A Candidate for Quark Colour in the VERSF Fold

Keith Taylor / VERSF Theoretical Physics Programme / 25 November 2026

General reader summary

A quark feels the strong interaction that binds quarks into protons. An electron does not. Could a fold of the same underlying medium account for that difference? Picture four players choosing partners for a doubles match. The first player has three possible partners, so there are exactly three ways to form the teams. The four outcomes of a fold can be paired in the same three ways. Treating the four names as interchangeable gives each comparison the same energy cost. An earlier VERSF result uses a symmetry of the fold to single out one comparison. A theory of quark colour must explain how the three can still have equal energy.

What happens when the surroundings keep a record? Each of the three comparisons gives the same chances for the four elementary outcomes. If the surroundings record exactly which outcome occurred and we lose access to that record, the fold alone can no longer tell us which comparison we prepared. The paper gives another possible recording rule. The surroundings could register that one of these comparisons is in play while leaving their combination intact.

The proposal makes a testable prediction within the model. Prepare two interacting folds in three different ways. In each preparation, swapping the two folds leaves the pair unchanged. If the folds have the proposed colour symmetry, all three preparations must have the same energy. An energy difference would rule out that symmetry for the interaction being tested.

Abstract

The three complementary-pair contrasts of the four VERSF outcomes are the nontrivial Klein characters. The fold symmetry used for R4 distinguishes a line and a plane. We identify the additional relabelling premise that protects a degenerate triplet and derive the exact pullback from each relation link to an outcome exchange. A cyclic rim retains a connected star of exchanges but splits the triplet by 3κ at unit inertia. With zero net phase around each triangle, independent exchanges generate su(3) after trace removal and preserve the fourth ray. A nonzero net phase on a cycle extends the four-amplitude control algebra to u(4). On two triplets, diagonal relabelling permits four invariant couplings, whereas diagonal SU(3) permits two. We give a pairing test for the difference. Repeated partial atom records suppress internal coherence geometrically. An alternative coherent occupancy record preserves the triplet for all interaction times. These results specify tests of a candidate colour carrier. Its physical selection, recording rule and gauge transport remain to be derived from the VERSF action.

Keywords quark colour · fold geometry · bath transport · quantum recording · SU(3)

1 Three contrasts from four outcomes

The Primitive Selection paper [1, §28E] begins with four elementary outcomes A, B, C and D. Their six unordered pairs are the relation supports. Two distinct pairs are adjacent when they share one outcome. Each pair then has four neighbours and a unique disjoint complement. This gives the octahedral graph. The Chiral Matter Representation paper [2, §7] separately assumes a threefold confined support for the quark sector. We test whether the first construction supplies a candidate for that assumed multiplicity.

An exact map

In the outcome order A, B, C, D, let u = (1,1,1,1)/2. Write C for the matrix whose rows are the three unit vectors below. The entries act on changes in outcome weights. When we later use these vectors as coherent amplitudes, we make an additional choice of Hilbert space and relative phases.

$$c_1 = \tfrac{1}{2}(1,1,-1,-1) \quad c_2 = \tfrac{1}{2}(1,-1,1,-1) \quad c_3 = \tfrac{1}{2}(1,-1,-1,1)$$

$$CC^{\mathrm{T}} = I_3, \quad C^{\mathrm{T}}C = Q = I_4 - uu^{\mathrm{T}}.$$

The rows record AB versus CD, AC versus BD and AD versus BC. Orthogonality gives $\|Cx\|^2 = \|Qx\|^2$. Thus every redistribution of fixed total weight is captured with the same normalisation. A uniform change gives zero. The geometry singles out a three-dimensional real space before assigning any particle label.



Figure 1. Opposite nodes are complementary pairs. Keeping only the six boundary links distinguishes distances that the full relation graph treats equally.

Why the three costs agree

Let B be the six-by-four incidence matrix, with a 1 wherever an outcome belongs to a pair. Then $B^{\mathrm{T}}B = 2I_4 + 11^{\mathrm{T}}$, with 1 the all-ones column. The unit Laplacian $L = 6I_6 - BB^{\mathrm{T}}$ obeys $LBx = 4Bx$ for x orthogonal to u. The uniform relation vector has eigenvalue 0 and ker B^{T} has eigenvalue 6. The multiplicities are 1, 3 and 2. Reading these eigenvalues as stiffnesses assumes equal edge costs and common kinetic normalisation.

