Stationary Joint Response and the Force Normalisation Test in VERSF

Protection, allowed relaxation and sector dependence

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17 November 2026 |

Result in plain language

A force between two objects is carried by a field, and the strength of the force depends on how stiffly that field resists being disturbed. Earlier VERSF work gave a first estimate of that stiffness for the three forces of particle physics. In the comparison convention used here, the measured stiffnesses are close to half the corresponding baseline values. Section 10.5 distinguishes this baseline from the doubled locking benchmark. This paper asks what kind of hidden internal motion could soften a field by one common factor, and what kind could not.

Picture a rope with small weights hanging from it. If the weights only dangle and jiggle, a slow wave on the rope travels as it did before, however wildly they jiggle. The rope becomes softer only when something gives way along with every stretch of the rope itself, like a spring fitted in line with each link. We show which arrangement is which and solve an example of each. We also show when the softening is the same for all three forces and leaves the speed of the waves unchanged, as measurement requires.

Once the softening is reduced to a single number, we try one simple and clearly labelled assumption about the smallest repeating cell of the VERSF construction. It returns a factor of 0.525, close to the declared comparison value of 0.52. Applying that comparison requires the baseline mapping in Section 10.5. The calculation is a conditional derivation: its assumptions still need justification on a physically admissible VERSF branch. VERSF need not admit only this construction. A softer field is also not the same thing as the Yang–Mills mass gap, which remains a separate problem.

1. Sources, contribution and physical input boundary

The calculation uses the nonlinear paper [1], the twelve-direction matter–record calculation [2], and the master-action, history and gauge-response sources [3–5] in the versions preserved in its archive. Their common-action structure and stationary-transfer construction are retained. It also takes account of the later force-strength paper's input boundary [6], the exact translated joint kinetics [7], and the latest supplied wheel-construction report [8]. Section 11 states precisely which later findings are used; this is not a claim to have re-audited the entire corpus.

The propositions apply block inversion, orthogonal projection and spectral response theory to the remaining calculation. They distinguish a correct conditional reduction from an identification of the physical parent. The ordinary derivative-expansion framework [9] supplies context, not a VERSF coefficient or a claim of mathematical novelty.

No measured force strength chooses a model parameter. The number 0.523163 is the previously declared comparison target in the baseline of Section 10.5, which states its composition, resolution and unresolved mapping to the locking benchmark. Sections 2–6 give the controls. Sections 7–9 derive and test the response criteria. Section 10 reduces the target condition to one number under stated premises, and evaluates that number under one further declared premise. Sections 11–13 explain the current physical boundary and the Yang–Mills connection.

The result of Section 10 has two parts with different standing. Propositions 5 and 6 are mathematical statements about a stated class of parents. The value 31/59 follows from them only together with premises C4 and C5, which are declared here and await justification within a physically admissible VERSF realization. Establishing one such realization suffices for the conditional result; uniqueness is not required.

2. Exact elimination of an auxiliary current

The bosonic master action has constitutive and continuity defects of the form [3]

$$D_J = J - \kappa\, d_U \rho, \qquad D_T = \partial_\sigma \rho + d_U^\dagger J. \tag{1}$$

An independent configuration variable does not thereby acquire an independent canonical momentum. On this restriction there is no $\partial_\sigma J$ term. The current should first be eliminated under its actual action and measure.

At a fixed remaining configuration z, suppose the retained defects are affine in real current coordinates,

$$\boldsymbol{D} = P(z)J + d_0(z, \dot{z}), \qquad S_J = \tfrac{1}{2}\boldsymbol{D}^T W(z)\boldsymbol{D}, \qquad A = P^T W P > 0. \tag{2}$$

Here all relevant mixed blocks of the supplied W are retained. Flat conditional current volume and current independence of W are hypotheses. The conditional integral is exact:

$$\int \mathrm{d}J \exp(-S_J) = (2\pi)^{\wedge}(N_J/2)(\det A)^{-1/2} \exp(-d_0^T W_\perp d_0/2), \tag{3}$$

$$W_\perp = W - WP(P^T W P)^{-1} P^T W. \tag{4}$$

Consequently, the reduced action contains both

$$S_{\mathrm{red}} = \tfrac{1}{2} d_0^T W_\perp d_0 + \tfrac{1}{2} \log\det A \tag{5}$$

up to a field-independent constant. The log determinant is per regulated integration slice. It may depend on the gauge source and must then be differentiated. For complex current coordinates interpreted as pairs of real coordinates, the determinant is counted with the corresponding doubled real multiplicity.

Writing $d_0 = Q\dot{z} + r$ gives an explicit induced velocity matrix and mixed term,

$$M_z = Q^T W_\perp Q, \qquad S_{\mathrm{red}} = \tfrac{1}{2}\dot{z}^T M_z \dot{z} + \dot{z}^T Q^T W_\perp r + \tfrac{1}{2} r^T W_\perp r + \tfrac{1}{2} \log\det A. \tag{6}$$

This is a derived reduction for the supplied parent, not a new choice of current inertia. Singular blocks require an explicitly justified quotient and the appropriate range condition, and not the silent removal of null directions.

Scope. A current-dependent metric, a nonflat conditional current measure, nonlinear current-dependent history terms, or further time derivatives can invalidate the Gaussian integral. The fully lifted source must decide whether (2) applies. In addition, identifying σ with physical Euclidean time is a separate attachment. Equation (6) does not establish it. These qualifications are material: an arbitrary static Hessian cannot be relabelled as the physical kinetic metric.

3. A stationary state with the current reduction retained

An explicit check uses a fixed real incidence matrix d, $L = d^T d$, a declared Euclidean time τ, and the orthogonal two-defect branch

$$S = \tfrac{1}{2} \int \mathrm{d}\tau [a\|J - \kappa d\rho\|^2 + b\|\dot{\rho} + d^T J\|^2 + \mu^2 \|\rho\|^2], \qquad a, b, \kappa, \mu^2 > 0. \tag{7}$$

The weights in this check are supplied, not derived physical values. The independent cycle currents are included in J, not replaced by endpoint gradients. Applying (3) gives

$$A = aI + b\,dd^T, \qquad M = ab\,(aI + bL)^{-1}, \tag{8}$$

$$S_{\mathrm{red}} = \tfrac{1}{2}\int \mathrm{d}\tau[(\dot{\rho} + \kappa L\rho)^T M(\dot{\rho} + \kappa L\rho) + \mu^2\rho^T\rho] + \tfrac{1}{2}\,\mathrm{Tr}_\tau \log A. \tag{9}$$

For the fixed background, M commutes with L. The mixed velocity term is a total derivative and drops from a periodic Euclidean trace. The corresponding symmetric, flat-coordinate canonical Hamiltonian is

$$H_\rho = \tfrac{1}{2}p^T M^{-1}p + \tfrac{1}{2}\rho^T K_\rho \rho, \qquad K_\rho = \kappa^2 ML^2 + \mu^2 I. \tag{10}$$

The source-dependent determinant and boundary/measure conventions must be restored when comparing different gauge backgrounds. They do not disappear from a physical response calculation.

Define $\Omega = (M^{-1/2}K_\rho M^{-1/2})^{1/2}$. The normalized ground state of (10) is the usual Gaussian with exponent $-\rho^T M^{1/2}\Omega M^{1/2}\rho/(2\hbar)$, and

$$C_\rho = (\hbar/2)M^{-1/2}\Omega^{-1}M^{-1/2}, \qquad C_p = (\hbar/2)M^{1/2}\Omega M^{1/2}. \tag{11}$$

Thus the covariance follows from the coupled reduced action. It is not separately prescribed. For a Laplacian eigenvalue λ,

$$M_\lambda = ab/(a + b\lambda), \qquad \Omega_\lambda^2 = \kappa^2\lambda^2 + \mu^2/M_\lambda. \tag{12}$$

The positive mass term makes the finite state normalizable, including the constant mode. With $\mu^2 = 0$, that mode needs separate treatment. Euclidean current correlations are recovered from the conditional Gaussian in (3); this does not identify J with the canonical matter gauge current.

