# Physical Gauge Normalisation and a One-Calibration Force Test in VERSF

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## General reader summary

A model can get the balance between forces roughly right while getting their individual strengths wrong. Think of an instrument whose strings are in tune with one another, but all play too high. This paper uses the measured strong force to set the scale of a VERSF construction, then calculates the other strengths.

The original construction makes electromagnetism about seven per cent too strong. A proposed shared cost brings the individual strengths much closer to measurement, with the electromagnetic error below one per cent. However, the weak-mixing angle, which describes the balance between the electroweak forces, becomes less accurate. The shared-cost rule is inherited from a paper seeking to explain electromagnetism; it remains a hypothesis.

The force fields' own quantum fluctuations give a small further improvement within the modified construction, but unfinished matter corrections could outweigh it. The paper also calculates how forming records move and contribute to resistance to acceleration, under a specified rule for their motion. This supplies a concrete, testable model. Completing the corrections and connecting the whole response to physical forces remain necessary.

## Abstract

We construct and test a compact-group completion of the VERSF wheel response, with exact auxiliary elimination and one force calibration conditional on its inherited construction premises. The global-cost extension implements the inherited equal-cost premise as $z_1 = (15/14)(31/59) = 465/826$. With one-loop finite conversion and the calculated pure-gauge two-loop block, the low-energy outputs are $\hat{\alpha}^{-1}(M_Z) = 128.84309$ and $\hat{s}_W^2(M_Z) = 0.22802766$, against the fixed 2024 benchmarks $(127.930, 0.231290)$. The convention-matched weak and hypercharge couplings differ by +0.355% and −0.566%; the strong coupling is calibrated. The individual strengths improve, but the weak-mixing residual worsens from −0.21% on the original branch to −1.41%. The omitted two-loop matter terms need not be smaller than the calculated gauge term. An explicit Gaussian extension within the VERSF history-action form returns $W_{\mathrm{eff}}(\omega) = [1 - (\omega^2 + 1 + \lambda)^{-1}]W_0$, retaining inherited cross-defect terms. We calculate record correlations and stable reversible completions. A dynamical Schur identity gives the effective inertia, including auxiliary motion, and a bound on the low-frequency remainder. For the primitive wheel with unit retained and bath inertias in the stated canonical units, the record bath supplies 46.85%, 52.02% and 18.65% of the temporal coefficients. These shares depend on the kinetic choice. The results establish quadratic compatibility, while the physical gauge-source map, retained-inertia rule and nonlinear heat-kernel identification remain conditions. External mass and Yukawa inputs are retained; a matter regulator-conversion check is unresolved, and finite Higgs and Yukawa matching is unevaluated. The global-cost and kinetic rules remain construction choices. The three physical force strengths are not derived.

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Notation. $A \in \{C, W, Y\}$ labels colour, weak isospin and conventional hypercharge. $K_A$ is the charge trace metric and $C_A$ the adjoint Casimir; $N$ in Section 9 denotes an $SU(N)$ factor. $a$ is the unrelaxed action amplitude, $z$ its relaxed coefficient, and $a_{\mathrm{lat}}$ a lattice spacing. $\lambda_{\mathrm{hr}}$ is the original hub–rim eigenvalue; $\lambda_0, \lambda_1$ distinguish the global-cost test. $t$ is heat-kernel diffusion time, $\sigma$ a sequential coordinate and $t_{\mathrm{phys}}$ physical time. $E_A, B_A$ are electric and magnetic coefficients, while $\mathcal{B}_A = g_A^{-2}$ is an inverse squared coupling and $v_A$ the matter vertex. $\delta_A^{(1)}$ denotes the finite one-loop correction (18). $C_W = 0.16995599$ is an inherited conversion constant, not $C_{A=W} = 2$. $a_R$ is the numerical integral constant in (33). Hatted observables use the fixed PDG convention of Section 7.

In Section 10, $W_0$ is the resolved local defect metric, $C_Q$ and $\Omega_Q^2$ are the bath coupling and restoring operator, $\tau_E$ is a dimensionless Euclidean evolution coordinate, and $\omega$ is its Fourier frequency. $J_D$ is the complete defect Jacobian. The projected record coordinate is $r_{\mathrm{hr}}$, $q_\perp$ is a transverse field amplitude, $\mathfrak{m}$ its retained inertia and $\lambda_{\mathrm{cyc}}$ a primitive edge-cycle eigenvalue, abbreviated $\nu$ in Section 10.6. There $\mathsf{H}$ and $\mathsf{M}$ are static and kinetic matrices, and $\mathcal{I}_F$ is the reduced temporal coefficient. These are distinct from the hypercharge address $q$, particle masses and the matching scale $\mu_*$.

# 1 Purpose and main results

The finite-matching paper [1] obtained a close weak-mixing comparison after calibrating the strong and electromagnetic inputs. Its result establishes a useful conditional relation among the forces, but an electromagnetic calibration removes a common normalisation test. The present paper restores that test. A concrete joint gauge–auxiliary weight is specified, integrated and converted to continuum couplings. Only the strong coupling fixes the boundary scale. Weak and electromagnetic outputs are calculated without either entering that solve.

The construction need not be unique. What matters is whether its choices are explicit, whether its dynamics are well posed and compatible with the intended VERSF sector, and whether its calculated responses survive comparison with the physical quantities. Compatibility of the complete primitive system is a stronger claim than compatibility of the effective sector constructed here.

Five propositions organise the structural results. Proposition 1 isolates the normalisation that survives changes of field coordinates and clock units. Proposition 2 shows why a conditional transition law loses an absolute gauge-response term. Proposition 3 completes the wheel's series response on the compact gauge group. Sections 7–8 test the original unit-amplitude branch, and Section 9 constructs the global-cost branch and calculates its pure-gauge two-loop matching. Proposition 4 gives an explicit Gaussian history-action connection. Proposition 5 supplies the dynamical reduction and its remainder bound; the primitive wheel application calculates both static and temporal response without fitting a force.

**Status convention.** Exact statements concern the explicitly defined finite-regulator construction. Perturbative matching and continuum running have their stated truncations. Experimental values are either calibration inputs or comparison targets, as marked. A diagnostic fit measures a missing response and is not counted as a prediction. None of these labels requires unique selection of the construction by VERSF.

The original branch's close mixing angle conceals an error in the individual electroweak strengths, visible in Table 3. The global-cost branch substantially improves their magnitudes, while the mixing residual worsens from −0.21% to −1.41% in the partial two-loop row. The calculated gauge correction improves both comparisons relative to the global-cost one-loop row only; the missing matter terms need not preserve that direction. Table 7 distinguishes the branches and matching orders. Neither branch establishes all three physical strengths.

# 2 Source commitments and the construction choices

The sources provide more structure than a free three-parameter force fit. The master action [2] treats the primitive residuals in a common positive metric, including curvature, density transport, current continuity, record composition, bond compatibility and scalar–completion stretch. Its typed projections are not independent sector weights to be adjusted after observing the answer. The gauge-action theorem [3] identifies the reduced curvature Hessian, canonical connection normalisation and the same connection in the matter vertex and gauge self-interaction. The history-measure paper [4] separates a raw transfer weight from the stochastic process obtained after normalisation. Its Fisher-to-action bridge requires the raw weighting and the physical tangent directions, not just the transition probabilities.

The stationary-response paper [5] supplies the holding-step wheel and its relaxation factor. The sequential-time paper [6] supplies the distinction between opportunity counting and physical time. The ledger [7] asks for a common physical response, a declared clock and distance conversion, and sector responses that survive physical matching. The interacting-response paper [8] provides a restricted uniform-current correction used only as a sensitivity test below. These are the relevant source dependencies; they do not by themselves establish the specific compact completion chosen here.

The effective construction uses the faithful compact group and inherited trace census

$$G = [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6, \quad K = (6,13/2,21/2). \qquad (1)$$

The matter and Higgs contributions are $K_F = (6,6,10)$ and $K_H = (0,1/2,1/2)$. Matter generators use conventional hypercharge $Y$, not the integer address $q = 6Y$. No additional charged record precursor is included. The fermion and scalar discretisations and their finite conversions are inherited from [1]. The bosonic compact weight is specified in Section 5, rather than borrowing a Wilson plaquette action with a different nonlinear expansion.

Table 1. Construction choices and their status.

| Item | Value or rule | Status |
|---|---|---|
| Charge census | $K_F$ and $K_H$ above | Inherited premise |
| Wheel | Holding-step seven-site wheel | Inherited effective sector |
| Source profile | Normalised mean-zero hub–rim mode | Declared symmetry choice |
| Unrelaxed amplitude | $\mathfrak{a} = 1$ | Declared benchmark in action units |
| Compact completion | Heat kernel for the $K$ metric | Explicit additional dynamical choice |
| Charged matter | Inherited overlap fermions and nearest-neighbour Higgs | Effective regulator choice |
| Force calibration | $\alpha_s^{(5)}(M_Z) = 0.1180$ | Sole force input to prediction solve |
| Masses and Yukawa data | Frozen external SMDR reference inputs | Conditional Standard Model inputs |

The unit value of $\mathfrak{a}$ is a testable construction choice until the raw primitive weight has been projected onto the gauge and source coordinates. The compact heat-kernel law is motivated by isotropic diffusion and exact composition. Section 10 realises its quadratic wheel response within the Gaussian history-action form, retaining the mixed defect terms, but does not derive its full nonlinear plaquette law or its complete primitive source projection.

