

SGRI-1

The Scale- and Common-Normalisation-Free Gauge Relation in VERSF

*An exact one-loop invariant of the frozen PCN Standard-Model gauge branch,
with a transverse correction theorem and its first two-loop deformation*

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General Reader Summary

The earlier gauge comparison seemed to depend on two unknown settings: the high-energy point at which the VERSF pattern should be imposed and one overall conversion between the primitive response and familiar continuum couplings. That made it look as though another scale search was unavoidable.

This paper shows that one prediction survives however those two settings are chosen. At the declared one-loop level, any two of the three gauge-force strengths fix the third. The relation is determined by the frozen VERSF gauge pattern together with the Standard Model running coefficients; it is not fitted to the measured weak angle.

Against the same post-freeze comparison used in the earlier papers, the simplest prediction is about three per cent low. Standard gauge two-loop running moves it in the correct direction and removes about two fifths of that gap. The remaining difference is more than an order of magnitude larger than the shift caused by the quoted strong-coupling uncertainty, so it is a real residual at this comparison grade rather than an input-precision artefact.

The main practical advance is that everything still missing can be recorded in one transverse number. Future source physics, finite matching, thresholds and higher-order running can therefore be tested against one common ledger instead of reopening the matching scale and overall normalisation each time.

Abstract

SGRI-1 specialises the one-loop determinant condition already established in CLSN-2 to the frozen PCN gauge branch and turns it into an explicit primitive-integer relation among the three gauge couplings. The construction removes both the unknown common boundary normalisation and the unknown matching scale from the relative gauge comparison.

Using the electromagnetic and strong couplings as the only low-energy comparator inputs, the frozen branch predicts a weak mixing angle of 0.2244 at one loop. Standard gauge-only two-loop transport raises that prediction to 0.2273, compared with 0.2312 in the frozen comparator. The remaining discrepancy is encoded in one transverse scalar, so source response, finite matching, thresholds and further running can be accumulated in a single correction ledger. The paper also gives exact reconstruction formulas and a complete quotient coordinate for that ledger. It does not claim an absolute derivation of all three gauge strengths, and it does not use the residual to select source physics.

Contribution and non-overlap boundary

Item	Status in this paper	Boundary
Frozen PCN gauge triple and one-loop beta census	Consumed	Inherited from CLSN-1 / VERSF gauge census; not refitted.

Item	Status in this paper	Boundary
Unique scale- and normalisation-free relation	EXPLICIT SPECIALISATION / EXACT	CLSN-2 Theorem 9 already gives the determinant condition; SGRI evaluates the primitive normal, observable form and frozen-branch relation.
Exact match equivalence and reconstruction	STRENGTHENED / EXACT	Gives explicit coordinates and physical side conditions for the two- nuisance one-loop problem.
Transverse quotient and correction ledger	NEW / EXACT	Shows all omitted effects enter the relative test only through one quotient coordinate.
Gauge-gauge two-loop deformation	NEW / POST-FREEZE DIAGNOSTIC	First explicit calculated deformation of the invariant.
D41 physical typing and universal factor s	Imported provenance only	Handled in CLSN-2; not re-derived or ranked here.
Complete physical 144-link response	OPEN	Successor calculation shared with the gauge-source programme.

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1 Objective, scope and data firewall

The objective is to extract the strongest exact gauge statement available from the frozen PCN branch without selecting either the unknown continuum matching scale or an unknown common boundary conversion. This is deliberately narrower than a complete gauge-coupling derivation. It is a theorem about the relative geometry of one frozen branch and its one-loop Standard-Model flow.

Novelty boundary. SGRI-1 does not attempt to derive R_{phys} , choose a D41 operational reading, derive the universal primitive-to-continuum factor s, or select a physical matching scale. Those source-typing questions are treated in CLSN-2 and the physical-response programme. SGRI-1 asks what survives after the common factor and one-loop scale direction are quotiented out exactly.

