} , the complete physical reduction and the common gauge normalisation are open.

The record portion of the paper distinguishes FC-relative completion, inherited renewal and population writing from independently physical field variations. Exact Kraus intertwining describes preservation of an already written sharp fact. The canonical writer can alter working states while maintaining the old spectator ledger, but that retention alone neither freezes nor reopens a MASD record-phase degree of freedom. A separately protected, conditional same- Φ alternating holonomy supplies one local record restriction. Carrying it through the native 18-coordinate H4 parent gives exact stiffness increments $P_3/6$ and $P_3/287$ on two alternative retained-coordinate reductions, without altering the uniform mode. This supports a specific constrained-action mechanism on that model; it does not show that the corresponding physical matter and Higgs projection changes either gauge contrast. The sharp-pullback theorem rules out a nontrivial exact continuation cut obtained only by output processing of the unchanged classically resolved writer, conditional on the reported scalar commutant; a source-controlled input split and a coherent protected-record carrier remain different possibilities.

The remaining physical gate is explicit: verify the off-equilibrium record map, construct its transported history constraints, evaluate the whole constrained MASD action at the committed background, and project the resulting operator onto the 47-dimensional matter and Higgs carrier. A zero pair $J = T$ would exclude this mechanism as a source of relative gauge splitting despite its native stiffness; a nonzero pair would require physical work insertion, complete normalisation and post-freeze testing. The paper closes with an exact structural result and a conditional native action result, not a completed prediction of Standard Model gauge strengths.

Appendix A. Representation-trace arithmetic and the weighting convention

The Abelian generator is $Y/2$ in the convention $Q = T_3 + Y/2$. Dynkin indices are $1/2$ for the fundamental of $SU(3)$ and of $SU(2)$. For three generations:

- Q_L : $\text{Tr } T_C^2 = 3 \times 2 \times (1/2) = 3$; $\text{Tr } T_3^2 = 3 \times 3 \times (1/2) = 9/2$; $\text{Tr}(Y/2)^2 = 18 \times (1/6)^2 = 1/2$.
- u_R : $\text{Tr } T_C^2 = 3 \times (1/2) = 3/2$; $\text{Tr } T_3^2 = 0$; $\text{Tr}(Y/2)^2 = 9 \times (2/3)^2 = 4$.
- d_R : $\text{Tr } T_C^2 = 3/2$; $\text{Tr } T_3^2 = 0$; $\text{Tr}(Y/2)^2 = 9 \times (1/3)^2 = 1$.
- L_L : $\text{Tr } T_C^2 = 0$; $\text{Tr } T_3^2 = 3 \times (1/2) = 3/2$; $\text{Tr}(Y/2)^2 = 6 \times (1/2)^2 = 3/2$.
- e_R : $\text{Tr } T_C^2 = 0$; $\text{Tr } T_3^2 = 0$; $\text{Tr}(Y/2)^2 = 3 \times 1^2 = 3$.
- H : $\text{Tr } T_C^2 = 0$; $\text{Tr } T_3^2 = 1/2$; $\text{Tr}(Y/2)^2 = 2 \times (1/2)^2 = 1/2$.

The normalised K7 six-face closed-walk identity multiplies each quadratic representation trace by six, giving the F_r vectors of Section 5. Each state is counted once. If instead the Higgs components were weighted by one half, as a complex scalar is in a perturbative one-loop count relative to a Weyl fermion, F_H would become $(0, 3/2, 3/2)$ and F_{total} would be $(36, 75/2, 123/2)$, which is not a multiple of Z . The balance $F_{\text{total}} = 6Z$ is therefore a statement about the state-counting rule, and the Higgs statistics weight is carried by w_H .

