Finite Matching and the Physical Force Strengths in VERSF

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General reader summary

Calculating the strengths of nature's forces is like predicting how an instrument will sound from its construction. Knowing its shape gets us part of the way, but the movement of its parts changes the final notes. This paper calculates several of those changes for an explicit candidate within the VERSF programme and tests whether its motion obeys VERSF's rules.

Two measured quantities set the overall strength and scale, leaving one independent comparison. The main example misses that comparison by about two tenths of one per cent, without needing an extra record-like field whose connection to VERSF remains unproved. Adding that field makes little difference. However, changing the level of approximation moves the answer by more than the remaining gap, so the close match does not yet establish that level of accuracy.

The tested part of the model supports consistent quantum motion and conserves charge. A separate recording example also preserves information and accounts for the energy needed to write a record. These results establish important parts of compatibility with VERSF. The remaining task is to connect the complete interacting system to its underlying action and establish the accuracy of the force calculation; one successful compatible construction is sufficient.

Abstract

We calculate finite gauge matching for explicit lattice completions of the VERSF charge metric and test their dynamical compatibility. The main benchmark is a centre-projected Wilson construction without an extra charged record precursor. With the strong and electromagnetic inputs calibrated, it gives $\hat{s}_W^2 = 0.23089034$ against the fixed comparison 0.231290, and $(g_s, g, g') = (1.217716, 0.652253, 0.357375)$ at M_Z. Its weak-mixing residual is -0.172794%. Adding the precursor changes the angle by only 0.00003582. The benchmark's one-loop to two-loop running shift is 0.00270376, about 6.8 times its remaining discrepancy; this order comparison is a sensitivity diagnostic, not a truncation error estimate. Scalar and gauge finite conversions, the coupled precursor determinant and nonlinear gauge-action dependence are explicit. A positive finite-volume bosonic transfer supplies a self-adjoint generator, charge conservation and reversible admissibility. The inherited Weyl census passes anomaly tests, a finite recording witness balances information and energy, and uniform fixed-density current fluctuations shift only the calibrated common coefficient. The radial completion defect alone does not supply full precursor alignment, while the isolated varying density–current block lacks the required relativistic kernel. The paper establishes a close calibrated comparison and constructive bosonic compatibility at finite regulator. It neither derives three independent physical couplings nor completes the primitive reduction, native recording dynamics or global chiral transfer. Physical mass thresholds and higher-order matching remain uncalculated.

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Notation. z is the bare common coefficient; K_A is the charge Gram; C_A is the adjoint Casimir; Q_A is the mixed fourth-order contraction; δ_A is a finite inverse-coupling correction; γ is the precursor mixing coefficient; γ_E is Euler's constant; ζ interpolates the two plaquette actions; $\mathcal{J}(m)$ is the massive scalar response. The letters C, W, Y label the three gauge factors. The vector $\mathbf{r}$ in (26) is a response derivative, distinct from the precursor field r.

1 Purpose and main results

The preceding force-response paper [1] converted the fermion contribution but left the corresponding scalar and gauge conversions incomplete. Its apparent improvement in the weak-mixing result therefore depended on partial matching. The present calculation supplies a scalar conversion, a coupled charged auxiliary threshold and a gauge-action-dependent finite conversion within one explicitly specified effective construction. The three sectors use the same links and charge conventions throughout.

Status convention. A source result is a statement inherited with its stated hypotheses. A construction choice specifies the effective model calculated here. A calculated result follows from that model at the stated perturbative order. A physical return would additionally require the common primitive action, measure, current attachment and scale conversion to reproduce this model. Section 9 proves finite-regulator bosonic compatibility and gives explicit balance and current tests; the full primitive identification remains open. One physically justified admitted construction would suffice; no uniqueness requirement is imposed.

The main benchmark is the centre-projected Wilson construction with the light Higgs and no extra charged record precursor. The logarithmic and faithful trace actions are controls, while the coupled precursor is a tested extension. This organisation uses fewer unproved field identifications; it does not establish that the full primitive action reduces to the benchmark. The main results connect finite matching to a concrete compatibility test. The complete Gaussian scalar–precursor determinant is integrated without losing an eliminated fluctuation. Equal quadratic charge metrics nevertheless give different finite gauge conversions, through a mixed fourth-order charge contraction. The resulting coupling triples can be compared with a held-out electroweak quantity. For the projected Wilson branch, the same bosonic action also supplies admissible motion, while an exactly integrated current extension leaves that comparison invariant. The close result survives removal of the unproved charged precursor. These statements have distinct restrictions, set out in Table 1 and proved in Sections 3–9.

Table 1. Calculated results and their restrictions

Result	Value or identity	Restriction
Scalar conversion	$c_S = -0.000603016862$	Nearest-neighbour complex scalar
Coupled heavy threshold	0.001176881836	Two charged doublets, $\gamma = 7/32$
Difference of gauge conversions	$-Q_A/4 + C_A/24$	Trace minus logarithmic plaquette, one loop
Main weak-mixing benchmark	0.23089034, residual -0.172794%	Projected Wilson without precursor; two calibrations
Added precursor	0.23092617, shift $+0.00003582$	Full extra charged determinant retained
Running-order sensitivity	0.00270376	One-loop to two-loop running; not an error estimate
Bosonic dynamical compatibility	Positive transfer and unitary evolution	Finite volume, regulated scalar sector
Independent-current correction	$\delta_A^J = c_J K_A$	Uniform complex carrier and fixed charged density; calibration invariant

Result	Value or identity	Restriction
Three physical strengths from primitives	Not established	Full source attachment remains to be supplied

The use of measured calibration inputs is explicit. Displaying three coupling values does not create three independent predictions. The strong coupling is an input; the electromagnetic combination is also calibrated; the weak-mixing value tests the remaining degree of freedom.

2 Corpus inputs and the declared construction

The public paper inventory, retained corpus archives and relevant website papers were searched for the complete scalar operator, nonlinear closure action, joint metric, finite matching, physical clock and threshold prescriptions. The dependency passages in [2–9] were then read in detail. Appendix C records coverage and the reproducible search manifest. Full-text corpus coverage is distinct from independently proving every result in that corpus.

The master action defines a vacuum-relative logarithmic holonomy defect and a common quadratic defect metric [3, §§9–13]. Its history-density relation also contains higher-order defect terms. It therefore motivates the logarithmic branch below, but does not justify setting all primitive mixed or higher-order terms to zero. The running paper supplies the covariant full Higgs doublet at two-derivative order and proves cancellation of a constant kinetic residue [4, §§2.4–3.2]. The refinement theorem fixes the leading spatial metric, not the dispersion at every regulator momentum [5]. The chiral theorem supplies the shared-link overlap representative under its hypotheses [6]. None of these statements turns a leading symbol into a complete finite conversion by itself.

The ordered factors are colour, weak isospin and conventional hypercharge, denoted $A = C, W, Y$. The charge metric and its matter decomposition are

$$K = (6,13/2,21/2), \qquad K_F = (6,6,10), \qquad K_H = (0,1/2,1/2). \qquad (1)$$

The faithful group is $[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6$, with integer charge $q = 6Y$. The carrier contains three Standard Model families, three inert singlets and one complex Higgs doublet. The singlets change the address dimension from 47 to 50 but contribute no charge trace. They introduce no gauge correction in this calculation.