The calculation is source-first. Z and b are fixed before any measured gauge coupling is opened. All measured quantities enter only in Sections 8 and 10 as downstream diagnostics. No discrepancy is allowed to alter Z , b , n or the theorem.

2 Frozen source vectors and conventions

Use conventional hypercharge Y defined by $Q=T_3+Y$. The primitive VERSF charge convention is $q=6Y$, so the frozen CLSN-1 branch

$$(Z_3, Z_2, Z_q) = (6, 13/2, 378)$$

becomes, in the conventional- Y coupling basis,

$$Z \equiv (Z_3, Z_2, Z_Y) = (6, 13/2, 21/2), \quad Z_Y = Z_q/36.$$

Write the inverse squared couplings as

$$A(\mu) \equiv (A_3, A_2, A_Y) = (g_s^{-2}, g^{-2}, g'^{-2}).$$

The one-loop Standard-Model beta vector on the declared fixed census is

$$b \equiv (b_3, b_2, b_Y) = (-7, -19/6, 41/6).$$

Before thresholds, finite matching and factor-dependent source response are inserted, the most general continuation of the frozen branch is

$$A(\mu) = s Z + \tau b, \quad \tau \equiv \ln(\mu^*/\mu)/(8\pi^2).$$

Here s and μ^* are intentionally left unknown. The entire point of SGRI is that neither is needed to obtain one nontrivial relation among the three components of A .

3 The affine one-loop gauge plane

Varying s and μ^* moves A only in $\text{span}\{Z, b\}$. The two source directions are linearly independent because, for example,

$$\det [[6, -7], [13/2, -19/6]] = 53/2 \neq 0.$$

Affine-plane theorem. At the declared one-loop, fixed-census, no-threshold grade, the frozen PCN branch occupies a rank-two plane in the three-dimensional space of inverse squared gauge couplings. Therefore there is exactly one independent linear obstruction to exact matching by a common boundary conversion and one matching scale.

This statement already explains why the earlier three-sector minimax scan could reduce, but not generically eliminate, its residual: two adjustable downstream coordinates span a plane, leaving one transverse degree of mismatch.

4 The unique nuisance-free normal

The normal to the plane is fixed by the cross product of the two frozen directions. Exact rational arithmetic gives

$$Z \times b = (233/3, -229/2, 53/2).$$

Multiplying to primitive integers gives

$$n = (466, -687, 159), \quad \gcd(466, 687, 159) = 1.$$

The cancellation identities are exact:

$$\begin{aligned} 466 \cdot 6 - 687 \cdot (13/2) + 159 \cdot (21/2) &= 0, \\ 466 \cdot (-7) - 687 \cdot (-19/6) + 159 \cdot (41/6) &= 0. \end{aligned}$$

Because $\text{span}\{Z, b\}$ has codimension one, every linear functional that kills both nuisance directions is proportional to n . The primitive integer representative is therefore unique up to sign.

5 Main theorem: the SGRI gauge relation

Provenance note. CLSN-2 Theorem 9 already established the one-loop exact-matching condition in determinant form, $\det[Z, c, A] = 0$, with c proportional to the one-loop beta direction. The present theorem is the explicit specialisation of that condition to the frozen PCN branch. What is added here is the primitive integer normal, the observable electroweak form, the reconstruction formulas and the one-dimensional transverse quotient used below.

