Mixed Matter Matching and the One-Calibration Force Test in VERSF

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General reader summary

A force does not act in an empty world. The particles it acts on also change how strongly it appears to act. Some particles respond to more than one force, so a disturbance involving one force can alter the response to another. Calculating these shared effects is part of turning VERSF's underlying construction into numbers that can be compared with experiments.

This paper calculates a definite part of those effects without adjusting it to obtain the desired answer. It improves the electromagnetic comparison by about fifteen per cent of the gap left by the preceding calculation. However, it slightly worsens the comparison that tests the balance between the two electroweak forces. The calculation therefore does not supply the final three force strengths.

The paper connects the calculation to VERSF's earlier work and explains how to count each disturbance once. The corrections calculated so far are much smaller than the changes needed to remove the remaining difference. That makes the construction's basic choices the more promising place to investigate next: how it assigns costs, how it is probed and how strongly it responds. This is a conclusion about where to look, not a proof that the uncalculated effects cannot help.

Abstract

We evaluate the distinct-factor fermion exchanges and the Abelian fermion self-exchange in the heat-kernel, overlap-fermion branch of the VERSF one-calibration force test. The calculation uses published two-loop overlap matching integrals and derives their product-group charge contractions, with a factor of one half for each Weyl species. For overlap parameter $\rho=1$, the commuting-generator coefficient is $u_F=-0.0019399628$, with a quoted-source numerical half-width of 0.0000020. At $z=465/826$, the resolved contribution to the inverse squared colour, weak and hypercharge couplings is $\Delta Z^{F,\mathrm{res}}=(-0.001493705,-0.003610139,-0.006739206)$. Its transfer from the Wilson reference to the specified heat-kernel action is justified for these terms by their common leading gauge propagator and absence of non-Abelian gauge vertices. A fundamental-Dirac recovery check and recovery of the complete Standard Model two-loop gauge coefficient matrix verify the representation and spin counting. Added to the inherited one-loop boundary and pure-gauge two-loop block, the result is $\hat{\alpha}^{-1}(M_Z)=128.70324$ and $\hat{s}_W^2(M_Z)=0.22798807$, using $\alpha_s^{(5)}(M_Z)=0.1180$ as the sole force input to the numerical solve. Against the fixed benchmarks (127.930,0.231290), the weak and hypercharge couplings differ by $+0.419\%$ and -0.514%. We also derive the normalisation identity $J_P=h_2$ under the archived contraction's stated operator conventions; its numerical failure cannot be repaired by an arbitrary conversion factor. Non-Abelian same-factor conversion, finite Higgs and Yukawa terms, and the complete primitive physical attachment remain open. The required local remainder is substantially larger than the evaluated matter terms, favouring a test of the construction's normalisation and source assumptions over an expectation of closure by ordinary finite corrections. The corpus connection is made explicit through the common-action reduction, exact fluctuation accounting and a Casimir-contraction recovery check. The inherited global-cost hypothesis is held fixed, without claiming its historical independence from electromagnetic data. The three physical force strengths are not derived.

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1 Main results and scope

The purpose is to calculate the missing matter response in the force construction of [1]. A charged matter loop can involve both the force being probed and a second force exchanged internally. Its contribution is consequently a matrix of representation weights, rather than a common scalar multiplying all three couplings. This paper evaluates the entries that transfer to the existing heat-kernel action without its unresolved non-Abelian same-factor conversion.

The main result is the resolved matrix in Table 3 and its boundary increment in Eq. (12). The full hypercharge row of the gauge-exchange fermion block is evaluated, together with the four distinct-factor entries in the colour and weak rows. Two non-Abelian diagonal entries remain outside that result. Finite Higgs and Yukawa matching are separate contributions. “Resolved” refers to this specified perturbative subset, not to the full physical matter response.

A second result is a convention identity for the independent plaquette check. When the archived polarisation is the Hessian of the parity-even, unit-charge Weyl determinant, its vacuum contraction must equal the published fundamental-Dirac coefficient h_2. Section 7 derives this statement from the partition function. The available numerical contraction fails it. Internal determinant tests pass, but they do not establish the failed external comparison or licence a rescaling of its answer.

Status convention. Exact algebra concerns the declared representation census and finite effective construction. Matching numbers inherit the specified regulator, published numerical integrals and perturbative truncation. The forward calculation uses no electroweak target in its boundary solve. Inverse diagnostics use those targets only to quantify the required remainder. A compatible VERSF construction would suffice; unique selection of the construction is not required. Compatibility of the complete primitive theory is not asserted merely because an effective calculation is well posed.

Table 1. Results and their restrictions.

Result	What is established	Restriction
Product-group fermion matrix	Fixed representation and Weyl counting	Overlap regulator at $\rho=1$
Resolved heat-kernel entries	Six distinct-factor exchanges and Abelian self-exchange	Common leading propagator and unchanged matter vertices
Forward comparison	Two electroweak outputs after one strong calibration	Partial finite two-loop boundary
Plaquette identity	$J_P=h_2$ in the stated conventions	Numerical agreement remains absent
Required remainder	Two independent response combinations	An inverse diagnostic, not supplied dynamics

2 The fixed construction and physical normalisation

The construction retains the faithful gauge group, conventional hypercharge and the census used in [1]:

$$G=[SU(3)\times SU(2)\times U(1)]/\mathbb{Z}_6, \quad K=(6,13/2,21/2). \tag{1}$$

The fermion and Higgs traces are $K_F=(6,6,10)$ and $K_H=(0,1/2,1/2)$. The adjoint Casimirs are $C_{adj}=(3,2,0)$. The three family copies are already included in the fermion traces. Sterile singlets make no gauge contribution. We include one complex Higgs doublet and no additional charged record doublet.