Independent check. On the seven-vertex, twelve-edge wheel, all 19 real coordinates were assembled before elimination. With $(a, b, \kappa, \mu^2, \omega) = (1.3, 0.8, 0.7, 0.4, 0.37)$, direct inversion and reduction agree to 6.5×10^{-15} in the retained covariance; determinant factorization agrees to 2.7×10^{-15}. These are solver checks on (7), not physical error bars.

4. Nonlinear locking and an unchanged long-distance residue

The two-field branch of [2, Section 16.11] has a useful exact nonlinear extension. On a commuting transverse continuum restriction with fixed flat measure, take

$$\begin{aligned} S_A = K_A \int \mathrm{d}\tau\, \mathrm{d}^d x[&(m_x/2)\dot{x}^2 + (m_y/2)\dot{y}^2 + (a_w/2)|\nabla x|^2 \\ &+ (b_w/2)|\nabla y|^2 + U(x - y)]. \end{aligned} \tag{13}$$

An external canonical current couples to x. The current is a probe here, not a dynamically integrated chiral matter field. The continuum attachment and coefficients are hypotheses. The interaction U can be nonlinear; it must define a stable relative sector. Boundary conditions and the integration domain must allow the following linear change of variables without additional identifications.

Let $M = m_x + m_y$, $\alpha = m_y/M$, $\beta = m_x/M$, and

$$q = (m_x x + m_y y)/M, \qquad r = x - y, \qquad x = q + \alpha r, \qquad y = q - \beta r. \tag{14}$$

The temporal action separates into $M\dot{q}^2/2 + \mu_r\dot{r}^2/2$, where $\mu_r = m_x m_y/M$. The spatial coefficients are

$$A_s = a_w + b_w, \qquad B_s = (a_w m_y - b_w m_x)/M, \qquad C_s = (a_w m_y^2 + b_w m_x^2)/M^2. \tag{15}$$

If $a_w/m_x = b_w/m_y = v^2$, then $B_s = 0$. The full action, measure and Hamiltonian separate into a free common sector and an interacting relative sector. At finite volume with the common zero mode removed or fixed, the ground state is $\Psi_q \otimes \Psi_r$. Neither sector has been frozen.

For the connected Euclidean response, use $\hbar = 1$. Exact factorization gives

$$\mathcal{X}_A(i\omega, \mathbf{k}) = 1/(K_A M(\omega^2 + v^2k^2)) + \alpha^2 G_{rr}(i\omega, \mathbf{k}). \tag{16}$$

If the relative sector is gapped and its connected susceptibility remains finite at zero momentum, the second term has no massless pole. Therefore

$$E_A = K_A M, \qquad B_A = K_A(a_w + b_w), \qquad Z_A = \sqrt{(E_A B_A)} = K_A M v. \tag{17}$$

These leading coefficients are unchanged by *any* interaction confined to $U(r)$ that preserves the stated assumptions. This is an exact extension beyond harmonic locking. Its proof is the factorization, not a small-fluctuation expansion. It does not apply automatically to a compact global quotient with coupled boundary conditions, a changing measure, charged non-Abelian relative modes, or a full matter determinant.

5. Unequal speeds, stationary correlations and numerical checks

The leading conclusion has a useful extension when $B_s \neq 0$. In Fourier space, set $D = M\omega^2 + A_s k^2$ and complete the Gaussian common-field square:

$$q' = q + (B_s k^2/D)\, r, \qquad x = q' + (\alpha - B_s k^2/D) r. \tag{18}$$

The shifted common field is independent of the remaining relative action, whose quadratic kernel gains $-B_s^2 k^4/D$. The exact connected two-point function is

$$\mathcal{X}_A = 1/(K_A D) + (\alpha - B_s k^2/D)^2 \tilde{G}_{rr}. \tag{19}$$

For Euclidean approaches to the origin, the coefficient in brackets is bounded. If the full shifted relative susceptibility remains bounded, it cannot change the leading $1/p^2$ term. A gapless relative sector or singular critical susceptibility removes this argument and must be calculated, not assumed. Equation (19) is a statement about this action and source, not a universal identity for VERSF.

Quantum check of (16). A two-oscillator mode restriction with $K_A = 1$ and unit derivative coefficients has canonical coordinates $Q = (x + y)/\sqrt{2}$, $R = (x - y)/\sqrt{2}$ and

$$H = \tfrac{1}{2}(P_Q^2 + k^2 Q^2) + H_R, \qquad H_R = \tfrac{1}{2}P_R^2 + \tfrac{1}{2}(k^2 + 2c)R^2 + uR^4. \tag{20}$$

The positive quartic is a verification fixture, not an additional proposed VERSF action term. Its exact common-mode separation tests the theorem for nonlinear stationary states. At $k = 0$, the table concerns the gapped relative Hamiltonian only; the full state and response use $k > 0$ before the limit.

c	u	$E_1 - E_0$	$\langle R^2 \rangle$	$\chi_R(0)$	Pole residue
1	0	1.41421356	0.35355339	0.50000000	0.5
0.01	0	0.14142136	3.53553391	50.00000000	0.5
0.01	0.001	0.21316873	2.33734471	21.90794017	0.5
0.01	1	1.72993814	0.28664932	0.33068763	0.5

Here $\chi_R(0) = 2\,\Sigma_{n>0}\, |\langle n|R|0\rangle|^2/(E_n - E_0)$ and the full response is $\mathcal{X} = 1/[2(\omega^2 + k^2)] + G_R(i\omega, k)/2$, where $G_R(0, 0) = \chi_R(0)$. A hundredfold susceptibility increase does not change its coefficient. The limit keeps $c > 0$ fixed; sending the relative gap to zero first is a different limit.

Hermite bases of 70 and 110 states agree within 4.4×10^{-12} in the table's susceptibilities. Separate runs at $K_A = 1$, 6, 13/2, 21/2 and $u = 0$, 1, 10 verify the $1/(2K_A)$ residue and reproduce the quadratic two-pole limit. Five-point ground-energy derivatives check the spectral susceptibility within 7.5 ×

10^{-10} in that latter set. The complete code, inputs and returns are archived. Numerical accuracy does not establish physical admission.

6. What a successful remaining calculation must return

On a selected reversible positive-transfer branch, a Perron eigenfunction $h > 0$ and eigenvalue λ_0 give a stationary law without inserting a separate covariance [3,4]:

$$Th = \lambda_0 h, \qquad P(z, \mathrm{d}z') = (T(z, z')h(z')/(\lambda_0 h(z)))\, \nu(\mathrm{d}z'),$$
$$\pi(\mathrm{d}z) = h(z)^2 \nu(\mathrm{d}z)/(\int h^2 \mathrm{d}\nu). \tag{21}$$

This construction determines the state *once the complete transfer and reference measure are fixed*. It does not determine them from a desired stationary state. The fermionic positivity/continuation condition must be certified on the actual carrier. A driven or irreversible physical branch may require a different stationary construction; (21) cannot be imposed on it by convenience.

For a genuine shared-source quantum calculation, first put the source-dependent measure and constrained carrier on a common Hilbert space. The derivatives of the transformed Hamiltonian then include metric, density, support and constraint contributions. At a nondegenerate ground state the static matrix is

$$R_{AB} = \langle H_{AB} \rangle - 2\,\mathrm{Re}\,\Sigma_{n>0}\, \langle 0|H_A|n\rangle\langle n|H_B|0\rangle/(E_n - E_0). \tag{22}$$

All fields allowed to respond must be present in that same state. The stationary state used must be physically admissible for the recording dynamics on the branch under study.

The required transverse response is a matrix kernel with the physical matter-current insertion, satisfying the appropriate gauge identities. In an unbroken isotropic continuum limit write

$$\Gamma^{(2)}_{T,A}(\omega, k) = E_A \omega^2 + B_A k^2 + O(p^4), \qquad c_A^2 = B_A/E_A, \qquad Z_A = \sqrt{(E_A B_A)},$$
$$g_A^2 = Z_A^{-1}. \tag{23}$$

This conversion requires common length and time units, a common limiting speed and the canonical current matching discussed in [2,5]. Independent sector time rescalings cannot supply that common attachment.