Theorem 1: Frozen-branch Scale- and Common-Normalisation-Free Gauge Relation. On the frozen unscreened PCN branch, with the inherited one-loop Standard-Model beta census, fixed field content, and no threshold or finite matching insertion, the gauge couplings satisfy

$$466 g_s^{-2} - 687 g^{-2} + 159 g'^{-2} = 0.$$

Proof. Substitute $A = sZ + \tau b$. Then $n \cdot A = s(n \cdot Z) + \tau(n \cdot b) = 0$ by the exact identities of Section 4. Neither s nor μ^* appears. ■

Equivalently, with $\alpha_3 = g_s^2/(4\pi)$, $\alpha_2 = g^2/(4\pi)$ and $\alpha_Y = g'^2/(4\pi)$,

$$466 \alpha_3^{-1} - 687 \alpha_2^{-1} + 159 \alpha_Y^{-1} = 0.$$

Differentiating at one loop gives $d(n \cdot A)/d \ln \mu = -(n \cdot b)/(8\pi^2) = 0$. The invariant is exact only at the declared fixed-census grade. Thresholds, finite scheme shifts and higher-loop terms are not assumed to vanish; they are treated as corrections in Section 9.

6 Exact matching equivalence and reconstruction

Theorem 2: Exact Matching Equivalence with Physical Side Conditions. Let A be any proposed inverse-coupling triple in the declared one-loop basis. Algebraically, because $\text{rank}[Z, b] = 2$, there exists a unique pair (s, τ) satisfying $A = sZ + \tau b$ if and only if $n \cdot A = 0$. A physical matching interpretation additionally requires $s > 0$ and the reconstructed τ , equivalently μ^* , to lie in the admitted scale interval.

Necessity is Theorem 1. For sufficiency, $n \cdot A = 0$ places A in the two-dimensional plane orthogonal to n , which is exactly $\text{span}\{Z, b\}$. Linear independence then gives a unique coordinate pair.

Solving from the strong and weak components gives

$$s = (14/53)A_2 - (19/159)A_3,$$

$$\tau = (12A_2 - 13A_3)/53,$$

and the hypercharge component is then fixed by

$$A_Y = (229/53)A_2 - (466/159)A_3.$$

Reconstruction control. Applying the strong and weak components of the frozen M_Z comparator to these formulas gives $s = 0.5412951$ and $\ln(\mu^*/M_Z) = 29.0266$, corresponding to $\mu^* \approx 3.68 \times 10^{14}$ GeV. This reproduces the strong-weak crossing reported in CLSN-1 near 3.7×10^{14} GeV. Because the full comparator has $n \cdot A \neq 0$, the reconstructed hypercharge component does not equal the measured one; that mismatch is exactly the transverse obstruction quantified below.

No-escape corollary. If $n \cdot A \neq 0$, no alternative choice of the common boundary conversion and no alternative one-loop matching scale can make the frozen branch exact. Any repair must contain a contribution outside $\text{span}\{Z, b\}$.

7 Observable electroweak form and prediction protocol

Using $e=g \sin\theta_W=g' \cos\theta_W$ and $\alpha=e^2/(4\pi)$,

$$A_2 = \sin^2\theta_W/(4\pi\alpha), \quad A_Y = \cos^2\theta_W/(4\pi\alpha), \quad A_3 = 1/(4\pi\alpha_s).$$

Substitution into Theorem 1 gives the observable SGRI relation

$$846 \sin^2\theta_W - 159 = 466 \alpha/\alpha_s,$$

or

$$\sin^2\theta_W = [159 + 466(\alpha/\alpha_s)]/846.$$

This is a two-input prediction, not a zero-input derivation of all gauge strengths. Any two independent gauge quantities determine the third at this grade. In particular, α and α_s determine $\sin^2\theta_W$ without a matching-scale choice and without a common-normalisation choice. Conversely, α and $\sin^2\theta_W$ determine α_s , and α_s and $\sin^2\theta_W$ determine α .

The relation should therefore be read as the strongest relative prediction of the frozen branch, not as a claim that the absolute gauge couplings or the physical high-energy scale have already been derived.

8 Post-freeze comparator

Only after the theorem is frozen do we reopen the same comparator values already used in the CLSN chain: $\alpha_s(M_Z)=0.1180$, $1/\alpha(M_Z)=127.95$ and $\sin^2\theta_W(M_Z)=0.23122$. The weak angle is a comparator only; the SGRI prediction uses α and α_s .