Which symmetry equates the three characters

Assign A, B, C, D to (0,+), (0,−), (1,+), (1,−). Then $2c_1$, $2c_2$ and $2c_3$ are the values of the Klein-group characters $\chi_b = (-1)^b$, $\chi_\omega = \omega$ and $\chi_{b\omega} = (-1)^b\omega$. Orientation reflection preserves χ_b and reverses the other two. Exchanging the two binary coordinates interchanges χ_b and χ_ω and preserves $\chi_{b\omega}$. Accordingly [1, §28H] selects $\chi_{b\omega}$ for the R4 population difference. The bit flips and coordinate exchange generate an order-eight group. It preserves the line spanned by c_3 and the plane spanned by c_1 and c_2. An invariant stiffness may give the line and plane different costs.

Equal-overlap premise. Assume that the colour stiffness is invariant under all four-atom relabellings. Adjoining a threefold cycle of the character rays to the order-eight group generates S_4, of order 24. This group acts transitively on overlapping relation pairs and protects equal costs. Tuning could also produce equality. A physical model must explain how this stiffness coexists with the R4 selector on the same atoms.

2 What survives the cyclic restriction

VERSF introduces a cyclic boundary under an additional two-neighbour locality premise [1, §28E]. To test the effect of this choice, take the six rim positions of the scalar wheel [3, §5] in the order shown in Figure 1. Its hub is coupled equally to all positions. The antipodal-odd sector, where opposite coordinates have equal magnitudes and opposite signs, is the same three-dimensional relation space as the image of the contrasts in Section 1. Its rim sum vanishes and it leaves the hub unexcited.

A test that distinguishes the two geometries

Allow rim links at cyclic distances one and two to have stiffnesses κ_1 and κ_2. This interpolation is a diagnostic choice. With unit rim inertia and spoke stiffness β, a Fourier mode k has squared frequency

$$\omega_k{}^2 = \beta + 2\kappa_1[1 - \cos(\pi k/3)] + 2\kappa_2[1 - \cos(2\pi k/3)].$$

Odd rim mode	Squared frequency	Multiplicity
k = 1, 5	$\beta + \kappa_1 + 3\kappa_2$	2
k = 3	$\beta + 4\kappa_1$	1

The signed splitting is $3(\kappa_1 - \kappa_2)$. Equal treatment of overlapping pairs forces $\kappa_1 = \kappa_2$ and protects the common value against a change of overall stiffness. On the bare cyclic rim $\kappa_2 = 0$, so a uniform spoke correction cannot close the split. For $\beta = \kappa_1 = 1$ the squared frequencies are 2, 2 and 5. This difference comes entirely from which comparisons the dynamics retains.

Each boundary link is an outcome exchange

Across relations ij and ik the difference of incidence coordinates is $(Bx)_{ij} - (Bx)_{ik} = x_j - x_k$. Its squared magnitude is the quadratic form of $H_{jk} = (e_j - e_k)(e_j - e_k)\dagger$. On the full octahedron each exchange occurs twice. Since B multiplies lengths in the contrast space by $\sqrt{2}$, the isometric lift is $T = BQ/\sqrt{2}$. Thus $B^TLB = 8Q$, whereas $T\dagger LT = \Sigma H_{ij} = 4Q$, with the sum over the six unordered pairs.

Around Figure 1 the relation labels are AB, AC, AD, CD, BD, BC. The six successive differences measure BC, CD, AC, BC, CD, AC. The shared label is never C. After isometric pullback the rim stiffness is the star $H_{AC} + H_{BC} + H_{CD}$. Its eigenvalues on the contrasts are 1, 1 and 4. The stiff ray has amplitudes $(1,1,-3,1)/\sqrt{12}$. The doublet redistributes the three leaves at fixed sum. Any antipodal ordering has the same form. The three opposite pairs can be ordered and oriented in $3! \times 2^3$ ways. Up to rotation and reversal there are four rims. Relabelling Figure 1 produces four distinct centres and exhausts them.