## 3 Physical normalisation and the clock

Throughout, $\Gamma$ is the action divided by $\hbar$, so its coefficients specify dimensionless quantum weights. For one factor, let its long-wavelength quadratic form in physical coordinates be

$$\Gamma_A^{(2)} = \frac{1}{2}\int dt_{\mathrm{phys}}\, d^3x \left[\mathcal{E}_A|F_{0i}|^2 + \mathcal{B}_A \sum_{i<j} |F_{ij}|^2\right], \qquad (2)$$

with the spatial index in the electric term summed. The same connection enters matter transport through $D_i = \partial_i + iv_A A_i^a T_a$. The coefficients are assumed positive. This is a local, two-derivative description; frequency-dependent dissipation and nonlocal response must first be reduced to the intended physical regime.

**Proposition 1.** After the clock has been chosen to make this factor's propagation isotropic, its dimensionless coupling satisfies

$$g_A^{-2} = \frac{\sqrt{\mathcal{E}_A \mathcal{B}_A}}{v_A^2}. \qquad (3)$$

**Proof.** Set the propagation speed to $c_A = \sqrt{\mathcal{B}_A/\mathcal{E}_A}$ and use $x^0 = c_A t_{\mathrm{phys}}$, transforming the temporal connection as a one-form. Both quadratic coefficients become $\sqrt{\mathcal{E}_A \mathcal{B}_A}$. Canonical field normalisation then turns the vertex coefficient into $v_A/(\mathcal{E}_A \mathcal{B}_A)^{1/4}$. Squaring and inverting gives (3). A field-coordinate change $A'_i = sA_i$ changes both quadratic coefficients by $s^{-2}$ and the vertex by $s^{-1}$, leaving (3) unchanged. □

With dimensionless lattice coefficients $c_{E,A}, c_{B,A}$, time spacing $a_t$ and spatial spacing $a_s$, the corresponding relations are

$$\mathcal{E}_A = c_{E,A}\frac{a_t}{a_s}, \quad \mathcal{B}_A = c_{B,A}\frac{a_s}{a_t}, \quad c_A = \frac{a_s}{a_t}\sqrt{\frac{c_{B,A}}{c_{E,A}}}. \qquad (4)$$

Thus the geometric mean is independent of the coordinate conversion. A common physical light cone additionally requires the ratios $c_{B,A}/c_{E,A}$ to agree across the sectors. Separate factor-dependent clock rescalings cannot establish one shared physical spacetime. The present isotropic construction satisfies this requirement at its declared quadratic level; its origin from the complete sequential dynamics is still to be established.

The integer hypercharge convention illustrates the same point. If $q = 6Y$ and the matter connection is held fixed, then $A_Y = 6A_q$, $Z_Y = Z_q/36$ and $g_Y = 6g_q$. Changing that convention consistently does not change a force. Nor can a factor be recovered by counting both orientations of a face after the action has already used the sum over $\mu < \nu$.

The matching scale fixes a coordinate conversion only after a convention such as $\mu_* = 1/a_{\mathrm{lat}}$ is declared. In ordinary units,

$$a_{\mathrm{lat}} = \frac{\hbar c}{\mu_*}, \qquad \frac{a_{\mathrm{lat}}}{c} = \frac{\hbar}{\mu_*}. \qquad (5)$$

Neither quantity identifies a native irreversible event interval without the additional clock map. This distinction prevents an inferred high-energy scale from being presented as an independently derived VERSF tick.

## 4 What normalised histories cannot determine

A positive raw transfer family $\mathsf{T}_F$ depends on a fixed external probe $F$. Let its leading eigenvalue and positive right eigenvector be $\Lambda_F$ and $h_F$. Its conditional transition law is

$$P_F(x,y) = \frac{\mathsf{T}_F(x,y)h_F(y)}{\Lambda_F h_F(x)}. \qquad (6)$$

**Proposition 2.** Conditional transition probabilities, their source derivatives and their conditional Fisher information do not determine a source-only scalar contact term in the raw transfer weight.

**Proof.** For a scalar function $u(F)$, replace the raw weight by

$$\mathsf{T}'_F = e^{-u(F)}\mathsf{T}_F. \qquad (7)$$

The right eigenvector is unchanged and the leading eigenvalue becomes $e^{-u(F)}\Lambda_F$. The two factors cancel in (6), for every value of $F$. All derivatives of the conditional transition law therefore agree. The raw free energy per step, $-\log\Lambda_F$, instead gains $u(F)$. Taking $u(F) = \frac{1}{2}cF^2$ changes its quadratic response by $c$. □

This is an information-loss statement about conditioning on a fixed external probe. If $F$ is itself integrated as a dynamical field, the multiplier changes the joint physical theory. It is then not an equivalent realisation. If the full source-dependent raw action is known, its contact term can be calculated and the ambiguity disappears.

The relevance to VERSF is specific. A stochastic wheel kernel and its Fisher metric can constrain relaxation and motion without, on their own, proving the absolute raw gauge cost. The master action may supply that cost through its primitive metric and source embedding. That calculation must retain the normalising eigenvalue or equivalent raw source weight. It cannot be reconstructed from the normalised kernel alone. This explains why an exact wheel eigenvalue does not complete the force normalisation.

# 5 Exact compact completion of the wheel response

## 5.1 The quadratic wheel sector

The hub row of $P$ is uniform on all seven sites. Each rim row has probability $1/4$ on the hub, itself and its two rim neighbours. Its stationary measure is $\pi = (7,4,4,4,4,4,4)/31$. The operator $L = I - P$ is self-adjoint in the $\pi$ inner product and has spectrum

$$\mathrm{spec}(L) = \{0,1/2,1/2,1,1,5/4,31/28\}. \qquad (8)$$

For a unit-normalised source profile $\chi$ and scalar probe $f$, the joint quadratic cost is

$$\Gamma(f,r) = \frac{\mathfrak{a}}{2}\| f\chi - r \|_\pi^2 + \frac{\mathfrak{a}}{2}\langle r, Lr\rangle_\pi. \qquad (9)$$

Eliminating $r$ gives the Schur response $\mathfrak{a}\langle\chi, L(I+L)^{-1}\chi\rangle_\pi$. On the $D_6$-invariant mean-zero hub–rim profile

$$\chi_{\mathrm{hr}} = \frac{(24,-7,-7,-7,-7,-7,-7)}{\sqrt{168}}, \quad \lambda_{\mathrm{hr}} = 31/28, \quad z = \mathfrak{a}\frac{31}{59}. \qquad (10)$$

The Gaussian integral also gives (10), because its determinant is independent of $f$ under the stated flat measure. This is why a source-dependent fluctuation determinant must not be added again to the same restricted Gaussian sector. The absolute coefficient is $z$, whereas $31/59$ is its ratio to the unrelaxed coefficient.

## 5.2 A positive group weight with exact elimination

Let $k_t$ be the heat kernel on $G$ for the invariant Laplacian

$$\Delta_K = \sum_{A,a} K_A^{-1} X_{A,a}^2, \qquad \partial_t k_t = \Delta_K k_t. \qquad (11)$$

Here $X_{A,a}$ are the left-invariant generator derivatives, with the usual negative Laplacian spectrum. Haar measure has total mass one. The heat kernel is a positive central density. On the quotient group its character expansion contains precisely the representations that descend to the quotient. Heat-kernel or Villain lattice actions are established constructions [9,10]; their use here is an explicit completion of the wheel sector, not a new general construction of lattice gauge theory.

For a plaquette holonomy $U$ and auxiliary group variable $V$, set

$$w(U,V) = k_{t_1}(UV^{-1})k_{t_2}(V), \quad t_1 = \frac{1}{2\mathfrak{a}}, \quad t_2 = \frac{1}{2\mathfrak{a}\lambda_{\mathrm{hr}}}. \qquad (12)$$

**Proposition 3.** Haar elimination of $V$ yields a positive compact plaquette weight with leading coefficient (10):

$$\int_G w(U,V)\, dV = k_{t_1+t_2}(U), \qquad t_1 + t_2 = \frac{1}{2z}. \qquad (13)$$

**Proof.** The first equality is the heat-kernel semigroup identity. The second follows from the times in (12). Locally, the two short-time quadratic costs are $\mathfrak{a} \parallel F - R \parallel_K^2/2$ and $\mathfrak{a}\lambda_{\mathrm{hr}} \parallel R \parallel_K^2/2$, giving the same series reduction as (9) on the selected mode. □

The group integral fixes more than a Hessian: it specifies the full compact nonlinear weight. All representation coefficients of $k_t$ are nonnegative, so the pure-gauge plaquette theory has a positive transfer at finite regulator. On a spatial lattice, write its spatial plaquette weight as $W_s$ and its electric heat operator as $T_E = \exp\left[t \sum_\ell \Delta_{K,\ell}\right]$. On the gauge-invariant Hilbert space the symmetric transfer is $T = W_s^{1/2} T_E W_s^{1/2}$, with generator $H = -\hbar a_t^{-1} \log(T/\Lambda_0)$. Here $\Lambda_0$ is fixed at zero external probe; it is not removed separately at each probe value. This specifies the pure-gauge motion as well as the static weight. Gauge-invariant matter uses the same connection, but this construction does not establish a nonperturbative chiral fermion transfer or irreversible recording. The auxiliary $V$ is a reversible constitutive variable, not automatically a permanently retained record.

The equality in (13) integrates its fluctuations once. Counting the same $V$ as an additional independent charged field afterwards would change the theory. Additional current, matter or record sectors require distinct operators and their actual couplings to this block.

### 5.3 The range supplied by source choice

For any mean-zero unit profile, the spectral theorem gives

$$\frac{1}{3} \le \frac{z}{\mathfrak{a}} = \langle \chi, L(I+L)^{-1}\chi \rangle_\pi \le \frac{5}{9}. \qquad (14)$$

This bound concerns the quadratic wheel family. Extending a general profile by a heat kernel whose time reproduces its $z$ is an additional effective convention, rather than a proof that every multimode compact elimination reduces to that kernel. The profile comparison in Section 8 adopts this convention explicitly. A uniform component lowers the response because it lies in the zero eigenspace. Removing the holding step changes the wheel itself and gives $4/7$ on its hub–rim mode, outside the family in (14).

