A History Mediator for Quark Mixing in VERSF

Stationary, Quantum and Interacting Tests

Keith Taylor
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General reader summary

Quarks come in different types, and the weak force can turn one type into another. VERSF already provides a proposed pattern for their masses and for how often these changes happen. This paper tests an extra connection between that pattern and the history stored by the surrounding system, rather like adding a coupling between musical instruments and asking how it changes the notes they produce together.

For one way of attaching this connection, the calculation brings two difficult mixing numbers most of the way towards their measured values, even after allowing for quantum motion. Before the extra interactions are included, the relative pattern of quark masses changes by less than one part in a hundred thousand. The successful attachment is one of six equally well-posed possibilities, however, and the paper does not yet explain why nature would use that one. The answer also depends on how freely the surrounding system moves.

The interaction tests show why a very light quark need not acquire a large mass simply because it is coupled to its surroundings. They also show that the relative masses can still shift: keeping those shifts small requires the interaction to be sufficiently weak. The advance is a worked example linking history, quantum motion and quark mixing, with its choices and limits exposed. It does not yet reproduce the complete measured mixing pattern or derive the three force strengths.

Abstract

On a declared algebraic-completion branch of a Hermitian history-mediator construction, nonlinear quantum matching reduces the inherited $|V_{td}|$ shortfall by 84% and the $|J|$ shortfall by 66%. The six inherited Yukawa singular values retain their relative pattern to about eight parts per million, with a 1.13% common decrease. These are leading effective-action results, conditional on the source attachment, unit source inertia and the auxiliary interpretation of the completion coordinates. The displayed attachment is the most favourable of six tested alignments for four stationary observables; the separate oscillator-control zero-point energy favours 300 degrees. The construction was not blind to the earlier favourable AX1 branch. We report the full alignment family, marginal experimental pulls and source-inertia thresholds. The same elimination supplies the full mixed 36×36 second-order response. Finite CAR, spatial and bulk contractions distinguish charging gaps from chirality-changing masses. A right-handed spurion symmetry makes the latter corrections multiplicative in a symmetry-preserving analytic expansion; it does not keep all mass ratios fixed. At the unit Higgs–bath benchmark, the top-to-up ratio has first derivative −0.7343 with respect to the contact multiplier, giving a roughly 0.14 linear budget for a ten-percent change. The physical history embedding, source metric and complete dressed decay response remain to be attached. The paper supplies a solved, testable extension of the inherited conditional flavour result, not a completed physical flavour or force-strength derivation.

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Notation

Table 1. Main symbols; construction coordinates are dimensionless.

Symbol	Meaning
z, w	Prepared source and responsive complex bath coordinate
Ω, d_f; U_f, Y_f	Inherited orientation and singular values; mass frames and Yukawa matrices
W, g, L_h; μ, ν	History weights and Laplacian; mediator and completion costs
h, $\mathbb{H}$; T_f	Three-pair history mediator, its Hermitian lift and completion matrices
ϑ, m_w, $\varepsilon_\hbar$	Relative source alignment, source inertia and quantum action parameter
V, J; $\bar{Y}_f$, V_{eff}	Stationary mixing and signed CP invariant; quantum matched coefficients
κ_H, ε_I, a_{lat}	Higgs–bath scale, contact multiplier and spatial coordinate spacing
K^{ct}, $K(\omega_E)$; $\mathcal{J}$	Instantaneous and full response kernels; interacting weak current

1 Results, provenance and status

The result is a solved history-mediated correction to an inherited conditional quark-flavour return. At rest it improves four of the five mixing comparisons tested. On its algebraic-completion quantum branch, the two largest inherited deficits are substantially reduced and the relative six-mass pattern is nearly unchanged. Interactions generated by the same action admit explicit finite and spatial tests, and a symmetry explains why chirality-changing corrections carry the inherited mass itself. The numerical comparisons and the conditions supporting them are stated together below.

Status convention. Exact identities concern the displayed finite actions. Stationary, leading quantum matched and interacting results are distinct approximations, labelled in their tables. The inherited masses are inputs to this enlargement. No measured CKM value appears in the forward action or solver, but the construction was developed with knowledge of the favourable historical AX1 branch. Lack of an empirical optimisation objective is not evidence that the attachment or inertia was derived from the primitive source. One compatible construction can advance VERSF without being unique; its physical installation must still be demonstrated.

Table 2. Main results and their conditions.

Result	Calculated advance	Condition or remaining attachment
Stationary mediator	All-pair, feedback-inclusive improvement; exact preservation of inherited singular values	$\vartheta = 0$ is the best of six tested attachments, not selected by the energy test
Quantum matched branch	84% and 66% reduction of the $\lvert V_{td}\rvert$ and $\lvert J\rvert$ deficits; about eight ppm shape change	Leading bilinear expansion; auxiliaries have no inertia; $m_w = 1$
Mixed interaction response	Full 36×36 contact plus connected source kernel	Specified bilinear coordinates and operator prescription
Mass naturalness	Chirality-changing corrections carry their own inherited mass spurion	Analytic, symmetry-preserving perturbation theory; ratios still change
Bulk stability budget	Top/up first-order ratio change $-0.7343\ \varepsilon_I$ at $\kappa_H = 1$	Finite cubic cutoff; six coherent copies; not a finite-coupling bound
Physical flavour	Recovery conditions for one compatible joint installation	History embedding, induced inertia, dressed current and recorded-decay normalisation remain

The November ledger [1] and the subsequent weak-current construction [2] are the programme context. The representation-resolved history return in [5] supplies the conditional charged tensor, six singular values and baseline mixing frame. Its parent-action and CAR/Fock routes agree at the reported source-stack grade. The finite master-action paper [13, Corollary 17.1] closes the full completion compatibility test because $T_f = c_H \otimes B_f$: its full residual is exactly $\|(I - P_{full})c_H\|$, reported as 1.149×10^{-15}. An added interaction must pass its own descent test; that baseline result is not reopened.

Source-level shared-link fermion kinetics and the normalised spatial Dirac/Weyl symbol are also available [13, Sections 5–8 and 14.3], together with positive current stiffness [7] and the conditional local time attachment [8]. They do not assign an inertia to an independently introduced history mediator. The remaining task is to install the correction in the same typed mass, kinetic, history and current construction. The unsuccessful controls in Section 7 and Appendix B test particular extra assumptions, not the cumulative VERSF derivation.

2 Inherited data and the nonlinear source

The bare input is the reconstructible branch used in [2,5], with a = 9/40, b = 81/2000, c = 243/100000 and φ = 2π/3:

$$\Omega = [0,\ a,\ c\exp(i\varphi);\ -a,\ 0,\ b;\ -c\exp(-i\varphi),\ -b,\ 0],\quad U_u^0 = \exp(-\Omega/2),\quad U_d^0 = \exp(\Omega/2). \tag{1}$$

$$Y_u^0 = U_u^0\ \mathrm{diag}(7.214053900982\times 10^{-6},\ 0.003704884912807,\ 0.9725186481825). \tag{2}$$

$$Y_d^0 = U_d^0\ \mathrm{diag}(0.00001563045011879,\ 0.0003127500251071,\ 0.01620450741891). \tag{3}$$

These dimensionless singular values retain the inherited calibration and scale convention of [5]. They are not refitted here. In all comparisons, use the fixed PDG 2025 global-fit centres [6, Eqs. (12.27)–(12.28)], including $|V_{td}| = 0.00858$; direct single-process extractions are not mixed into that reference.

Let $\theta_j = \pi j/3$ for j = 0,…,5. On each of three identical density copies take the real perturbation below, with z = x + iy and w = u + iv. The factor 1/3 combines the six-rim harmonic normalisation $1/\sqrt{3}$ with the normalised three-copy diagonal $1/\sqrt{3}$.

$$\delta\rho_j = [x\cos\theta_j + y\sin\theta_j + u\cos(2\theta_j) + v\sin(2\theta_j)]/3. \tag{4}$$

$$\Gamma_s(z,w) = 3\,\Sigma_j\,\{[(1+\delta\rho_j)^2 - 1]^2/4\} + (2/3)|z|^2 + (8/5)|w|^2. \tag{5}$$

The two positive quadratic terms complete the published unit-branch stiffnesses. Retaining the displayed quartic density potential is an explicit finite nonlinear extension. It is not asserted to equal the nonlinear reduction over all 183 complementary primitive coordinates. That distinction matters beyond the cubic jet, even though the quadratic and cubic recoveries are exact.

$$\Gamma_s = (5/3)|z|^2 + (13/5)|w|^2 + (1/2)\mathrm{Re}(w^*\,z^2) + (1/6)\mathrm{Re}(w^3) + P_4(z,w). \tag{6}$$

$$P_4(z,w) = (|z|^4 + |w|^4)/48 + |z|^2|w|^2/12 + \mathrm{Re}(z^2w^2)/24. \tag{7}$$

P_4 is the exact fourth-order part of the displayed density action. At the origin the E_1 and E_2 Hessians are $(10/3)I_2$ and $(26/5)I_2$. The mixed third-derivative components are (1,0,−1,0,1,0), so the canonical cubic coefficient is 1/2. This explicit embedding demonstrates how the three-copy normalisation can already be included in that coefficient. It does not retrospectively prove the exact coordinate convention of an archived execution, and no further $1/\sqrt{3}$ is applied to this cubic.

Proposition 1. For every fixed prepared z, $\Gamma_s(z,w)$ has a unique global minimiser in the two real w coordinates, and its Hessian is bounded below by $(11/5)I_2$.

Proof. Write $B_{j1} = \cos(2\theta_j)/3$ and $B_{j2} = \sin(2\theta_j)/3$. Then $3B^TB = I_2$. The second derivative of $[(\rho^2 - 1)^2]/4$ is $3\rho^2 - 1 \geq -1$. Thus $\partial^2\Gamma_s/\partial w^2 \geq -3B^TB + (16/5)I_2 = (11/5)I_2$. Coercivity follows from the positive quadratic term. ▫

The leading source response is $w = -(5/52)z^2$. At the declared preparation z = 1, its exact source-only minimum is −0.0925162643. The amplitude z = 1 is a preparation input, not an action-selected vacuum amplitude.