In the AB−CD, AC−BD, AD−BC convention, the star has strength differences (−1,+1,−1). It therefore already supplies the connectivity of §4 and the three imbalances of §5. A rim with a single stiffness supplies one fixed sum. Full control requires separately adjustable terms. On a unit-inertia bare rim of strength κ the star gives the 3κ splitting above.

The cyclic doublet is the k = ±1 rim pair. Reference [1, §28AZ, Corollary 6.85, p. 90] identifies the fold-access and completed R4 projectors on that same Fourier pair, up to exchange of the two labels. Counting these modes again gives no independent colour degrees of freedom. A physical embedding must explain the two uses. Arbitrary SU(3) rotations on a single triplet cannot preserve a fixed R4 plane. The normal to this rim plane singles out C. The product character selected in §28H defines a different ray. Section 5 identifies the controls that preserve each.

The published link and record interaction

The master-action wheel benchmark uses link and record variables x and y on a transverse cycle mode of eigenvalue λ [4, §15.5]. Its potential is $\frac{1}{2}[a\lambda x^2 + b\lambda y^2 + c(x - y)^2]$. At a = b = c = 1 and identity inertia its in-phase and out-of-phase squared frequencies are λ and λ + 2. The λ = 2 doublet gives 2 and 4. The λ = 5 singlet gives 5 and 7. The two sets stay separated on each corresponding branch.

There is a stronger test. For fixed positive a and b the two-by-two stiffness block increases by 3 diag(a,b) from λ = 2 to λ = 5. Both ordered eigenvalues increase by at least 3 min(a,b) for any common bond strength $c \geq 0$. A crossing between differently ordered branches can occur as c varies, but cannot establish equivalence of the full responses. The source benchmark acts on link cycles. Matching Fourier labels lets us test this mechanism without identifying those cycles with microscopic quark amplitudes.

Coherent driving changes the test

On the unit wheel, let each opposite pair of spokes have an independent stiffness perturbation. Restricted to the three odd coordinates these are the three diagonal projectors. Under first-order complex evolution they and the wheel stiffness generate eight trace-free directions. Under the actual second-order oscillator equations their position–momentum generators instead have dimension 21, the symplectic algebra in six dimensions. The choice of evolution law is therefore essential when interpreting an eight-generator result.

3 What the bath can record

Take four complex amplitudes. Write $P = uu^\dagger$ for the uniform ray and $Q = I_4 - P$ for its orthogonal complement. The atom projectors alone do not determine their relative phase frame. In a model with six nonzero pair couplings, write each edge vector as $e_i - \exp(i\varphi_{ij})e_j$ and take $\varphi_{ji} = -\varphi_{ij}$. A common null amplitude must satisfy $\psi_j = \exp(i\varphi_{ij})\psi_i$ on every edge. All four magnitudes are then equal.

$$\varphi_{AB} + \varphi_{BC} + \varphi_{CA} \equiv 0 \pmod{2\pi}$$
$$\varphi_{AB} + \varphi_{BD} + \varphi_{DA} \equiv 0 \pmod{2\pi}$$
$$\varphi_{AC} + \varphi_{CD} + \varphi_{DA} \equiv 0 \pmod{2\pi}$$

These are three independent triangle tests. Fix the phase at A and use AB, AC and AD to fix the others. The three remaining edges are consistent exactly when their respective triangle sums vanish. In that case the common ray is unique and has representative u in the transported frame. A nonzero loop phase excludes such a ray for the whole set of exchanges. A tree alone cannot test the three missing edges.

When a loop phase permits mixing with the fourth direction

Assume independent control of the rank-one edge Hamiltonians. On a spanning tree, rephase the four amplitudes so every tree edge has zero phase. The tree then generates u(3) on the contrast triplet and fixes u (§§4–5). This holds for the bare rim's exchange star at any assignment of phases to its edges. One added edge tests its own loop. The full six-edge graph tests all three independent loops.