## 6 Finite matching fixed by the completion

Near the identity, write $U = \exp(iF)$ and $\parallel F \parallel_K^2 = \sum_A K_A |F_A|^2$. The short-time group heat kernel has the local form

$$k_t\left(e^{iF}\right) = N(t) j(F)^{-1/2} \exp[-\parallel F \parallel_K^2/(4t)]\,[1 + \text{image terms}], \qquad (15)$$

where source-independent curvature factors are absorbed into $N(t)$. The exponential-map Haar Jacobian satisfies

$$\log j(F) = -\frac{1}{24}\sum_A C_A |F_A|^2 + O(F^4), \qquad C = (3,2,0). \qquad (16)$$

At $t = 1/(2z)$ the local action therefore contains

$$-\log k_t\left(e^{iF}\right) = \text{constant} + \frac{1}{2}\sum_A \left(zK_A - \frac{C_A}{24}\right)|F_A|^2 + O(F^4). \qquad (17)$$

Equation (17) is the perturbative local expansion; nontrivial compact images are not represented by a power series in the weak coupling. The displayed shift is a property of the plaquette density, separate from the link Haar measure already included in a lattice gauge calculation. Character checks in Appendix A verify its local non-Abelian curvature at the benchmark to floating-point precision.

The preceding finite-matching paper derives, for its logarithmic plaquette completion, the gauge-plus-ghost correction $-(C_W - 1/12)C_A$ [1,11]. The heat-kernel action has the same leading logarithmic quadratic action and no quartic term proportional to $z$. Its additional quadratic term in (17) is of one-loop order. Terms of order $F^4$ with coefficient of order one contribute only at the next order after canonical field normalisation. Consequently the one-loop heat-kernel conversion is

$$\delta_A^{\mathrm{HK}} = c_F K_{F,A} + c_S K_{H,A} - \left(C_W - \frac{1}{24}\right)C_A. \qquad (18)$$

This adds the plaquette correction to the inherited logarithmic-action result; it neither adds a second link measure nor imports the Wilson quartic contribution. The constants are $c_F = -0.014356773308$ for the declared overlap regulator and $c_S = -0.000603016862$ for the nearest-neighbour scalar, both in the conventions of [1]. Their underlying integrals and source references are retained in the executable package.

Table 2. Finite inverse-coupling contributions at $\mu_* = 1/a_{\mathrm{lat}}$.

| Contribution | Colour | Weak | Hypercharge |
|---|---|---|---|
| Fermion | −0.086140640 | −0.086140640 | −0.143567733 |
| Higgs | 0 | −0.000301508 | −0.000301508 |
| Heat-kernel gauge and ghost | −0.384867970 | −0.256578647 | 0 |
| Total $\delta_A^{\mathrm{HK}}$ | −0.471008610 | −0.343020795 | −0.143869242 |

The continuum boundary condition is

$$g_A^{-2}(\mu_*) = zK_A + \delta_A^{\mathrm{HK}} + O(g_0^2). \qquad (19)$$

The omitted term is a higher-order lattice-to-continuum conversion, not a numerical integration error. Specifying a complete local weight does not make its one-loop matching exact. This uncertainty is particularly relevant to precision claims based on a small residual mixing-angle difference.

# 7 The one calibration calculation

## 7.1 Inputs and physical conventions

The main solve fixes $\mathfrak{a} = 1$ and hence $z = 31/59$. Its sole calibrated force datum is the five-flavour strong coupling $\alpha_s^{(5)}(M_Z) = 0.1180$. The energy $M_Z = 91.1876$ GeV specifies the comparison point. The fixed comparison targets are the 2024 PDG values $\hat{\alpha}^{-1} = 127.930$ and $\hat{s}_W^2 = 0.231290$ [12]. They are not labelled as the latest values and do not enter the prediction solve.

SMDR [13] runs the full Standard Model parameters and supplies low-energy conversions. The main run uses its three-loop running option for the high-scale evolution. At the low end, its published gauge-conversion routines and five-loop option for the strong decoupling path are retained. These options use the

terms implemented for each parameter; they do not mean every beta function or threshold is known at the same loop order. The reported hatted quantities decouple the top quark but retain the weak gauge bosons, whereas the strong input has five active quark flavours. Directly comparing full-theory electroweak couplings with those hatted targets would mix conventions.

The external Higgs, Yukawa and vacuum inputs are frozen at $Q_0 = 172.4$ GeV from the archived SMDR reference model. All three gauge inputs are overwritten for each trial. The three trial gauge couplings at $Q_0$, together with $\mu_*$, solve three boundary equations (19) and one low-energy strong equation. The electromagnetic and weak-mixing data are absent from these four equations.

This is one force calibration, not a derivation of the mass and Yukawa sector. The reference non-gauge parameters were themselves determined in a Standard Model input analysis and can carry indirect electroweak information. The two outputs are therefore conditional predictions of this boundary construction, not statistically independent first-principles predictions of the entire Standard Model. No claim is made that the candidate also reproduces the measured Higgs, weak-boson or fermion pole spectrum with these frozen parameters.

## 7.2 Calculated strengths

Table 3. One-force-calibration results in the stated low-energy conventions.

| Quantity | Calculation | Fixed comparison | Difference |
|---|---|---|---|
| $\alpha_s^{(5)}(M_Z)$ | 0.118000 | 0.118000 | Calibration |
| $\hat{\alpha}^{-1}(M_Z)$ | 119.44483 | 127.930 | −6.633% |
| $\hat{s}_W^2(M_Z)$ | 0.23079927 | 0.231290 | −0.2122% |
| $g_s$ | 1.217716 | 1.217716 | Calibration |
| $\hat{g}$ | 0.675156 | 0.651689 | +3.601% |
| $\hat{g}'$ | 0.369829 | 0.357468 | +3.458% |

The comparison couplings in the last two rows are calculated from the stated $\hat{\alpha}$ and $\hat{s}_W^2$, not separate experimental inputs. Their definitions are

$$\hat{g}^2 = \frac{4\pi}{\hat{\alpha}^{-1}\hat{s}_W^2}, \qquad \hat{g}'^2 = \frac{4\pi}{\hat{\alpha}^{-1}(1-\hat{s}_W^2)}. \qquad (20)$$

For the original branch, the inverse electromagnetic strength is low by 6.633%; the electromagnetic strength itself is high by about 7.10%. The individual gauge couplings are high by about 3.5%. Those percentages describe different quantities. The small relative mixing-angle residual is not the accuracy of the individual force strengths.

The matching scale and coordinate conversions are

$$\mu_* = 4.15034 \times 10^{11}\ \text{GeV}, \quad \log(\mu_*/M_Z) = 22.23871, \qquad (21)$$

$$a_{\text{lat}} = 4.75448 \times 10^{-28}\ \text{m}, \quad a_{\text{lat}}/c = 1.58592 \times 10^{-36}\ \text{s}. \qquad (22)$$

The latter are the conversions in (5), not an independently measured distance or physical recording period. Changing the matching convention would require its compensating logarithmic conversion and running; shifting $\mu_*$ alone is not a free repair of the prediction.

### 7.3 Checks of running and thresholds

Table 4. Running-order and scheme checks at the same fixed boundary.

| Treatment | Electromagnetic inverse strength | Weak-mixing ratio |
|---|---|---|
| Two-loop gauge-only running, no physical low-scale conversion | 119.69782 | 0.23056736 |
| SMDR two-loop high-scale option, hatted outputs | 119.42724 | 0.23075378 |
| SMDR three-loop high-scale option, hatted outputs | 119.44483 | 0.23079927 |
| SMDR five-loop high-scale option, hatted outputs | 119.44380 | 0.23080298 |

The first row reproduces the simpler running convention used to expose the boundary dependence, but is not a like-for-like physical comparison. Within the SMDR variants, the low-energy conversion routines are held fixed. The small change between the last two rows supports numerical stability of this running step. It does not bound the missing two-loop lattice matching, unknown interacting sectors or the primitive reduction.

For the main solution the corresponding non-decoupled outputs are $\alpha_{\mathrm{full}}^{-1} = 119.62146$ and $s_{W,\mathrm{full}}^{2} = 0.23101220$. Their difference from the hatted values is large enough to matter when quoting the mixing-angle residual. Varying the sole force input from $0.1171$ to $0.1189$ gives $\hat{\alpha}^{-1} = 119.40891$ to $119.48022$ and $\hat{s}_W^2 = 0.23109674$ to $0.23050659$. This is a stated input sensitivity, not a combined uncertainty interval.

## 8 Dependence on choices and the required correction

### 8.1 Changing the wheel coefficient

Table 5 keeps the compact heat-kernel convention and matter discretisations fixed while changing the coefficient entering (19). The $5/9$ row is the maximal mean-zero quadratic response of the holding-step wheel at $\alpha = 1$. The $4/7$ row is a distinct no-holding wheel.

Table 5. Coefficient sensitivity with strong calibration only.

| Source coefficient $z$ | Meaning | $\hat{\alpha}^{-1}$ | $\hat{s}_W^2$ |
|---|---|---|---|
| 1/2 | Holding-wheel eigenmode | 112.99918 | 0.23311647 |
| 31/59 | Hub–rim benchmark | 119.44483 | 0.23079927 |
| 5/9 | Holding-wheel upper bound | 127.08309 | 0.22834956 |
| 4/7 | No-holding hub–rim mode | 131.10643 | 0.22717091 |

Within this fixed matching prescription, electromagnetic agreement would require $z = 0.55889671$, beyond the $5/9$ upper endpoint. Thus source redistribution alone in the declared holding-wheel quadratic family cannot reach that electromagnetic target with unit amplitude. This is a restriction of this prescription, not an exclusion of all compact completions of the wheel or of the complete VERSF dynamics. Omitted matching terms can change the endpoint comparison.