Appendix B. Rank, kernel lattice and scalar-slice checks

With columns (Q, u, d, L, e, H), $R = \begin{bmatrix} 18 & 9 & 9 & 0 & 0 & 0 \\ 27 & 0 & 0 & 9 & 0 & 3 \\ 3 & 24 & 6 & 9 & 18 & 3 \end{bmatrix}$. A nonzero 3×3 minor is obtained from columns Q, u, H, so $\text{rank } R = 3$. Direct multiplication gives $Ru_1 = Ru_2 = Ru_3 = 0$ for $u_1 = (0, 0, 0, 1, 0, -3)$, $u_2 = (0, -1, 1, 0, 1, 0)$, $u_3 = (-1, -2, 4, 3, 0, 0)$. The 3×6 integer matrix with these rows has Smith normal form $\text{diag}(1, 1, 1)$, so the three vectors are a basis of the lattice $\ker R \cap \mathbb{Z}^6$. The alternative triple $(-1, -2, 4, 3, 0, 0)$, $(0, -1, 1, 0, 1, 0)$, $(-1, -2, 4, 0, 0, 9)$ spans the same rational kernel but has Smith form $\text{diag}(1, 1, 3)$, index three in the lattice, since $(-1, -2, 4, 3, 0, 0) - (-1, -2, 4, 0, 0, 9) = 3u_1$. The Smith form of R itself is $\text{diag}(3, 3, 54)$. Contracting $n = (466, -687, 159)$ with R gives $(-9684, 8010, 5148, -4752, 2862, -1584)$; summing the first five coefficients yields 1584, so the matter-universal slice gives $1584\kappa(m - h)$. Contracting $m = (-2, -3, 3)$ gives $(-108, 54, 0, 0, 54, 0)$. On the two-price slice the image vectors $F_m = (36, 36, 60)$ and $F_H = (0, 3, 3)$ are independent, so the slice meets the kernel only at zero.

Appendix C. Same-source Schur algebra

Let x denote the retained representation-valued source coordinate and η the complete auxiliary coordinate. For $S^{(2)} = \frac{1}{2}x^\dagger W_0 x + \text{Re}(x^\dagger B \eta) + \frac{1}{2}\eta^\dagger C \eta$, stationarity in η gives $\eta^* = -C^\dagger B^\dagger x$ on the supported quotient, and substitution returns $S_{\text{eff}}^{(2)} = \frac{1}{2}x^\dagger (W_0 - B C^\dagger B^\dagger) x$. If C is positive on the retained auxiliary quotient and the full parent is positive, W_{eff} is positive semidefinite. Gauge covariance of the parent implies gauge covariance of W_{eff} . Any retained nulls must be quotient-handled before a sector price is called physical. This is the only Schur algebra used by SGW-2; no history covariance is silently promoted to energetic stiffness.

Appendix D. Frozen SGW-1 baseline and Stage-B handoff

The exact SGW-1 canonical-history baseline is inherited unchanged [1]. In the D_6 orbit basis,

$$(d_0, d_1, d_2, d_3) = (25638856, 1125824, 394240, 229376)/57254613,$$

with $Z_3 = 5616/2201$, $Z_2 = 2444/899$, $Z_q = 136080/713$ and $\mathcal{T}_{\text{CH}} = 241223808/1468067 \approx 164.313895755$. The factor-blind scalar-wheel T–B Schur refinement replaces c by $(195/196)c$ and returns $(d_0, d_1, d_2, d_3)_N \approx (0.447554457, 0.019598775, 0.006869360, 0.003999315)$, with $Z_3^N \approx 2.550513824$, $Z_2^N \approx 2.717626018$, $Z_q^N \approx 190.697235740$ and $\mathcal{T}_N \approx 163.776491722$. These values remain source-derived baselines, not the final physical gauge boundary. The typed SGW-2 correction must be returned from the common parent and inserted before any measured comparison is opened.

Appendix E. Primitive metric, local no-go and rooted-loop normalisation

E.1 Primitive operational metric. MASD defines $W_{op} = \text{diag}(\pi) \otimes I_{\text{orientation}} \otimes I_{\text{internal}}$ and $W_{XY} = \iota_X^* W_{\text{primly}}$. Representation dependence is not supplied by arbitrary particle-species coefficients in the primitive metric; it can only arise through typed embeddings and reduction.