Quantity	Value
$\alpha_s(M_Z)$	0.1180
$1/\alpha(M_Z)$	127.95
$\sin^2\theta_W$ comparator	0.2312200000
SGRI one-loop prediction from α and α_s	0.2244264903
Absolute weak-angle residual	0.0067935097
Relative residual	2.938%

In inverse-coupling coordinates the same comparator point is

$$A(M_Z) = (0.6743853521, 2.3542675851, 7.8276698992),$$

$$I_Z \equiv n \cdot A = 466A_3 - 687A_2 + 159A_Y = -58.51874294.$$

The nonzero value does not modify the theorem. It states that the physical comparator point is off the frozen one-loop plane. Because the two nuisance directions have already been removed, the discrepancy is necessarily transverse at this grade.

9 Transverse quotient and correction theorem

Restore every effect excluded from the reference theorem and write

$$A_{\text{phys}}(\mu) = sZ + \tau b + \Delta(\mu).$$

Theorem 3: Transverse Quotient Theorem. Two corrections Δ and Δ' are indistinguishable to SGRI if $\Delta' - \Delta$ lies in $\text{span}\{Z, b\}$. The quotient space $\mathbb{R}^3/\text{span}\{Z, b\}$ is one-dimensional, and $n \cdot \Delta$ is a complete coordinate on it. Consequently

$$I_{\text{phys}} \equiv n \cdot A_{\text{phys}} = n \cdot \Delta = 466\Delta_3 - 687\Delta_2 + 159\Delta_Y.$$

Thus common boundary shifts and one-loop scale shifts vanish automatically. Factor-dependent source response, finite scheme conversion, threshold matching and higher-loop transport contribute only through their component along the unique transverse direction.

Corollary: Additive correction ledger. If $\Delta = \Sigma_k \Delta^k(k)$, then $I_{\text{phys}} = \Sigma_k n \cdot \Delta^k(k)$. Independently calculated corrections may therefore be accumulated as a one-number budget without reconstructing the nuisance coordinates after every step.

For geometric diagnostics one may use the Euclidean projector onto the transverse line,

$$P_{\perp} = nn^T / (n \cdot n), \quad n \cdot n = 714406.$$

The minimum-Euclidean-norm representative of a required transverse correction is

$$\Delta_{\perp} = [I / (714406)] n.$$

For the one-loop comparator, $||\Delta_{\perp}|| = |I_Z| / ||n|| = 0.0692345$. This Euclidean representative is a geometric diagnostic only; the physical correction need not be parallel to n . What is physical and basis-fixed in the declared coupling coordinates is the scalar $n \cdot \Delta$.

10 First calculable deformation: two-loop gauge transport

The exact one-loop invariant is expected to deform beyond one loop because the beta functions are no longer affine in the inverse couplings. In the conventional-Y ordering (3,2,Y), the standard gauge-gauge two-loop matrix used in the post-freeze diagnostic is

$$B = [[-26, 9/2, 11/6], [12, 35/6, 3/2], [44/3, 9/2, 199/18]].$$

Contracting with the SGRI normal gives the exact row

$$n^T B = (-18028, -1195, 4745/3).$$

With the sign convention $d(g_i^{-2})/d \ln \mu = -b_i/(8\pi^2) - [\Sigma_j B_{ij} g_j^2]/(128\pi^4)$, the one-loop term cancels and

$$dI/d \ln \mu = [18028 g_s^2 + 1195 g^2 - (4745/3)g'^2]/(128\pi^4) + \text{Yukawa terms}.$$

The sign is important: running from the high coordinate to M_Z drives I negative, which is the direction required by the post-freeze comparator.