In the tree frame, an added edge with a nonzero loop phase has a defining vector d with both Pd and Qd nonzero. Its Hamiltonian $dd^\dagger$ therefore has a nonzero block between the uniform line and the triplet. Commuting with iQ removes the diagonal blocks. Commutation with the tree algebra then spans all six real mixing directions, since U(3) rotates a nonzero triplet vector into every direction, including its independent phase quadrature. Commutators of opposite quadratures supply the missing diagonal on the uniform line after subtraction of a triplet diagonal. The real dimension is 9 + 6 + 1 = 16, giving u(4).

For a direct stiffness check, give all six edges unit strength and set only $\varphi_{BC} = \theta$. The characteristic polynomial of their sum is

$$(\lambda - 4)[\lambda(\lambda - 4)^2 - 4(1 - \cos\theta)].$$

At θ = 0 it gives 0 and a triple eigenvalue 4. At θ = 0.3 radians its roots are approximately 0.01123, 3.78268, 4 and 4.20609. The zero mode moves and the triplet splits. These are eigenvalues of a particular static sum. The dimension 16 refers to the independently driven edges.

Resolving an elementary outcome

Reference [1, §28F] specifies a complete, repeatable atom-grain instrument as the closing operation. Discarding its record gives $\Delta(\rho) = \Sigma_i P_i \rho P_i$, with $P_i = |i\rangle\langle i|$. The three pure contrasts all give the same output $I_4/4$. For every normalised input, $\mathrm{Tr}[P\Delta(\rho)] = 1/4$ because the diagonal of P is constant. This includes the uniform input u. This probability measures leakage only when preparation in Q has been established. A record of the atom followed by classical erasure retains the dephasing.

A continuous diagnostic makes the loss measurable. Let an atom i leave a normalised pointer state b_i and suppose all distinct pointer overlaps equal a real η between 0 and 1. Tracing the pointer multiplies every off-diagonal element by η. For two different contrast basis states, their post-event trace distance is η, down from 1. The probability of leakage into P for any triplet input is $(1 - \eta)/4$. Distinguishability and leakage therefore probe different aspects of the same recording event.

An interaction that preserves all three amplitudes

Introduce a two-state pointer initially in $|0\rangle$ and take $H = gQ \otimes X$, with $\hbar = 1$ and X the pointer flip. At every time t the joint unitary is $P \otimes I + Q \otimes \exp(-igtX)$. On Q the pointer follows exactly the same evolution for every internal state. The internal state, including its correlations with a reference, is therefore preserved at every t. At $t = \pi/(2g)$ the two pointer records first become orthogonal and

$$U = P \otimes I - iQ \otimes X.$$

At this time tracing out the pointer implements $P\rho P + Q\rho Q$. The timing determines whether the pointer perfectly distinguishes the two sectors. Preservation of the encoded state needs no timing adjustment. This is a decoherence-free subspace [7].

Colour survival thus calls for a second physical commitment grain, capable of recording occupancy coherently. Summing diagonal atom effects cannot give Q, which has off-diagonal terms. A spatial record of motion would additionally need a position variable. Section 6 supplies a test of recording with a conjugate reference.

Which sharp records respect every exchange

In the zero-loop-phase frame of §3, for a sharp observable R on the four-amplitude model, demand that its value be conserved by every connected exchange flow. The pulse argument of Section 5 gives every permutation. Commutation with all permutations makes all four diagonal entries equal and all twelve off-diagonal entries equal. Hence $R = \alpha P + \beta Q$. Its only nontrivial two-outcome projective measurement distinguishes P from Q, up to interchanging the labels. In particular Q equals one quarter of the sum of the six pair Hamiltonians. Within the exchange model the coherent sum can therefore couple a common pointer to occupancy. This singles out a possible observable under the stated conservation requirement. A physical commitment instrument also specifies the pointer coupling and the state change for each record, both of which require a separate derivation.