The compact auxiliary integral is the heat-kernel convolution of [1, Section 5]. It is carried out once, before the remaining gauge and matter fluctuations are expanded. The global-cost branch fixes

$$z=465/826, \quad Z_{0,A}=zK_A, \quad g_{0,A}^2=1/(zK_A). \tag{2}$$

Here Z_A always denotes an inverse squared coupling; it is distinct from the magnetic coefficient $\mathcal{B}_A$ below. The value of z incorporates the inherited 15/14 equal-cost hypothesis. Its source paper sought to account for electromagnetism. Holding it fixed in this calculation establishes one force input to the numerical solve, not a historically independent one-input derivation of all force data. Neither that factor nor the source profile is adjusted here.

The leading physical identification remains the invariant established in [1]:

$$g_A^{-2}=(\mathcal{E}_A\mathcal{B}_A)^{1/2}/v_A^2. \tag{3}$$

The electric coefficient $\mathcal{E}_A$, magnetic coefficient $\mathcal{B}_A$ and matter vertex v_A must be evaluated for the same connection and physical clock. The isotropic effective branch assumes $v_A=1$ and the common leading coefficient in Eq. (2). Its embedding into the complete primitive VERSF action remains a condition, as it was in [1]. Finite matching does not supply that embedding by itself.

At matching scale $\mu_*=1/a_{lat}$, the boundary used for the numerical result is

$$Z_A=zK_A+\delta_A^{(1)}+\Delta Z_A^{G}+\Delta Z_A^{F,res}. \tag{4}$$

The inherited one-loop term is $\delta^{(1)}=c_FK_F+c_SK_H-(c_g-1/24)C_{adj}$, with $c_F=-0.01435677308$, $c_S=-0.000603016862$ and $c_g=0.16995599$. The pure-gauge two-loop block is $\Delta Z^G=(-0.00974545894,-0.00399265669,0)$. We use the unrounded values retained by the calculation files. These terms and their heat-kernel action conversion were evaluated in [1].

The finite matching convention is $g_0=Z_g\,g_{MS}$ with $Z_g^2=1+l_0g_0^2+l_1g_0^4+\cdots$ at $a_{lat}\mu_*=1$. Hence $g_{MS}^{-2}=g_0^{-2}+l_0+l_1g_0^2+\cdots$. No additional $l_0^2g_0^2$ term belongs in that inverse-coupling boundary.

3 From the overlap reference to a product gauge group

3.1 The two reference coefficients

Constantinou and Panagopoulos give the Wilson-gauge, overlap-fermion two-loop matching for fundamental Dirac fermions [2, Eq. (50)]. Its fermion part can be collected as

$$l_1^F=N_f(a_F/N+b_FN). \tag{5}$$

With $d_\pi=(16\pi^2)^2$, the two coefficients are

$$a_F=(55/12-4\zeta(3))/d_\pi-c_-, \quad b_F=-481/(36d_\pi)-k_1/(8\pi^2)-c_+. \tag{6}$$

At $\rho=1$, the published integrals are $k_1=0.00732057894(3)$, $c_-=-0.000979(1)$ and $c_+=0.000872(3)$. The resulting values are $a_F=0.000969981404$ and $b_F=-0.001500516722$. These are imported regulator integrals, not results inferred from measured forces and not an independent repetition of the original two-loop integration.

3.2 The representation reduction

Let $t_A(r)=\mathrm{Tr}_r(T_A^2)$ for one normalised generator of factor A, including multiplicity from the other factors and generations. Let $C_B(r)$ be the quadratic Casimir of factor B on that irreducible block, with $C_Y(r)=Y_r^2$. Define

$$M_{AB}^F=\sum_{r\ \mathrm{Weyl}} t_A(r)C_B(r). \tag{7}$$

For a fundamental Dirac fermion of SU(N), $t=1/2$ and $C_r=(N^2-1)/(2N)$. The arbitrary-representation colour identities of [3, Eqs. (36)–(40)] give

$$a_F/N+b_F N \to t_r[-4a_F C_r+2(a_F+b_F)C_{\mathrm{adj}}]. \tag{8}$$

Proposition 1. Product-group fermion coefficient. In the parity-even perturbative sector, using the same overlap regulator for each Weyl block, the published Wilson-reference contribution has coefficient matrix

$$L_{AB}^{F,W}=u_F M_{AB}^F+v_F C_{\mathrm{adj},A}K_{F,A}\delta_{AB},\quad u_F=-2a_F,\quad v_F=a_F+b_F. \tag{9}$$

Proof. For one Dirac block, Eq. (8) separates the matter Casimir from the adjoint term. Each Weyl block contributes half the Dirac parity-even result. An internal gauge line of factor B supplies $C_B(r)g_{0,B}^2$. For distinct factors the generators commute, so there is no adjoint commutator term. The adjoint term survives only when the internal and external factors coincide. Summing the external trace $t_A(r)$ over the Weyl census gives Eq. (9). For two Weyl fundamentals, Eq. (9) returns Eq. (5) exactly. □

The statement concerns perturbative gauge response around the index-zero branch. Halving a parity-even Dirac coefficient does not construct the global chiral phase or prove a nonperturbative chiral measure.

Table 2. Representation data with family multiplicities included.