A two-input boundary-value calculation uses only $\alpha_s(M_Z) = 0.1180$ and $1/\alpha(M_Z) = 127.95$ at low energy, together with the frozen high-coordinate boundary $A(\mu^*) = sZ$. The gauge-only integration returns

Quantity	Gauge-only two-loop return
s	0.52142506
μ^*	6.7319469×10^{13} GeV (diagnostic coordinate)
$A_3(M_Z)$	0.67438535
$A_2(M_Z)$	2.31469092
$A_Y(M_Z)$	7.86724657
$\sin^2 \theta_W$	0.2273330516
$I(M_Z)$	-25.03688346

Relative to the one-loop prediction, the weak angle moves upward by 0.0029065612. This removes 42.784% of the original 0.0067935097 weak-angle residual without changing Z , fitting the weak angle, or inserting a factor-specific source correction.

Comparator floor. Repeating the same gauge-only two-input calculation at $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ moves the predicted weak angle only to 0.22707 at $\alpha_s = 0.1189$ and 0.22760 at $\alpha_s = 0.1171$, a shift of about ± 0.00027 . The remaining 0.00389 gap to 0.23122 is therefore about 14.6 times larger than that input-induced shift. The residual is not a quoted- α_s precision artefact at this grade.

The dominant omitted Yukawa term is too small to supply the remaining transverse budget by itself. For $c_t = (2, 3/2, 17/6)$, $n \cdot c_t = 352$. Even the deliberately conservative bound $y_t^2 \leq 1$ over the full interval gives $|\Delta I_t| < 0.772$. A precision treatment still requires the complete coupled Standard-Model RG system; the bound is used only to show that top-Yukawa transport cannot account for a remainder of order 33.

10A Stage-B boundary-reading controls

The SGRI coordinate also gives a compact downstream cross-check of the three CLSN-2 boundary readings. These calculations are Stage B only: no row is permitted to select the source response, and the clamped and eigenvalue rows are counterfactual boundary controls rather than physical Stage-A returns. Each row solves the same gauge-only two-loop boundary-value problem using only α_s and α at low energy.

Boundary reading	s	μ^* (GeV)	$\sin^2\theta_W$ predicted	$I(M_Z)$
Unscreened	0.5214	6.7×10^{13}	0.22733	-25.04
Relaxed compliance (1,1,0.9866)	0.5251	8.6×10^{13}	0.22871	-36.86
D41 clamped diagonal	0.5584	7.9×10^{13}	0.23437	-85.64
Eigenvalue triple (1,1,0.9191)	0.5443	3.1×10^{14}	0.23594	-99.19

The frozen comparator requires $I(M_Z) = -58.52$. The relaxed-compliance reading remains below the measured weak angle and removes about 63% of the original one-loop weak-angle gap. The clamped diagonal reading crosses to the other side of the comparator, and the eigenvalue control overshoots further. Thus the two operational readings bracket the comparator in SGRI's own coordinate, but none closes the budget and none is selected. This is the same physical message as the CLSN-2 spread comparison expressed in the one-dimensional SGRI quotient.

11 The correction budget

The transverse theorem turns the remaining gauge problem into a ledger rather than another multi-parameter scan.

Contribution	Projected I	Status
Required by frozen M_Z comparator	-58.51874294	Post-freeze target
Gauge-gauge two-loop transport	-25.03688346	Calculated diagnostic
Fraction of original weak-angle residual removed	42.784%	Diagnostic
Remaining transverse budget	-33.48185948	Open
Conservative top-Yukawa magnitude bound	< 0.772	Too small alone

The remaining -33.48185948 must be supplied by some combination of source-derived representation-dependent response, finite native-to-continuum matching, threshold corrections and the remaining coupled/higher-loop transport. These contributions are not interchangeable physically, but SGRI provides a common scalar account in which their net transverse effect must close.

For orientation only, the minimum-Euclidean-norm representative of the remaining correction is

$$\Delta_{\perp, \text{rem}} = (-0.0218399, 0.0321974, -0.00745181), \quad ||\Delta_{\perp, \text{rem}}|| = 0.0396129.$$

Again, this is not a proposed physical sector correction. It is the unique shortest vector carrying the required transverse scalar in the declared coordinates and is useful only as a scale for future source returns.