E.2 Local neutral no-go. On the executed neutral branch the current and matter gauge mixed Hessian is zero. For the local faithful-link completion parent, $b_{A,c} = 0$ and the Gaussian determinant is independent of the open-link gauge phase, so $\chi_c = 0$. The exact representation ratios 1 and 18 therefore do not constitute nonzero physical susceptibilities on that branch.

E.3 Rooted-loop normalisation. The closed-walk identity has the form $\text{Tr } L_U^3 = C_{\text{graph}} - 6\sum_p \text{Re Tr } U_p$, the minus sign arising because the off-diagonal entries of L_U are $-U$. For the once-counted positive loop cost $\Phi_{\text{loop}} = \sum_p [\dim(V) - \text{Re Tr } U_p]$ this is $\text{Tr } L_U^3 = C'_{\text{graph}} + 6\Phi_{\text{loop}}$. Hence an action term $\alpha_{\text{loop}}\Phi_{\text{loop}}$ is represented in the SGW spectral normalisation by $\kappa = \alpha_{\text{loop}}/6$.

E.4 Common component. Since $R1 = 6Z$, $(\alpha_{\text{loop}}/6)R1 = \alpha_{\text{loop}}Z$. The common price component is parallel to the primitive gauge vector and SGRI-null. If $\alpha_{\text{loop}} = 1$ and the loop is the same PCN physical work term, it reconstructs the bare gauge vector and may not be added again as an independent Stage-A correction.

E.5 Bath-Schur frontier. HMGBG reduces the unresolved physical pricing to $W_{\text{bath}} = W_{\text{core}}(0) - C^+(\Omega_Q^2)^*C$. The SGW and MCFR release is the representation-valued projection of this operator after the full typed Defect-Lift, quotient and terminal-record projection. That execution remains open.

E.6 Selected-current carrier typing. MCFR selects one shared representation-valued current rather than three orthogonal C/W/Y ledgers. On the minimal 45-Weyl branch its normalised support pullback is G_{45} with $\det G_{45} = 11/120$; any factor-label-preserving rescaling of I_3 remains diagonal, so it cannot reproduce G_{45} , and $\|G_{45} - I_3\|_F^2 = 41/12$.

E.7 D41 central-price no-go. On one E2 quadrature D41 returns $M_1 = g_{\text{cyc}}^2 [dH_{\text{CWY}} - \Delta\mu\mu^T]$ with $\mu = (36, 26, 47)^T/49$. For any gauge-covariant central sector operator the factor pullback $K_{XY} = \text{Tr}(P_X W P_Y)$ satisfies $K_{CC} = K_{CY}$ and $K_{WW} = K_{WY}$ because $P_C P_Y = P_C$ and $P_W P_Y = P_W$. The D41 rank-one term fails both because $36^2 \neq 36 \cdot 47$ and $26^2 \neq 26 \cdot 47$. No diagonal sector-price ledger reproduces D41, independently of d , Δ , g_{cyc} and all gauge data.

E.8 Central-projection release rule. Derive the typed W_{bath} from one physical history parent and retain the gauge-covariant representation structure before factor compression. Sector averages followed by R give the full loop vector; for the non-common part, Section 12 gives an equivalent central-trace contraction directly on the properly reduced operator. Gauge averaging may be omitted inside these central traces, but it may not be moved ahead of Schur reduction without proof. Factor-space history matrices remain source checks rather than inverse price maps.

Appendix F. Inherited record results used by Part C

These finite results are retained because Sections 13 to 18 cite them. None supplies a sector price or a gauge coefficient; each is graded in the ledger of Section 23.