Block	t_C	t_W	t_Y	C_C	C_W	Y^2
Q	3	9/2	1/2	4/3	3/4	1/36
u^c	3/2	0	4	4/3	0	4/9
d^c	3/2	0	1	4/3	0	1/9
L	0	3/2	3/2	0	3/4	1/4
e^c	0	0	3	0	0	1
H complex scalar	0	1/2	1/2	0	3/4	1/4

Consequently the rows of M^F are (8,9/4,11/12), (6,9/2,1/2) and (22/3,3/2,95/18), in colour, weak and hypercharge order. The scalar matrix M^H has zero colour row and identical weak and hypercharge rows (0,3/8,1/8). These matrices are not gauge-kinetic mixing between distinct simple factors. Entry AB describes how an internal exchange of force B changes the coefficient of the external force A.

Numerically, $u_F=-0.001939962807$ and $v_F=-0.000530535318$. The published numerical half-width on u_F is 0.0000020. A conservative sum of the quoted c_- and c_+ half-widths gives 0.0000040 on v_F. These are source-integration sensitivities, not uncertainties covering the incomplete physical calculation.

4 Which matter contributions transfer to the heat kernel

Equation (9) is a Wilson-reference result. Importing its complete diagonal into a different gauge action without conversion is not justified. The distinct-factor entries have a narrower and sufficient transfer argument.

Proposition 2. Transfer of the commuting-generator entries. Suppose the matter operator and its shared-link source vertices are unchanged, each gauge factor has the same leading free propagator as the Wilson reference, and there are no additional cross-factor gauge interactions at this order. The entries with $A \neq B$ transfer unchanged to the heat-kernel action. The Abelian same-factor entry transfers to its locally quadratic Abelian heat-kernel sector.

Proof. A two-loop term with external factor A and internal factor $B \neq A$ contains a fermion loop and one internal B propagator. Independent direct-product factors have no gauge vertex joining A to B. The relevant diagrams therefore use only the common matter vertices and the common leading B propagator. Non-Abelian commutators between the two factors vanish. The commuting-generator part of Eq. (9) supplies their coefficient. For external and internal hypercharge, the local Abelian heat-kernel action has no gauge self-vertices; the same commuting-generator reduction applies. The retained terms include the contact vertices of the overlap matter operator, not only its two three-point vertices. □

The proposition is a statement about the specified effective regulator. If an unresolved record sector changes the leading propagator at momenta throughout the integration region, or supplies additional matter vertices, its conclusion must be re-evaluated. Equality of low-momentum poles alone would not prove equality of a finite matching constant. This distinction is also explicit in the VERSF regulator-matching theorem [6, Section 9, especially Theorem 9G].

For this branch, define the resolved mask by $m_{CC}=m_{WW}=0$ and all other $m_{AB}=1$. The evaluated increment is

$$\Delta Z_A^{F,res}=u_F\sum_B m_{AB}M_{AB}^F g_{0,B}^2. \tag{10}$$

Table 3. Resolved coefficients $L_{AB}^{F,res}$. The boundary increment in Eq. (12) is each row summed with weights $g_{0,B}^2$.

External response	Internal colour	Internal weak	Internal hypercharge
Colour	not included	−0.004364916	−0.001778299
Weak	−0.011639777	not included	−0.000969981
Hypercharge	−0.014226394	−0.002909944	−0.010238693

For clarity, “not included” is not a zero result. It denotes the complete non-Abelian same-factor block still requiring its action conversion. The complete Wilson-reference diagonal contributions, before that conversion, would be -0.007421962 in colour and -0.004125561 in weak isospin at Eq. (2). They are useful reference checks and are excluded from Eq. (4).

An independent spin-and-representation check uses the known continuum two-loop gauge matrix. In the same order of factors,

$$B^{gauge}=2M^F+4M^H+\mathrm{diag}[(10/3)C_{adj}K_F+(2/3)C_{adj}K_H-(34/3)C_{adj}^2]. \tag{11}$$

This gives rows $(-26,9/2,11/6)$, $(12,35/6,3/2)$ and $(44/3,9/2,199/18)$, matching the standard running matrix used in the inherited calculation. Equation (11) verifies counting and the logarithmic structure. It does not determine a missing finite scalar or Yukawa constant.

5 The evaluated correction and the forward force test

5.1 Boundary response

Substitution of Eq. (2) into Eq. (10) gives

$$\Delta Z^{F,res}=(-0.001493705395,-0.003610138672,-0.006739206029). \tag{12}$$

The distinct-factor part of the hypercharge entry is -0.005007068788; its Abelian self-exchange contributes -0.001732137242. All seven evaluated entries have a negative sign. These signs follow from the published overlap integral and positive charge contractions; none is selected to remove an experimental residual.

The physical comparison retains the pinned SMDR implementation and fixed non-gauge inputs of [1,7]. Four variables are solved for: three running couplings at $Q_0=172.4$ GeV and the common scale μ_*. The four conditions are

$$g_A^{-2}(\mu_*)=Z_A \ (A=C,W,Y), \quad \alpha_s^{(5)}(M_Z)=0.1180. \tag{13}$$

The electromagnetic and weak-mixing targets do not enter these equations. The quoted quantities use the same PDG-compatible conventions and fixed 2024 benchmarks as [1,8]. This preserves a meaningful comparison between matching orders. It does not update the benchmark edition or claim prediction of the frozen masses and Yukawa inputs.

Table 4. Forward outputs at fixed construction and strong calibration.

Boundary	$\hat{\alpha}^{-1}(M_Z)$	$\hat{s}_W^2(M_Z)$	μ_* in GeV
One-loop plus pure-gauge two-loop block [1]	128.84309	0.22802766	4.62040×10^{12}
Plus resolved fermion terms in Eq. (12)	128.70324	0.22798807	4.54396×10^{12}
Fixed comparison targets	127.930	0.231290	not applicable

The inverse-electromagnetic residual falls from 0.91309 to 0.77324, a reduction of 15.32% of that residual. The relative error in inverse electromagnetic strength changes from +0.714% to +0.604%. The weak-mixing residual becomes slightly larger: its relative error changes from −1.410% to −1.428%. These two observations must be reported together.