Table 2. Fixed choices and their provenance

Ingredient	Choice used here	Source status
Charge census	Equation (1)	Inherited effective census [4, 7]
Geometry	Isotropic four-dimensional Euclidean lattice	Explicit continuation and regulator choice
Closure metric	The common quadratic metric (1)	Logarithmic, faithful trace and projected Wilson completions
Scalar dispersion	Covariant nearest-neighbour Laplacian	Completion of the leading symbol [4, 5]
Fermions	Overlap representative with $r = \rho = 1$	Conditional representative [6]; conversion [10]
Record precursor	Absent in the main benchmark; one extra doublet in the extension	Declared typed effective extension

Ingredient	Choice used here	Source status
Bond coefficient	$\gamma = 7/32$ in the extension only	Borrowed positive wheel coefficient [1, Eq. (4)], tested over a range
Infrared transport	One-loop matching and two-loop gauge running	Declared perturbative approximation

The extra doublet represents a coherent charged precursor only. It is not identified with a permanent record or an event-counting process. Transferring the wheel bond coefficient into this charged doublet block is a declared choice. Its representation can occur in a completion-map carrier, but its dynamics have not been derived from the primitive record operation. Section 9.5 shows why the radial stretch defect alone is insufficient; Section 6 uses the construction without this extra field as the main benchmark and reports the extension alongside it. The address trace defining the closure metric remains (1); the extra propagating auxiliary field is accounted for through its determinant, not by adding another Higgs to that trace.

Let U_p be a face holonomy and $\Omega_p = -i\mathrm{Log}_0 U_p$ on the frozen identity neighbourhood. The logarithmic and faithful trace control actions are

$$S_g^{\log} = \frac{z}{2}\sum_p \mathrm{Tr}_R\, \Omega_p^2, \qquad S_g^{\mathrm{tr}} = z\sum_p \mathrm{Tr}_R\left(1-\cos\Omega_p\right). \qquad (2)$$

Both have tree stiffness zK_A. Both are functions of the faithful representation. Section 5 also constructs a centre-projected Wilson weight with the same perturbative quadratic metric. Their local perturbative expansion is well defined. Global branch-cut behaviour and reflection positivity of the logarithmic action are not established by the one-loop calculation; consequently its Euclidean response is not offered as a complete Lorentzian physical evolution.

The gauge-response programme already separates the physical boundary from its finite scheme bridge [9]. The calculation here evaluates that bridge for specified completions. The common phase parent of [8, §19.5] motivates a coupled block, but does not itself assign its variables to the charged scalar representation.

The finite result also depends on the regulator. The centred derivative in the refinement analysis has the correct small-momentum symbol, but using its squared sine symbol unchanged over a full lattice momentum zone introduces additional low-energy branches. We therefore specify the nearest-neighbour Laplacian rather than silently treating the two operators as identical.

3 The joint scalar and precursor response

This section evaluates the optional precursor extension. The main benchmark retains only one light Higgs doublet; its scalar conversion is calculated in Section 4. Write Δ_U for the positive covariant nearest-neighbour Laplacian, with unit regulator spacing. Let h and r be complex weak doublets of hypercharge $1/2$, transforming on the same links. The complete quadratic action of this sector is

$$S_{hr} = \frac{1}{2}\langle h, \Delta_U h\rangle + \frac{1}{2}\langle r, \Delta_U r\rangle + \gamma \parallel h - r \parallel^2, \qquad \gamma > 0. \qquad (3)$$

A small positive common mass regulates the light mode when needed. It is removed after the infrared logarithm is subtracted. The unbroken background has no scalar expectation value. A gauge-invariant positive quartic can stabilise large amplitudes without changing the one-loop background gauge response at that background.

Proposition 1. Exact coupled determinant. For (3), a constant orthogonal rotation in the two-field space separates one light doublet and one massive doublet. Up to background-independent constants,

$$\mathrm{logdet} P_{hr}[U] = \mathrm{logdet}\Delta_U + \mathrm{logdet}(\Delta_U + 4\gamma). \qquad (4)$$

Proof. The quadratic matrix and its eigenoperators are

$$P_{hr} = \begin{pmatrix} \Delta_U/2 + \gamma & -\gamma \\ -\gamma & \Delta_U/2 + \gamma \end{pmatrix}, \qquad P_+ = \Delta_U/2, \quad P_- = \Delta_U/2 + 2\gamma. \qquad (5)$$

The rotation is independent of the gauge background. Its Jacobian is constant and the factors of two cancel in the background-subtracted determinant. This gives (4). ■

Eliminating r first gives a different-looking, momentum-dependent light operator:

$$P_{\mathrm{eff}} = \frac{\Delta_U(\Delta_U + 4\gamma)}{2(\Delta_U + 2\gamma)}. \qquad (6)$$

Its determinant must be accompanied by that of the eliminated block, $\Delta_U/2 + \gamma$. The latter cancels the denominator in (6). Keeping only the reduced operator would count the negative intermediate determinant while omitting its positive partner. This is a concrete instance of the common-measure reduction rule in [4].

Let $\mathcal{I}(M)$ denote the gauge stiffness from one unit-charged complex scalar of dimensionless mass M. The extra threshold supplied by (4) is $\mathcal{I}\left(\sqrt{4\gamma}\right)$. For the chosen bond coefficient,

$$M = \sqrt{7/8}, \qquad \mathcal{I}(M) = 0.001176881836, \qquad \mathcal{I}\left(\sqrt{2\gamma}\right) = 0.002057227795. \qquad (7)$$

The Schur-only answer would therefore assign a heavy correction of -0.000880345959 instead of the positive value in (7). This is an error in mode counting, not a physical screening effect. Equation (4) includes each charged fluctuation once.

The massive doublet is integrated out in matching onto the one-Higgs effective Standard Model. Its response is evaluated at external momenta small compared with M/a. A separate threshold logarithm must not be added to the full lattice quantity in (7): the conversion and decoupling logarithms are already combined in this zero-momentum heavy contribution.

4 The scalar conversion constant

The unit-charge scalar operator has momentum denominator

$$D_m(k) = m^2 + 4\sum_{\mu=1}^{4} \sin^2\left(k_\mu/2\right), \qquad k_\mu \in [-\pi, \pi]. \qquad (8)$$

The gauge response includes both the two-vertex loop and the contact term. Omitting the contact term violates the zero-momentum cancellation. The transverse small-momentum coefficient reduces to

$$\mathcal{I}(m) = 8\int_{[-\pi,\pi]^4} \frac{d^4k}{(2\pi)^4} \frac{\sin^2 k_1 \sin^2 k_2}{D_m(k)^4}. \qquad (9)$$

An exact proper-time reduction makes the infrared subtraction stable:

$$\mathcal{I}(m) = \int_0^\infty e^{-m^2 t} f(t)\, dt, \quad f(t) = \frac{t}{3}[e^{-2t} I_1(2t)]^2 [e^{-2t} I_0(2t)]^2. \qquad (10)$$

Here I_n is the modified Bessel function. Since $f(t) \sim (48\pi^2 t)^{-1}$ at large t, the modified minimal-subtraction conversion at $\mu = 1/a$ is

$$c_S = \lim_{m\to 0}\left[\mathcal{I}(m) + \frac{\log m^2}{48\pi^2}\right] = -0.000603016862. \qquad (11)$$

Appendix A supplies the subtraction and derivation. The numerical integral error is below 2×10^{-14}; that precision applies to the specified integral, not to the physical approximation. The negative scalar term is small. Adding it to the preceding fermion-only calculation shifts the weak-mixing value from 0.22777618 to 0.22775782.

The coupled block has a total scalar conversion

$$c_{hr} = c_S + \mathcal{I}\left(\sqrt{4\gamma}\right) = 0.000573864974 \quad \text{at } \gamma = 7/32. \qquad (12)$$

A constant scalar kinetic residue cannot replace this calculation. With a consistently transformed measure, the background subtraction obeys $\log\det[Z_H P(U)] - \log\det[Z_H P(1)] = \log\det P(U) - \log\det P(1)$. Thus multiplying the scalar kinetic term alone does not supply an independently adjustable Higgs gauge weight. Field-dependent metrics, masses and extra charged modes can change the response, but they require their own operator calculation.

5 Gauge fluctuations and the nonlinear closure law

The distinction between the two actions in (2) begins at fourth order in the local holonomy. This is precisely the order at which a pair of quantum fluctuations can correct a quadratic background response. Agreement of the tree charge metric is therefore insufficient for finite one-loop matching.