F.1 Exact finite phase response of the bond term. With $M = \Pi_j \Phi^{\text{tr}} \Pi_i = BV$ and $D_B = \Pi_j U \Pi_i - V$, take on one fixed, aligned, occupied scalar channel $U = e^{i\theta}$ and $M = be^{i\psi}$ with $b > 0$, so $V = e^{i\psi}$. The isolated bond contribution with its native positive weight is $\Gamma_B(\theta, \psi) = (w_B/2) |e^{i\theta} - e^{i\psi}|^2 = w_B[1 - \cos(\theta - \psi)]$, with $\partial \Gamma_B / \partial \psi = w_B \sin(\psi - \theta)$ and $\psi_{\min} = \theta$ modulo 2π . A supplied link phase $\theta_\sigma = \sigma \theta_0$ favours a same-sign record phase. This is a stationary alignment rule for the isolated scalar bond term, not a derived temporal update or an instruction to impose $D_B = 0$ after each event. The same scalar phase does not change $MM^\dagger = M^\dagger M = b^2$, so with all other Gram contributions fixed $G_+ = G_-$ and $\Pi_+ = \Pi_-$: this phase-only route does not encode the sign in the occupied projector. If the link returns to $\theta = 0$ the isolated bond action has its stable minimum at $\psi = 0$; persistence of the old phase requires additional source structure. This is not an impossibility theorem for the complete action.

F.2 Exact support criterion for the write operation. PJCK supplies the marked transition architecture $K(dY', de|Y) = [R(Y')/(\lambda R(Y))]T(dY', de|Y)$ with $R > 0$ and $\lambda > 0$ [19]. At a source state with fixed sign σ , let A_σ be terminal records decoding that sign, with A_+ and A_- disjoint. For any measurable set E of wrong-record transitions, multiplication by the strictly positive finite Perron factor gives $K_\sigma(E|Y) = 0 \Leftrightarrow T_\sigma(E|Y) = 0$, because a strictly positive function has a strictly positive integral against any nonzero positive measure (the Perron and Doob transforms preserve zero sets [50]). Normalisation therefore cannot create absent sign-selective support or eliminate a nonzero erasure transition. This does not require that a completed record be written in a single opportunity; a source-derived multistep stopping rule may matter, but its terminal support must be evaluated explicitly. An irreducible finite kernel with positive transitions throughout one communicating class cannot have two disjoint absorbing sign sectors; an appended ledger enlarges the state architecture and is not excluded.

F.3 Orientation sensitivity of the existing terminal instrument. The inherited finite-field source [18] defines $j(\theta) = \psi_\gamma - U(\theta)\psi_\gamma$, $p_j = jj^\dagger/(j^\dagger j)$, $K_a = i[T_a, p_j]$, $S = \sum_a K_a^2$, $\eta = \lambda_{\max}(S)^{-1}$, $C_0 = v(I - \eta S)$ and $C_a = v\eta K_a$; on its nonzero-current, regular square-root branch the retained record operation is $I_{e,r}(\theta)[q] = \sum_\alpha Q_r C_{e\alpha}(\theta) U_e(\theta) q U_e(\theta)^\dagger C_{e\alpha}(\theta)^\dagger Q_r$, and the current-dependent C must vary with the link probe. The published edge-9, terminal-6 witness at neutral preparation returns $p_r(0) = 0.0414565421$ and $p'_r(0) = -0.0824840109$ (inherited, not rerun), so conditional on a physical attachment $\theta = \sigma\epsilon$, $p_r(+\epsilon) - p_r(-\epsilon) = -0.1649680218\epsilon + o(\epsilon)$. Terminal outcomes can respond statistically to opposite link perturbations, but this is not a faithful sign-write certificate: the attachment of σ to this probe is unproved, the outcome has positive probability under both signs near zero, and the instrument is a declared finite-field family, not the complete MASD/CAR/Fock evolution.