The convention-matched couplings are reconstructed by $g^2=4\pi/(\hat{\alpha}^{-1}\hat{s}_W^2)$ and $g'^2=4\pi/[\hat{\alpha}^{-1}(1-\hat{s}_W^2)]$.

Table 5. Force comparison after the resolved fermion contribution.

Quantity	Calculated	Reference	Relative difference
g_s from the five-flavour strong input	1.217716	1.217716	calibrated
Weak g	0.654416	0.651689	+0.419%
Hypercharge g'	0.355630	0.357468	−0.514%

The weak coupling moves farther from its reference; hypercharge moves closer. Thus this matter subset does not deliver the two signs required to improve both individual electroweak couplings simultaneously.

5.2 Numerical sensitivity and physical precision

Varying c_* over its quoted numerical half-width changes $\hat{\alpha}^{-1}$ by about 0.000144 and $\hat{s}_W^2$ by about 0.000000041. The two-loop boundary truncation and omitted sectors are not bounded by those small numbers.

Table 6. Sensitivity tests with the same resolved matching terms.

Variation	$\hat{\alpha}^{-1}(M_Z)$	$\hat{s}_W^2(M_Z)$
SMDR running option 2	128.68540	0.22794566
Main option 3	128.70324	0.22798807
SMDR running option 5	128.70220	0.22799153
Strong input 0.1171 with option 3	128.66731	0.22826340
Strong input 0.1189 with option 3	128.73863	0.22771717

The option numbers are the inherited SMDR running settings, not a claim that every boundary and threshold contribution has been calculated to the same loop order. The strong-input range is a sensitivity interval, not a fitted improvement. Extra printed digits support reproduction of the calculation.

With the convention $\mu_* = 1/a_{lat}$, the main scale corresponds to $a_{lat} = 4.34262 \times 10^{-29}$ m and $a_{lat}/c = 1.44854 \times 10^{-37}$ s. These are coordinate conversions of the fitted matching scale. They are not an independently derived irreversible recording interval.

6 What the remaining response must supply

Let $R = (R_C, R_W, R_Y)$ denote the omitted finite response added to Eq. (4). It includes the non-Abelian same-factor fermion conversion and the Higgs and Yukawa terms for the same action. It is an unevaluated function of that construction, not permission to choose three free counterterms.

The measured-output sensitivity after recalibrating the strong input is

$$\delta o = JR + O(\|R\|^2), \quad o = (\hat{\alpha}^{-1}, \hat{s}_W^2). \tag{14}$$

Table 7. Rows of the numerical Jacobian J at the resolved partial boundary.

Output	R_C	R_W	R_Y
$\delta\hat{\alpha}^{-1}$	6.704718	12.519435	12.558876
$\delta\hat{s}_W^2$	−0.05516448	0.07533908	−0.02225025

The two target requirements are therefore

$$6.704718R_C + 12.519435R_W + 12.558876R_Y = -0.77323656, \tag{15}$$

$$-0.05516448R_C + 0.07533908R_W - 0.02225025R_Y = 0.00330193. \tag{16}$$

They expose why “all couplings soften” is not a sufficient mechanism. The strong recalibration and the relative weak–hypercharge response both matter. They also quantify the required sizes before any missing integral is evaluated.

For example, if the omitted colour response were zero, the linear diagnostic requires $(R_W, R_Y) = (0.0198115, -0.0813182)$. If the omitted hypercharge response were zero, it requires $(R_C, R_W) = (-0.0832890, -0.0171579)$. These restrictions are illustrative slices of the requirement, not assumptions used in Table 4. The full nonlinear inverse solves give $(R_W, R_Y) = (0.0196078, -0.0811163)$ in the first slice and $(R_C, R_W) = (-0.0830866, -0.0172630)$ in the second. These diagnostic fits are excluded from every forward row.

The two displayed slices require a colour or hypercharge remainder near 0.08, but that is not a lower bound when all three sectors may change. Eliminating R_W from the two response equations gives $0.49401374R_C + 0.50598626R_Y = -0.04114589$, using the unrounded Jacobian. Since both weights are positive and sum to one, every solution of this linear diagnostic has $\max_A |R_A| \geq 0.04114589$. Equality is attained at $R = (-0.04114589, 0.00154809, -0.04114589)$. This is the smallest possible largest correction within the local linear model; it is not a bound on the full nonlinear inverse problem.

The evaluated matter increments have magnitudes 0.0015 to 0.0067, and the excluded Wilson-reference non-Abelian diagonals have magnitudes 0.0041 and 0.0074. Even the unrestricted local minimum is about six times the largest evaluated matter increment. The fixed-sector examples require about ten times the largest reference diagonal. On these magnitudes, closure by the remaining ordinary two-loop finite matching terms alone looks unlikely. The more plausible next place to look is the construction’s inherited 15/14 cost premise, source profile and unit response amplitude. This is an inference from the observed scale, not an exclusion theorem: the uncomputed scalar, Yukawa and action-conversion coefficients could be enhanced or add coherently, and no upper bound on their sum has been established.

A separate issue is the sign of the small partial shift. The omitted non-Abelian blocks are comparable to or larger than the included colour and weak increments. After strong recalibration they could reinforce or reverse the weak-coupling change, depending on their converted values and relative weights. The worsening in Table 5 is therefore a result for the evaluated subset, not a stable sign prediction for the complete two-loop correction.