3 The history mediator and stationary solution

This finite candidate specifies its source, completion, canonical kinetics and tree weak current together. Its history coefficients are inherited from [11], its Hermitian-to-frame convention from [4, Sections 6–9], and its ordered placement from [14]. Assigning that history operator to the family-pair mediator and aligning it with the inherited Ω remain constitutive choices. A positive finite action and an equivariant map establish mathematical consistency; they do not, by themselves, establish admission by the primitive source or the independent CY3′ descent test.

3.1 The mediator, its symmetry and its declared alignment

Write $q = (1, \exp(-2\pi i/3), \exp(2\pi i/3))/\sqrt{3}$ and associate its components with the ordered pairs (12, 23, 31). Introduce h in $\mathbb{C}^3$ and its Hermitian lift $\mathbb{H}$. The source injection is normalised so that $\|\mathbb{H}(h_s)\|_F = |w|$:

$$h_s = wq/\sqrt{2}, \quad \mathbb{H}(h) = \Sigma_{(ij)}(h_{ij}E_{ij} + h_{ij}^*E_{ji}), \quad X = i\mathbb{H}(h). \tag{8}$$

Let S cyclically permute the three families and let P_{12} exchange the first two. For the stated source reflection and rotation, the map obeys

$$S\mathbb{H}(wq/\sqrt{2})S^\dagger = \mathbb{H}(e^{2\pi i/3}wq/\sqrt{2}). \tag{9}$$

$$P_{12}\mathbb{H}(wq/\sqrt{2})P_{12}^\dagger = \mathbb{H}(w^*q/\sqrt{2}). \tag{10}$$

These identities give an explicit dihedral-quotient intertwiner. The correspondence of this source reflection to P_{12}, relative to the fixed Ω, is part of the construction. The full fixed-seed action is not claimed to possess an unbroken family-permutation symmetry. Simultaneously changing all family coordinates leaves the observables unchanged; changing only the source attachment changes the model. Nor does the failure of the anti-Hermitian cyclic lift of the Section 7 control under this particular reflection identify a universal error in that lift: another representation convention can use another reflection axis.

3.2 A common action and its exact stationary solution

Use the zero-holonomy canonical quotient coefficients from [11], W = 75/434 and g = 8/93, on the declared three-pair carrier. They are fixed here before comparison with observations. The history paper does not identify this carrier with the complete physical quark occupancy map. Define

$$L_h = g(2I - S - S^\dagger), \quad A_h = WI + L_h, \quad L_h q = (8/31)q. \tag{11}$$

$$\Gamma_h = \mu[W\|h - wq/\sqrt{2}\|^2 + h^\dagger L_h h], \quad \mu > 0. \tag{12}$$

$$U_u(h) = \exp[i\mathbb{H}(h) - \Omega/2], \quad U_d(h) = \exp[i\mathbb{H}(h) + \Omega/2]. \tag{13}$$

$$\Gamma_* = \Gamma_s(z,w) + \Gamma_h + (\nu/2)\Sigma_f\|T_f - U_f(h)d_f\|^2_F, \quad \nu > 0. \tag{14}$$

The matrices Ω and d_f are the inherited inputs in Section 2; $\mu = \nu = 1$ is the benchmark. The potential admits different kinetic completions. The main quantum result in Section 5 constrains h and T_f algebraically and gives only w a kinetic term. The separate 44-coordinate oscillator control assigns canonical inertias also to $\sqrt{2}$(Re h, Im h) and the 36 real completion entries. A common stationary potential does not make these quantum theories equivalent.

Proposition 2. At fixed z and positive μ, ν, this finite potential has a unique global minimiser. The stationary completion, retention and additional source stiffness are

$$T_{f,*} = U_f(h_*)d_f, \quad h_* = r_h w_* q/\sqrt{2}, \quad r_h = W/(W + 8/31) = 75/187. \tag{15}$$

$$\Gamma_{*,\mathrm{red}} = \Gamma_s(z,w) + \mu d_h|w|^2/2, \quad d_h = W(1 - r_h) = 4200/40579. \tag{16}$$

Proof. The T_f costs are nonnegative and vanish uniquely at the displayed values. The h cost has positive Hessian A_h; solving its linear stationary equation gives the displayed retention and reduced cost. Proposition 1 then bounds the reduced source Hessian below by $(11/5 + \mu d_h)I$. Thus the source has a unique global minimum, and so do the successive completion variables. The full potential need not be globally convex in its original coordinates. Its Hessian at the minimum is positive because both completion blocks and the final Schur complement are positive. ▫

At z = 1 the reflection symmetry and uniqueness force w to be real. Its value follows from one cubic, including the mediator reaction:

$$w^3/12 + w^2/2 + (109/20 + \mu d_h)w + 1/2 = 0. \quad (17)$$

For $\mu = 1$, $w_* = -0.0907637627$, $r_h = 0.4010695187$ and $d_h = 0.1035018113$. The source-only value is -0.0925162643. Thus the positive mediator cost slightly reduces the source magnitude; the calculation has not discarded backreaction. The flat static Gaussian determinant of the T_f elimination is independent of h, as is that of the h elimination at fixed W and L_h. This exact classical statement does not remove the quantum kinetic coupling introduced by the shifted coordinates.

The resulting X_{23} is $-0.0128702551 + 0.0074306452i$, with argument 150 degrees and modulus 0.0148612905. That phase follows from the declared alignment, the negative source solution and $X = +i\mathbb{H}$. The other two pair components have the same magnitude and remain in the calculation. No $b/\sqrt{3}$ amplitude or $2\leftrightarrow3$ projection is imposed. The AX1 row of [4] is therefore not being relabelled as this result.

3.3 The canonical weak current and the numerical return

With canonical quark and Higgs kinetics and T_f as the Yukawa coefficient, the minimal covariant doublet current has $J_{raw} = Z_Q = I$. Its mass-basis matrix is $V = U_u^\dagger U_d$. Appendix A gives the common kinetic-frame formula and its projector check.

Table 3. Stationary return and fixed PDG 2025 comparison [6, Eqs. (12.27)–(12.28)]. The final column contains global-fit central values, not inputs to the calculation.

Quantity	Inherited baseline	Hermitian mediator	Reference centre
$\lvert V_{us}\rvert$	0.22306945	0.22277617	0.22501
$\lvert V_{ub}\rvert$	0.00393119	0.00376323	0.003732
$\lvert V_{cb}\rvert$	0.04028487	0.04186598	0.04183
$\lvert V_{td}\rvert$	0.00610817	0.00804205	0.00858
$\lvert J\rvert \times 10^5$	1.843506	2.914086	3.12

The $|V_{td}|$ and $|J|$ central-value gaps shrink by 78.2% and 83.9%. $|V_{ub}|$ and $|V_{cb}|$ also improve, while the $|V_{us}|$ gap grows by 15.1%. These are comparisons of correlated global-fit quantities; the corresponding marginal pulls appear with the quantum return in Section 5.3.

The sign convention is $J = \mathrm{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*)$, with rows (u,c,t), columns (d,s,b) and the W^+ current as in [6, Eqs. (12.2)–(12.3)]. It equals $c_{12}c_{23}c_{13}^2s_{12}s_{23}s_{13}\sin\delta$ in the standard PDG parametrisation. The inherited and mediator branches give negative J, respectively $-1.8435063 \times 10^{-5}$ and $-2.9140863 \times 10^{-5}$. Matching the measured positive sign requires conjugating the entire construction, including source, orientation and current attachment. This is a declared CP branch choice, not eigenvector rephasing. Tables of $|J|$ test the magnitude only; neither the sign attachment nor its source selection is derived here.

All six singular values are preserved exactly by the unitary construction. They are still inputs, not mass predictions. The numerical unitarity residual is 9.4×10^{-16}. Independent projector, passive-coordinate and simultaneous-family-permutation checks agree below 1.4×10^{-15}. These controls ensure that the reported change is a relative frame response of the declared action rather than an omitted kinetic redefinition.

4 The six alignment choices

The alignment energy test belongs to the independent 44-coordinate oscillator completion. Let H_8 be the (w,h) Hessian and J_Y the 36×8 derivative of the completion target. At ν = 1 its Hessian is

$$H_{44} = [[H_8 + J_Y^T J_Y, -J_Y^T], [-J_Y, I_{36}]]. \quad (18)$$

Replace q by exp(iϑ)q while holding Ω fixed. The six choices ϑ = 0, 60, 120, 180, 240 and 300 degrees describe the reflection attachments and their opposite source directions. Their classical minimum energy is identical, 1.66486995524018. At small $\varepsilon_\hbar$, the leading quantum splitting is determined by the full Hessian:

$$E(\vartheta) = \Gamma_{*,\min} + \varepsilon_\hbar E_1(\vartheta) + O(\varepsilon_\hbar^2), \quad E_1(\vartheta) = \mathrm{Tr}(\sqrt{H_{44}(\vartheta)})/2. \quad (19)$$

Table 4. Leading quantum energy and stationary alignment response. Energy differences are coefficients of $\varepsilon_\hbar$, relative to ϑ = 0.

ϑ	$10^5\Delta E_1$	$\lvert V_{ub}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{td}\rvert$	$J \times 10^5$
0	+0.000000	0.00376323	0.04186598	0.00804205	-2.914086
60	+6.841749	0.00205980	0.04049458	0.00758858	-1.148239
120	+8.582971	0.00292226	0.03886451	0.00584904	-0.214727
180	+3.464149	0.00474210	0.03868917	0.00422574	-0.820050
240	-3.387962	0.00584088	0.04012461	0.00479170	-2.429025
300	-5.111471	0.00547974	0.04167113	0.00680836	-3.589338

The lowest of these six leading energies is at 300 degrees, the attachment associated with P_{23}, not the primary 0-degree attachment. Thus a lowest-zero-point-energy prescription does not explain the favourable stationary choice. The expansion is controlled as $\varepsilon_\hbar$ tends to zero; Table 4 is not a controlled unit-quantum branch prediction. Nor has the source alignment been made into a dynamical coordinate: choosing the lowest-energy attachment would itself require a physical preparation or selection argument. The continuous-angle diagnostic in the archive is not asserted to be an admitted family of source states.