F.4 Exact ordered history sign test. Enumerate paths $h = (0, v_1, \dots, v_6, 0)$ with 0 the hub, $(v_1, \dots, v_6)$ a permutation of the rim and successive vertices adjacent in W_7 ; assign $\omega_c = +1$ to clockwise rim steps, -1 to anticlockwise steps and 0 to spokes, and set $S(h) = \sum_{e \in h} \omega_c(e)$. Of the 720 permutations exactly twelve are admissible, six with $S = +5$ and six with $S = -5$, and reversal flips every sign [19]. With $\sigma_{\text{path}}(h) = S(h)/5$, all twelve paths share the final vertex 0 and the occupied set $\{0, \dots, 6\}$, so no deterministic function of either datum recovers σ_{path} on both classes, while an ordered directed history does. If the histories are retained as orthogonal ledger states, $P_\pm = \sum_{h: S(h) = \pm 5} |h\rangle\langle h|$ satisfy $P_+ P_- = 0$, $\Sigma_L = P_+ - P_-$ has $\Sigma_L |h\rangle = \sigma_{\text{path}}(h) |h\rangle$, opposite-sign diagonal states have trace distance one, compression to the endpoint or occupied set gives trace distance zero, and $[\Sigma_L \otimes I, I_L \otimes U_{\text{work}}] = 0$ so the spectator-ledger continuation preserves the observable once installed. A complete-path label or an equivalent circulation encoding would suffice; assuming one is not its derivation. A biased preparation law and the realised circulation are different random variables unless the source proves otherwise, since opposite finite biases can assign positive probability to the same histories [21].

F.5 Finite record write and the established polar bond response. The occupied-address term $G_i^{\text{term}} = \sum_{\omega \in \Omega} \omega P_{(\omega,i)}^{\text{rec}}$ is set-based: a repeated address adds no term under fixed weights, and a positive addition supported in the old occupied band does not change Π , so support motion is neither necessary nor

sufficient for bond mismatch [17]. A physical write $W_a: (\rho, \Phi, \Omega, U, J; h) \rightarrow (\rho', \Phi', \Omega', U', J'; h||a)$ must return every field change, and on a rank-stable patch its first-order response is $\delta D_B = \delta \Pi_j U \Pi_i + \Pi_j \delta U \Pi_i + \Pi_j U \delta \Pi_i - \delta V$ with spectral-gap projector derivatives and the ambient polar derivative $\delta V = B^+(\delta M - \delta B V) + \delta P_{out} V$; at a rank-changing event projectors and polar maps are recomputed rather than continued through an uncertified gap [17]. All induced defect cross terms stay in the action variation, $\delta \Gamma_D = \Sigma_X \langle \tau_X, \delta D_X \rangle + \frac{1}{2} \Sigma_{X,Y} \langle D_X, (\delta W_{XY}) D_Y \rangle$ with $\tau_X = \Sigma_Y W_{XY} D_Y$. GNC-EX3A inherits the nonzero bond-record mechanism from MASD §47A and RPC-1/D31 [23]: at a bond-compatible background with aligned frames, $U_* = I$, $V_* = P$, $M_* = BP$ with B positive on P , the internal polar rotation satisfies $\delta D_B = P \delta U P - \Omega_{pol}$ with $B \Omega_{pol} + \Omega_{pol} B = S - S^\dagger$, $S = P \delta M P$; a pure positive stretch with fixed polar orientation gives zero and an intrinsic relative rotation gives a response. On the inherited wheel-mode benchmark after current elimination, $H_\lambda = [[a\lambda + c, -c], [-c, b\lambda + c]]$ and $k_{red}(\lambda) = a\lambda + bc\lambda/(b\lambda + c)$, unit-weight branch $a = b = c = 1$. These quantify the declared mechanism without selecting the physical background or metric; the scalar HM4 record-phase carrier and a weak-adjoint record rotation are different source choices whose responses cannot be combined as one vacuum.