For a residual target $s = Z_A/Z_A^{(0)}$ at the same matched scale,

$$s^2 = (E_A/E_A^{(0)})\, B_A/B_A^{(0)}. \tag{24}$$

If the speed is unchanged, both ratios must equal s. In the comparison baseline of Section 10.5, s = 0.523163 would require each matched coefficient to retain about 52.3 percent of its baseline. A variance of that size, a pole residue of one half in a different normalization, or a delayed spatial-loop inertia is insufficient. The value is a target comparison, not an input to (2)–(20).

The calculation in Section 4 instead returns $Z_A/Z_{A,\mathrm{harmonic}} = 1$ exactly on its branch. Its unit-coefficient values $Z_A = (12, 13, 21)$, in the inherited conventional hypercharge normalization, are the same benchmark as [2]. This is neither a numerical derivation of 0.52 nor a prediction that the full VERSF correction is zero.

7. Which couplings can change the leading response?

The locking result can be sharpened into a test of the coupling itself. Work at fixed regulator and on the physical or properly gauge-fixed transverse carrier. Suppose the inverse connected two-point function of retained common coordinates q and other coordinates r is

$$\mathcal{Q}(p) = (K(p), \quad V(p)\ ;\ V(p)^{\dagger}, \quad H(p)), \qquad H(p) \succeq h_0 I, \qquad h_0 > 0. \tag{25}$$

Here $p = (\omega, \mathbf{k})$. For an interacting theory, $\mathcal{Q}$ means the *exact* inverse connected kernel, not a bare static Hessian. For a Gaussian theory it is the quadratic action. The marginal connected q response is the inverse of

$$K_{\text{eff}}(p) = K(p) - V(p)H(p)^{-1}V(p)^{\dagger}. \tag{26}$$

Proposition 1 (order of the relaxation). If $\|V(p)\| \le C|p|^r$, then

$$0 \preceq K(p) - K_{\text{eff}}(p), \qquad \|K(p) - K_{\text{eff}}(p)\| \le C^2|p|^{2r}/h_0. \tag{27}$$

Consequently a two-derivative mixing, $r = 2$, cannot change the order-p^2 coefficient *through this block elimination*. A one-derivative mixing, $r = 1$, can. Proof: block inversion gives (26); $0 \preceq H^{-1} \preceq h_0^{-1}I$ gives (27). If the blocks have regular expansions, the one-derivative contribution is explicitly

$$V(p) = p_\mu V_\mu + O(|p|^2) \quad \Longrightarrow \quad \delta K_{\text{eff}} = -p_\mu p_\nu V_\mu H(0)^{-1} V_\nu^{\dagger} + O(|p|^3). \tag{28}$$

Symmetries can remove the odd-order remainder. Their presence must be checked and not presumed.

The unequal-speed locking model has $V \propto k^2$ and a bounded relative susceptibility. It therefore belongs to the protected case. Increasing that bounded susceptibility can enlarge a p^4 correction without changing the limiting p^2 coefficient. If H^{-1} becomes singular, the order count changes. For example, $H^{-1} = O(p^{-2})$ makes two-derivative mixing capable of producing an order-p^2 term.

What this does not exclude. The leading part of K can itself change through source-dependent contact terms, determinants, nonlinear common-field interactions, or other integrated fields. Equation (27) does not protect such changes. It extends the diagnostic, not the exact factorization theorem beyond its premises. A large bare covariance is insufficient evidence for either route. Ordinary effective-field-theory elimination and derivative counting provide the methodological context [9]; the equations above state the precise finite-block claim used here.

For increasing volume or refinement the constants C and h_0 must be uniform in common physical units. A bound available only on one finite wheel does not justify interchanging those limits with $p \to 0$.

7.1 The sign of the temporal coefficient

Proposition 1 does not distinguish the two derivative sectors. They behave differently. Let the eliminated coordinates be canonical, with positive inertia μ and positive stiffness $\mu^{1/2}\Omega^2\mu^{1/2}$, so that $H(\omega) = \mu^{1/2}(\omega^2 + \Omega^2)\mu^{1/2}$ at fixed $\mathbf{k}$, and let the mixing V be independent of ω.

Proposition 2 (temporal coefficient). Under these conditions

$$K - K_{\text{eff}} = V\mu^{-1/2}(\Omega^{-2} - \omega^2\Omega^{-4})\mu^{-1/2}V^{\dagger} + O(\omega^4). \tag{29}$$

The static part lowers the stiffness by $V\mu^{-1/2}\Omega^{-2}\mu^{-1/2}V^{\dagger} \succeq 0$. The coefficient of ω^2 in K_{eff} is raised by $V\mu^{-1/2}\Omega^{-4}\mu^{-1/2}V^{\dagger} \succeq 0$. A dynamical positive-energy channel with frequency-independent coupling can therefore lower the spatial coefficient, but it cannot lower the temporal one. Proof: expand $(\omega^2 + \Omega^2)^{-1}$ in (26).

The model of Section 8 shows the effect: at fixed k the ω^2 coefficient of (35) is $E_0 + g^2k^2/(m^2 + u^2k^2)^2 \ge E_0$.

One way to lower the temporal coefficient through block elimination is a mixing that is odd in the frequency, $V = i\omega V_\tau + \dots$ with real V_τ, so that $VH^{-1}V^{\dagger} = \omega^2 V_\tau H^{-1} V_\tau^{T}$. That is the structure of a variable which enters a defect together with $\partial_\tau q$ and has no canonical momentum of its own, such as the

current in (1). Its elimination is the exact integral (3), and because $W_\perp \preceq W$ in (4) the induced velocity metric of (6) obeys

$$M_z = Q^T W_\perp Q \preceq Q^T W Q. \tag{30}$$

The two routes are distinct: static susceptibility of dynamical modes acts on the spatial coefficient, and per-slice elimination acts on the temporal one. Section 10 treats both within one parent.

8. An explicit stable joint model that changes the response

To demonstrate that the allowed route is nonempty, consider a real common coordinate q coupled to a real spatial vector $\mathbf{P}$. On a periodic regulated space, with the constant q mode fixed, take the canonical Hamiltonian

$$H = \int \mathrm{d}^d x[\pi_q{}^2/(2E_0) + |\boldsymbol{\pi}_P|^2/2 + (B_0/2)|\nabla q|^2 + (u^2/2)|\nabla \mathbf{P}|^2 + (m^2/2)|\mathbf{P}|^2 + g\,\mathbf{P}\cdot\nabla q]. \tag{31}$$

All of E_0, B_0, u^2, m^2 are positive. The new term is a *declared diagnostic coupling*; it has not been derived from the VERSF master action. Completing its potential square gives

$$(m^2/2)|\mathbf{P} + (g/m^2)\nabla q|^2 + \tfrac{1}{2}(B_0 - g^2/m^2)|\nabla q|^2 + (u^2/2)|\nabla \mathbf{P}|^2. \tag{32}$$

Thus $g^2 < B_0 m^2$ ensures stability and a normalizable joint Gaussian ground state at every finite regulator and finite volume after fixing that zero mode. Both fields fluctuate. The ordinary positive canonical kinetic energy specifies a self-adjoint Hamiltonian; no independent stationary covariance is inserted.