12 Relation to CLSN-1 and CLSN-2

CLSN-1 and SGRI-1 answer different mathematical questions. CLSN-1 allowed the common inverse-coupling factor and the matching scale to vary downstream and found the nearest one-loop three-sector match. SGRI-1 proves that this search necessarily leaves one irreducible direction: the minimax scan moves within $\text{span}\{Z, b\}$, while $I = n \cdot A$ measures the component normal to that plane. Different fit norms can choose different nearest points; none can change I .

CLSN-2 addresses a different layer again: the physical legitimacy of the same-source gauge response, the absolute primitive-to-continuum conversion and the operational typing of candidate screening mechanisms. It also supplied the determinant form of the one-loop two-nuisance matching condition that SGRI specialises explicitly. SGRI-1 does not repeat the source-typing derivations. Its distinctive additions

are the primitive normal, the observable relation, explicit reconstruction and the transverse quotient/ledger into which any frozen physical response can be projected.

Division of labour. CLSN-2 asks “what physical source response and absolute conversion are legitimate?” SGRI-1 asks “what exact gauge relation and correction coordinate survive even while those objects are unknown?” The former controls physical promotion; the latter removes nuisance freedom and supplies the invariant test.

This separation also prevents an attractive candidate response from being selected because it improves a gauge comparator. Source candidates such as D41 retain only the status established in CLSN-2 until the complete physical parent is differentiated and reduced. SGRI will consume the resulting frozen coefficients; it does not choose them.

13 Falsifiers and successor calculation

F1: Frozen-input violation. Altering Z after measured couplings are opened converts the theorem into a fit.

F2: Hypercharge convention mismatch. The relation uses conventional Y with $Q=T_3+Y$ and $Z_Y=Z_q/36$. Any other normalization must transform both Z and the beta functions consistently.

F3: Census change. Any additional gauge-charged state changes b and therefore changes n. A gauge singlet does not.

F4: Threshold erasure. Crossing a threshold without its beta-function change and finite matching cannot be hidden inside the one-loop invariant.

F5: Scheme-by-notation. A factor-dependent finite scheme bridge is a real Δ and contributes through $n \cdot \Delta$.

F6: Higher-loop overclaim. Exact constancy of I is a one-loop statement. Beyond one loop its deformation must be calculated.

F7: Target feedback. The measured residual and the Stage-B boundary-control table may not select a source response, a matching scale, a D41 reading or a universal factor.

F8: Absolute-coupling overclaim. Cancellation of the common s in SGRI does not mean s has been physically derived; it means this particular relative test is insensitive to it.

Successor calculation. Construct the complete same-source physical gauge parent on the faithful 144-link quotient and form $K_g=H_{aa}-H_{a\eta} H_{\eta\eta}^+ H_{\eta a}$ with every same-order matter, current, record, completion and bath auxiliary included. Pass positivity, gauge-gradient nulls, simple-factor isotropy, Ward covariance, reciprocity and regulator stability. Then freeze the resulting Z_3, Z_2, Z_q before comparison, convert to conventional Y, and project the returned correction directly onto $n=(466, -687, 159)$. The primary SGRI question is whether the physical source supplies the remaining transverse budget. Any CLSN-2 source fingerprint is a separate, downstream cross-check rather than an SGRI input.