For one generator T_A of a factor, define

$$Q_A = \sum_{B=1}^{12} \frac{\mathrm{Tr}_R(T_A^2 T_B^2)}{K_B}, \qquad C_A = C_2^{\mathrm{adj}}(A) = (3,2,0). \qquad (13)$$

The sum runs over all eight colour, three weak and one hypercharge generators. On each irreducible address block, it reduces to its normalised Casimir load, as in [7, §16.8]:

$$Q_A = \sum_r \mathrm{Tr}_r\,(T_A^2)\left[\frac{C_{2,C}(r)}{6} + \frac{C_{2,W}(r)}{13/2} + \frac{Y_r^2}{21/2}\right]. \qquad (14)$$

Table 3. Contributions to the mixed contraction

External factor	Colour fluctuations	Weak fluctuations	Hypercharge fluctuations	Total Q_A
Colour	4/3	9/26	11/126	1447/819
Weak	1	3/4	5/84	38/21
Hypercharge	11/9	15/52	389/756	4976/2457

These contractions are representation data. Their physical coefficient depends on the fluctuation law. The oscillator slopes of [7] and the Euclidean plaquette corrections here contain the same charge contraction but different dynamics; their numerical coefficients are not added.

Proposition 2. One-loop change of plaquette completion. For the local actions (2), with identical quadratic metric, links, gauge fixing and Haar measure, the finite gauge-plus-ghost conversions to the same continuum scheme satisfy

$$c_{g,A}^{\mathrm{tr}} - c_{g,A}^{\log} = -\frac{Q_A}{4} + \frac{C_A}{24}. \qquad (15)$$

Proof. The action difference through the required order is $-z\mathrm{Tr}\Omega^4/24$. In four dimensions, the free flux variance of generator B on a specified face is $1/(2zK_B)$. The two possible trace orderings satisfy

$$\sum_B \frac{\mathrm{Tr}(T_A T_B T_A T_B)}{K_B} = Q_A - \frac{C_A}{2}. \qquad (16)$$

Contracting the four adjacent and two alternating background–fluctuation placements gives $-(6Q_A - C_A)/24$, which is (15). The remaining one-loop diagrams are identical between the two actions because their quadratic and cubic link vertices agree. ■

The known fundamental Wilson conversion provides the absolute gauge-plus-ghost reference [10]. Removing its quartic contribution and then using (15) gives

$$c_{g,A}^{\log} = -(c_W - 1/12)C_A, \qquad c_{g,A}^{\mathrm{tr}} = -Q_A/4 - (c_W - 1/8)C_A, \quad c_W = 0.16995599. \qquad (17)$$

The absolute universal part is inherited from the published one-loop calculation; this paper calculates the action-dependent difference, rather than claiming to have independently recomputed every gauge and ghost diagram. For a single fundamental $SU(N)$ carrier, (17) recovers $1/(8N) - c_W N$ exactly at the printed precision. The logarithmic Abelian action has no perturbative gauge self-interaction and its corresponding entry vanishes.

Table 4. Finite gauge-plus-ghost conversion

Action	Colour	Weak	Hypercharge	Global status
Logarithmic faithful trace	−0.25986797	−0.17324531	0	Quotient-compatible local branch
Cosine faithful trace	−0.57656516	−0.54229293	−0.50630851	Faithful trace action
Centre-projected Wilson	−0.46820130	−0.27741198	−0.00694444	Quotient-compatible positive gauge kernel

5.1 Completing the Wilson branch on the faithful quotient

A product of separate fundamental Wilson characters has the last row of Table 4 locally, but is not itself a function on the faithful quotient. That defect can be repaired explicitly. Write a representative as $g = (g_C, g_W, e^{i\varphi})$, with φ acting on the integer charge $q = 6Y$, and define the lifted face action

$$S_{\mathrm{lift}}(g) = 12z[3 - \mathrm{ReTr}_3 g_C] + 13z[2 - \mathrm{Tr}_2 g_W] + 378z[1 - \cos\varphi]. \qquad (18)$$

Its three coefficients are fixed by K and one common z: they are $2zK_C$, $2zK_W$ and $36zK_Y$. The Abelian factor corresponds to minimum conventional hypercharge $1/6$. There are no independently fitted sector multipliers. With $\omega = (e^{2\pi i/3}, -1, e^{i\pi/3})$, set

$$w_{\mathrm{quot}}([g]) = \frac{1}{6}\sum_{j=0}^{5} \exp\left[-S_{\mathrm{lift}}(\omega^j g)\right], \qquad S_{\mathrm{quot}} = -\sum_p \log w_{\mathrm{quot}}([U_p]). \qquad (19)$$

Proposition 3. Centre projection and the local matching. The weight (19) is positive, invariant under the faithful centre identification, and has nonnegative character coefficients on the quotient. Its perturbative expansion around the identity has the same finite one-loop conversion as the separate-factor Wilson action.

Proof. Multiplying a representative by ω permutes the six terms, proving descent to the quotient. The exponential Wilson weights of each factor have nonnegative character coefficients: expand the exponential of the fundamental and conjugate characters and use nonnegative tensor-product multiplicities; for the Abelian factor the coefficients are the positive Bessel coefficients. The centre average removes precisely those representations on which ω acts nontrivially. It leaves the other coefficients nonnegative. Thus the associated convolution kernel is positive; the usual character factorisation across a reflection plane gives the pure-gauge reflection-positive lattice construction [13]. This statement concerns the gauge weight, not a proof of positivity for the interacting chiral measure.

At the identity, the six lifted costs divided by z are

$$z^{-1}S_{\mathrm{lift}}(\omega^j) = (0,295,621,808,621,295). \qquad (20)$$

The identity term therefore dominates in its neighbourhood. Corrections from the other centre terms are exponentially small in z, which scales as inverse squared coupling in the perturbative limit. They do not enter the power-series one-loop conversion. This establishes the last assertion. ■

At the main benchmark's calibrated value $z = 0.56146595$, the nonidentity weight relative to the identity term at zero background is 2.33×10^{-72}. At the fixed test value $31/59$ it is 9.67×10^{-68}. These are local saddle weights, not bounds on the full nonperturbative theory. Direct finite-group tests verify centre invariance at generic holonomies. Every matter and Higgs census representation also descends to the quotient, so the same projected links can be used in their local operators.

This is a completed gauge-action construction with a positive pure-gauge kernel. Section 9 extends the positive transfer to the scalar sector and proves reversible admissibility. It does not establish that the full VERSF history measure supplies (19), or prove the complete chiral, current and native record dynamics. The centre projection was introduced to enforce the global group condition after the local result was calculated. Its action weights were not adjusted to improve the weak-mixing output; nevertheless, the comparison is retrospective and is not presented as a blinded branch selection.

This comparison does not imply that measurable couplings depend on a mere change of renormalisation convention. If one describes the same physical theory in a different regulator, its boundary data must transform consistently. Here the nonlinear action is changed while its leading boundary metric is held fixed. The different finite answers measure the additional microscopic information that this leading metric does not contain.

6 Matching and the three coupling values

At $\mu = 1/a$, use the matching convention

$$g_A^{-2}(\mu) = zK_A + \delta_A, \qquad \delta_A = c_F K_{F,A} + c_S K_{H,A} + c_{g,A}. \qquad (21)$$

Equation (21) defines the main benchmark without the extra charged precursor. For the extension, replace c_S by c_{hr}, adding the full heavy determinant in (4). Removing that extension retains the light Higgs and does not claim that all native record contributions vanish.

The local parity-even overlap-fermion conversion at $r = \rho = 1$ is [10, Eq. (3.6), Table I]

$$c_F = -\frac{5}{72\pi^2} - (-0.01161402 + 0.01893460) = -0.014356773308. \qquad (22)$$

The Weyl trace convention in (1) supplies the appropriate count: two Weyl fundamentals have the same trace weight as one Dirac fundamental. The extension to the anomaly-free Standard Model census concerns this local parity-even response. It does not settle the global chiral determinant measure.