### 8.2 A common adjustment does not repair both outputs

Allowing the electromagnetic measurement to determine $z$ is useful as a diagnostic, provided it is labelled as a second force calibration. The result is

$$z_{\mathrm{EM}} = 0.55889671, \quad \mathfrak{a}_{\mathrm{EM}} = 1.06370665, \quad \hat{s}_W^2 = 0.22809547. \qquad (23)$$

The common amplitude would increase by 6.371%, but the remaining angle residual would become $-1.381\%$. A common amplitude correction therefore cannot make both electroweak observables agree in this branch. This conclusion follows from solving the coupled equations, rather than from treating the weak and hypercharge responses as separate adjustable numbers.

To quantify a second direction without presenting a fit as a derivation, consider the restricted extension

$$g_A^{-2}(\mu_*) = zK_A + \delta_A^{\mathrm{HK}} + hK_{H,A}. \qquad (24)$$

Fitting both electroweak targets, while retaining strong calibration, gives

$$z = 0.55370365, \quad h = 0.10497537, \quad \mathfrak{a} = 1.05382308. \qquad (25)$$

These numbers define an inverse problem. They are excluded from Table 3. In this two-direction parametrisation the required change from the benchmark is

$$\Delta Z_A = (0.55370365 - 31/59)K_A + 0.10497537K_{H,A}. \qquad (26)$$

Other response directions could satisfy the same two measured constraints; the $K_H$ direction is a transparent inherited basis vector, not an assertion that only Higgs fluctuations can repair the result. A physically calculated correction must include its own action, determinant, source vertices and matching. Multiplying a scalar kinetic residue by a constant while transforming its measure consistently does not generate this extra finite gauge response [1].

### 8.3 A known restricted current contribution

The fixed-density uniform-current calculation inherited from [1,8] gives the common shift $c_J K_A$ with $c_J = 0.00103611031$. Adding that separate restricted block would give $\hat{\alpha}^{-1} = 119.70749$ and $\hat{s}_W^2 = 0.23070998$. It slightly improves the absolute electromagnetic response and worsens the mixing comparison. Its common nature cannot supply the independent direction in (26).

This is an extension test, not a claim that all independent circulating currents have been integrated into the heat-kernel construction. The auxiliary variable in (12) has already been eliminated. A future current calculation must establish that its retained variables are distinct and must include the cross terms with density, matter and recording fields. The restricted $c_J$ result must not be added a second time if it is already contained in that joint determinant.

## 9 Global closure and two-loop gauge matching

### 9.1 The fixed global-cost branch

The fine-structure source [14, Section 5.2 and Appendix D.4] counts fourteen local contributions and one global contribution with the same cost as one local term. This gives $15/14$. Reference [14, Section 6.2] uses the measured low-energy electromagnetic strength as a comparison, rather than an algebraic input fixing this factor, and reports no fitted continuous parameter. This does not establish that the equal-cost premise was selected without knowledge of that measurement. Equality remains a construction premise: symmetry among local channels does not fix the distinct global weight. Appendix E.4 of [14] also allows its dynamical coefficient to change with the microscopic model. This branch tests the inherited hypothesis before

continuum evolution. The phrase one calibration counts fitted force inputs to the forward solve, conditional on that hypothesis; it does not certify blind or target-independent model selection.

With $a = 1$, the retained compact construction has

$$z_0 = \frac{31}{59}, \qquad z_1 = \frac{15}{14} z_0 = \frac{465}{826}, \qquad \lambda_0 = \frac{31}{28}, \qquad \lambda_1 = \frac{465}{361}. \tag{27}$$

An explicit positive realisation on the same wheel variables adds the relative hub penalty

$$\begin{aligned} \Delta\Gamma_h &= \frac{J_c}{2}\left(b_h^T r\right)^2, \qquad J_c = \frac{354}{11191}, \\ b_h &= (1, -1/6, -1/6, -1/6, -1/6, -1/6, -1/6)^T. \end{aligned} \tag{28}$$

The Riesz vector $\pi^{-1} b_h$ lies along the hub–rim source, and $b_h^T \pi^{-1} b_h = 961/168$. The penalty therefore increases that mode's restoring eigenvalue by $\kappa = (961/168) J_c = 1829/10108 = \lambda_1 - \lambda_0$. Substituting $\lambda_1$ into $z = \lambda/(1+\lambda)$ gives (27). The cost is positive, invariant under rim symmetries and gauge invariant when the squares use the common-base-point trace norm. Its coefficient is obtained from the inherited factor, not derived independently from the primitive action. For comparison, assigning the penalty the existing spoke conductance $1/31$ would give $z = 31/55$, a different declared choice.

The nonlinear completion changes the second heat-kernel time in (12) to $1/(2\lambda_1)$. Exact elimination (13) still holds. The same links enter the overlap fermion operator and nearest-neighbour Higgs action; this minimal branch has no additional charged record doublet. The compact completion is an explicit choice beyond the quadratic penalty.

### 9.2 What the auxiliary integration contributes

Write $\mathcal{B}_A = g_A^{-2}$ for the inverse squared coupling, distinct from the magnetic coefficient $B_A$ in (2). At the retained one-loop order, the closure on/off difference is

$$\delta_A^{(1)}(\lambda_1) - \delta_A^{(1)}(\lambda_0) = 0, \qquad \mathcal{B}_A(\lambda_1) - \mathcal{B}_A(\lambda_0) = (z_1 - z_0) K_A + O(g_0^2). \tag{29}$$

The auxiliary integration is already complete in (13). At fixed background links the matter operators are independent of the heat-kernel time, so their one-loop determinants cancel in this difference. The quadratic heat-kernel shift in (17) is independent of $z$, as is the one-loop gauge conversion at $\mu_* a_{\mathrm{lat}} = 1$. Thus (18) remains unchanged. Quantum gauge exchange introduces further dependence at higher order, which is the calculation below.

Equation (29) does not remove the leading closure response. It prevents the same auxiliary integral from being counted again as a separate late electroweak multiplier. Multiplying the original low-energy electromagnetic inverse directly by $15/14$ would give $127.97660$ and preserve its mixing angle. That is a different finite-matching hypothesis whose interaction and activation scale have not been supplied by this compact parent. The forward branch here instead runs (27) from the boundary and gives the one-loop row in Table 7.

### 9.3 Pure-gauge two-loop conversion

Use $g_{0,A}^2 = (z_1 K_A)^{-1}$. In the published matching convention [15], at $\mu_* a_{\mathrm{lat}} = 1$,

$$\begin{aligned} g_0 &= Z_g g_{\overline{\mathrm{MS}}}, \qquad Z_g^2 = 1 + l_0 g_0^2 + l_1 g_0^4 + O(g_0^6), \\ g_{\overline{\mathrm{MS}}}^{-2} &= g_0^{-2} + l_0 + l_1 g_0^2 + O(g_0^4). \end{aligned} \tag{30}$$

There is no additional $l_0^2 g_0^2$ in this convention. For a Wilson $SU(N)$ gauge action the pure-gauge coefficient is [15]

$$l_{1,W}^{G} = -\frac{3}{128N^2} + 0.018127763034 - 0.007910118514N^2. \qquad (31)$$

It must be converted to the specified plaquette action. The Wilson-to-Manton conversion from the general one-plaquette result [16,17] is

$$g_M^{-2} = g_W^{-2} + A_1 + A_2 g_W^2 + O(g_W^4), \qquad A_1 = -\frac{2N^2-3}{24N}, \qquad (32)$$

$$\begin{aligned} A_2 = \quad & -a_R \frac{2N^2-3}{24} + \frac{N^4-3N^2+3}{384N^2} - \frac{9N^4-30N^2+36}{1152N^2}, \qquad (33) \\ a_R = \quad & -0.0011 \pm 0.0002. \end{aligned}$$

For $SU(3)$, these expressions give $A_1 = -5/24$ and $A_2 = -0.0305625$, reproducing the Wilson row of [17, Table V]. The numerator $2N^3 - 3$ printed in [17, Eq. (11)] does not reproduce that table; (32) uses $2N^2 - 3$, as in the original expression and the table check. Reference [16] has an erratum. The implementation uses the later reproduction [17], which cites it; the original erratum text has not been independently inspected.

The order-one quartic term in the local heat-kernel action is

$$\begin{aligned} -\log k_{1/(2z)}\left(e^{iF}\right) = \text{const} + \frac{1}{2}\sum_A \left(zK_A - \frac{C_A}{24}\right)|F_A|^2 \\ -\frac{\mathrm{Tr}_{\mathrm{adj}}(\mathrm{ad}F)^4}{5760} + O(F^6). \end{aligned} \qquad (34)$$

For a non-Abelian factor,

$$\mathrm{Tr}_{\mathrm{adj}}(\mathrm{ad}F)^4 = 2N\mathrm{tr}_F F^4 + 6(\mathrm{tr}_F F^2)^2. \qquad (35)$$

Contracting with the four-dimensional free plaquette variance $g_0^2/2$ gives $-N^2 g_0^2/1152$ in $\mathcal{B}$. The quadratic shift $-C_A/24$ is already included in (18). Combining the Wilson-to-Manton conversion and the heat-kernel quartic term yields

$$l_{1,HK}^{G} = l_{1,W}^{G} - A_2 - \frac{N^2}{1152}. \qquad (36)$$

The corresponding change of $g_M^2$ in this last term contributes only at the following order. Equations (31)–(36) include the pure-gauge, ghost, measure and plaquette-conversion contributions through the stated order.