F.6 Intrinsic polar rotation and the complete writer derivative. On aligned, equal-rank occupied support write $M = BV$ with B positive, $V = P$ and $U = I$ at the reference point. For $M(t) = B \exp(t\Omega)$, Ω anti-Hermitian, holding the link fixed gives $\delta V = \Omega$ and $\delta D_B = -\Omega$: an intrinsic change with bond cost, not a gauge transformation; $U = \text{Pol}(\Phi)$ is not imposed as a kinematic identity. For the finite-field family of F.3, the directional derivative along an attached field variation is $j = \dot{\psi}_Y - \dot{U} \psi_X - U \dot{\psi}_X$, $\dot{\rho}_J = (P_\perp j j^\dagger + j j^\dagger P_\perp)/N$ with $P_\perp = I - \rho_J$, $\dot{K}_a = i[T_a, \dot{\rho}_J]$, $\dot{S} = \Sigma(\dot{K}_a K_a + K_a \dot{K}_a)$, $\dot{\eta} = -\eta^2 v^\dagger \dot{S} v$ for a simple maximal eigenvalue with eigenvector v , the constant-rank square-root derivative $C_o \dot{C}_o + \dot{C}_o C_o = -\dot{\eta} S - \eta \dot{S}$, $\dot{C}_a = v \eta \dot{K}_a + (\dot{\eta}/(2v\eta)) K_a$, and $\dot{L}_{r\alpha} = Q_r(\dot{C}_\alpha U + C_\alpha \dot{U})$; moving readout adds $\dot{Q}_r C_\alpha U$ and a positive edge weight $v h_e$ contributes $v h_e [\dot{L}_{r\alpha} + (\hbar_e/2\hbar_e) L_{r\alpha}]$. Freezing C while varying U omits a real source response. There is no independent Φ argument in this family, so $\partial_\Phi L_{r\alpha} = 0$ at fixed U , endpoints, T_a and Q_r ; this partial zero does not imply zero total response to a physical Φ change, and a declared static relaxation gives $\dot{z} = -H_{zz}^{-1} H_{z\Phi} \dot{\Phi}$ on the allowed quotient, which is not an event-time law. The archived edge-9 witness (probability 0.0414565421 into terminal 6 carrying $q = +1$, link-probe slope -0.0824840109 [18]) shows that these working terminal sectors cannot be identified with already written fact sectors, since that would give leakage at the reference field; the slope describes a future mark, not the HX1 phase.

Appendix G. Verification record

G.1 Exact arithmetic. Rational arithmetic verifies $R1 = 6Z$, $n \cdot Z = m \cdot Z = 0$, nR and mR , the sector-dimension conversion to A_I and $A_T/3$, $\text{Tr } A_I = \text{Tr } A_T = 0$, the uniqueness of m from the three invisibility conditions, and the independence of F_m and F_H . The determinant of the W/Y minor of the two-normal matrix is nonzero. The reconstruction $v = (0, (53J - T)/528, (229J - T)/528)$ is checked on all six sector basis vectors. The Smith forms $\text{diag}(1, 1, 1)$ for (u_1, u_2, u_3) and $\text{diag}(3, 3, 54)$ for R are computed over $\mathbb{Z}$.

G.2 Matrix controls for the direct contraction. Two synthetic complex Hermitian parents use a 47-dimensional retained carrier and nine auxiliary coordinates, with a fixed random seed 20261013. The first auxiliary block has rank nine and the second rank six. In both, L annihilates the auxiliary null space, K_0 is formed as a positive retained block plus $LC^+L^\dagger$, and the full parent is positive to numerical tolerance. Direct traces of K_{eff} and the contracted parent expressions differ by less than 3×10^{-14} . The rank-six case tests the pseudoinverse range condition. The common identity gives $J = T = 0$; the matter-universal slice gives $J = 0$ and $T = 264\alpha_{\text{loop}}(m - h)$. No measured gauge target is used.