Only the longitudinal component of $\mathbf{P}$ couples to a given momentum. After a harmless Fourier phase rotation, its mode stiffness and inertia are

$$V_k = (B_0 k^2, \quad gk\,;\, gk, \quad m^2 + u^2 k^2), \qquad M = \mathrm{diag}(E_0, 1). \tag{33}$$

With $F_k = M^{-1/2} V_k M^{-1/2} > 0$, the exact coordinate covariance is

$$C_k = \tfrac{1}{2} M^{-1/2} F_k^{-1/2} M^{-1/2}. \tag{34}$$

It is generally correlated in q and P. Integration of P is exact and gives

$$\Gamma^{(2)}{}_q(\omega, k) = E_0\omega^2 + B_0 k^2 - g^2 k^2/(\omega^2 + m^2 + u^2 k^2). \tag{35}$$

Its leading coefficients are therefore

$$E = E_0, \qquad B = B_0 - g^2/m^2, \qquad Z/Z_0 = c/c_0 = \sqrt{(1 - g^2/(B_0 m^2))}\,. \tag{36}$$

This is a genuine change of the long-distance derivative coefficient in a stable joint quantum model. It is distinct from the relative-locking examples. But it changes only B at leading order and changes the speed. It does not meet the unchanged-speed test requiring both E/E_0 and B/B_0 to equal the target. A temporal correction must come from the same admitted parent; an independently added temporal analogue is not justified by (31). Section 8.2 states the condition under which a temporal partner belongs to the same parent.

8.1 Exact spectrum, numerical checks and finite-size behaviour

The squared normal frequencies of the longitudinal pair are

$$\omega_\pm{}^2(k) = \tfrac{1}{2}[(B_0/E_0)k^2 + m^2 + u^2 k^2 \pm \sqrt{(((B_0/E_0)k^2 - m^2 - u^2 k^2)^2 + 4g^2 k^2/E_0)}\,]. \tag{37}$$

The determinant of V_k is $k^2(B_0 m^2 - g^2 + B_0 u^2 k^2) > 0$ for $k \neq 0$. At small momentum,

$$\omega_{-}^{2}(k) = ((B_0 - g^2/m^2)/E_0)\, k^2 + O(k^4), \qquad \omega_{+}^{2}(k) = m^2 + O(k^2). \tag{38}$$

Hence the joint model has a massive internal branch and a gapless collective branch despite its changed response. The same exact calculation establishes both statements.

The following round parameters were fixed independently of the comparison target: $E_0 = B_0 = 1$, $m^2 = 2$, $u^2 = 0.7$, and $g = 0.3, 0.6, 0.9$.

g	B	Z/Z_0	c/c_0	$B_0m^2 - g^2$
0.3	0.955	0.97724101	0.97724101	1.91
0.6	0.820	0.90553851	0.90553851	1.64
0.9	0.595	0.77136243	0.77136243	1.19

Direct inversion of the full frequency matrix agrees with (35) to relative error below 3.7×10^{-16} over the archived checks. The stationary covariance satisfies $C_kV_kC_k = M^{-1}/4$ with residual below 2.6×10^{-16}. Symbolic inversion independently proves (35) for arbitrary positive parameters satisfying the stability inequality.

For an explicit spatial lattice use matched forward differences and their adjoints, lattice spacing one, and $\hat{k} = 2\sin(\pi/N)$ for the smallest nonzero momentum. The forward-difference phase is absorbed in the polarization coordinate; both the gradient and mixing use the same symbol. For $g = 0.6$:

N	lowest frequency $\omega_{-}(\hat{k})$	$\omega_{-}/\hat{k}$
16	0.35297085	0.90463444
32	0.17747171	0.90530958
64	0.08885970	0.90548110
128	0.04444535	0.90552415
256	0.02222461	0.90553492

The analytic limit is $\sqrt{0.82} = 0.90553851$, while the gap tends to zero as N^{-1}. This is a finite-size verification of the declared model, not a numerical study of the full non-Abelian theory or of its continuum limit.

8.2 A temporal partner from the same parent

Suppose the coupling in (31) is the spatial part of one form that is isotropic under rotations of the scaled gradient

$$\tilde{\partial} q = (c_0^{-1}\partial_\tau q, \nabla q), \qquad c_0^2 = B_0/E_0.$$

The parent then contains one further auxiliary component P_τ. Take it without a τ-derivative term, like the current of Section 2. In Euclidean form the completed action is

$$\begin{aligned} S = \int \mathrm{d}\tau\, \mathrm{d}^d x\, [&(E_0/2)(\partial_\tau q)^2 + (B_0/2)|\nabla q|^2 + \tfrac{1}{2}|\partial_\tau \mathbf{P}|^2 + (u^2/2)|\nabla \mathbf{P}|^2 \\ &+ (m^2/2)(|\mathbf{P}|^2 + P_\tau^2) + g(\mathbf{P}\cdot\nabla q + c_0^{-1} P_\tau\, \partial_\tau q)]. \end{aligned} \tag{39}$$

With $P_\mu = (P_\tau, \mathbf{P})$, the potential square (32) extends to

$$(m^2/2)|P_\mu + (g/m^2)\tilde{\partial}_\mu q|^2 + \tfrac{1}{2}(B_0 - g^2/m^2)|\tilde{\partial} q|^2, \tag{40}$$

with the terms in $\partial_\tau \mathbf{P}$ and $\nabla \mathbf{P}$ unchanged, so the same inequality $g^2 < B_0m^2$ gives a positive action and a normalizable joint measure. Since P_τ has no canonical momentum, its elimination on each slice is the Gaussian integral (3), here with a field-independent determinant. It leaves the Hamiltonian (31)

with E_0 replaced by $E_0(1-\eta)$, where $\eta = g^2/(B_0 m^2)$. That Hamiltonian is self-adjoint and positive under the same inequality. The exact inverse response is

$$\Gamma^{(2)}_q(\omega, k) = E_0(1-\eta)\,\omega^2 + B_0 k^2 - g^2 k^2/(\omega^2 + m^2 + u^2 k^2), \tag{41}$$

with leading coefficients

$$E/E_0 = B/B_0 = Z/Z_0 = 1 - g^2/(B_0 m^2), \qquad c = c_0. \tag{42}$$

Both coefficients are reduced by the same factor and the speed is unchanged. The isotropy premise is what fixes the temporal mixing at g/c_0. A temporal strength g_τ in its place gives $E/E_0 = 1 - g_\tau^2/(E_0 m^2)$ and changes the speed unless $g_\tau = g/c_0$.

With the parameters of the previous table, and the same lattice convention:

g	Z/Z_0	c/c_0	$\omega_-/\hat{k}$, $N = 16$	$N = 64$	$N = 256$
0.3	0.955	1	0.99945131	0.99996592	0.99999787
0.6	0.820	1	0.99747593	0.99984137	0.99999008
0.9	0.595	1	0.99242665	0.99950914	0.99996925

The collective branch remains gapless, $\omega_-^2 = c_0^2 k^2 + O(k^4)$. The completed model relaxes the response in both derivative sectors at unchanged speed and still supplies no gap. It is a declared diagnostic like (31): neither the isotropy premise nor the coupling has been derived from the VERSF master action. Proposition 2 does not restrict P_τ, which is not a canonical coordinate.

9. The susceptibility must have the right sector structure

There is a second independent requirement: a correction must act on the right source directions. On a common Hilbert space, let a differentiable family with a nondegenerate ground state be

$$H(f) = H_0 + f_A L_A + \tfrac{1}{2} f_A f_B Q_{AB} + O(|f|^3). \tag{43}$$

Assume $L_A = \Sigma_{i=1}^{r} C_{Ai} O_i$ with real C_{Ai} and self-adjoint operators O_i. Ground-state perturbation theory, with convergent sums and controlled domains, gives

$$R_{AB} = \langle Q_{AB} \rangle - (C \chi C^T)_{AB}, \tag{44}$$

$$\chi_{ij} = 2\,\mathrm{Re}\,\Sigma_{n>0}\,\langle 0|O_i|n\rangle\langle n|O_j|0\rangle/(E_n - E_0). \tag{45}$$

This is the explicit operator form of (22), not an additional fluctuation correction to add to it. Write $\langle Q \rangle = R_0 + D$, where $R_0 > 0$ is the comparison baseline and D includes every change of the quadratic contact term. Define the dimensionless net relaxation matrix

$$\mathcal{L} = R_0^{-1/2}(C \chi C^T - D) R_0^{-1/2}, \qquad R = R_0^{1/2}(I - \mathcal{L}) R_0^{1/2}. \tag{46}$$

Thus a common fractional response $R = s R_0$ is equivalent to $\mathcal{L} = (1-s)I$. Stability of this static response requires $I - \mathcal{L} > 0$; it does not alone establish stability of the whole Hamiltonian.