The two outputs constrain two combinations of three boundary increments. A direction in the kernel of J leaves these outputs unchanged to first order after strong recalibration. Consequently an inverse fit cannot identify the microscopic correction uniquely, even if it matches both targets. Forward evaluation of the action is the discriminating test.

7 The plaquette reference check in fixed conventions

7.1 What must be compared

The published coefficient h_2 describes the average Wilson plaquette, whereas l_1 is a background-field matching coefficient [2,4]. They are different observables. Nevertheless, the vacuum contraction called J_P in [1] has a definite relation to h_2 when its operator conventions are those stated there.

Write the unit-charge, parity-even Weyl effective action as

$$\Gamma_W[A]=\Gamma_W[0]+(1/2)\sum_q A_\mu(-q)\Pi_{\mu\nu}^W(q)A_\nu(q)+O(A^3). \tag{17}$$

For one fundamental Dirac flavour, the factor of two from chirality and the trace $\mathrm{Tr}(T^aT^b)=\delta^{ab}/2$ cancel. Its polarisation per adjoint component is therefore Π^W. With the Feynman-gauge leading covariance $g_0^2\delta_{\mu\nu}/\hat{q}^2$, Gaussian contraction gives

$$J_P=-(1/2)\int_{BZ} d^4q/(2\pi)^4\ \mathrm{Tr}_\mu\Pi^W(q)/\hat{q}^2, \tag{18}$$

$$[\log Z/V]_{F,2}=(N^2-1)N_f g_0^2 J_P. \tag{19}$$

The subscript F,2 denotes the fermion contribution of order g_0^2. Here $\hat{q}^2=4\sum_\mu \sin^2(q_\mu/2)$, and Π^W has its charge and coupling stripped as in Eq. (17). The common zero-mode prescription is understood; the integral is its infinite-volume limit.

The published convention [4, Eqs. (9)–(12)] is $\beta=2N/g_0^2$, $\log Z/V=\cdots+d_1/\beta+\cdots$, $c_2=d_1/(24N^2)$ and $c_2^F=(N^2-1)N_f h_2/(12N)$. Substituting these relations into Eq. (19) proves

$$J_P=h_2. \tag{20}$$

This identity does not turn a plaquette observable into a coupling-matching coefficient. It fixes the reference test for a vacuum contraction used in an action conversion. Under the stated definitions there is no unspecified minus sign, Dirac factor or generator normalisation left to fit.

7.2 Numerical status and independent derivative checks

The archived eight-scramble calculation with 2^{19} momentum pairs per scramble gives $J_P=+0.0084792$, with between-scramble standard error 0.0000291. Its unsubtracted estimator is +0.0084800. The published value at $\rho=1$ is $h_2=-0.01934341(1)$. A direct rerun with four scrambles of 2^{18} pairs gives $+0.0084787\pm0.0000510$, consistent with the archived contraction. The signs of the contraction and published reference differ. The result therefore does not pass Eq. (20).

The convention check uses $\Gamma_F=-\log\det D$ for the Dirac effective action, its parity-even Weyl half, and the minus sign relating the effective-action insertion to log Z. Reference [4] defines the free-energy density as $-\log Z/V$ but expands $+\log Z/V$ in its Eq. (10); Eqs. (18)–(20) use that latter convention. The archived estimator contains the corresponding negative contraction factor, and the finite-block test below differentiates $-\log|\det D|$. These checks remove a free choice of overall convention sign; they do not rule out a sign or contact-term error elsewhere in the full implementation. A sign flip alone would give -0.0084792, still 0.0108642 away from h_2, so it cannot explain the magnitude mismatch.

We compare the implemented first overlap derivative with the independent expression in [4, Appendix A], and also compare the nonlocal second derivative in that appendix. On random nonzero momentum pairs the matrix identities agree at floating-point precision. A separate two-momentum block test compares the analytical determinant Hessian with central differences of an explicitly diagonalised Hermitian kernel. The curvature converges to the analytical value. These tests are retained as operator checks, not presented as a replacement for Eq. (20).

The discrepancy is unresolved at the level of the complete observable evaluation. The evidence does not identify a justified numerical repair, nor does it establish an error in the published coefficient. In particular, multiplying the archived contraction by the ratio needed to obtain the target is not an admissible check. Both numbers remain excluded from a non-Abelian heat-kernel matter conversion in this paper.

The completed entries in Eq. (12) use the published background matching coefficient through Proposition 2. They do not use the failed J_P integral. Resolving Eq. (20) is therefore still necessary for the proposed conversion route, but is not a prerequisite for those distinct-factor and Abelian entries.

8 The VERSF connection and the remaining calculation

8.1 What the corpus already supplies

The force calculation consumes a sequence of results with different roles. The master action supplies a common parent; gauge descent specifies how its relaxed response is read; coarse graining preserves the contribution of removed variables; regulator matching relates a chosen finite operator to physical comparison conventions. Table 8 makes those dependencies explicit. Each result retains its source hypotheses. Establishing compatibility on one physical realisation is sufficient; none of these links requires a uniqueness theorem for all possible VERSF realisations.

Table 8. Corpus findings used in the force calculation.