4 Can the fold generate eight rotations

The octahedron has a finite relabelling group. Continuous colour transport asks more of the dynamics. We therefore test a specific interaction built from the four outcome labels without inserting the eight standard colour matrices. Work in the flat frame of §3. For each unordered pair ij, put $d_{ij} = e_i - e_j$ and take the candidate Hamiltonian

$$H_{ij} = d_{ij}\, d_{ij}\dagger = I - S_{ij},$$

where S_{ij} exchanges outcomes i and j. A continuously variable duration generates $\exp(-itH_{ij})$. A discrete permission to relabel two outcomes does not by itself supply this Hamiltonian. The assumptions under test are coherent pair exchange and enough independent interaction settings to compose the resulting evolutions.

Lie closure on the triplet

Each H_{ij} annihilates u and preserves Q. The six restrictions span all real symmetric matrices on the real contrast space. To see independence, lift any vanishing linear combination back to four dimensions. It annihilates both Q and u. Its ij off-diagonal entry is the negative of the coefficient of H_{ij}, so every coefficient vanishes. The dimension of the symmetric matrices in three dimensions is six, completing the proof.

Their sum is the normalised octahedral pullback 4Q of §2. Removing the trace on Q leaves five independent real symmetric traceless matrices. In complex amplitude evolution these enter multiplied by i. Their commutators span the three real skew-symmetric directions. For example, the commutator of a diagonal matrix with a symmetric off-diagonal pair yields the corresponding skew matrix whenever the two diagonal entries differ. These two spaces are independent and exhaust the eight-dimensional Lie algebra su(3). Before the trace is removed the dimension is nine, giving u(3). The uniform direction stays inert under the original pair exchanges.

Three independent exchanges already suffice if their pairs connect all four outcomes. A spanning tree on four labels has only two shapes, a star and a path. Conjugating along tree edges generates every pair exchange, as the proof below shows. Every connected pair graph contains such a tree. For a disconnected graph a nonzero contrast that is constant on each component and has zero total is annihilated by every exchange, so the full mixing group is impossible. This gives a connectivity test for any proposed set of physical exchange terms.

Compatibility with recording

Each exchange commutes with Q. Consequently, any sequence of exchanges can operate during the occupancy measurement in Section 3. The bath can register an event while a superposition undergoes coherent internal mixing.

An isolated fold with exact colour symmetry must preserve Q and have scalar restriction hQ there, by irreducibility. Within the six exchange terms, equal coefficients give precisely such a restriction. Any static imbalance breaks that symmetry. A nonscalar local background would have to transform as an additional degree of freedom. The available controls instead provide candidate generators for a connection. A link map between triplets at x and y must transform as $V_{yx} \mapsto U_y\, V_{yx}\, U_x\dagger$ under local frame changes. The fold action must supply such links [5]. On one triplet S_4 alone already enforces a scalar invariant Hamiltonian. Section 6 tests the distinction on two folds.

5 Which exchanges the fold must supply

There is an operational distinction between addressing each of the six outcome pairs and addressing the three partitions AB | CD, AC | BD and AD | BC. In a partition, compare the strengths assigned to the exchange on its first pair and the exchange on its complementary pair. Their common part and their difference have different actions on the triplet. The following statements assume the same coherent lift of the outcome labels used in Section 4.

An analytic proof of the connectivity criterion

At pulse duration $\pi/2$, in units with $\hbar = 1$ and unit strength, $\exp(-i\pi H_{ij}/2) = S_{ij}$. This follows by evaluating the two eigenvalues 0 and 2 of H_{ij}. On Q a swap has determinant −1. Subtracting $(2/3)I_3$ from its Hamiltonian gives the SU(3) pulse $\exp(i\pi/3)S_{ij}$ on Q. The phase cancels in conjugation. Swaps along a connected graph generate every outcome permutation by moving transpositions along paths. Conjugation by such a permutation therefore supplies $\exp(-itH_{kl})$ for every k,l from the admitted edge flows. Section 4 then gives all eight traceless generators. Conversely, if the graph is disconnected, choose a nonzero vector constant on each component with coordinate sum zero. Every admitted flow fixes its ray, which excludes full SU(3) mixing. This proves the criterion for independently available edge pulses of this form. It makes no assertion about a generic time-independent Hamiltonian with the same nonzero entries.