Table 6. Pure-gauge two-loop coefficients for the heat-kernel boundary.

| **Factor** | $l_{1,W}^{G}$ | $A_2$ | **HK quartic term** | $l_{1,HK}^{G}$ |
|---|---|---|---|---|
| Colour | −0.05566747 | −0.03056250 | −0.00781250 | −0.03291747 |
| Weak | −0.01937209 | −0.00823438 | −0.00347222 | −0.01460993 |
| Hypercharge | not applicable | not applicable | 0 | 0 |

Multiplying by the respective bare squared couplings gives

$$\Delta\mathcal{B}^{(2),G} = (-0.00974545894,\ -0.00399265669,\ 0). \qquad (37)$$

The Abelian heat kernel is locally quadratic, so its pure-gauge perturbative correction vanishes. Charged matter can still correct hypercharge. The quotient modifies compact image contributions, not these local

power-series coefficients. The quoted uncertainty of $a_R$ gives coefficient sensitivities 0.000125 for colour and 0.00004167 for weak; these do not estimate omitted matter contributions.

### 9.4 One calibration with the gauge correction

The boundary includes (37), retains matter matching at its existing order and is solved with the unchanged SMDR option 3 and external inputs of Section 7. The three boundary equations and the low-energy strong calibration determine the three trial gauge couplings and the matching scale. Neither electroweak comparison target enters these equations.

Table 7. Baseline, global-cost branch and partial two-loop results. Hatted comparison conventions are unchanged.

| Quantity | Original $z_0$, one loop | Global cost $z_1$, one loop | $z_1$ plus gauge two loop only | Fixed comparison |
|---|---|---|---|---|
| $\hat{\alpha}^{-1}(M_Z)$ | 119.44483 | 128.95841 | 128.84309 | 127.930 |
| $\hat{s}_W^2(M_Z)$ | 0.23079927 | 0.22779128 | 0.22802766 | 0.231290 |
| $g_s$ | 1.217716 | 1.217716 | 1.217716 | 1.217716, calibrated |
| $g$ | 0.675156 | 0.654051 | 0.654004 | 0.651689 |
| $g'$ | 0.369829 | 0.355233 | 0.355446 | 0.357468 |
| $\mu_*$ (GeV) | $4.1503 \times 10^{11}$ | $5.1533 \times 10^{12}$ | $4.6204 \times 10^{12}$ | not an input |

Relative to the $z_1$ one-loop row, the pure-gauge correction removes 11.21% of the inverse-electromagnetic residual and 6.76% of the weak-mixing residual. Both comparisons include recalibration of the scale. These are reductions of two particular residuals, not percentages of completion of the Standard Model derivation. In the partial row, $g$ is 0.355% above and $g'$ is 0.566% below the comparison values. The strong entry is calibrated. These coupling residuals are not residuals of their squares. Against the original branch, the inverse-electromagnetic residual changes from −6.633% to +0.714%, while the weak-mixing residual changes from −0.2122% to −1.4105%. The main improvement in the individual strengths therefore coexists with a worse mixing comparison.

The partial matching scale corresponds, under (5), to $a_{\mathrm{lat}} = 4.27078 \times 10^{-29}$ m and $a_{\mathrm{lat}}/c = 1.42458 \times 10^{-37}$ s. They remain coordinate conversions, not a derived irreversible event interval. The maximum boundary-equation residual is $1.91 \times 10^{-14}$.

Varying only $a_R$ over its quoted numerical uncertainty gives $\hat{\alpha}^{-1} = 128.842695$ to 128.843476 and $\hat{s}_W^2 = 0.22802648$ to 0.22802885. Separately, SMDR running options 2 and 5 give (128.825254,0.22798534) and (128.842045,0.22803112). Neither sensitivity bounds the missing finite two-loop sectors. The forward result is therefore labelled partial, rather than assigned a total theoretical error bar. The omitted matter block includes three fermion generations as well as Higgs and Yukawa contributions. No bound makes it smaller than the pure-gauge block, so the favourable direction of this partial step is not evidence that the complete two-loop correction has the same sign.

## 9.5 The open matter conversion and exact representation weights

The published overlap two-loop calculation uses Wilson gluons [15,18]. Its transfer to the heat-kernel action requires the matter-dressed gauge contribution to be converted as well. An attempted plaquette-contraction check evaluates

$$J_P = -\frac{1}{2}\int_{\mathrm{BZ}} \frac{d^4q}{(2\pi)^4} \frac{\sum_\mu \Pi_{\mu\mu}^{\mathrm{Weyl}}(q)}{\hat{q}^2}, \qquad \hat{q}^2 = 4\sum_\mu \sin^2\left(q_\mu/2\right). \qquad (38)$$

Here the matter polarisation is itself a four-dimensional momentum integral. Eight independently scrambled Sobol runs, each with $2^{19}$ momentum pairs, give $J_P = +0.0084792 \pm 0.0000291$, with uncertainty from between-scramble variance. The unsubtracted estimator is $+0.0084800$. The proposed identification with the published $h_2(1) = -0.01934341$ [19] does not reproduce that reference.

Finite differences of the actual 16-site overlap determinant agree with the analytic derivatives within $2 \times 10^{-10}$ after Richardson extrapolation, including a pure-gauge zero check. A full-Dirac determinant contraction is also consistent with the internal projector calculation. These tests verify internal derivative relations, not the identification with [19]. The ratio of the published coefficient to the estimated integral is about −2.2813, so a sign reversal or a factor of two alone cannot reconcile them. Their observable definitions, contractions and regulator conventions must be matched before treating them as the same quantity; a substantive calculation or implementation difference remains possible. Neither the published $h_2$ nor the unmatched $J_P$ has been used to generate a two-loop matter boundary term; no contradiction with the published calculation is claimed.

The representation contractions can be completed independently. With rows labelling an external generator and columns an internal summed Casimir, define

$$M_{AB}^F = \sum_{r\in\mathrm{Weyl}} \mathrm{Tr}_r\,(T_A^2)C_B(r), \qquad M_{AB}^H = \mathrm{Tr}_H(T_A^2)C_B(H). \qquad (39)$$

In the order $(C, W, Y)$, the inherited census gives

$$M^F = \begin{pmatrix} 8 & 9/4 & 11/12 \\ 6 & 9/2 & 1/2 \\ 22/3 & 3/2 & 95/18 \end{pmatrix}, \qquad M^H = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3/8 & 1/8 \\ 0 & 3/8 & 1/8 \end{pmatrix}. \qquad (40)$$

The matrices need not be symmetric because their two indices have different definitions. No fermion spin factor is hidden in (39). At $z_1$, their contractions with the bare squared couplings are $M^F g_0^2 = (3.13842478, 3.09070857, 3.47388414)$ and $M^H g_0^2 = (0, 0.12362834, 0.12362834)$. The finite regulator integrals and the appropriate spin factors still have to multiply these weights. The matrices alone are not finite matching coefficients.

## 9.6 Specification of complete matching

The full two-loop increment must contain

$$\Delta\mathcal{B}_A^{(2)} = \Delta\mathcal{B}_A^{(2),G} + \Delta\mathcal{B}_A^{(2),F} + \Delta\mathcal{B}_A^{(2),H} + \Delta\mathcal{B}_A^{(2),\mathrm{Yukawa}}. \qquad (41)$$

Table 8. Contributions required at the same matching order.

| Contribution | Status | Acceptance check |
|---|---|---|
| Gauge, ghosts, measure and action conversion | Calculated in (31)–(37) | Wilson-to-Manton table; adjoint quartic contraction; recovery of (18) |
| Overlap fermions and mixed gauge exchanges | Representation weights calculated; regulator conversion unresolved | Resolve (38)'s reference discrepancy and reproduce the Wilson result before conversion |
| Higgs and gauge exchange | Finite two-loop integrals unevaluated | Shared-link Ward identities, consistent scalar mass and infrared subtraction, recovery of the one-loop scalar conversion |
| Yukawa interactions | Reference couplings fixed; finite lattice matching unevaluated | Specify chiral lattice vertices and match the same local action and counterterms |

The missing terms in (41) have not been established to vanish. Continuum running fixes logarithms, not the regulator-dependent finite constants. Resolving the matter comparison and evaluating these remaining integrals are the next acceptance gates for this specified branch. A complete result must then repeat the same one-force-calibration solve without selecting corrections from either electroweak target.

# 10 Connection to the VERSF history action

## 10.1 The source action and the explicit extension

The history-measure source [4, Sections 7–10 and 15–16] couples unresolved record coordinates to the complete primitive defect vector of the master action [2]. At quadratic order its raw influence action has the metric

$$W_{\mathrm{eff}}(\omega) = W_{\mathrm{core}}(\omega) - C_Q^{\dagger}\left(\omega^2 I + \Omega_Q^2\right)^{-1} C_Q. \qquad (42)$$

Here $C_Q$ is the defect–bath coupling and $\Omega_Q^2$ is the bath restoring operator. Their physical values and the resolved local metric remain to be evaluated on a specified primitive branch. Equation (42) concerns the raw action. It is not identified with a Fisher metric after conditional normalisation: [4, Corollary 16.2] leaves that equivalence as a separate obligation, consistent with Proposition 2.

An explicit construction within this form recovers the wheel response. Let $\mathcal{D}(X) = \sum_\alpha \iota_\alpha \, \mathcal{D}_\alpha(X)$ denote the complete typed defect vector, with $X$ the primitive configuration and $\alpha$ labelling closure, current realisation, transport conservation, record composition, bond compatibility and scalar–completion stretch. Fix a common-defect-zero branch and a positive resolved local metric $W_0$. The primitive variables remain independent and every inherited cross-defect overlap is retained. For each residual coordinate introduce a seven-component unresolved record coordinate $r$. These independent bath coordinates are distinct from the resolved spatial record transport already in the master action; their identification with particular substrate modes remains a physical condition of the construction.