G.3 History enumeration. Hub 0, rim 1 to 6, rim adjacency $b - a = \pm 1$ modulo 6; for each permutation form $h = (0, p, 0)$, retain it if every consecutive pair is an edge, and compute S from signed rim steps. Of 720 candidates, 12 are accepted; S distribution $\{+5: 6, -5: 6\}$; one final vertex; one occupied set; no ordered history shared between sign classes; reversal flips S for every accepted path. Orthogonality of ledger labels gives $P_+P_- = 0$ and unit trace distance; compression to the endpoint or occupied set gives zero; tensor-factor separation gives conservation under $I_L \otimes U_{\text{work}}$; positivity of the Perron multiplier gives the raw and normalised zero-set equivalence.

G.4 Population writer. On each internal matrix unit the canonical adjoint channel multiplies by $\Sigma_y P_{\text{even}}(y|x) = 1$. A rational seven-site diagnostic $p(y|x) = [1 + ((x + 2y) \bmod 7)]/28$ has exact unit row sums; for a correlated complex state on 7×49 dimensions the internal-marginal and A_I errors were 6.94×10^{-18} and the $A_T/3$ error 2.23×10^{-15} . The diagnostic kernel is not a physical D36 replacement.

G.5 Native action and constrained solution. Expansion of $2\Gamma = (v + \theta)^2 + \ell\theta^2 + (\theta - y)^2 + \ell y^2$ reproduces the source H4 slice; exact rational arithmetic verifies the completed-square formula, the inherited stationary slopes and both spectra of Section 14. In the native test with $x = \theta$, $\eta = (v, y)$, $A = \ell + 2$, $B = (1, -1)$, $C = \text{diag}(1, \ell + 1)$, $R_{\text{rec}} = (0, 1)$, $F_{\text{rec}} = s$, the bordered stationary system $[C, R_{\text{rec}}^\dagger; R_{\text{rec}}, 0](\eta, \lambda)^T = (-B^\dagger, s)^T$ at $x = 1$ agrees with the closed formula across $\ell = 1, 2, 4, 5$ and slopes $-2, 0, 1/6, 1/3, 1, 5/2$ to maximum absolute error 1.43×10^{-14} .

G.6 Fact preservation, writer derivative and selector. Exact Pauli multiplication returns the rank-two real constraint on (a_0, a_x, a_y, a_z) ; finite unitary checks verify the leakage formula and pure-phase preservation; a working-unitary example changed the full density while retaining the old projector, as did an intra-sector amplitude-damping channel [28]. A deterministic 3×3 test (NumPy seed 731, four Hermitian generators, simple-eigenvalue branch) gave central-difference amplitude errors 8.28×10^{-8} at step 10^{-4} and 2.07×10^{-8} at half-step, completeness residuals below 3×10^{-15} and zero old-record intertwining residual for the append map and its derivative [29]. Exact arithmetic verifies relaxed stiffness $5/2$ and zero additional explicit-constraint cost; scalar rotation of a nonnormal morphism preserved both Gram factors to below 5×10^{-16} ; for qubit effects $(I \pm 0.6\sigma_z)/4$ and $(I \pm 0.6\sigma_x)/4$ the commutator matrix had rank three in the Hermitian basis, leaving only I ; the input split satisfied completeness and export pullback to below 5×10^{-16} and its history-controlled extension preserved the old record exactly [30]. The native 84-effect scalar-commutant certificate is consumed from PSAC-1-R19, not recalculated.