Proposition 3 (rank requirement with unchanged contacts). If $D = 0$, then $\mathcal{L} \succeq 0$ and $\mathrm{rank}\,\mathcal{L} \le \mathrm{rank}\,C \le r$. A nonzero common reduction on an n-dimensional source space therefore requires at least n independent linearly coupled operator channels. If $r < n$ and $0 < s < 1$,

$$\|\mathcal{L} - (1-s) I_n\|_{\mathrm{op}} \ge 1 - s. \tag{47}$$

Proof: (45) is positive semidefinite on real operator combinations; matrix multiplication cannot increase rank. A normalized null vector of $\mathcal{L}$ gives (47).

For one independent transverse component, the unbroken $SU(3) \times SU(2) \times U(1)$ algebra has $8 + 3 + 1 = 12$ directions. In the unchanged-contact, linear-response-channel setting, one scalar channel cannot supply a nonzero common relaxation in all twelve. A multiplet can supply several channels; this is not a claim that twelve distinct fields, new particles or adjustable parameters are needed. Nonlinear source couplings can also produce full-rank D, outside this restricted rank conclusion.

9.1 A quantitative test for the factor near 0.52

An internal symmetry can make each adjoint block proportional to its invariant metric. It does not relate the proportionality constants of three different gauge factors. Even a shared susceptibility therefore needs the correctly normalized couplings and contact terms in each sector.

For example, take the diagnostic model (31) in three independent sectors with $E_A^{(0)} = B_A^{(0)} = K_A$, the same $m^2 = 2$ and $g = 0.6$, and $K_A = (6, 13/2, 21/2)$. This uses the familiar ratios in a new declared unit convention; it is not a replacement for the locking benchmark $Z_A = (12, 13, 21)$. Then

$$B_A/B_A^{(0)} = 1 - 0.18/K_A, \qquad Z_A/Z_A^{(0)} = \sqrt{(1 - 0.18/K_A)}\,. \tag{48}$$

Sector	K_A	fractional reduction of B_A	$Z_A/Z_A^{(0)}$
Colour	6	0.03000000	0.98488578
Weak	6.5	0.02769231	0.98605664
Hypercharge	10.5	0.01714286	0.99139152

These values demonstrate sector dependence even with identical internal masses and couplings. Their speeds differ, so they are diagnostic response ratios, not three physically matched force strengths.

For the illustrative target in the baseline of Section 10.5, the unchanged-speed condition takes the following form. Its physical use requires the common-current, common-clock and common-scale matching of Section 6:

$$\mathcal{L}_E = \mathcal{L}_B = (1 - s)I, \qquad s = 0.523163, \qquad 1 - s = 0.476837. \tag{49}$$

Here the matrices are formed separately from the physical electric and magnetic coefficients; identifying the source parameters of (43) with them is a physical matching step. With $D = 0$, (49) requires a 47.6837 percent normalized relaxation in *each* relevant direction and in both derivative sectors. By Proposition 2 the temporal part cannot be supplied by frequency-independent coupling to dynamical modes; per-slice elimination or contact terms can supply it. More general frequency-dependent couplings require their own response calculation. With $D \neq 0$, it is the *net* matrix in (46) that must meet this requirement. This translates the declared target into a calculation test. Section 10 reduces that test to one number under stated premises.

A useful bound follows directly from the spectrum. If every transition coupled by $O_v = \Sigma_i\, v_i O_i$ has energy at least $\Delta > 0$, then

$$v^T \chi v \leq (2/\Delta)\ \mathrm{Var}_0(O_v). \tag{50}$$

For one source direction with unchanged contact, achieving δR_0 relaxation therefore needs $\mathrm{Var}_0(L) \geq \delta R_0 \Delta/2$. Large variance alone is not sufficient: transition energies and source overlaps still enter (45). All quantities must carry the same finite-volume or intensive normalization before a uniform bound is claimed.

The seeded 12×4 channel check in the archive has rank four and eight null directions. Its operator-norm distance from the target is 0.476837, saturating (47). No model parameter was selected using that target.

9.2 When the fractional correction is sector independent

The sector dependence in the table above has a simple origin: the auxiliary stiffness and the mixing were held fixed while the field coefficient carried K_A. The opposite case is equally simple.

Proposition 4 (one carrier trace). In sector A let the field, the auxiliary and the mixing all be Lie-algebra valued, and let the quadratic action have the same geometric coefficients in every sector and be a single trace over the common carrier, on which $\mathrm{Tr}(T_a T_b) = K_A \delta_{ab}$. Then $E_A^{(0)} = K_A e_0$, $B_A^{(0)} = K_A b_0$, $m_A^2 = K_A \hat{m}^2$ and $g_A = K_A \hat{g}$, and

$$\eta_A = g_A^2/(B_A^{(0)} m_A^2) = \hat{g}^2/(b_0 \hat{m}^2) \qquad \text{for every sector.} \tag{51}$$

Proof: each term is bilinear in Lie-algebra-valued quantities and so carries one factor K_A, which cancels in the ratio.

With (g, m^2) replaced by K_A (0.6, 2) in the example above, all three reductions of B_A equal 0.18. An adjoint-valued auxiliary supplies 8 + 3 + 1 = 12 channels for each derivative component, which is the count Proposition 3 requires. This is the mechanism by which the prefactor K_A drops out of every ratio in (13). It is a sufficient condition for a common factor. Whether an admitted physical branch has both this trace weighting and common geometric coefficients is a source question.

10. Conditional evaluation of the common factor

Sections 7–9 give necessary conditions. This section gives sufficient ones. It shows that in a least-squares parent one projector controls both derivative sectors (Section 10.1), states premises under which the common-factor form of (49) holds and only one number remains (Section 10.2), puts that number in a series form (Section 10.3), and evaluates it on the $K = 7$ cell under one further declared premise (Section 10.4). Sections 10.5 and 10.6 compare the result with the target and state its standing.

10.1 One projector for both coefficients

Let $u \in \mathbb{R}^N$ collect the components of the scaled gradient ∂q in every source direction, so that $N = n(1 + d)$ for n directions. Suppose the part of the parent action that responds at leading order has the least-squares form of Section 2, with the eliminated variables J entering the retained defects affinely:

$$\boldsymbol{D} = PJ + Qu, \qquad S = \tfrac{1}{2}\boldsymbol{D}^T W \boldsymbol{D}, \qquad W > 0, \tag{52}$$

under the hypotheses stated after (2), with a positive-definite baseline matrix. For the response identity below, use fixed leading blocks as in C1 and assume the conditional determinant and measure supply no additional quadratic source term; otherwise include their contributions as in Section 11. Holding J at zero gives the baseline $R_0 = Q^T W Q$. Eliminating J by (3) and (4) gives $R = Q^T W_\perp Q$. The relaxation matrix of (46) is then

$$\mathcal{L} = R_0^{-1/2}(R_0 - R)R_0^{-1/2} = \hat{Q}^T \Pi_P \hat{Q}, \qquad \hat{Q} = W^{1/2} Q R_0^{-1/2},$$
$$\Pi_P = W^{1/2} P (P^T W P)^{-1} P^T W^{1/2}. \tag{53}$$

Here $\hat{Q}^T \hat{Q} = I$ and Π_P is an orthogonal projector. In the notation of (44), the contact term is $Q^T W Q$ and $C \chi C^T$ is $Q^T W P (P^T W P)^{-1} P^T W Q$.

Proposition 5 (principal angles). The eigenvalues of $\mathcal{L}$ are $\cos^2 \theta_i$, where θ_i are the principal angles between the subspaces $W^{1/2}$ ran Q and $W^{1/2}$ ran P of defect space. If rank P is smaller than N, complete the list to N angles with right angles, corresponding to zero relaxation eigenvalues. Consequently:

(a) $0 \preceq \mathcal{L} \preceq I$, and $I - \mathcal{L} > 0$ exactly when the two subspaces meet only at zero;

(b) rank $\mathcal{L} \leq$ rank P, which recovers the channel count of Proposition 3;

(c) the full-matrix common-factor condition $\mathscr{L} = (1 - s)\mathrm{I}$ holds if and only if all N angles are equal, $\theta_i = \theta$, and then

$$s = \sin^2 \theta. \tag{54}$$

Proof: $\hat{Q}^T \Pi_P \hat{Q}$ is the compression of an orthogonal projector to a subspace with orthonormal frame $\hat{Q}$. Its eigenvalues are the squared cosines of the principal angles between the two subspaces [11].