Let $\Pi = \mathrm{diag}(\pi)$, and use the normalised hub–rim profile $\chi$ of (10). With a dimensionless Euclidean evolution coordinate $\tau_E$, define the fixed-matrix quadratic extension

$$\Gamma_{\mathrm{ext}} = \Gamma_V + \frac{1}{2}\int d\tau_E \, [\, \| \, \partial_{\tau_E} r \, \|^2_{\Pi\otimes W_0} + \| \, \chi \otimes \mathcal{D} - r \, \|^2_{\Pi\otimes W_0} + \langle r, (L_\kappa \otimes I) r \rangle_{\Pi\otimes W_0}]. \qquad (43)$$

The retained-variable kinetic term is not specified by (43). The bath kinetic coefficient is one in the displayed canonical units. The operator $L_\kappa$ is $L$ for the original branch, or $L + J_c \Pi^{-1} b_h b_h^T$ for the global cost (28). The cell labels are internal profiles carrying the residual's transformation law, not additional particle species. This construction is on the fixed physical tangent carrier; moving supports and nonlinear gauge-dependent bath kinetics require their own terms.

After whitening $\Pi$ and $W_0$, the blocks of (42) are $W_{\mathrm{core}} = I$, $C_Q = -\hat{\chi} \otimes I$ and $\Omega_Q^2 = \left(I + \hat{L}_\kappa\right) \otimes I$, with $\hat{\chi} = \Pi^{1/2}\chi$ and $\hat{L}_\kappa = \Pi^{1/2} L_\kappa \Pi^{-1/2}$. The unit core matrix is a coordinate expression of the supplied $W_0$; it does not derive its absolute physical size.

## 10.2 Exact reduction with the mixed terms retained

**Proposition 4. Gaussian history extension.** If $L_\kappa \chi = \lambda\chi$, $\lambda > 0$ and $\| \chi \|_\Pi = 1$, integrating the entire bath once gives

$$W_{\mathrm{eff}}(\omega) = w_\lambda(\omega) W_0, \qquad w_\lambda(\omega) = 1 - \frac{1}{\omega^2 + 1 + \lambda}, \qquad w_\lambda(0) = z = \frac{\lambda}{1+\lambda}. \tag{44}$$

**Proof.** Expanding (43) gives the negative profile lift as the mixed block and $\omega^2 + I + L_\kappa$ as the bath block. Its inverse acts on $\chi$ by multiplication by $(\omega^2 + 1 + \lambda)^{-1}$. Substitution into (42) gives (44). At zero frequency, completing the square gives $r_* = (1+\lambda)^{-1} \chi \otimes \mathcal{D}$. The determinant is source independent under the fixed-matrix and flat-measure hypotheses. □

Let $J_D$ be the differential of the complete defect vector at the common zero. Then

$$W_{\alpha\beta}^{\mathrm{eff}} = z\, \iota_\alpha^\dagger W_0 \iota_\beta, \qquad H_{\mathrm{prim}}(z) = H_V + z J_D^\dagger W_0 J_D. \tag{45}$$

Thus every inherited cross block is retained. For $H_V \succcurlyeq 0$, the null space of the second matrix is $\ker H_V \cap \ker J_D$, independent of positive $z$. The bath is strictly positive, and a stable primitive quadratic branch remains stable on its quotient. This is a finite-regulator tangent result, not a continuum or complete chiral-theory certificate.

The two eigenvalues $\lambda_0 = 31/28$ and $\lambda_1 = 465/361$ reproduce $z_0 = 31/59$ and $z_1 = 465/826$ exactly. The second uses $J_c = 354/11191$ from (28), so the inherited equal-global/local-cost premise is still explicit. Choosing the spoke conductance $J_c = 1/31$ would instead give $z = 31/55$. Neither coefficient is adjusted in this forward calculation; the provenance of the inherited equal-cost choice is stated in Section 9.1.

The extension replaces a resolved local cost by its bath-reduced cost. It must not be applied again to the heat-kernel weight in Section 5 or to a primitive metric that already includes the same fluctuations. All remaining primitive auxiliaries still require the common reduction. Source-dependent spectra, measures or supports would add contributions absent from (44).

## 10.3 Record motion and correlations

For a fixed, canonically normalised scalar defect source $d$, the coupled hub–rim coordinate $r_{\mathrm{hr}}$ has the explicit oscillator Hamiltonian

$$H_r(d) = \frac{1}{2} p_r^2 + \frac{1}{2}(d - r_{\mathrm{hr}})^2 + \frac{\lambda}{2} r_{\mathrm{hr}}^2. \tag{46}$$

Translation of the oscillator gives its mean, source-energy response and zero-temperature connected Euclidean covariance:

$$\begin{aligned} \langle r_{\mathrm{hr}}\rangle_d &= \frac{d}{1+\lambda}, & E_0(d)-E_0(0) &= \frac{z}{2}d^2, \\ \Omega_{\mathrm{hr}} &= \sqrt{1+\lambda}, & \langle \delta r_{\mathrm{hr}}(\tau_E)\delta r_{\mathrm{hr}}(0)\rangle &= \frac{e^{-\Omega_{\mathrm{hr}}|\tau_E|}}{2\Omega_{\mathrm{hr}}}. \end{aligned} \tag{47}$$

The real-time ground-state Wightman covariance is $e^{-i\Omega_{\mathrm{hr}}t}/(2\Omega_{\mathrm{hr}})$ in the same canonical units. The other six wheel modes remain in the determinant. They are not omitted merely because their overlap with this source vanishes.

Table 9. Conditional record response in canonical dimensionless units.

| Quantity | Original branch | Global-cost branch |
|---|---|---|
| Static factor $z$ | 0.525423729 | 0.562953995 |
| Mean response $\langle r_{\mathrm{hr}}\rangle_d/d$ | 28/59 | 361/826 |
| Frequency $\Omega_{\mathrm{hr}}$ | 1.451600102 | 1.512642933 |
| Covariance at $\tau_E = 0$ | 0.344447482 | 0.330547275 |
| Covariance at $\tau_E = 1$ | 0.080667965 | 0.072828456 |

Equation (44) also gives $w_\lambda(\omega) = z + \omega^2/(1+\lambda)^2 + O(\omega^4)$. These are calculated time correlations and a frequency response on the constructed branch. A dynamical source changes the joint normal modes. Neither the displayed frequency nor the Euclidean coordinate identifies a permanent-record event rate in seconds.

## 10.4 A stable propagation completion and its dependence on choice

Let $q_\perp$ be one transverse spatial Fourier amplitude, with curvature amplitude $d = kq_\perp$. A family of explicit quadratic completions is

$$H_{\mathfrak{m},\lambda}(k) = \frac{p_\perp^2}{2\mathfrak{m}} + \frac{p_r^2}{2} + \frac{1}{2}(kq_\perp - r_{\mathrm{hr}})^2 + \frac{\lambda}{2}r_{\mathrm{hr}}^2, \qquad \mathfrak{m} > 0. \tag{48}$$

Each member is positive and self-adjoint on its oscillator domain, generates unitary motion and has a positive Euclidean heat kernel. At $k = 0$ the retained free coordinate uses a finite-volume regulator or the relevant gauge quotient. This supplies the reversible-generator property discussed in [20, Sections 2 and 6]. Permanent-record production and its event rate are additional requirements.

Eliminating the record coordinate gives the exact Euclidean kernel

$$\begin{aligned} \mathcal{K}_\perp(\omega,k) &= \mathfrak{m}\omega^2 + k^2 - \frac{k^2}{\omega^2+1+\lambda} \\ &= \mathfrak{m}\omega^2 + zk^2 + \frac{k^2\omega^2}{(1+\lambda)^2} + O(k^2\omega^4). \end{aligned} \tag{49}$$

If the inherited trace $K_A$ weights this whole sector and the same-link matter vertex is one, the leading coefficients and invariant (3) are

$$\mathcal{E}_A = \mathfrak{m}K_A, \quad \mathcal{B}_A = zK_A, \quad c_A^2 = \frac{z}{\mathfrak{m}}, \quad g_A^{-2} = K_A\sqrt{\mathfrak{m}z}. \tag{50}$$

Table 10. Two positive completions of the same static global-cost response.

| **Retained kinetic rule** | **Leading speed squared** | $g_A^{-2}/K_A$ |
|---|---|---|
| Unit inertia, $\mathfrak{m} = 1$ | $z_1$ | 0.750302602 |
| Matched inertia, $\mathfrak{m} = z_1$ | 1 | 0.562953995 |

The second row is a stable quadratic completion preserving the common electric and magnetic coefficient intended in the force calculation. It introduces no independently fitted numerical value, but the rule $\mathfrak{m} = z$ remains an additional constitutive choice. A common light cone and the static result do not imply it. In particular, [4] distinguishes reference operational geometry from the induced defect metric. Changing clock coordinates cannot turn the first row into the second because the product in (50) is invariant.

## 10.5 The full primitive curvature response

An explicit bath metric does not complete the primitive source projection. The unit orthogonal-defect wheel branch of [2, Section 47A] contains the closure, composition and bond terms

$$\Gamma_{\text{wheel}}(u, y) = \frac{1}{2} \| B_C u \|^2 + \frac{1}{2} \| B_C y \|^2 + \frac{1}{2} \| u - y \|^2. \qquad (51)$$

The first term is a direct curvature cost and must remain. For an edge-cycle mode with eigenvalue $\lambda_{\text{cyc}} > 0$, eliminating $y$ gives

$$k_u = \lambda_{\text{cyc}} + \frac{\lambda_{\text{cyc}}}{1 + \lambda_{\text{cyc}}}, \qquad k_F(z) = z\left(1 + \frac{1}{1 + \lambda_{\text{cyc}}}\right). \qquad (52)$$

The second expression uses the unit curvature amplitude $f = \sqrt{\lambda_{\text{cyc}}}\, u$ and applies the bath extension to all three raw defects. It is not a rescaling of a physical gauge generator. The source's canonical isotypic assignment has $\lambda_{\text{cyc}} = (4,5,1)$ in colour, weak and Abelian order [2, Section 47B].