For the main projected Wilson benchmark, the complete correction at this order is

$$\delta = (-0.5543419432, -0.3638541283, -0.1508136860). \qquad (23)$$

Only the part independent of a common rescaling affects the held-out comparison. One may set $\tilde{\delta}_A = \delta_A - (K_A/K_C)\delta_C$ while replacing z by $\tilde{z} = z + \delta_C/K_C$. This changes no boundary coupling. The bare and colour-subtracted common coefficients must not be conflated.

Let $a_A = \alpha_A^{-1} = 4\pi/g_A^2$ and $t = \log(\mu/M_Z)$. The gauge-only running used for direct comparison with [1] is

$$\frac{da_A}{dt} = -\frac{b_A}{2\pi} - \frac{1}{8\pi^2}\sum_B \frac{B_{AB}}{a_B}, \quad b = (-7, -19/6, 41/6), \qquad (24)$$

$$B = \begin{pmatrix} -26 & 9/2 & 11/6 \\ 12 & 35/6 & 3/2 \\ 44/3 & 9/2 & 199/18 \end{pmatrix}. \qquad (25)$$

The fixed comparison set is $M_Z = 91.1876$ GeV, $\alpha_s(M_Z) = 0.118$, $\hat{\alpha}^{-1}(M_Z) = 127.930$ and $\hat{s}_W^2(M_Z) = 0.231290$ [11, 12]. The 2024 electroweak convention is retained throughout to compare with [1]; it is not described as the latest experimental fit. The strong and electromagnetic values determine z and the matching scale. The weak-mixing value is withheld from that solve. The running keeps the full massless three-family Standard Model census from M_Z to μ; the five-to-six-flavour strong threshold and electroweak mass thresholds are not applied. This shared approximation is retained for comparison with [1], not assigned sub-per-cent accuracy.

Table 5. Two-input calibrated results in the same running convention

Included finite terms	$\hat{s}_W^2$	μ **in GeV**	**Bare** z
No finite terms	0.22733892	6.6964×10^{13}	0.521346
Fermions only	0.22777618	6.3836×10^{13}	0.534993
Fermions and light scalar	0.22775782	6.3964×10^{13}	0.535023
Logarithmic gauge without precursor	0.22789434	9.8114×10^{12}	0.550460
Faithful trace gauge without precursor	0.22493634	9.1208×10^{12}	0.602152
Main projected Wilson benchmark	0.23089034	1.9911×10^{12}	0.561466
Projected Wilson with precursor	0.23092617	1.9833×10^{12}	0.561408

The main benchmark and its controls return the force values in Table 6. The reference weak and hypercharge couplings use the same electromagnetic input and withheld weak-mixing value, so all quantities share one scheme.

Table 6. Calibrated coupling triples at the Z mass

Coupling	Main benchmark	Added precursor	Logarithmic control	Reference
g_s	1.217716	1.217716	1.217716	1.217716
g	0.652253	0.652202	0.656526	0.651689
g'	0.357375	0.357384	0.356681	0.357468
$\hat{s}_W^2$	0.230890	0.230926	0.227894	0.231290

The main benchmark and logarithmic control have no extra precursor. The strong value is a calibrating input in all three model columns. Weak and hypercharge are tested jointly after their electromagnetic combination is fixed. Their relative errors are $+0.0865\%$ and -0.0260% for the main benchmark, $+0.0787\%$ and -0.0237% with the precursor, and $+0.7423\%$ and -0.2201% for the logarithmic control. The corresponding weak-mixing residuals are -0.172794%, -0.157306% and -1.468139%.

Adding the charged precursor changes the weak-mixing value by only 0.00003582227. Most of the projected branch's improvement therefore comes from gauge and fermion matching rather than that auxiliary field. The main benchmark has $z = 0.5614659483$ and $\mu = 1.9911242 \times 10^{12}$ GeV. Section 9 explains the identification issue motivating the comparison; omitting the extra field does not settle the actual native record response.

Against the no-finite-term row, the logarithmic control removes about 14% of the weak-mixing residual, and the projected benchmark removes about 90%. This is an improvement in the stated numerical comparison, not a percentage closure of the Standard Model programme. No extra sector coefficient was fitted to the weak angle. The remaining perturbative and physical-attachment uncertainties exceed the numerical integration error.

7 Dependence on the declared choices

7.1 Record mixing and nonlinear action

The benchmark has no precursor mixing coefficient. Adding the explicit two-field block tests this additional assumption with the same two calibration inputs. Table 7 reports both the benchmark and the tested positive bond range.

Table 7. Charged precursor extension of the projected benchmark

γ	$M = \sqrt{4\gamma}$	Heavy threshold	$\hat{s}_W^2$
No extra field	Not applicable	0	0.23089034
7/128	0.467707	0.003145483	0.23098609
7/64	0.661438	0.002057228	0.23095296
7/32	0.935414	0.001176882	0.23092617
7/16	1.322876	0.000556848	0.23090729

The added field shifts the result by 0.00001695 to 0.00009574 over this range, less than the benchmark's 0.00039966 discrepancy. The four extended cases span less than 8×10^{-5}. Taking the heavy mass arbitrarily close to zero would eventually invalidate the one-Higgs low-energy census; it is not a legitimate way to generate an unlimited finite threshold while keeping the same running system.

At fixed absence of the precursor, changing the gauge action gives 0.23089034 for projected Wilson, 0.22789434 for the logarithmic action and 0.22493634 for the faithful trace action. The logarithmic-to-trace interpolation $S_g(\zeta) = (1-\zeta)S_g^{\log} + \zeta S_g^{\mathrm{tr}}$ remains affine in its one-loop conversion for $0 \le \zeta \le 1$. All three actions share the tree metric. Their different finite answers make the nonlinear closure law a consequential part of physical attachment; the close projected result is not universal across these completions.

7.2 Numerical agreement and physical accuracy

Table 8 measures changes from the projected benchmark without the extra precursor. Each test recalibrates the same strong and electromagnetic inputs; the optional precursor extension is labelled separately. The third measurement remains outside each solve. With the same finite matching, one-loop running gives 0.22818658, two-loop gauge-only running gives 0.23089034, and adding a running top Yukawa with externally supplied $y_t(M_Z) = 0.99$ gives 0.23092729. The extra top input is a sensitivity test, not a predicted coupling.

Table 8. Benchmark sensitivities and unresolved accuracy

Test	$\hat{s}_W^2$	Change from benchmark	Interpretation
Main benchmark	0.23089034	0	One-loop matching and two-loop gauge running
Fixed comparison	0.23129000	+0.00039966	Remaining discrepancy, equivalent to a −0.172794% residual
One-loop running	0.22818658	−0.00270376	Order sensitivity; not a truncation error estimate
Added top input	0.23092729	+0.00003694	Same benchmark with external $y_t(M_Z) = 0.99$
Strong input range	0.23062767 to 0.23115716	−0.00026267 to +0.00026681	$\alpha_s = 0.118 \pm 0.0009$ only
Electromagnetic input range	0.23088779 to 0.23089290	−0.00000256 to +0.00000256	$\hat{\alpha}^{-1} = 127.930 \pm 0.008$ only
Joint input rectangle	0.23062513 to 0.23115973	−0.00026521 to +0.00026939	Four corners; not a confidence interval
Added precursor at $\gamma = 7/32$	0.23092617	+0.00003582	Optional field extension
Physical thresholds and higher matching	Not calculated	Not estimated	No combined physical uncertainty assigned

The one-loop to two-loop running shift is 0.00270376, about 6.8 times the remaining discrepancy. This is a comparison between successive approximations, not an estimate of the next omitted order, an error bar or evidence that the discrepancy is covered by uncertainty. It does establish that the close match alone cannot demonstrate sub-per-cent physical accuracy. The input rectangle also remains below the fixed comparison throughout the tested range. Strong-input variation dominates electromagnetic-input variation here; these ranges are kept separate from approximation and construction dependence.

The missing five-to-six-flavour strong threshold, electroweak mass thresholds, full physical mass census and two-loop finite matching have not been assigned a numerical error. Their effects must be calculated before a statistical agreement claim is justified. The nonlinear gauge-action dependence in Section 7.1 is a choice of microscopic completion and is likewise not combined into a probabilistic uncertainty band.