Because u contains the temporal and the spatial components together, $\mathcal{L}_E$ and $\mathcal{L}_B$ are diagonal blocks of one matrix. Equality of these blocks alone does not exclude mixed temporal–spatial terms. Proposition 5(c) applies to the full matrix; the block condition (49) is equivalent only when the mixed blocks vanish, as under C3. The requirement of Section 8, that a temporal correction come from the same parent, is met by construction. For the target, $\sin^2 \theta = 0.523163$ means $\theta = 46.33°$. Equal weight on the two subspaces, $\theta = 45°$, gives $s = 1/2$ exactly.

10.2 Premises under which one number remains

C1 **Least-squares parent.** The leading response is governed by (52), with a flat conditional measure and W independent of J. The leading source expansion uses fixed P, Q and W, and the conditional determinant and measure have no additional quadratic source contribution. All leading contact terms must be included in (52); otherwise the full response is not given by (53) alone.

C2 **One carrier trace.** Defect space factorizes into a geometric factor and the internal carrier, with $W = W_{\mathrm{geo}} \otimes G$, $P = P_{\mathrm{geo}} \otimes I$ and $Q = Q_{\mathrm{geo}} \otimes T$. The geometric factors are the same in every sector. Here G is the carrier trace form and T inserts the generator of the source direction.

C3 **Isotropy at the common speed.** The geometric data (W_{geo}, P_{geo}, Q_{geo}) are invariant under rotations of the $1 + d$ components of $\vec{c}q$.

Proposition 6 (reduction to one number). Under C1 and C2, $\mathcal{L} = \mathcal{L}_{\mathrm{geo}} \otimes I_n$, where $\mathcal{L}_{\mathrm{geo}}$ is the $(1 + d) \times (1 + d)$ matrix of (53) built from the geometric data alone. It is the same in every sector. Under C3 in addition, $\mathcal{L}_{\mathrm{geo}} = \eta I$, so that

$$\mathcal{L}_E = \mathcal{L}_B = \eta\, I_n, \qquad s = 1 - \eta, \qquad c = c_0. \tag{55}$$

Proof: under C2, $Q^T W Q = (Q_{\mathrm{geo}}{}^T W_{\mathrm{geo}} Q_{\mathrm{geo}}) \otimes \mathsf{K}$ with $\mathsf{K}_{ab} = \langle T_a, T_b \rangle_G$, and

$$Q^T W P (P^T W P)^{-1} P^T W Q = [Q_{\mathrm{geo}}{}^T W_{\mathrm{geo}} P_{\mathrm{geo}} (P_{\mathrm{geo}}{}^T W_{\mathrm{geo}} P_{\mathrm{geo}})^{-1} P_{\mathrm{geo}}{}^T W_{\mathrm{geo}} Q_{\mathrm{geo}}] \otimes \mathsf{K}.$$

The factor K, which equals $K_A \delta_{ab}$ on sector A, cancels in $\mathcal{L}$. Under C3, $\mathcal{L}_{\mathrm{geo}}$ commutes with an irreducible action of the rotation group on $\mathbb{R}^{1+d}$ and is a multiple of the identity by Schur's lemma.

The model of Section 8.2 with the weighting of Proposition 4 is an instance, with $\eta = \hat{g}^2/(b_0 \hat{m}^2)$. Under C1 to C3 the common-factor form of (49) holds, including vanishing mixed blocks, but its numerical value remains to be calculated. The declared comparison target corresponds to $\eta = 0.476837$.

This paper has not yet established C1 to C3 on a physically admissible VERSF branch. C1 is the hypothesis already examined in Section 2. C2 and C3 are the same-carrier and common-clock attachments of Section 6, expressed as conditions on the mixing.

10.3 The series form

A further structural premise fixes the form of η.

C4 **Constitutive entry.** The scaled gradient enters the eliminated sector only through one constitutive defect, in the manner of D_J in (1). Each component u contributes

$$(a/2)\,|u - r|^2 + (h/2)\,|r|^2, \tag{56}$$

with r the auxiliary paired with that component, a the constitutive weight and h the restoring weight of the auxiliary.

Minimizing over r gives $r = au/(a + h)$ and

$$s = h/(a + h), \qquad 1 - s = a/(a + h), \qquad s/(1 - s) = h/a. \tag{57}$$

The relaxed coefficient is the series combination $ah/(a + h)$ of the two weights. In the variables of (31) this is the case $g = B_0 = a$ and $m^2 = a + h$, for which the stability inequality $g^2 < B_0 m^2$ holds for every $h > 0$. In the language of Proposition 5, $\tan^2 \theta = h/a$.

The comparison target corresponds to $h/a = 1.097$. Equal weights give $s = 1/2$. C4 has one consequence that can be tested without a number: the whole leading coefficient must pass through the constitutive defect. A separate direct term for the gradient, outside the defect, raises s toward 1.

10.4 Evaluation on the $K = 7$ cell

The weight ratio h/a is a property of the cell on which the constitutive defect lives. For the $K = 7$ construction that cell is the seven-state wheel: a hub joined to six rim states, each rim state joined to its two neighbours. The canonical benchmark of [5] uses a reversible transfer $\hat{T}$ on this wheel with stationary weights

$$\pi = (1/31)(7, 4, 4, 4, 4, 4, 4). \tag{58}$$

The declared input here is the transfer in which a state holds or moves to each neighbour with equal weight, $\hat{T}_{xy} = 1/(\deg x + 1)$ for $y = x$ or y adjacent to x. It is reversible with the stationary weights (58). The nonzero eigenvalues of $I - \hat{T}$ are

$$½, ½, 1, 1, 5/4, 31/28. \tag{59}$$

The first five belong to the rim modes. The last belongs to the hub–rim contrast f, equal to 24 on the hub and −7 on each rim state, which is orthogonal to the constant in the stationary metric and has $\hat{T}f = -(3/28)f$.

Source note. The declared transition rule reproduces these stationary weights and the benchmark gap ratios 1, 1, 2, 2, 31/14, 5/2. Stationary weights alone do not determine the transfer. Identifying this rule with the transfer of the proposed physical branch requires confirmation against [5].

C5 **Wheel-resolved series form.** On one cell the auxiliary for each component u is a function r on the seven states, and the cost is

$$\Gamma = (a/2)\,\|u\chi - r\|_\pi^2 + (a/2)\,\langle r, (I - \hat{T})\,r\rangle_\pi, \tag{60}$$

with three conditions on its ingredients.

(a) *Mean-zero cell profile.* The profile χ has $\|\chi\|_\pi = 1$ and $\langle \chi, 1\rangle_\pi = 0$. This is an additional restriction on the internal cell profile. Since u is already a spacetime derivative, zero mixing at zero spacetime momentum does not by itself exclude a uniform cell profile. A physical realization must justify the mean-zero restriction separately.

(b) *Cell symmetry.* χ is invariant under the rotations and reflections of the rim.

(c) *Same metric, no added pinning.* The restoring cost of r is the Dirichlet form of $\hat{T}$ in the stationary metric, with the same weight a as the constitutive term. No independent pinning term is added.

Proposition 7 (cell evaluation). Under C5, χ is the normalized hub–rim contrast, and

$$s = \lambda_{hr}/(1 + \lambda_{hr}) = 31/59 = 0.525424, \qquad \lambda_{hr} = 31/28. \tag{61}$$

Proof: the functions on the wheel that are invariant under its symmetries are spanned by the constant and f, and (a) removes the constant. $\hat{T}$ is self-adjoint in the stationary metric, so for an eigenfunction χ with gap λ the minimum of (60) is at $r = u\chi/(1 + \lambda)$ with value $(a/2)u^2\lambda/(1 + \lambda)$. This is C4 with $h/a = \lambda$.