Source finding	Role in this paper	Condition retained
Master action, Theorem 9A and Sections 21–26 [5]	One residual metric fixes the cross blocks and the common Hessian	Specified occupied carrier and metric; Gauss–Newton form requires a common defect zero
Gauge-action descent, Sections 5–6 and Theorem 1 [11]	Relax the auxiliary modes of the same physical connection	Quotient exact null directions; include quantum determinants at their proper integration stage
Exact fermion blocking, Proposition 7 and Section 9 [12]	Retained and removed determinants together preserve the link response	Differentiate the source-dependent block map and retain induced interactions
Gauge running, Propositions 2–4 [13]	Fix the census and statistics weights; cancel constant matter pairings	Same physical regulator, transformed measure and operator; finite local terms remain separate
Matter–record response, Proposition 16.2 and Section 16.8 [14]	Independent exact check of the representation contractions in Table 2	Its compact quantum model has different dynamics and is not an additive correction here
Stationary response, Propositions 5–7 [15]	Explain how a common electric and magnetic factor can arise	Least-squares parent, common trace, isotropy and the declared wheel/source profile
Chiral matching, Theorems 9F–9G; Higgs refinement [6,9]	Connect the leading particle and spatial symbols to continuum conventions	A common leading symbol does not fix the full finite-momentum vertices or finite matching
Finite matching, Section 9; parameter closure, Sections 7–8 [16,17]	Supply compatibility tests and a rule for tracing inputs into outputs	Finite bosonic compatibility does not complete the primitive reduction or global chiral measure

The stationary construction returns 31/59 under its C1–C5 premises [15, Sections 10.2–10.4]. Equation (2) instead uses the global-cost extension of [1]. These are explicit related constructions, not a second independent derivation of the 15/14 hypothesis. The current boundary inherits that hypothesis with the provenance stated in Section 2.

8.2 Exact recovery of the earlier charge contractions

There is a direct algebraic connection to the twelve-direction response of [14]. Sum over the generators of each factor on an irreducible block: the sum of their squares is the quadratic Casimir times the identity.

With the same charge normalisation, its Casimir load is therefore obtained from the two matrices already used in Eq. (11):

$$Q_A=\sum_B(M_{AB}{}^F+M_{AB}{}^H)/K_B,\quad Q=(1447/819,38/21,4976/2457). \tag{21}$$

Substitution into $\gamma_A=Q_A/(4K_A)+11C_{adj,A}/(576K_A)$ recovers exactly the earlier fractions (26155/314496,989/13104,2488/51597). The calculation package checks this with rational arithmetic. Thus the mixed charge contractions are the same representation data that previously generated sector-dependent compact response. They are not three independent sector weights.

The coefficients multiplying those contractions are different. The earlier γ_A is a small-quantum-fluctuation slope for a three-link compact Hamiltonian. Equation (10) is a regulated fermion matching contribution weighted by internal gauge propagators and the Weyl determinant. The algebraic agreement validates the shared charge bookkeeping; it gives no prescription for adding the earlier slope to Eq. (12).

8.3 One parent response and one integration of each fluctuation

The master action [5, Theorem 9A] forms each cross-defect metric as $W_{XY}=\iota_X{}^\dagger W_{prim}\iota_Y$. These overlaps come from one metric and fixed embeddings. They are not independent coefficients for the three force factors. At a simultaneous zero of the defects, restricting every operator to the fixed physical quotient gives the Hessian and auxiliary reduction

$$H=G_V+J_{def}{}^\dagger W_{prim}J_{def},\quad H_{red}=H_{aa}-H_{a\xi}H_{\xi\xi}{}^{-1}H_{\xi a}. \tag{22}$$

Here J_{def} is the derivative of the complete defect map, a denotes retained gauge-source coordinates and ξ all eliminated coordinates; the inverse is taken on the physical quotient where $H_{\xi\xi}>0$. The source symbol a is distinct from the lattice spacing a_{lat}. Away from a common defect zero, the full second derivative of the action is required. Equation (22) cannot be extended there by dropping the additional terms.

The gauge-action theorem itself distinguishes constitutive minimisation from quantum integration [11, Section 6]. For a real Gaussian auxiliary with precision C[a], linear source j[a] and a source-independent reference measure, exact integration gives

$$S_{eff}[a]=S_{ret}[a]-(1/2)j[a]^TC[a]^{-1}j[a]+(1/2)\log\det C[a]+\text{constant}. \tag{23}$$

The determinant contributes whenever its source derivatives do. If the parent is already a quantum effective action at the chosen scale, the fluctuations absorbed into its coefficients are not integrated again. In the present branch, the exact compact record convolution of [1] is already included before Eq. (4) is assembled.

The corresponding fermion statement is the exact blocking identity of [12]. On a nonsingular smooth branch, with eliminated block $D_e[a]$ and retained Schur operator $D_c[a]$,

$$\log|\det D[a]|=\log|\det D_e[a]|+\log|\det D_c[a]|. \tag{24}$$

Two source derivatives preserve this equality only when the block map is differentiated as well. In the stated index-zero parity-even sector, the Weyl effective action is minus one half of this logarithmic magnitude. This identity explains both the statistics factor and why deleting an eliminated determinant loses response. This parity-even identity does not construct the global Weyl phase.

There is also an interaction consequence. Section 9 of [12] shows that averaging a fluctuating bilinear produces a connected four-fermion term, together with a contact correction when the link dependence is nonlinear. Therefore a primitive record covariance that changes the matter action must enter through its common vertices before loop integration. A record variance, a compact response slope and a determinant correction cannot be counted as independent additions merely because they were computed in separate papers.

8.4 Constant matter normalisation is not an extra correction parameter

The running paper proves cancellation of constant positive matter pairings in the leading response, including five Hermitian generation matrices and one Higgs scalar, a class with 46 real parameters [13,

Section 3]. A stronger statement applies whenever a proposed change is only an invertible, source-independent change of integration variables in the complete finite action.