Among these six stationary alignments, ϑ = 0 gives the smallest absolute discrepancy for each of $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and |J|. Its X_{23} phase is exactly the 150-degree phase of the earlier favourable AX1 branch [4]. The amplitude and full three-pair generator are calculated here, but the phase coincidence must not be presented as an independently selected success. No source argument in this construction fixes P_{12} relative to Ω. The reported result is a six-member family, with stationary $|V_{td}|$ spanning 0.00422574 to 0.00804205. The prominent 0-degree return is a conditional benchmark, not a prediction averaged over or selected from that family.

For the primary attachment, varying μ from 1/2 to 2 changes $|V_{td}|$ only from 0.00806066 to 0.00800586 and |J| from 2.9245 to 2.8938 × 10^{-5}. The improvement is therefore insensitive to that particular cost ratio. By contrast, changing W by factors 1/2 and 2 gives $|V_{td}|$ = 0.00732291 and 0.00885353. This is sensitivity to the history weighting; those multipliers are not proposed physical corrections.

The canonical holonomy control uses $L_{h,k} = 2gI - g[\exp(2\pi ik/21)S + \exp(-2\pi ik/21)S^\dagger]$, for k = 0, ±1, ±2, ±3. At k = ±1 the $|V_{td}|$ values are 0.00787445 and 0.00828825, so the improvement is not confined to zero holonomy. A separate preparation sweep at |z| = 1/2 and 1 and arguments 0, 30, 60, 90, 120 and 150 degrees shows strong directional dependence; all rows are retained in the companion calculation files. At

z = i, for example, $|V_{td}| = 0.00420036$ and $|J| = 0.8062 \times 10^{-5}$. The unit real preparation is thus an explicit input. These sweeps are tests of dependence, not a search that licenses a best-fit branch.

5 Quantum matching and source inertia

5.1 The branch and its source connection

The distinction between dynamical and algebraic coordinates is supplied by the corpus. The bounded-fold paper [15, Sections 6.1, 6.3 and 9.4] proves the Schur reduction when completion coordinates have no inertia and no derivative cross-coupling, and identifies such a sector in the continuity-only benchmark. This motivates a different branch of the same finite potential (14): keep the two w coordinates dynamical, while h and T_f are constrained auxiliary coordinates. Their zero momenta and stationarity constraints are eliminated before quantising the retained variables. This is a constrained Hamiltonian construction; it is not the small-inertia limit of the quantum oscillator model, and it does not add a separate flat Gaussian path measure for the auxiliaries.

The branch uses $z = \mu = \nu = 1$, the 0-degree attachment and source inertia $m_w = 1$. The native auxiliary-sector result motivates this reading but does not identify these particular mediator coordinates with its physical embedding or derive their source metric. Section 5.4 measures this dependence without selecting an inertia from the observations.

$$h_0(w) = r_h wq/\sqrt{2}, \quad T_{f,0}(w) = U_f(h_0(w))d_f. \tag{20}$$

$$H_b = -(1/2m_w)\nabla^2_w + \Gamma_s(1,w) + d_h|w|^2/2. \tag{21}$$

Equation (14) supplies both reductions. The nonlinear source polynomial is retained in full. The resulting two-dimensional confining Hamiltonian has a ground state and a positive gap. At $m_w = 1$ the calculated gap is 2.3477915877 in the model energy unit. Auxiliary response to matter is not set to zero: Section 5.5 returns the contact interactions generated when those same stationarity constraints include the fermion bilinears.

5.2 The mass and current must be projected together

Retain the shared-link fermion kinetic construction of [13] in its normalised spatial infrared representative, and use the common left-doublet weak vertex. A local mass-frame change introduces $A_{f,\mu} = U_f^\dagger \partial_\mu U_f$. On a finite source-space kinetic stencil the corresponding off-diagonal matrix element is $K_{ij}U_f(q_i)^\dagger U_f(q_j)$. This is exactly the transformed kinetic operator, including the connection. Dropping it while retaining the fluctuating vertex $U_u^\dagger U_d$ changes the model.

For external energies below the bath gap, expand the action in the quark–Higgs bilinears and project onto the bath ground state. At first order in those bilinears the effective Yukawa matrix is the ground-state matrix element of Y itself. The kinetic term and the source-frame weak current are unchanged at this order. If L_f denotes the left singular frame of the resulting matrix, the leading matched mixing matrix is therefore

$$\bar{Y}_f = \langle 0|U_f(h_0(w))d_f|0\rangle, \quad V_{eff} = L_u^\dagger L_d. \tag{22}$$

This is neither $\langle U_u^\dagger U_d\rangle$ nor a matrix obtained by taking square roots of averaged transition probabilities. It is the first-order Wilson coefficient of the fermion action after the quantum bath has been integrated out. Its identification with the complete experimental CKM matrix would additionally require the higher-order terms below, canonical residues and the recording/decay attachment. The small-bilinear or scale-separation expansion is stated explicitly; the source has not fixed a physical hierarchy making all later terms negligible.

A finite 5×5 source-space test with both chiral fermion components verifies this basis change: the source- and mass-frame spectra agree within 1.8×10^{-13}, and the transformed weak-current matrix element within 5.2×10^{-17}. Omitting the induced connection instead changes that test operator by norm 1.29025. These are implementation checks of an exact coordinate identity, not additional empirical evidence.

5.3 The quantum matched return

The ground state is obtained in a tensor Hermite basis. Cutoffs of 16, 24 and 32 functions per coordinate and quadrature orders eight higher agree in the reported matrix entries to better than 10^{-12}. The basis test supports the printed digits; it is not a certified error bound on the physical model. Table 5 uses the same fixed reference comparison as Table 3.

Table 5. Leading matched coefficients on the algebraic-completion branch.

Quantity	Stationary	Quantum matched	Reference
$\lvert V_{us}\rvert$	0.22277617	0.22365304	0.22501
$\lvert V_{ub}\rvert$	0.00376323	0.00344212	0.003732
$\lvert V_{cb}\rvert$	0.04186598	0.04224407	0.04183
$\lvert V_{td}\rvert$	0.00804205	0.00818989	0.00858
$\lvert J\rvert \times 10^5$	2.914086	2.689944	3.12

The hierarchy retention has a structural explanation. Writing $A_f = \langle U_f \rangle$ gives $\bar{Y}_f = A_f d_f$. For ordered singular values, $\sigma_{min}(A_f)d_{fi} \le \sigma_i(\bar{Y}_f) \le \sigma_{max}(A_f)d_{fi}$. A well-conditioned common left factor therefore retains a pre-existing hierarchy; it does not add independent noise of equal absolute size to every mass entry. This inequality does not guarantee the relative up/down mixing, which must still be evaluated.

The quantum return improves $\lvert V_{us}\rvert$ and $\lvert V_{td}\rvert$ relative to the stationary calculation, while $\lvert V_{ub}\rvert$, $\lvert V_{cb}\rvert$ and $\lvert J\rvert$ move away from their reference centres. Relative to the inherited baseline, the $\lvert V_{td}\rvert$ shortfall falls by about 84% and the $\lvert J\rvert$ shortfall by about 66%. The six mass retention factors lie between 0.9886731 and 0.9886804: roughly eight parts per million of relative shape change and a 1.13% common scale decrease. That scale change is reported, not absorbed into a new Higgs calibration. V_{eff} is unitary at this order; full decay rates need not have the same normalisation.

Table 6. Marginal global-fit pulls, with no uncertainty assigned to this construction. The CP column compares magnitudes; the unconjugated J sign is opposite to the reference.

Return	$\lvert V_{us}\rvert$	$\lvert V_{ub}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{td}\rvert$	$\lvert J\rvert$
Inherited	-2.9	+2.2	-2.2	-14.5	-10.6
Stationary	-3.3	+0.3	+0.0	-3.2	-1.7
Quantum matched	-2.0	-3.4	+0.5	-2.3	-3.6

A pull is (calculated value − reference centre) divided by the quoted lower or upper one-standard-deviation error on the side of the calculated value. The reference errors are 0.00068 for $\lvert V_{us}\rvert$, +0.000090/−0.000085 for $\lvert V_{ub}\rvert$, +0.00079/−0.00069 for $\lvert V_{cb}\rvert$, +0.00019/−0.00017 for $\lvert V_{td}\rvert$, and $+0.13/-0.12 \times 10^{-5}$ for J [6]. They are marginal errors of an SM-constrained global fit, including its input uncertainties. These pulls omit the uncertainty of this construction and must not be added as independent chi-squared contributions. The reduction is substantial, but the quantum row is not uniformly better than the stationary row or consistent with all five central comparisons.

Table 7. Quantum matched return for all six relative alignments at $m_w = 1$. The signed J uses the same orientation convention throughout.

ϑ (degrees)	$\|V_{ub}\|$	$\|V_{cb}\|$	$\|V_{td}\|$	$J \times 10^5$
0	0.00344212	0.04224407	0.00818989	-2.689944
60	0.00176549	0.04087426	0.00782481	-0.920894
120	0.00272206	0.03926621	0.00615267	+0.016140
180	0.00449181	0.03910531	0.00449283	-0.587449
240	0.00553156	0.04053792	0.00490667	-2.200460
300	0.00514939	0.04206343	0.00689061	-3.362867

On the algebraic-completion branch, H_b at zero matter source is independent of ϑ: all six attachments have the same ground-state energy and gap. Their matched mixing differs, with $|V_{td}|$ spanning 0.00449283 to 0.00818989. At 300 degrees |J| is closer to its reference than at zero degrees, while $|V_{td}|$ is much worse. Thus the zero-point ranking in Section 4 belongs to a different kinetic completion, and neither energy calculation selects the primary quantum attachment.

5.4 How much does the source inertia matter?

The history metric would give m_w only after the embedding and eliminated coordinates are identified: $G_{ind} = D_ι^T G_X D_ι$, with any induced memory terms retained. The wheel coefficients W and g do not determine that embedding. Here m_w is varied alone; the source potential, attachment, inherited masses and bilinear map are fixed. This is a sensitivity calculation, not an empirical determination of m_w.