Uniform-component check. Let $L = \mathrm{I} - \hat{\mathrm{T}}$ and let $\chi = \alpha\chi_{hr} + \beta 1$ be a normalized symmetric profile, with $\alpha^2 + \beta^2 = 1$. The constant function 1 has unit norm in the stationary metric. With r unrestricted as in (60), minimization gives $r = u(\mathrm{I} + L)^{-1}\chi$ and

$$s(\chi) = \langle \chi, L(\mathrm{I} + L)^{-1}\chi \rangle_\pi = (1 - \beta^2)\ 31/59. \tag{61a}$$

The uniform component therefore lowers the residual factor: it relaxes completely because $L1 = 0$. A constraint preventing the uniform part of r from responding would change the result and would be an additional premise, absent from (60). The mean-zero value (61) is unchanged.

Profile on the wheel	gap λ	$s = \lambda/(1 + \lambda)$	symmetric
Rim doublet, one wave around the rim	1/2	1/3 = 0.3333	no
Rim doublet, two waves	1	1/2 = 0.5000	no
Rim alternating	5/4	5/9 = 0.5556	no
Hub–rim contrast	31/28	31/59 = 0.5254	yes

Among normalized mean-zero profiles, only the hub–rim contrast, up to sign, satisfies (b). The other rows show what a non-symmetric profile would return.

Under C1 to C5 the common factor is therefore 31/59 in all twelve directions and in both derivative sectors. The value depends on the declared transfer. The same wheel without the holding step has hub–rim gap 4/3 and returns 4/7 = 0.5714. Measuring the gap in units of the smallest gap returns 31/45 = 0.6889. The one-step generator $I - \hat{T}$ of the canonical transfer is the choice declared in (c). It is a premise, and these alternatives show that the comparison tests it.

10.5 The comparison target and its resolution

The declared target can be reproduced as follows. Run the measured couplings from the Z mass with the one-loop Standard Model coefficients $(b_3, b_2, b_Y) = (-7, -19/6, 41/6)$, starting from $\alpha_s = 0.1180$, $\sin^2\theta_W = 0.23122$ and $\alpha^{-1} = 127.95$, and form the sector factors $s_A(\mu) = g_A^{-2}(\mu)/K_A$ against $K_A = (6, 13/2, 21/2)$. Their relative spread is least at $\mu_0 = 8.03 \times 10^{13}$ GeV, where

$$(s_C, s_W, s_Y) = (0.518795, 0.531899, 0.518795), \qquad \text{mean } 0.523163. \tag{62}$$

With gauge-only two-loop running the minimum moves to 5.72×10^{13} GeV and the factors are (0.519011, 0.526530, 0.519011), with mean 0.521517. In this baseline convention the common comparison value is therefore about 0.52, with the sector values spanning 0.519 to 0.532 at one loop. Its later digits reflect the inputs and the loop order.

Evaluation	s	minus one-loop mean	inside sector range
Cell evaluation, 31/59	0.525424	+0.002261	yes
Equal weights in the series form	0.500000	−0.023163	no
Rim alternating profile, 5/9	0.555556	+0.032393	no
Wheel without the holding step, 4/7	0.571429	+0.048266	no

The value 31/59 is 0.43 percent above the one-loop mean and 0.75 percent above the two-loop mean, and it lies inside the sector range in both cases. The common-factor result returns equal sector factors. In the comparison the weak factor is higher than the other two, by 2.5 percent at one loop and 1.4 percent at two loops. That difference is of the size of the change between loop orders and is not addressed here.

Baseline matching. The reproduction uses the baseline $K_A = (6, 13/2, 21/2)$ with $g_A^2 = Z_A^{-1}$. Against the locking benchmark $Z_A = (12, 13, 21)$ of Section 6 with the same rule (23), the same measured couplings give half these factors: the one-loop mean is about 0.26158 and the gauge-only two-loop mean about 0.26076. Identifying 31/59 with the required physical residual therefore needs an explicit mapping of the source normalization, field coefficients and common clock between the two baselines. The agreement near 0.52 is provisional until that mapping is established; it cannot be assumed from a change of notation. Propositions 5 to 7 are statements about a ratio to a declared baseline and do not depend on that choice.

10.6 Standing of the evaluation and failure conditions

Propositions 5 and 6 are proved for their stated class. Proposition 7 is a calculation under C5. Together they give a conditional value: on any physically admissible VERSF realization satisfying C1 to C5, the common residual factor is 31/59. It is sufficient to establish one such realization, with its dynamics, measure, source coupling and baseline justified on that branch. VERSF need not uniquely select it or exclude other realizations. The physical admission and baseline matching of this proposed realization remain to be established here.

C5 was written down with the comparison target known. No input to it uses the target, and each ingredient is fixed by a stated rule and not by an adjustable number, but its selection was not blind. Agreement at the present resolution is weak evidence on its own. With a window of about 0.013, simple closed forms unrelated to VERSF fall inside it, and $1 - (25/32)^3 = 0.523163$ matches the one-loop mean to six digits by coincidence.

For a proposed realization, the following conditions invalidate this derivation, require a recalculation, or prevent the stated comparison.

1. The admitted parent is not of least-squares form in its eliminated variables, or its conditional measure is not flat. Alternatively, its determinant or measure contributes additional quadratic source terms. Proposition 5 alone then does not give the full response; the contributions discussed in Section 11 must be included.
2. The source-derived mixing is not isotropic at the common speed. The electric and magnetic corrections can then differ, or mixed terms can survive; unchanged propagation speed requires a separate check, as illustrated in Section 8.
3. The auxiliary sector is not weighted by the carrier trace of the field. The geometric coefficients may also differ between sectors. Either change removes the sufficient condition of Proposition 4 and can produce sector dependence, as in Section 9.1.
4. The leading coefficient has a direct part outside the constitutive defect, or the admitted profile has a uniform component, or it is not symmetric on the cell. These changes require a new evaluation. A positive direct contribution raises the residual factor relative to the total unrelaxed coefficient. A uniform component lowers it under (60), as shown in (61a); a nonsymmetric profile samples other eigenvalues. Any source condition that fixes a uniform component of the same profile, for example through current universality, must be shown compatible with C5(a).

5. The restoring weight of the auxiliary is not the Dirichlet form of the canonical transfer in the same metric. An admitted pinning term, a different holding weight or a different normalization of the generator changes λ_{hr}.
6. A matched comparison with a controlled error excludes 0.5254 as the common factor.

Conditions 1 to 5 are tests on the source equations. They can be tested for the proposed branch without reference to any measured coupling. Failure on a different allowed branch does not invalidate a conditional result on a branch where the premises hold.

11. Determinants, physical input and the current source boundary

The contact matrix D must not be discarded to make the rank test applicable. For instance, differentiating the exact current determinant in (5), on one regulated slice and for smooth source parameters f_A, gives

$$\partial_A \partial_B(\tfrac{1}{2} \log\det A) = \tfrac{1}{2}\,\mathrm{Tr}(A^{-1}A_{AB} - A^{-1}A_B A^{-1}A_A). \tag{63}$$

This is an action contribution before taking the joint expectation and connected subtraction. It can have either sign. Adding it after a full parent calculation that already contains the current integral would double count it. A two-dimensional positive-matrix fixture verifies (63) by a five-point derivative with error below 7.0×10^{-11}.

Source boundary. The later force-strength paper [6] retains coupled circulation and continuity, inherited current memory and recording constraints. Its stated remaining operand is the enlarged operational kinetic rule and its physical recording attachment. It does not replace that rule with a free overall rescaling. The newer wheel construction report [8, Section 10.24] attempts an explicit joint operator and identifies a more specific obstruction in that attempted construction: the history measure of link-holonomy increments, including its quadratic link metric and its globally valid temporal completion.