Numerical accuracy is tested separately. An independent integration in the g_A variables reproduces the benchmark couplings within 1.4×10^{-12}; an analytic one-loop calibration agrees with the shooting

calculation within 9×10^{-17} in $\hat{s}_W^2$. Direct scalar quadrature and source differentiation test the finite matching numerics. These checks support the reported numbers for the declared approximation without supplying the missing physical uncertainty.

7.3 A close angle can conceal an incorrect overall strength

A second test fixes $z = 31/59$ and uses only the strong input to determine the scale. This value is a borrowed candidate from the seven-site construction [2], not a proven bare coefficient for the present finite action.

Table 9. One-input test with the common coefficient fixed

Gauge action without precursor	Predicted $\hat{\alpha}^{-1}(M_Z)$	Predicted $\hat{s}_W^2$
Logarithmic faithful action	121.5817	0.22987608
Faithful trace action	108.4717	0.23119214
Centre-projected Wilson	118.7893	0.23403136
Comparison values	127.9300	0.23129000

The trace branch almost reproduces the angle while missing the inverse electromagnetic strength by about 15%. This is why a close ratio is not enough to establish the actual force strengths. The two-input calibration in Table 5 makes this distinction explicit rather than allowing the ratio to obscure the absolute normalisation.

7.4 The remaining direction in coupling space

Differentiating the calibrated main benchmark with respect to the three boundary corrections gives

$$d\hat{s}_W^2 = \mathbf{r} \cdot d\delta, \qquad \mathbf{r} = (-0.05351337, 0.07953102, -0.01865442), \qquad \mathbf{r} \cdot K = 0. \qquad (26)$$

The null relation is the expected invariance under a common boundary rescaling. If an additional correction were confined to the Higgs trace direction, the full nonlinear calibration would require

$$d\delta = xK_H, \qquad x = 0.01313008 \quad \text{to reach } 0.231290. \qquad (27)$$

Equation (27) is an excluded fit to the third datum, recorded only to quantify the remaining demand. It is about eleven times the optional heavy threshold in (7). With that threshold already included, the additional coefficient required is 0.01195320, consistently smaller by 0.00117688. Neither coefficient is used in a prediction table. More generally, the benchmark requires $\mathbf{r} \cdot d\delta \simeq 0.00039966$ at linear order; a successful correction need not be confined to the Higgs direction. Constant kinetic rescaling cannot supply it, and selecting an action or threshold because it reproduces (27) would not derive it.

8 Physical scales and the November ledger

On the isotropic relativistic interpretation used here, the main benchmark's calibrated matching energy corresponds to a regulator length and time conversion

$$a_{\mathrm{conv}} = \frac{\hbar c}{\mu} = 9.910 \times 10^{-29}\ \mathrm{m}, \qquad t_{\mathrm{conv}} = \frac{\hbar}{\mu} = 3.306 \times 10^{-37}\ \mathrm{s}. \qquad (28)$$

Adding the precursor gives $a_{\mathrm{conv}} = 9.949 \times 10^{-29}$ m and $t_{\mathrm{conv}} = 3.319 \times 10^{-37}$ s. These are coordinate conversions conditional on $\mu = 1/a$ in natural units and a common propagation speed. The matching

energy was calibrated. Equation (28) therefore does not derive a primitive event duration, identify one ordering step with a number of seconds, or identify this regulator length with the mesoscopic coherence length. A physical history-clock calculation must establish that attachment separately.

The November ledger [2] separates force matching from joint dynamical admission, flavour and strong observables. Table 10 records the specific contribution of this paper without changing its planning score.

Table 10. Relationship to the November ledger

Ledger task	Result supplied here	Work still required
Gauge normalisation and finite matching	Explicit conversions for declared operators	Primitive operator and common metric attachment
Simultaneous weak and hypercharge response	A calibrated triple and one held-out test	Remove the residual without target fitting
Coupled physical evolution	Exact two-field determinant; positive bosonic transfer; balanced recording witness	Full varying primitive currents, native records and physical clock
Higgs sector	A scalar kinetic and heavy-threshold calculation	Physical potential, vacuum and pole mass
Flavour and strong observables	No new closure claimed	Same-action masses, mixing and QCD observables

The calculation makes finite matching executable and proves part of the dynamical compatibility required for one realisation. It does not by itself satisfy the ledger's route to 90%, which also requires the common physical dynamics, flavour returns and a strong-sector observable. The global chiral measure and regulator-removal questions likewise remain separate from this local perturbative result.

9 Dynamical compatibility

Compatibility is tested by constructing motion and interactions for one realisation. The Hamiltonian admissibility axioms require composition, strong continuity and preservation of transition probabilities; they leave the particular Hamiltonian model dependent [14, Sections 2 and 6]. Physical admissibility similarly states necessary constraints without selecting a unique law [17]. The common-action identification is more specific: its independently varied density, links, currents, spatial record transport and completion maps must reduce to the retained fields with the same measure [3]. The following tests establish part of that identification and locate the remaining obstruction to a literal precursor interpretation.

9.1 Motion from the same bosonic weight

Proposition 3 supplies a pointwise positive gauge weight with a positive convolution operator. These are distinct forms of positivity, and both enter the transfer construction.

For the projected gauge weight (19), retain the nearest-neighbour covariant kinetic terms and, when testing the original precursor branch, the positive local term $\gamma \parallel h - r \parallel^2$. A small common mass $m_0 > 0$ regulates the scalar zero mode. On a finite spatial lattice, write $\mathcal{C}_G$ for the product of the temporal gauge

convolution operators, $\mathcal{C}_s$ for the scalar Gaussian temporal kernel, V for the spatial and local potential, and P_G for the Haar projection onto gauge-invariant states. Define

$$\mathbb{T} = P_G e^{-V/2}(\mathcal{C}_G \otimes \mathcal{C}_s)e^{-V/2}P_G. \qquad (29)$$

All factors use the same links and representations. The scalar temporal kernel is the ordinary positive Gaussian kernel in the real components of the complex fields. Gauge projection imposes the Gauss constraint; it is not a replacement of the independent link variables by endpoint frames.

Proposition 4. Bosonic dynamical compatibility. At finite spatial volume with $m_0 > 0$, the transfer (29) is positive and self-adjoint on the gauge-invariant Hilbert space. On its nonzero spectral support, normalising by its largest eigenvalue λ_0 defines

$$H = -a_t^{-1}\log(\mathbb{T}/\lambda_0), \qquad U(s) = e^{-isH}, \qquad H \geq 0. \qquad (30)$$

The evolution $U(s)$ is a strongly continuous unitary group, preserves the Gauss constraint and satisfies the three reversible admissibility axioms of [14].

Proof. The nonnegative character coefficients make $\mathcal{C}_G$ positive. Gaussian convolution is positive by its nonnegative Fourier multiplier. For any ψ, the quadratic form of (29) is that of the positive product kernel evaluated on $e^{-V/2}P_G\psi$, so it is nonnegative. The symmetric ordering makes the transfer self-adjoint. The positive mass confines the scalar coordinates; at finite volume the integral kernel is compact with a finite leading eigenvalue. Gauge invariance makes every factor commute with the gauge action. The spectral logarithm on the nonzero support is self-adjoint and nonnegative after division by λ_0. Spectral calculus gives the group law, strong continuity and preservation of inner products. Gauge invariance gives $[H, P_G] = 0$. These are the claimed properties. ■

This supplies motion for the same bosonic action used in the loop calculation. The parameter s is a composition label. Identifying it with physical seconds still uses a declared clock conversion. The argument establishes the finite-regulator result; no uniform mass gap or regulator-removal theorem is inferred. The scalar mass is removed only in the infrared-subtracted matching calculation of Section 4. The source's history-positivity theorem [15, Sections 20–20.2] explicitly makes the same separation between a bosonic positivity proof and a full fermion-including transfer.