14 Conclusion

The frozen PCN gauge branch and the inherited one-loop Standard-Model beta census contain more predictive content than a scale scan alone reveals. Together they define a rank-two plane in the three-dimensional space of inverse squared gauge couplings. Its primitive normal is fixed exactly and uniquely as

$$n = (466, -687, 159).$$

The resulting relation

$$466 g_s^{-2} - 687 g'^{-2} + 159 g'^{-2} = 0$$

eliminates both the unknown matching scale and the unknown common continuum conversion at the declared one-loop grade. In observable form,

$$846 \sin^2 \theta_W - 159 = 466 \alpha / \alpha_s.$$

The determinant existence condition has an antecedent in CLSN-2. SGRI-1 makes that condition explicit for the frozen PCN branch, fixes its primitive integer normal and observable form, proves the reconstruction/no-escape statements with their physical side conditions, and turns every remaining correction into one complete transverse coordinate.

The post-freeze M_Z comparator gives $I = -58.51874294$. Standard gauge-gauge two-loop transport moves I to -25.03688346 and raises the two-input weak-angle prediction from 0.22442649 to 0.22733305 , removing 42.784% of the original weak-angle residual. The remaining transverse budget is -33.48185948 . Future source, threshold, finite-scheme and coupled-running calculations can therefore be judged by one additive scalar before any new global fit is attempted.

Programme handoff. Do not return to searching over μ^* or an arbitrary common rescaling. The next physical calculation is the complete source-first 144-link Hessian and same-order Schur reduction. Freeze its gauge coefficients, then project its correction onto the SGRI normal. That calculation, rather than another candidate ranking exercise, decides whether the remaining transverse budget can be closed.

Appendix A: Exact arithmetic

Frozen vectors:

$$Z = (6, 13/2, 21/2), \quad b = (-7, -19/6, 41/6).$$

Cross product and primitive normal:

$$Z \times b = (233/3, -229/2, 53/2), \quad n = (466, -687, 159).$$

Orthogonality:

$$n \cdot Z = 0, \quad n \cdot b = 0, \quad n \cdot n = 714406.$$

The primitive integer normal is unique up to sign because $\text{rank}[Z, b] = 2$ and the orthogonal complement is one-dimensional.

Appendix B: Reconstruction and transverse projector

For $A = sZ + \tau b$, the source-plane coordinates reconstructed from A_3 and A_2 are

$$s = (14/53)A_2 - (19/159)A_3, \quad \tau = (12A_2 - 13A_3)/53.$$

The corresponding hypercharge prediction is

$$A_Y = (229/53)A_2 - (466/159)A_3.$$

For a general corrected point $A = sZ + \tau b + \Delta$, the transverse projection is

$$P_{\perp} \Delta = [n \cdot \Delta / (n \cdot n)] n = [I / 714406] n.$$

The quotient coordinate $I = n \cdot A$ is invariant under $A \rightarrow A + aZ + cb$ for arbitrary real a, c at this one-loop grade.

Appendix C: Two-loop comparator arithmetic

Gauge-gauge matrix in the conventional $(3, 2, Y)$ ordering:

$$B = [[-26, 9/2, 11/6], [12, 35/6, 3/2], [44/3, 9/2, 199/18]].$$

$$n^T B = (-18028, -1195, 4745/3).$$

The two-input low-energy conditions are

$$A_3(M_Z) = 1 / (4\pi\alpha_s) = 0.6743853521,$$

$$A_2(M_Z) + A_Y(M_Z) = 1/(4\pi\alpha) = 10.1819374843.$$

With $A(\mu^*) = sZ$, gauge-only integration gives

$$\begin{aligned} s &= 0.52142506, \quad \ln(\mu^*/M_Z) = 27.32755167, \quad \mu^* = 6.7319469 \times 10^{13} \text{ GeV}, \\ A &= (0.67438535, 2.31469092, 7.86724657), \\ \sin^2\theta_W &= 0.2273330516, \quad I = -25.03688346. \end{aligned}$$

For $c_t = (2, 3/2, 17/6)$, $n \cdot c_t = 352$. Taking $y_t^2 \leq 1$ across the full interval gives the conservative diagnostic bound $|\Delta I_t| < 352 \times 27.32755167 / (128\pi^4) = 0.772$. A precision result requires the complete coupled running and threshold ledger.

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