The report also distinguishes the two levels of calculation. Its local quadratic elimination is well defined on the specified positive block. Its proposed slice-by-slice integral of unrestricted stretches in the raw nonlinear action is not normalizable. Therefore the finite Gaussian state in Section 3 cannot be promoted to the complete nonlinear stationary state merely by removing its regulator or mass term. A physically justified history law, measure, domain or additional admitted dynamics must supply the missing confinement. The history rate discussed elsewhere in the report is a different object from the bare per-slice potential.

The report further finds that its specified colour-blind vertex-record probabilities have zero first-order score along unitary link-phase changes at the flat reference. That particular observation law consequently returns no quadratic Fisher weight in those directions. It is not a theorem that every VERSF record operation is blind to them. The required improvement is to identify and evaluate a physically admitted record operation that resolves the increments concerned.

These are attributed findings from the inspected construction package, not independently rerun non-Abelian results of this paper. They narrow the work to do without asserting that no admissible realization exists. The diagnostic coupling (31), its completion (39) and the cell cost (60) are not offered as that missing realization.

12. Consequences for the Yang–Mills work

The calculation separates three properties that can coexist in different combinations: a normalizable finite-volume stationary state, a changed force-response coefficient, and a positive energy gap uniform in volume. Equations (31)–(38) give the first two while failing the third. The completed model of Section 8.2 sharpens the example: it relaxes both derivative coefficients by a common factor at unchanged speed, and its collective branch is still gapless. This is a direct controlled example, not just a verbal distinction.

At volume L^d the low common mode has frequency $c(2\pi/L) + o(L^{-1})$, after removing the exact zero mode. Thus the gap of that declared Gaussian model tends to zero while its internal branch remains massive and its response coefficient remains relaxed. Removing the one constant coordinate does not remove the sequence of nonzero long-wavelength modes.

The later joint-kinetics paper [7] establishes a complementary result on its own stated branch. Its conditional translation gives

$$H_{\text{pull}} = U^{*}(H_G \otimes I + I \otimes H_R)U. \tag{64}$$

Under its gauge-equivariance and domain assumptions, a gauge-singlet excitation of H_G tensored with the invariant record vacuum remains a physical excitation after the unitary transformation. Consequently this construction cannot remove a sequence of gauge-singlet colour excitations whose energies tend to zero. Correlations in the translated ground state do not change that conclusion. This result is already part of the Yang–Mills work; it is cited here as a consistency check and is not counted as a new proof.

The full non-Abelian target concerns the spectrum of physical gauge-invariant states, not the pole of a chosen gauge coordinate [10]. The massless collective coordinate in our commuting diagnostic is therefore neither a disproof of a non-Abelian mass gap nor evidence that a gauge-fixed propagator settles it. Likewise, a changed coefficient of $\operatorname{tr} F^2$ is not by itself a gap theorem.

The connection is operationally useful: derive the same admitted joint dynamics, then perform two separate calculations. Extract its source-normalized electric and magnetic response for the force problem. Establish its physical excitation spectrum with uniform volume and regulator control for the mass-gap problem. Neither answer substitutes for the other.

13. What is established and what remains

The exact auxiliary-current reduction and the nonlinear locking theorem stand. The response criteria supply a derivative-order test, a sign rule for the temporal coefficient, a stable correlated quantum counterexample to unchanged response together with its speed-preserving completion, a sector-rank and normalization test, a sufficient condition for sector independence, and a quantitative susceptibility budget. Section 10 adds three statements. In a least-squares parent the residual factor is the squared sine of one angle whenever the full-matrix common-factor condition holds. C1 to C3 supply that form, while C4 and C5 give the numerical factor 31/59. The comparison near 0.52 uses the declared baseline and remains subject to the factor-of-two mapping in Section 10.5. The tools are standard block inversion, orthogonal projection and spectral response theory; their application makes the remaining VERSF calculation more explicit.

The next physical return is unchanged in kind and sharper in content: one common, normalizable current–matter–record dynamics and measure with resolved link increments. Its transverse kernel must include mixed terms, determinants and all responding variables. It must reproduce the protected control when those interactions are removed. It now has five specific questions to answer, C1 to C5, each decidable on the source equations. Two regulator levels are a useful diagnostic, not a proof of

convergence; the final claim needs a controlled limit and a stable error estimate. Establishing C1 to C5 on one physically admissible VERSF realization is sufficient for the conditional force result; uniqueness is not required. That admission, the baseline mapping and residual sector difference of Section 10.5, and the full Yang–Mills gap remain open here.

Sources and reproducibility

[1] K. Taylor, *Nonlinear Record Dynamics and the Force Strength Gap in VERSF*, version 16, 8 November 2026, Appendices F–G. Preserved source snapshot.

[2] K. Taylor, *Physical Matter Record Dynamics and Gauge Response in VERSF*, version 13, 8 November 2026, Sections 11, 13, 16–17. Preserved source snapshot. Public paper.

[3] K. Taylor, *The VERSF Master Substrate Action and Constitutive-Descent Theorem*, public 95-page PDF downloaded 8 November 2026; Sections 8A–10.1.1, 13–17 and 47J. Preserved source PDF; the later report [8] provides its own source-version register.

[4] K. Taylor, *The VERSF History Measure Generation Geometry and Branch Closure Theorem*, public 110-page PDF downloaded 8 November 2026; Sections 7–10 and 56A.1–56A.11. Preserved source PDF.

[5] K. Taylor, *The Gauge Response and Coupling Evaluation Theorem in VERSF*, public 104-page PDF downloaded 8 November 2026; Sections 22–27 and status ledger. Preserved source PDF.

[6] K. Taylor, *From Primitive Dynamics to Physical Force Strengths in VERSF*, current document version 1, modified 9 November 2026; general-reader summary, abstract and source/input boundary. Statements here are limited to that scope.

[7] K. Taylor, *Joint Kinetics, Record Clocks and the Remaining Yang–Mills Target*, 10 November 2026; Section 2. Current nine-page PDF, version 0. The metric, conditional translation and exact tensor-sum Hamiltonian were checked in the source text.

[8] *Record flat directions, restoring response and one enlarged transfer on the canonical wheel*, `REPORT.md`, Section 10.24 and lead ledger, in the supplied `cfb1_dynamics_and_records(25).zip`. Read 13 November 2026. The package is preserved in the companion, with its own scripts, source register and proofs. Its further results are attributed and not presented as independently reproduced here.

[9] A. V. Manohar, *Introduction to Effective Field Theories*, Les Houches lectures (2018), arXiv:1804.05863. General methodological context for elimination, matching and derivative expansions; no numerical input is taken from it.

[10] A. Jaffe and E. Witten, *Quantum Yang–Mills Theory*, Clay Mathematics Institute problem description, Section 4. Official problem statement. Used only to distinguish the physical spectrum target from a coordinate response.

[11] Å. Björck and G. H. Golub, Numerical methods for computing angles between linear subspaces, *Mathematics of Computation* 27 (1973) 579–594. Used only for the characterization of principal angles in Proposition 5.

Reproduction. The companion contains the complete calculation and source snapshots, the LaTeX source, `calculate.py`, `strengthening_checks.py`, `strengthening_results.json`, `conditional_factor_checks.py`, the inspected construction package, and a manifest. Run the scripts with Python, NumPy, SciPy and SymPy. `strengthening_checks.py` verifies symbolic block inversion and stability, stationary covariance, low-momentum coefficients, finite-size spectra, sector ratios, the channel-rank obstruction and determinant derivatives. `conditional_factor_checks.py` verifies the principal-angle identity on a random instance, the completed model of Section 8.2 and its lattice spectrum, the sign rule of Proposition 2, the trace weighting of Proposition 4, the wheel spectrum and the cell evaluation in exact arithmetic, and the reproduction of the comparison target at one and two loops. Analytic propositions supply the general statements; the numerical fixtures only check selected instances. The comparison target enters only the rank-distance diagnostic, the reproduction of Section 10.5 and the comparison lines, after the model inputs are fixed.

No experimental force data fixes any model parameter, and no percentage completion is inferred from these mathematical results.