The elementary scalar check is exact. For one real mode,

$$T_m(x', x) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x'-x)^2}{2} - \frac{m^2}{4}(x'^2 + x^2)\right], \qquad E(m) = 2\text{arsinh}(m/2). \qquad (31)$$

The transfer eigenvalue ratio is $e^{-E(m)}$. The two-field rotation gives masses $m_+ = m_0$ and $m_- = (m_0^2 + 4\gamma)^{1/2}$. With $m_0 = 0.3$ and $\gamma = 7/32$, the gaps are 0.2988862404 and 0.9466039491. Direct quadrature reproduces both. Thus the Euclidean weight carries a definite evolution spectrum; the Hamiltonian is not an unrelated extra choice.

A positive configuration kernel also gives a reversible history kernel by the ground-state transform. If $\mathbb{T}\psi_0 = \lambda_0\psi_0$ with $\psi_0 > 0$, then

$$P(y|x) = \frac{\mathbb{T}(x, y)\psi_0(y)}{\lambda_0\psi_0(x)}, \qquad \pi(x) \propto \psi_0(x)^2. \qquad (32)$$

The kernel is normalised and satisfies detailed balance. Its path weights telescope back to the raw transfer, with endpoint factors and powers of λ_0. The latter must be retained in an action response. This stochastic history representation is not identified with Lorentzian unitary evolution; both are derived objects with different meanings.

9.2 Conservation and fermion consistency

At the leading Lorentzian derivative order, the scalar equations are

$$D_\mu D^\mu h + 2\gamma(h - r) = 0, \qquad D_\mu D^\mu r + 2\gamma(r - h) = 0. \qquad (33)$$

For a Hermitian gauge generator T_A, define $j^\mu_{h,A} = \mathrm{Im}\big(h^\dagger T_A D^\mu h\big)$ and similarly for r. The covariant divergences obey

$$D_\mu j^\mu_{h,A} = 2\gamma \mathrm{Im}\big(h^\dagger T_A r\big), \qquad D_\mu j^\mu_{r,A} = -2\gamma \mathrm{Im}\big(h^\dagger T_A r\big). \qquad (34)$$

The bond transfers charge between the two fields while preserving their total. This Noether current is distinct from the independently varied primitive current of Section 9.4. On the exact lattice, the corresponding link-variation identity gives the discrete continuity equation. The Gauss constraint is preserved by (30). A seven-site Abelian check gives local residuals below 1.4×10^{-15} and zero total charge derivative.

Gauge charge conservation and BCB information conservation [16] are distinct statements. The latter follows here for reversible evolution from unitarity: the global state retains its distinguishability and von Neumann entropy. A subsystem can acquire entropy by becoming correlated with a record store. Section 9.3 accounts explicitly for such a store.

The inherited left-handed Weyl census has hypercharges $(1/6, -2/3, 1/3, -1/2, 1, 0)$ with multiplicities $(6,3,3,2,1,1)$. Exact arithmetic gives

$$\mathcal{A}_{SU(3)^3} = \mathcal{A}_{SU(3)^2 Y} = \mathcal{A}_{SU(2)^2 Y} = \mathcal{A}_{Y^3} = \mathcal{A}_{\mathrm{grav}^2 Y} = 0. \qquad (35)$$

There are twelve weak doublets across the three families, so the ordinary $SU(2)$ Witten parity test also passes. The extra scalar, when retained, is bosonic and introduces no chiral anomaly. These checks establish the relevant perturbative representation consistency. They do not prove the global chiral determinant measure or reflection positivity of the complete interacting overlap-fermion theory. That distinction is already required by [15, Section 20.2].

9.3 A balanced recording operation

An explicit recording witness tests whether this architecture contradicts BCB. Consider a finite gauge-invariant one-cell sector with orthonormal energy states $|n\rangle$, energies E_n and any mutually commuting conserved charges. A neutral record bit costs energy $\epsilon > 0$, supplied by a two-level work store initially in its excited state. For declared probabilities $0 \le p_n \le 1$, define

$$W|n\rangle = \sqrt{1 - p_n}\,|n, 0, 1\rangle + \sqrt{p_n}\,|n, 1, 0\rangle. \qquad (36)$$

The last two entries are the record and store occupations. Directly,

$$W^\dagger W = I, \qquad H_{\mathrm{out}} W = W(H + \epsilon I), \qquad Q_{\mathrm{out}} W = WQ. \qquad (37)$$

Proposition 5. Balanced finite recording. Equation (36) extends to a unitary operation on a larger carrier, preserves all inner products and the displayed total energy and charge, and writes an informative record whenever the p_n differ. If subsequent operations leave old pointer registers untouched, their readout projectors are retained.

Proof. Different n remain orthogonal, and the squared amplitudes for each n sum to one. Both output branches have energy $E_n + \epsilon$ and the same system charge. Hence (37) holds and the isometry extends to a unitary after completing bases within the conserved sectors. Old pointer projectors commute with all later operations acting on other registers. Tracing inaccessible partners gives a completely positive, trace-

preserving reduced operation. Copying the pointer to an inaccessible environmental register can make the retained readout classical without destroying global information. ■

For $p = (0.1,0.4,0.8)$, $\epsilon = 0.4$ and an equal coherent input, the mean record energy is 0.1733333333, exactly equal to the store's mean energy loss. The reduced system entropy is 0.4619944139 bits although the enlarged state remains pure. The record is a finite resource: extending the history requires further blank registers and supplied work. No unlimited erasure or zero-cost resetting is assumed.

Because the corresponding Kraus operators are functions of the one-cell energy, this example preserves that cell's thermal density matrix. This fact does not license inserting it into the full interacting gauge theory without recalculation. A spectral operation on the total many-body Hamiltonian need not be local. The native local marked instrument, its event rate and its effect on dynamical gauge response remain separate compatibility data. Equation (36) is an existence witness for balance, not a derivation of those data.

9.4 Adding independent current fluctuations

The master action independently varies a target-based current J and includes both its constitutive defect and covariant divergence [3, Sections 13–14 and 49.2]. The simplest nontrivial extension retains these current fluctuations at fixed substrate density. Let D_U map vertex fields to oriented edge differences on a finite Euclidean regulator, and set $\Delta_U = D_U^\dagger D_U$. On a fixed-density background with vanishing charged density, so that this restriction does not constrain the gauge source, the tested current action is

$$S_J = \frac{a}{2} \| J \|^2 + \frac{b}{2} \| D_U^\dagger J \|^2, \qquad M_J = aI_E + bD_U D_U^\dagger, \qquad a, b > 0. \qquad (38)$$

All edge components are retained, including independent circulating currents. For a complex current carrier, integration contributes $+\log\det M_J$ to the effective action. The factor $1/2$ in the complex quadratic form changes only a background-independent normalisation.

Proposition 6. Current determinant and calibration invariance. The exact finite-dimensional identity is

$$\det M_J = a^{E-V} b^V \det(\Delta_U + a/b). \qquad (39)$$

If every address in the inherited carrier uses the same a, b, its one-loop gauge correction is $\delta_A^J = \mathcal{J}\left(\sqrt{a/b}\right) K_A$. It is absorbed by the common calibration, leaving the two-input weak-mixing prediction unchanged at this order.

Proof. Apply Sylvester's determinant identity to $I_E + (b/a) D_U D_U^\dagger$. The gauge-independent powers give (39). The remaining determinant is that of a massive complex scalar on the complete address representation. A common scalar operator gives the same scalar response per unit charge trace, yielding $\delta_A^J = c_J K_A$. In the matching relation $g_A^{-2} = zK_A + \delta_A$, adding $c_J K_A$ and replacing z by $z - c_J$ leaves every boundary coupling unchanged. The subsequent running and both calibrating measurements are therefore identical. ■

At $a = b = 1$,

$$c_J = 0.001036110310774, \qquad z: 0.561465948348 \mapsto 0.560429838038. \qquad (40)$$

The main benchmark's weak-mixing value remains 0.230890344142; the numerical change is zero at the reported precision. The precursor extension obeys the same common-shift identity. Direct determinants on a 3^4 lattice, with 81 vertices and 324 complex edge variables, agree with (39) within 1.8×10^{-13} at three

gauge-source amplitudes. This is a substantive current extension with a definite response, not the deletion of the current block.