G.7 History restriction and the complete native action. The verification assembles $G = 3I - S - S^T$ and the 18×18 H4 matrix of Section 18 with six coordinates in each of v, θ, y ; its smallest eigenvalue is 0.467911138. The single constraint row is $(0, 0, f_3^T)$. For each retained six-dimensional coordinate the bordered auxiliary stationarity system is solved and the full parent pulled back through the returned lift; the differences from $\Delta K_\theta = P_3/6$ and $\Delta K_v = P_3/287$ have norms 4.59×10^{-16} and 1.49×10^{-17} , the largest native reduced-action residual is 2.67×10^{-15} , constraint residuals are below 2×10^{-17} , and the uniform θ

increment is below 10^{-16} . Exact rational arithmetic gives $6 - 35/6 = 1/6$, $6/7 - 35/41 = 1/287$ and relative increment $1/245$. All six nonzero principal $\mathbb{Z}_7$ residues satisfy the conditional holonomy equality to machine precision, and a central difference with step 10^{-6} gives maximum derivative error 5.51×10^{-10} . A dense positive 15-coordinate parent with all cross couplings, four retained coordinates and three generic constraint rows agrees with the independent constrained solve to 1.24×10^{-14} ; adding a dependent row changes the reduced operator by less than 7.10×10^{-15} , which directly verifies that dependent facts add no artificial independent penalty. The deterministic control uses NumPy seed 20261013 and is recorded in [37]. The checks establish the algebra; they do not prove the same- Φ premise, enumerate the complete continuous move law, derive a physical trap-activation time or evaluate the full MASD Hessian.

Appendix H. External verification and provenance boundaries

H.1 Exact algebra. The K7 sector matrix, its kernel lattice, the normal contractions, the J/T reconstruction and the constrained-Schur algebra are reproducible directly from the displayed matrices and source assumptions. Appendix G supplies the native 18-coordinate numerical cross-check with declared unit conventions. These are internal mathematical checks, not independent empirical confirmation of VERSF.

H.2 Inherited VERSF premises. Fact Commitment, the Fold-to-K7 attachment, phase-faithfulness and trap protection, the MCFR population writer, the MASD metric and the reported 84-effect scalar commutant are sourced to the programme papers listed in the References. Their derivations are not reproduced in full here. They must remain separately auditable; the present conclusions carry their stated premise grades and are not upgraded by citation.

H.3 Open physical tests. No source-returned off-equilibrium same- Φ Jacobian, full committed-background MASD Hessian on the admitted carrier, 47-dimensional matter and Higgs action lift, numerical J/T contraction, absolute loop coefficient or final physical gauge boundary is supplied here. The calculations required to fill these gaps are stated in Sections 19 and 20.

H.4 Attached verification package. The executable file `sgw2_verify.py` accompanies this paper. It uses only the Python standard library and exact rational arithmetic, and it reproduces, with a PASS or FAIL line for each item: the trace table from Dynkin indices and the F_r vectors; $F_{\text{total}} = 6Z$ and the perturbative-weight alternative; R , its rank and the Smith forms $\text{diag}(1, 1, 1)$ and $\text{diag}(1, 1, 3)$ of the two kernel triples and $\text{diag}(3, 3, 54)$ of R ; the normals n and m , the uniqueness of m and $m \cdot Z = 0$; A_J , A_T , their zero traces and the reconstruction formula on all six sector vectors; the independence of F_m and F_H ; $\det G_{45} = 11/120$ and the mismatch $41/12$; the D41 pullback identities and their failure for $\mu\mu^T$; $k_e(s)$, both spectra and the Theorem 11 formula against an independent constrained solve; the full 18×18 H4 parent with the exact constrained Schur reductions giving $P_3/6$ and $P_3/287$ and an unchanged uniform mode; and the twelve ordered wheel histories. Its SHA-256 is 615a105c745faf86c006d7255637b3c62d8b00a4dcd5bc27dd373642e929bf1. Items that consume archived source executions (the EX2E arrays, the 84-effect commutant, the edge-9 witness, the MCFR covariance and portal values) are not reproduced by it and remain cited at their source grade.

H.5 Status of references. The programme papers listed below are source documents for the VERSF construction and for calculations inherited by SGW-2. Their appearance documents internal provenance, not independent peer review or experimental validation. Standard representation trace identities and general quantum-channel and Schur algebra are distinguished from VERSF physical assumptions and are cited to independent mathematical sources [38 to 53].

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