Complementary exchanges can cancel the mixing

Let $D_1 = H_{AB} + H_{CD}$, $D_2 = H_{AC} + H_{BD}$ and $D_3 = H_{AD} + H_{BC}$. In the contrast basis of Section 1 a direct multiplication gives

$$D_j = 2(Q - |c_j\rangle\langle c_j|), \quad j = 1, 2, 3.$$

These operators commute. Their sum is 4Q. With the shared phase removed, their generators span a two-dimensional diagonal algebra. Any sequence of these controls changes only the three relative phase factors subject to the common-phase redundancy. The adjacency pattern of the six pairs is therefore insufficient if the dynamics ties complementary edges together.

The AB | CD imbalance $H_{AB} - H_{CD}$ couples c_2 to c_3 with matrix element 2 and annihilates c_1. The AC | BD and AD | BC differences similarly couple c_1 to c_3 and c_1 to c_2. With independent diagonal controls, two distinct imbalances connect the three modes. A diagonal splitting gives each symmetric coupling its skew partner by commutation. Commutators on adjoining edges give the missing edge, and each symmetric–skew pair gives a diagonal difference. Together these generate su(3). One imbalance leaves a mode isolated.

With no diagonal controls, any two imbalances generate a three-dimensional algebra. Their commutator gives the skew coupling on the third edge, and commuting it back returns the initial pair. All three imbalances supply three symmetric and three skew couplings together with two independent diagonal differences, giving dimension eight.

Controls that preserve the R4 character

In the amplitude encoding of §1 the R4 product character spans the c_3 line. The balanced controls preserve that line. Adding $H_{AD} - H_{BC}$ mixes c_1 and c_2 and still preserves it. After trace removal the generators span the full four-dimensional line stabiliser $s(u(2) \oplus u(1))$. It contains two diagonal differences and the symmetric and skew couplings within the plane. Retaining the common phase gives dimension five. The AB | CD imbalance sends c_3 to $2c_2$, and the AC | BD imbalance sends it to $2c_1$. Each moves the selected line.

For the cyclic Fourier plane of Figure 1 the normal is instead the C-versus-rest vector. The three exchanges among its leaves, H_{AB}, H_{AD} and H_{BD}, preserve that plane. Each individual exchange joining C to a leaf mixes the normal with the plane. Preserving a plane still allows rotations of the two readout directions inside it.

6 What this establishes about quarks

The fold supplies three independent contrasts. A physical identification with quark colour would require the common VERSF action to select a spinorial carrier with the required degeneracy, preserve its amplitudes in a bath record and transport them coherently between folds [2, 5]. The following tests separate two of those requirements. The first tests a record on one triplet. The second tests the symmetry of an interaction between two triplets. The proton raises a further question. Even if a fold carries quark colour, a theory of proton mass must also account for the energy of a bound state of quarks and gluons.

A relational test for the recording rule

As a finite benchmark, introduce a conjugate triplet reference and prepare the normalised singlet $\Omega = \Sigma_j c_j \otimes \bar{c}_j / \sqrt{3}$. Conjugation refers to the chosen coherent frame. A matched change of triplet frame U and reference frame U* leaves Ω fixed. The measurement asks whether the two systems still return to their prepared singlet after the proposed recording event. This gives the contrast test a relational form once the conjugate preparation and its readout are supplied.

For the partial atom pointer in Section 3, the reduced channel on the four-outcome system is $E_\eta(\rho) = \eta\rho + (1 - \eta)\Delta(\rho)$. The singlet return for an operation with Kraus operators K_a on this system is $\Sigma_a |\mathrm{Tr}(QK_aQ)|^2/9$, obtained by taking its overlap with Ω. For Δ each of the four atom projectors has triplet trace 3/4. Thus the complete atom record gives a return of 1/4. Ideal occupancy recording has an identity action within Q and gives unity. Linearity yields

$$F(\eta) = (1 + 3\eta)/4.$$

Triplet survival is $(3 + \eta)/4$. It counts every component left in Q. Singlet return instead tests the correlations with the reference. Any separable system–reference state has singlet overlap at most 1/3. The largest squared Schmidt coefficient of Ω is 1/3, and convex mixtures obey the same bound. One event certifies entanglement by this witness exactly when $\eta > 1/9$. At or below threshold this witness is inconclusive.