The fixed-density restriction matters. Restoring a density fluctuation ρ in the isolated constitutive and continuity block gives, for one spatial Laplacian eigenvalue $\lambda = k^2$ and Euclidean frequency ω_E,

$$S_{\rho J} = \frac{a}{2}|j - i\kappa k\rho|^2 + \frac{b}{2}|i\omega_E\rho + ikj|^2. \qquad (41)$$

Its exact Schur kernel and full determinant are

$$P_\rho = \frac{ab(\omega_E^2 + \kappa^2\lambda^2)}{a + b\lambda}, \qquad \det P_{\rho J} = ab(\omega_E^2 + \kappa^2\lambda^2). \qquad (42)$$

Keeping the eliminated determinant is essential. This isolated block has $\omega \propto \lambda$ at small spatial momentum, rather than the relativistic $\omega \propto \sqrt{\lambda}$. Thus it cannot simply be declared identical to the scalar kinetic operator in (3). Other master-action blocks can change this result; (42) is not a no-go theorem for the complete action. It identifies the precise additional dynamical work needed when density, current and completion all vary.

9.5 Testing the record precursor identification

A charged weak doublet can be typed as a completion operator from a right-handed charged-lepton address, with $Y = -1$, to a left-handed lepton doublet, with $Y = -1/2$. The operator then has $Y = +1/2$. This establishes that such a representation can occur inside a completion-map carrier. It does not identify it with the positive polar stretch, which transforms by conjugation on the right carrier.

The master action's stated scalar defect is

$$\mathcal{D}_H = X_H - (\phi_H/\Lambda_\star)P_H^R, \qquad X_H = \log B_H - \log B_{H,\star}. \qquad (43)$$

On the frozen support, its differential depends on the positive stretch B_H and Higgs modulus. The local completion's polar orientation enters the fermion bilinear separately [3, Sections 47F and 49.3–49.5]. Consequently, an independent charged field cannot be identified with X_H merely because both have been called records.

The local distinction can be calculated. Around aligned nonzero doublets, write $h = vn + \eta_h$ and $r = vn + \eta_r$, with n a unit real four-component direction. The postulated alignment and a canonically rescaled radial-only penalty have quadratic Hessians

$$H_{\text{align}} = 2\gamma\begin{pmatrix} I_4 & -I_4 \\ -I_4 & I_4 \end{pmatrix}, \qquad H_{\text{rad}} = 2\gamma\begin{pmatrix} P_n & -P_n \\ -P_n & P_n \end{pmatrix}, \qquad P_n = nn^{\mathsf{T}}. \qquad (44)$$

The first has four massive relative real modes; the second has one. At $\gamma = 7/32$, their spectra are respectively $\{0^4, (7/8)^4\}$ and $\{0^7, 7/8\}$. The three missing directions are relative orientations, not a simultaneous gauge rotation of both fields. The rank distinction is independent of a rescaling of radial coordinates.

Proposition 7. Limit of the literal stretch identification. The radial completion defect alone cannot reproduce the full alignment Hessian in (44). A full common-metric cross term cannot lift a tangent that all retained defect differentials annihilate.

Proof. At a common defect zero the residual Hessian is $J_D^\dagger W_{\text{prim}} J_D$. If $J_D v = 0$, its quadratic form on v vanishes for every W_{prim}. Relative orientation changes that keep the positive polar stretch fixed lie in the

kernel of the local radial block. The full alignment assigns them positive cost. Therefore an additional nonzero defect differential, an induced interaction or a justified constraint is required. ■

This is a failure of the proposed direct identification, not an exclusion of every possible VERSF completion. Spatial bond and support motion, or a calculated interaction after integrating other fields, could supply the missing orientation response. Its coefficient and determinant would then have to be evaluated on that construction. The full master action is not replaced by the radial-only surrogate in (44). Nor does this broken-background tangent test determine an analytic two-doublet mass matrix at the unbroken origin; that origin requires its own regular variables.

9.6 Scope of the compatibility result

Table 11. What the test establishes

Requirement	Result	Scope
Faithful gauge group and shared links	Pass	Projected gauge kernel and inherited representations
Positive bosonic transfer	Pass	Finite volume, regulated scalar zero mode
Reversible admissibility	Pass	Self-adjoint generator obtained from that transfer
Gauge charge balance	Pass	Scalar exchange and Gauss invariance
Perturbative anomaly tests	Pass	Standard Model Weyl census
Balanced physical recording	Explicit witness	Finite one-cell instrument and work store
Independent-current integration	Exact return	Uniform weighting and fixed density
Full doublet alignment from radial stretch alone	Insufficient	Three relative orientation costs are missing
Fully varying primitive current and density	Conditional	Isolated block gives (42), not the required relativistic kernel
Complete chiral and native recording dynamics	Not established	Requires the full physical carrier and local operation

The strongest supported statement is that the effective gauge and scalar construction has a positive finite-regulator dynamics compatible with VERSF's reversible admissibility principles, a charge-consistent matter census, and explicit balanced-record witnesses. This is more than a statement that its formulas are mathematically writable. The statement that it is the reduction of the complete VERSF common action remains conditional on the actual identification of the primitive variables and interactions.

10 Required common action return

The required return is a local, representation-typed map from the varying primitive density, currents, spatial record transport and completion fields to the retained effective fields. Its reduction must give the same background-dependent operator and measure for one complete branch. If the charged precursor is retained, this map must supply the three relative orientation costs in (44). If it is not retained, the actual completion modes must still be integrated or retained explicitly. The varying density–current block must

supply a justified physical low-momentum dispersion; the isolated result (42) is insufficient. Its gauge second derivative must include direct vertices, relaxation, the measure and every eliminated determinant. For a positive bosonic Hessian $P(A)$, its contribution has the form

$$\partial_A \partial_B \mathrm{logdet} P = \mathrm{Tr}\left(P^{-1} P_{,AB}\right) - \mathrm{Tr}\left(P^{-1} P_{,A} P^{-1} P_{,B}\right). \qquad (45)$$

Real fields carry the usual factor $1/2$; complex fields and fermions carry their corresponding determinant multiplicities and signs. Equation (45) must be evaluated on the physical quotient. A zero mode is not made harmless by an unrecorded inverse: gauge fixing, constraints and retained massless fields must be handled explicitly.

Table 12. Required return and acceptance tests

Return	Where it enters	Acceptance test
Full nonlinear gauge law and metric	Equations (2), (15), (17)	Recover the two faithful-trace limits, their difference and the centre-projected kernel
Full scalar and charged record operator	Equations (3)–(12), (43), (44)	Recover the scalar limit and account for every completion mode; supply relative orientation costs if the precursor is retained
Primitive current and bond blocks	Equations (38)–(42), (45)	Recover the fixed-density determinant and common-shift invariance; obtain justified low-momentum motion when density varies
Fermion operator and determinant prescription	Equations (21), (22)	Recover the local overlap conversion on its reference branch
Physical spectrum and thresholds	Equations (21), (24)	Decouple heavy modes and recover the stated light census
Temporal and spatial attachment	Equations (28)–(32)	Recover the positive bosonic transfer; establish one physical light cone and the clock conversion
Native record operation	Equations (36), (37)	Information and energy balance, local record retention and the actual event rate

At one-loop order around a fixed background, the gauge Hessian and its background derivatives encode the required cubic and quartic information. The master action's quadratic history metric alone does not determine every such derivative, especially when its measure and cross blocks vary with the background. This is a concrete computational target: a matrix-valued function of the background source, together with its measure and physical identification.