Table 8. Source-inertia sensitivity on the same quantum branch. Pulls compare |J| and $|V_{ub}|$ with the fixed reference.

m_w	$\|V_{td}\|$	$\|J\| \times 10^5$	$\|V_{ub}\|$ pull	|J| pull
0.04	0.00900935	1.784269	-18.3	-11.1
1/12	0.00865527	2.133884	-12.6	-8.2
0.25	0.00836225	2.464952	-7.2	-5.5
1	0.00818989	2.689944	-3.4	-3.6
4	0.00811280	2.802117	-1.5	-2.6
16	0.00807662	2.858127	-0.6	-2.2
256	0.00805054	2.900101	+0.1	-1.8

At $m_w = 1/12$, the $|V_{ub}|$ and |J| pulls deteriorate to −12.6 and −8.2. This value is taken only as an inertia control from the unit-weight continuity example in [15]; its full history law is not imported. At large inertia the quantum result approaches the stationary one. Unit inertia is therefore a substantive condition of the headline result, not a harmless coordinate convention.

A logarithmic scan over $0.01 \le m_w \le 256$, with refined threshold solves, finds both $|V_{td}|$ and |J| closer to their reference centres than the inherited baseline for m_w above approximately 0.04450 within the scanned interval. Requiring at least 80% reduction of the $|V_{td}|$ gap and 60% reduction of the |J| gap gives approximately $0.5419 \le m_w \le 10.1858$. Requiring all four of $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and |J| to improve in absolute

distance to their reference centres gives m_w above approximately 1.9448 on that scan; this condition excludes $|V_{us}|$ and differs from comparing asymmetric pulls. These are numerical crossing tests on the stated interval, not a proof of monotonicity or a global bound. No threshold is used to change the benchmark.

The ground-state solver contains no reference values. A separate comparison routine applies the stated criteria. At the threshold crossings, 24 and 32 Hermite functions per coordinate agree in the observables to below 10^{-12}; checks at both scan endpoints give the same stability. This numerical precision does not include uncertainty in the source metric, auxiliary interpretation or omitted interactions.

5.5 The interactions generated by the same elimination

A leading Yukawa matrix is not the whole effective action. Let t collect the 36 real completion entries, $x = \sqrt{2}(\mathrm{Re}\ h, \mathrm{Im}\ h)$, F(x) the corresponding real vector of $U_f(h)d_f$, and b the 36 real sources coupled as $b^T t$. For fermions these sources represent the Hermitian quadratures of the quark–Higgs bilinears. Algebraic elimination at fixed w gives $t = F(x) - b/\nu$; expanding the h stationarity equation gives the following exact coefficient through second order in the commuting sources. Promoting these sources to fermion bilinears requires a coincident-time product prescription; Section 6 states and tests that additional choice.

$$\Gamma_{aux,red}(w,b) = \Gamma_{red}(w) + b^T F(x_0) - b^T D_{aux}(w) b/2 + O(b^3). \tag{23}$$

$$D_{aux}(w) = \nu^{-1} I_{36} + J_F(x_0)(\mu A_R)^{-1} J_F(x_0)^T. \tag{24}$$

Here A_R is the real form of $W\,I + L$ and J_F is the full 36×6 derivative, including every family-pair component. The first term is the completion-matrix contact response; the second is the mediator contact response. Both would be lost by freezing the auxiliaries at their no-matter solutions in every order. A direct minimisation with opposite small sources reproduces the predicted second derivative to 5×10^{-8}.

Quantum fluctuations of w add a distinct time-dependent contribution. Write $O_a(w) = F_a(x_0(w))$, $\Delta_n = E_n - E_0$ and $A_{na} = \langle n|O_a|0\rangle$. All functions and source wavefunctions can be chosen real in these quadratures. The connected spectral kernel and the complete second-order coefficient on this branch are

$$C_{ab}(t_E) = \Sigma_{n>0}\, A_{na} A_{nb} \exp(-\Delta_n |t_E|). \tag{25}$$

$$K_{ab}(\omega_E) = \langle D_{aux,ab} \rangle + \Sigma_{n>0}\, 2\Delta_n A_{na} A_{nb} / (\Delta_n^2 + \omega_E^2). \tag{26}$$

After bath integration the effective action contains $-\tfrac{1}{2}\int b_a K_{ab} b_b$, with the appropriate two-time integral for its nonlocal part. This is a joint up/down response, not separately assigned sector noise. The algebraic contact and the connected source spectrum count different effects once each. Every term in the real-frequency Euclidean kernel is positive semidefinite. The 36×36 arrays, all mixed entries and their spectral weights are supplied in the calculation archive.

Table 9. Second-order kernel at zero Euclidean frequency, $m_w = \mu = \nu = 1$.

Contribution	Trace	Largest eigenvalue
Completion contact	36.00000000	1.00000000
Mediator contact	6.54061144	2.19543905
Connected source response	0.01803571	0.00907489
Total	42.55864715	3.19757763

The Frobenius norm of the mixed up/down block is 0.05729373; its mediator-contact part is 0.05717516 and its source-spectrum part 0.00021126. These are dimensionless response coefficients in the declared bilinear coordinates, not gauge couplings. At frequencies 0, 1, 2 and 4 the largest eigenvalue of the

connected source part is respectively 0.00907489, 0.00768223, 0.00526063 and 0.00232717. The retained spectral sum reproduces the directly evaluated equal-time covariance to relative error below 1.5 × 10^{-13}. The contact coefficients are not numerically tiny. Their physical effect depends on the bilinear normalisation, energy scale and fermion contractions; it cannot be declared negligible because the leading mixing table looks favourable.

6 Interactions and stability of the mass pattern

6.1 An explicit finite matter realisation

The kernel in (26) acts on fermion bilinears. Its entries cannot be added directly to Table 5. To contract them, retain one spatial momentum and one colour and helicity copy of the canonical chiral Dirac carrier, with three left and three right modes in each up/down sector. The anticommutation relations and common weak vertex follow the CAR and shared-link construction of [13, Sections 2.2–2.7]. The one-mode projection, neutral occupation $(N_u,N_d) = (3,3)$, Higgs amplitude and coincident-time product prescription are additional declared choices. This is a finite contraction test, not the full coloured spatial theory.

Let κ_H multiply the quark–Higgs bilinear in the bath Hamiltonian energy convention. It exposes the relative Higgs/bath magnitude which was absent from the leading singular-frame calculation. Source inertia one does not set $\kappa_H = 1$. Connecting κ_H to a physical Higgs amplitude also requires the cell or mode normalisation and the same-action energy conversion. The existing time attachment [8] is retained; a gauge matching scale or a geometric traversal frequency is not substituted for this bath scale.

$$H_0(p) = \Sigma_f\, c_f^\dagger\, [[pI, \kappa_H \bar{Y}_f], [\kappa_H \bar{Y}_f^\dagger, -pI]]\, c_f. \tag{27}$$

$$B^R_{fij} = c_{fLi}^\dagger c_{fRj} + c_{fRj}^\dagger c_{fLi}, \quad B^I_{fij} = i(c_{fLi}^\dagger c_{fRj} - c_{fRj}^\dagger c_{fLi}). \tag{28}$$

$$H_{ct} = -\kappa_H^2 \Sigma_{ab} K^{ct}{}_{ab}\{B_a,B_b\}/4, \quad K^{ct} = I_{36} + \langle J_F(\mu A_R)^{-1} J_F^T\rangle. \tag{29}$$

The primary prescription is the symmetric product of Hermitian CAR bilinears. A second Hamiltonian discards the one-body term generated on rewriting that product in empty-vacuum normal order. Section 6.7 distinguishes this counterterm change from equivalent normal-order notation. Their attachment to the native ordered interacting transfer remains explicit. No prescription is selected by its agreement with measured mixing. The neutral sector has dimension 400 and each charged addition/removal sector dimension 300. All states are retained. The lowest three energy levels are not automatically three quark poles: the code identifies free poles by their creation/removal residues and follows their continuation.

6.2 The exact completion contact and the charge gap

For the completion block alone, the anticommutation relations give an exact reduction, separately in each up/down sector:

$$\Sigma_{ij}\, [(B^R_{ij})^2 + (B^I_{ij})^2] = 6N - 4N_L N_R, \quad H_T = \kappa_H^2(-3N + 2N_L N_R). \tag{30}$$

Proof. For $A = c_{Li}^\dagger c_{Rj}$, $A^2 = (A^\dagger)^2 = 0$ and $AA^\dagger + A^\dagger A = n_{Li} + n_{Rj} - 2n_{Li}n_{Rj}$. Summing $2(AA^\dagger + A^\dagger A)$ over the nine pairs proves the identity. ▫

Empty-vacuum normal ordering removes the term $-3\kappa_H^2 N$, a common chemical shift. It does not remove the particle-hole symmetric half charge gap. With the other sector held fixed, define

$$\Delta^{ch}_f = [E_0(N_f+1) + E_0(N_f-1) - 2E_0(N_f)]/2. \tag{31}$$

At zero Yukawa amplitude, the contact alone gives $\Delta^{ch}_f = 3\kappa_H^2$ at $N_f = 3$. This is a finite-cell charging energy. It is not automatically a running current-quark mass or an experimental quark pole mass, and its

presence does not by itself constitute a forbidden Dirac mass. It does show why the hierarchy of averaged Yukawa singular values cannot determine the interacting addition spectrum on its own.

There is also a bound which retains the full nonlinear $Y_f(w)$, without freezing or harmonically approximating the bath. For the T-only ablation, the norm of the neutral Yukawa operator is at most $\kappa_H S$, where $S = \Sigma_{fi} d_{fi} = 0.9927636350$. In either charged sector it is at most $\kappa_H(S - d_{f,min})$. Applying these norm bounds to the ground energies gives

$$\Delta^{ch}{}_f \geq 3\kappa_H{}^2 - \kappa_H(2S - d_{f,min}) \quad [\text{T-only ablation}]. \tag{32}$$

At $\kappa_H = 1$ this lower bound is 1.01447994 in the up sector and 1.01448836 in the down sector. The bound applies to this exact completion-contact ablation, with its specified finite matter carrier. It is not a bound after adding the mediator contact or replacing the carrier by the full spatial theory.