For N successive records in the same atom basis with fresh pointers and no intervening mixing, $\Delta^2 = \Delta$ gives $E_\eta^N = \eta^N \mathrm{id} + (1 - \eta^N)\Delta$. Thus $F_N = (1 + 3\eta^N)/4$. If event n has common overlap η_n, replace η^N by the product over events. With any fixed $0 \le \eta < 1$, the off-diagonal entries of the atom density matrix decay geometrically, whereas populations persist. Exact indefinite preservation under this atom-pointer model requires $\eta = 1$. The coherent occupancy record preserves the triplet even when its sector labels are perfectly distinguishable.

What an exchange pulse changes

The ¼ limiting return above assumes no intervening mixing. For an exact counterexample, place the common exchange pulse $U = \exp(-i\pi Q) = P - Q$ between two complete atom records. It is generated by $\Sigma H_{ij} = 4Q$ for time $\pi/4$ at unit strength. Every atom-basis entry of U has magnitude ½. The second record therefore makes each final atom equally likely for every first outcome. The channel on any input ρ is $\mathrm{Tr}(\rho)I_4/4$. Applied to one side of the prepared singlet, it leaves $I_4/4 \otimes I_3/3$ and gives return 1/12 exactly. The same pulse acts as −I on the triplet. Two occupancy records with this pulse retain unit singlet return.

For partial records with the same overlap η and this pulse between every adjacent pair of records, the two atom bases alternate in the frame following the pulses. Their complete dephasings compose in either order to the same completely depolarising channel. Expanding the two partial channels over m pairs gives

$$F_{2m} = (1 + 4\eta^m + 7\eta^{2m})/12.$$

For $0 < \eta < 1$ the return approaches 1/12 with repetition. The number of events needed depends on η and on the pulse protocol.

Two triplets distinguish relabelling from colour symmetry

Let V = ran Q and put two copies in a common frame, or parallel-transport one to the other. Both now transform by the same U. For the diagonal S_4 action, $V \otimes V$ decomposes as $1 \oplus 2 \oplus 3 \oplus 3'$. The first three summands are symmetric, the last antisymmetric. Each occurs once, giving four independent real invariant energies. Under diagonal SU(3), the decomposition is $6 \oplus \bar{3}$, symmetric and antisymmetric. Thus an invariant Hamiltonian has only two coefficients, equivalently aI + b Swap. All three symmetric energies must agree. This counts scalar interactions in a common frame, without additional fields carrying colour indices.

A direct discriminator is the normalised symmetric pairing $s = \Sigma_j c_j \otimes c_j / \sqrt{3}$. The real relabelling matrices fix s, so a projector onto s is allowed by S_4. In the colour frame $U = \mathrm{diag}(1, \exp(i\pi/3), \exp(-i\pi/3))$, the transformed self-pair $s' = (U \otimes U)s$ is orthogonal to s because $1 + 2\cos(2\pi/3) = 0$. Colour symmetry forbids an energy difference between the two. In the contrast basis s spans the scalar line, s′ lies in the traceless diagonal doublet and $t = (c_1 \otimes c_2 + c_2 \otimes c_1)/\sqrt{2}$ lies in the symmetric three-dimensional summand. Given S_4 invariance, equality of the energies of s, s′ and t tests the full symmetric degeneracy. A single equality tests only one splitting.

The conjugate-reference singlet Ω uses $U \otimes U^*$ and is invariant. The two-fold pairing s uses $U \otimes U$ and moves under a colour rotation. Their component arrays coincide in the chosen real basis. The transformation on the second system determines which measurement is being performed.