Proposition 3. Invariance under a consistent constant field rescaling. Let T be an invertible, source-independent linear change of matter variables. Transform every kinetic term, mass, interaction, source vertex and measure consistently, keeping the same physical regulator. The background-normalised partition functional and all of its source derivatives are unchanged:

$$\mathcal{Z}_T[a]/\mathcal{Z}_T[0]=\mathcal{Z}[a]/\mathcal{Z}[0]. \tag{25}$$

Proof. Substitute the transformed variables in the regulated integral. The ordinary or Grassmann Jacobian is independent of a and is included in the transformed measure; any overall constant cancels in the displayed ratio. Differentiating the identical functionals proves the statement. □

This is a change-of-variables identity, not a claim that all VERSF metrics describe the same physics. Changing only a kinetic coefficient while leaving the other raw terms fixed changes the canonically normalised masses or interactions. A source-dependent pairing also produces derivative vertices and possibly a nonconstant measure contribution. Such changes can affect finite matching, but their complete action must be supplied. An unknown constant Higgs residue alone is therefore not a free multiplier for ΔZ^H.

8.5 From the coupled response to the physical coefficient

The primitive calculation must return the transverse kernel as a function of both frequency and spatial momentum on the same connection. Its low-frequency and low-momentum derivatives give $\mathcal{E}_A$ and $\mathcal{B}_A$ in Eq. (3); the shared matter source gives v_A. The stationary response paper supplies explicit conditions under which the two derivative coefficients acquire the same factor [15]. It also demonstrates why a static reduction alone does not determine that factor physically.

Differentiating the invariant in Eq. (3) makes the required comparison quantitative. For a small change on a common physical branch,

$$\delta Z_A/Z_A=(1/2)\delta\mathcal{E}_A/\mathcal{E}_A+(1/2)\delta\mathcal{B}_A/\mathcal{B}_A-2\delta v_A/v_A. \tag{26}$$

Thus a proposed primitive correction must account for its temporal, spatial and vertex responses together before it is identified with a boundary increment. The isotropic convention used here has the common speed and $v_A=1$ fixed. A different attachment requires its own matching map. The coordinate conversions in Section 5.2 do not replace the commitment-time attachment required by [9, Section 9.1, p. 33].

8.6 The remaining return and the November ledger

The next complete boundary has the form

$$Z^{complete}=Z^{partial}+(d_C,d_W,0)+\Delta Z^H+\Delta Z^{Yuk}. \tag{27}$$

The quantities d_C,d_W denote the full remaining same-factor fermion blocks for the heat-kernel action, including their conversion. The Higgs term includes the scalar-gauge diagrams and the counterterms required by the declared scalar mass and interaction prescription. The Yukawa term requires the same chiral and scalar source operators. If an additional primitive interaction is retained, it must be displayed as an additional term with a specified origin. Equation (27) is not a claim that all primitive VERSF sectors have already been reduced into this effective action.

Table 9. Executable return required to complete this matching calculation.

Quantity	Where it enters	Acceptance test
Non-Abelian same-factor fermion blocks	d_C, d_W in Eq. (27)	Recover the Wilson reference when its gauge vertices are restored; preserve the evaluated distinct-factor entries
Plaquette conversion check	Action-dependent contraction	Reproduce Eq. (20), including sign, chirality and generator trace
Scalar finite response	ΔZ^H	Use the declared Higgs operator and critical-mass prescription; recover the scalar logarithmic weights in Eq. (11)
Yukawa finite response	ΔZ^{Yuk}	Use the same chiral source and Higgs vertices; vanish when the corresponding Yukawa matrices vanish
Full force comparison	Eq. (13) with Eq. (27)	Keep the strong calibration and all construction rules fixed; report both electroweak outputs
Primitive physical attachment	Eqs. (3), (22)–(26) and the regulator assumptions	Use one physical link and clock; preserve determinants, shared-link vertices and the common quotient

The public corpus supplies the common-action structure, descent conditions, regulator identities and several explicit restricted executions. The source search and direct examination described in Appendix B did not supply an evaluated replacement for the finite scalar and Yukawa constants required by this particular heat-kernel branch. A universal running coefficient or a formula for a Yukawa operator is not such a constant. This is a statement about the quantities found and used, not a proof that no other construction can complete the task.

Four tests make the result falsifiable. First, the product-group transfer fails if the actual retained operator changes the leading internal propagator or adds vertices omitted by Proposition 2. Second, the proposed plaquette conversion remains unvalidated until Eq. (20) is reproduced under the same conventions. Third, a complete force comparison must satisfy both measured-output tests without selecting its finite coefficients from Eqs. (15)–(16). Fourth, a claim of primitive VERSF force normalisation must evaluate Eq. (3) with the same physical connection and time map.

The scale test in Section 6 changes the research priority. A construction-level calculation should determine the cost factor, source profile and physical response amplitude from a common compatible VERSF realisation, then repeat both electroweak comparisons with the strong input alone calibrated. Changing an amplitude must mean changing the physical response in Eq. (3), not merely rescaling matter coordinates, which Proposition 3 leaves invariant. The remaining finite terms remain necessary for precision and for the complete sign, but there is no demonstrated basis for expecting them to supply the entire residual. Selecting construction parameters from the electroweak targets would be a fit, not this forward test.

The result advances the finite-matching part of ledger entry SM-26 and makes the normalisation audit in SM-9 more explicit [10, pp. 8, 10, 14–15]. It does not establish the global chiral completion in SM-29, the mass-and-flavour programme, or a confinement observable. The 90% roadmap requires a common finite construction to pass several tests together. Seven evaluated matching entries are progress within that programme, not seven closed ledger entries.