6.3 The controlled current correction

As κ_H tends to zero, fermion energies are small relative to every nonzero bath gap. Eliminating virtual bath excitations then gives $H_{eff} = \kappa_H H_Y - \kappa_H{}^2 \Sigma_{ab} K_{ab}(0) B_a B_b/2 + O(\kappa_H{}^3)$. This uses the full mixed kernel once, including the source spectrum. The same effective-space transformation must act on the weak current [16]. Direct bath admixture changes that current at order $\kappa_H{}^2$; the order-κ_H change comes from the eigenstate response to the displayed quadratic interaction.

$$\Delta_{fi} = \kappa_H \bar{d}_{fi} + \kappa_H{}^2 c_{fi} + O(\kappa_H{}^3), \quad \mathcal{J}_{ij} = 2\langle u_i | \Sigma_a c_{uLa}{}^\dagger c_{dLa} | d_j \rangle. \tag{33}$$

Here Δ_{fi} follows the addition/removal pole associated with family i at zero interaction, and $\mathcal{J}$ is the rest-frame current with its free spinor factor removed. At $\kappa_H = 0$ it reproduces V_{eff}. It is not an assumed unitary CKM matrix at nonzero interaction. The factor two is fixed by the free chiral spinors, not by a fitted decay normalisation.

Table 10. Coefficients c in the finite pole-gap expansion at p = 0.

Family	Completion contact	Mediator contact	Source spectrum	Total
u	1	0.0000120563	0.0000000337	1.0000120900
c	1	0.0061773913	0.0000172065	1.0061945978
t	1	0.0233382042	0.0000448410	1.0233830452
d	1	0.0000005890	0.0000000021	1.0000005910
s	1	0.0000116637	0.0000000407	1.0000117044
b	1	0.0005245975	0.0000008848	1.0005254823

The completion contact adds $c_{fi} = 1$ for every family but leaves the normalised weak current unchanged at first order in κ_H. In the free mass basis its off-diagonal action flips two occupied family chiralities; one current leg cannot connect that correction to an unperturbed pole. The other two contributions do change the current. With the symmetric prescription their leading result is

$$|\mathcal{J}_{us}| = 0.22365303647 + 0.01408646321\kappa_H + O(\kappa_H{}^2). \tag{34}$$

$$|\mathcal{J}_{us}/\mathcal{J}_{ud}|^2 = 0.05265515109 + 0.00387460907\kappa_H + O(\kappa_H{}^2). \tag{35}$$

The ratio is independent of a common weak normalisation. It is not yet a ratio of complete semileptonic decay rates. The full 3×3 derivative, pole residues and all mixed entries are returned in the archive. The coefficient of the ratio becomes approximately −0.72704 if the induced one-body term is discarded after

normal ordering. This is a change of Hamiltonian. Keeping that term restores exact equivalence, as Section 6.7 demonstrates; an ordering convention alone cannot change the answer.

For this regulator and gap definition, keeping the light up-sector charge gap within 1% of its free value requires approximately $\kappa_H < 7.13 \times 10^{-8}$ at leading order. The associated change in $|\mathcal{J}_{us}|$ is then only about 10^{-9}. This is a quantitative stability test of the one-cell construction, not an observational bound on VERSF or a physical mass-calibration result. Section 6.5 shows why it cannot be exported to an extended system. The expansion is not extrapolated to $\kappa_H = 1$.

6.4 Temporal propagation and the unit benchmark

To retain the frequency dependence, introduce a finite spectral bath with levels Δ_n and ground-to-excited operators $O_n = \Sigma_a A_{na} B_a$. Its Hermitian Hamiltonian is

$$H_{sp} = H_0 + H_{ct} + \Sigma_{n>0}\Delta_n|n\rangle\langle n| + \kappa_H\Sigma_{n>0}(|n\rangle\langle 0| + |0\rangle\langle n|) \otimes O_n. \quad (36)$$

This reproduces the calculated two-point kernel with causal unitary propagation. The source term $K^{source}(0)$ is not also added as a contact. At finite κ_H this is a declared completion of the quadratic response: it does not reconstruct higher connected correlators of the nonlinear bath or higher orders of h elimination. Table 11 reports the unit benchmark as a test of that completion, not as a controlled continuation of the small-amplitude expansion.

Table 11. Half charge gaps with both contact blocks at $\kappa_H = 1$ and p = 0.

Temporal treatment	Up sector	Down sector
Static contacts only	2.95459932	2.99978176
Two spectral modes	2.95383829	2.99978080
Five spectral modes	2.95383317	2.99978079
Eight spectral modes	2.95383315	2.99978079

The eight modes carry 99.9999788% of the static source trace, and a fourteen-mode check changes the up-sector gap by less than 10^{-9}. The largest reported eigensolver residual is below 4×10^{-10}. The temporal correction is small compared with the contact effect in this benchmark. At p = 0.25, retaining the same kinetic term, the corresponding eight-mode gaps are 3.29627989 and 3.24999376. Pole weight fragments at unit strength, and the near-degenerate neutral states make a chosen three-vector current basis unreliable. The complete energy-resolved current arrays are therefore retained instead of publishing a selected 3×3 matrix as physical mixing.

The CAR contact identity holds exactly in the finite matrices; free-current recovery is below 3×10^{-16}. An independent Schur-complement continuation checks the perturbative current derivative, with its largest discrepancy decreasing quadratically to below 9×10^{-7}. Inclusive current strengths satisfy the full finite-space sum rule. Energy differences and all final-state strengths are supplied, but no lepton phase space, common muon normalisation or record acceptance is invented.

6.5 Spatial enlargement of the interacting test

The normalised spatial Dirac symbol is inherited from [13, Sections 5–8 and 14.3]. To test the one-cell contraction, place the same local contact on an odd periodic chain of N_s cells, with spectral momenta $p_n = 2\pi n/(N_s a_{lat})$. One colour and helicity copy is retained here. The spectral derivative is an explicit finite discretisation of the inherited infrared symbol; its exact finite edge table is not claimed to follow from that

symbol alone. At fixed $\kappa_H = a_{lat} = 1$ introduce ε_I as a multiplier of the contact, separate from the quantum parameter $\varepsilon_\hbar$ and from the small-κ_H expansion above:

$$H_{ct}(\varepsilon_I) = -\varepsilon_I \kappa_H^2 \Sigma_x \Sigma_{ab} K_{ab}^{ct} B_a(x) B_b(x)/2, \quad K_{ct} = K_T + K_h. \tag{37}$$

The first calculation returns derivatives at $\varepsilon_I = 0$ of the zero-momentum addition/removal poles and the same canonical weak current. Every virtual Slater determinant that contributes at this order is included. It is not a self-energy-only replacement of the mixing matrix. If β_a denotes the one-body matrix underlying B_a and P_- the occupied negative-energy projector averaged over momenta, the vacuum self-energy coefficient is

$$S_{ct} = -\kappa_H^2 \Sigma_{ab} K_{ab}^{ct} \{\mathrm{Tr}(P_- \beta_b)\beta_a + \beta_a \beta_b/2 - \beta_a P_- \beta_b\}. \tag{38}$$

This follows by differentiating the Slater expectation $-\kappa_H^2 \Sigma_{ab} K_{ab}^{ct} [\mathrm{Tr}(P_- \beta_a)\mathrm{Tr}(P_- \beta_b) + \mathrm{Tr}(P_- \beta_a \beta_b) - \mathrm{Tr}(P_- \beta_a P_- \beta_b)]/2$. The current also receives the connected two-particle contribution. The source spectrum is not replaced by $K_{source}(0)$ in these spatial loops: their internal energies need not be small relative to its gaps. Table 12 therefore tests the entire instantaneous mixed contact, while the retarded source remains the separate calculation of Section 6.4.

Table 12. Spatial contact derivatives at $\varepsilon_I = 0$, $\kappa_H = a_{lat} = 1$.

Cells N_s	Up pole derivative from T	Up pole derivative from T+h	Derivative of $\lvert \mathcal{J}_{us}/\mathcal{J}_{ud} \rvert^2$
1	1.000000000	1.000012056	0.003893423
3	0.333335604	0.333342963	0.002786787
9	0.111115841	0.111122836	0.002950328
33	0.030310706	0.030317691	0.003057085
129	0.007762708	0.007769692	0.003079742

The T contribution to the current derivative vanishes: its self-energy is diagonal in the free family frames and changes no rest-frame chiral spinor, and it has no mixed up/down vertex. Numerical residuals are below 10^{-9}. The h block gives the reported nonzero current response. The one-cell contact-only coefficient differs from (35), which also includes the source spectrum. No infinite-volume limit of the full weak current is asserted from this finite sequence.

For the T block the mass coefficient has an exact first-order expression. Write $m_{fi} = \kappa_H \bar{d}_{fi}$. At zero external momentum,

$$\partial \Delta_{fi}/\partial \varepsilon_I|_0 = \kappa_H^2 F_N(m_{fi}), \quad F_N(m) = N_s^{-1} \Sigma_p \, m/\sqrt{(p^2+m^2)}. \tag{39}$$

The zero mode contributes $1/N_s$ for positive m. It is the origin of the unit one-cell coefficient. At fixed cutoff and positive m, the one-dimensional bulk limit is

$$F_\infty^{(1)}(m) = (a_{lat}\, m/\pi)\, \mathrm{asinh}[\pi/(a_{lat}\, m)]. \tag{40}$$

It tends to zero as m tends to zero, although the fractional correction grows logarithmically. Thus the one-cell constant cannot be read as an additive bulk Dirac mass. At a fixed finite box, the limit $m \to 0$ changes the free zero-mode degeneracy; it cannot be interchanged silently with the bulk limit.

A second test diagonalises the T-only spatial Hamiltonian at $\varepsilon_I = \kappa_H = 1$, without perturbation theory. Its constant family rotations preserve each family number. At three cells the neutral sector has dimension 8,000; all 37 neutral family-number sectors are searched and the balanced (3,3,3) sector is lowest in both factors. Addition and removal sectors for all three families are retained. Table 13 prints the light-family half charge gaps.

Table 13. Exact spatial T-contact ablation at unit interaction.

Cells	a_{lat}	Up half charge gap	Down half charge gap
1	1	2.82932197	2.99993086
3	1	1.26046082	1.54893516
3	0.5	0.84832083	1.04403667
3	2	2.13729538	2.36106592

The gaps decrease substantially when spatial motion is included, but remain large at this unit benchmark. Changing a_{lat} here changes hopping relative to the fixed cell interaction; it is not a continuum refinement at fixed physical four-fermion coupling. This exact ablation does not include h or the retarded source, and its charge gaps are not relabelled as current-quark masses. Removing the contact recovers all six free masses to better than 10^{-12}; the exact three-cell finite-difference derivative agrees with the first-order formula within 2×10^{-8}.