The fourth direction and the hypercharge test

Pati and Salam [10] placed a lepton in a fourth colour direction. The branching $4 \to 3 \oplus 1$ is shared by that construction and this one. In su(4) the adjoint splits under SU(3) as $8 \oplus 1 \oplus 3 \oplus \bar{3}$. When the three triangle phases of §3 vanish, one may choose a common frame in which every H_{ij} annihilates u. Consequently all six real generators connecting the triplet to u are absent, even after taking commutators. The ninth Hermitian direction before trace removal is the central projector Q. Modulo a scalar on the full four-space, its traceless representative is $Q - \tfrac{3}{4}I_4 = (Q - 3P)/4$. This representative acts on u. The exchange Hamiltonian Q itself annihilates u. A nonzero loop phase instead admits all six mixing directions under independent exchange control. Thus the triangle sums test whether the candidate quark triplet can mix with the proposed fourth direction. Calling that process quark–lepton conversion would require a physical lepton assignment for the fourth direction.

Normalise the traceless direction as $G = (Q - 3P)/3$. Its triplet and uniform eigenvalues are ⅓ and −1, the pattern of B − L if the uniform ray is assigned to a lepton. In the convention of [2, §§9–14], electric charge is $T_{3L} + Y$. The values of G/2 match Y for Q_L and L_L, respectively ⅙ and −½. For u_R, d_R and e_R the required Y values are ⅔, −⅓ and −1. Their offsets from G/2 are +½, −½ and −½. They have the form of a right-isospin contribution. The exchange model supplies neither that generator nor a lepton identification. The left-handed agreement is a consistency check.

What the proton mass would require

In the modified minimal subtraction scheme (MS-bar) at μ = 2 GeV, the quoted central current masses are $m_u = 2.16$ MeV and $m_d = 4.70$ MeV [11]. Their valence sum is 9.02 MeV, about 0.96% of the proton mass in units with c = 1. The entire proton has rest mass. Contributions to that mass are obtained from proton matrix elements of quark and gluon operators [6]. The valence sum alone does not measure them. A VERSF mass prediction would require a colour-singlet baryon state, its rest energy and a scale-setting prescription. The dimensionless stiffnesses here provide none of these quantities.

Mathematical context and verification

Relation to established results

Marin [8, Lemma 1] gives both generating families used here. They are neighbouring transpositions for the path and transpositions through one letter for the star. For an irreducible reflection representation, [9, Proposition 2.15] gives the full image sl(V) of the derived reflection algebra. The unitary three-dimensional realisation here takes the compact real form su(3). The VERSF-specific result is the incidence pullback, its survival as a cyclic star and the tests of recording and colour symmetry on this proposed carrier.

Reproducing the finite checks

Form B by listing the six two-element subsets of four labels. Diagonalise $6I_6 - BB^T$ and restrict the six H_{ij} using $CH_{ij}C^T$. Vectorising the trace-removed matrices gives rank 5. Vectorising their pairwise commutators gives rank 3. Iterating commutators of the anti-Hermitian generators stabilises at 8. Each statement was checked by exact rational or complex-rational matrix arithmetic. Sections 1–4 give the definitions needed to reproduce the calculation.

The reduced three-by-three forms of the traceless sums for AB | CD, AC | BD and AD | BC are diagonal. Before trace removal the three diagonals are (0,2,2), (2,0,2) and (2,2,0). The differences have a single symmetric off-diagonal pair of entries equal to 2, respectively at (2,3), (1,3) and (1,2). These matrices reproduce the control test without any choice of a fitted parameter.

For independent replication of the tree claim, use AB, BC, CD for a path and AB, AC, AD for a star. Subtract Tr(H)/3 from each restricted generator times the identity, multiply by i, and stack the real and imaginary parts of each flattened matrix into an eighteen-component real column. Adjoin commutators that increase real rank and stop when rank stabilises. Both trees give rank eight. The pulse argument in Section 5 explains why both finite computations suffice for every connected graph on four labels.

For the two-fold count, the S_4 action in the contrast basis permutes the character axes with signs. Symmetric tensors split into the scalar diagonal, the traceless diagonal plane and the three off-diagonal symmetric pairs. The antisymmetric pairs give the sign twist of the standard triplet. Equivalently, for the five conjugacy classes with sizes (1,6,3,8,6), the standard characters are (3,1,−1,0,−1). Their fourth-power average is (81+6+3+0+6)/24 = 4. The symmetric square of the SU(3) fundamental is irreducible, and its exterior square is the conjugate fundamental.

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