9 Conclusion

The calculation replaces seven previously unevaluated fermion entries with fixed coefficients and propagates their effect through the same one-calibration force solve. It completes the gauge-exchange fermion contribution to hypercharge and the exchanges between distinct force factors. The inverse electromagnetic comparison improves by about fifteen per cent of the remaining residual, while the weak-mixing comparison slightly worsens.

The required remainder is substantially larger than the matter corrections evaluated here. Even with all three sectors free, the local response equations require at least one correction of magnitude 0.04115, about six times the largest evaluated matter increment. Ordinary missing finite terms alone are therefore an unlikely explanation on the observed scale. The next priority is to test the construction's cost premise, source profile and physical response amplitude against the common VERSF action. The omitted terms still matter for precision and may reverse the partial weak-coupling shift; their total size has not been bounded.

The non-Abelian conversion check has a definite normalisation: the stated vacuum contraction must equal the published h_2. The available computation fails that test, and a sign flip alone cannot reconcile the magnitudes. The corpus supplies the common-source and fluctuation-accounting conditions for the physical test, while exact Casimir recovery connects the charge data to the earlier matter–record calculation. These results identify both the calculated contribution and the construction-level questions most likely to control the remaining discrepancy.

Appendix A Numerical verification and inputs

A1 Representation and matching checks

The calculation reconstructs Eq. (5) from two Weyl fundamentals for $N=2,3,4,5$, with floating-point residuals below 10^{-16}. Summing the block traces in Table 2 gives $K_F=(6,6,10)$; adding the scalar row gives Eq. (1). Equation (11) recovers the complete two-loop gauge coefficient matrix within 10^{-12}. Equation (21) and all three earlier compact-response fractions are recovered in exact rational arithmetic by `check_corpus_bridge.py`. These are checks of the derived contractions, not independent evaluations of the published finite integrals.

The inherited baseline is reproduced as $\hat{\alpha}^{-1}=128.843085267$ and $\hat{s}_W^2=0.228027663234$. The main partial solve has maximum boundary residual below 2×10^{-14}. Central differences with boundary steps 0.0002, 0.0001 and 0.00005 check the Jacobian in Table 7. The calculation keeps the inverse diagnostics separate from every forward result. The weighted-mean identity and saturating remainder in Section 6 are checked algebraically and by an independent linear programme in `check_remainder_scale.py`; the two minima agree within 10^{-12}. This check uses the existing Jacobian and makes no additional forward-force claim.

The derivative checks in Section 7 use the Wilson-overlap kernel $X(k)=i\sum_\mu \gamma_\mu \sin k_\mu+\sum_\mu(1-\cos k_\mu)-1$, $H=\gamma_5 X$ and $D=I+\gamma_5\mathrm{sign}(H)$. The analytical first and second spectral-projector derivatives are compared with the published X-operator formulas. In the two-momentum finite test, the analytical curvature is 0.056673564835; central differences at steps 0.01 and 0.001 give 0.056674177209 and 0.056673567173. Reducing the step further eventually exposes floating-point cancellation. These checks exercise nonzero momentum transfer and do not close the separate vacuum-integral discrepancy.

A2 Fixed comparison inputs

The benchmark edition is PDG 2024 [8]. The fixed targets are $\hat{\alpha}^{-1}(M_Z)=127.930\pm0.008$ and $\hat{s}_W^2(M_Z)=0.231290\pm0.000040$. The calculation is not within those experimental errors, and the small integration sensitivities reported here are not full theory errors. The strong input is fixed at $\alpha_s^{(5)}(M_Z)=0.1180$ for the main row.

At $Q_0=172.4$ GeV, the external SMDR inputs are $v=246.60029586365829$ GeV, $\lambda_H=0.1262969232875777$, $y_t=0.9309715465595599$, $y_b=0.0155080080991419$, $y_c=0.003400275709585654$, $y_s=0.00029291581783326 2$, $y_d=0.0000147240913707 2013$, $y_u=0.0000067321435653851 0$, $y_\tau=0.0099949323705356$, $y_\mu=0.00058838465029327$ and $y_e=0.0000027929943407 0567$. The low-energy routines use $M_t=172.4$ GeV, $M_h=125.20$ GeV, $M_W=80.34918252735155$ GeV and $M_Z=91.1876$ GeV. The bridge retains $\Delta\alpha_{had}^{(5)}=0.02783$ for the additional Sommerfeld output, which is not either comparison target. Every trial overwrites all three gauge inputs.

Appendix B Reproducibility and sources consulted

The accompanying calculation package contains the contraction and forward-solve scripts, numerical results, derivative checks, the inherited SMDR bridge and pinned source archive, and the public-source manifest. The principal commands are `python calculate_matching.py` and `python add_checks.py`, after

building the included SMDR bridge. The first script evaluates the coefficients and forward rows; the second evaluates the explicitly separated inverse diagnostics and sensitivity checks. The retained unmatched plaquette script can be run independently. A failed reference test is recorded as unresolved, not converted into a passing test.

The sources in Table 8 were read at the cited theorem and equation locations. The indexed public-paper corpus was searched for the common action, shared links, fluctuation accounting, normalisation cancellation, finite matching, physical clocks, representation response and BCB's scope; the live physics catalogue and the cited public pages were checked. The package records search coverage, source hashes and the passages supporting each dependency. The mathematical connection in Section 8 rests on those passages and the displayed identities, not on the number of documents searched.

The archive has no assigned public repository identifier. Its files should accompany any public release that claims executable reproduction. All references below identify public website or literature locations. For [14], the package retains the consulted public-paper snapshot because its website download currently resolves to a different paper; Eq. (21) is self-contained and its exact check is included.

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