6.6 Three dimensional contact response and colour and spin

The bulk completion coefficient can also be evaluated with the inherited three-dimensional Dirac dispersion. For a cubic spectral cutoff $|p_j| \le \pi/a_{lat}$, use one canonical mode per cell: $F_{cube}(m) = (a_{lat}/2\pi)^3 \int d^3p\, m/\sqrt{(p^2+m^2)}$. This normalisation fixes the loop weight before comparison. In contrast to one dimension, $F_{cube}(m)/m$ has a finite massless limit, 0.3788010774 at $a_{lat} = 1$. An equal-volume spherical cutoff $\Lambda = (6\pi^2)^{1/3}/a_{lat}$ gives the analytic control

$$F_{sphere}(m) = [3m/(2\Lambda^3)]\, \{\Lambda\sqrt{(\Lambda^2+m^2)} - m^2\, \mathrm{asinh}(\Lambda/m)\}. \qquad (41)$$

The source bilinear is taken to be the unnormalised coherent sum over three colours and two spin states. With $n_{rep} = 6$ identical copies in the isotropic vacuum, only the trace term in S_{ct} acquires the factor n_{rep}; the exchange and one-body terms act within each copy. This follows directly from $\beta_a \to I_{nrep} \otimes \beta_a$ and $P_- \to I_{nrep} \otimes P_-$. It is checked independently in the full 72×72 one-body matrices. A normalised average over copies would define another interaction and is not inserted. The $n_{rep} = 1$ control is retained in the archive.

Table 14. Bulk fractional mass derivatives $(1/m_{fi})\partial m_{fi}/\partial \varepsilon_I$ at zero interaction, $\kappa_H = a_{lat} = 1$ and six coherent copies.

Family	T only cube	h only cube	T+h cube	T+h sphere
u	2.272806	0.568428	2.841234	2.886635
c	2.272792	0.569163	2.841955	2.887368
t	2.048924	0.058034	2.106958	2.141055
d	2.272806	0.018785	2.291591	2.328097
s	2.272806	0.018849	2.291655	2.328162
b	2.272586	0.062801	2.335387	2.372604

These are derivatives at $\varepsilon_I = 0$. Finite fractional corrections protect a large hierarchy but need not preserve its ratios. The two cutoff shapes change the coefficients by about 1.6%. The retarded spatial response and full three-dimensional weak vertex, gauge/Higgs dressing and recorded decay are not included in this bulk table. Coefficients of order two to three do not justify extrapolating the derivative to unit interaction.

For the cubic benchmark, the top-to-up ratio gives a direct measure of relative-pattern stability:

$$[m_t/m_u](\varepsilon_I)/[m_t/m_u](0) = 1 - 0.734276\,\varepsilon_I + O(\varepsilon_I^2), \quad \kappa_H = a_{lat} = 1. \tag{42}$$

A ten-percent first-order budget therefore gives $\varepsilon_I \lesssim 0.1362$, approximately 0.14. This is a local diagnostic, not a rigorous finite-interaction upper bound: higher-order terms and a change of vacuum have not been bounded. The common quantum scale reduction in Section 5 is separate from this interacting ratio change.

Table 15. Dependence of the top/up linear coefficient on the Higgs–bath scale, retaining the same bath contact and cubic cutoff.

κ_H	$\partial \ln(m_t/m_u)/\partial\varepsilon_I$	Coefficient divided by κ_H^2
0.1	-0.005703	-0.570250
0.25	-0.036604	-0.585666
0.5	-0.156501	-0.626005
1	-0.734276	-0.734276
2	-3.893698	-0.973425

The κ_H^2 interaction prefactor does not make −0.734276 a scale-independent coefficient. Internal fermion masses are themselves $\kappa_H \bar{d}_{fi}$, so the loop integrals and fractional response change with κ_H. In particular, the divided coefficient is approximately −0.5703 at $\kappa_H = 0.1$ and −0.9734 at $\kappa_H = 2$. Thus $\varepsilon_I \kappa_H^2 \lesssim 0.14$ is only a rough benchmark extrapolation near unit scale, not a general derived bound. A physical hierarchy claim must supply κ_H and remain within a controlled interaction regime.

6.7 Phase protection and equivalent operator ordering

Proposition 3 (multiplicative mass protection). In the finite algebraic-completion model, regard each inherited mass coefficient d_{fi} as a spurion with the phase opposite to its right-handed field. On a symmetry-preserving branch with a symmetry-preserving regulator, every analytic perturbative chirality-changing correction into that column carries the same spurion. It is therefore proportional to d_{fi}, with coefficients that can depend on the other masses and interactions. A consistently zero column is protected exactly; multiplicative protection alone does not preserve mass ratios or bound the fractional coefficient.

$$\Sigma_{LR,fi} = d_{fi}\, F_{fi}(|d_{fi}|^2, \text{other invariant data}) \quad \text{[analytic expansion]}. \tag{43}$$

$$d_{fi} = 0 \;\Rightarrow\; [H, Q_{Rfi}] = 0, \quad \langle T\, \psi_L(t)\, \psi_{Rfi}^{\dagger}(0)\rangle = 0 \quad \text{[phase-invariant state]}. \tag{44}$$

Proof. Transform $\psi_{Rfi} \to \exp(i\alpha)\psi_{Rfi}$, the corresponding completion column $T_{fi} \to \exp(-i\alpha)T_{fi}$, and $d_{fi} \to \exp(-i\alpha)d_{fi}$, with the other data unchanged. Both the Yukawa coupling and $\|T_f - U_f d_f\|^2$ are invariant; the source and mediator potentials, Gaussian measure and kinetic terms preserve this charge. Eliminating auxiliaries or integrating the nonlinear bath therefore preserves the spurion identity. An analytic correction into this column must have the charge of d_{fi}. Every allowed monomial contains at least one d_{fi}; additional factors involving this coefficient pair as $|d_{fi}|^2$. Factoring it out gives the stated form, which is columnwise and may mix left-family directions. At $d_{fi} = 0$ the spurion is absent and an exact right-handed phase symmetry remains, forcing the charged two-point function to vanish in an invariant state. The finite CAR zero-column and vertex checks verify this symmetry for all six columns. ▫

This is a naturalness result for the declared construction. Logarithmic mass dependence can accompany multiplicative corrections in an infrared limit, while nonanalytic strong-coupling behaviour and a

spontaneously broken vacuum require separate treatment. A finite charging gap is compatible with the symmetry; it is not itself a chirality-changing Dirac mass. The proposition does not supply an anomaly analysis of the fully gauged theory or a uniform bound on all perturbative coefficients. Table 14 exhibits the finite first-order coefficients and their unequal family dependence.

Finally, normal ordering must retain the same operator. At unit contact multiplier let $O_K = (\kappa_H^2/2)\Sigma_{ab}\, K_{ab}^{ct}\, c^\dagger\beta_a\beta_b\, c$. Then

$$H_{ct} = :H_{ct}:_0 - O_K. \tag{45}$$

The right-hand side is exactly the original symmetric Hamiltonian. Discarding $-O_K$ gives the alternative Hamiltonian used for the sensitivity row in Section 6.3; it is not an innocuous notation change. Keeping it restores identical spectra and currents, with matrix agreement below 4×10^{-18} in the finite test. The native transfer must determine the physical contact and any actual counterterm, but a change of operator notation creates no new physical freedom.

7 Control constructions

The same bath source admits well-posed constructions that move the mixing in the wrong direction. They are controls on the proposed attachment. They neither replace the inherited conditional mass return nor determine the physical history metric.

The conjugated-stiffness control uses $D_g = \mathrm{diag}(1,2,4)$, $R_0 = [I + \sigma(D_g - I)]^{-1}$, the anti-Hermitian full cyclic lift $X(w) = \Sigma(wq_aE_{ij} - w^*q_a{}^*E_{ji})/\sqrt{2}$, and $R(w) = \exp(X)R_0\exp(-X)$. Its seeds are $B_f = R_0^{-1}Y_f^0$. Minimising its positive quadratic completion cost gives

$$Y_f(w) = R(w)B_f, \quad \Gamma_{red} = \Gamma_s + (\mu/2)\mathrm{Tr}[C(I - R(w))], \quad C = \Sigma_f B_f B_f^\dagger. \tag{46}$$

With $\mu = \sigma = z = 1$, the multistart minimum is $w = -0.1687273667 + 0.0058265444i$. Its $|V_{td}|$ and $|J|$ fall by 8.97% and 14.88% from the inherited baseline, worsening their deficits. Each sector preserves the product of its three singular values because $\det[R(w)R_0^{-1}] = 1$, although individual masses change. Common-unitary cancellation, mass-product recovery and finite-difference checks are implementation controls, not extra findings. Source-angle and stiffness sweeps retain the same qualitative failure; their full numerical returns are in the companion files.

A distinct ordered-transport control sets $U_{u,d} = \exp[X(w) \mp \Omega/2]$ and penalises $\|T_f - U_f d_f\|^2$. Its completion cost vanishes at the stationary target, leaving w at its source-only minimum. Its $|V_{ub}|$ more than doubles. A common generator inside exponentials need not cancel, because $\log(U_u^\dagger U_d)$ begins with $\Omega - [X,\Omega]/2$. The successful Hermitian mediator is therefore not explained by ordered placement alone.

Table 16. Control constructions and all historical AX1 branches. J has the fixed PDG sign convention.

Construction	$\lvert V_{ub}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{td}\rvert$	$J \times 10^5$
Inherited	0.00393119	0.04028487	0.00610817	-1.843506
Conjugated stiffness	0.00367523	0.03872589	0.00556028	-1.569128
Full cyclic ordered	0.00893039	0.04211414	0.00618366	-5.217943
AX1 30°	0.00654066	0.04002690	0.00485936	-2.997358
AX1 150°	0.00355688	0.04053772	0.00868990	-2.996768
AX1 270°	0.00337194	0.04026954	0.00576338	+0.465619

The historical AX1 generator has only its 2↔3 component, amplitude $b/\sqrt{3}$ and phases 30, 150 and 270 degrees [4]. The mediator retains all three components and calculates its amplitude with feedback. Its favourable phase nevertheless coincides with AX1 and remains an attachment choice, as Section 4 states.