Table 11. Geometric coefficients per unit curvature in the primitive wheel test.

| **Channel** | **Before the bath** | **After the global-cost bath** |
|---|---|---|
| Colour isotype | $6/5$ | $279/413 = 0.675544794$ |
| Weak isotype | $7/6$ | $155/236 = 0.656779661$ |
| Abelian isotype | $3/2$ | $1395/1652 = 0.844430993$ |

These are restricted geometric coefficients, not matched force strengths. They show explicitly that a common multiplier of the history metric does not remove the primitive closure and bond response. They do not exclude a different allowed primitive branch. In general the complete reduction of (45), over all same-order auxiliaries $\eta$, is

$$\mathcal{K}_g(z) = H_{FF}(z) - H_{F\eta}(z) H_{\eta\eta}(z)^{-1} H_{\eta F}(z). \qquad (53)$$

Only if the full relevant Hessian scales by $z$ does its Schur response automatically scale by $z$. The unscaled substrate potential and the actual gauge-source embedding can change that conclusion. Moreover, the Gaussian construction fixes a quadratic kernel, not the compact heat-kernel law or its higher vertices. Table 7 therefore retains the nonlinear completion and matching assumptions under which it was calculated. Establishing one compatible primitive realisation with the required raw weighting, kinetic rule, source pullback and nonlinear completion would suffice; no uniqueness proof is required.

## 10.6 Dynamical reduction and the completed wheel test

The temporal response can be calculated before identifying a reduced coefficient with a force. The relevant operation is elimination of the coupled coordinates in the frequency-dependent quadratic action, rather than elimination of their potential alone. The following finite-dimensional identity applies to a conservative, time-reversal-even quadratic branch. It allows mixed kinetic terms and uses the inverse only on the declared physical quotient.

**Proposition 5. Dynamical Schur reduction.** Let the retained coordinates be $x$ and the auxiliary coordinates be $\eta$. Suppose the Euclidean kernel is $\mathsf{H} + \omega^2\mathsf{M}$, with $\mathsf{H} \succcurlyeq 0$, $\mathsf{M} \succ 0$ and $\mathsf{H}_{\eta\eta} \succ 0$. Define

$$\mathsf{R} = \mathsf{H}_{\eta\eta}^{-1}\mathsf{H}_{\eta x}, \qquad \mathsf{Q} = \begin{pmatrix} I \\ -\mathsf{R} \end{pmatrix}, \qquad \mathsf{C} = \mathsf{M}_{\eta x} - \mathsf{M}_{\eta\eta}\mathsf{R}, \tag{54}$$
$$\mathsf{H}_{\mathrm{red}} = \mathsf{Q}^{\dagger}\mathsf{H}\mathsf{Q}, \qquad \mathsf{M}_{\mathrm{red}} = \mathsf{Q}^{\dagger}\mathsf{M}\mathsf{Q}.$$

The exact reduced kernel and its controlled low-frequency remainder are

$$\mathcal{K}_x(\omega) = \mathsf{H}_{\mathrm{red}} + \omega^2\mathsf{M}_{\mathrm{red}} - \omega^4\mathsf{C}^{\dagger}\big(\mathsf{H}_{\eta\eta} + \omega^2\mathsf{M}_{\eta\eta}\big)^{-1}\mathsf{C}. \tag{55}$$

$$0 \preccurlyeq \mathsf{H}_{\mathrm{red}} + \omega^2\mathsf{M}_{\mathrm{red}} - \mathcal{K}_x(\omega) \preccurlyeq \omega^4\mathsf{C}^{\dagger}\mathsf{H}_{\eta\eta}^{-1}\mathsf{C}. \tag{56}$$

**Proof.** Substitute $\eta = \tilde{\eta} - \mathsf{R}x$. The static mixed block vanishes; the retained static and kinetic blocks become the two pullbacks in (54), while the mixed kinetic block is $\omega^2\mathsf{C}$. Completing the square in $\tilde{\eta}$ gives (55). The inverse ordering $0 \prec \big(\mathsf{H}_{\eta\eta} + \omega^2\mathsf{M}_{\eta\eta}\big)^{-1} \preccurlyeq \mathsf{H}_{\eta\eta}^{-1}$ gives (56). Since $\mathsf{Q}$ has full column rank, $\mathsf{M}_{\mathrm{red}} \succ 0$. □

Thus the leading inertia includes the motion of the auxiliaries along their static response. With no mixed kinetic block it is $\mathsf{M}_{xx} + \mathsf{R}^{\dagger}\mathsf{M}_{\eta\eta}\mathsf{R}$, rather than $\mathsf{M}_{xx}$ alone. Gaussian elimination in one step or successive block eliminations gives the same kernel when all determinants are retained once. A static Schur complement therefore supplies only one part of the physical normalisation test in Proposition 1.

For the explicit wheel branch (51), write $\nu = \lambda_{\mathrm{cyc}} > 0$ and complete the retained motion by the positive kinetic term $\big(\mathfrak{m}_u\dot{u}^2 + \mathfrak{m}_y\dot{y}^2\big)/2$. This is a declared kinetic choice in the same canonical units as (43); no force measurement fixes it. The three raw residuals in one cycle are $\big(\sqrt{\nu}\,u, \sqrt{\nu}\,y, u - y\big)$. Give each its independent seven-profile bath from (43), and eliminate those baths and $y$ for the prescribed curvature coordinate $f = \sqrt{\nu}\,u$. With $w = w_\lambda(\omega)$ from (44), the exact answer is

$$\mathcal{K}_f(\omega) = \frac{1}{\nu}\left[\mathfrak{m}_u\omega^2 + (\nu+1)w - \frac{w^2}{\mathfrak{m}_y\omega^2 + (\nu+1)w}\right]. \tag{57}$$

Its expansion $\mathcal{K}_f(\omega) = k_F(z) + \omega^2\mathcal{I}_F + O(\omega^4)$ reproduces (52) and gives the explicit temporal coefficient

$$\mathcal{I}_F = \frac{\mathfrak{m}_u + \mathfrak{m}_y/(\nu+1)^2}{\nu} + (1-z)^2\,\frac{\nu+2}{\nu+1}. \tag{58}$$

For unit retained inertias, the two eigenvalues of the raw $(u, y)$ potential are $h = \nu$ and $h = \nu + 2$. Put $\Omega^2 = 1 + \lambda$. The four coupled normal frequencies satisfy

$$\omega_{\pm}^2(h) = \frac{\Omega^2 + h \;\pm\sqrt{(\Omega^2 - h)^2 + 4h}}{2}, \qquad h \in \{\nu, \nu+2\}. \tag{59}$$

Both roots are positive because their sum is $\Omega^2 + h > 0$ and their product is $h\lambda > 0$. In the three-dimensional projected bath, one further mode has frequency $\Omega$ and is uncoupled to $(u, y)$. The other wheel

profiles remain in the source-independent determinant. Equations (57)–(59) complete the reversible dynamics of this declared cycle test; they do not specify permanent-record production.

Table 12. Temporal response of the primitive wheel test at $z = 465/826$ and $\mathfrak{m}_u = \mathfrak{m}_y = 1$.

| Channel | Temporal coefficient $\mathcal{I}_F$ | Record share of $\mathcal{I}_F$ | Lowest coupled frequency |
|---|---|---|---|
| Colour isotype, $\nu = 4$ | 0.489211052 | 46.85% | 0.984162856 |
| Weak isotype, $\nu = 5$ | 0.428399634 | 52.02% | 1.014381387 |
| Abelian isotype, $\nu = 1$ | 1.536513816 | 18.65% | 0.674235546 |

The exact temporal coefficients are $2086106/4264225$, $3757973/8772120$ and $2096653/1364552$ in the table's order. The record share is the second term of (58) divided by the total. It is appreciable and sector dependent even though the history multiplier is common. The dependence on the retained kinetic choice is also explicit: $\partial\mathcal{I}_F/\,\partial\mathfrak{m}_u = 1/\nu$ and $\partial\mathcal{I}_F/\,\partial\mathfrak{m}_y = 1/[\nu(\nu+1)^2]$, while $k_F$ is unchanged.

These are dimensionless responses of internal cycle coordinates. The cycle eigenvalue is not a physical spatial wave number, the listed frequency is not a continuum mass gap, and the curvature coordinate has not been identified with the matter-normalised gauge source. Consequently $k_F$ and $\mathcal{I}_F$ cannot yet be substituted for $\mathcal{B}_A$ and $\mathcal{E}_A$ in (3). The calculation supplies a complete check for that later identification: it must recover both (52) and (58), with the same physical source and clock. The error bound (56) additionally controls use of the leading temporal coefficient once the full matrices are supplied.

## 11 Consequences for the November ledger

The effective candidate has an explicit compact weight, auxiliary integral, one-loop conversion and one-force-calibration comparison. The global-cost branch adds a calculated pure-gauge two-loop block. Equations (39)–(40) fix the mixed representation weights; Table 8 separates the completed gauge calculation from the unresolved matter conversion and unevaluated Higgs and Yukawa matching. Section 10 supplies a history-action extension and completes a reversible primitive-wheel test, including its temporal response. Propositions 4–5 give the exact reduction rules; the physical kinetic and source identifications remain conditions of the intended force calculation.

The ledger's physical normalisation gates [7] therefore have constructive quadratic examples, but remain open at the level of the complete primitive system. The raw core metric and source lift in (43), the retained kinetic rule in (48)–(50), the joint reductions (53)–(55) and the nonlinear completion must be identified on one physical branch. Bosonic transfer positivity does not complete the interacting chiral theory. Masses, mixing and a refinement-stable strong-sector observable remain separate obligations. No overall completion percentage is assigned.