Five failure criteria make the result testable. **F1:** a scalar implementation fails this benchmark if its contact-inclusive nearest-neighbour limit does not reproduce (9)–(11). **F2:** a proposed joint reduction fails if it retains (6) but omits the eliminated determinant required by (4). **F3:** an effective branch fails the present numerical confrontation if its calibrated weak and hypercharge outputs retain the residual of Table 6 beyond its justified uncertainty. That statement rejects the branch at its claimed accuracy, not every possible VERSF construction. **F4:** a literal radial-stretch identification fails if it claims the full rank-four relative alignment without supplying the three missing orientation costs. **F5:** a primitive current reduction fails this reference test if its fixed-density uniform limit does not recover (39), or if it drops the eliminated determinant in (42).

The target was known before this research. The construction choices are documented and their dependence tested, but this is not a blinded source-admission exercise. A future physical return must fix and publish its complete branch and numerical prescription before using the withheld force comparison to judge it. Multiple justified branches may be reported; disagreement is not a reason to silently discard one.

11 Conclusion

The paper supplies finite scalar and gauge matching for an explicit construction and establishes part of its compatibility with VERSF dynamics. Its main benchmark is the centre-projected Wilson branch without the unproved extra charged precursor. Two measured calibration inputs give $(g_s, g, g') = (1.217716, 0.652253, 0.357375)$ at M_Z and one independent weak-mixing comparison, 0.23089034 against 0.231290. The relative discrepancy is -0.172794%. Adding the precursor changes the angle by only 0.00003582, so the close comparison does not rely on that field identification.

The input and approximation checks refer to this same benchmark. The one-loop to two-loop running shift, 0.00270376, exceeds the remaining discrepancy by a factor of about 6.8. It is an order-sensitivity diagnostic, not an error estimate. Physical thresholds and higher-order matching remain uncalculated, so the numerical agreement is not yet a demonstrated precision for the physical forces.

The same bosonic weight supplies a positive finite-regulator transfer, a self-adjoint generator and charge-conserving quantum motion. The Weyl census passes the stated anomaly tests; a separate finite recording witness balances information and energy. A uniform fixed-density current extension integrates exactly into the common calibration. Together these are constructive compatibility results. The complete interacting primitive action, measure, varying currents and native records must still be connected to one effective construction, and its force uncertainty established. One successful physically compatible realisation is sufficient; uniqueness is not required.

Appendix A Scalar response derivation

For a background with momentum p in direction 1 and polarisation in direction 2, the contact-inclusive scalar polarisation is

$$\Pi_{22}(p) = \int_{\mathrm{BZ}} \frac{d^4k}{(2\pi)^4} \left[\frac{2\cos k_2}{D_m(k)} - \frac{4\sin^2 k_2}{D_m(k) D_m(k + pe_1)} \right]. \qquad (46)$$

At $p = 0$, periodic integration by parts cancels the two terms. Expanding through p^2 and integrating by parts in k_1 gives (9). This calculation explicitly retains the contact contribution before the expansion. On a finite periodic box a constant vector potential can encode a nontrivial Wilson line, so the infinite-volume zero-momentum argument is not substituted for an exact finite-volume identity.

Using $D^{-4} = \int_0^\infty t^3\, e^{-tD}\, dt/6$ and the one-dimensional identity $\langle \sin^2 k\, e^{2t\cos k} \rangle = I_1(2t)/(2t)$ gives (10). The convergent expression actually integrated is

$$c_S = \int_0^\infty \left[f(t) - \frac{1}{48\pi^2(1+t)} \right] dt - \frac{\gamma_E}{48\pi^2}. \qquad (47)$$

Here γ_E is Euler's constant, distinct from the bond coefficient γ. The continuum massive scalar at zero external momentum contributes $\log(\mu^2/m^2)/(48\pi^2)$ in the stated scheme; subtracting it at $\mu a = 1$ yields (11). The massless limit is taken only after this subtraction.

Appendix B Gauge matching derivation

The commutator identity for a generator of factor A is

$$\sum_{B\in A}\left[T_B,[T_B,T_A]\right]=C_AT_A. \qquad (48)$$

Generators from the other factors commute. Multiplying by T_A, tracing and dividing by the common Gram within the factor gives (16). The flux variance follows by integrating $\left(\hat{k}_\mu^2+\hat{k}_\nu^2\right)/\hat{k}^2$ over the symmetric four-dimensional zone. It equals $1/2$; the constant gauge mode has zero flux and is removed before inversion.

For a fundamental $SU(N)$ reference, $Q=C_F=(N^2-1)/(2N)$. Its quartic contraction is $-N/12+1/(8N)$. Subtracting that term from the published Wilson conversion $1/(8N)-c_WN$ gives the logarithmic value $-(c_W-1/12)N$. Restoring the general faithful trace quartic yields (17). This comparison uses identical local measures and gauge fixing; it does not transfer a finite constant across unrelated primitive metrics.

The general scalar–precursor reduction is also a test of field normalisation. Constant nonsingular changes of integration coordinates modify determinants by background-independent factors, which cancel. A background-dependent transformation instead changes the measure response and must be differentiated alongside the transformed Hessian in (45).

Appendix C Numerical verification and reproducibility

The reproducibility package contains `calculate.py`, `verify.py`, `verify_centre.py`, `check_compatibility.py`, `check_benchmark.py`, `construction.json`, the numerical result files, corpus coverage and topic-search manifests, and the manuscript source. Calculations require Python, NumPy and SciPy. Running the five calculation scripts reproduces the tables without access to a private account or to unpublished numerical arrays. The website retrieval script additionally uses PyMuPDF for PDF text extraction. Public source URLs and byte hashes identify the editions consulted.

The public inventory yielded 698 distinct PDF byte streams through 726 functioning URLs, together with the text of all 559 public posts and 13 pages returned by the website index. Retained archived source texts were also screened. Four inherited PDF URL strings had escaped spellings with functioning counterparts. Coverage and topic-search manifests record the editions and search results; text-variant counts are not counts of independent papers. This is a full-inventory dependency search, not a claim that all 26,126 current PDF pages were independently reviewed line by line.

Table 13. Independent numerical checks

Check	Result
Direct four-dimensional scalar quadrature at $M=\sqrt{7/8}$	0.00117688183636 at 48 nodes per direction
Proper-time integral at the same mass	0.00117688183624
Finite-source second derivative, step 0.002	1.06887279 against analytic 1.06887654
Pure-gauge change of the finite scalar log determinant	Below 2×10^{-13}
Full joint determinant against its light/heavy factorisation	Below 10^{-12}
Constant scalar normalisation cancellation	Below 3×10^{-13}

Check	Result
Explicit 50-address charge contractions and central action	Agreement at floating-point precision
Projected weight under all six centre representatives	Maximum tested difference below 2×10^{-13}
Independent evolution in the g_A variables	Coupling agreement below 2×10^{-12}
Scalar transfer gaps, 80-node quadrature	Errors below 2×10^{-15}
Coupled charge continuity	Local residual below 1.4×10^{-15}
Current determinant identity, three backgrounds	Error below 1.8×10^{-13}
Dynamical current Schur formula	Error below 6×10^{-17}
Record isometry and energy balance	Isometry residual below 2×10^{-16}; exact energy intertwiner in the tested matrices
Common-current recalibration	Zero change at reported precision for the main benchmark
Benchmark one-loop analytic calibration	Weak-mixing agreement within 9×10^{-17}
Benchmark response derivatives at three step sizes	Component variation below 2.1×10^{-11}
Alignment versus radial relative-mode rank	Four versus one

The finite-source error decreases quadratically with its step. The scalar integral was also checked with mass regulators from 0.2 to 0.005 and converges to (11) after logarithmic subtraction. Exact identities such as the field rotation and common-rescaling null direction are implementation checks, not separate physical findings. The positivity proofs for the projected gauge kernel and its bosonic transfer are analytic. Small negative transfer eigenvalues of order 10^{-16} in the largest quadrature are floating-point roundoff, not a failure of that proof. The numerical checks do not establish the logarithmic action's global positive transfer, the full chiral measure or higher-order perturbative accuracy. The image-only corpus overview was inspected visually; the inventory also contains three trivial test PDFs, which are not research evidence.

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