8 Physical attachment and recovery tests

The public source chain supplies the weighted completion $R_\Omega = (W + \Delta_\Omega)^{-1}W$, its polar orientation V_H, occupancy selectors and canonical kinetic factors [10–13]. Typed up/down compressions $S_{fL}{}^\dagger V_H{}^\dagger R_\Omega S_{fR}$ can have different mass frames while sharing one left-doublet geometry. The zero-holonomy wheel weights used here are a conditional benchmark, not a complete physical occupancy map. Substituting their eigenvalues alone cannot install the mediator in that construction.

Retaining the primitive Higgs leg $c_H \otimes B_f(w)$ preserves the published full-tensor residual pointwise and after quantum averaging. That recovery is already available. What it does not prove is the new terminal-history embedding or its induced source inertia. The physical metric must be pulled back with that embedding; eliminating dynamical coordinates can additionally generate memory and determinant terms, as Appendix B demonstrates.

The conditional clock theorem [8] gives $\Delta t_{phys}(\delta) = \delta\, a_t/c\star$ and $\lambda_F{}^{phys} = (c\star/a_t)[-\log(1 - h_F)]$ under its homogeneous opportunity and common-Higgs metric assumptions. If the model ordering coordinate is attached to that clock, a dimensionless bath gap Δ becomes $E = \hbar(c\star/a_t)\Delta$. This conversion does not determine the dimensionless source inertia, Higgs–bath amplitude or CKM ratios. Neither an oscillator frequency nor a gauge matching scale is substituted for the physical event hazard.

Table 17. Required return for one compatible physical installation.

Returned object	Where it enters	Recovery or acceptance test
Typed history embedding and source reflection	q, ϑ, W, L_h; quantum source measure	Recover the published history conductances and oriented occupancy maps; specify P_{12} relative to Ω or retain the family spread
Induced source metric and auxiliary constraints	H_b, m_w and any memory terms	Establish zero auxiliary inertia/cross-coupling; recover the metric from the same action
Higgs–bath and cell normalisation	κ_H, contact and loop weights	Same-action mass, energy and spatial conversion; stable regulator comparison
Spatial self-energy and weak vertex	Dressed masses and $\mathcal{J}$ in one kinetic frame	Recover the inherited shared-link symbol and coordinate invariance; retain the current identity
Recording, acceptance and lepton response	Complete measured weak rates and CP attachment	Recover [2] when the correction is removed; one common lepton normalisation; specified conjugate orientation
Added mixed tensor and reduced Hessian	Primitive source compatibility of the extension	Independent CY3′ descent check; preserve the closed baseline primitive-leg test

One compatible realisation with these returns is sufficient; uniqueness is not required. The physical test must use the same propagating fermion states in its mass and current. In particular, a moving mass frame introduces $U_f{}^\dagger \partial_\mu U_f$ in the kinetic term. A fluctuating vertex with that connection omitted is the control observable of Appendix B, not the dressed recorded weak response. The source-frame formulation and the leading effective-action matching retain this distinction explicitly.

9 Conclusions

A history mediator with calculated feedback can substantially improve the inherited mixing magnitudes while preserving the stationary mass singular values. Its algebraic-completion branch survives nonlinear quantum averaging, gives the two headline deficit reductions and produces a complete mixed second-order kernel. These are material constructive results beyond a favourable isolated phase insertion.

The numerical success is conditional. The primary attachment is the most favourable of six tested alignments, its phase coincides with the historical AX1 phase, its sign requires a CP-conjugate physical attachment, and the quantum result depends on source inertia. The scan quantifies that dependence rather than choosing a better-fitting inertia. The stationary and quantum returns improve different observables; neither completes the measured mixing pattern.

The interaction analysis supplies multiplicative mass protection on a symmetry-preserving branch and finite coefficients that can be tested. It also quantifies a limit on the claim that the relative pattern is unchanged: at the unit Higgs–bath benchmark, the top/up ratio has a ten-percent linear budget only for a contact multiplier of roughly 0.14 or less. Establishing the physical history embedding, metric and complete dressed decay response is the remaining calculation, with the inherited conditional VERSF results kept as recovery conditions.

Appendix A Weighted completion and canonical response

A.1 Weighted completion and feedback

Let $A_\Omega = W + \Delta_\Omega$, with $W > 0$ and $\Delta_\Omega \geq 0$. The weighted resolvent $R_\Omega = A_\Omega^{-1}W$ is similar to the positive Hermitian contraction $W^{1/2}A_\Omega^{-1}W^{1/2}$; it need not be Hermitian in the ordinary Euclidean metric.

For a differentiable source variation, direct differentiation of $A_\Omega R_\Omega = W$ gives an executable response formula even when W and Δ_Ω do not commute:

$$\delta R_\Omega = A_\Omega^{-1}[\delta W(I - R_\Omega) - \delta\Delta_\Omega R_\Omega]. \tag{47}$$

Variations of V_H, selectors and kinetic factors must accompany this response. Keeping only $-A_\Omega^{-1}\delta\Delta_\Omega R_\Omega$ silently assumes that the local history weight is fixed. At fixed seeds B_f, a finite weighted completion is

$$\Gamma_c = (\mu/2)\Sigma_f \{\mathrm{Tr}[(T_f - B_f)^\dagger W(T_f - B_f)] + \mathrm{Tr}(T_f^\dagger \Delta_\Omega T_f)\}. \tag{48}$$

$$T_{f,*} = R_\Omega B_f, \quad \Gamma_{c,\mathrm{red}} = (\mu/2)\mathrm{Tr}[C(W - WA_\Omega^{-1}W)], \quad C = \Sigma_f B_f B_f^\dagger. \tag{49}$$

Proposition 4. For fixed seeds, the source force of this weighted completion is the following envelope derivative:

$$\delta\Gamma_{c,\mathrm{red}} = (\mu/2)\mathrm{Tr}\{C[(I - R_\Omega)^\dagger \delta W(I - R_\Omega) + R_\Omega^\dagger \delta\Delta_\Omega R_\Omega]\}. \tag{50}$$

Proof. Substitute $T_{f,*}$ into Γ_c. Differentiating at fixed $T_{f,*}$ gives the displayed expression; terms containing $\delta T_{f,*}$ vanish by stationarity. Equivalently, differentiate $W - WA_\Omega^{-1}W$ and use $R_\Omega^\dagger = WA_\Omega^{-1}$. No commutativity assumption is needed. ▫

For the two complex 3 × 3 completion matrices and flat static measure, Gaussian integration gives a factor equal to the determinant of A_Ω raised to power −6. At a dimensionless temperature θ_T, the effective free energy therefore contains $+6\theta_T \log\det(A_\Omega)$, with derivative $6\theta_T\mathrm{Tr}[A_\Omega^{-1}(\delta W + \delta\Delta_\Omega)]$. This term vanishes from the source force only when the determinant is source independent, as in the conjugated-stiffness control. It is an alternative static elimination, not a second contribution to add to a full-coordinate fluctuation calculation. A physical quantum influence functional also requires the kinetic operator and its

frequency dependence. Source-dependent seeds or integration measures supply further derivatives and are not covered by the fixed-seed formula.

A.2 Mass and current in one kinetic frame

Let M_f denote the raw Yukawa coefficient, Z_Q the common left-doublet kinetic metric, Z_{fR} its sector-specific right metric and Z_H the scalar residue. Define $C_Q = Z_Q^{-1/2}$ and $C_{fR} = Z_{fR}^{-1/2}$. The physical-bath and finite-kernel papers require the mass and current to be transported together [9, Sections 2 and 7; 12; 13]:

$$Y_{f,c} = C_Q M_f C_{fR} Z_H^{-1/2}, \quad J_c = C_Q J_{raw} C_Q. \tag{51}$$

At a reference background where all kinetic metrics are identity, differentiation gives

$$\delta Y_{f,c} = \delta M_f - (\delta Z_Q Y_f + Y_f \delta Z_{fR} + \delta Z_H Y_f)/2. \tag{52}$$

$$\delta J_c = \delta J_{raw} - (\delta Z_Q J_0 + J_0 \delta Z_Q)/2. \tag{53}$$

For the minimal local gauge-covariant kinetic insertion, $J_0 = I$ and $\delta J_{raw} = \delta Z_Q$, so $\delta J_c = 0$. A nonlocal or energy-dependent vertex need not obey that simple cancellation; its residual must be calculated. If P_i^u and P_j^d are the spectral projectors of $Y_{f,c} Y_{f,c}^\dagger$, the squared weak transition coefficient is

$$p_{ij} = \mathrm{Tr}(P_i^u J_c P_j^d J_c^\dagger). \tag{54}$$

Its derivative has four terms, one for each varied factor. The projector derivative for a nondegenerate eigenvalue λ_i is $\Sigma_{j \neq i}(P_j \delta H\, P_i + P_i \delta H\, P_j)/(\lambda_i - \lambda_j)$. This evaluates an observable without an arbitrary choice of eigenvector phases. A centred finite-difference test reproduces the canonical matrix derivative to 9×10^{-12} and the transition-probability derivative to 9×10^{-10}. These are checks of the response map, not evaluations of a physical history metric.

An exact control illustrates why the kinetic attachment matters. Suppose F is only a nonsingular change of left coordinates and $M_f = F Y_f^0$. In that case $Z_Q = F F^\dagger$ and $J_{raw} = Z_Q$. With $Q_F = (F F^\dagger)^{-1/2} F$,

$$Y_{f,c} = Q_F Y_f^0, \quad Q_F^\dagger Q_F = I, \quad J_c = I. \tag{55}$$

All masses and mixing moduli are then unchanged. Taking the numerical F of the Section 7 control reproduces the inherited moduli within 8×10^{-16}. This does not derive $Z_Q = F F^\dagger$ for the action of Section 7, which explicitly has canonical kinetics. Imposing that metric solely to undo an unfavourable result would change the physical model. The control separates a coordinate deformation from a physical completion deformation; a time-dependent coordinate change would also require its connection terms.