Table 13. Required return from the complete joint-action calculation.

| Quantity to return | Where it enters | Acceptance check |
|---|---|---|
| Raw source weight and measure | (6)–(7), (42)–(44) | Retain the raw contact response; recover (44) on the declared fixed-matrix bath |
| Primitive core metric, bath carrier and source lift | (43)–(45), (53) | Include all same-order primitive modes and their mixed terms; recover the stated wheel factors on the restricted branch |

| Quantity to return | Where it enters | Acceptance check |
|---|---|---|
| Electric, magnetic and matter-vertex coefficients | (2)–(4), (48)–(50), (54)–(58) | Evaluate the invariant coupling and retained kinetic rule with one common physical clock; recover (58) on the unit-inertia wheel test |
| Nonlinear completion and full fluctuation operator | (12)–(19), (39)–(41), (53) | Recover the minimal heat-kernel weight when additional interactions are absent; count each joint fluctuation once |
| Matching at the claimed accuracy | (30)–(41); Table 8 | Retain the gauge block, resolve the matter reference check and evaluate the remaining finite terms in consistent schemes |
| Two predicted electroweak outputs | (20); Table 7 | Use neither target to select continuous response coefficients |

A compatible construction can satisfy Table 13 without being uniquely selected. Proposition 4 supplies the bath reduction for one explicit class, and Table 10 exhibits stable propagation under a declared kinetic rule. Tables 11–12 give the static and temporal responses of a primitive wheel test, showing why both must enter the physical source identification. The diagnostic fits in (25) remain excluded from the forward results; the inherited global-cost premise and partial matching status of Table 7 are unchanged.

Three explicit tests distinguish progress from relabelling. First, any proposed clock or field rescaling that claims to fix the strengths must change the invariant (3) through a physical change in the action or vertex; otherwise it cannot work. Second, a claimed primitive normalisation must reproduce the raw source free-energy response, including the contact freedom exposed in Proposition 2. Third, a claimed correction for this branch must improve both electroweak outputs after the sole strong calibration and a consistent matching calculation. A single close ratio does not pass the third test.

## 12 Conclusion

The compact wheel completion supplies a calculable raw weight and counts its auxiliary fluctuations once. The original unit-amplitude branch misses the individual electroweak strengths. A positive global-cost extension implementing the inherited 15/14 premise brings their magnitudes much closer, but changes the mixing comparison as well. Its finite one-loop closure correction is zero beyond the leading coefficient change; a separate late electroweak multiplier does not follow from that integral.

Relative to the global-cost one-loop branch, the pure-gauge two-loop matching improves both comparisons. With the same strong calibration, the inverse electromagnetic strength and weak mixing become 128.84309 and 0.22802766, compared with 127.930 and 0.231290. The corresponding weak and hypercharge couplings differ by +0.355% and −0.566%. The mixing residual is nevertheless worse than on the original branch, changing from −0.21% to −1.41%. These are partial-order results: the unresolved fermion conversion and unevaluated Higgs and Yukawa contributions need not be smaller than the gauge block or share its favourable direction.

The history-action connection makes the relaxation mechanism explicit at quadratic order. A seven-profile unresolved record bath returns the frequency-dependent metric in (44), preserves mixed defect terms and gives calculable motion and correlations. Proposition 5 shows how that motion contributes to the effective inertia and bounds the low-frequency remainder. In the primitive wheel test with the declared unit inertias, the record contribution is appreciable and sector dependent; its percentage shares change with the kinetic choice. The static and temporal responses are therefore both available for checking a later physical identification. A stable propagation completion can retain the intended force coefficient, but its kinetic rule and physical source map remain additional conditions.

The finite-matching task is specified in Table 8: reproduce the matter reference in consistent conventions and complete the remaining finite integrals before repeating the forward solve. The primitive task in Table 13 is to identify the bath, raw weighting, retained kinetics and matter source on one complete branch, and to establish the nonlinear completion. This requires an evaluated compatible construction, without requiring a uniqueness proof or selecting corrections from the target forces.

## Appendix A Character checks and numerical verification

For an $SU(2)$ factor, write $n = 2j + 1$, $C_2 = j(j + 1)$ and $\tau = t/K_W$. At the identity the heat-kernel Hessian along a generator is

$$\kappa_2(\tau) = \frac{\sum_{n\geq 1} n^2\,[(n^2-1)/4]e^{-\tau(n^2-1)/4}}{3\sum_{n\geq 1} n^2\,e^{-\tau(n^2-1)/4}}. \qquad (60)$$

For $SU(3)$, representations are indexed by $p, q \geq 0$, with $d_{pq} = (p + 1)(q + 1)(p + q + 2)/2$ and $C_2(p, q) = (p^2 + q^2 + pq + 3p + 3q)/3$. The analogous Hessian is

$$\kappa_3(\tau) = \frac{\sum_{p,q\geq 0} d_{pq}^2\, C_2(p,q)e^{-\tau C_2(p,q)}}{8\sum_{p,q\geq 0} d_{pq}^2\, e^{-\tau C_2(p,q)}}. \qquad (61)$$

At $z = 31/59$, the sums give $\kappa_2 = 3.331920903954803$ and $\kappa_3 = 3.027542372881356$. Their local expansion values $zK_W - 2/24$ and $zK_C - 3/24$ agree within $5 \times 10^{-16}$. The sums use $n \leq 100$ and $p, q < 60$. These are checks on the individual covering factors; the quotient projection changes global image terms, not the perturbative coefficient used in (18).

The wheel is independently diagonalised in $\pi$-orthonormal coordinates. Its stationary measure, detailed balance, source norm and Schur response are checked directly. The raw-weight counterexample is also evaluated on this finite kernel: multiplying by $e^{-0.0315}$ leaves its Doob transition matrix unchanged within $10^{-16}$ while increasing the raw free energy by 0.0315.

The SMDR boundary residual in the main solve is below $10^{-12}$. A reference-control invocation with the original reference gauge inputs reproduces $\alpha_s^{(5)} = 0.1180$ and gives $\hat{\alpha}^{-1} = 127.91042$, $\hat{s}_W^2 = 0.23133054$. Those are the archived reference model's outputs, not a fit to the separate fixed targets used here. This control tests the wrapper and prevents accidental inheritance of a calibrated gauge input. Numerical solver precision is much better than the physical approximation; printed extra digits support reproduction, not physical precision.

For the global-cost branch, exact auxiliary elimination reproduces (27)–(29). Independent adjoint-trace contractions verify (35)–(36). Rebuilding the pinned SMDR source from a clean extraction of the two-loop calculation archive reproduces its forward rows to relative tolerance $10^{-12}$. The finite-determinant checks in Section 9.5 pass, while the matter reference comparison remains explicitly open in the verification output. Solver tolerances and these checks do not bound omitted physical terms.

For Section 10, exact wheel arithmetic reproduces both relaxation factors, while full seven-coordinate elimination and spectral covariance checks reproduce (44) and (47). A nonorthogonal six-coordinate test metric retains all its cross terms after the same reduction; it is a test fixture, not a claimed primitive metric. The two propagation choices agree with their exact normal modes, and the 24-coordinate primitive wheel preserves its six gauge null modes. Matrix-identity residuals are below $10^{-13}$. These checks validate the displayed constructions and do not discharge their physical identification conditions.

For Section 10.6, differentiating the exact kernel (57) at $\omega^2 = 0$ gives the fractions in Table 12 directly. A joint five-coordinate matrix for $(u, y)$ and the three coupled bath projections reproduces (57) and (59); the

remaining bath profiles decouple from this source. Direct block elimination also checks (55) with nonzero mixed kinetic blocks. The matrix-identity residuals are below $10^{-12}$. The finite-dimensional formulas are self-contained and do not establish the physical source identification.

## Appendix B External inputs and reproducibility

At $Q_0 = 172.4$ GeV the frozen external values are $v = 246.60029586365829$ GeV, $\lambda_H = 0.1262969232875777$, $y_t = 0.9309715465595599$, $y_b = 0.0155080080991419$, $y_c = 0.003400275709585654$, $y_s = 0.000292915817833262$, $y_d = 0.00001472409137072013$, $y_u = 0.00000673214356538510$, $y_\tau = 0.0099949323705356$, $y_\mu = 0.00058838465029327$ and $y_e = 0.00000279299434070567$. The low-energy routines receive $M_t = 172.4$ GeV, $M_h = 125.20$ GeV and the reference weak-boson mass 80.34918252735155 GeV. These are fixed external comparison inputs. Their use is not a prediction of those masses. The wrapper retains $\Delta\alpha^{(5)}_{\mathrm{had}} = 0.02783$ for SMDR's additional Sommerfeld output, which is not used in either reported hatted target. The package records the full reference input file.

The source manifest records public URLs, content hashes and indexed-text coverage for the VERSF normalisation, raw-measure, clock and joint-response sources. The technical dependencies are identified by the cited sections; the history-action construction uses the master-action, history-measure and admissibility papers.

The reproduction materials retain the finite integrals, SMDR bridge, pinned public SMDR source and licence, frozen inputs and numerical checks. In VERSF_Two_Loop_Matching_Reproducibility.zip, gauge_matching.py reproduces (31)–(37) and the partial forward rows; check_plaquette.py and check_dirac.py retain the unresolved matter comparison, which verify.py reports as open. That archive includes running dependencies and build instructions. Global_Closure_15_14_Reproducibility.zip and Global_Closure_Fluctuation_Calculation.zip contain the global-cost and auxiliary-integral calculations. No electroweak target enters a forward solve; diagnostic fits are isolated.

VERSF_History_Action_Connection_Reproducibility.zip reproduces Section 10. Its calculate.py returns the history factors, covariances and static wheel checks; calculate_dynamics.py evaluates (54)–(59) and Table 12, with output in dynamics_results.json. The archive includes both calculation notes and public source URLs. No measured force enters these calculations.

The calculation packages accompany the manuscript and have no assigned public repository identifier. References use public sources. Table 8 states the remaining matching tasks.

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