Let a common small left deformation act as $\delta Y_f = M\, Y_f$, and split $M = A + S$ with A anti-Hermitian and S Hermitian. With $H_f = Y_f Y_f^\dagger$, the induced response is

$$\delta H_f = [A, H_f] + \{S, H_f\}. \tag{56}$$

Proposition 5. For nondegenerate singular values, a common anti-Hermitian deformation leaves CKM moduli and its CP invariant unchanged to first order. The Hermitian part controls the physical first-order changes in masses and relative mixing, when the quark kinetic term and source-frame weak current are kept canonical.

$$\delta \ln d_{fi} = (U_f^\dagger S\, U_f)_{ii}. \tag{57}$$

$$(\Xi_f)_{ij} = (U_f^\dagger A\, U_f)_{ij} + [(d_{fi}^2 + d_{fj}^2)/(d_{fj}^2 - d_{fi}^2)](U_f^\dagger S\, U_f)_{ij}, \quad i \neq j. \tag{58}$$

$$\delta V = -\Xi_u V + V \Xi_d, \quad \delta |V_{ij}|^2 = 2\mathrm{Re}(V_{ij}^* \, \delta V_{ij}). \tag{59}$$

Proof. Differentiate the two Hermitian eigenvalue problems. Their off-diagonal eigenframe changes are $(U_f^\dagger \delta H_f U_f)_{ij}/(d_{fj}^2 - d_{fi}^2)$. A acts as the same left unitary on both frames and cancels in $U_u^\dagger U_d$, up to unobservable diagonal rephasings. Substitution gives the remaining expressions. ▫

Appendix B Quantum controls

The independent-completion control assigns canonical kinetic terms to all 44 real coordinates of Γ_*. The full Hessian has minimum eigenvalue 0.12386333, but unit quantum fluctuations are much larger than the mean mediator. The following exact argument makes that warning quantitative for a specified instantaneous-projector observable.

Let F(h) collect the 36 real components of $U_f(h)d_f$. Conditional on the entire Euclidean path $h(t_E)$, the completion path integral is Gaussian even though F is nonlinear. Allow a common completion inertia m_T for the formula; the evaluated construction has $m_T = \nu = \varepsilon_\hbar = 1$. With time integration understood in the inner products, define

$$\mathcal{K}_{osc} = -m_T \partial^2/\partial t_E^2 + \nu I, \quad G_T = \mathcal{K}_{osc}^{-1}. \tag{60}$$

$$S_{eff}[w,h] = S_{wh} + \langle F,(\nu I - \nu^2 G_T)F\rangle /2 + \text{constant}. \tag{61}$$

$$\nu - \nu^2 G_T(\omega) = \nu m_T \omega^2/(\nu + m_T\omega^2). \tag{62}$$

The determinant is independent of h. The added action vanishes for constant F but penalises motion: its low-frequency limit is $m_T\omega^2$. This is why the static elimination in Section 3 cannot simply be used as the quantum action. Conditional completion fluctuations have mean $\nu G_T F$ and covariance $\varepsilon_\hbar G_T$. They must not be added again after the Gaussian integration.

$$\sigma_T^2 = \varepsilon_\hbar/[2\sqrt{(m_T\nu)}] \quad \text{(ground-state coincident-time variance per real coordinate).} \tag{63}$$

For a periodic path the variance acquires the usual factor $\coth[\beta_E\sqrt{(\nu/m_T)}/2]$ when the Euclidean extent is β_E and the path weight is $\exp(-S/\varepsilon_\hbar)$. The ground-state value therefore gives the strongest bound below. At unit parameters $\sigma_T^2 = 1/2$, and the total completion variance has trace at least 18, irrespective of the nonlinear mediator motion.

Proposition 6. For this bosonic Hamiltonian, take the equal-time current observable $p_{ij} = \langle \mathrm{Tr}(P_i^u P_j^d)\rangle$, where P_i^f is the ordered rank-one spectral projector of $T_f T_f^\dagger$. Then every entry satisfies

$$|p_{ij} - 1/3| \le \sinh[\|d_d\|_F^2/(2\sigma_T^2)]/\sqrt{2}. \tag{64}$$

Proof. Conditional on the full mediator path, the up and down completion noises are independent isotropic complex Gaussians. The positive Green function obeys $\nu\int G_T = 1$, so the down conditional mean has Frobenius norm at most $\|d_d\|_F$. For a centred isotropic down matrix, unitary invariance gives $\langle P_j^d\rangle = I/3$, even after conditioning on any up projector. The projector is unchanged by $T_d \to -T_d$; its shifted-Gaussian expectation therefore equals that for the symmetric mixture of shifts $\pm\mu_d$. Writing $a = \|\mu_d\|^2/\sigma_T^2$, this mixture has chi-squared divergence $\cosh(a) - 1$ from the centred Gaussian. Its total variation distance is at most $\sqrt{[\cosh(a) - 1]/2} = \sinh(a/2)/\sqrt{2}$. Since $0 \le \mathrm{Tr}(P_i^u P_j^d) \le 1$, this bounds the expectation difference. Averaging over up noise and the complete nonlinear bath path preserves the bound. Degeneracies have Gaussian measure zero. ▫

$$\|d_d\|_F^2 = 0.000262684118, \quad 0.333147587 \le p_{ij} \le 0.333519080. \tag{65}$$

This is a nonperturbative exclusion of the quark hierarchy for this equal-time observable at the declared unit kinetics. It is uniform in the source alignment and does not use the harmonic covariance. Even permitting the construction's own stationary $p_{ud} = 0.9503566$ would require $\sqrt{(m_T\nu)}/\varepsilon_\hbar \ge 3001.18$ by this

necessary bound. With $m_T = \varepsilon_h = 1$ that means ν of order 9×10^6; this is neither sufficient nor a VERSF-derived correction.

The scope matters. This calculation does not yet include fermion propagation and its backreaction. For a real completion source j_F supplied by fermion bilinears, a coupling $+ \langle T, j_F \rangle$ produces, on the same Gaussian integration, the additional terms

$$\nu \langle F, G_T j_F \rangle - \langle j_F, G_T j_F \rangle / 2. \qquad (66)$$

The retarded bilinear and induced four-fermion term come together. Identifying the instantaneous projectors with dressed external quark states would discard that structure. Proposition 6 excludes that shortcut for the displayed bosonic model, not every physical completion of VERSF.

A second control treats $T_f(h) = U_f(h) d_f$ as composite but gives all eight (w,h) coordinates flat canonical kinetics. Integrating the six mediator coordinates gives the source quadratic kernel $[\omega^2 + W - W^2/(\omega^2 + 187/434)]I_2$, with the full nonlinear source potential retained. The conditional mediator fluctuations are sampled once, not added again after integration.

For the sudden-current probe V(h), inclusive weight is $\langle |V_{ij}|^2 \rangle$ and coherent weight is $|\langle V_{ij} \rangle|^2$. Four chains at each of four temperature/slice regulators give, at $\beta_E = 64$ and 1024 slices, inclusive us and td weights 0.022536(19) and 0.0054270(79); the coherent weights are 0.019428(21) and 0.0001143(23). Errors are the larger of blocked and between-chain estimates. Effective-sample-size estimates span about 2,350–7,350 from 8,000 retained observations; the regulator results agree within their statistical errors. The stored time correlator approaches the coherent weight, separating bath excitations from elastic response.

Positivity bounds any acceptance rule that retains the elastic channel and weights other final states between zero and one: $P_{us}^{accepted}/P_{ud}^{accepted} \le P_{us}^{inc}/P_{ud}^{coh} \approx 0.023305$, versus the reference ratio 0.0533303. A conservative five-standard-error envelope is 0.02341, excluding neither model nor discretisation error. A common weak calibration or energy cut cannot repair this probe. Replacing it by the effective-mass observable on this same eight-coordinate branch also fails: twelve chains give $|V_{us}| = 0.34619(41)$, $|V_{ub}| = 0.01764(24)$, $|V_{cb}| = 0.05494(23)$, $|V_{td}| = 0.00693(24)$, $|J| = 1.127(62) \times 10^{-4}$ at the finest regulator. The success of Section 5 depends on the different auxiliary structure as well as the matched observable.

These controls do not quantify the native recorded weak response: the first promotes completion coefficients to independent oscillators, and the second uses a chosen flat mediator metric. Their failures exclude those constructions for the stated probes; they do not overturn the inherited conditional flavour return or its shared-link fermion kinetics.

Appendix C Reproduction and public sources

The companion calculation package is VERSF_Bath_Quark_Calculation.zip. Its top-level README locates the manuscript builder, construction inputs, forward solvers, comparison routines, stored arrays and file_hashes.json. The scientific dependencies are the public website papers linked in the references. The displayed actions, baseline matrices and analytic identities are self-contained. The package is distributed alongside the manuscript and is required for its full numerical reproduction; the referenced website papers are the public scientific sources.

The principal solver is bath_quark_reassessment/code/propagating_auxiliary.py, with auxiliary_response_kernel.py for the complete 36×36 response. The stationary action and all branch sweeps are in hermitian_history_completion.py and alignment_energy.py. Independent checks are in verify_hermitian_candidate.py and verify_propagating.py. inertia_and_pulls.py records the inertia scan, asymmetric marginal pulls and Higgs-scale sensitivity in source_sensitivity/inertia_and_pulls.json and mass_ratio_scale.json. alignment_quantum.py evaluates all six quantum attachments; sign_checks.py

verifies the PDG quartet convention and signed controls. The numerical forward routines do not contain measured mixing targets; comparison routines do.

The finite CAR and spatial calculations are reproduced by fermion_contact_test.py, fermion_mass_basis.py, fermion_spectral_test.py, spatial_contact_response.py, spatial_T_exact.py and spatial_bulk_response.py. Their construction manifests and verification results are in fermion_response and spatial_interaction. The exact spatial derivative is checked by a centred finite difference and free poles by direct eigensystem recovery. The mass-product identity of the control is checked both algebraically through the determinant and numerically by singular-value decomposition. These checks test implementation, not physical uncertainty.

The controls retain their complete Monte Carlo observations, block means, branch returns and original arrays. The quantum controls use Gaussian-preserving proposals, 3,000 warm-up proposals per chain and blocked/between-chain uncertainty estimates; production settings and seeds are printed in their manifests. Their statistical errors remain separate from the deterministic Hermite ground-state convergence errors.

Sources consulted. The public document catalogue, returned blog posts and linked content pages were checked. The full extracted corpus was screened for flavour, history completion, bath dynamics, kinetic normalisation and weak response; the relevant dependency sections were read in detail and are cited directly. corpus_reconsideration/coverage.json and the adjacent source manifest record retrieval coverage and hashes. This is a full-corpus search record, not a claim to have read every page or independently rerun the primitive 334-coordinate and RRHF source executions.

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